Group Field Theories for the Atoms of Space and their renormalisation

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Albert Einstein Institute

Workshop on “Strongly-Interacting Field Theories”
Jena, Germany, EU - 06/11/2015

Plan

- what are Group Field Theories
- relation with other QG approaches (and with GR/gravity)
- basics of RG set-up for GFTs
- perturbative renormalizability in GFTs - key results

Part I:

Group Field Theory

Group field theories

(Boulatov, Ooguri, De Pietri, Freidel, Krasnov, Rovelli, Perez, DO, Livine, Baratin, ……)

Quantum field theories over group manifold G (or corresponding Lie algebra) $\varphi : G^{\times d} \rightarrow \mathbb{C}$

QFT of spacetime, not defined on spacetime

relevant classical phase space for "GFT quanta": $\left(\mathcal{T}^* G\right)^{\times d} \simeq \left(\mathfrak{g} \times G\right)^{\times d}$

can reduce to subspaces in specific models depending on conditions on the field

d is dimension of "spacetime-to-be"; for gravity models, G = local gauge group of gravity (e.g. Lorentz group)

example: d=4 $\varphi(g_1, g_2, g_3, g_4) \leftrightarrow \varphi(B_1, B_2, B_3, B_4) \rightarrow \mathbb{C}$

can be defined for any (Lie) group and dimension d, any signature, ...

very general framework; interest rests on specific models/use
(most interesting QG models are for Lorentz group in 4d)

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Fock vacuum: “no-space” (“emptiest”) state $| 0 >$

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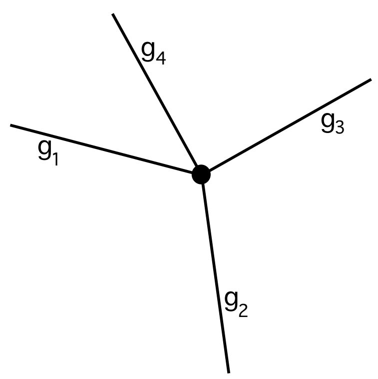

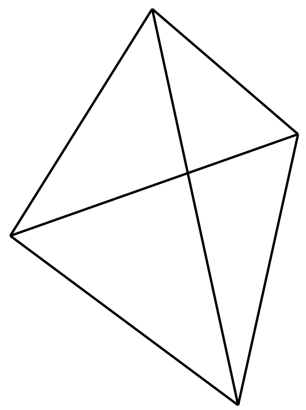

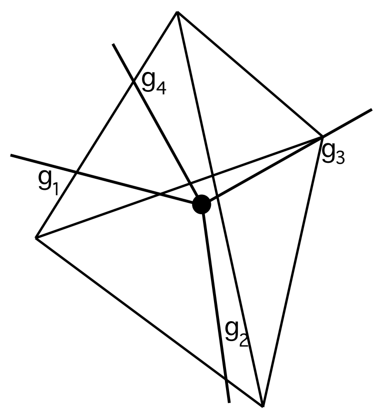

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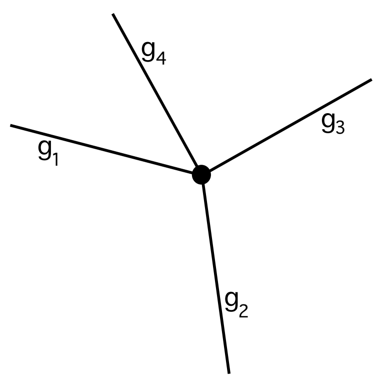

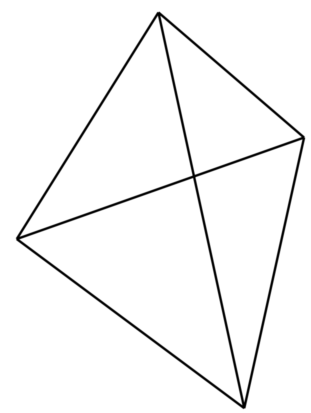

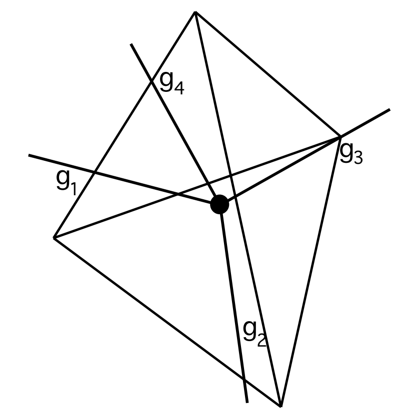

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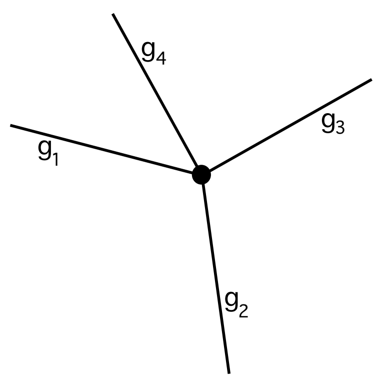

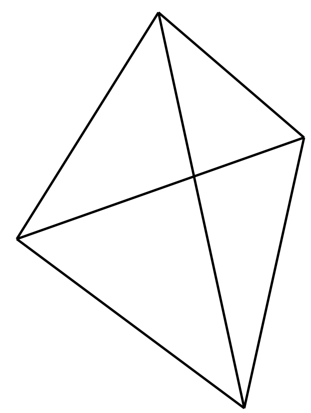

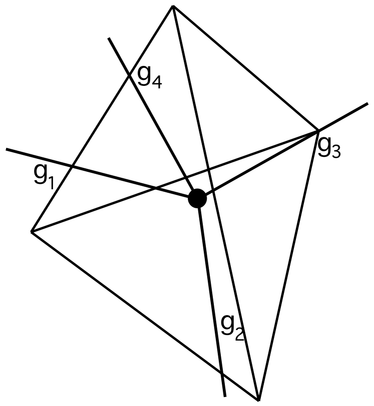

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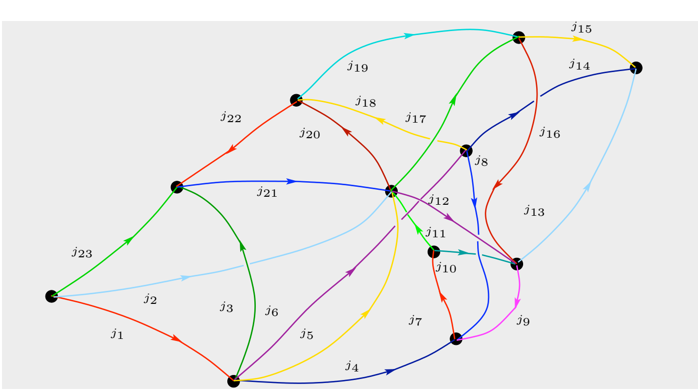

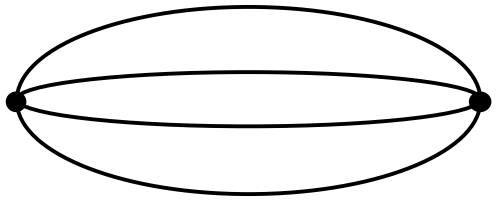

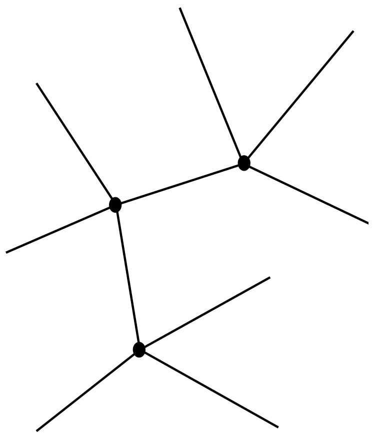

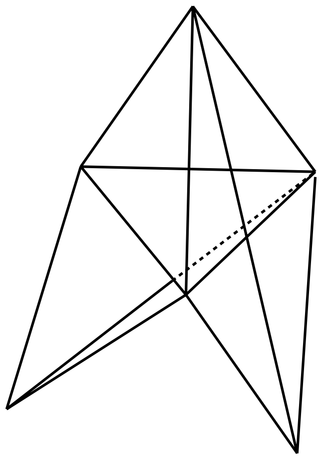

Group field theories

classical action: kinetic (quadratic) term + (higher order) interaction (convolution of GFT fields)

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

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combinatorics of field arguments in interaction: gluing of 5 tetrahedra across common triangles, to form 4-simplex ("building block of spacetime")

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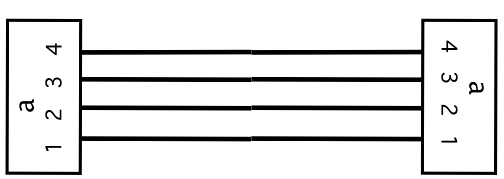

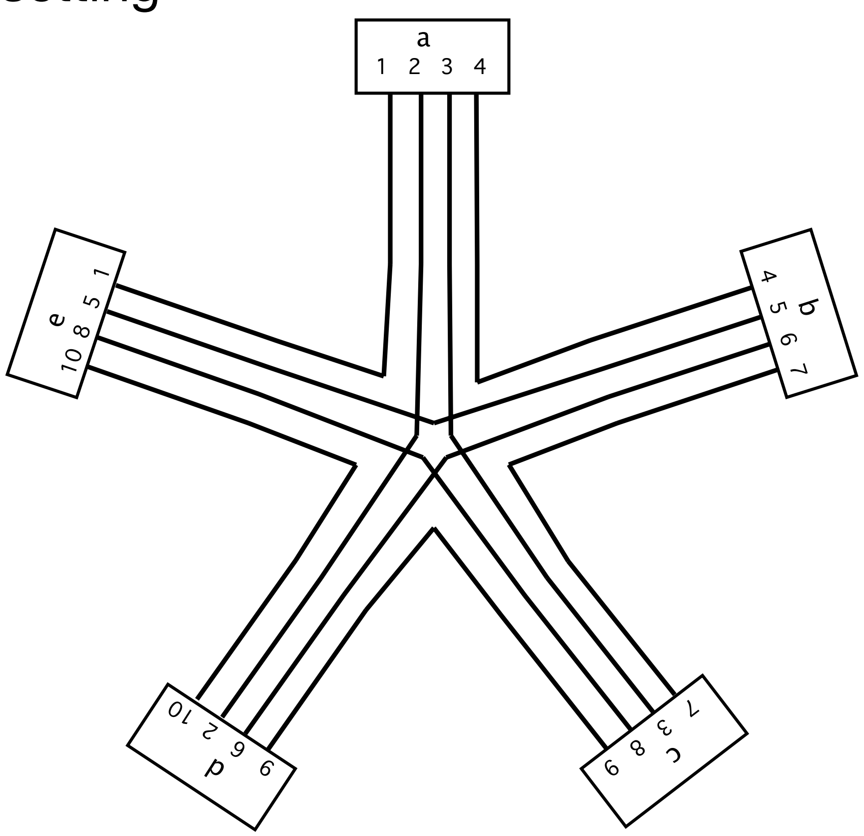

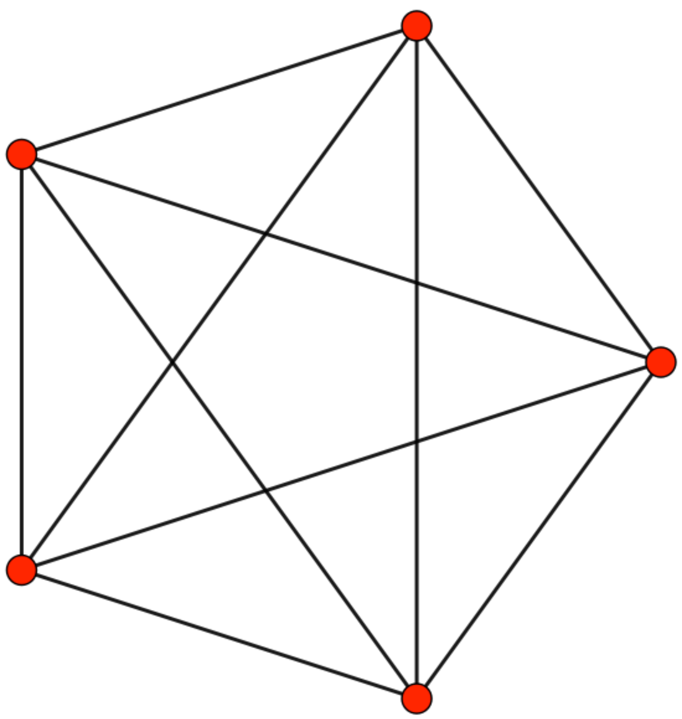

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Feynman perturbative expansion around trivial vacuum

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\; e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)} \mathcal{A}_\Gamma$$

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Feynman diagrams (obtained by convoluting propagators with interaction kernels) =

= stranded diagrams dual to cellular complexes of arbitrary topology

(simplicial case: simplicial complexes obtained by gluing d-simplices in arbitrary ways)

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Feynman amplitudes (model-dependent):

equivalently:

- spin foam models (sum-over-histories of spin networks)

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- lattice path integrals (with group+Lie algebra variables)

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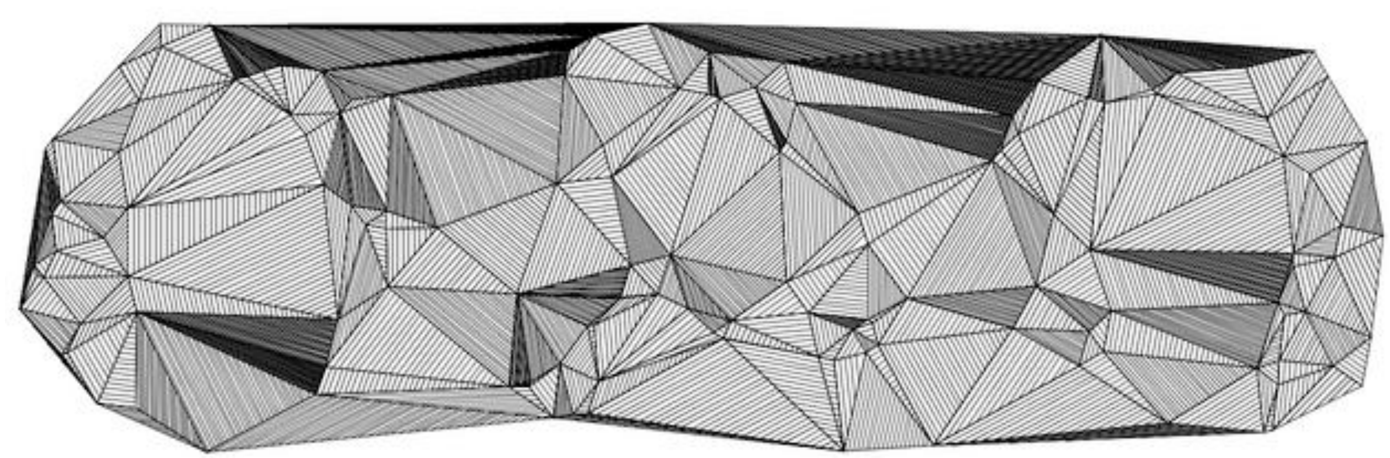

