HORAVA-LIFSHITZ GRAVITY FROM AN RG PERSPECTIVE

GIULIO D'ODORICO

SIFT 2015, 5-7 November, Jena, Germany

G. D., F. Saueressig, M. Schutten, Phys.Rev.Lett. 113 (2014) 171101, arXiv:1406.4366
G. D., J. W. Goossens, F. Saueressig, JHEP 1510 (2015) 126, arXiv:1508.00590

WHY HORAVA-LIFSHITZ GRAVITY?

FP Structure of Quantum Gravity

Theory Space: Quantum Einstein Gravity
Symmetry: Diffeomorphisms

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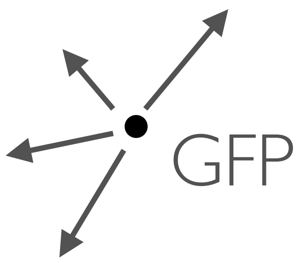

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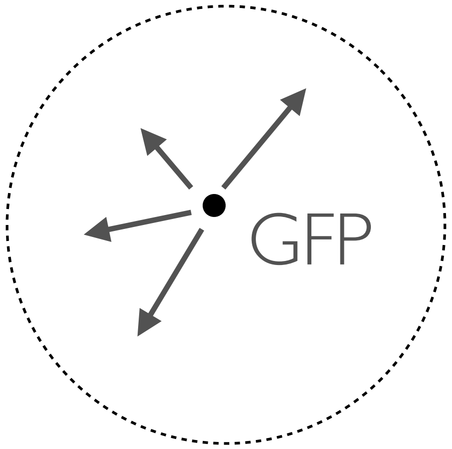

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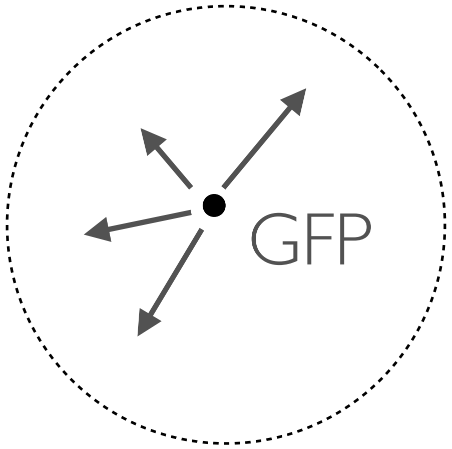

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- Unfortunately gravity is perturbatively nonrenormalizable

$$[G_N] = -2$$

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- Dynamics of General Relativity governed by Einstein-Hilbert action

$$S^{\rm EH} = \frac{1}{16\pi G_N} \int d^4x \sqrt{g} \big[-R + 2\Lambda \big]$$

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$$\ell\text{-loop-diagram diverges} \propto E^2 \, (G_N \, E^2)^\ell$$

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- Generalized, nonperturbative renormalizability requirement

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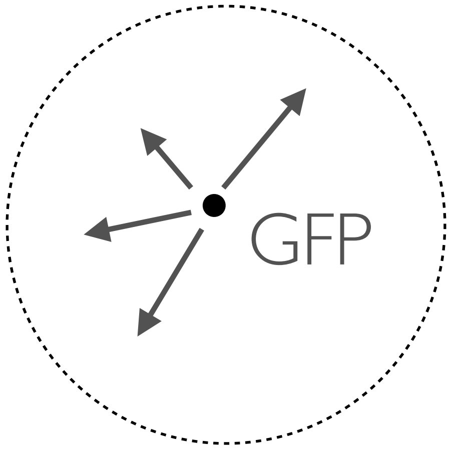

