asymptotic safety
from gauge theories to gravity

Daniel F Litim
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Strongly Interacting Field Theories 2015
U Jena, 5 Nov 2015

QFTs beyond asymptotic freedom

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Strongly Interacting Field Theories 2015
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standard model

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local QFT for fundamental interactions

- **strong** nuclear force
- **weak** force
- **electromagnetic** force

degrees of freedom

- spin 0 (the **Higgs** has finally arrived)
- spin 1/2 (quite a few)
- spin 1

perturbatively renormalisable & **predictive**

standard model

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local QFT for fundamental interactions

strong nuclear force
weak force
electromagnetic force

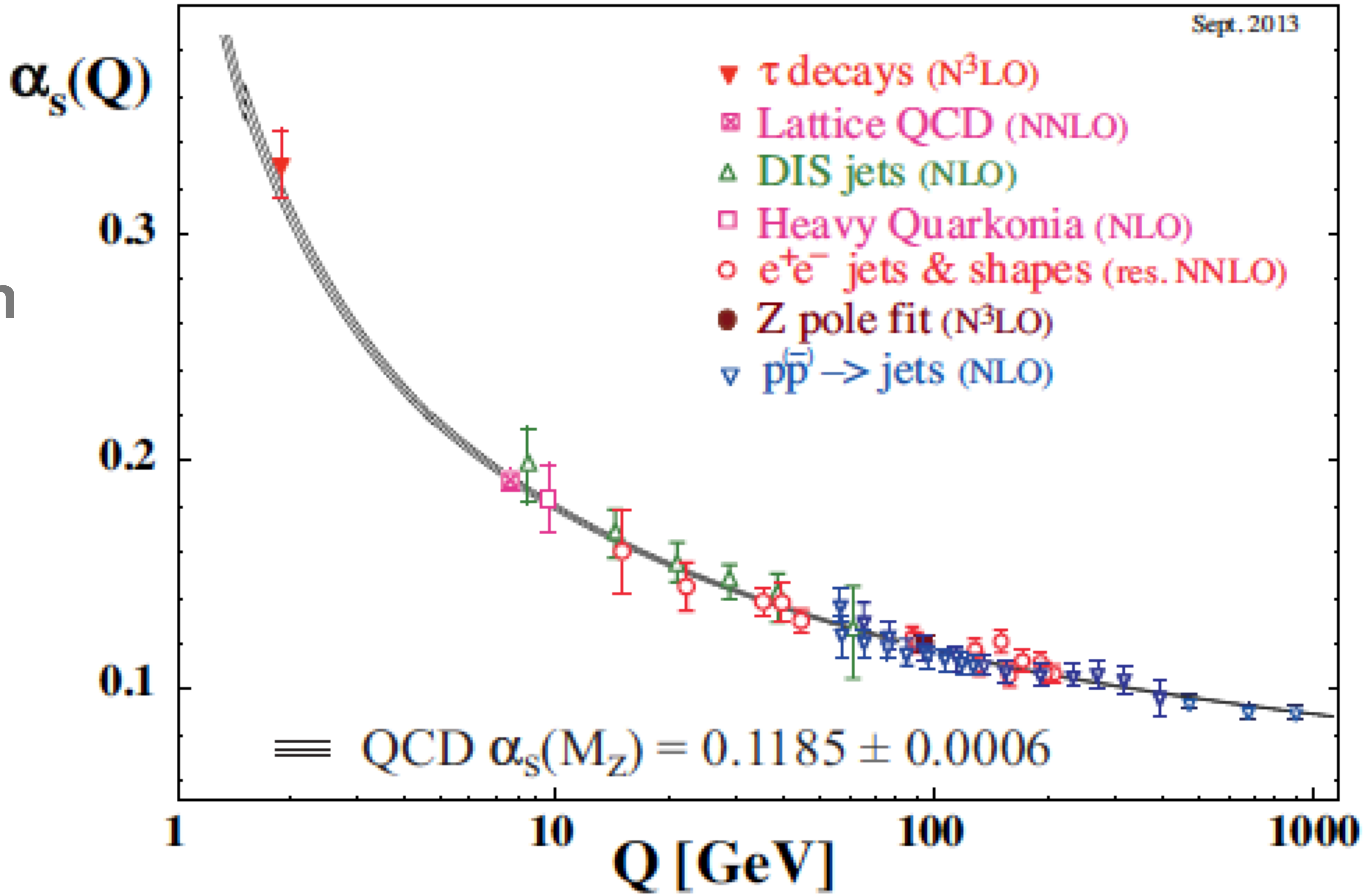
challenges

Higgs, U(1): maximal UV extension?
how does **quantum gravity** fit in?

(interacting) UV fixed points

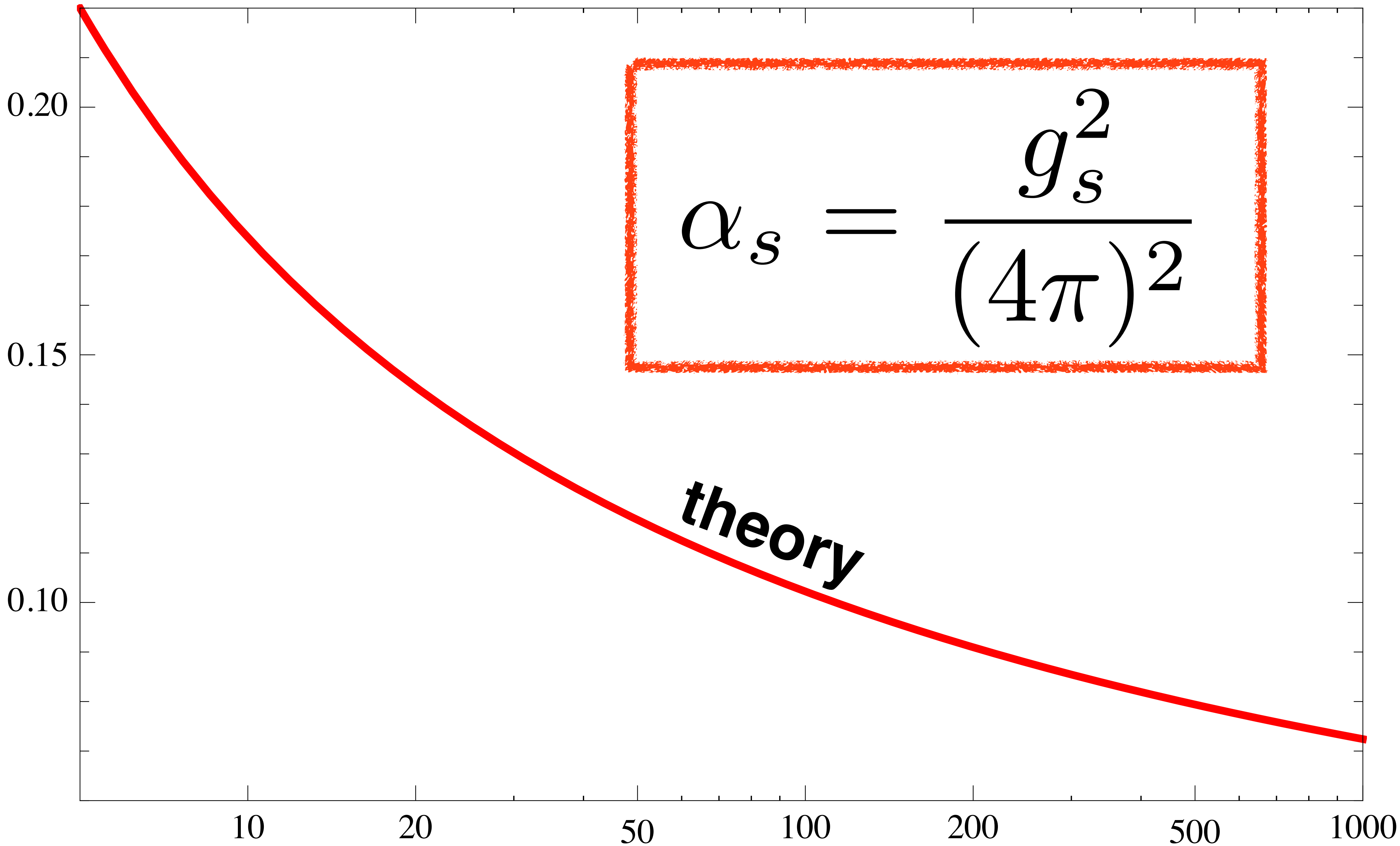
asymptotic freedom

triumph of QFT



asymptotic freedom

triumph of QFT



energy

Gross, Wilczek '74
Politzer '74

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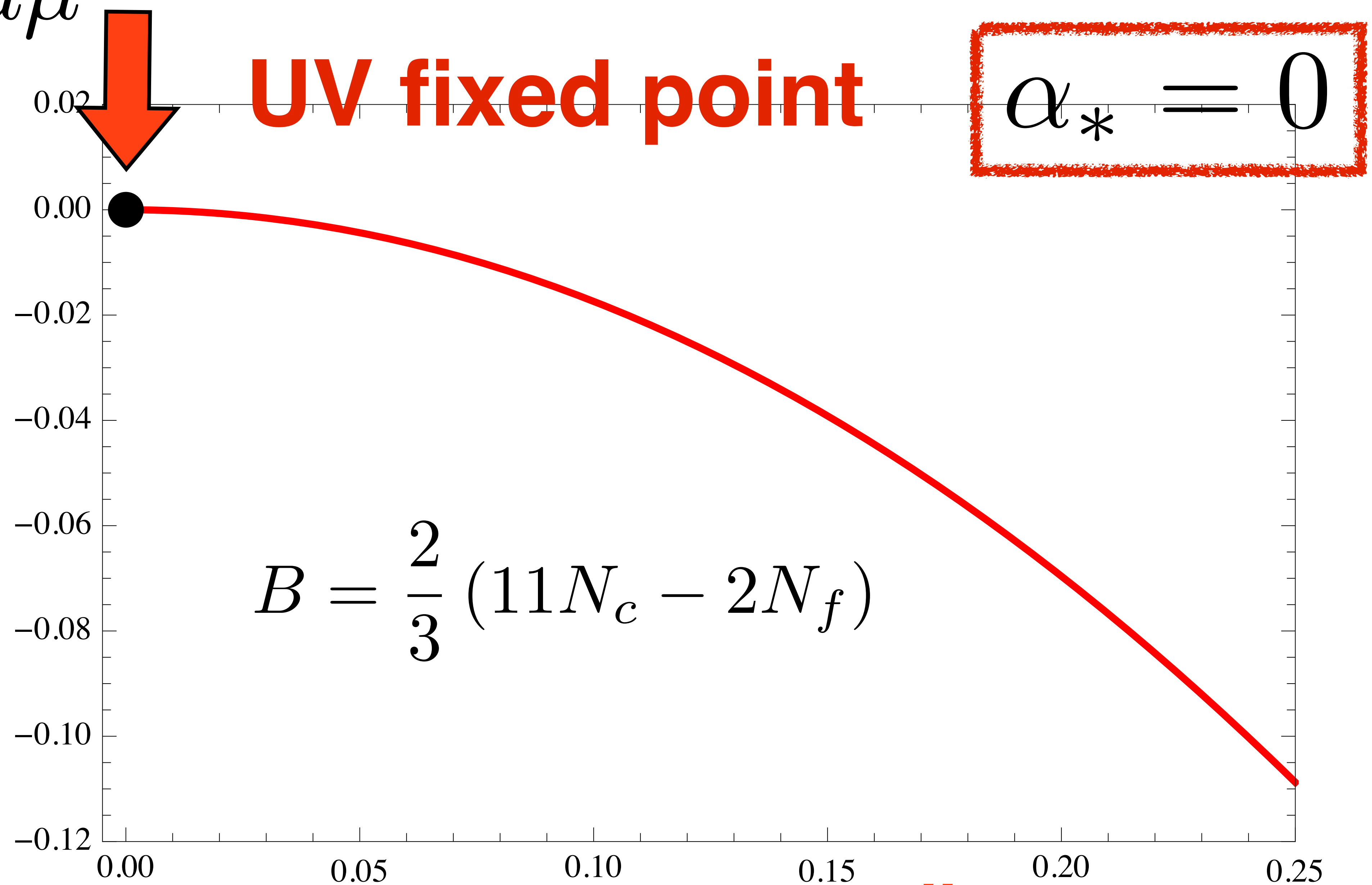
asymptotic freedom

$$\mu \frac{d\alpha_s}{d\mu} = -B\alpha_s^2 < 0$$

QCD beta function



gauge α_s coupling

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asymptotic freedom

$$\mu\frac{d\alpha_s}{d\mu} = -B\alpha_s^2 < 0$$

QCD beta function

UV fixed point

$$\alpha_* = 0$$

fundamental definition of QFT ⟺ **UV fixed point**

Wilson '71

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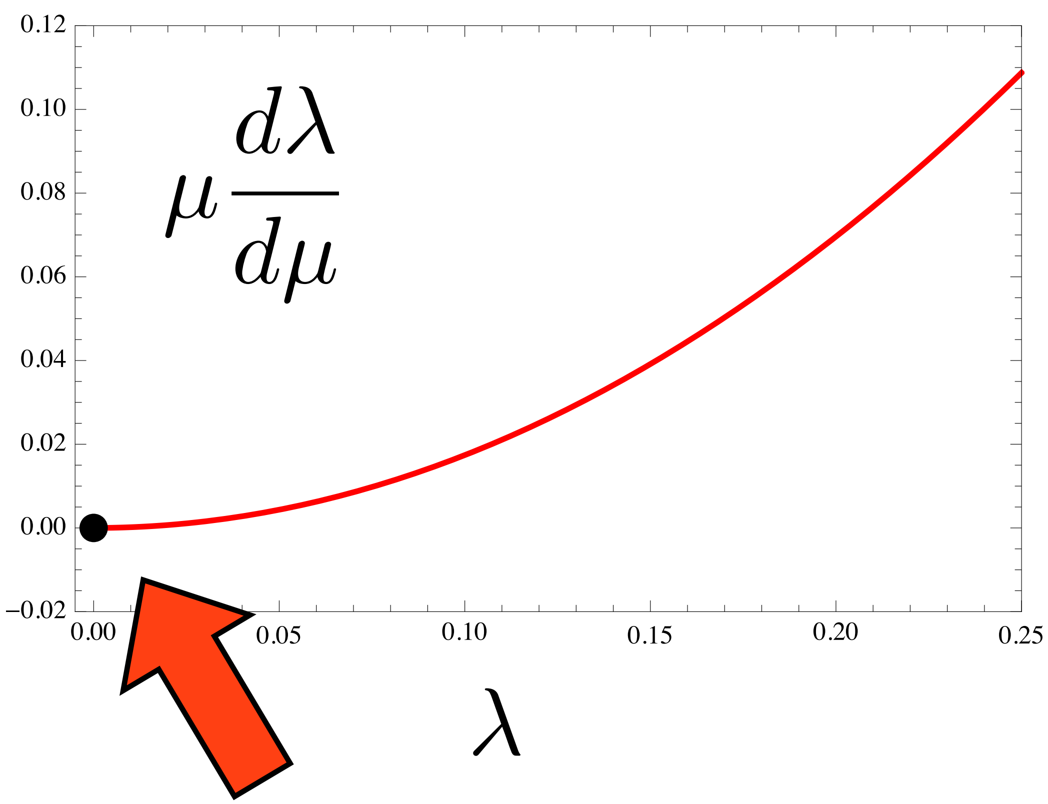
asymptotic freedom

