On higher-spin gravity in three dimensions

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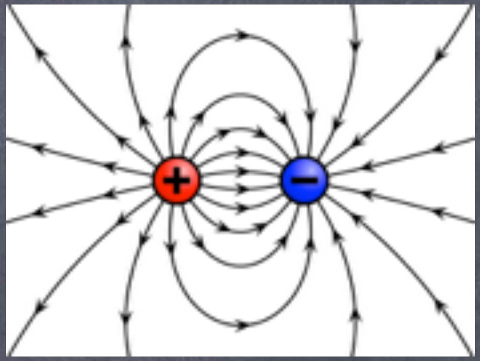
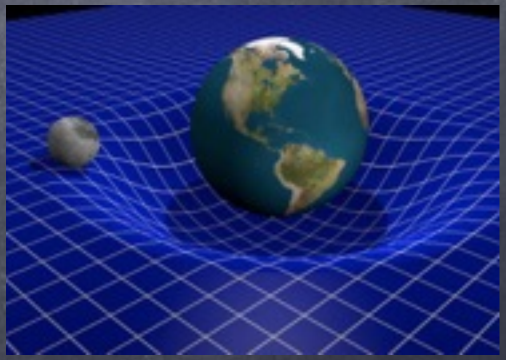
Higher spins

Gauge theories are a success story:

Spin 1: Electrodynamics,
Yang-Mills ... A_μ



Spin 2: Gravity $g_{\mu\nu}$



Why not go beyond? Spin 3, 4, 5, ...

$\phi_{\mu_1 \ldots \mu_s}$

Why higher spins?

1. <u>Generalisations of geometry:</u>

HS symmetries as generalised diffeomorphisms

$$g_{\mu\nu} \xrightarrow[\text{transformation}]{\text{HS gauge}} g'_{\mu\nu}$$

Curvature? Horizons? Causal structure?

Need new concepts!

Example in 3d:

Black hole $\xrightarrow[\text{transformation}]{\text{HS gauge}}$ wormhole

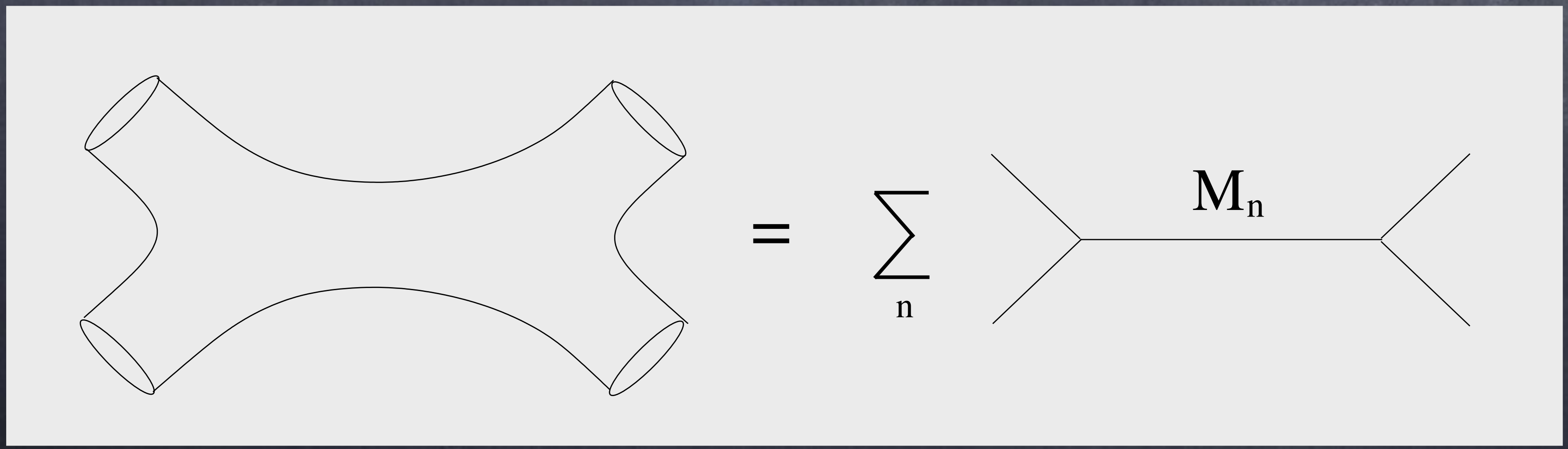
[Ammon,Gutperle,Kraus,Perlmutter]

Why higher spins?