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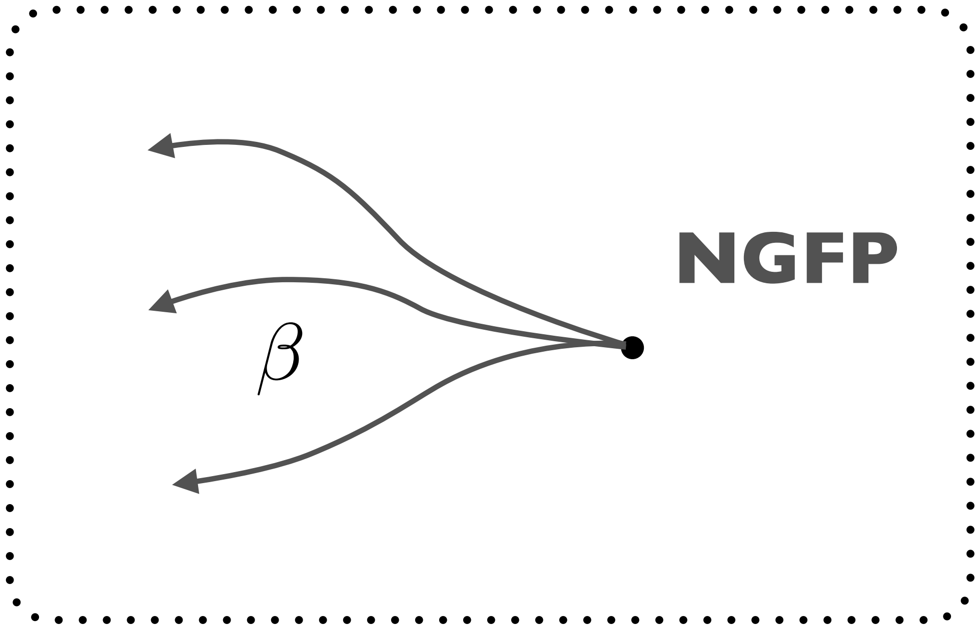

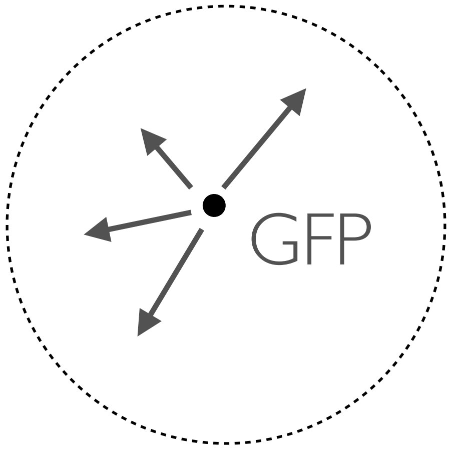

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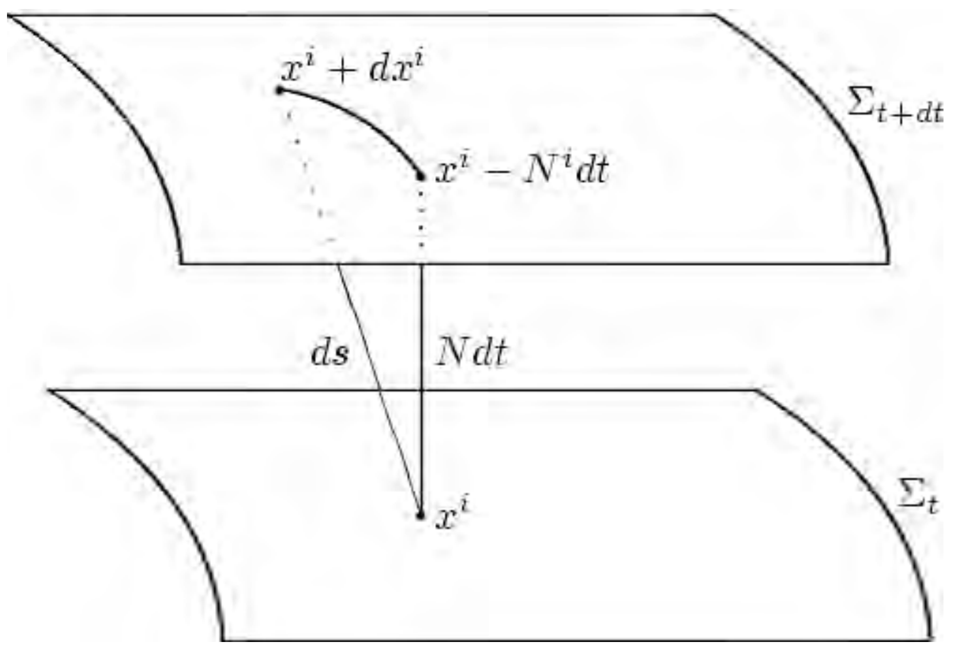