$$\mu \frac{d\alpha_{e.m.}}{d\mu} \propto \alpha_{e.m.}^2 > 0$$

QED beta function

$$\mu \frac{d\lambda}{d\mu} \propto \lambda^2 > 0$$

Higgs self-coupling
Yukawa couplings



IR fixed point

... but no UV fixed point

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asymptotic freedom

$$\mu \frac{d\alpha_{e.m.}}{d\mu} \propto \alpha_{e.m.}^2 > 0$$

QED beta function

$$\mu \frac{d\lambda}{d\mu} \propto \lambda^2 > 0$$

Higgs self-coupling
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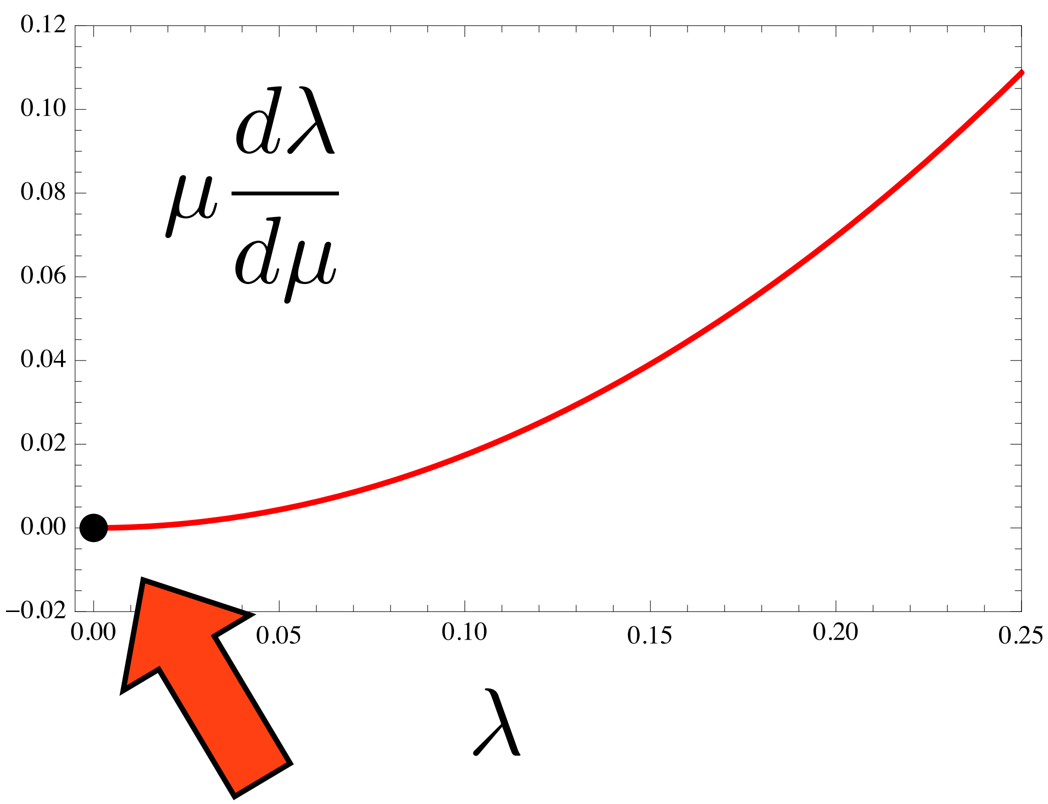
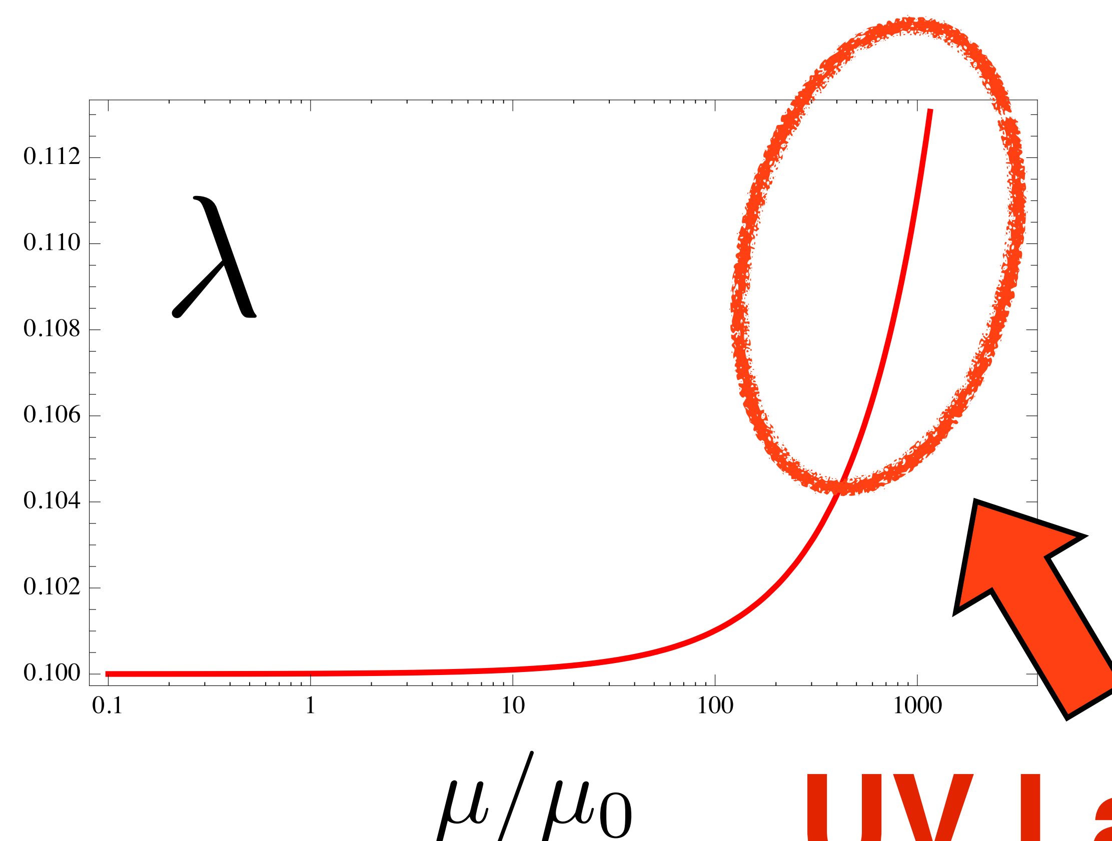


IR fixed point



UV Landau pole

asymptotic freedom

complete asymptotic freedom in 4D:

all couplings achieve non-interacting UV fixed point

fields	cAF?
scalars	no
scalars with fermions	no
non-Abelian gauge fields	yes
non-Abelian fields with fermions	yes*
non-Abelian fields, fermions, scalars	yes*

*) provided certain conditions hold true

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asymptotic freedom

complete asymptotic freedom in 4D:
all couplings achieve non-interacting UV fixed point

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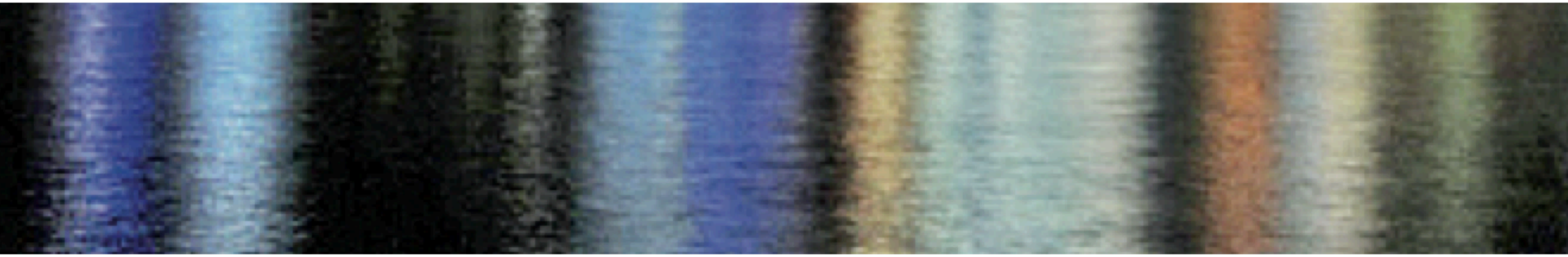
origin of
asymptotic safety



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origin of asymptotic safety

theory with coupling α**:**

$t = \ln \mu / \Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

perturbative non-renormalisability: $A > 0$

origin of asymptotic safety

theory with coupling α**:**

$t = \ln \mu/\Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

fixed points $\alpha_* = 0$ $\alpha_* = A/B$

origin of asymptotic safety

theory with coupling α:

$t = \ln \mu/\Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

fixed points if $A > 0\,,\ B > 0$:

$\alpha_* = 0$ **IR**

$\alpha_* = A/B$ **UV**

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origin of asymptotic safety

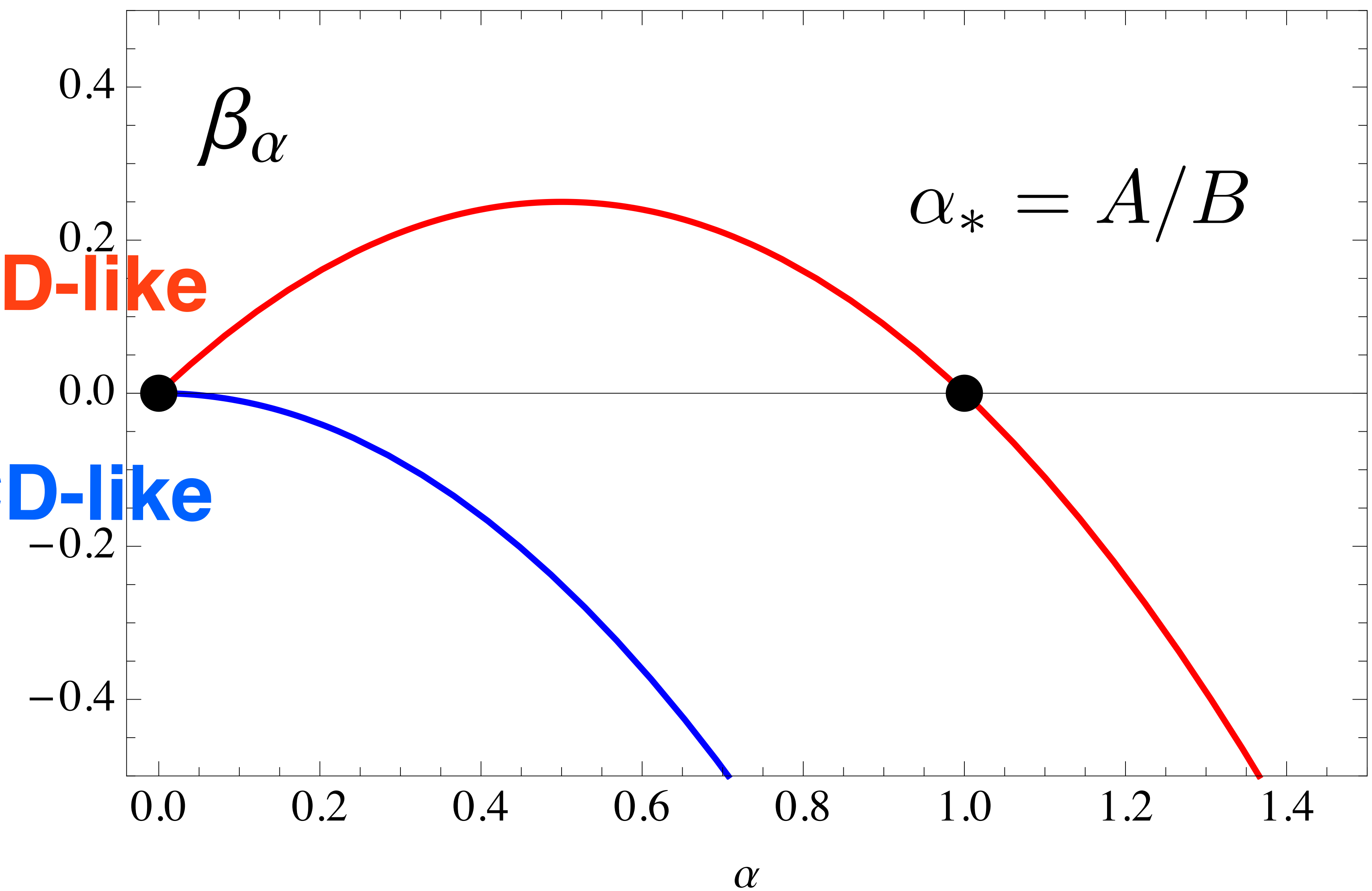
theory with coupling α **:** $t = \ln \mu / \Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

A>0: QED-like

A=0: QCD-like



origin of asymptotic safety

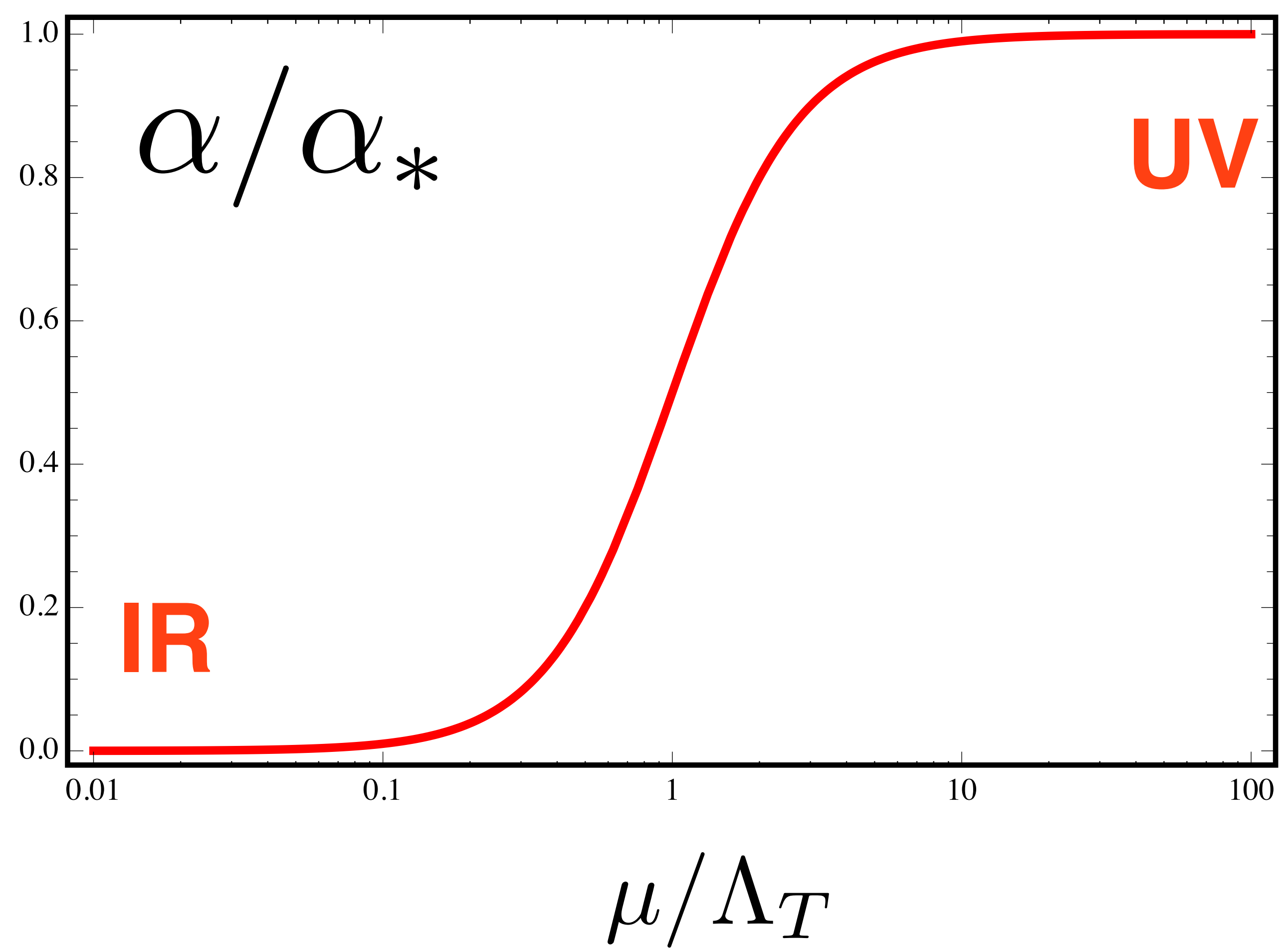
theory with coupling α**:**

$$t = \ln \mu/\Lambda$$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$$\alpha_* \ll 1$$

$$\alpha_* = A/B$$



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exact asymptotic safety

when is this reliable?

need $$\alpha_* = A/B \ll 1$$

epsilon expansion: $\epsilon = D - D_c$

large-N expansion: many fields

perturbation theory applicable

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asymptotic safety

how is this predictive?

UV: interactions are **softened by fluctuations**

UV behaviour characterised by

relevant, **marginal**, **irrelevant** invariants

predictivity ⟺ **finitely many** relevant invariants

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exact asymptotic safety

	dimension	coupling	
gravitons	$D = 2+\epsilon :$	$\alpha = G_N(\mu)\mu^{D-2}$	Gastmans et al '78 Christensen, Duff '78 Weinberg '79 Kawai et al '90
fermions	$D = 2+\epsilon :$	$\alpha = g_{\rm GN}(\mu)\mu^{2-D}$	Gawedzki, Kupiainen '85 de Calan et al '91
gluons	$D = 4+\epsilon :$	$\alpha = g^2_{\rm YM}(\mu)\mu^{4-D}$	Peskin '80 Morris '04
scalars	$D = 2+\epsilon :$	$\alpha = g_{NL}(\mu)\mu^{D-2}$	Brezin, Zinn-Justin '76 Bardeen, Lee, Shrock '76

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exact asymptotic safety

	dimension	coupling	
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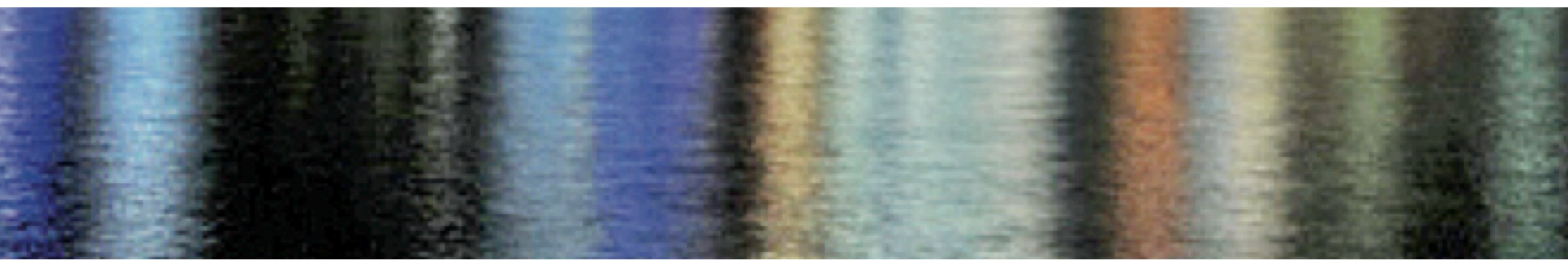
scalars	$D = 2 + \epsilon :$	$\alpha = g_{NL}(\mu)\mu^{D-2}$	Brezin, Zinn-Justin '76 Bardeen, Lee, Shrock '76
classes of gauge-Yukawa theories	$D = 4 :$ new!	several α_i new!	Litim, Sannino 1406.2337

exact asymptotic safety
of 4D gauge-Yukawa theories

Litim, Sannino 1406.2337



gauge theory + fermions

SU(NC) YM with NF fermions:

$$\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$$

$t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\, \alpha_g^2$$

$\alpha_* \ll 1$

gauge theory + fermions

SU(NC) YM with NF fermions: $\alpha_g = \frac{g^2 N_c}{(4\pi)^2}$ $t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\, \alpha_g^2$$

$\alpha_* \ll 1$

$B > 0$: asymptotic freedom

UV fixed point

$\alpha_* = 0$

$B < 0$: no asymptotic freedom

UV fixed point?

