Parametrisation dependence in non-perturbative approximations of quantum gravity

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Motivation I

- physical quantities should be independent of parametrisation and gauge fixing
  $\to$ why consider dependence on them?
- consider perturbation theory to given order N: cross sections etc. independent of these "subtleties"...
- ...up to order N, higher orders depend on gauge fixing and choice of parametrisation

Motivation II

- functional renormalisation: exact one-loop equation for effective action, but difficult to solve exactly
- nonperturbative approximations can depend on gauge fixing and parametrisation
- stability delivers measure for quality of approximation

Action, gauge fixing and ghosts I

- aim: construct functional integral over suitable integration variables that reproduces GR in the infrared
- ansatz for effective action:

$$\Gamma_{grav} = -\frac{1}{16\pi G_N} \int \sqrt{g}\,(R - 2\Lambda)$$

- assume that metric is fundamental degree of freedom

Action, gauge fixing and ghosts II

- use background field method: $g = \bar{g} + \ldots$ (specified soon)
- linear gauge fixing condition:

$$F_\mu = \left(\delta_\mu^\beta \bar{D}^\alpha - \frac{1+\beta}{4} \bar{g}^{\alpha\beta} \bar{D}_\mu \right) g_{\alpha\beta}$$

- implemented via gauge-fixing action:

$$\Gamma_{gf} = \frac{1}{32\pi G_N \alpha} \int \sqrt{\bar{g}} \bar{g}^{\mu\nu} F_\mu F_\nu$$

Action, gauge fixing and ghosts III

- gauge fixing gives rise to Faddeev-Popov ghosts:

$$\Gamma_{gh} = -\int \sqrt{\bar{g}}\, \bar{C}_\mu \mathcal{M}^\mu{}_\nu C^\nu \,, \qquad \mathcal{M}^\mu{}_\nu = \frac{\delta F^\mu}{\delta v^\nu}$$

- introduce transverse decomposition of vectors ($\delta v^\nu = \delta v^{\mathrm{T}\nu} + \bar{D}^\nu \delta\chi$ with $\bar{D}_\nu \delta v^{\mathrm{T}\nu} = 0$):

$$\begin{aligned}\delta F^\mu = &(\delta^\mu_\nu \bar{D}^2 + \bar{R}^\mu_\nu)\delta v^{\mathrm{T}\nu} \\ &+ \frac{1}{2}\left(\boxed{(3 - \beta)} \bar{D}^\mu \bar{D}_\nu + 4\bar{R}^\mu_\nu\right)\bar{D}^\nu \delta\chi + \mathcal{O}(g - \bar{g})\end{aligned}$$

$\to \beta = 3$ yields incomplete gauge-fixing

Metric parametrisations

- what we would like to do: geometric effective action (à la Vilkovisky-DeWitt)

$$g = \bar{g} - \sigma[\bar{g}; g]$$

- what we can do:
  - linear split:

$$g = \bar{g} + h$$

  - exponential parametrisation:

$$g = \bar{g}\left(e^{\bar{g}^{-1}h}\right) = \bar{g} + \bar{g}\left(e^{\bar{g}^{-1}h} - 1\right)$$

- here: study (mainly) one-parameter family of parametrisations:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} + \frac{\tau}{2} h_{\mu\rho} h^{\rho}_{\nu} + \mathcal{O}(h^3)$$

Graviton propagator

- central object in RG studies: $\Gamma^{(2)}$ (and its inverse)
- York TT-decomposition:

$$h_{\mu\nu} = h^{\mathrm{T}}_{\mu\nu} + 2\bar{D}_{(\mu}\xi^{\mathrm{T}}_{\nu)} + \left(2\bar{D}_{(\mu}\bar{D}_{\nu)} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{D}^2\right)\sigma + \frac{1}{4}\bar{g}_{\mu\nu}h$$

$$\bar{D}^\mu h^{\mathrm{T}}_{\mu\nu} = 0, \quad \bar{g}^{\mu\nu}h^{\mathrm{T}}_{\mu\nu} = 0, \quad \bar{D}^\mu \xi^{\mathrm{T}}_\mu = 0$$

- consider scalar sector in the limit of hard gauge fixing:

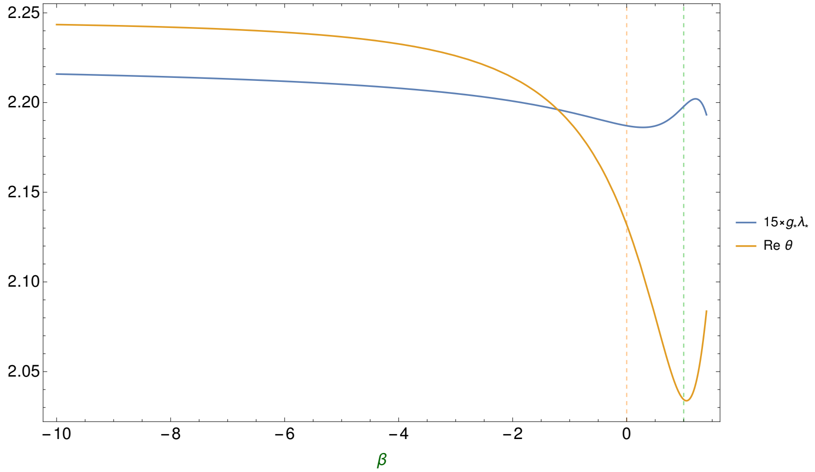
$$\lim_{\alpha\to 0}\left[\Gamma^{(2)}_{(\sigma h)}\right]^{-1} \sim \begin{pmatrix} \beta^2 & -6\beta \\ -6\beta & 36 \end{pmatrix} + \mathcal{O}(\bar{R})$$