2. Quantum Gravity: Exchange of higher spins could cure UV divergence.

String theory:

UV problems of perturbative quantum gravity solved by exchange of infinitely many massive higher-spin excitations.

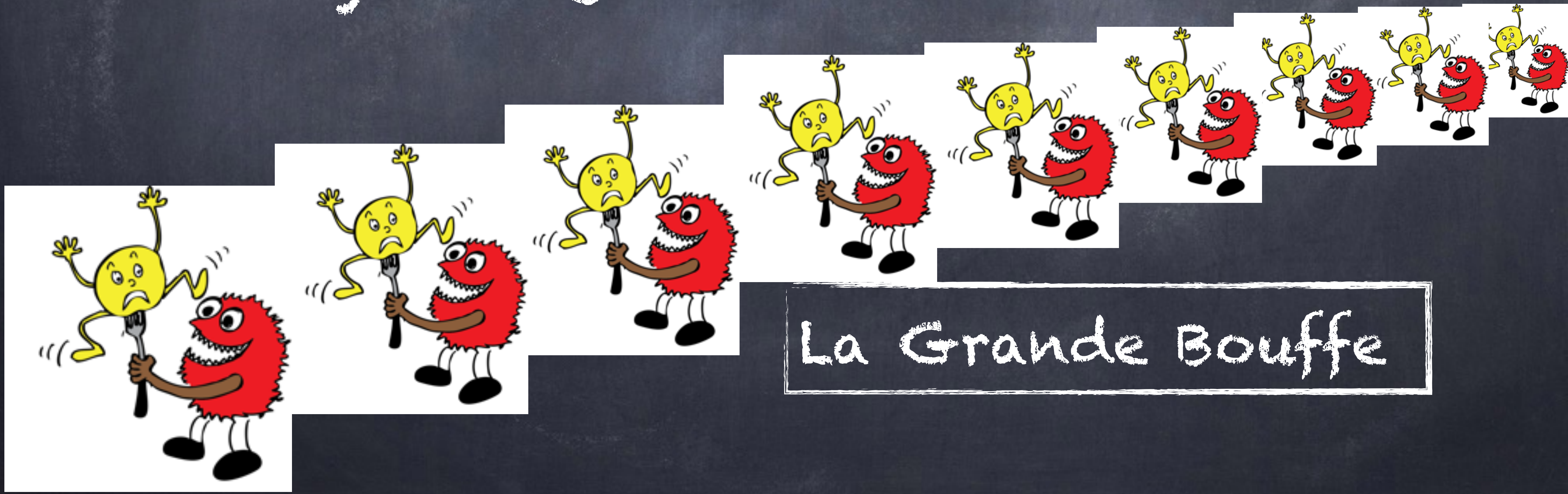


Are there other ways?

Why higher spins?

3. String theory: String theory could arise as a broken phase of HS theories – is HS symmetry the symmetry behind string theory?

Standard Higgs effect: Gauge bosons get massive by "eating" the Goldstone bosons.



La Grande Bouffe

Why higher spins?

4. AdS/CFT: HS theories as class of models in which holographic dualities can be studied (no SUSY required)

AdS	boundary
HS gauge theory	conformal field theory
HS gauge symmetry	global HS symmetry

If HS theory related to string theory:

extreme stringy limit of AdS/CFT.

Bumpy road to success

Many no-go results for d>3:

~~flat space~~ ~~minimal coupling~~ ~~finitely many HS~~

90's [Vasiliev et al.]: Consistent higher-spin gauge theories coupled to matter

- infinitely many HS
- non-zero cosmological constant

<u>Applications & Questions:</u>

AdS/CFT ? Solutions ? Uniqueness ?

Higgs ? Quantisation ?

Why 2+1 dimensions?

Gravity in 2+1 dimensions as toy model:

- No propagating degrees of freedom

$$R_{\mu\nu\lambda\sigma} = C_{\mu\nu\lambda\sigma} + \frac{2}{d-2}(g_{\mu[\lambda}R_{\sigma]\nu} - g_{\nu[\lambda}R_{\sigma]\mu})$$

$$= 0 \qquad - \frac{2}{(d-1)(d-2)} R\, g_{\mu[\lambda}g_{\sigma]\nu}$$

- Black holes exist for $\Lambda < 0$ [Bañados, Teitelboim, Zanelli]

  Black hole entropy can be studied.