$\alpha_* \neq 0$

gauge theory + fermions

SU(NC) YM with NF fermions:

$$\alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$t = \ln \mu / \Lambda$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3$$

$\alpha_* \ll 1$

2-loop

gauge theory + fermions

SU(NC) YM with NF fermions:

$$\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$$

$t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\, \alpha_g^2 + C\alpha_g^3$$

$\alpha_* \ll 1$

$$\alpha_* = 0 \qquad \alpha_g^* = B/C$$

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gauge theory + fermions

SU(NC) YM with NF fermions: $\alpha_g = \dfrac{g^2\, N_c}{(4\pi)^2}$ $\quad t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3$$

$\alpha_* \ll 1$

however: $\alpha_g^* = B/C$

no perturbative UV fixed point in gauge theories with fermionic matter $(C > 0)$

Caswell '74

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gauge theory + fermions

SU(NC) YM with NF fermions:

$$\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$$

$t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\, \alpha_g^2 + C \alpha_g^3$$

$\alpha_* \ll 1$

$$\alpha_g^* = B/C$$

scalar fields & Yukawa couplings required

gauge-Yukawa theory

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2} \qquad \alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$t = \ln \mu / \Lambda$

$\alpha_* \ll 1$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_g^2\,\alpha_y$$

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gauge-Yukawa theory

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2} \qquad \alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$t = \ln \mu/\Lambda$

$\alpha_* \ll 1$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_g^2\,\alpha_y$$

$$\partial_t \alpha_y = E\,\alpha_y^2 - F\,\alpha_g\,\alpha_y$$

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gauge-Yukawa theory

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2} \qquad \alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$t = \ln \mu / \Lambda$

$\alpha_* \ll 1$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_g^2\,\alpha_y$$

$$\partial_t \alpha_y = E\,\alpha_y^2 - F\,\alpha_g\,\alpha_y$$

→ **sensible interacting UV fixed point**

$$D\,F - C\,E > 0$$

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exact asymptotic safety

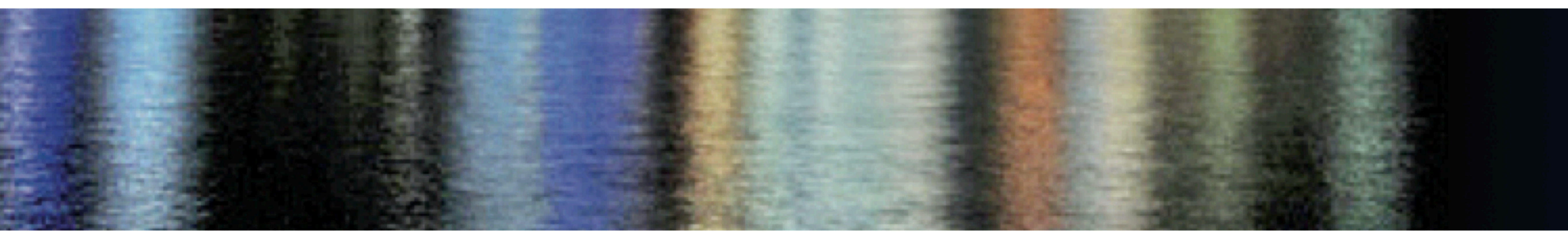
exact asymptotic safety in 4D:

couplings achieve interacting UV fixed point

fields	AS?
scalars	no
scalars with fermions	no
gauge fields	no
gauge fields with scalars or fermions	no
non-Abelian fields, fermions, scalars	yes*

*) provided certain conditions hold true

asymptotic safety and quantum gravity



quantum gravity

running coupling $g(k) = G_N(k)k^{D-2}$ $t = \ln k/\Lambda_c$

$$\partial_t g = (D - 2 + \eta_N)\, g$$

$g_* \neq 0$ $g_* = 0$

UV IR fixed points

4D:

large anomalous dimension $\eta_N = \eta_N(g, \text{all other couplings})$

large UV scaling exponents $\vartheta \approx \mathcal{O}(1)$

strong coupling effects $g_* \approx \mathcal{O}(1)$

relevant vs **irrelevant**

invariants not known a priori

quantum gravity

computational methods

continuum: non-perturbative renormalisation group

lattice: Monte Carlo simulations

simplicial gravity

dynamical triangulations

asymptotic freedom `the knowns'	vs	asymptotic safety `the unknowns'
$g_* = 0$		$g_* \neq 0$
anomalous dimensions		anomalous dimensions
$\eta_A = 0$		$\eta_N \neq 0$
canonical power counting		non-canonical power counting
$\{\vartheta_{\mathrm{G},n}\}$ are known		$\{\vartheta_n\}$ are not known
F^{256} irrelevant !		R^{256} relevant / marginal / irrelevant ?

K Falls, DL, K Nikolakopoulos & C Rahmede, 1301.4191

bootstrap search strategy

hypothesis **relevancy** of invariants **follows** their canonical **mass** dimension

K Falls, DL, K Nikolakopoulos & C Rahmede, 1301.4191

bootstrap search strategy

hypothesis **relevancy** of invariants **follows** their canonical **mass** dimension

strategy

Step 1 retain invariants up to mass dimension D

Step 2 compute $\{\vartheta_n\}$ (eg. RG, lattice, holography)

Step 3 enhance D, and iterate

convergence (**no convergence**) of the iteration:

hypothesis **supported** (**refuted**)

f(R)

$$\Gamma_k \propto f(R)$$

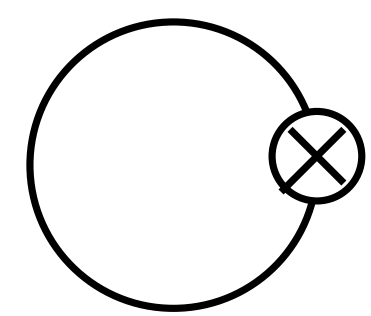
Ricci scalars

$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n \, k^{d_n} \int d^4x \sqrt{g}\, R^n$$

effective action with invariants up to mass dimension $D = 2(N-1)$

technicalities: **functional renormalisation**

$$k\frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} = \frac{1}{2}\,\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} k\frac{\mathrm{d}R_k}{\mathrm{d}k}\right] = \frac{1}{2}$$



here:

M Reuter hep-th/9605030

DL hep-th/0103195

hep-th/0312114

Falls, DL, Nikolakopoulos, Rahmede 1301.4191.pdf

Falls, DL, Nikolakopoulos, Rahmede 1410.4815

A Codello, R Percacci, C Rahmede 0705.1769, 0805.2909

P Machado, F Saueressig 0712.0445

identifying fixed points

$$f(R) = \sum_n \lambda_n R^n$$

polynomial expansion

generating function

$$\partial_t f + 4f - 2R\, f' = I[f]$$

$$I[f] = I_0[f] + I_1[f] \cdot \partial_t f' + I_2[f] \cdot \partial_t f''$$

recursive solution of $\beta_n \equiv \partial_t \lambda_n$ $\beta_{n-2} = 0$

family of FP candidates $\lambda_n = \lambda_n(\lambda_0, \lambda_1)$

`free' parameters (λ_0, λ_1)

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interlude: Wilson-Fisher FP

$$u(\rho) = \sum_{n=0} \frac{\lambda_n}{n!}\rho^n \qquad \rho = \frac{1}{2}\phi^a\phi_a$$

polynomial expansion

generating function

$$\partial_t u' = -2u' + (d-2)\rho u'' - A\frac{u''}{(1+u')^2} - B\frac{3u''+2\rho u'''}{(1+u'+2\rho u'')^2}$$

recursive solution $(\lambda_1 \equiv m^2)$

$$\boxed{\lambda_n = \lambda_n(m^2)}$$

Wilson-Fisher FP

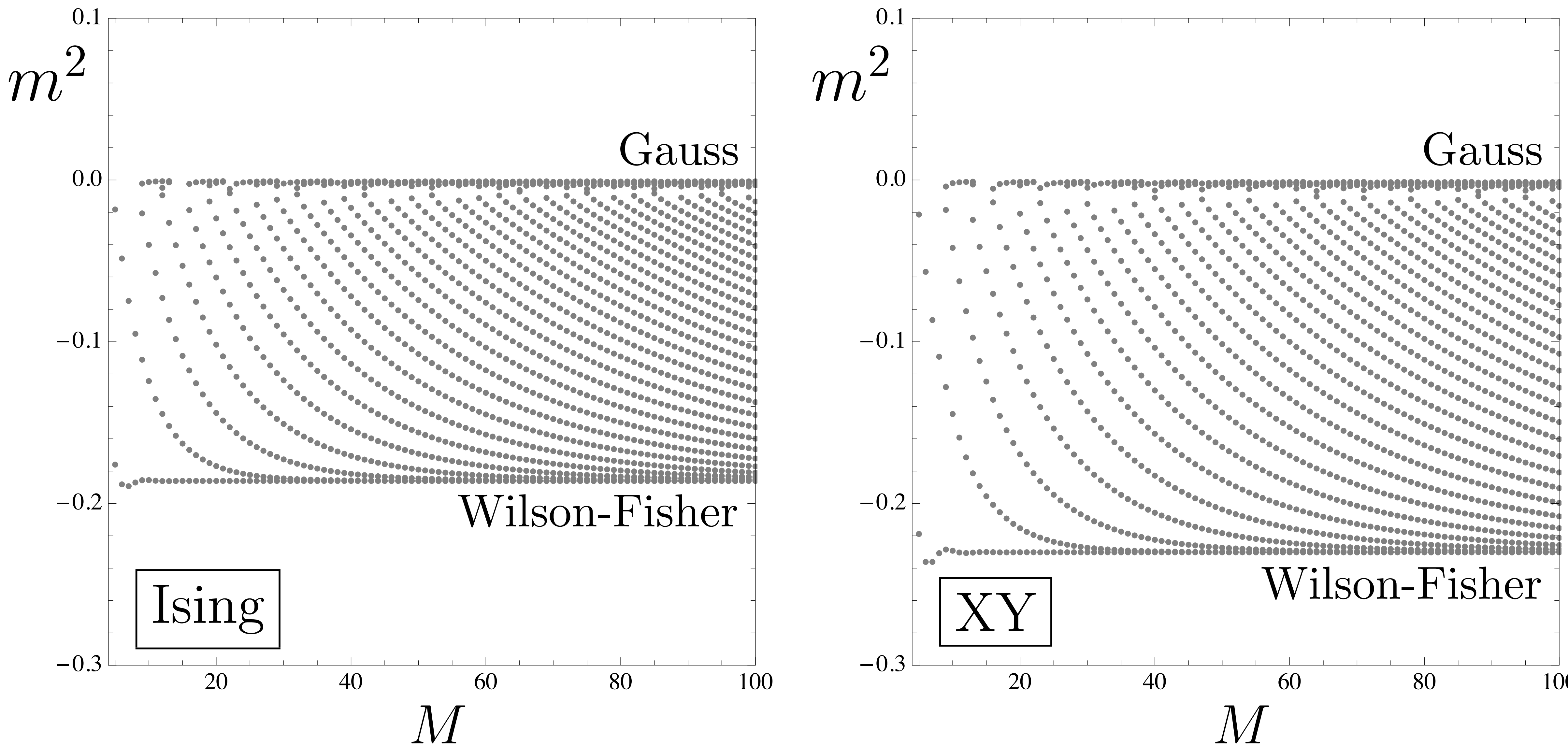
FP solutions with $\lambda_M = 0$

A Juettner, DL, E Marchais (2015)

Wilson-Fisher FP

FP solutions with $\lambda_M = 0$

A Juettner, DL, E Marchais (2015)



Wilson-Fisher FP

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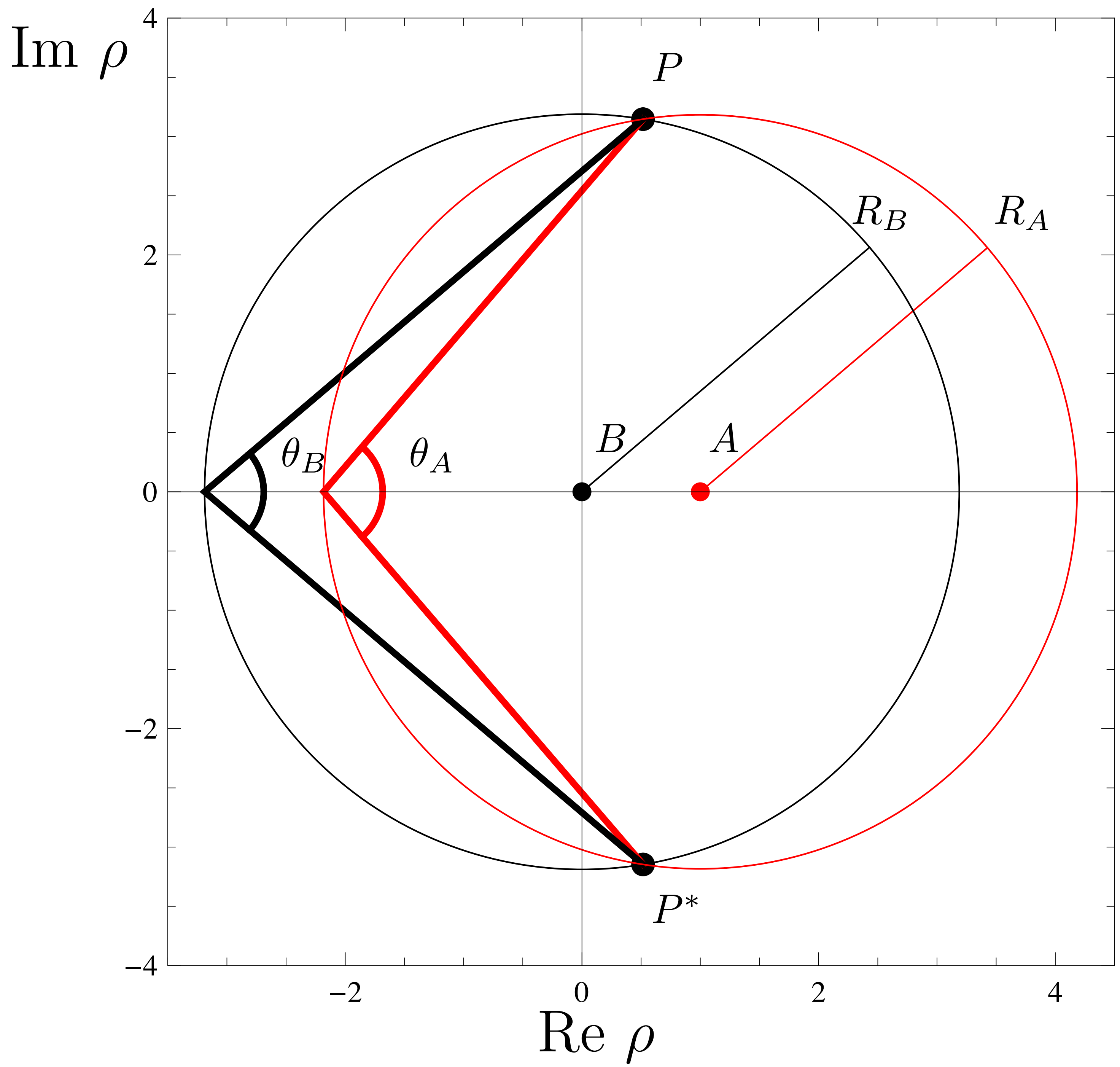
singularity in the complex plane

A Juettner, DL, E Marchais (2015)



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Wilson-Fisher FP

singularity in the complex plane

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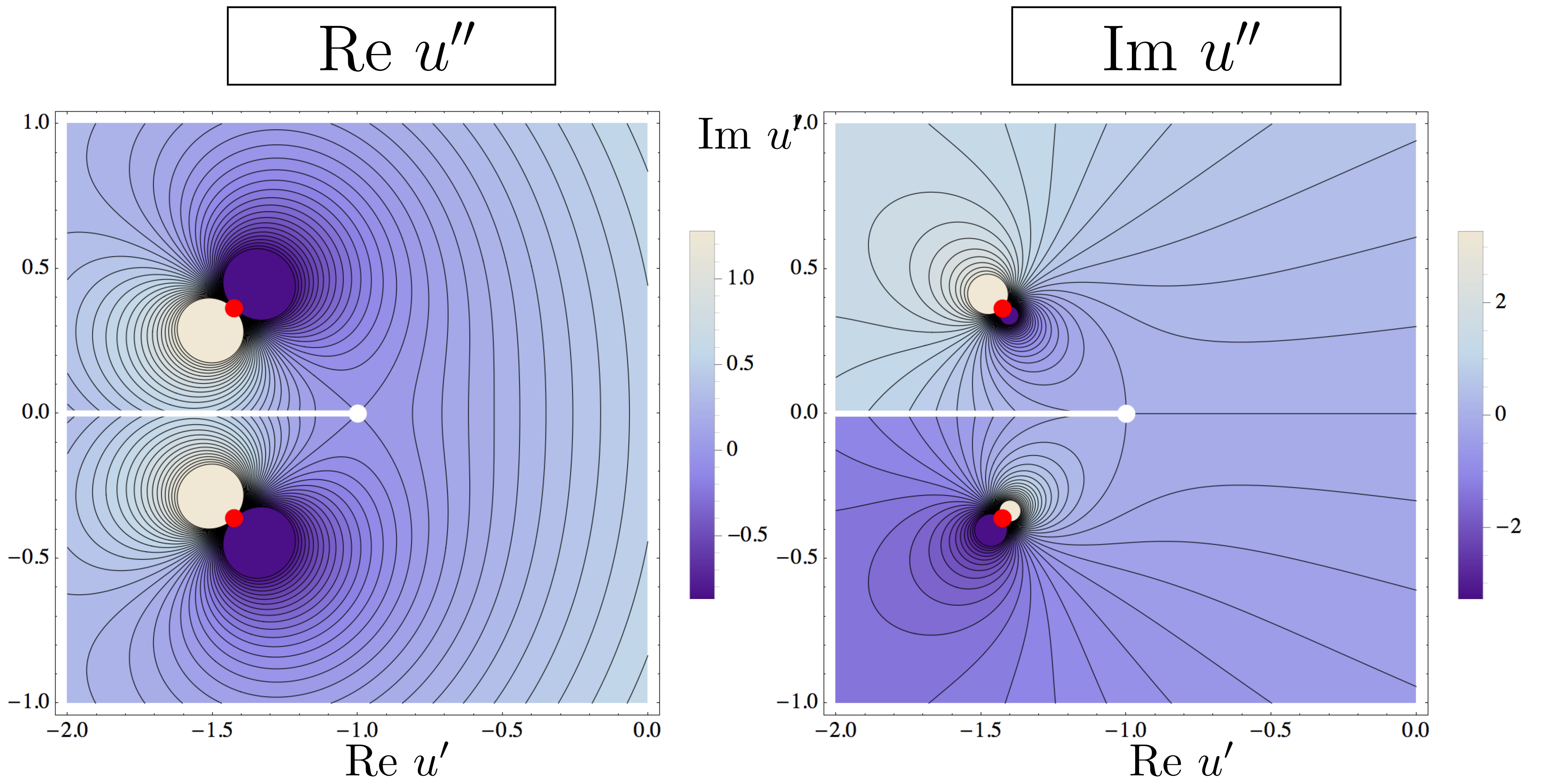