$\rightarrow$ degeneracy: only one scalar degree of freedom

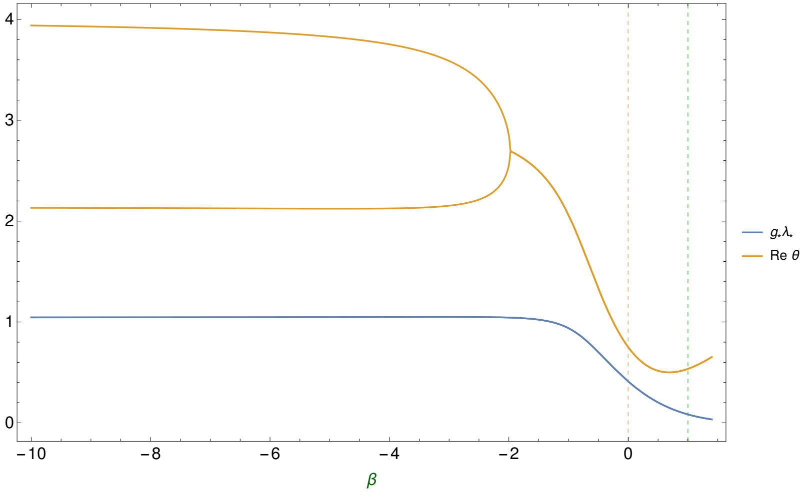
Asymptotic Safety

- perturbative renormalisability: too much to ask for in gravity
- Weinberg (1976): gravity might possess interacting, *non-perturbative* fixed point in the UV
- unstable manifold needs to be finite-dimensional
- same predictivity as asymptotic freedom
- almost all results up to now (first: Reuter 1996) find UV fixed point

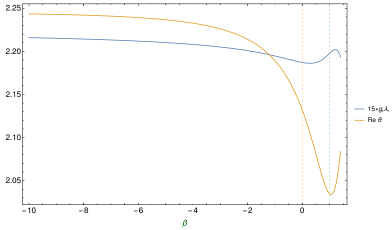
Linear parametrisation (τ=0, α=0)

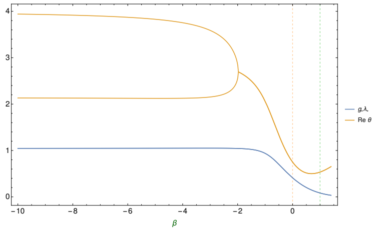


Exponential parametrisation (τ=1, α=0)



Comparison of linear and exp. parametrisation ($\alpha=0$)



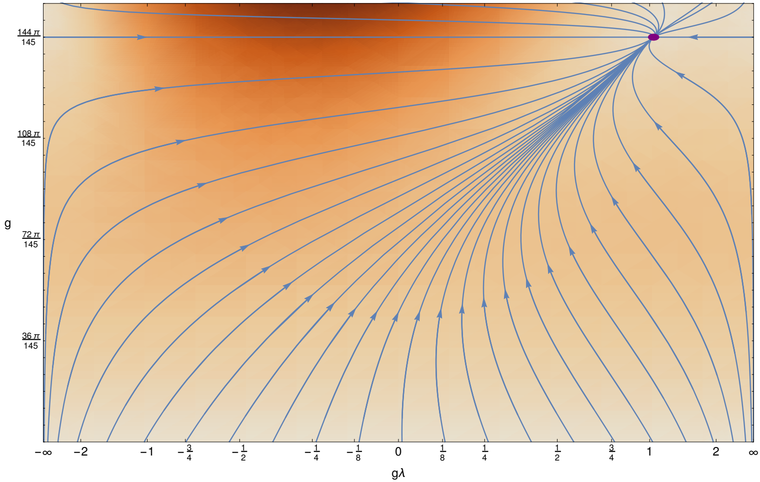


- interplay of gauge and parametrisation dependence
- stable region for both: $\beta \to \pm\infty$

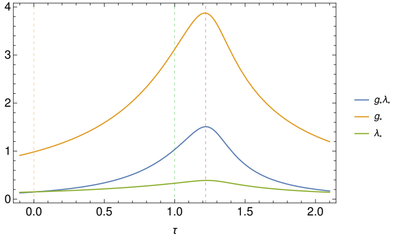
Global phase diagram

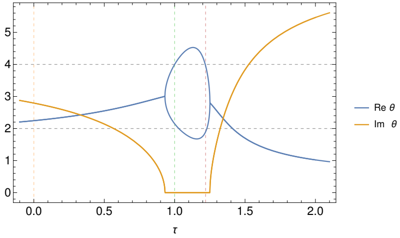
- for exponential split (τ=1) and $\beta \to \pm\infty$: α-dependence drops out of flow equations
- beta-functions can be integrated analytically
- complete picture of quantum gravity, both UV and IR limit are finite

Global phase diagram II



Parametrisation dependence (α=0, $\beta = \infty$)





Some more results

- include all four ultra-local h^2-terms in split:
  - only two linear combinations contribute to flow equations in our truncation
  - for special choice of these parameters: α-dependence drops out
- dimensional dependence:
  - linear split: UV FP seems to exist in all dimensions
  - exponential split: for "optimised" parameters $\alpha=0$, $\beta = \infty$, no UV FP above 5.731 dimensions

Summary

- gauge fixing in QG can be subtle issue in truncated RG flows
- interplay between parametrisation and gauge fixing
- for wide range of gauge fixings: UV FP
- there are regions of minimal sensitivity where universal quantities hardly change

Outlook

- check influence of transverse (traceless) decomposition (might not be well-defined on any manifold)
- broaden truncation to higher orders in curvature
- check stability of correlation functions
- include running gauge parameters