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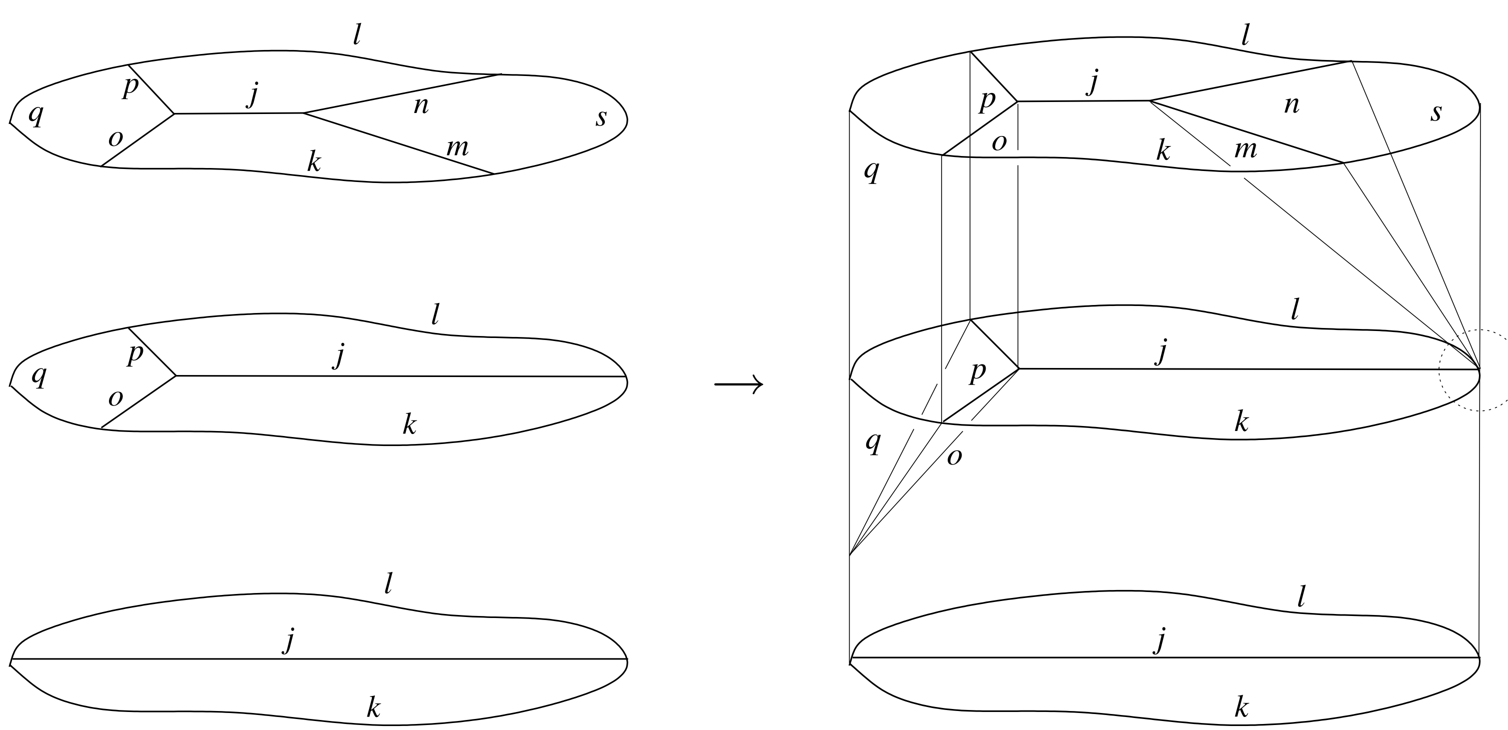
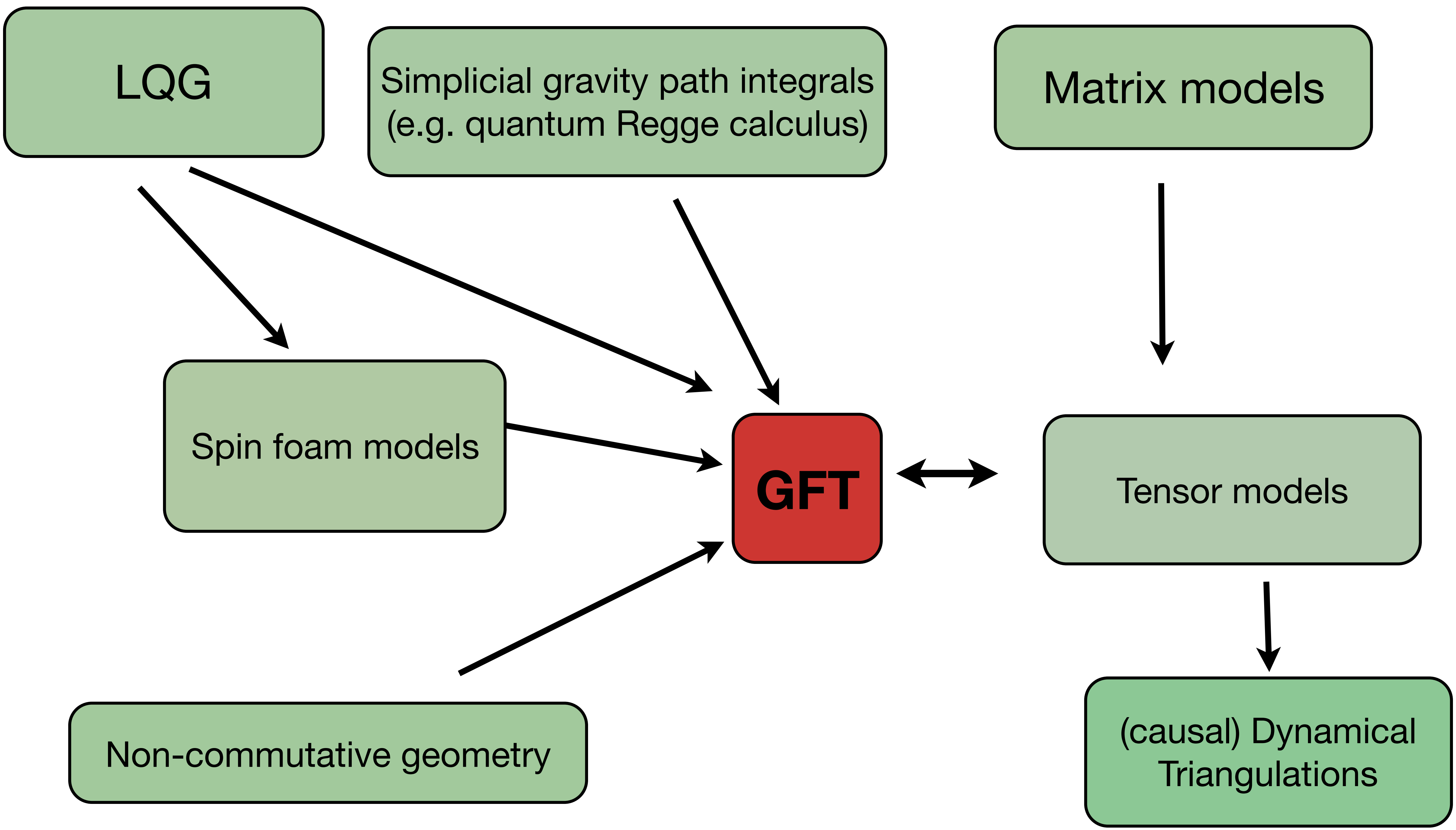


Part II:

Group Field Theory
and
other QG formalisms
(relation to discrete gravity)

Group Field Theory: convergence of approaches



GFT as 2nd quantisation of LQG

see talk by Hanno

the GFT proposal: $$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\, e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\, \mathcal{A}_\Gamma$$

spin networks as many-body systems and 2nd quantisation —-> GFT Fock space

DO, '13 ; Kittel, DO, Tomlin, to appear

(= space of "disconnected spin network vertices")

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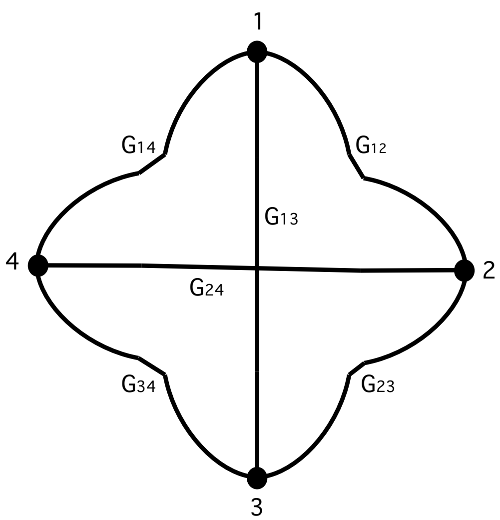

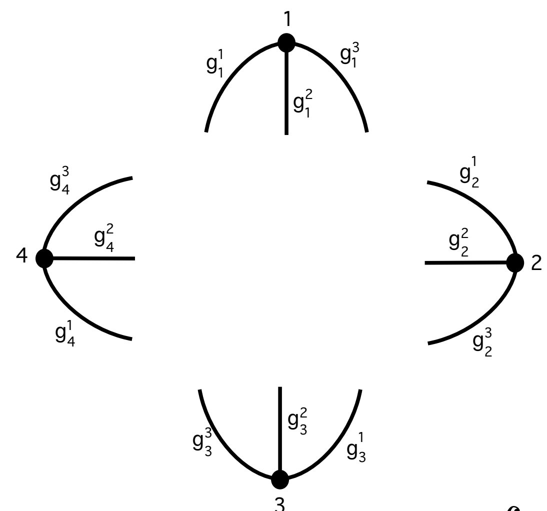

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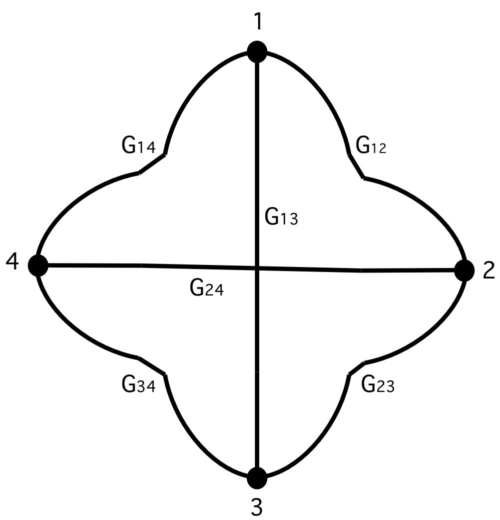

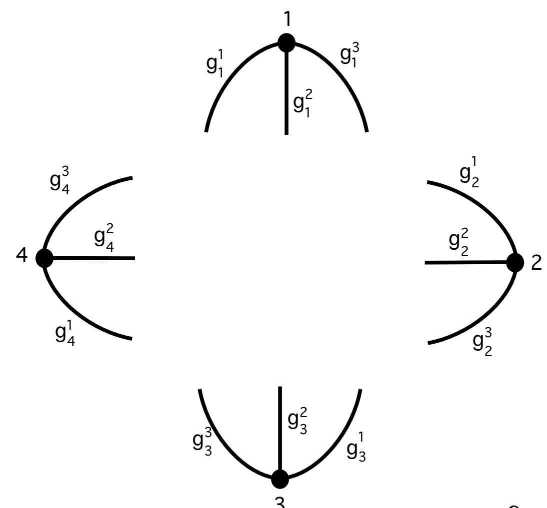

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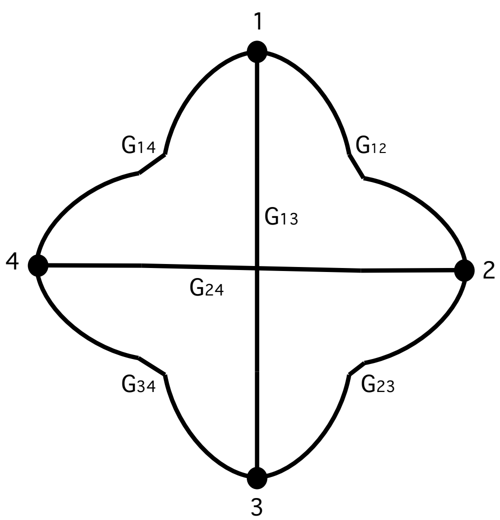

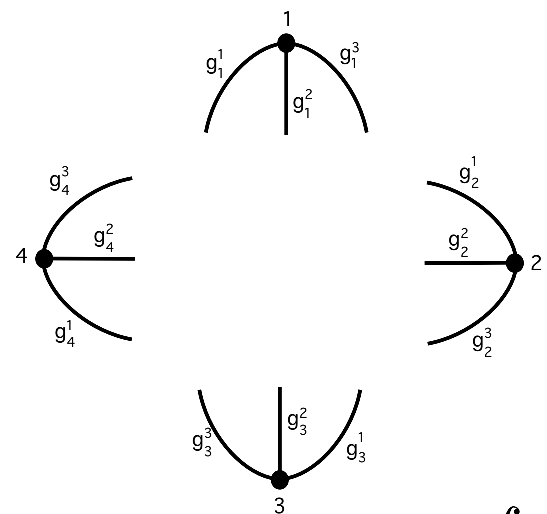

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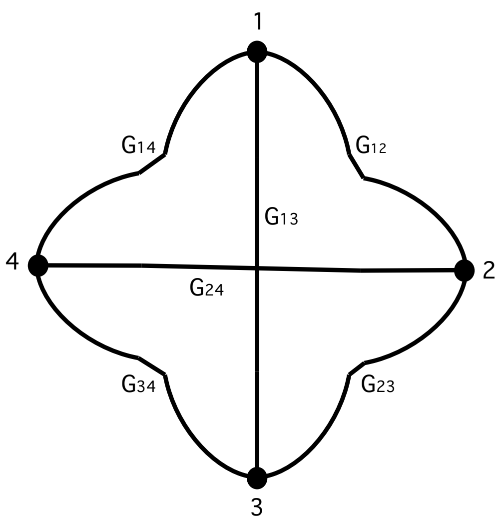