f(R)

recursive solution

$$\lambda_n(\lambda_0, \lambda_1) = \frac{P_n(\lambda_0, \lambda_1)}{Q_n(\lambda_0, \lambda_1)}$$

boundary condition

$$\lambda_N = 0 \quad \& \quad \lambda_{N+1} = 0$$

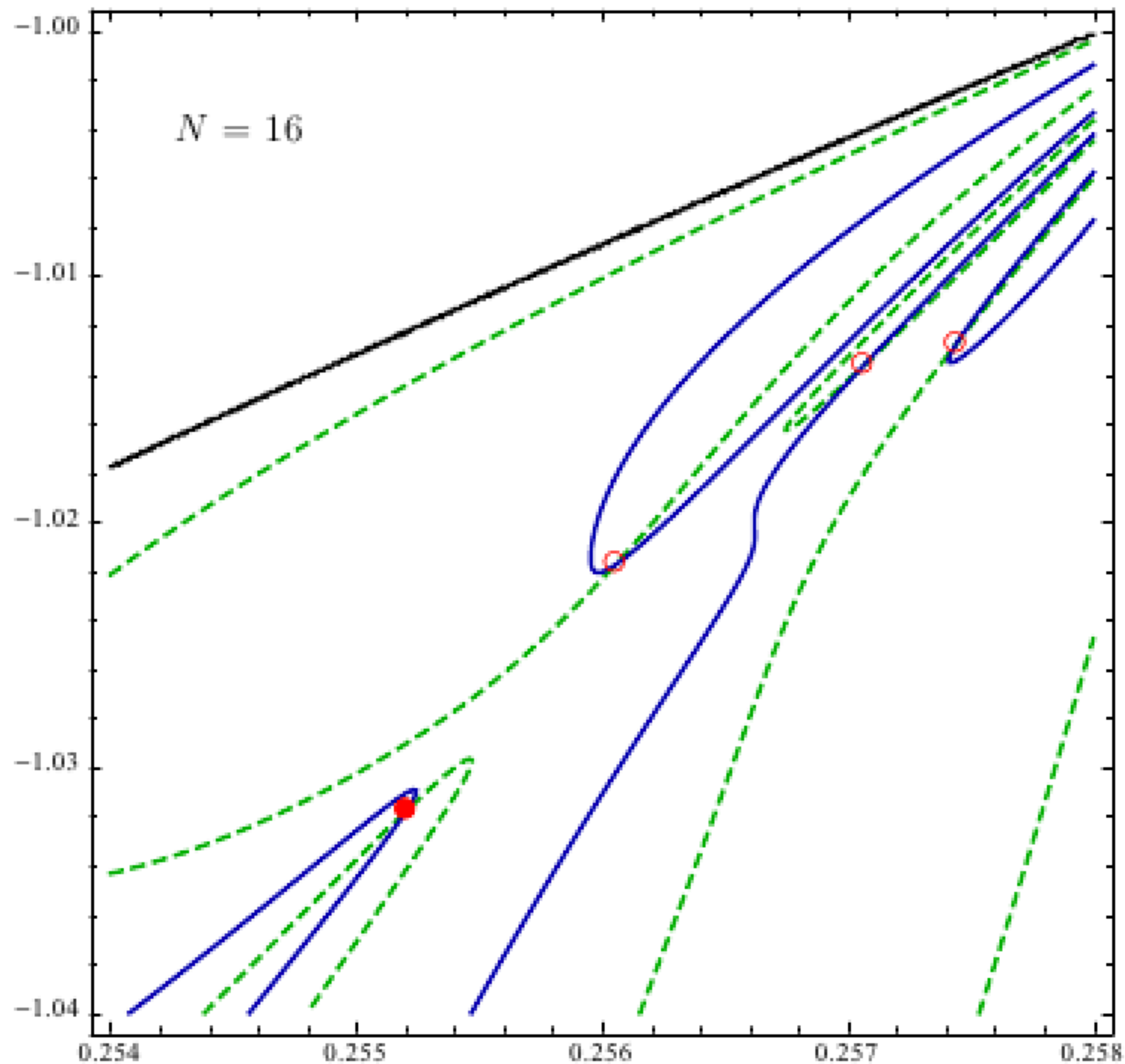
polynomials grow large, eg.

$P_{35} \approx 45.000$ terms

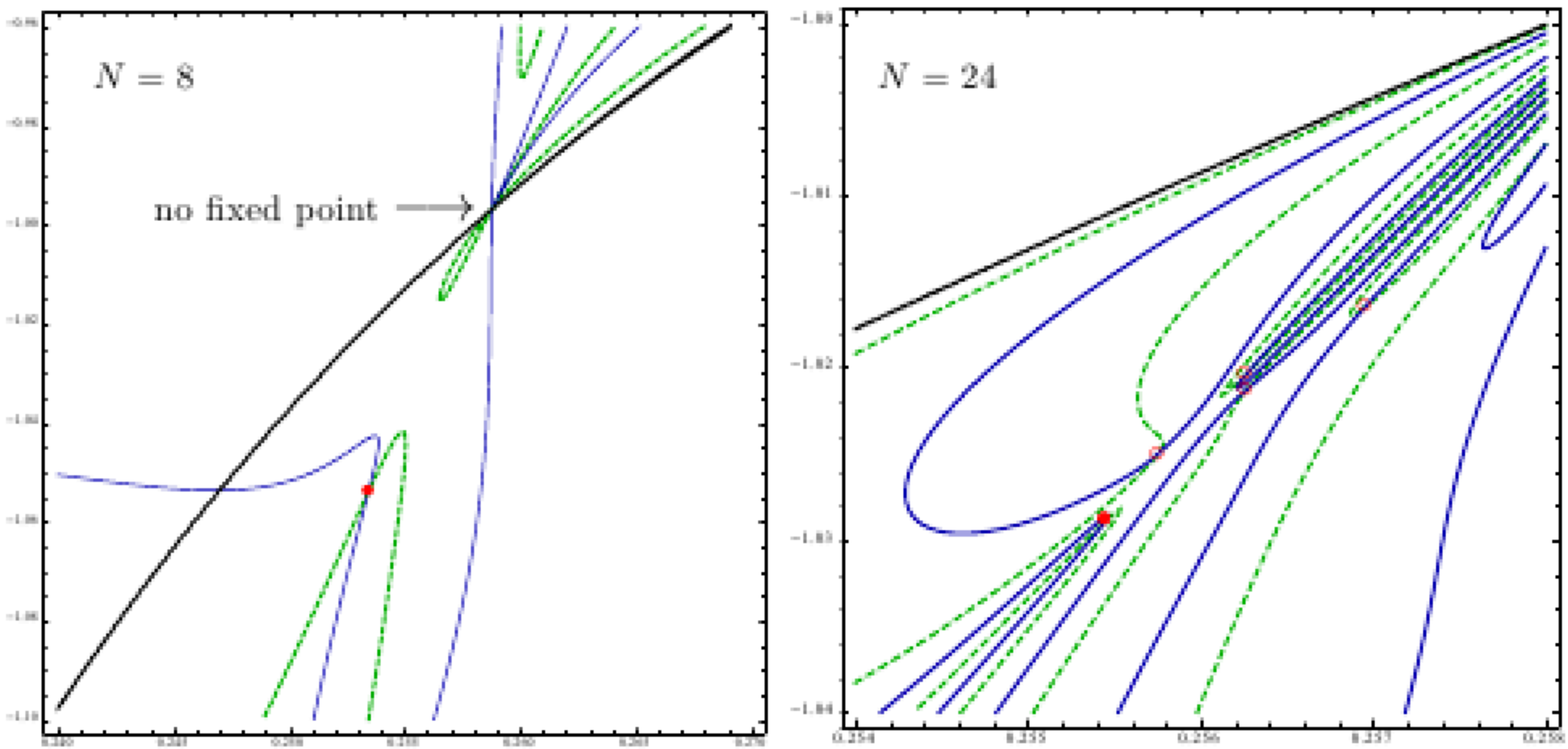
f(R)

boundary condition

$$\begin{aligned} \lambda_N &= 0 \\ \lambda_{N+1} &= 0 \end{aligned}$$



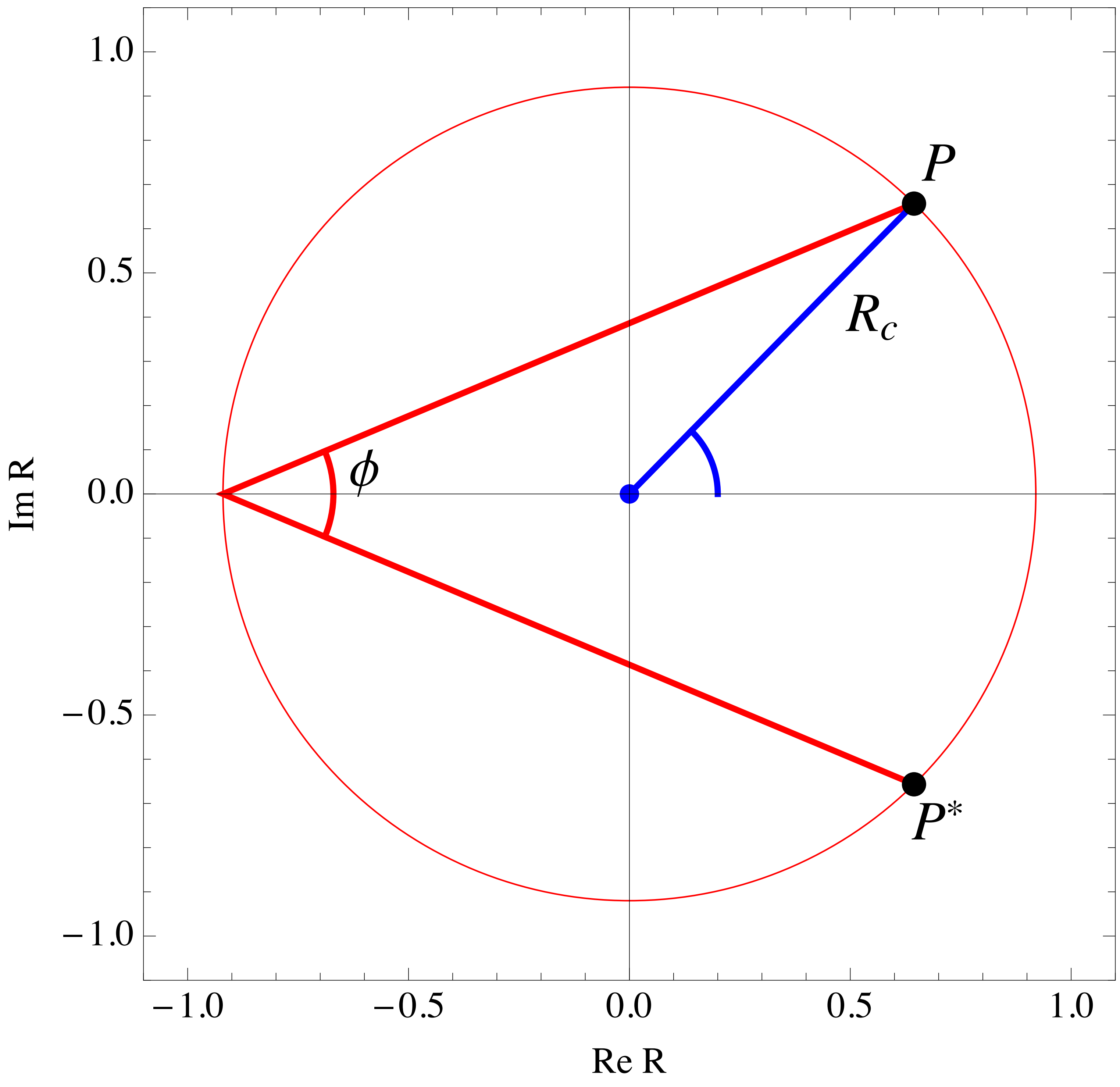
f(R)

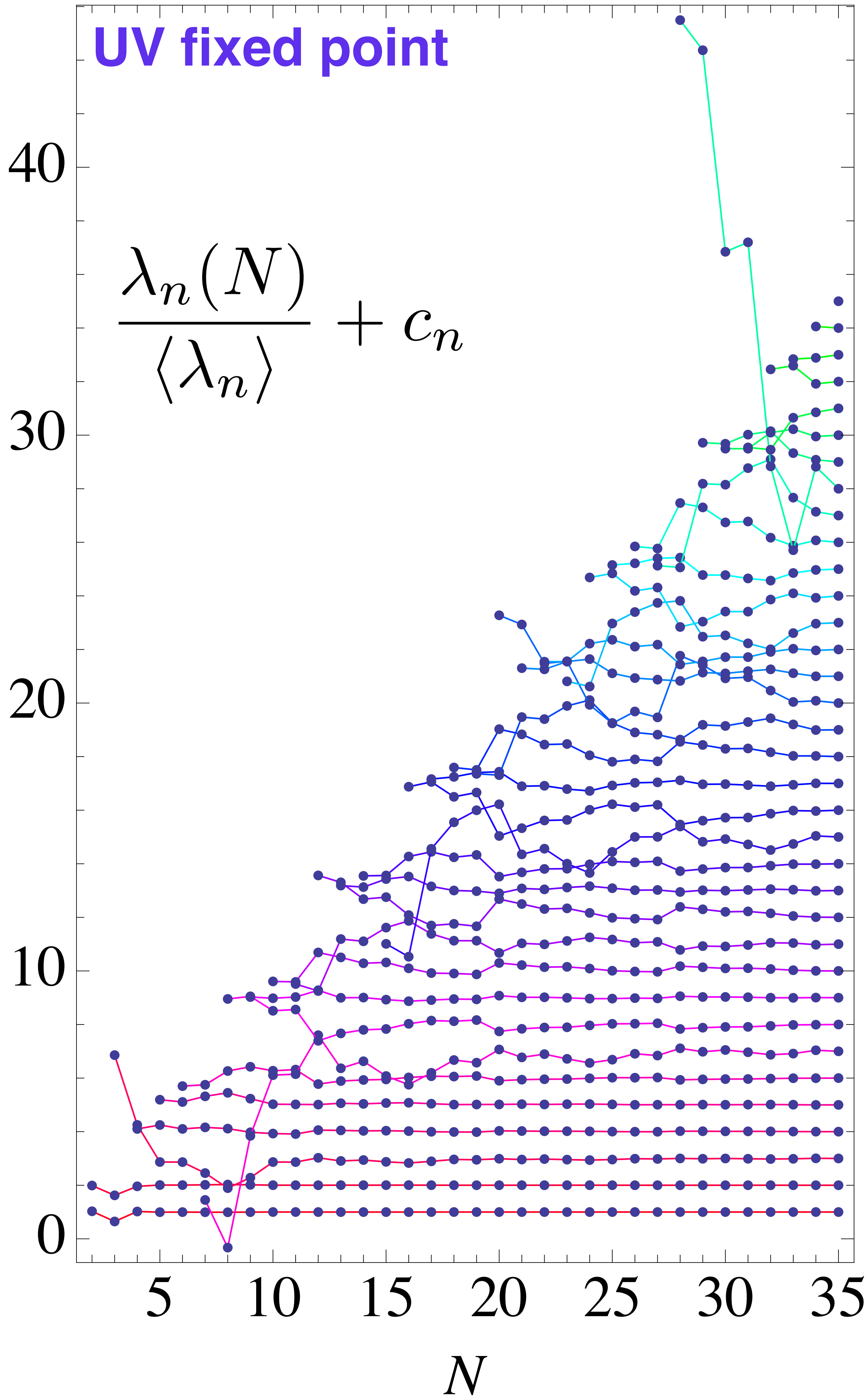


boundary condition $\lambda_N = 0 \quad \& \quad \lambda_{N+1} = 0$

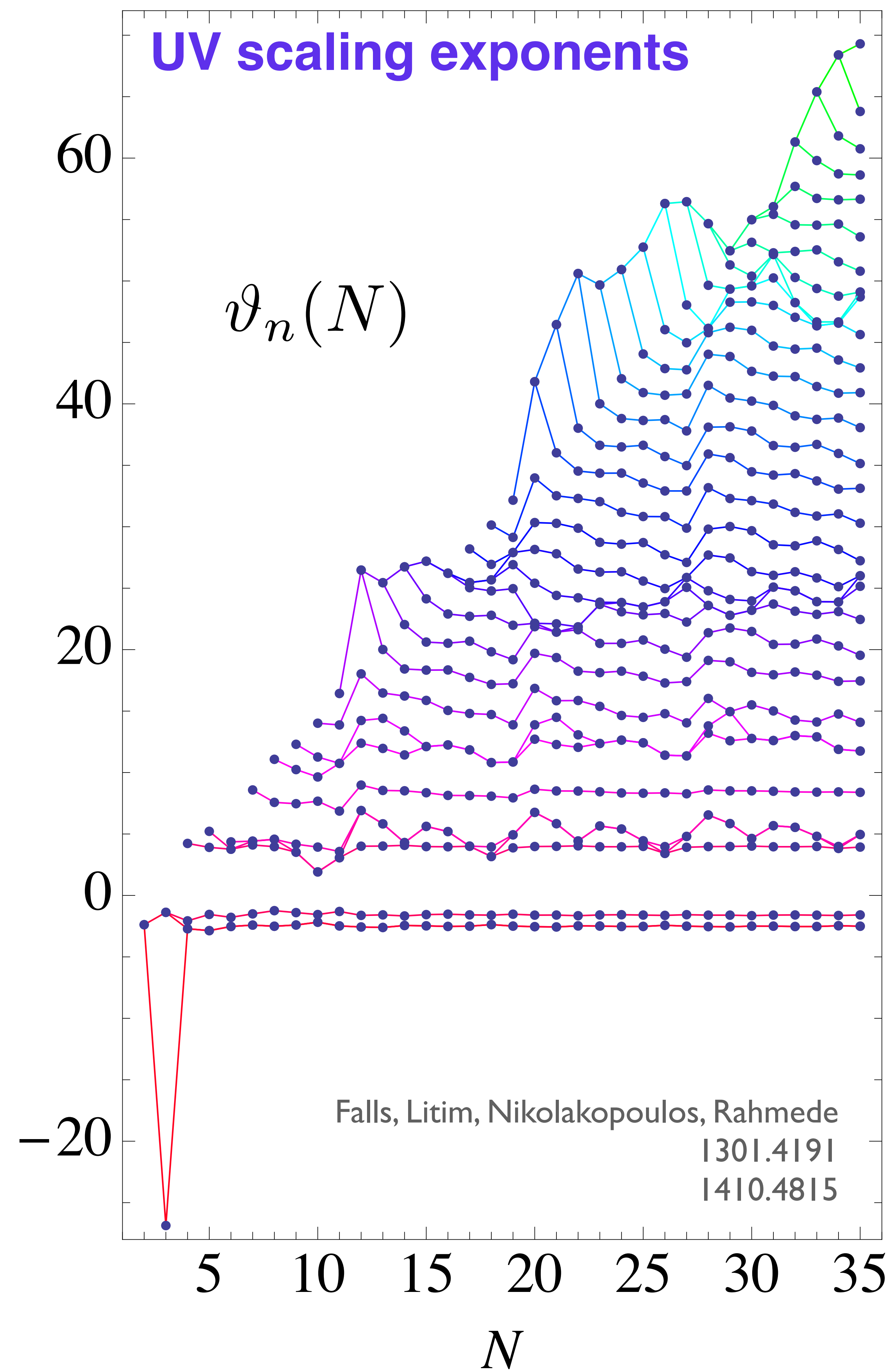
f(R)

radius of convergence





UV fixed point
$\frac{\lambda_n(N)}{\langle\lambda_n\rangle} + c_n$
0
10
20
30
40
5
10
15
20
25
30
35
N