  AdS/CFT concepts can be tested.

  Asymptotic symmetries: $\text{Virasoro} \oplus \text{Virasoro}$

  central charge $c = \frac{3\ell}{2G}$ (ℓ → AdS radius)

[Brown,Henneaux]

Higher spins

Higher spin extensions of gravity simpler in 2+1 than in higher dimensions: no propagating degrees of freedom

- asymptotic symmetries:
  Virasoro $\longrightarrow \mathcal{W}_N, \mathcal{W}_\infty$

  [Henneaux,Rey] [Campoleoni,S.F., Pfenninger,Theisen]

- generalisations of black holes

  [Gutperle,Kraus] [Ammon,Gutperle,Kraus,Perlmutter]

- AdS/CFT proposal: duality between HS gravity and minimal models [Gaberdiel, Gopakumar]

Outline

1. Higher-spin gravity in 3 dimensions

2. Asymptotic Symmetries

3. Holography

1) Gravity and higher-spins in 3d

Gravity:

$$S[e,\omega] = \frac{1}{8\pi G}\int \mathrm{Tr}\Big(e\wedge R + \frac{1}{3\ell^2} e\wedge e\wedge e\Big)$$

with $R = d\omega + \omega\wedge\omega$

Vielbeine e_μ^a with $e_\mu^a\, e_\nu^b\, \kappa_{ab} = g_{\mu\nu}$,

spin connections $\omega_\mu^{ab} \longrightarrow \omega_\mu^a = f^a{}_{bc}\,\omega_\mu^{bc}$.

For higher-spins: Two approaches

- Second order/Metric-like $g_{\mu\nu} \rightarrow \varphi_{\mu_1\ldots\mu_s}$
- First order/Frame-like $e_\mu^a \rightarrow e_\mu^{a_1\ldots a_{s-1}}$

Metric-like

Free massless spin s particle [Fronsdal]

$\longrightarrow$ fully symmetric tensor $\varphi_{\mu_1\ldots\mu_s}$

$$\varphi_{\mu_1\ldots\mu_{s-4}\lambda}{}^{\lambda}{}_{\nu}{}^{\nu} = 0 \quad \text{(double traceless)}$$

$$\Box\varphi_{\mu_1\ldots\mu_s} - \partial_{(\mu_1|}\partial^{\lambda}\varphi_{|\mu_2\ldots\mu_s)\lambda}{}^{\lambda} + \partial_{(\mu_1}\partial_{\mu_2}\varphi_{\mu_3\ldots\mu_s)\lambda}{}^{\lambda} = 0 \quad \text{(e.o.m.)}$$

$$\delta\varphi_{\mu_1\ldots\mu_s} = \partial_{(\mu_1}\xi_{\mu_2\ldots\mu_s)} \quad \text{(gauge invariance)}$$

where $\xi_{\mu_1\ldots\mu_{s-3}\lambda}{}^{\lambda} = 0$

Interactions: add term by term while retaining gauge invariance.

[Bengtsson,Bengtsson,Brink]
[Metsaev]
[Manvelyan,Mkrtchyan,Rühl]
[Sagnotti,Taronna]
[Joung,Lopez,Taronna] ...

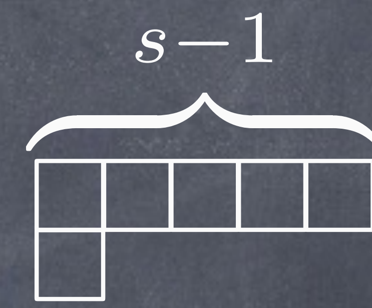
Frame-like

[Vasiliev]

Generalised vielbein $e_\mu^{a_1\ldots a_{s-1}}$ such that

$$\varphi_{\mu_1\ldots\mu_s} \;=\; \bar{e}^{a_1}_{(\mu_1}\ldots\bar{e}^{a_{s-1}}_{\mu_{s-1}}\,e^{b_1\ldots b_{s-1}}_{\mu_s)}\;\eta_{a_1 b_1}\ldots\eta_{a_{s-1}b_{s-1}}$$