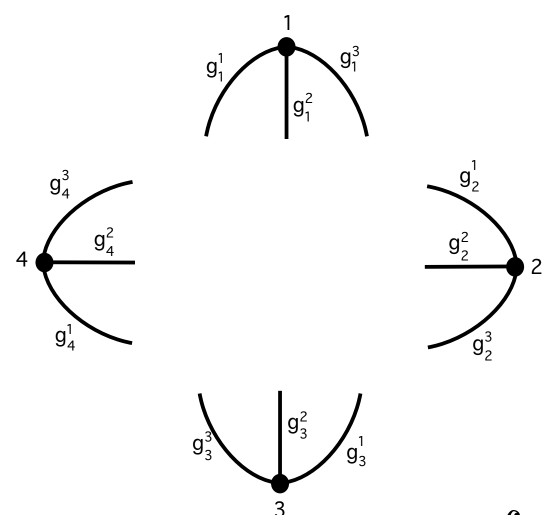

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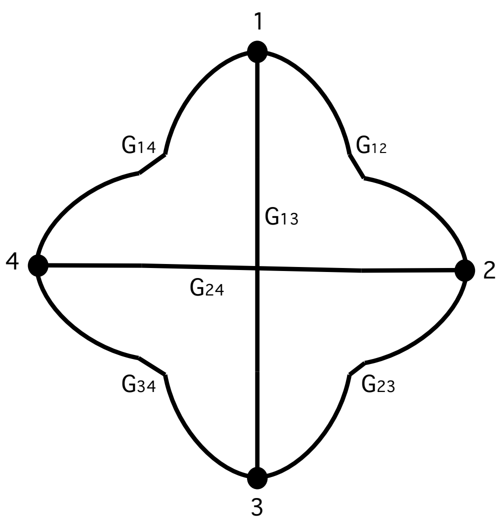

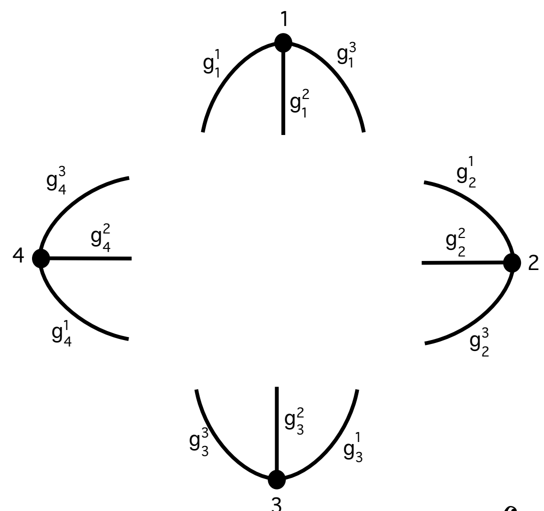

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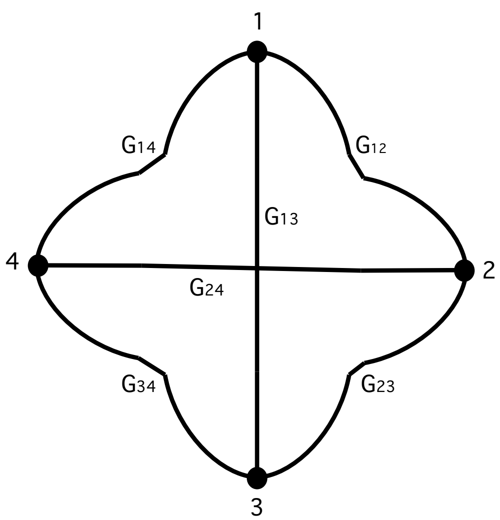

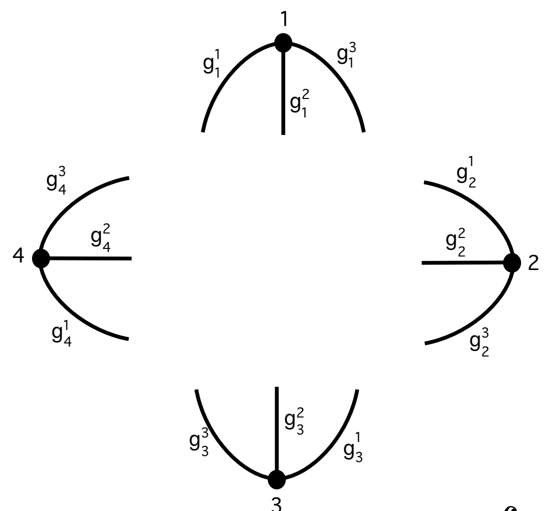

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$$\mathcal{H}_\gamma \subset \mathcal{H}_V \qquad \Psi_\gamma(G_{ij}^{ab}) \; = \prod_{[(ia),(jb)]} \int_G d\alpha_{ij}^{ab}\, \phi_V(\ldots, g_{ia}\, \alpha_{ij}^{ab}, \ldots, g_{jb}\alpha_{ij}^{ab}, \ldots) \; = \; \Psi_\gamma(g_{ia}(g_{jb})^{-1})$$

- same type of functions + same scalar product for given graph
- states for different graphs (same vertices) overlap
- no continuum embedding
- no cylindrical equivalence

←

need to accept technical differences

and change in perspective

——> fundamental discreteness

(not “quantising continuum fields”, not canonical GR)

GFT as 2nd quantisation of LQG

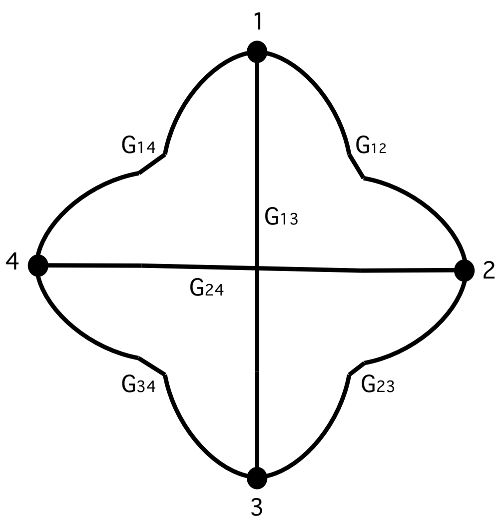
see talk by Hanno

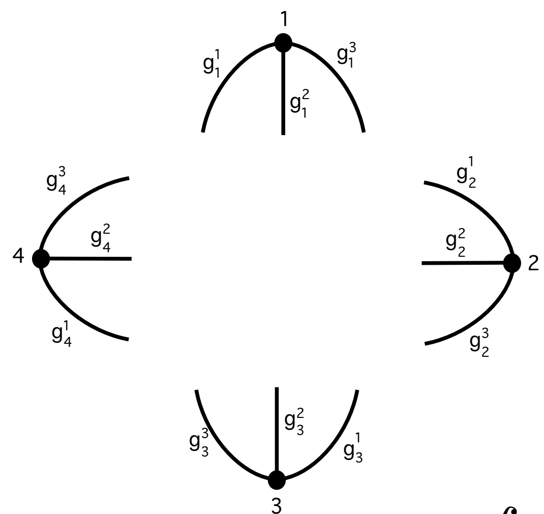
the GFT proposal: $$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\, e^{i\, S_\lambda(\varphi,\overline{\varphi})} \;=\; \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)} \mathcal{A}_\Gamma$$

spin networks as many-body systems and 2nd quantisation —-> GFT Fock space

DO, '13 ; Kittel, DO, Tomlin, to appear

(= space of "disconnected spin network vertices")





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for any canonical observable (incl. Hamiltonian constraint) -> GFT observable in 2nd quantisation

GFT as completion of spin foam models

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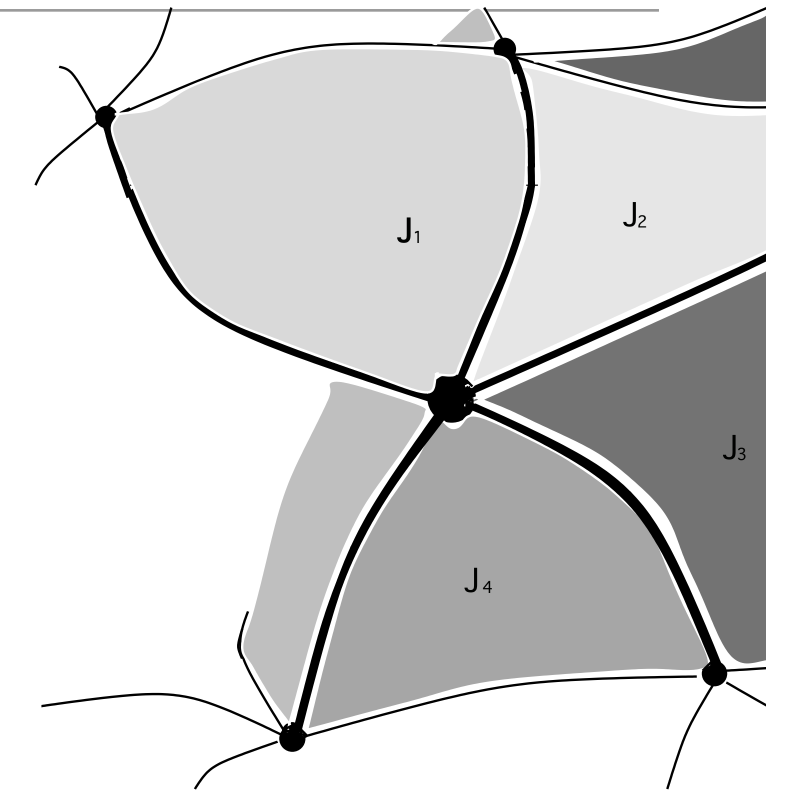
quantum spin network history = spin foam (complex with algebraic data)

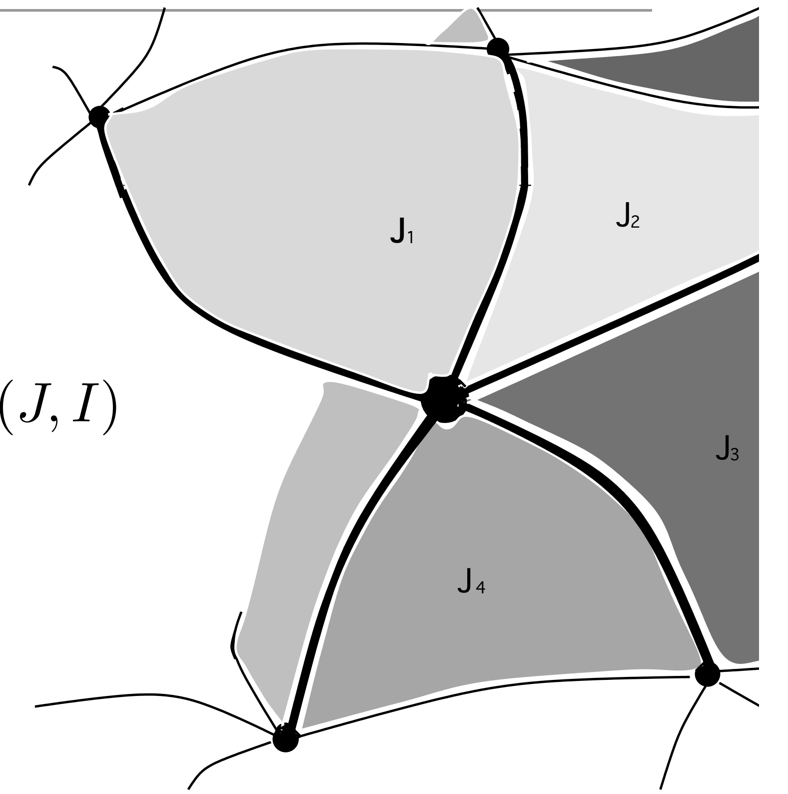


GFT as completion of spin foam models

quantum spin network history = spin foam (complex with algebraic data)

basic element of SF model: quantum amplitude for spin foam complex

$$\{\, \Gamma \,\} \qquad\qquad Z(\Gamma) = \sum_{\{J\},\{I\}|j,j',i,i'} \prod_f A_f(J,I) \prod_e A_e(J,I) \prod_v A_v(J,I)$$



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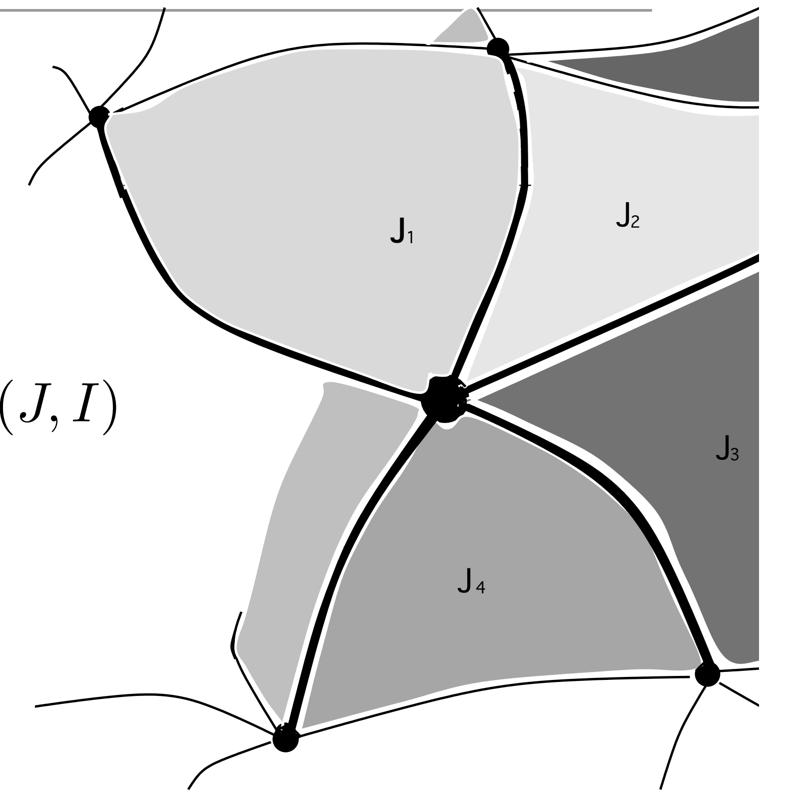
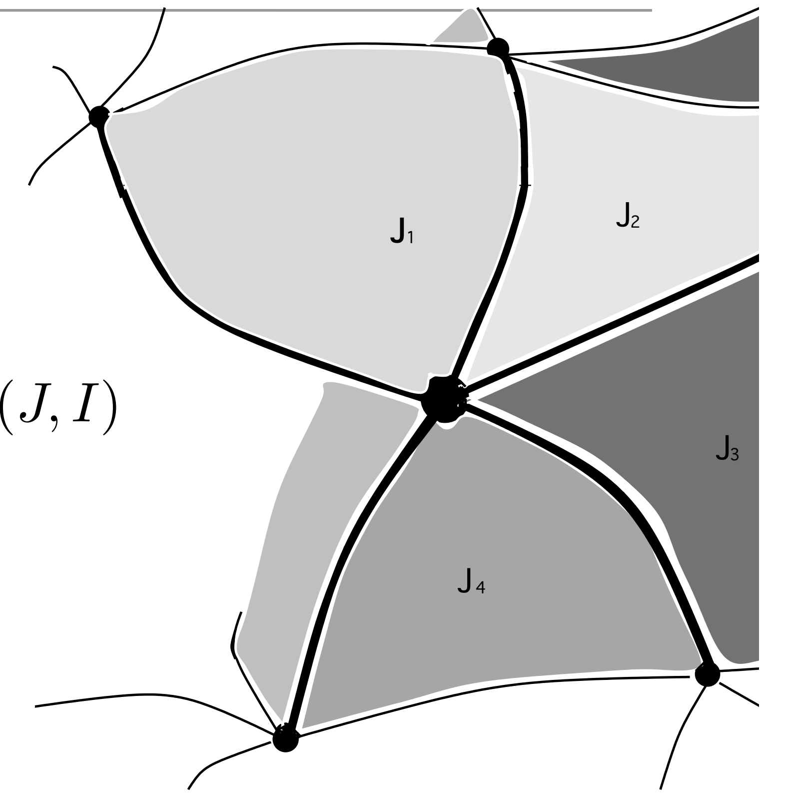


complete (formal) definition of SF model:

quantum amplitudes for all spin foam complexes + organization principle

GFT as completion of spin foam models



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the GFT proposal:

spin foam model

with sum over complexes

as GFT perturbative expansion

(valid for any SF model)