UV scaling exponents
$\vartheta_n(N)$
−20
0
20
40
60
5
10
15
20
25
30
35
N
Falls, Litim, Nikolakopoulos, Rahmede
1301.4191
1410.4815

scaling exponents

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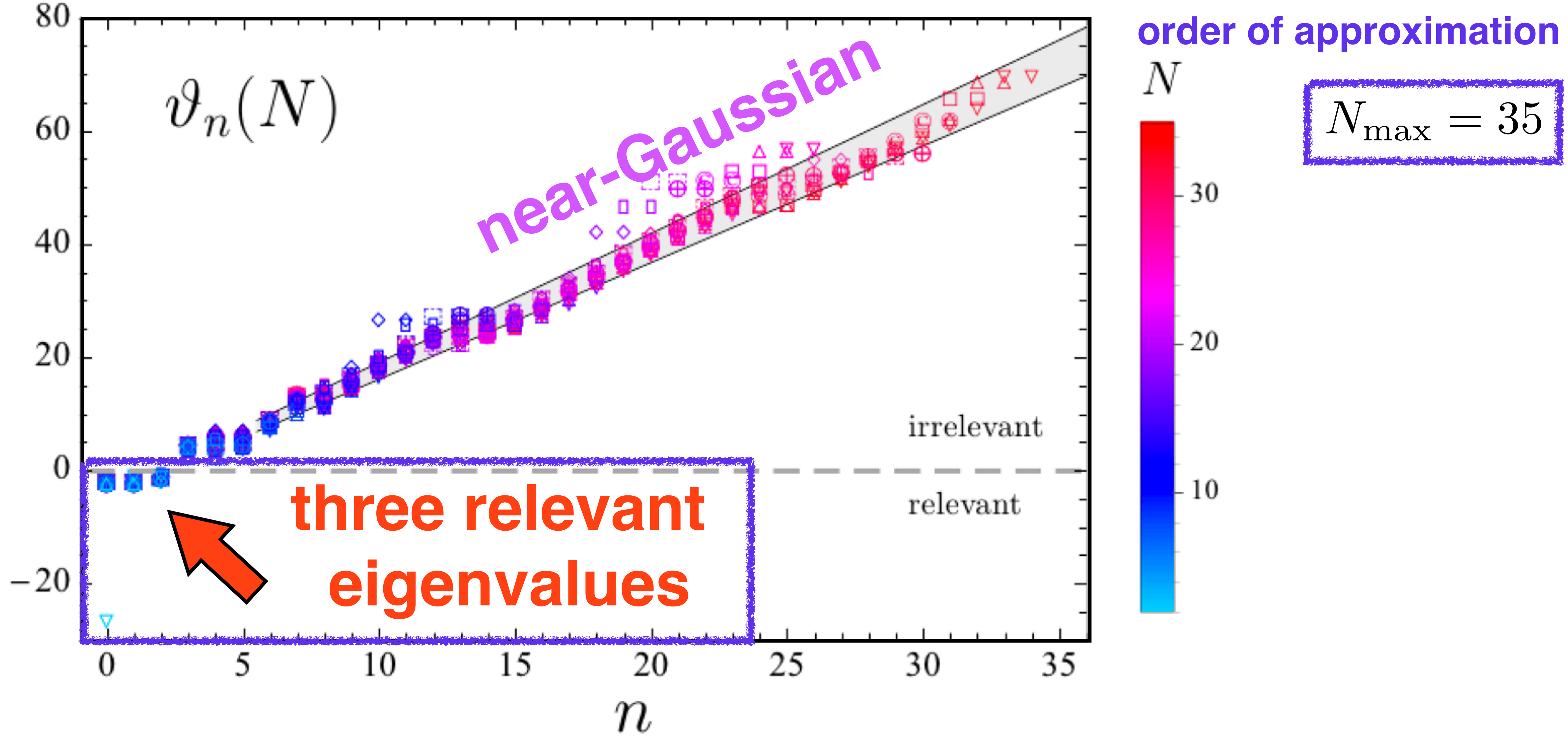
f(R)-type gravity

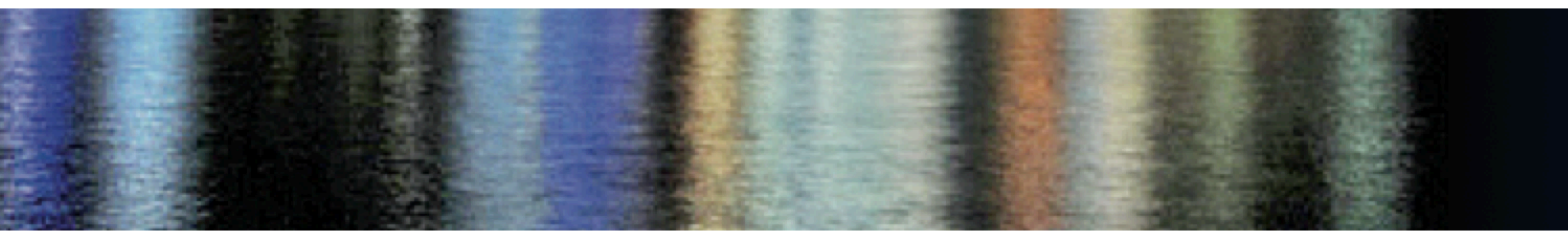
$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n \, k^{d_n} \int d^4x \sqrt{g} \, R^n$$



Falls, Litim, Nikolakopoulos, Rahmede
1301.4191
1410.4815

K Falls, C King, DL, K Nikolakopoulos & C Rahmede, (in prep.)

beyond Ricci scalars



f(Ricci)

$$\Gamma_k \propto \int d^d x \sqrt{g} \left[f_k(R_{\mu\nu} R^{\mu\nu}) + R \cdot z_k(R_{\mu\nu} R^{\mu\nu}) \right]$$

f(Ricci)

$$\Gamma_k \propto \int d^d x \sqrt{g}\,[f_k(R_{\mu\nu}R^{\mu\nu}) + R \cdot z_k(R_{\mu\nu}R^{\mu\nu})]$$

$$\begin{aligned}\partial_t \Gamma[\bar{g},\bar{g}] =& \frac{1}{2}\mathrm{Tr}_{(2T)}\left[\frac{\partial_t \mathcal{R}_k^{h^T h^T}}{\Gamma^{(2)}_{h^T h^T}}\right] + \frac{1}{2}\mathrm{Tr}'_{(1T)}\left[\frac{\partial_t \mathcal{R}_k^{\xi\xi}}{\Gamma^{(2)}_{\xi\xi}}\right] + \frac{1}{2}\mathrm{Tr}''_{(0)}\left[\frac{\partial_t \mathcal{R}_k^{\sigma\sigma}}{\Gamma^{(2)}_{\sigma\sigma}}\right] + \frac{1}{2}\mathrm{Tr}_{(0)}\left[\frac{\partial_t \mathcal{R}_k^{hh}}{\Gamma^{(2)}_{hh}}\right] \\ &+ \mathrm{Tr}''_{(0)}\left[\frac{\partial_t \mathcal{R}_k^{\sigma h}}{\Gamma^{(2)}_{\sigma h}}\right] - \mathrm{Tr}_{(1T)}\left[\frac{\partial_t \mathcal{R}_k^{\bar{C}^T C^T}}{\Gamma^{(2)}_{\bar{C}^T C^T}}\right] - \mathrm{Tr}_{(0)'}\left[\frac{\partial_t \mathcal{R}_k^{\bar{\eta}\eta}}{\Gamma^{(2)}_{\bar{\eta}\eta}}\right] - \mathrm{Tr}''_{(0)}\left[\frac{\partial_t \mathcal{R}_k^{\bar{\lambda}\lambda}}{\Gamma^{(2)}_{\bar{\lambda}\lambda}}\right] \\ &+ \frac{1}{2}\mathrm{Tr}''_{(0)}\left[\frac{\partial_t \mathcal{R}_k^{\omega\omega}}{\Gamma^{(2)}_{\omega\omega}}\right] - \mathrm{Tr}'_{(1T)}\left[\frac{\partial_t \mathcal{R}_k^{\bar{c}^T c^T}}{\Gamma^{(2)}_{\bar{c}^T c^T}}\right] + \frac{1}{2}\mathrm{Tr}'_{(1T)}\left[\frac{\partial_t \mathcal{R}_k^{\zeta^T \zeta^T}}{\Gamma^{(2)}_{\zeta^T \zeta^T}}\right] + \mathrm{Tr}'_{(0)}\left[\frac{\partial_t \mathcal{R}_k^{\bar{s}s}}{\Gamma^{(2)}_{\bar{s}s}}\right]\end{aligned}$$

f(Ricci)

$$\Gamma_k \propto \int d^dx \sqrt{g}\, [f_k(R_{\mu\nu}R^{\mu\nu}) + R \cdot z_k(R_{\mu\nu}R^{\mu\nu})]$$

generating function

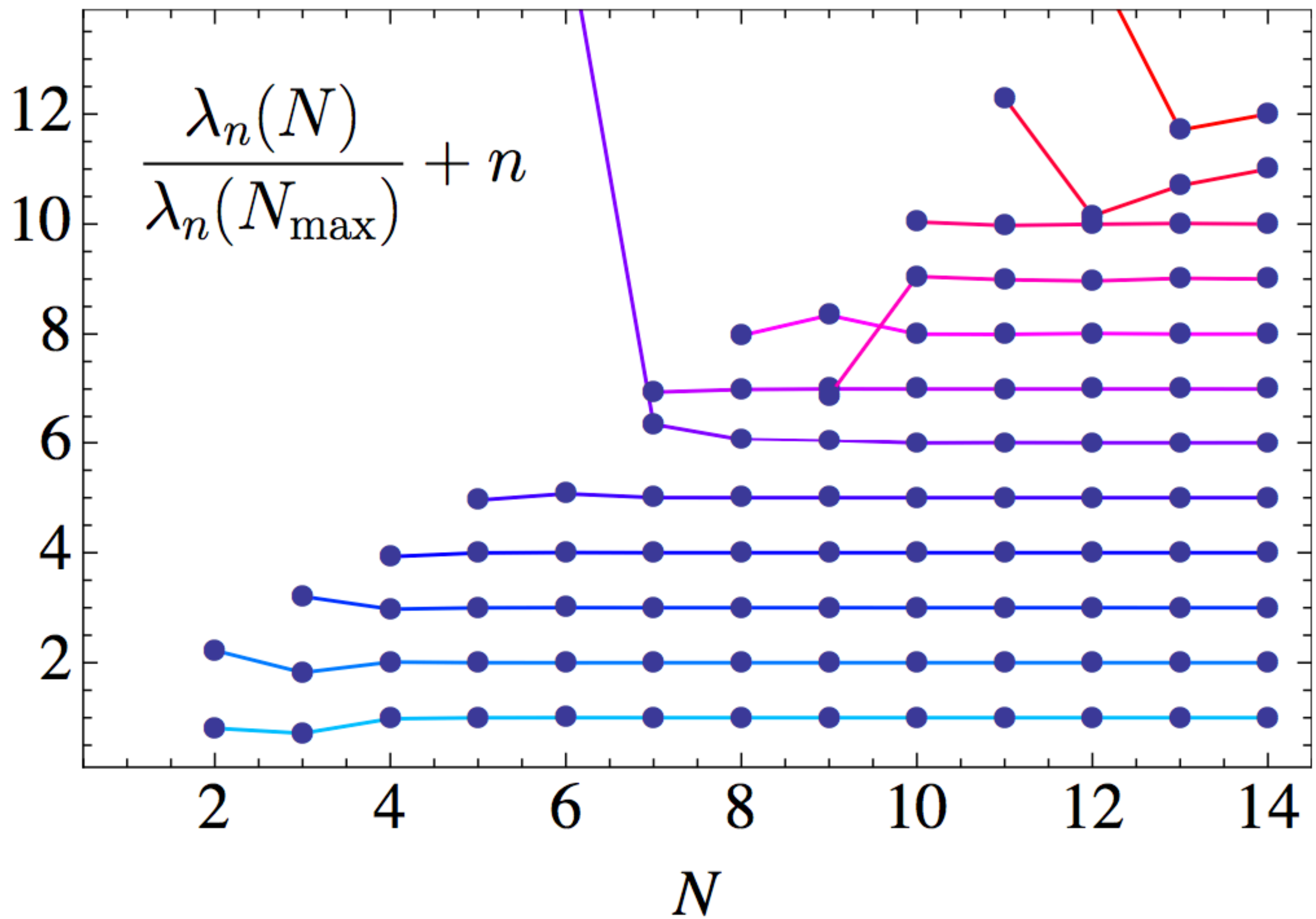
$$384\pi^2 \left[4f + 2\rho\, z - \rho^2\, (f' + \rho\, z') + \partial_t f + \rho\, \partial_t z\right] = I[f, z](\rho)$$

$$\begin{aligned} I[f,z](\rho) =& I_0[f,z](\rho) + \partial_t z\, I_1[f,z](\rho) + \partial_t f'\, I_2[f,z](\rho) + \partial_t z'\, I_3[f,z](\rho) \\ &+ \partial_t f''\, I_4[f,z](\rho) + \partial_t z''\, I_5[f,z](\rho)\ . \end{aligned}$$