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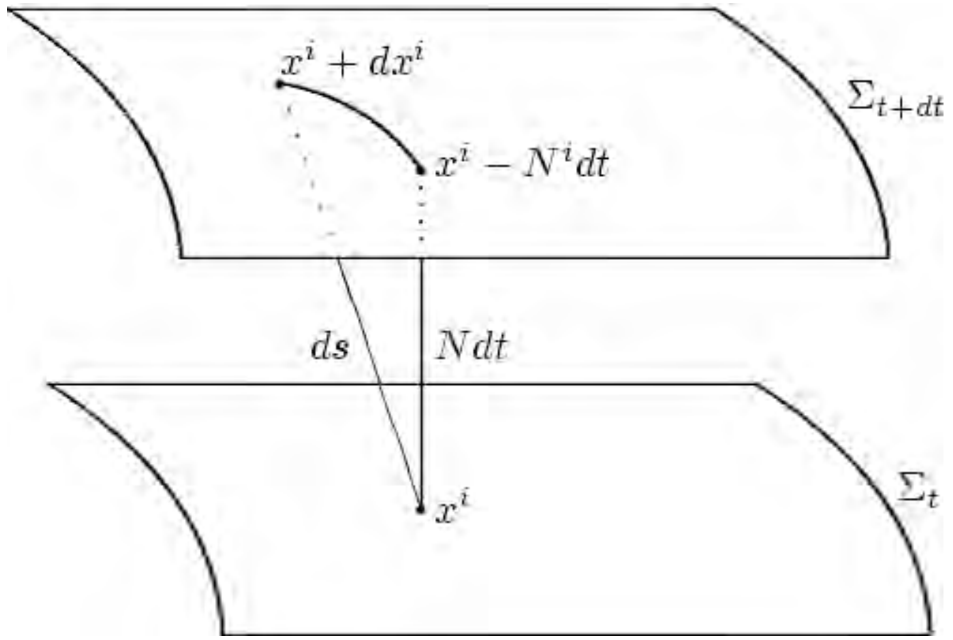

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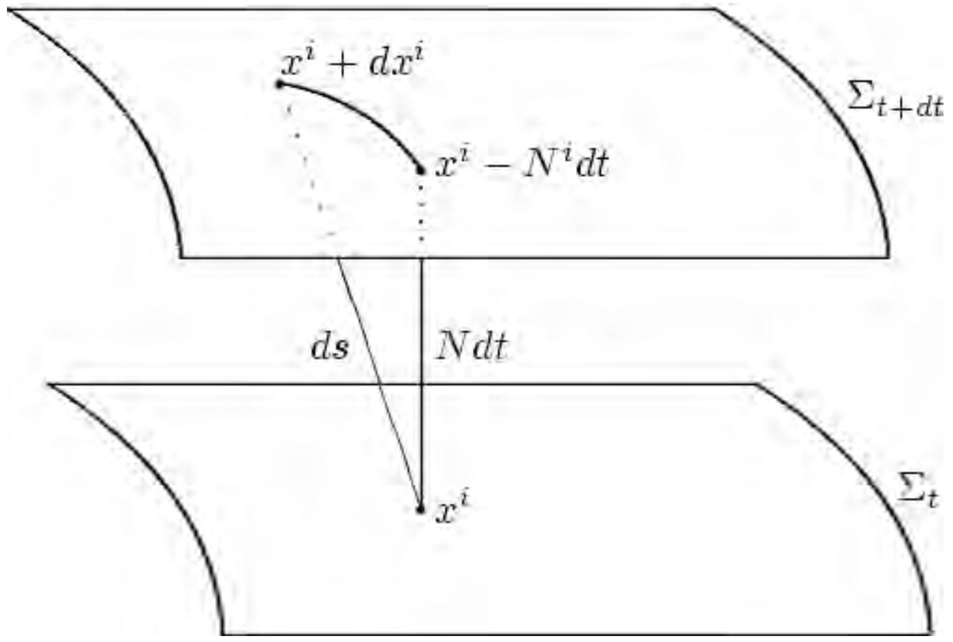