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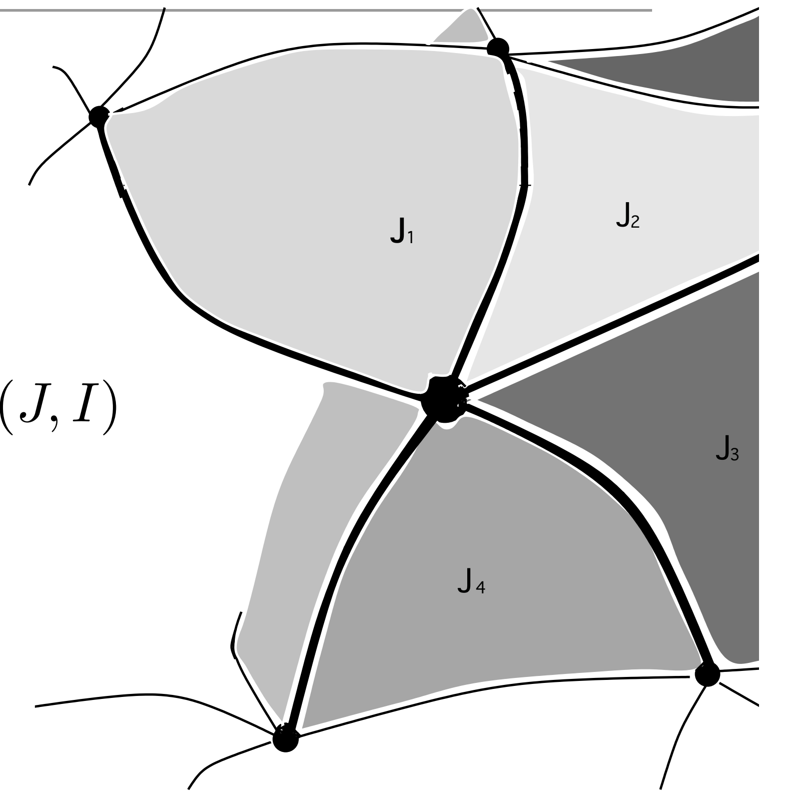

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complete (formal) definition of SF model:

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the GFT proposal:

$$Z(\Gamma) \leftrightarrow \begin{cases} A_f(J) \\ A_e(J,I) \\ A_v(J,I) \end{cases} \longleftrightarrow \begin{cases} \mathcal{K}(J,I) \sim \mathcal{K}(g) \\ \mathcal{V}(J,I) \sim \mathcal{V}(g) \end{cases} \leftrightarrow \; S(\varphi,\bar{\varphi})$$

spin foam model

with sum over complexes

as GFT perturbative expansion

(valid for any SF model)

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

$$\updownarrow$$

$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\; e^{i\,S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\mathcal{A}_\Gamma \qquad Z(\Gamma) \equiv \mathcal{A}_\Gamma$$

GFTs, loop quantum gravity, spin foam models

appropriate conditions on GFT fields or GFT dynamics (and choice of data) turn GFT Feynman amplitudes into lattice gauge theories/discrete gravity path integrals/spin foam models

e.g. gauge invariance of GFT fields under diagonal action of group G

example: d=3

$$\varphi_\ell \,:\, SO(3)^3/SO(3) \,\rightarrow\, \mathbb{R}$$

$$\forall h \in \mathrm{SO}(3), \qquad \varphi_\ell(hg_1, hg_2, hg_3) \,=\, \varphi_\ell(g_1, g_2, g_3)$$

\+ simplicial interaction with only delta functions

valid for GFT definition of BF theory in any dimension

Quantum 3d simplicial geometry (Riemannian)

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classical triangle in $\mathbb{R}^3$

3 edge vectors that close $x_1, x_2, x_3 \in \mathbb{R}^3 \quad s.t. \quad \sum_i x_i = 0$

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Phase space for triangle in discrete 3d gravity

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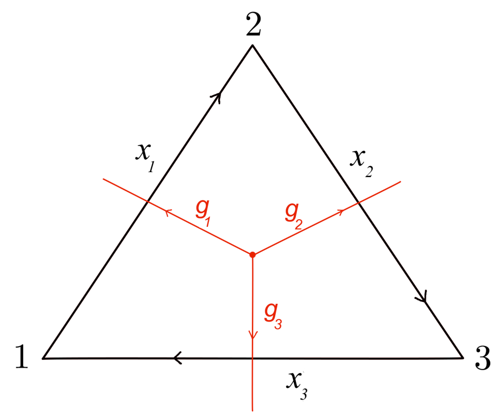

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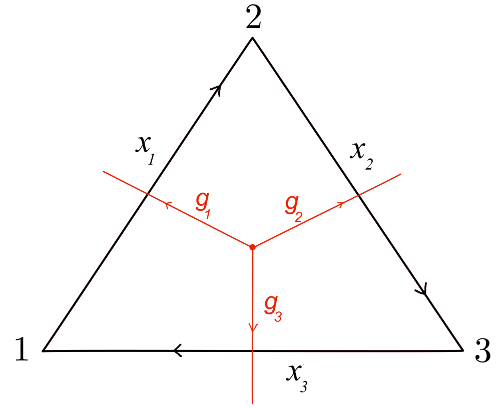

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GFTs, loop quantum gravity, discrete gravity

appropriate conditions on GFT fields or GFT dynamics (and choice of data) turn GFT Feynman amplitudes into lattice gauge theories/discrete gravity path integrals/spin foam models

e.g. gauge invariance of GFT fields under diagonal action of group G

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with only delta functions

valid for GFT definition of BF theory in any dimension

can be computed in different (equivalent) representations (group, spin, Lie algebra)

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$$S_{kin}[\varphi_\ell] = \int [\mathrm{d}g_i]^3 \sum_{\ell=1}^{4} \varphi_\ell(g_1, g_2, g_3)\overline{\varphi_\ell}(g_1, g_2.g_3),$$

$$\begin{aligned} S_{int}[\varphi_\ell] \quad = \quad & \lambda \int [\mathrm{d}g_i]^6 \, \varphi_1(g_1, g_2, g_3)\varphi_2(g_3, g_4, g_5)\varphi_3(g_5, g_2, g_6)\varphi_4(g_6, g_4, g_1) \\ & + \lambda \int [\mathrm{d}g_i]^6 \, \overline{\varphi_4}(g_1, g_4, g_6)\overline{\varphi_3}(g_6, g_2, g_5)\overline{\varphi_2}(g_5, g_4, g_3)\overline{\varphi_1}(g_3, g_2, g_1) \end{aligned}$$

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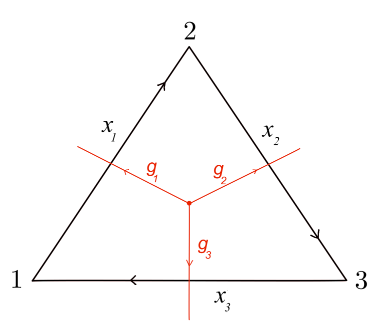

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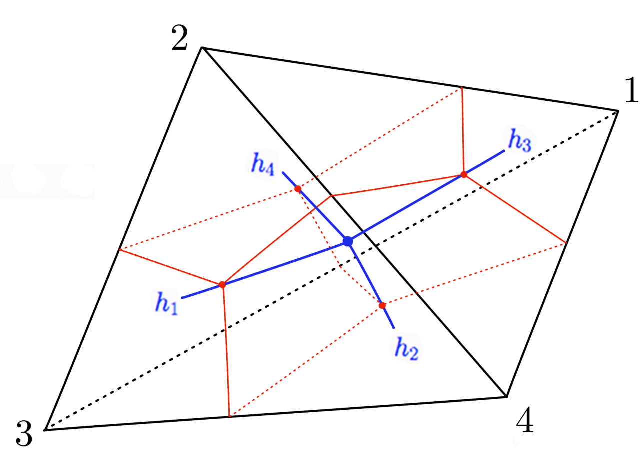

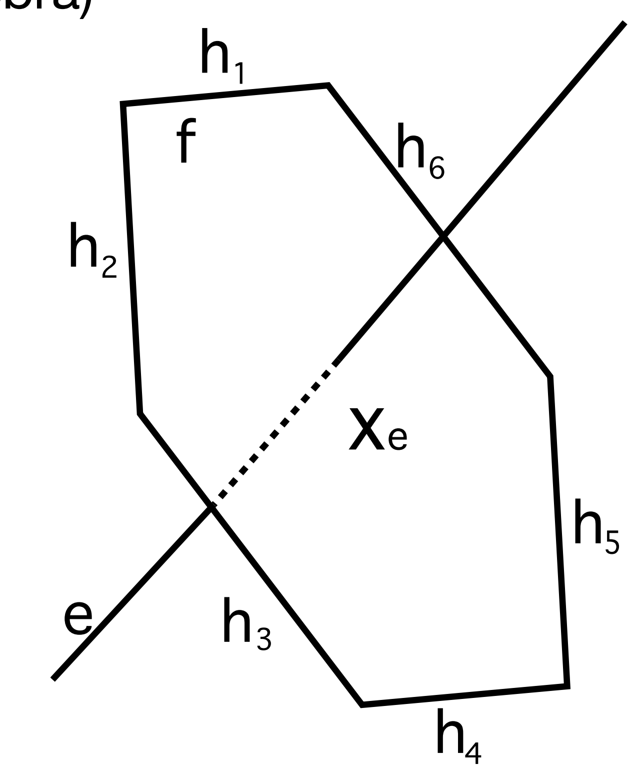

discretization of: $S(e,\omega) = \int Tr(e \wedge F(\omega))$

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$$= \sum_{\{j_e\}} \prod_e d_{j_e} \prod_\tau \left\{ \begin{array}{ccc} j_1^\tau & j_2^\tau & j_3^\tau \\ j_4^\tau & j_5^\tau & j_6^\tau \end{array} \right\} = \int \prod_l [\mathrm{d}h_l] \prod_e [\mathrm{d}^3 x_e] \, e^{i \sum_e \mathrm{Tr}\, x_e H_e}$$

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lattice gauge theory formulation of 3d gravity/BF theory

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lattice gauge theory formulation of 3d gravity/BF theory

discrete 1st order path integral for 3d gravity/BF theory on simplicial complex dual to GFT Feynman diagram

spin foam formulation of 3d gravity/BF theory

GFTs, loop quantum gravity,discrete gravity

GFT models of 4d gravity:

based on classical (Plebanski) formulation of GR as BF theory + (simplicity) constraints

start from GFT formulation of 4d BF theory
+ impose simplicity constraints (geometricity of simplicial structures)

(Barbieri, Baez, Barrett, Crane, Reisenberger, Perez, De Pietri, Engle, Pereira, Freidel, Krasnov, Rovelli, Livine, Speziale, Baratin, DO, ...)

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$$B \in \mathfrak{so}(3,1) \qquad \phi_{[IJ][KL]} = \phi_{[KL][IJ]}$$

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$$\delta\phi = 0 \Rightarrow \star B \wedge B = 0 \Rightarrow B \simeq e \wedge e$$

classically equivalent to Palatini-Holst gravity:

$$S_{Holst} = \frac{1}{G}\int_{\mathcal{M}} \left[\star e \wedge e \wedge F(\omega) + \frac{1}{\gamma}\, e \wedge e \wedge F(\omega) \right]$$

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concrete, well-defined GFT (spin foam) model(s) for 4d QG dynamics - nice discrete geometry, lots of results …..

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(Barbieri, Baez, Barrett, Crane, Reisenberger, Perez, De Pietri, Engle, Pereira, Freidel, Krasnov, Rovelli, Livine, Speziale, Baratin, DO, ...)

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$$B \in \mathfrak{so}(3,1) \qquad \phi_{[IJ][KL]} = \phi_{[KL][IJ]}$$

concrete, well-defined GFT (spin foam) model(s) for 4d QG dynamics - nice discrete geometry, lots of results …..

simplicity constraints =

= specific relation between SL(2,C) data and SU(2) data

decompose GFT field in SU(2) data + geometricity conditions

↓

GFT dynamics to LQG quantum states

Group field theories and tensor models

see talk by Razvan

same combinatorics (of states/observables and histories/Feynman diagrams), additional group-theoretic data

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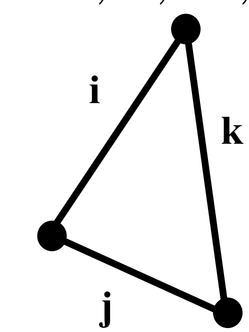

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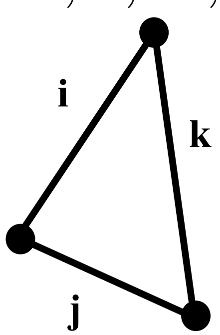

$$S(T) = \frac{1}{2} \sum_{i,j,k} T_{ijk} T_{kji} - \frac{\lambda}{4! \sqrt{N^3}} \sum_{ijklmn} T_{ijk} T_{klm} T_{mjn} T_{nli}$$

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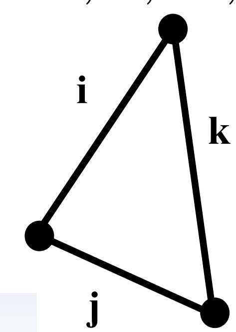

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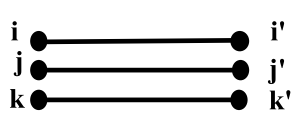