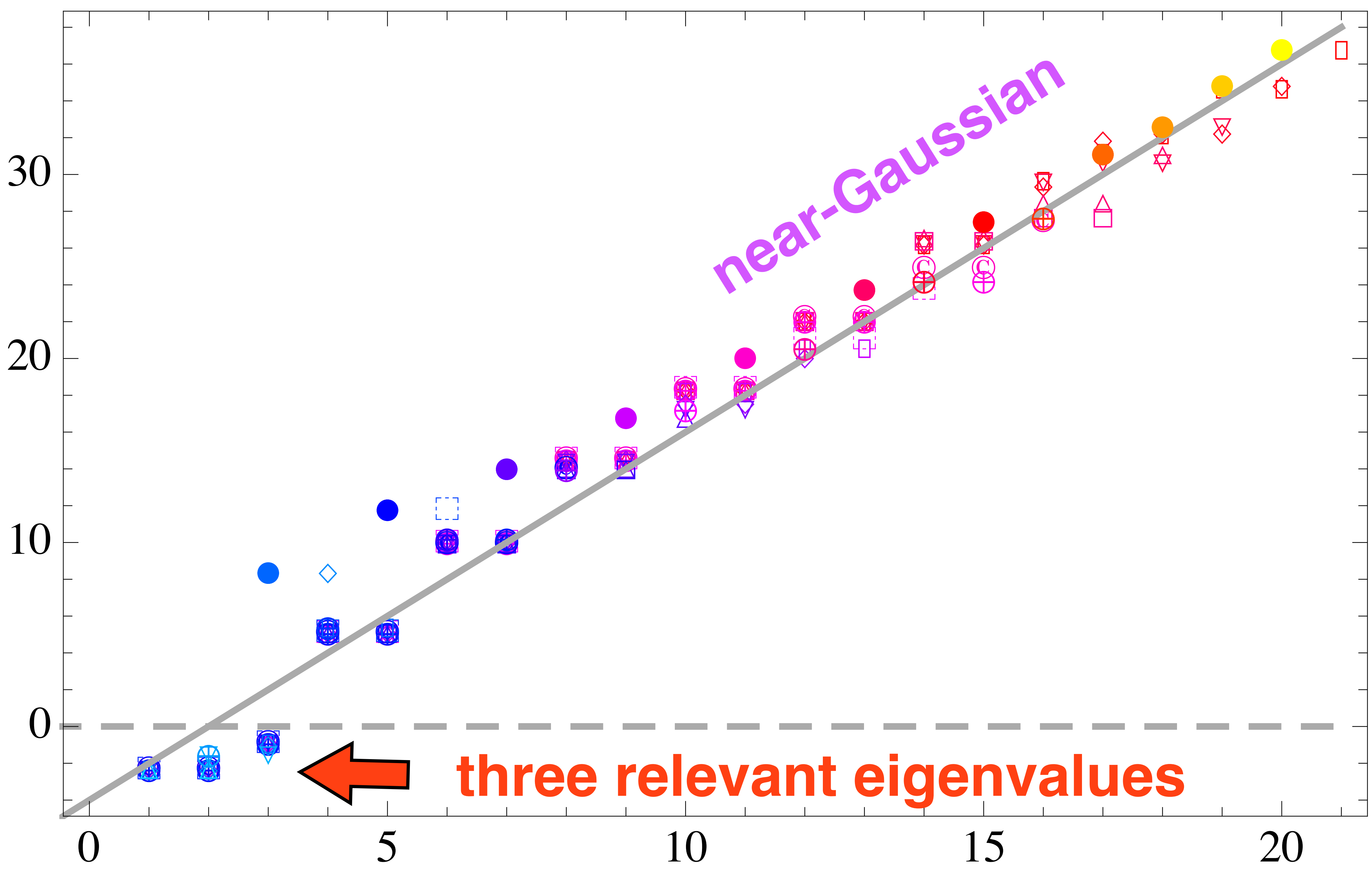
fixed points

recursive solution more demanding

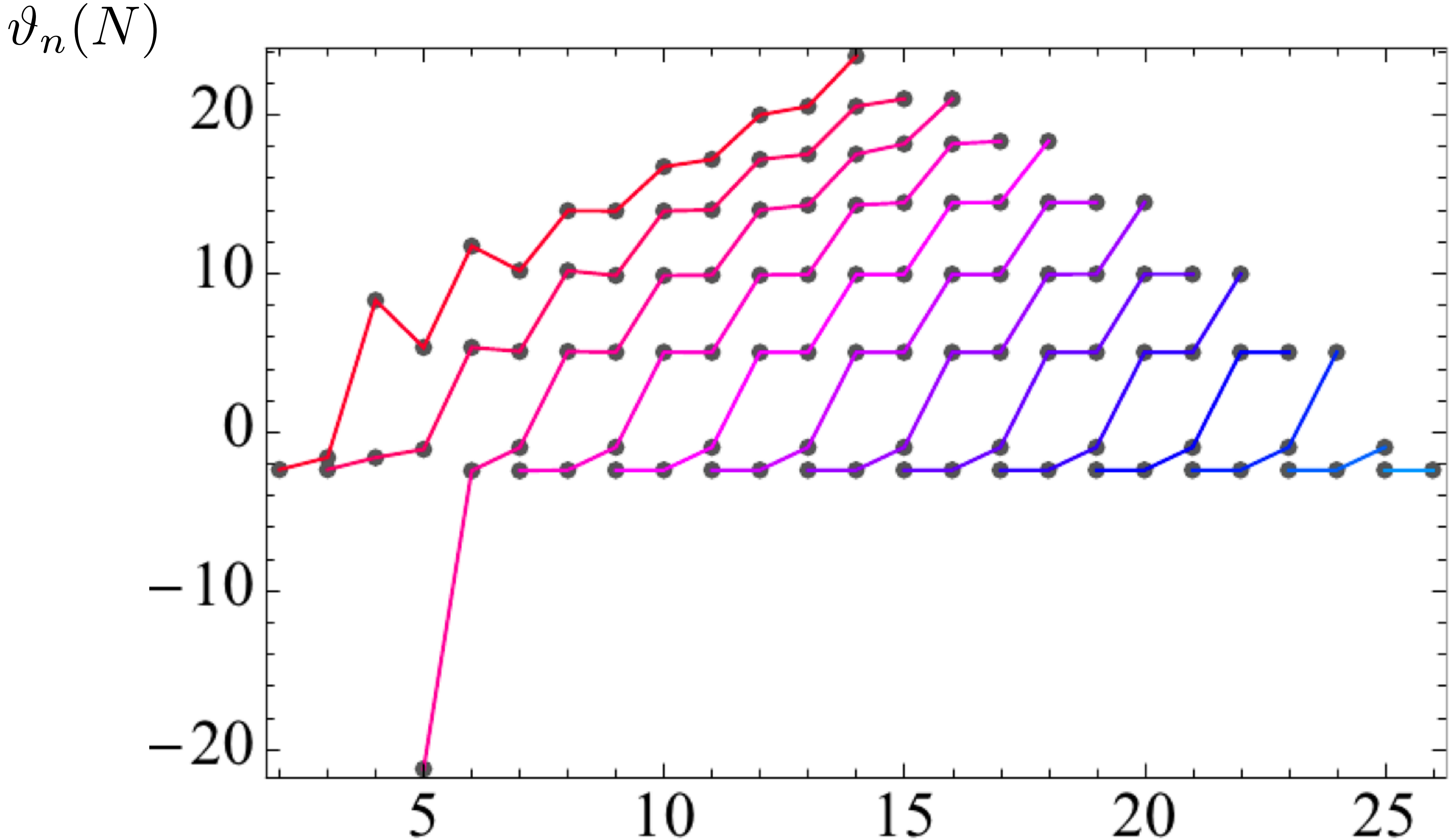
fixed points



scaling exponents



bootstrap test



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RG vs lattice

simplicial gravity

lattice fixed point in 4D
Hamber '00, '15

scaling exponent	lattice		RG	
ν	$0.335(9)$	Hamber '00	0.375	Litim '03
	$0.335(4)$	Hamber '15	0.3333	Falls 1503.06233

dynamical triangulations (casual vs euclidean)

lattice fixed point in 4D CDT
Ambjoern, Jordan, Jurkiewicz, Loll '11

spectral dimension $\mathcal{D}_s$	**CDT**	Ambjoern, Jurkiewicz, Loll '05
	EDT	Laiho, Coumbe '11
	RG	Reuter, Saueressig, '11

$$\mathcal{D}_s = \frac{2D}{2+\delta}$$

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conclusions

QFTs beyond asymptotic freedom

4D matter-gauge theories

- exact proof of asymptotic safety
- all types of fields required
  - sensible UV finite theory
  - no additional (super-)symmetry

4D quantum gravity

- systematic non-perturbative search strategies
- strong hints for interacting UV fixed point
  - intriguing near Gaussianity

54. Internationale Universitätswochen für Theoretische Physik
Schladming, Styria, Austria, February 21 - 26, 2016

New Trends in Particle Physics, Quantum Gravity & Cosmology

The focus of the 2016 edition of the Schladming Winter School in Theoretical Physics will be on new trends and open challenges in the understanding of the microcosm. With the discovery of a Standard Model-like scalar boson at the LHC and no further clear-cut observation of new phenomena at the electroweak scale the quest for a natural description of the microcosm has become even more pressing. Evaluating new theoretical frameworks which UV-complete the Standard Model or benchmark models beyond the Standard Model, fascinates physicists across the various communities including those working on model building, particle phenomenology, quantum gravity, cosmology and formal theory. Several key speakers will address this topic from different angles. Contributed seminars by participants and a poster session add to the vibrancy of the event.

Astrid Eichhorn (Imperial College)
Asymptotic Safety

Petr Horava (UC Berkeley)
Gravity and the Quantum

Hugh Osborn (tbc.) (U Cambridge)
Advances in Quantum Field Theory

Tilman Plehn (U Heidelberg)
Collider Physics: Tools and Techniques

Veronica Sanz (U Sussex)
Models for the LHC and Beyond

Martin Schmaltz (Boston U)
Fundamental Theory Beyond the SM

Mikhail Shaposhnikov (EPF Lausanne)
Particle Physics and Cosmology

Participation

Please access **http://physik.uni-graz.at/schladming2016/** if you wish to participate, and complete the registration form as soon as possible, but no later than **January 31, 2016**.

Contact

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Organising Committee

Reinhard Alkofer (U Graz)
Daniel Litim (U Sussex)
Jan Pawlowski (U Heidelberg)
Willibald Plessas (U Graz)
Hèlios Sanchis-Alepuz (U Graz)

exact asymptotic safety:
a gauge-Yukawa template

Litim, Sannino 1406.2337

University of Sussex

gauge-Yukawa theory

Lagrangean

$$L_{\rm YM} = -\frac{1}{2}\,{\rm Tr}\,F^{\mu\nu}F_{\mu\nu}$$

gauge — Nc colours

$$L_F = {\rm Tr}\left(\overline{Q}\, i\not{D}\, Q\right)$$

$$L_Y = y\,{\rm Tr}\left(\overline{Q}\, H\, Q\right)$$

Yukawa — Nf flavours

$$L_H = {\rm Tr}\,(\partial_\mu H^\dagger\, \partial^\mu H)$$

$$L_U = -u\,{\rm Tr}\,(H^\dagger H)^2$$

$$L_V = -v\,({\rm Tr}\,H^\dagger H)^2\,.$$

Higgs — Nf times Nf

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gauge-Yukawa theory

Lagrangean

$$L_{\rm YM} = -\frac{1}{2}{\rm Tr}\, F^{\mu\nu} F_{\mu\nu}$$

$$L_F = {\rm Tr}\left(\overline{Q}\, i \not{D}\, Q\right)$$

$$L_Y = y\, {\rm Tr}\left(\overline{Q}\, H\, Q\right)$$

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$$L_U = -u\, {\rm Tr}\left(H^\dagger H\right)^2$$

$$L_V = -v\left({\rm Tr}\, H^\dagger H\right)^2 .$$

couplings

$$\alpha_g = \frac{g^2\, N_C}{(4\pi)^2} \qquad \alpha_y = \frac{y^2\, N_C}{(4\pi)^2}$$

$$\alpha_h = \frac{u\, N_F}{(4\pi)^2} \qquad \alpha_v = \frac{v\, N_F^2}{(4\pi)^2} .$$

small parameter:

$$0 < \epsilon \ll 1 \qquad \epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

no asymptotic freedom

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gauge-Yukawa theory

$$\beta_g = \alpha_g^2 \left\{ \frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y \right\}$$

gauge

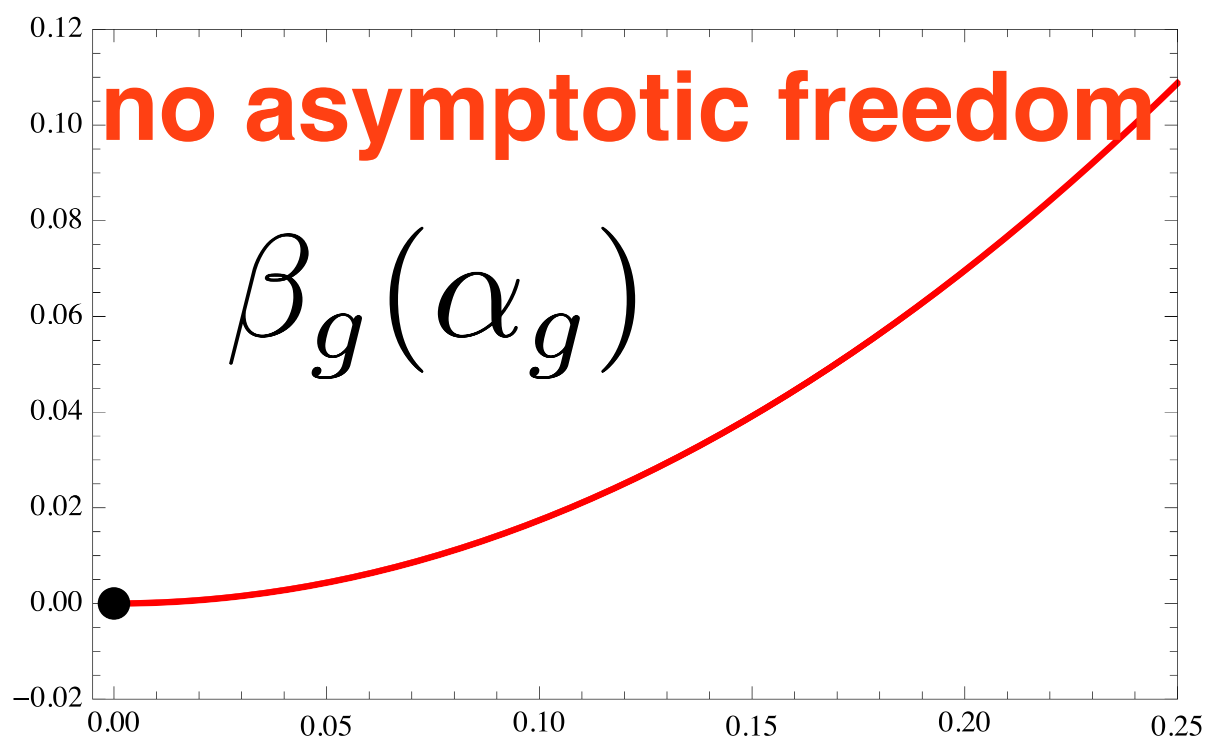
$$\beta_y = \alpha_y \left\{ (13 + 2\epsilon)\,\alpha_y - 6\,\alpha_g \right\} .$$

Yukawa

$$\beta_h = -(11 + 2\epsilon)\,\alpha_y^2 + 4\alpha_h(\alpha_y + 2\alpha_h)$$

$$\beta_v = 12\alpha_h^2 + 4\alpha_v\,(\alpha_v + 4\alpha_h + \alpha_y) .$$

Higgs



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gauge-Yukawa theory

$$\beta_g = \alpha_g^2 \left\{ \frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y \right\}$$

$$\beta_y = \alpha_y \left\{ (13 + 2\epsilon)\,\alpha_y - 6\,\alpha_g \right\} .$$

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gauge

Yukawa

Higgs

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gauge-Yukawa theory

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gauge

Yukawa

Higgs

gauge-Yukawa theory

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gauge

$$\beta_y = \alpha_y \left\{ (13 + 2\epsilon)\,\alpha_y - 6\,\alpha_g \right\} .$$

Yukawa

$$\beta_h = -(11 + 2\epsilon)\,\alpha_y^2 + 4\alpha_h(\alpha_y + 2\alpha_h)$$

$$\beta_v = 12\alpha_h^2 + 4\alpha_v\,(\alpha_v + 4\alpha_h + \alpha_y) .$$

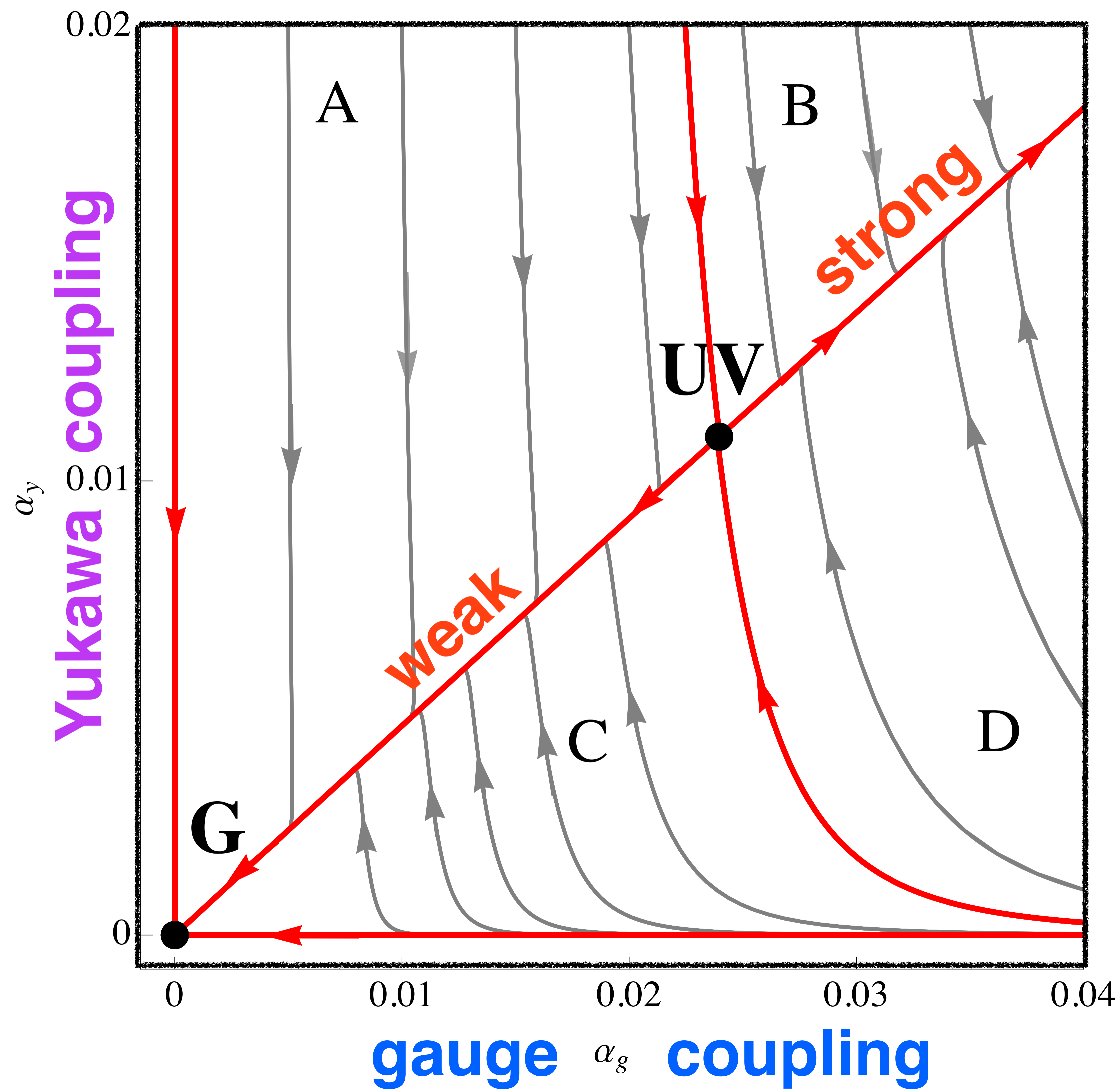
Higgs

exact
UV fixed point

$$\begin{aligned} \alpha_g^* &= 0.4561\,\epsilon + 0.7808\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_y^* &= 0.2105\,\epsilon + 0.5082\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_h^* &= 0.1998\,\epsilon + 0.5042\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_v^* &= -0.1373\,\epsilon + \mathcal{O}(\epsilon^2) \end{aligned}$$