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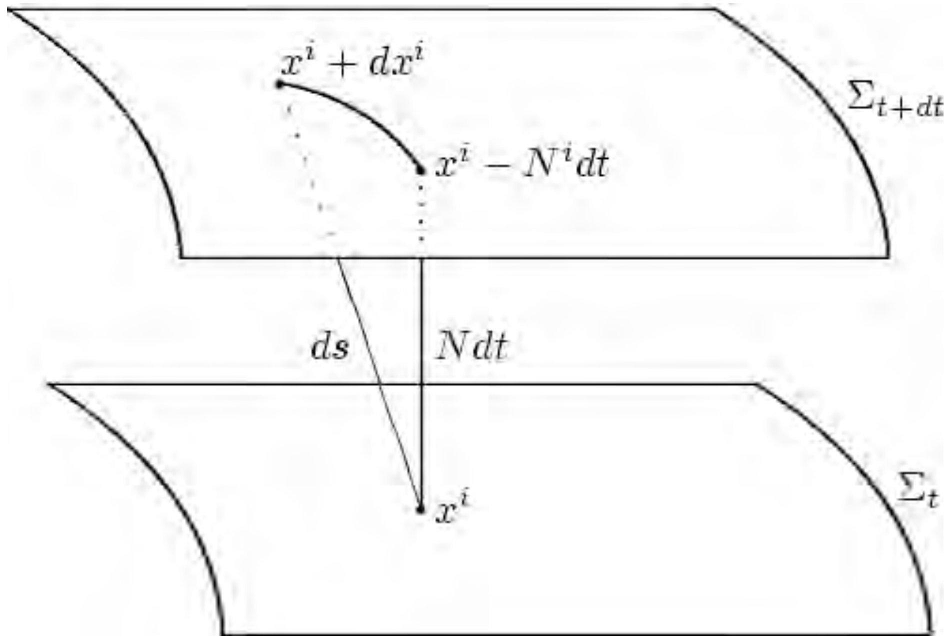

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- Projectable HL Gravity $N(t,x) = N(t)$

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Horava-Lifshitz gravity is power-counting renormalizable