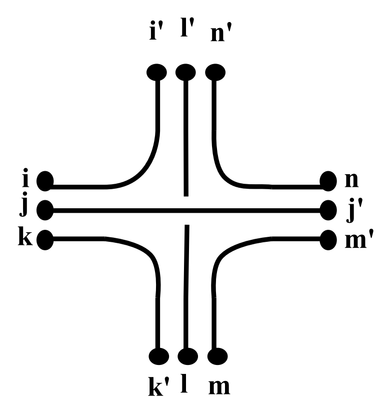

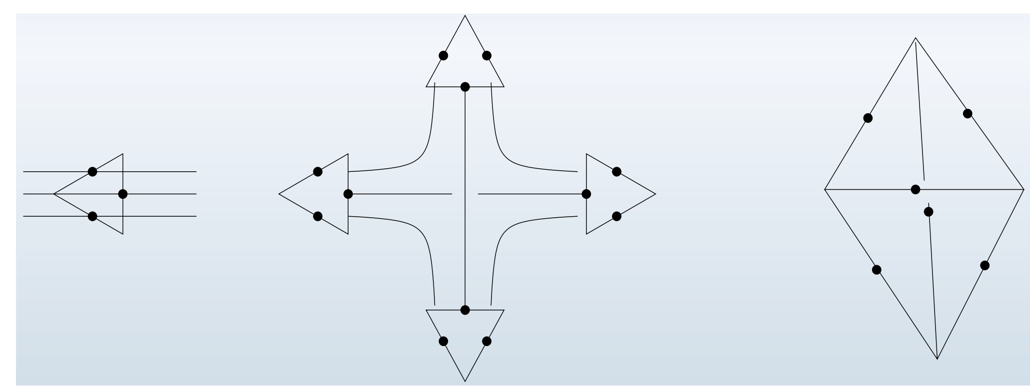

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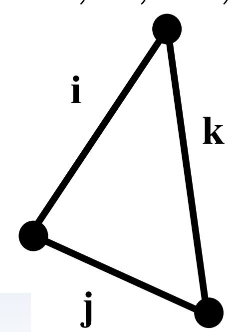

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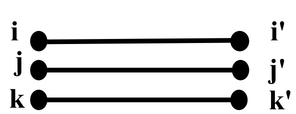

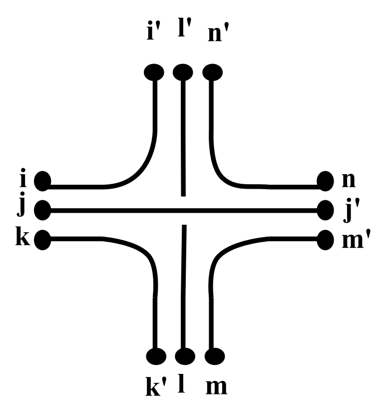

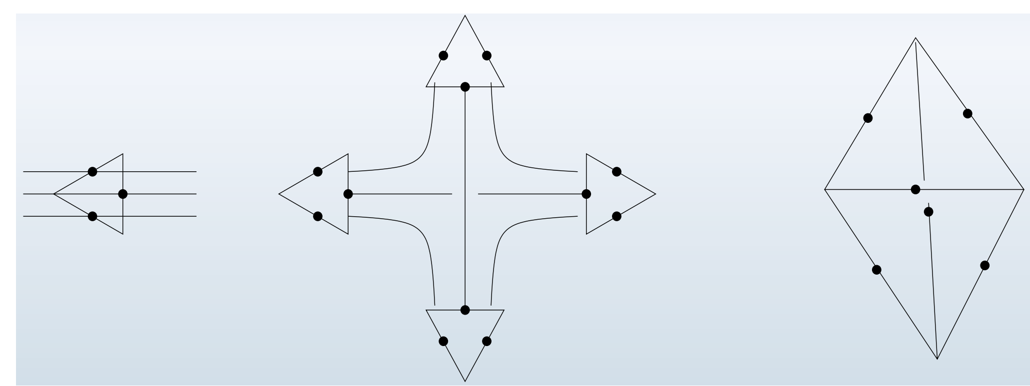

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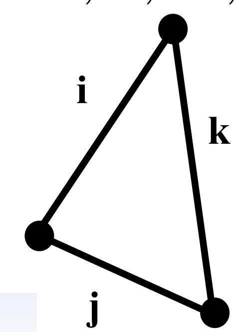

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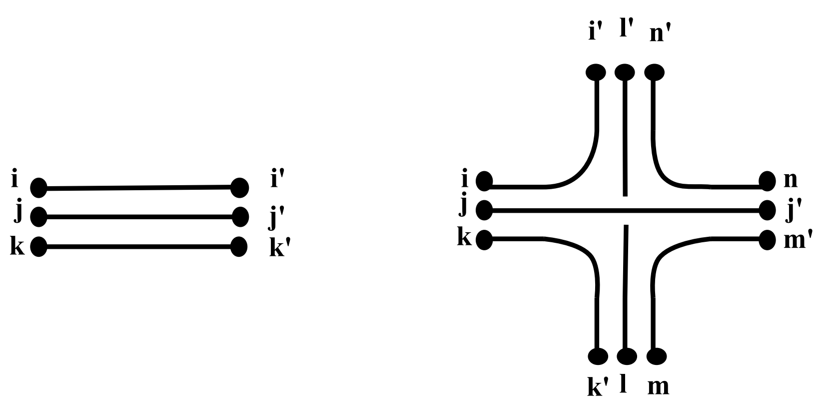

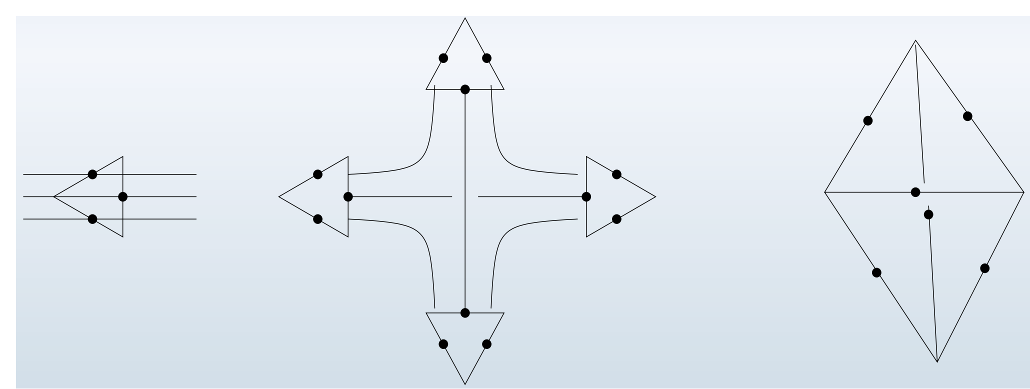

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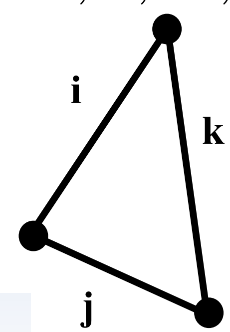

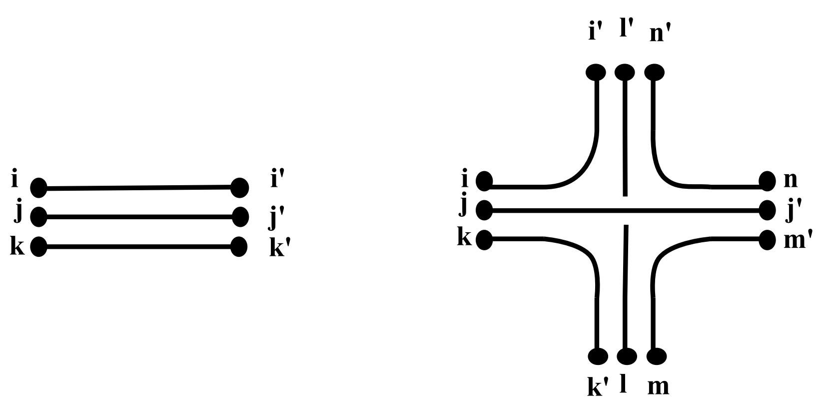

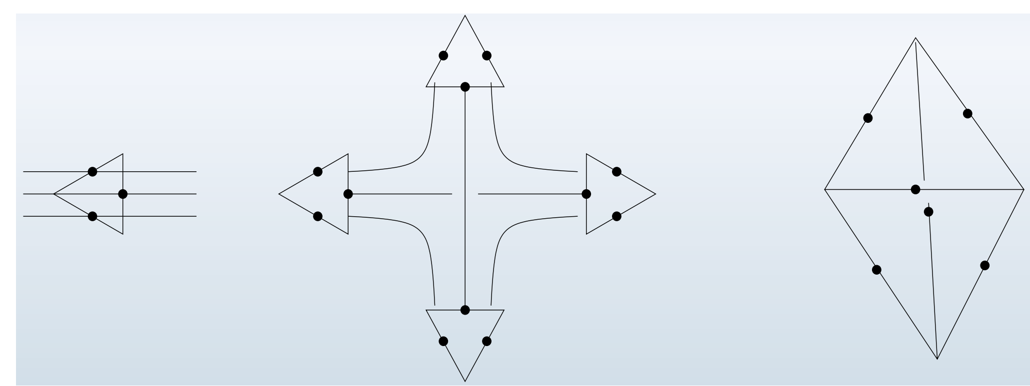

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many results on topology, scaling, constructive aspects, phase transitions, …

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- more interesting effective physics?
  - make use of geometric interpretation of data and field
  - easier to make contact with continuum physics

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- how to define the continuum limit (of the LQG/SF dynamics or, equivalently, of discrete gravity path integral)?

controlling quantum dynamics of more and more interacting degrees of freedom

new analytic tools from QFT embedding

- Non-perturbative GFT renormalization and phase diagram (see talk by Dario)
- Extraction of effective continuum dynamics in different phases

(as in QFT for condensed matter systems….)

Part III:

Group Field Theory renormalization: why? how?

The problem of the continuum limit in QG

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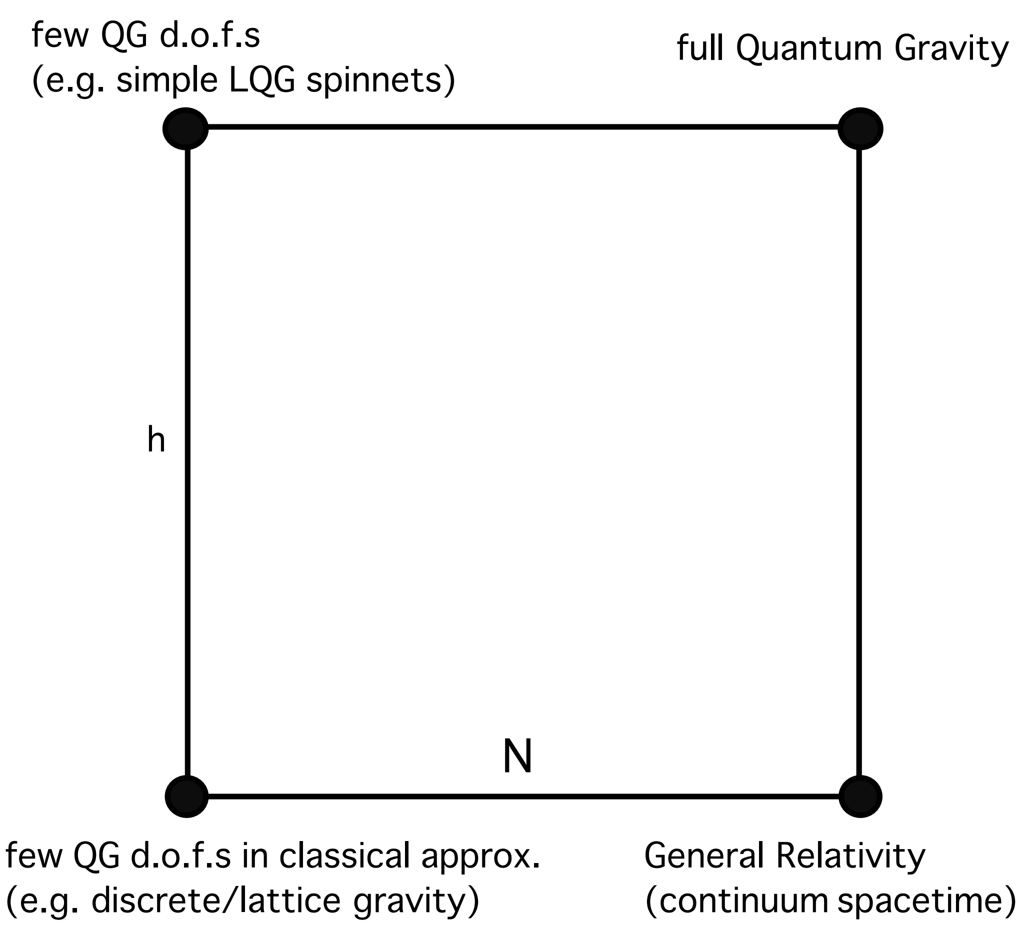

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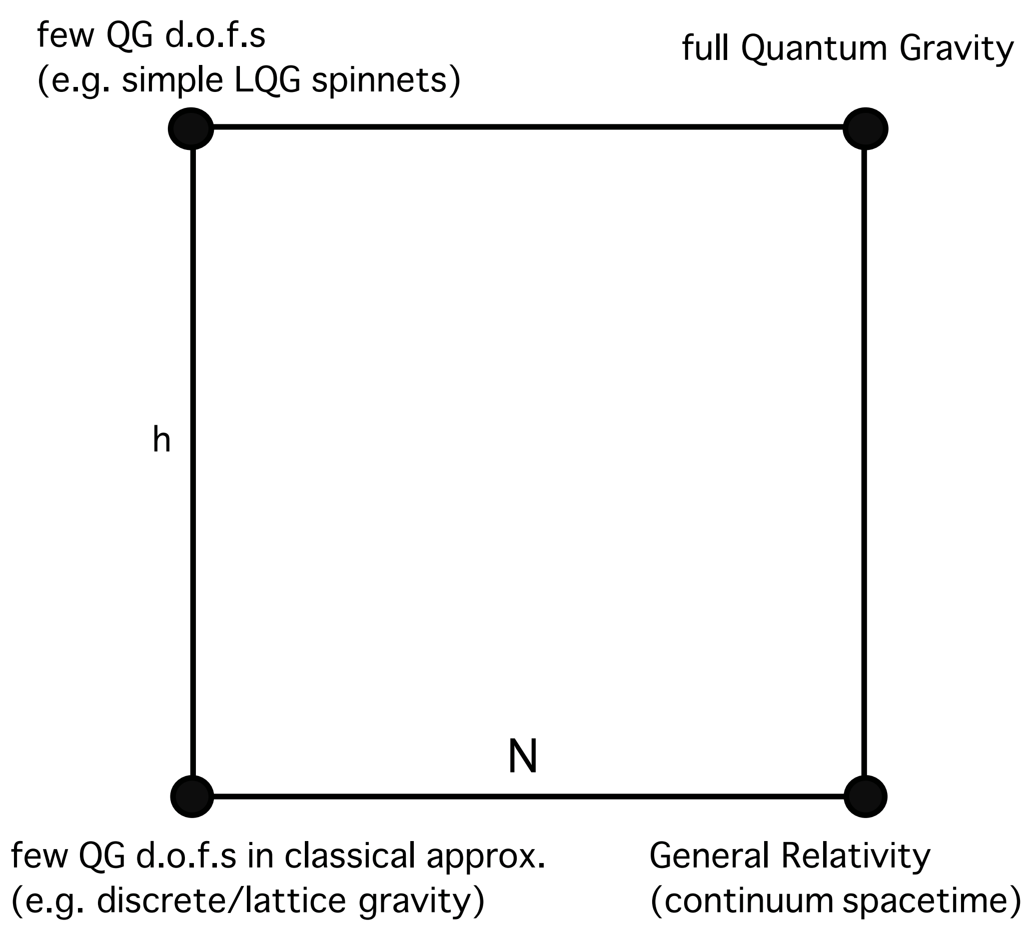