results

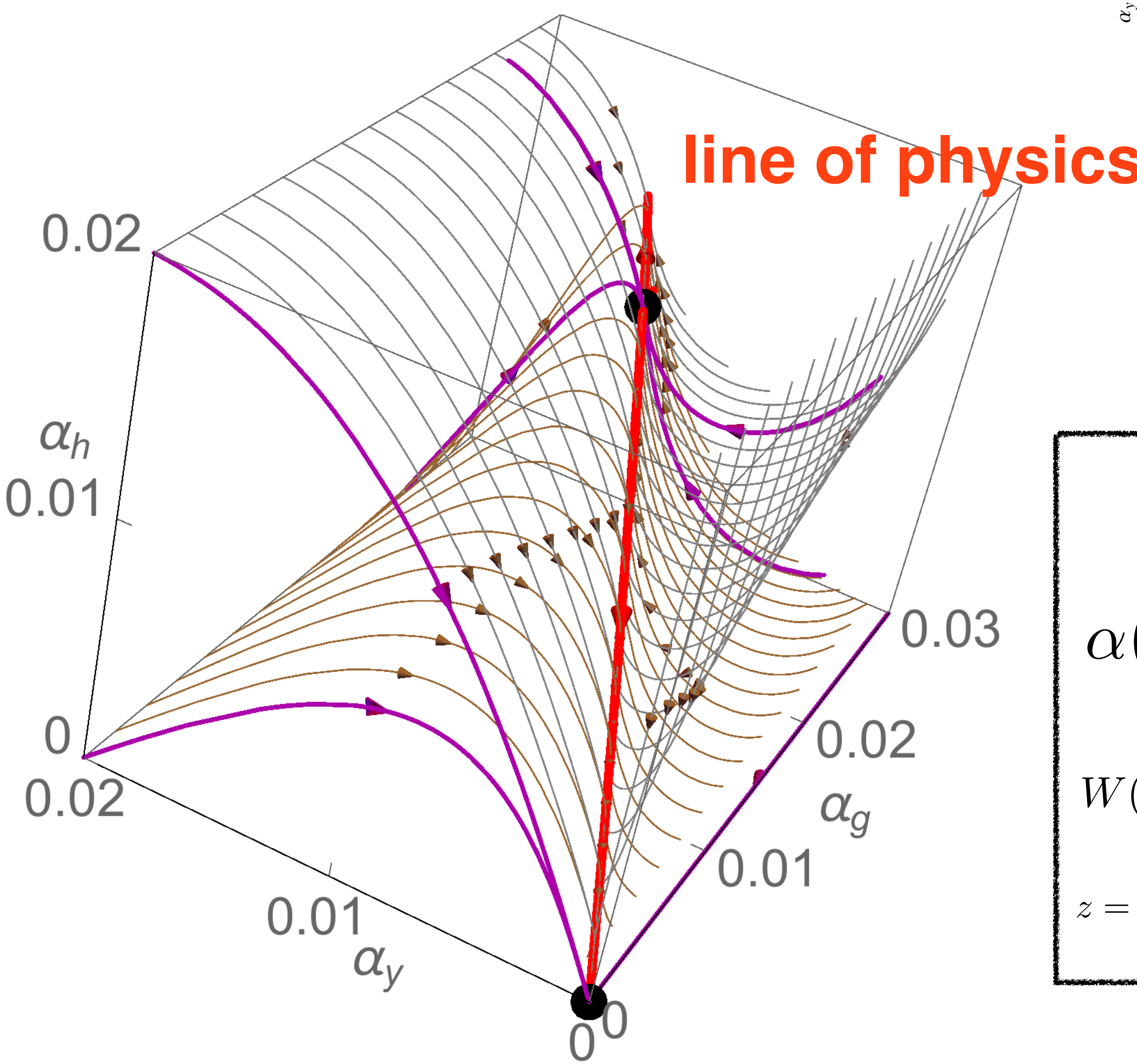
phase diagram

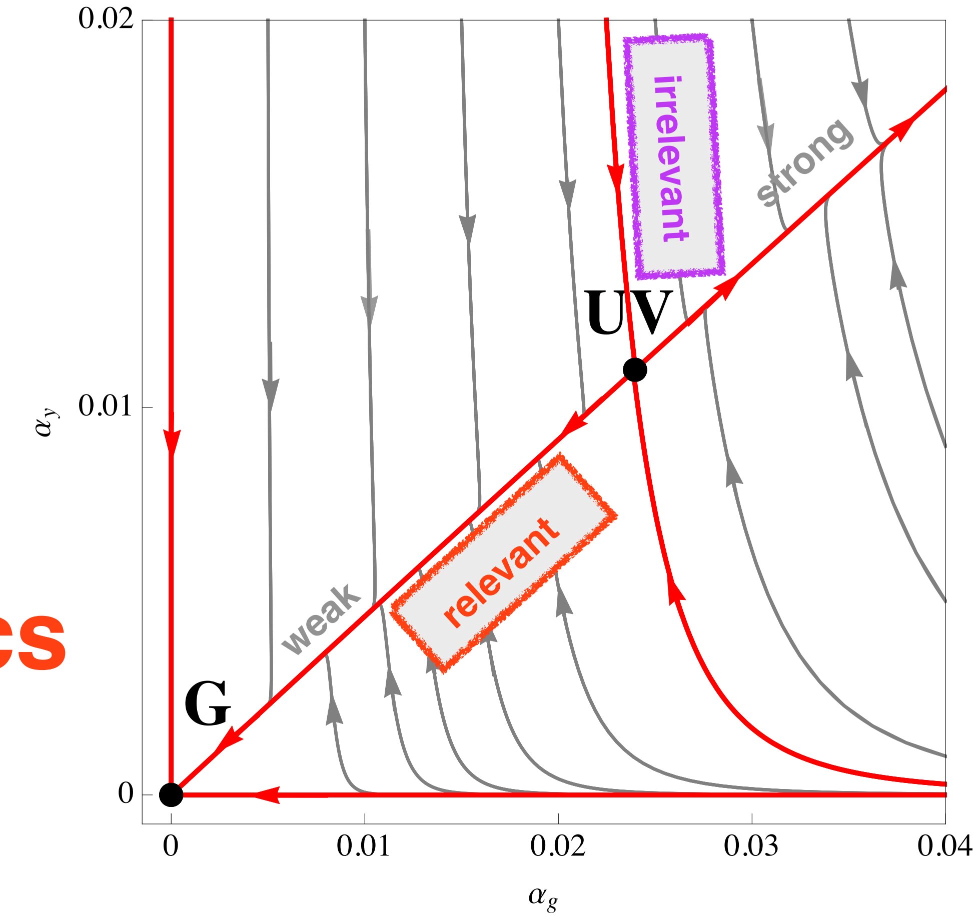


exact UV FP

strict perturbative control

phase diagram





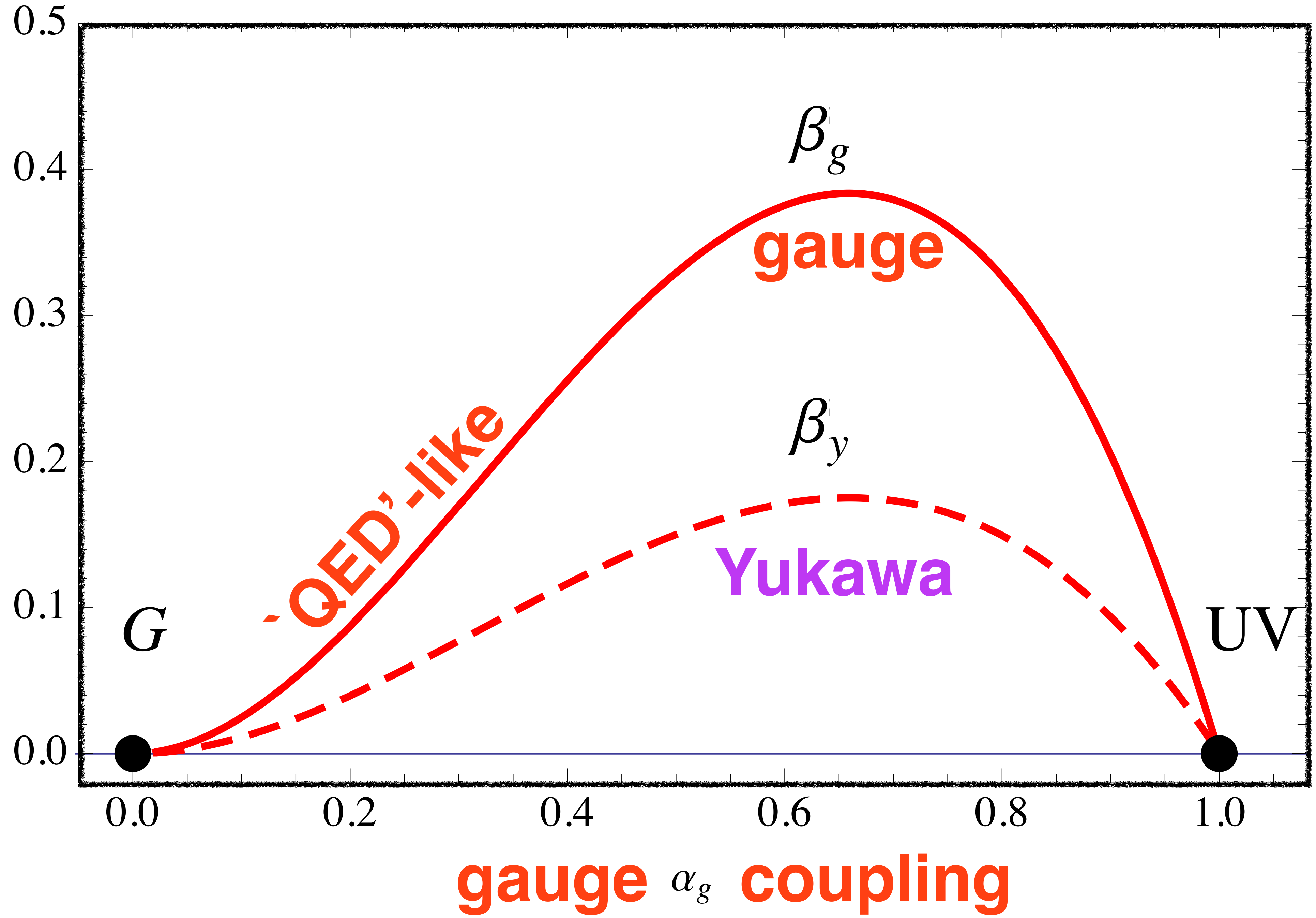
leading order

$$\alpha(\mu) = \frac{\alpha_*}{1 + W(\mu)}$$

$$W(\mu) = W_{\text{Lambert}}[z(\mu)]$$

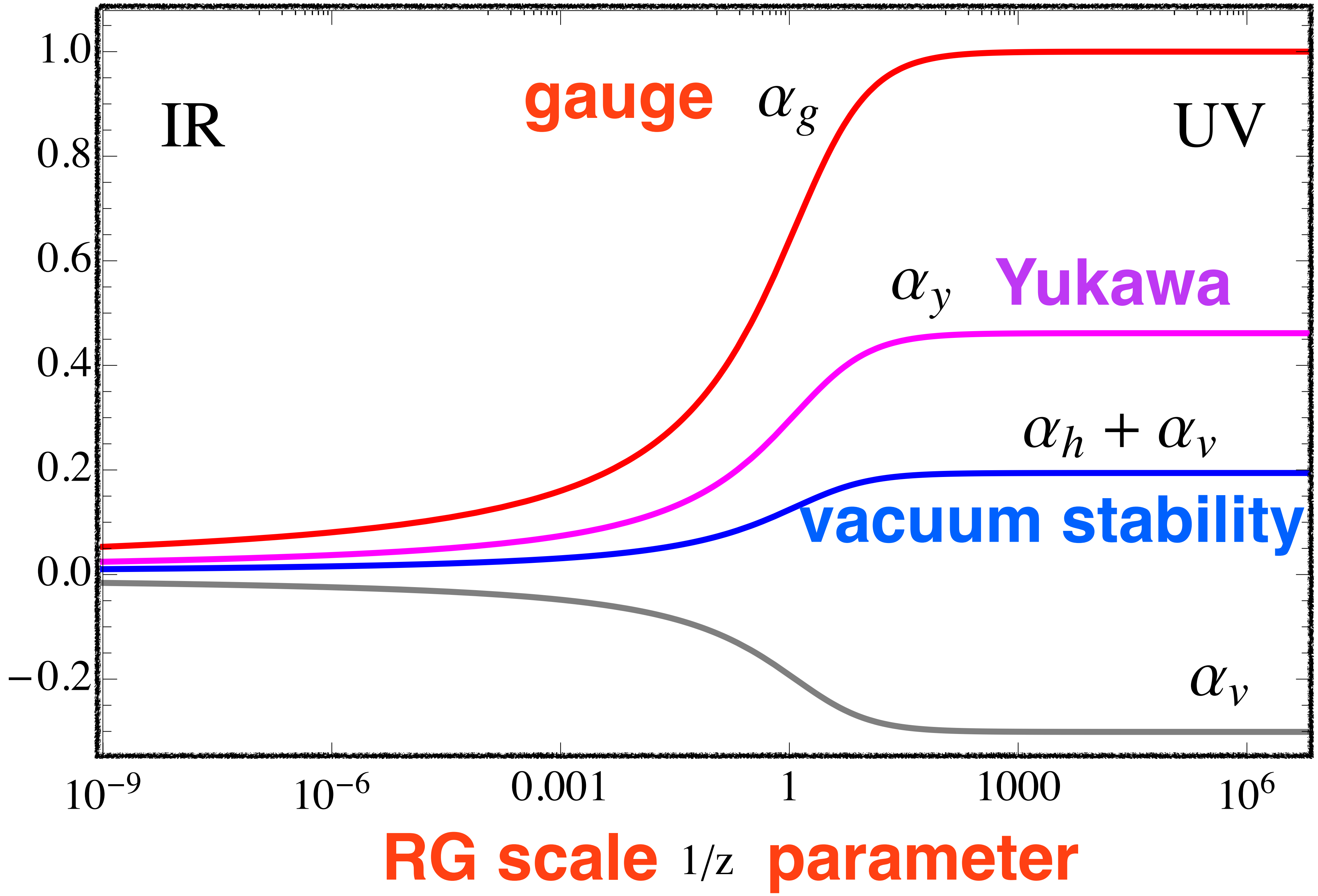
$$z = \left(\frac{\mu_0}{\mu}\right)^{-B\cdot\alpha_*} \left(\frac{\alpha_*}{\alpha_0} - 1\right) \exp\left(\frac{\alpha_*}{\alpha_0} - 1\right) .$$

results



interacting UV fixed point
entirely due to `fluctuations'

results



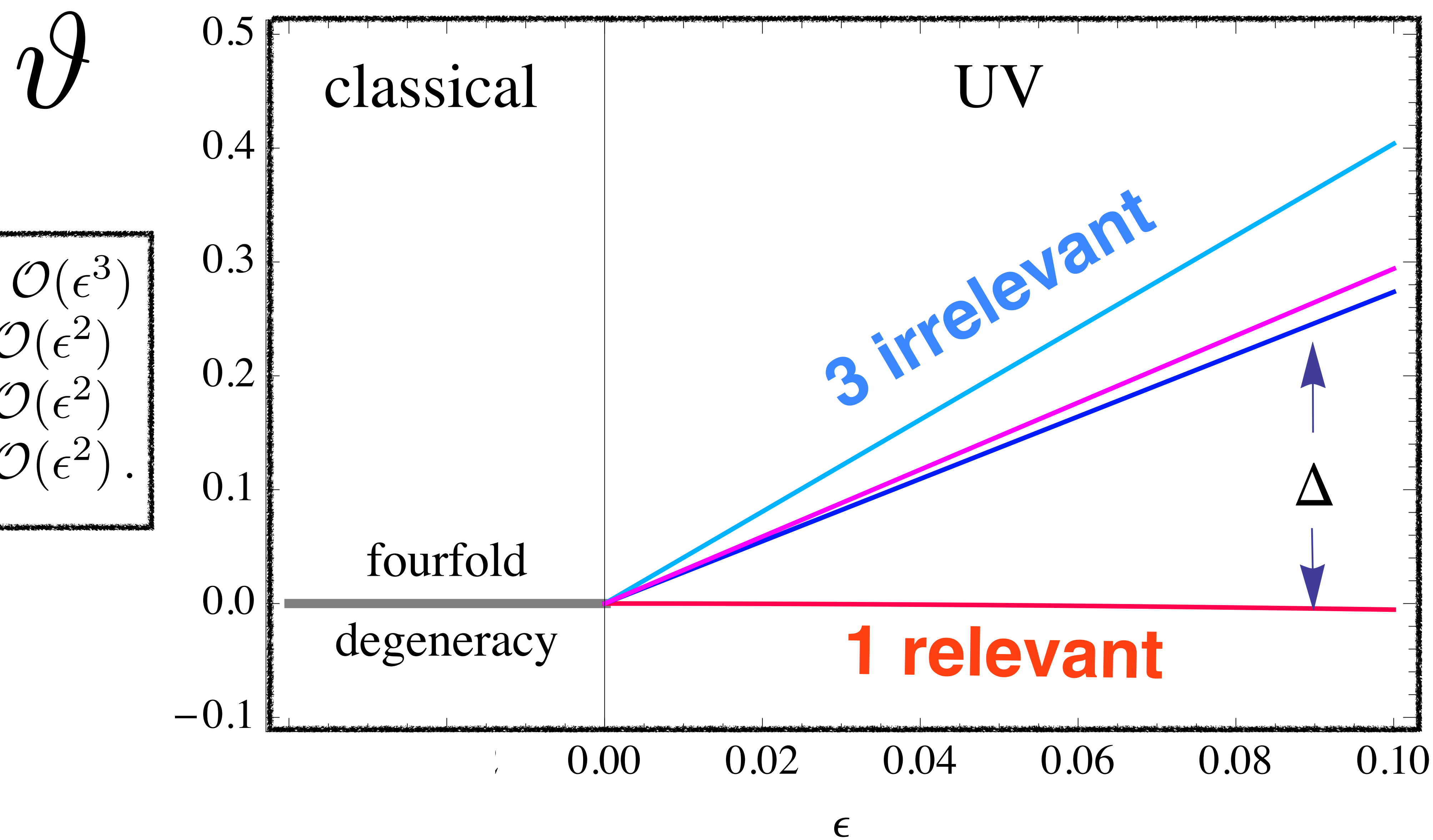
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
IR
gauge
α_g
UV
α_y
Yukawa
$\alpha_h + \alpha_v$
vacuum stability
α_v
10^{-9}
10^{-6}
0.001
1
1000
10^{6}
RG scale 1/z parameter