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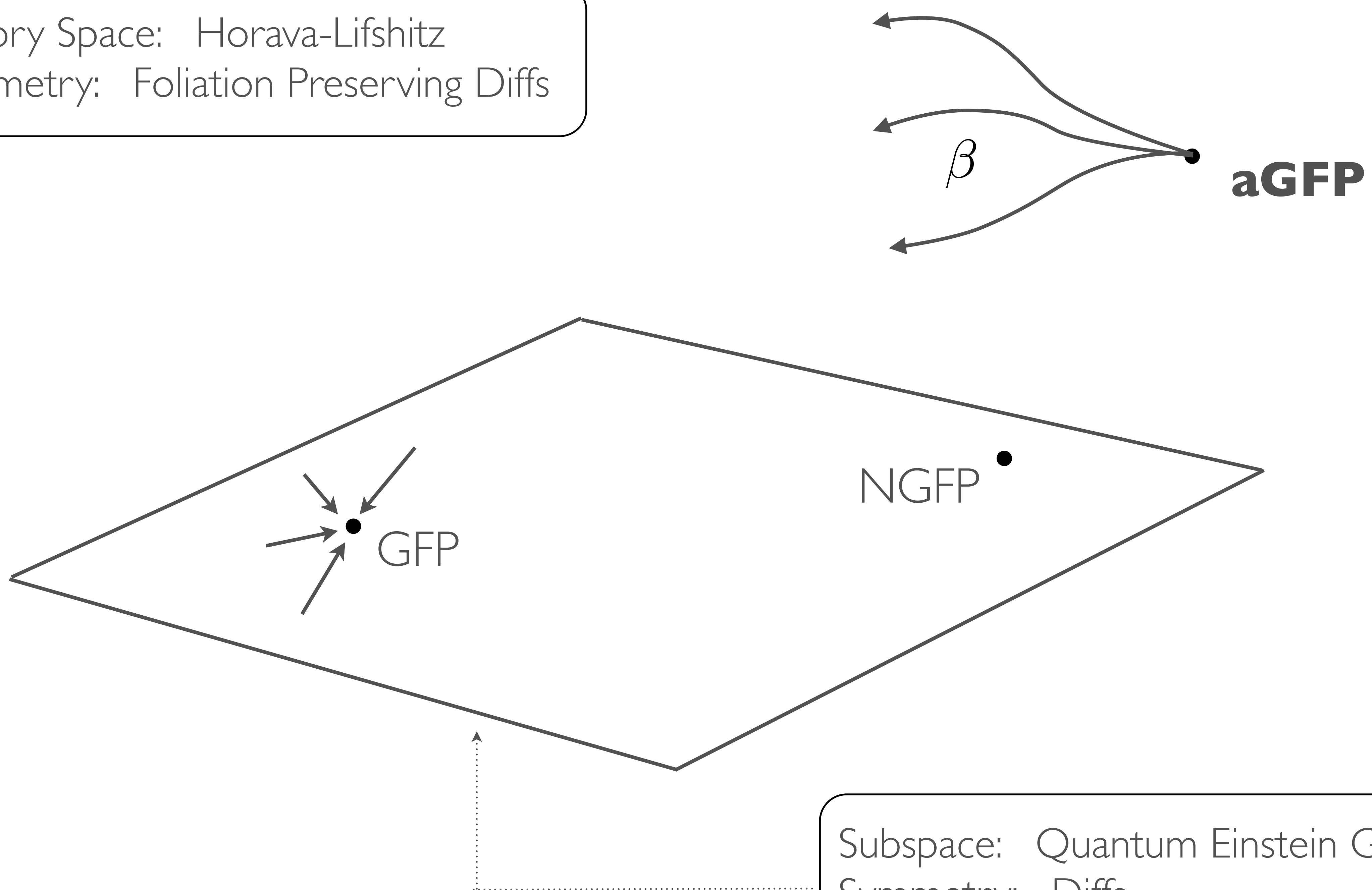