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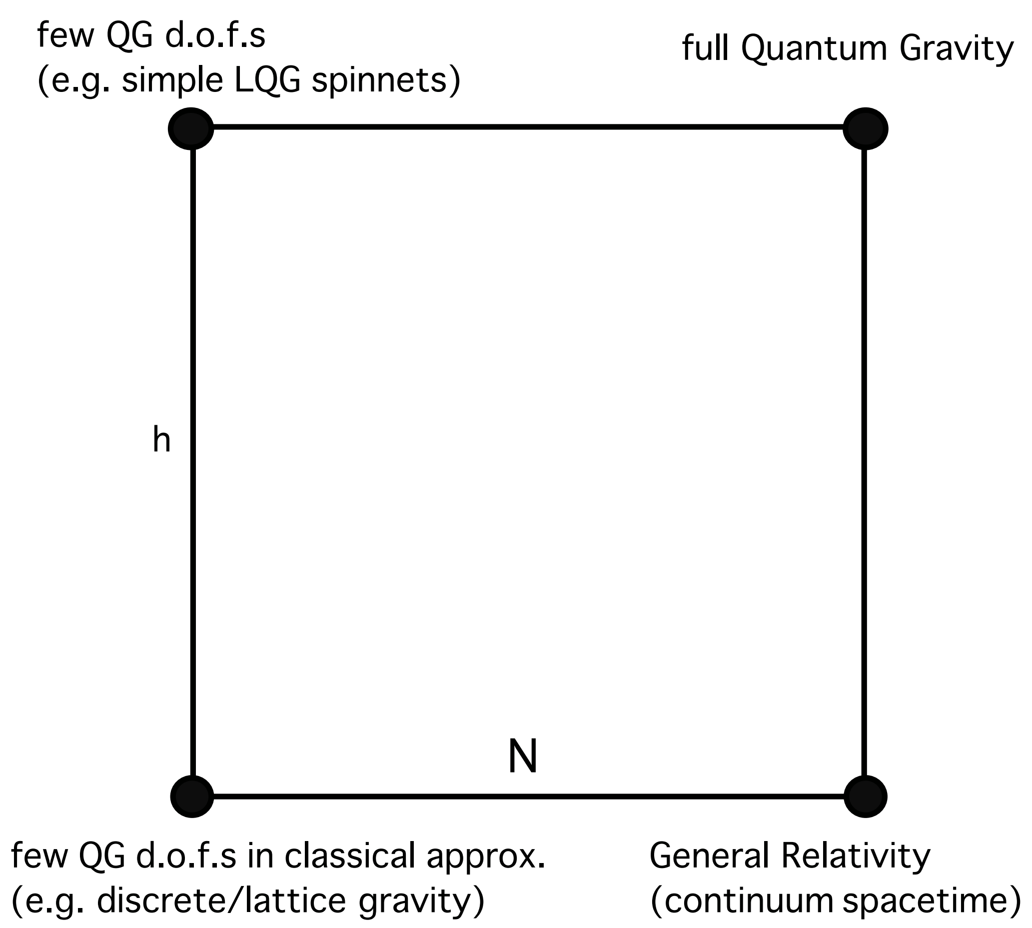
“well-understood” in spin foam models and discrete gravity



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in specific GFT case:

- treat GFT models as analogous to atomic QFTs in condensed matter systems
- fundamental formulation of QG = QFT, defined perturbatively around “no-space” (degenerate) vacuum
- need to prove consistency of the theory: perturbative GFT renormalizability
- need to understand effective dynamics at different “GFT scales”: RG flow of effective actions & phase structure & phase transitions

GFT renormalisation - general scheme

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\; e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\, \mathcal{A}_\Gamma$$

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general strategy:

treat GFTs as ordinary QFTs defined on Lie group manifold

use group structures (Killing form, topology, etc) to define notion of scale and to set up mode integration

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key difficulties:

- need to have control over “theory space” (e.g. via symmetries)
- main difficulty (at perturbative level):
  controlling the combinatorics of GFT Feynman diagrams to control the structure of divergences
  (more involved when gauge invariance is present)
  need to adapt/redefine many QFT notions: connectedness, subgraph contraction, Wick ordering, …..

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- RG flow: J_{max} ——> small J
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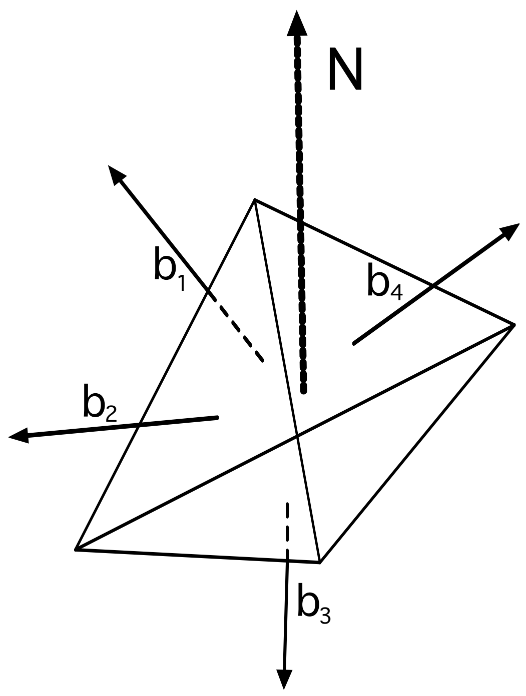
from LQG
from Regge calculus

arguments of GFT field: $b_i \in \mathfrak{su}(2)$ gravity case: d=4

| b | ~ J = irrep of SU(2) ~ “area of triangles”



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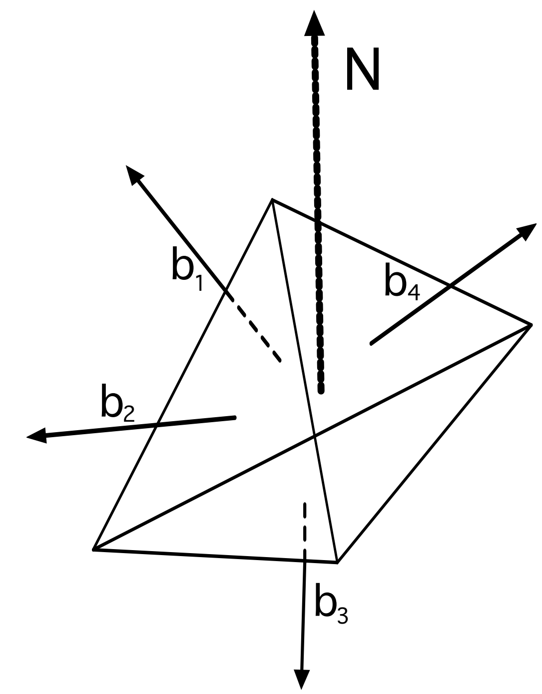
- GFT “UV” cut-off N ~ J_{max}
- RG flow: J_{max} ——> small J
- (perturbative) GFT renormalizability: UV fixed point as J_{max} ——> oo

from LQG
from Regge calculus

arguments of GFT field: $b_i \in \mathfrak{su}(2)$ gravity case: d=4



| b | ~ J = irrep of SU(2) ~ “area of triangles”

“geometric” interpretation of the RG flow?

- RG flow from large areas to small areas? not quite
- theory defined in non-geometric phase of “large” disconnected tetrahedra
- flow of coupling u to region of many interacting (thus, connected) “small” tetrahedra

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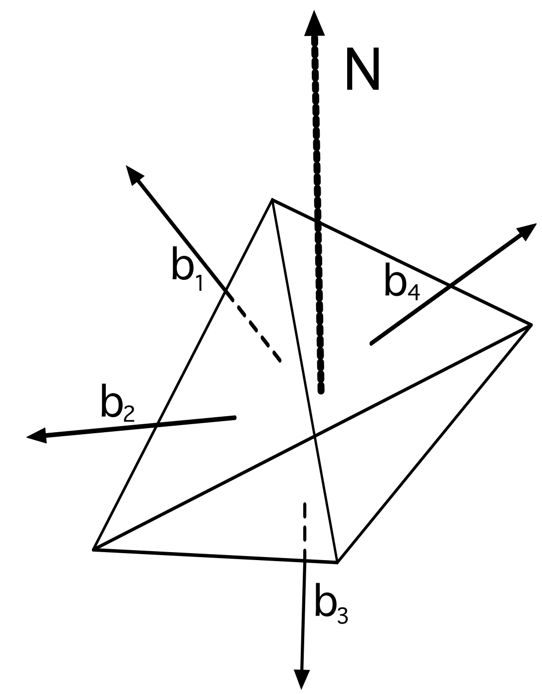

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- CAUTION in interpreting things geometrically outside continuum geometric approx

- expect “physical” continuum areas A ~ < J > < n >
- expect proper continuum geometric interpretation (and effective metric field) for < J > small, < n > large, A finite (not too small)

Part IV:

Group Field Theory renormalization (perturbative and non-perturbative):

a survey of results

Renormalization of GFTs: a brief review

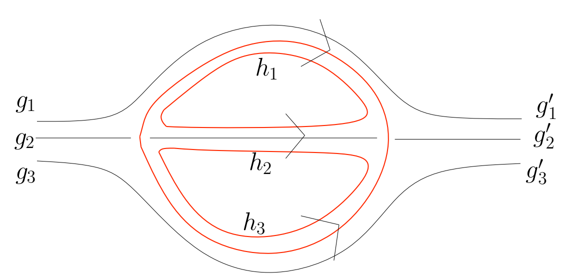
preliminary understanding:

power counting and radiative corrections in simplicial GFT models
(hard cut-off on fields, or heat-kernel regularisation of propagator, in representation space)

- 3d (non-abelian) (colored) Boulatov model (BF theory):
  - partial power counting and scaling theorems

L. Freidel, R. Gurau, DO, '09; J. Magnen, K. Noui, V. Rivasseau, M. Smerlak, '09; J. Ben Geloun, J. Magnen, V. Rovasseau, '10 ; S. Carrozza, DO, '11,'12

  - radiative corrections of 2-point function: need for Laplacian kinetic term



J. Ben Geloun, V. Bonzom, '11

  - super-renormalizability in abelian case (with Laplacian)

J. Ben Geloun, '13

- 4d gravity models
  - radiative correction of 2-point function in EPRL-FK model

J. Ben Geloun, R. Gurau, V. Rivasseau, '10; T. Krajewski, J. Magnen, V. Rivasseau, A. Tanasa, P. Vitale, '10; A. Riello, '13

GFT renormalisation - general scheme

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\; e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\, \mathcal{A}_\Gamma$$

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

general strategy:

treat GFTs as ordinary QFTs defined on Lie group manifold

use group structures (Killing form, topology, etc) to define notion of scale and to set up mode integration

subtleties of quantum gravity context at the level of interpretation

scales:

defined by propagator: e.g. spectrum of Laplacian on G = indexed by group representations

key difficulties:

- need to have control over "theory space" (e.g. via symmetries)
- main difficulty (at perturbative level):
  controlling the combinatorics of GFT Feynman diagrams to control the structure of divergences
  (more involved when gauge invariance is present)
  need to adapt/redefine many QFT notions: connectedness, subgraph contraction, Wick ordering, …..

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most results for “Tensorial Group Field Theories” (TGFTs)

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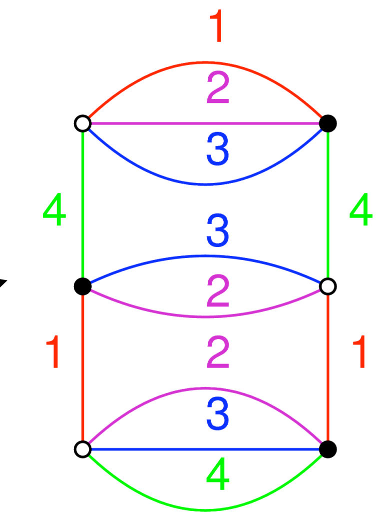

indexed by bipartite d-colored graphs ("bubbles")
dual to d-cells with triangulated boundary

$$\int [\mathrm{d}g_i]^{12} \varphi(g_1, g_2, g_3, g_4)\overline{\varphi}(g_1, g_2, g_3, g_5)\varphi(g_8, g_7, g_6, g_5)$$
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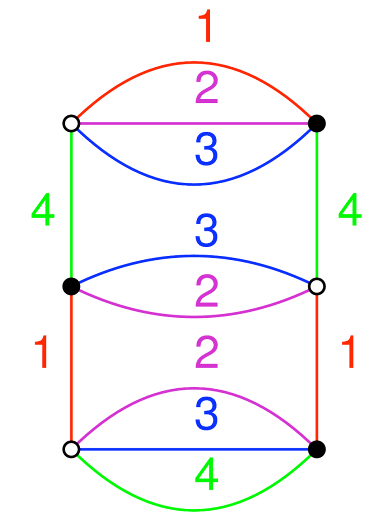

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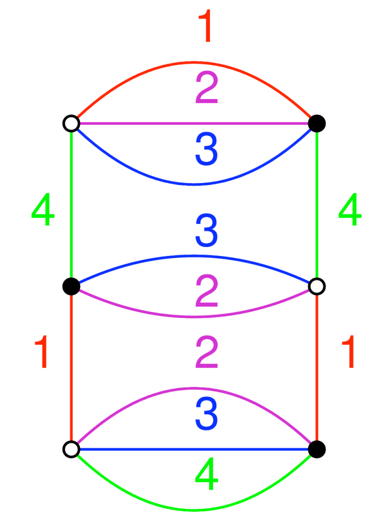

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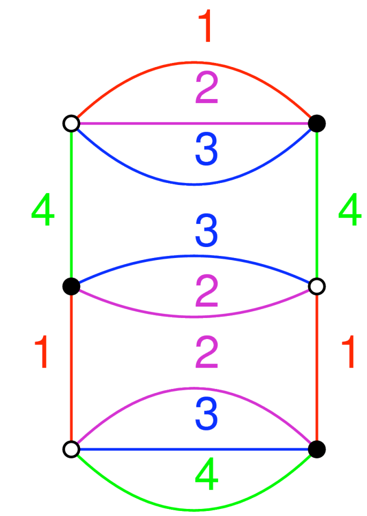