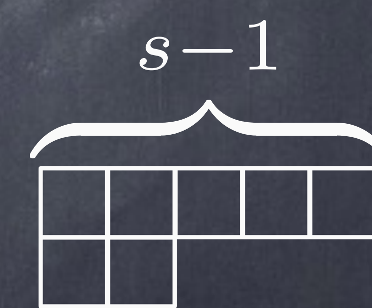
↑ background vielbein



Gauge transformations

$$\delta e_\mu^{a_1\ldots a_{s-1}} \;=\; \bar{D}_\mu \xi^{a_1\ldots a_{s-1}} + \bar{e}_{\mu,b}\,\Lambda^{b,a_1\ldots a_{s-1}}$$

with $\xi^{a_1\ldots a_{s-1}} \;=\; \xi^{\mu_1\ldots\mu_{s-1}}\,\bar{e}^{a_1}_{\mu_1}\ldots\bar{e}^{a_{s-1}}_{\mu_{s-1}}$.



Spin connections

$$\delta\omega_\mu^{b,a_1\ldots a_{s-1}} \;=\; \bar{D}_\mu \Lambda^{b,a_1\ldots a_{s-1}} + \bar{e}_{\mu,c}\,\Lambda^{bc,a_1\ldots a_{s-1}}$$

~~$\delta\omega_\mu^{bc,a_1\ldots a_{s-1}} \;=\; \bar{D}_\mu \Lambda^{bc,a_1\ldots a_{s-1}} + \bar{e}_{\mu,d}\,\Lambda^{bcd,a_1\ldots a_{s-1}}$~~

$\vdots$

vanish in 3 dimensions !

Action

Dualise: $\omega_\mu^{a_1 \dots a_{s-1}} = \omega_\mu^{b,c(a_2 \dots a_{s-1}} f^{a_1)}{}_{bc}$

$$S[e,\omega] = \frac{1}{8\pi G}\int \mathrm{Tr}\Big(e\wedge(d\omega+\omega\wedge\omega) + \frac{1}{3\ell^2}e\wedge e\wedge e\Big) \;\; ?$$

$$e = (e_\mu^a\, J_a + e_\mu^{a_1 a_2}\, J_{a_1 a_2} + \dots)dx^\mu$$

$$\omega = (\omega_\mu^a\, J_a + \omega_\mu^{a_1 a_2}\, J_{a_1 a_2} + \dots)dx^\mu$$

Lie algebra structure e.g. for spin 3:

$$[J_a, J_b] = f_{ab}{}^c\, J_c$$

$$[J_a, J_{bc}] = f^d{}_{a(b}\, J_{c)d}$$

$$[J_{ab}, J_{cd}] = -(\kappa_{a(c}\, f_{d)v}{}^f + \kappa_{b(c}\, f_{d)a}{}^f)J_f$$

sl(3)

Metric-like quantities: [Campoleoni,S.F.,Pfenninger,Theisen]

$$g_{\mu\nu} = e_\mu^{\mathcal{A}}\, e_\nu^{\mathcal{B}}\, \kappa_{\mathcal{AB}}\ , \quad \varphi_{\mu\nu\rho} = e_\mu^{\mathcal{A}}\, e_\nu^{\mathcal{B}}\, e_\rho^{\mathcal{C}}\, d_{\mathcal{ABC}}$$

Interacting higher spin theories

Generalisations to

sl(n): fields of spin 2 to spin n

hs_λ: fields of all spins s greater equal 2

⋮

[Blencowe]
[Bergshoeff,Blencowe,Stelle]
[Vasiliev]

Chern-Simons formulation:

$$S = S_{CS}[A] - S_{CS}[\bar{A}] \quad , \quad \left.\begin{array}{c} A \\ \bar{A} \end{array}\right\} = \omega \pm \frac{1}{\ell} e \; , \quad k = \frac{\ell}{4G}$$

with $S_{CS}[A] = \frac{k}{4\pi} \int \mathrm{Tr}\Big(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \Big)$

Coupling to massive scalar known in Prokushkin-Vasiliev theory (no action).