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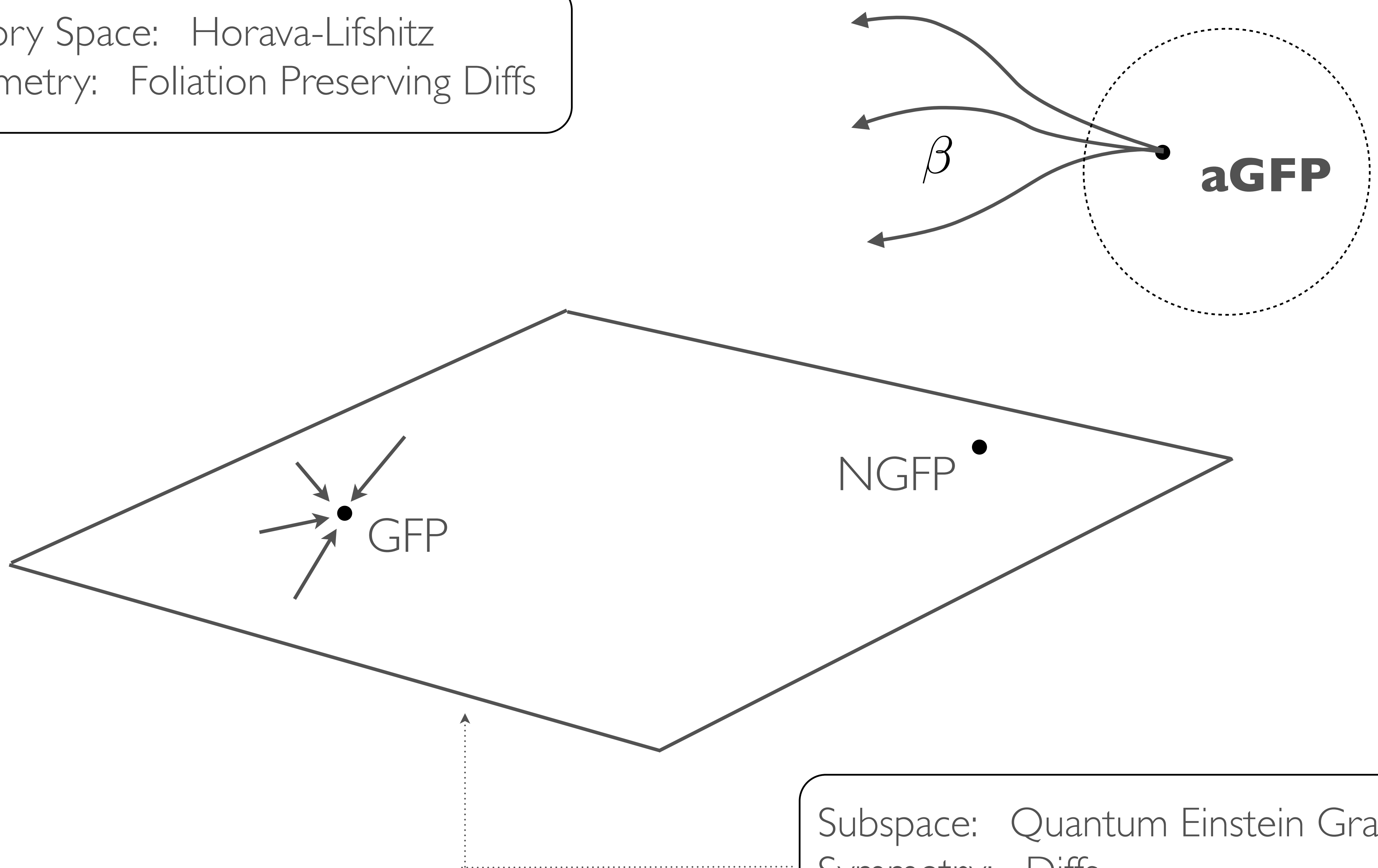