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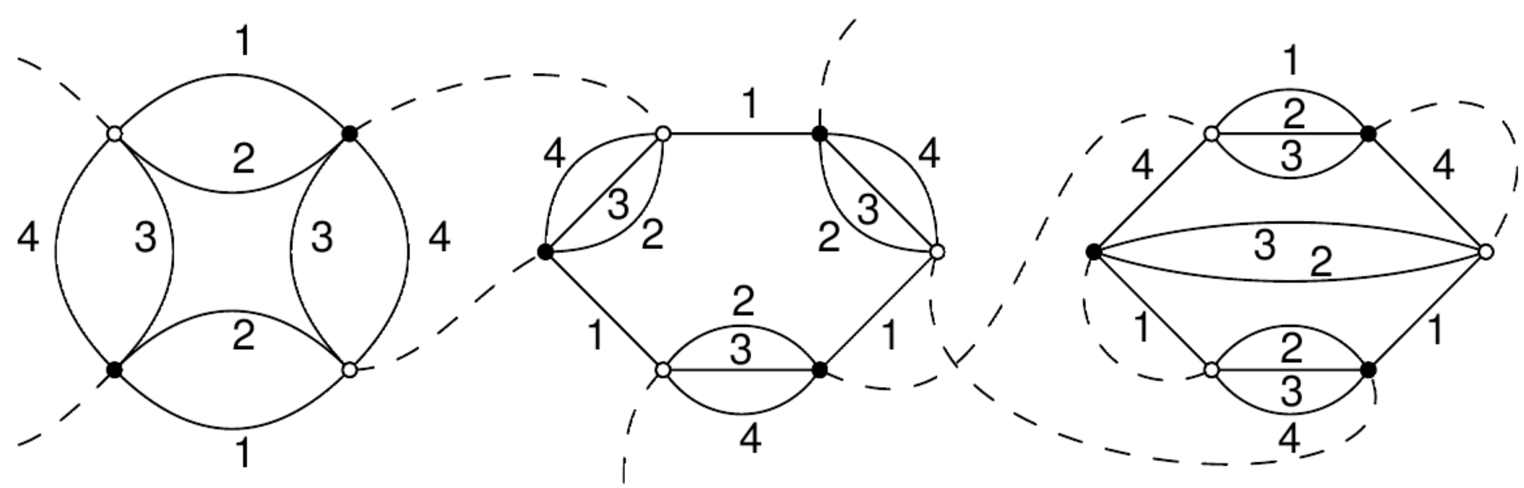

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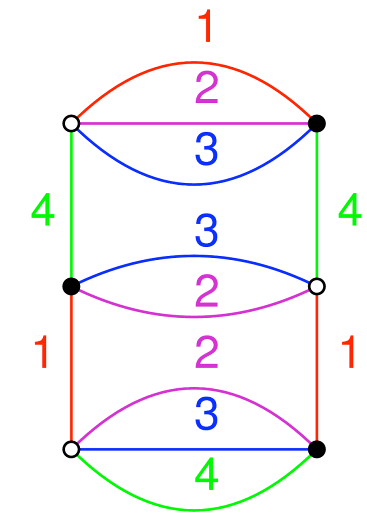

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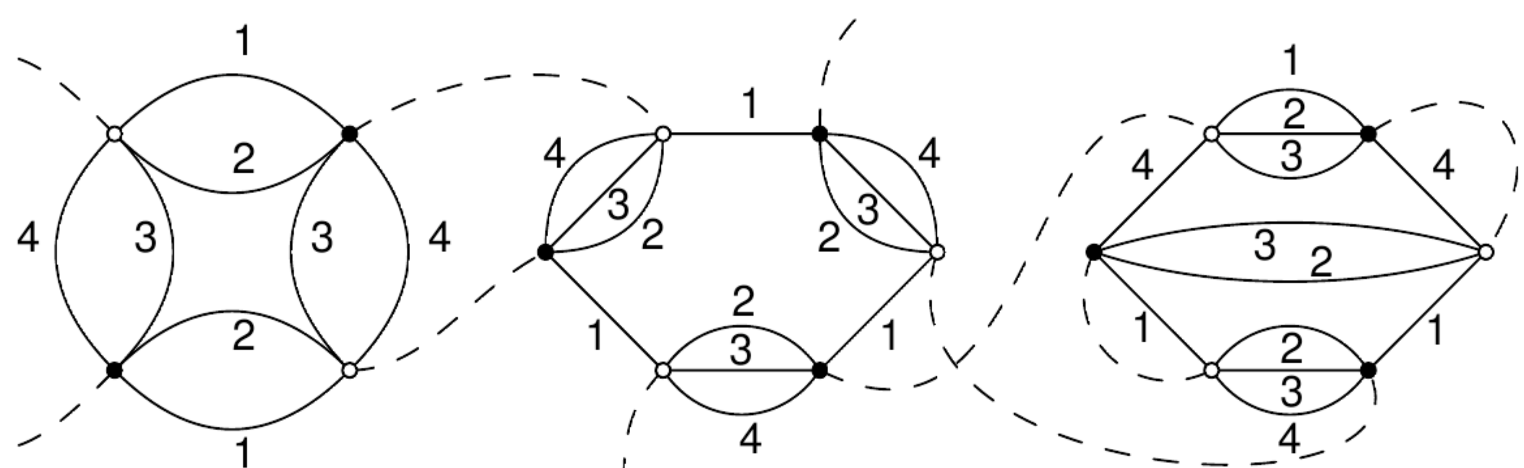
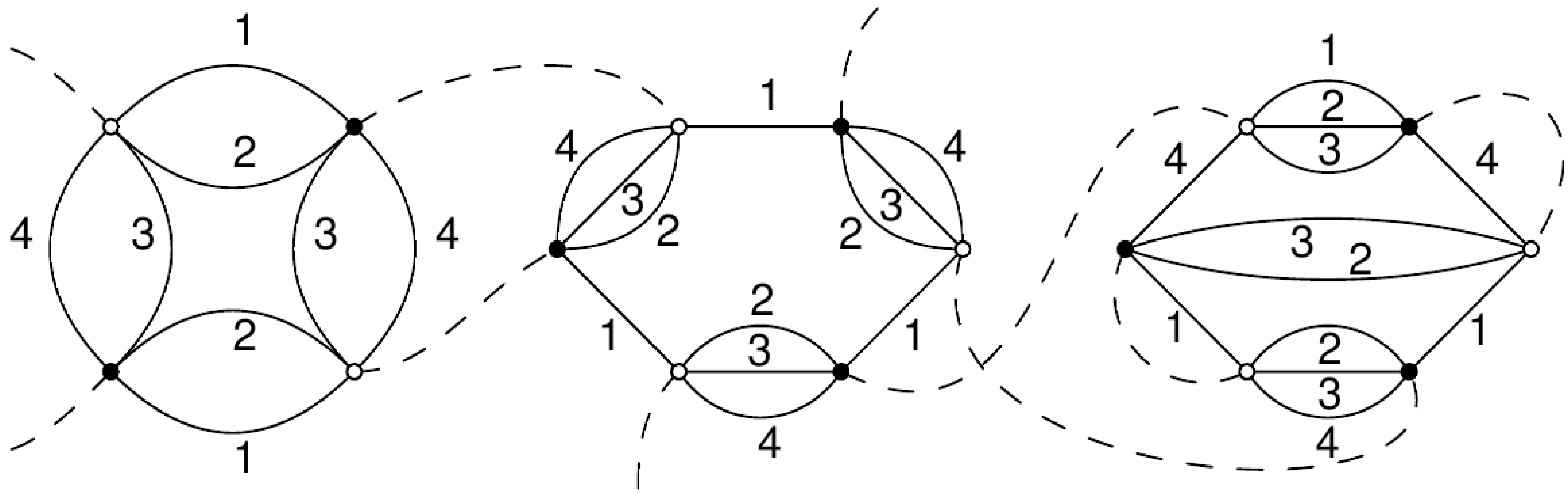


require generalization of notions of "connectedness", "contraction of high subgraphs", "locality", Wick ordering, ...

taking into account internal structure of Feynman graphs, full combinatorics of dual cellular complex, results from crystallization theory (dipole moves)

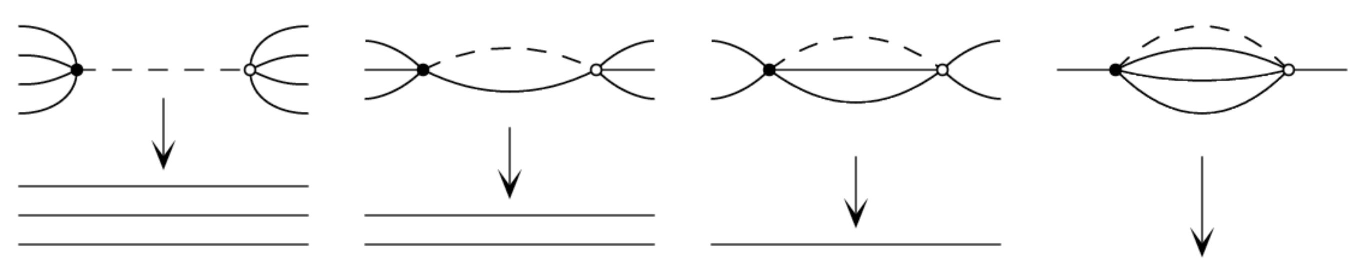
TGFT renormalization

example of Feynman diagram



- building blocks: coloured bubbles, dual to d-cells with triangulated boundary
- glued along their boundary (d-1)-simplices
- parallel transports (discrete connection) associated to dashed (color 0, propagator) lines
- faces of color i = connected set of (alternating) lines of color 0 and i

“contraction of internal line” ~ dipole contraction



TGFT example: SU(2), d=3, with gauge invariance

Carrozza, DO, Rivasseau, '13

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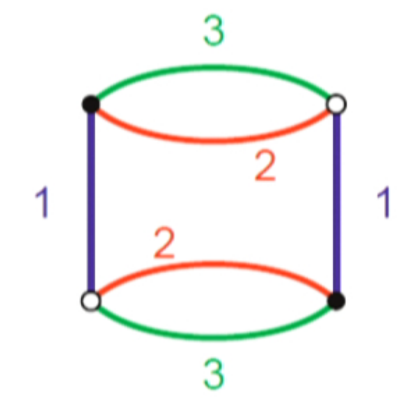

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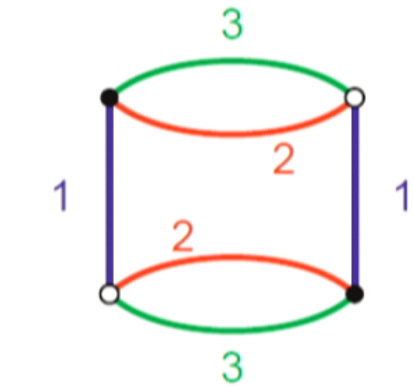

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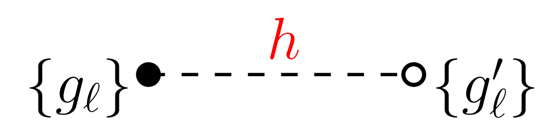

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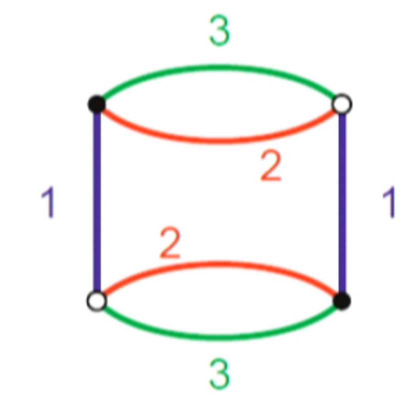

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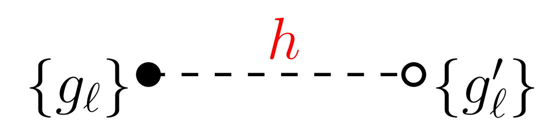

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introduce cut-off: $\Lambda \; (\sim \sum_\ell j_\ell(j_\ell + 1) \leq \Lambda^2)$

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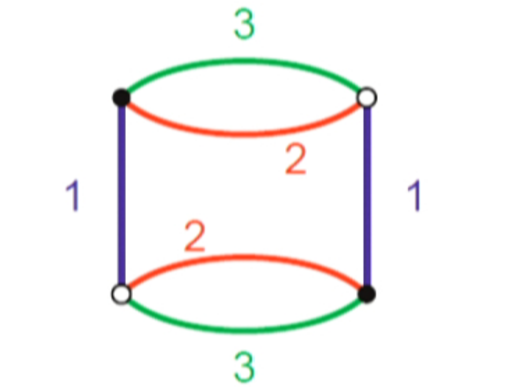

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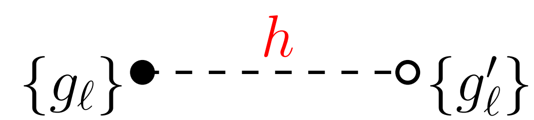

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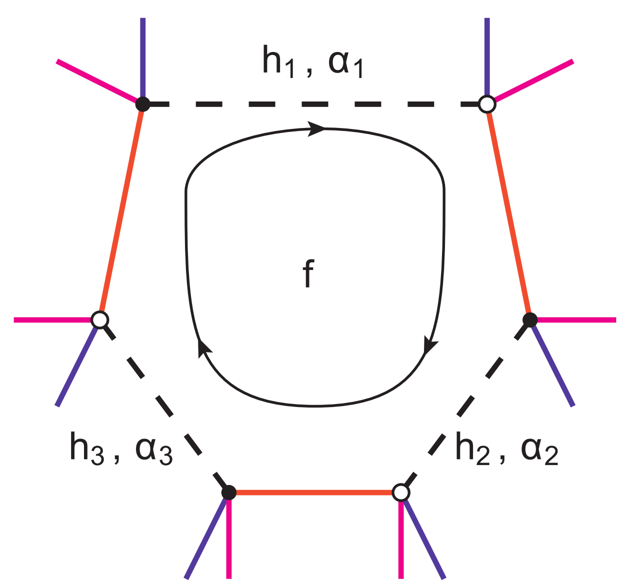
amplitudes factorise per face:



$\longleftrightarrow \qquad K_{\alpha_1+\alpha_2+\alpha_3}(h_1h_2h_3)$

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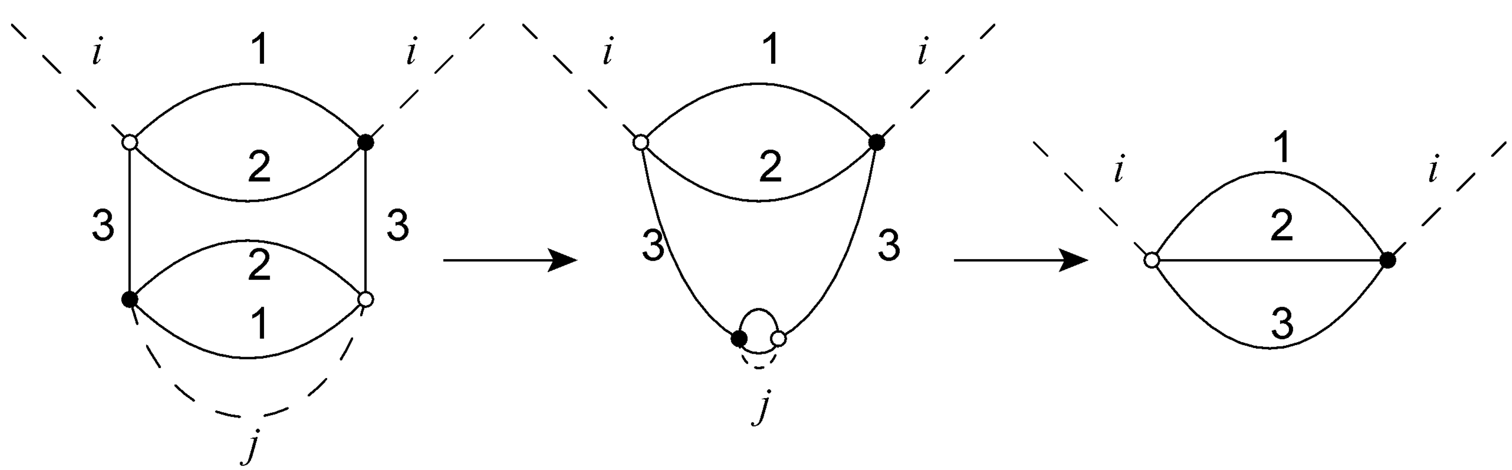

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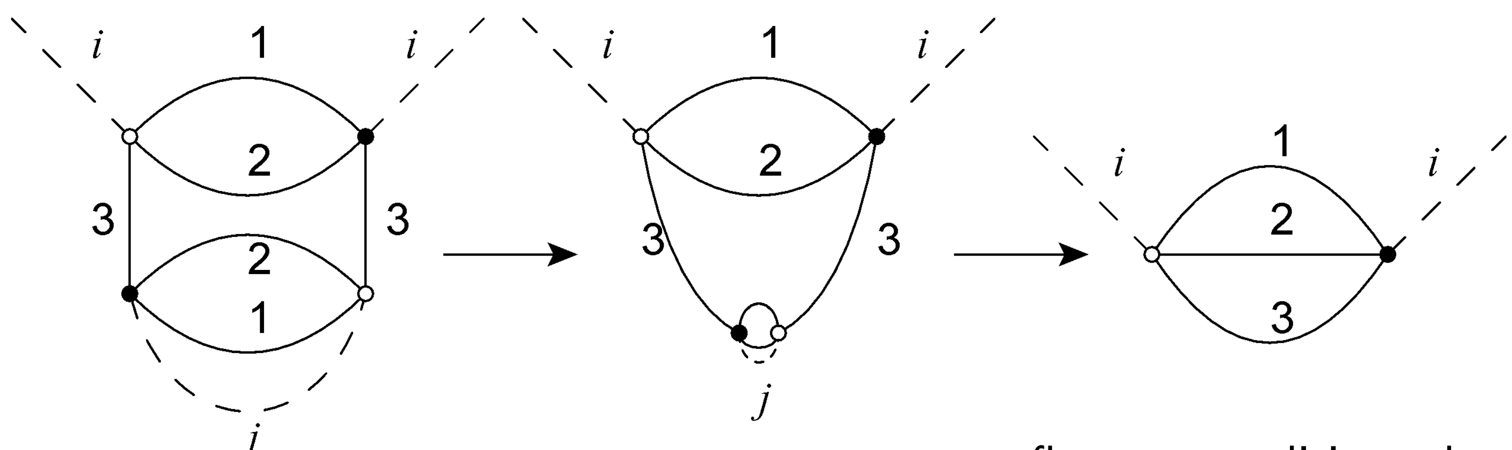

it requires a special property: "traciality"

- flatness condition: the parallel transports must peak around $\mathbf{1}$ (up to gauge)
- combinatorial condition: connected boundary graph.

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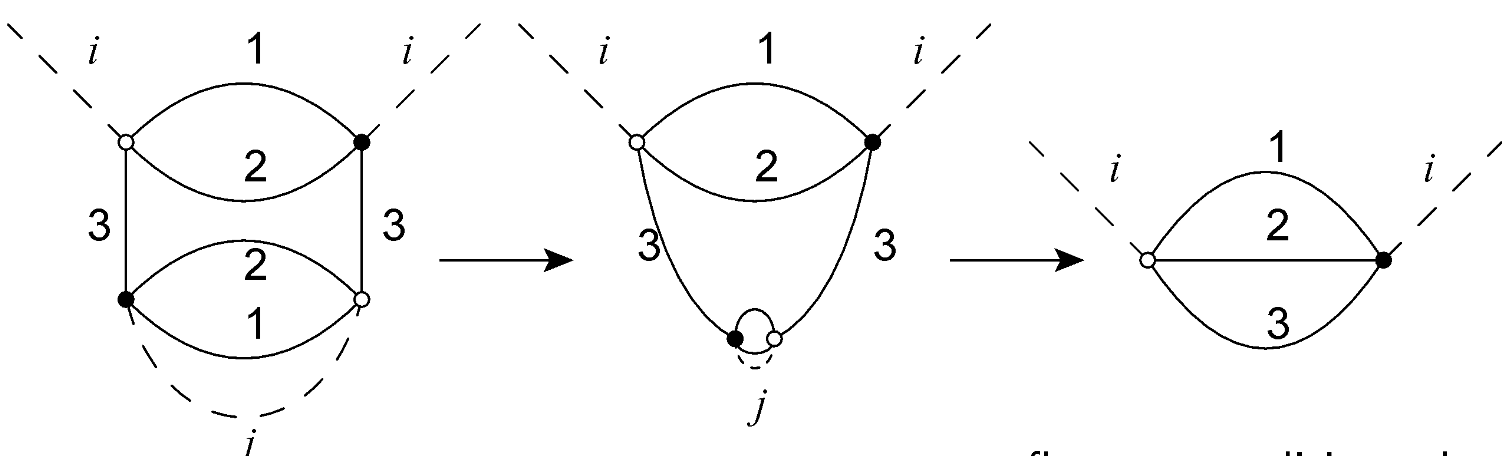
3	3	6	$G = \mathrm{SU}(2)$ [Oriti, Rivasseau, SC '13]
3	4	4	$G = \mathrm{SU}(2) \times \mathrm{U}(1)$ [SC '14]
4	2	4	
5	1	6	$G = \mathrm{U}(1)$ [Ousmane Samary, Vignes-Tourneret '12]
6	1	4	$G = \mathrm{U}(1)$ [Ousmane Samary, Vignes-Tourneret '12]

similar analysis for TGFTs on homogeneous space SU(2)/U(1) Lahoche, DO, '15

necessary condition: divergent subgraphs must be "quasi-local", i.e. tensor invariants



it requires a special property: "traciality"

- flatness condition: the parallel transports must peak around $\mathbf{1}$ (up to gauge)
- combinatorial condition: connected boundary graph.

true for models dominated by "melonic diagrams"

GFT perturbative renormalization

- systematic renormalisability group analysis of Tensorial Group Field Theory (TGFT) models:

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- several renormalizable abelian TGFT models (different groups and dimension, with/without gauge invariance)

  J. Ben Geloun, V. Rivasseau, '11; J. Ben Geloun, D. Ousmane-Samary, '11 S. Carrozza, DO, V. Rivasseau, '12

- first renormalizable non-abelian TGFT model in 3d with gauge invariance (3d BF + laplacian)

  S. Carrozza, DO, V. Rivasseau, '13

- first renormalizable TGFT model on homogeneous space (SU(2)/U(1))^d V. Lahoche, DO, '15

- proof of asymptotic freedom for abelian TGFT models without gauge invariance

  J. Ben Geloun, D. Ousmane-Samary, '11; J. Ben Geloun, '12

- study of asymptotic freedom/safety for non-abelian TGFT models with gauge invariance

  S. Carrozza, '14

- 4th order interactions: generic asymptotic freedom (strong wave function renorm.); higher orders: more subtle

- ……………

Nonperturbative GFT renormalisation (continuum limit)

see talk by Dario

the issue:

$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\; e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\,\mathcal{A}_\Gamma$$

controlling quantum dynamics of more and more (up to infinity) interacting degrees of freedom

~ evaluating GFT path integral (in some non-perturbative approximation = full sum over triangulations)

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IR cutoff N —-> 0

~ definition of full GFT path integral

~ full continuum limit

(all dofs of spin foam model/discrete gravity)

$$\mathcal{Z}_N[J] = e^{W_N[J]} = \int_M d\phi\, e^{-S[\phi]-\Delta S_N[\phi]+\mathrm{Tr}_2(J\cdot\phi)}$$

$$\Gamma_N[\varphi] = \sup_J \Big(\mathrm{Tr}_2(J\cdot\varphi) - W_N(J)\Big) - \Delta S_N[\varphi]$$

$$\partial_t \Gamma_N[\varphi] = \frac{1}{2}\overline{\mathrm{Tr}}(\partial_t R_N \cdot [\Gamma_N^{(2)} + R_N]^{-1})$$

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more or less standard set-up

main difficulty: combinatorial structure of interactions

Non-perturbative GFT renormalization

see talk by Dario

Functional RG approach to GFTs - recent results:

- Polchinski formulation based on SD equations   Krajewski, Toriumi, '14
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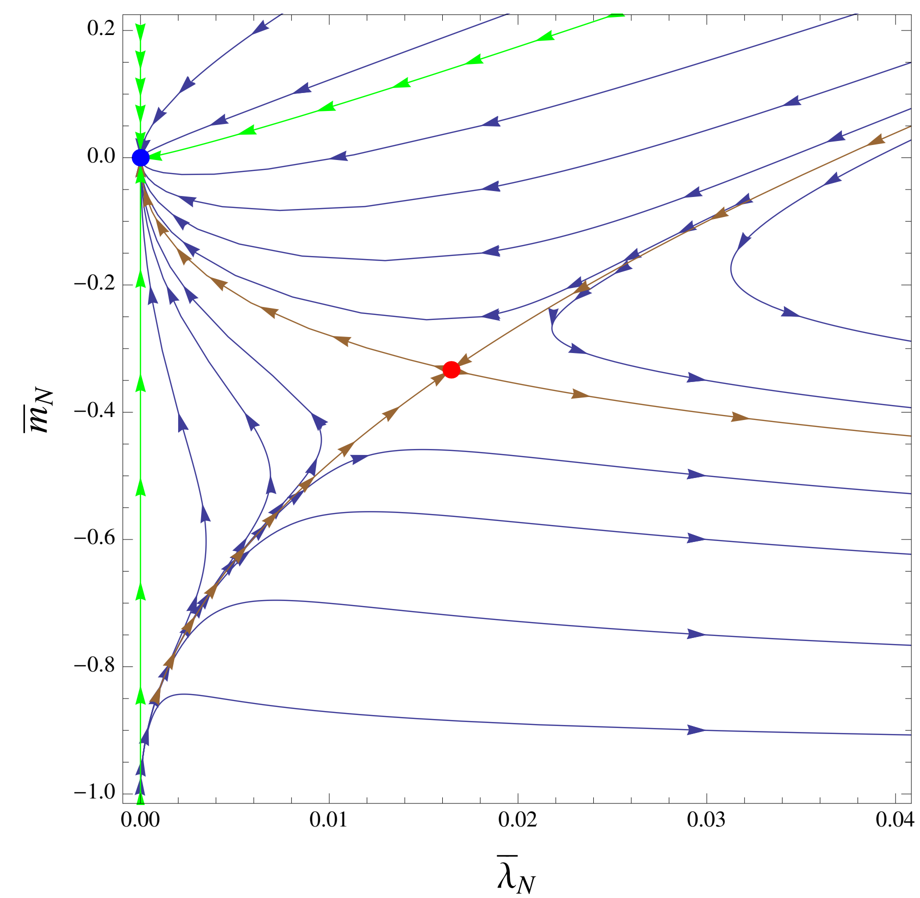

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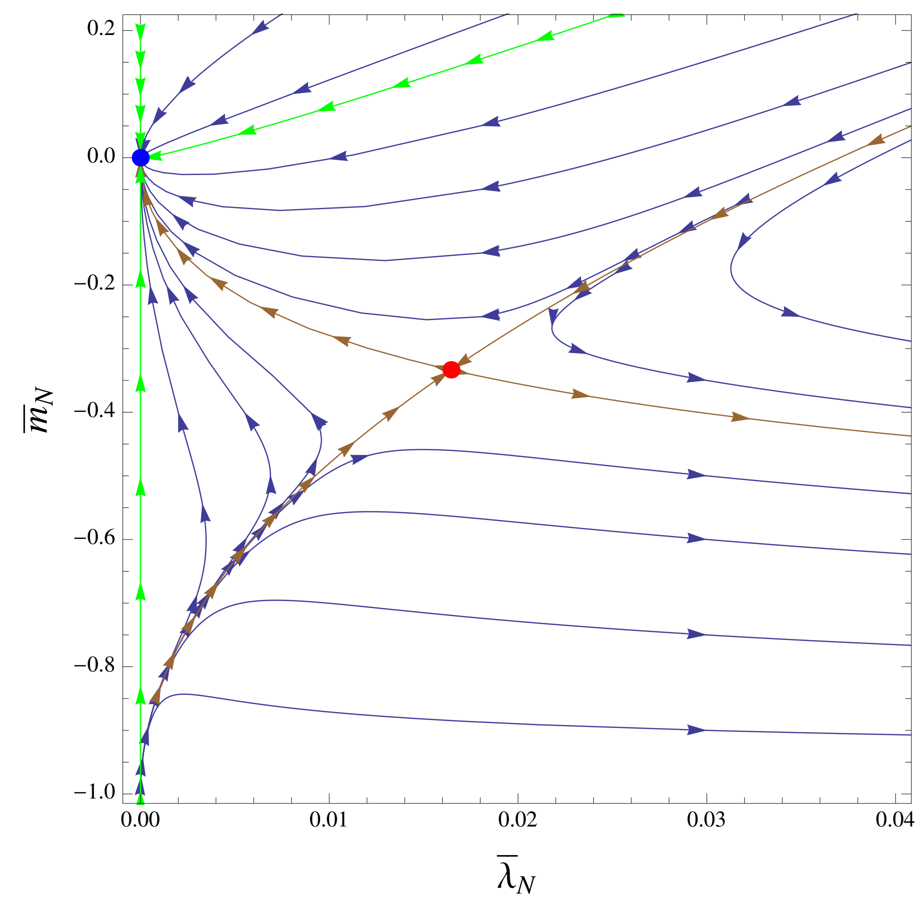

interesting for effective continuum physics:
cosmology from QG

(Quantum) Cosmology from GFT condensates

S. Gielen, DO, L. Sindoni, PRL, arXiv:1303.3576 [gr-qc]; JHEP, arXiv:1311.1238 [gr-qc]

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Thank you for your attention