results

UV scaling exponents

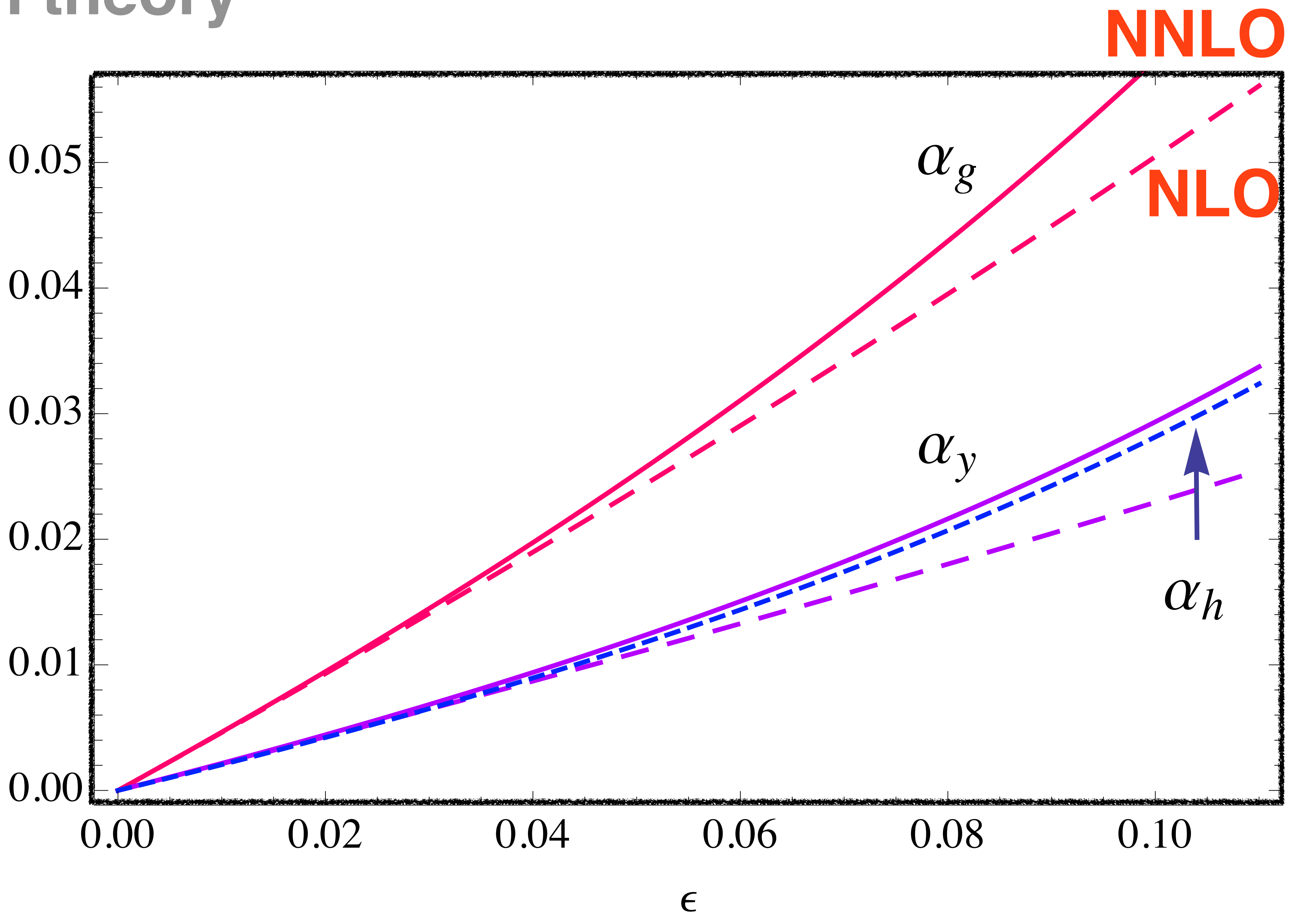
$$\vartheta_1 < 0 < \vartheta_2 < \vartheta_4 < \vartheta_3$$

$$\begin{aligned}
\vartheta_1 &= -0.608\,\epsilon^2 + \mathcal{O}(\epsilon^3)\\
\vartheta_2 &= 2.737\,\epsilon + \mathcal{O}(\epsilon^2)\\
\vartheta_3 &= 4.039\,\epsilon + \mathcal{O}(\epsilon^2)\\
\vartheta_4 &= 2.941\,\epsilon + \mathcal{O}(\epsilon^2)\,.
\end{aligned}$$



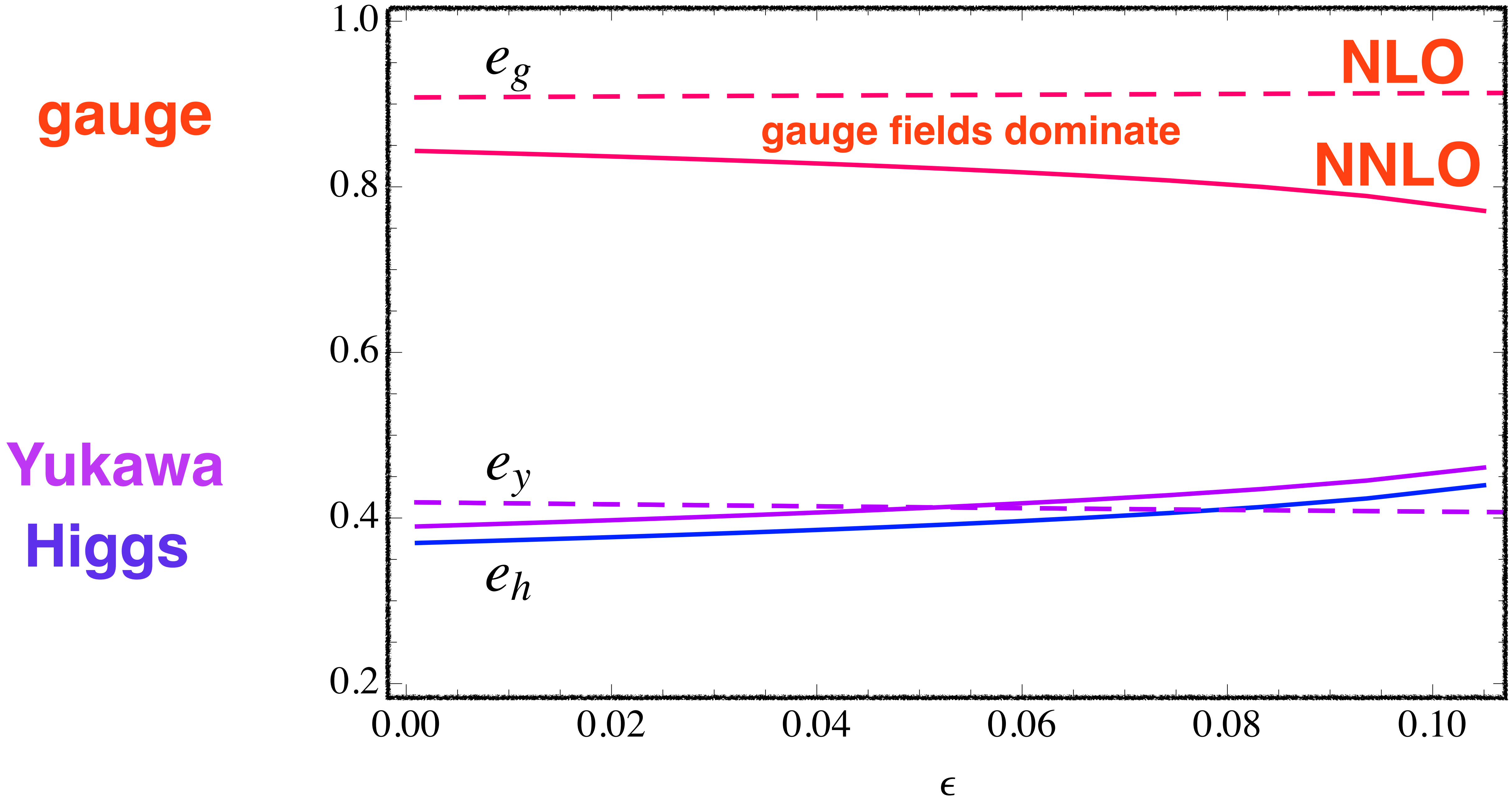
results

UV fixed point from perturbation theory



results

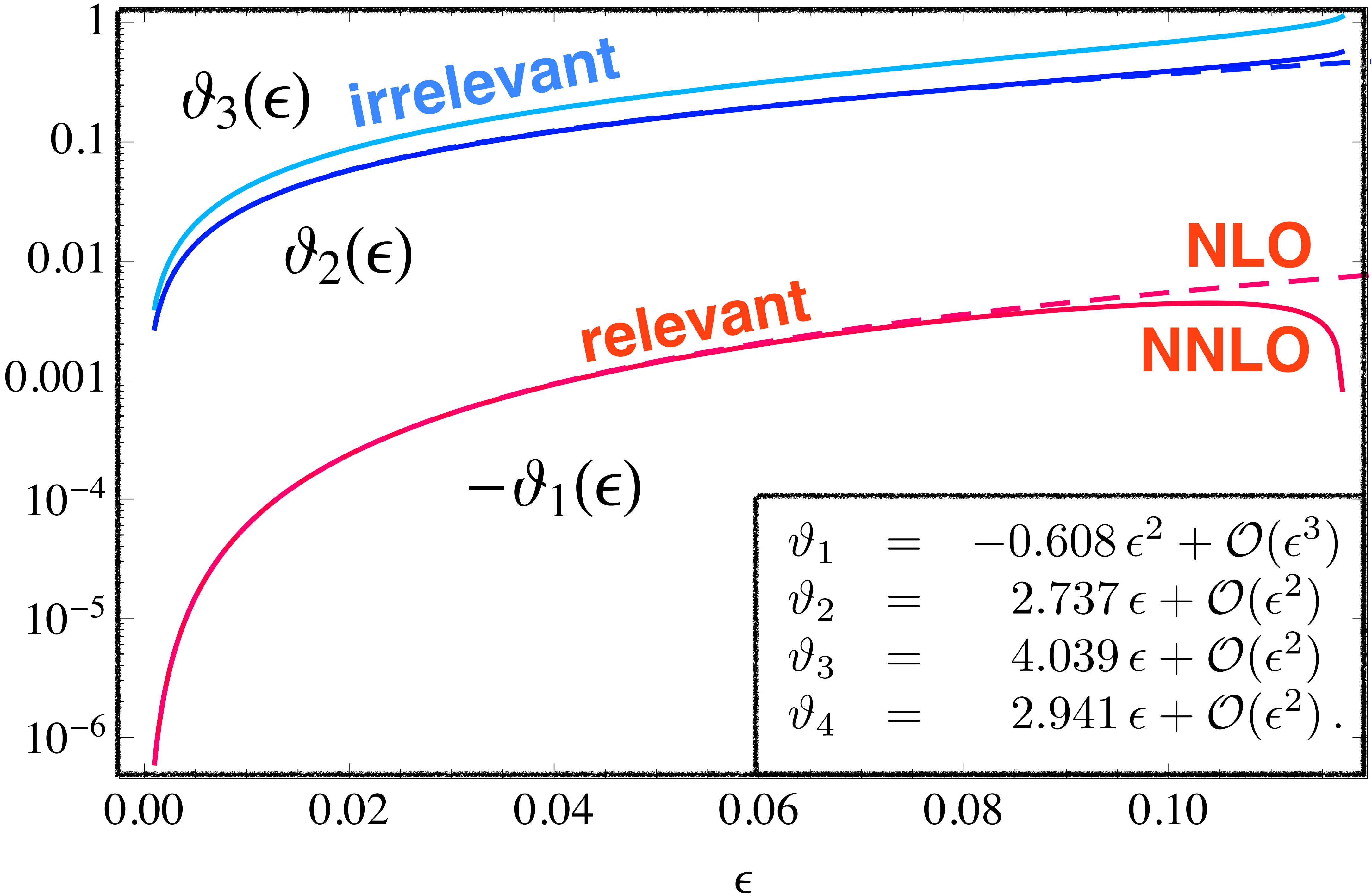
UV-relevant eigendirection



results

UV scaling exponents

$$\vartheta_1 < 0 < \vartheta_2 < \vartheta_4 < \vartheta_3$$



vacuum stability

Litim, Mojaza, Sannino
1501.03061

vacuum must be stable classically and quantum-mechanically

$$V \propto \alpha_v \mathrm{Tr}(H^\dagger H)^2 + \alpha_h (\mathrm{Tr} H^\dagger H)^2$$

stability

$$\alpha_h > 0 \quad \text{and} \quad \alpha_h + \alpha_v \geq 0 \qquad H_c \propto \delta_{ij}$$

$$\alpha_h < 0 \quad \text{and} \quad \alpha_h + \alpha_v / N_F \geq 0 \qquad H_c \propto \delta_{i1}$$

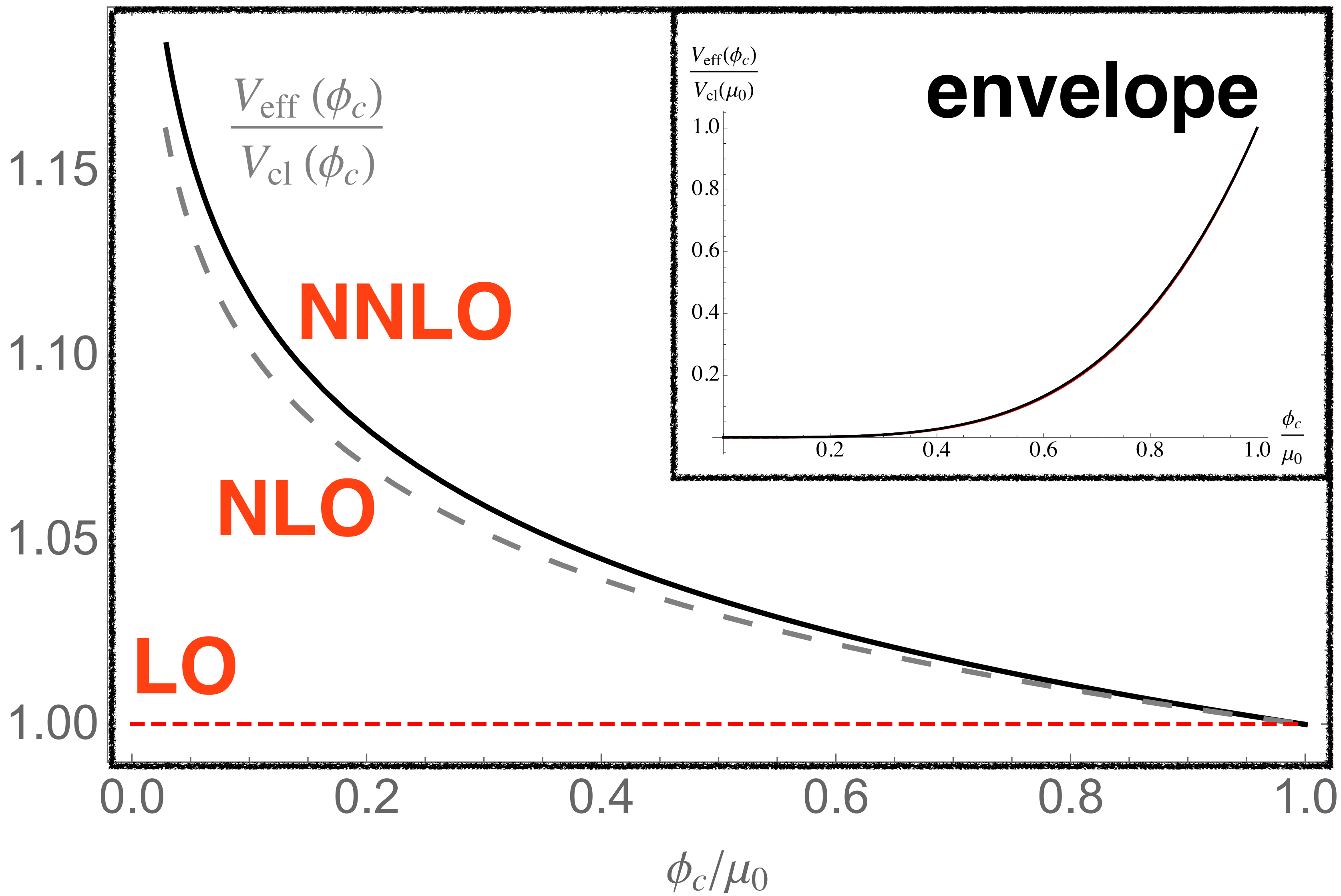
UV FP:

$$0 < \alpha_h^* + \alpha_v^*$$ **ok**

vacuum stability

quantum stability: Coleman-Weinberg type resummation of logs

$$\left(\mu_0 \frac{\partial}{\partial \mu_0} - \gamma(\alpha_j)\,\phi_c \frac{\partial}{\partial \phi_c} + \sum_i \beta_i(\alpha_j) \frac{\partial}{\partial \alpha_i}\right) V_{\rm eff}(\phi_c, \mu_0, \alpha_j) = 0$$



effective potential well-defined for all scales