2) Asymptotic symmetries

To describe AdS: consider CS theory on a cylinder and specify boundary conditions

$$\delta S\Big|_{\text{bound.}} = -\frac{k}{4\pi}\int dx^+ dx^- \,\mathrm{Tr}\Big(A_+\delta A_- - A_-\delta A_+\Big)$$

Boundary condition: $A_-\Big|_{\text{bound.}} = 0$

Gauge symmetries become global symmetries:

$\delta A = \partial\Lambda + [A,\Lambda]$ is generated by

$$G(\Lambda) = \int_{D^2} dx^i \wedge dx^j \,\mathrm{Tr}(\Lambda F_{ij}) - \frac{k}{2\pi}\int_{S^1} dx^i \mathrm{Tr}(\Lambda A_i)$$

constraint

Global charges satisfy affine Lie algebra $\hat{\mathfrak{g}}_k$.

AdS asymptotics

Coordinates: t, ρ, θ

sl(2)-part: L_0, L_1, L_{-1} with $[L_m, L_n] = (m-n)\, L_{m+n}$

Gauge choice: $A_\rho = b^{-1}(\rho)\, \partial_\rho b(\rho)$ with $b(\rho) = e^{\rho L_0}$

Constraint/e.o.m./boundary condition:

$$A_\theta = b^{-1}(\rho)\, a(t,\theta)\, b(\rho)$$

$$A_t = b^{-1}(\rho)\, a(t,\theta)\, b(\rho)$$

AdS: $a_{\text{AdS}} = L_1 + \frac{1}{4} L_{-1}$

Asymptotically AdS: $A - A_{\text{AdS}}\big|_{\text{bound.}} \text{finite}$

AdS asymptotics

Asymptotically AdS: $A - A_{\mathrm{AdS}}\big|_{\mathrm{bound.}}$ finite

Lie algebra generators $W_{\ell,m}$ in repr. of sl(2):

$$[L_n, W_{\ell,m}] = (\ell\, n - m)\, W_{\ell,m+n}$$

Then: $b^{-1}(\rho)\, W_{\ell,m}\, b(\rho) = \rho^m\, W_{\ell,m}$

$$\Rightarrow \; a = L_1 + w^{1,0}\, L_0 + w^{1,-1}\, L_{-1} + \sum_{\ell} \sum_{m \geq 0} w^{\ell,m}\, W_{\ell,m}$$

Equivalent to Drinfeld-Sokolov condition

(similar to the Hamiltonian reduction of WZW models) [Balog,L. Fehér,O'Raifeartaigh,Forgács, Wipf

AdS asymptotics

Asymptotically AdS: $A - A_{\rm AdS}\big|_{\rm bound.}$ finite

equivalent to Drinfeld-Sokolov condition:

The asymptotic symmetries are given by the Drinfeld-Sokolov reduction of $\hat{\mathfrak{g}}_k$. [Campoleoni, S.F., Pfenninger, Theisen]

sl(2) $\longrightarrow$ Virasoro $c = 6k$ [Brown, Henneaux]

sl(n) $\longrightarrow$ W_n [Campoleoni, S.F., Pfenninger, Theisen]

$hs(\lambda)$ $\longrightarrow$ $W_\infty(\lambda)$ [Henneaux, Rey] [Gaberdiel, Hartman] [Campoleoni, S.F., Pfenninger]

3) Higher-spin AdS/CFT

In AdS/CFT, the boundary operators correspond to sources for the bulk fields.

AdS | CFT

$$\text{“}\int \mathcal{D}\phi\, e^{-S_{\text{gravity \& more}}} = \left\langle e^{-\int d^dx\, \phi_0^i \mathcal{O}_i} \right\rangle_{\mathrm{CFT}_d}\text{”}$$

$$\phi^i\Big|_{\text{boundary of }\mathrm{AdS}_{d+1}} \sim \phi_0^i$$

$\mathcal{O}_i$ operator corresponding to ϕ^i

Higher-spin AdS/CFT

The string theoretic AdS/CFT correspondence has a higher-spin cousin:

Higher-spin gauge theories coupled to matter $\longleftrightarrow$ CFTs with classically conserved higher-spin currents $J^{\mu_1 \ldots \mu_s}$

bdy value of HS field ↘ CFT operator ↙

Coupling $\int d^d x\, \bar{\varphi}_{\mu_1 \ldots \mu_s}\, J^{\mu_1 \ldots \mu_s} \xrightarrow{\delta\bar{\varphi} = \partial\xi}$

$$\int d^d x\, \partial_{\mu_1} \xi_{\mu_2 \ldots \mu_s} J^{\mu_1 \ldots \mu_s} = - \int d^d x\, \xi^{\mu_2 \ldots \mu_s} \partial_{\mu_1} J^{\mu_1 \ldots \mu_s}$$

Higher-spin AdS/CFT

Prominent case: higher-spin AdS_4/CFT_3 :

Vasiliev theory

$\Updownarrow$

Free/critical bosons/fermions

[Klebanov,Polyakov]
[Sezgin,Sundell]
[Giombi,Yin] ...

If HS gauge symmetry is unbroken in the quantum theory, the boundary theories are essentially trivial for $d>2$.

In $d=2$, non-trivial quantum theories are known with extended symmetries: W-algebras.

Minimal model holography

The classical W-algebras have a quantum version $W_\infty(\lambda) \longrightarrow \hat{W}_\infty(\lambda)$.

We can then look for families of CFTs with W-symmetries as candidates for the boundary theory.

Prototype: W_n-minimal models

To compare to the classical higher-spin theories we look for families which admit a classical ($c \to \infty$) limit.

Minimal model holography

The minimal models $W_{n,k}$ come with central charges:

$$c_{n,k} = (n-1)\left(1 - \frac{n(n+1)}{(n+k)(n+k+1)}\right) \quad < n-1$$

$$= 2k\left(1 - \frac{(k+1)(k+(3n+1)/2)}{(k+n)(k+n+1)}\right) \quad < 2k$$

Two ways to achieve $c \to \infty$:

't Hooft limit: $n, k \to \infty, \ \lambda = \frac{n}{n+k}$ fixed

[Gaberdiel,Gopakumar]

semi-classical limit: $k \to -n-1$

[Castro,Gopakumar,Gutperle,Raeymaekers]

Minimal model holography

't Hooft limit:

- unitary
- Some CFT states match excitations of a scalar field coupled to HS fields
- many light states - interpretation?
- checks: partition function, 3-point functions, ...

[Gaberdiel,Gopakumar,Saha]
[Gaberdiel,Gopakumar,Hartman,Raju]
[Chang,Yin] ...

Minimal model holography

semi-classical limit:

- non-unitary
- light states match excitations of a scalar
- other states ~ non-perturbative solutions: conical defects: all HS charges match

  [Castro,Gopakumar,Gutperle,Raeymaekers]
  [Campoleoni,Prochazka,Raeymaekers] [Campoleoni,S.F.]

- further checks: partition function, correlation functions, ...

  [Perlmutter,Prochazka,Raeymaekers]
  [Hijano,Kraus,Perlmutter]

Developments

- HS black holes / entropy

[Gutperle,Kraus] [Ammon,Gutperle,Kraus,Perlmutter] [Pérez,Tempo,Troncoso] [Castro,Hijano,Lepage-Jutier,Maloney] [Bañados,Canto,Theisen] [Ammon,Castro,Iqbal] [de Boer,Jottar] [Henneaux,Pérez,Tempo,Troncoso] ...

- SUSY generalisations

[Creutzig,Hikida,Rønne] [Candu,Gaberdiel] [Henneaux,Lucena Gomez,Park,Rey] [Beccaria,Candu,Gaberdiel,Groher] [Candu,Peng,Vollenweider] ...

- Relation to string theory: relate HS-CFT dual to String-CFT dual

[Gaberdiel,Gopakumar] [Gaberdiel, Peng, Zadeh] [Creutzig,Hikida] [Hikida,Rønne] ...

- Improve understanding of HS theories: Metric-like formulation/interactions

[Campoleoni,S.F.,Pfenninger,Theisen] [Fujisawa,Nakayama] [S.F.,Kessel] [Kessel,Lucena Gomez,Skvortsov,Taronna] ...

...

Summary

HS AdS_3/CFT_2 duality:

- 2+1 dim. laboratory for HS and AdS/CFT
- W-symmetry of boundary CFT: Drinfeld-Sokolov reduction
- Minimal models candidates for CFT duals

HS gauge theories:

- fascinating extensions of gravity
- relation to string theory?
- rich possibilities for (non-SUSY) AdS/CFT