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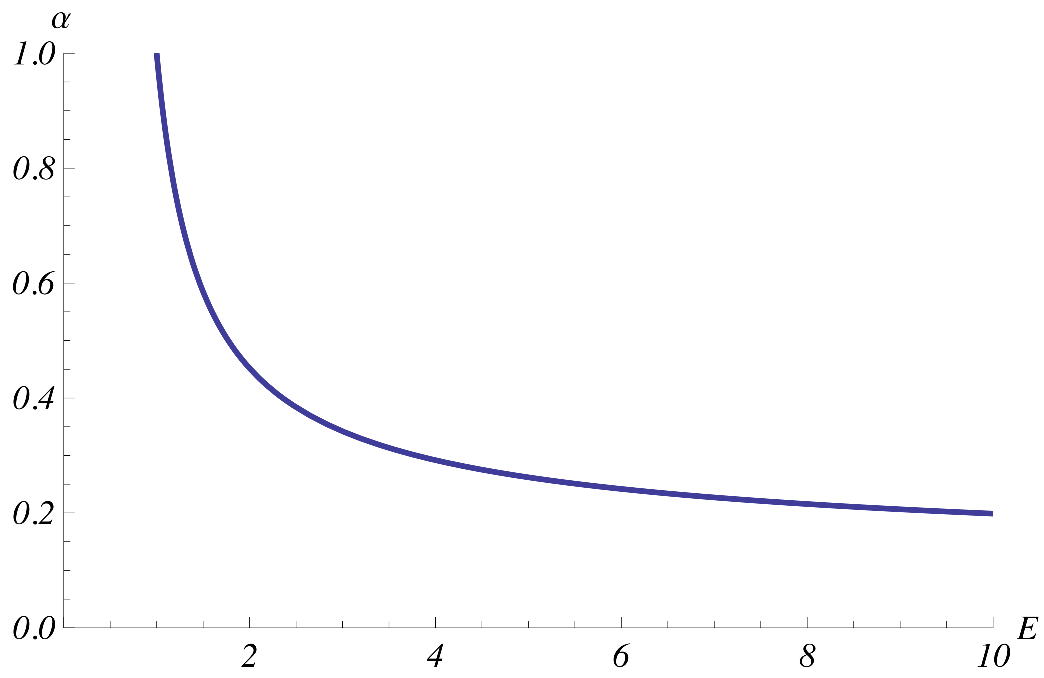

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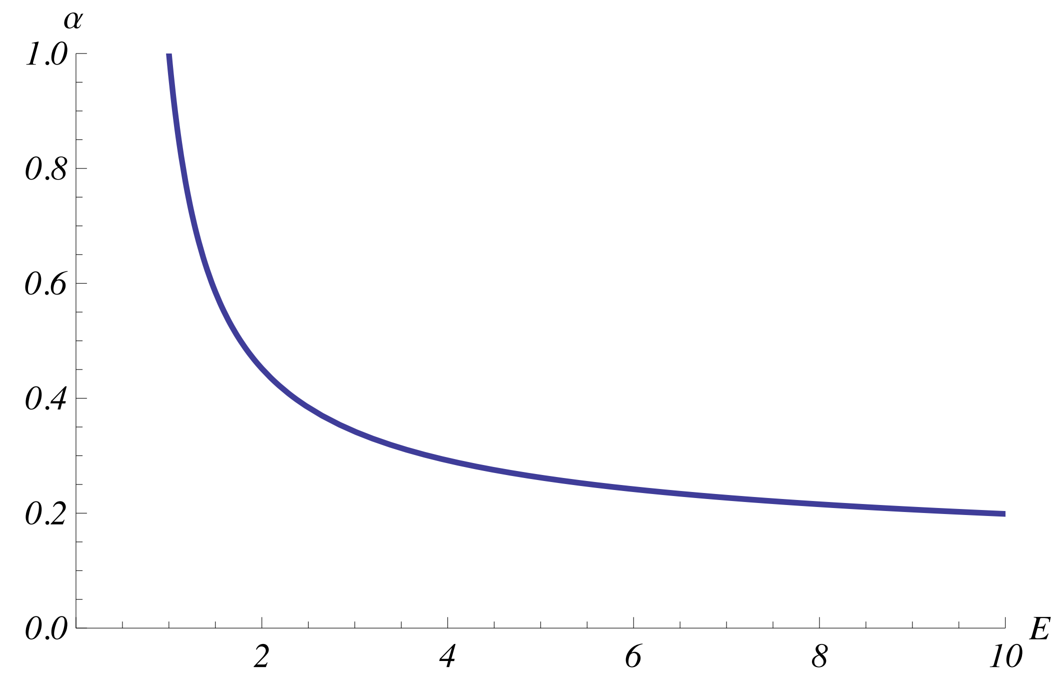

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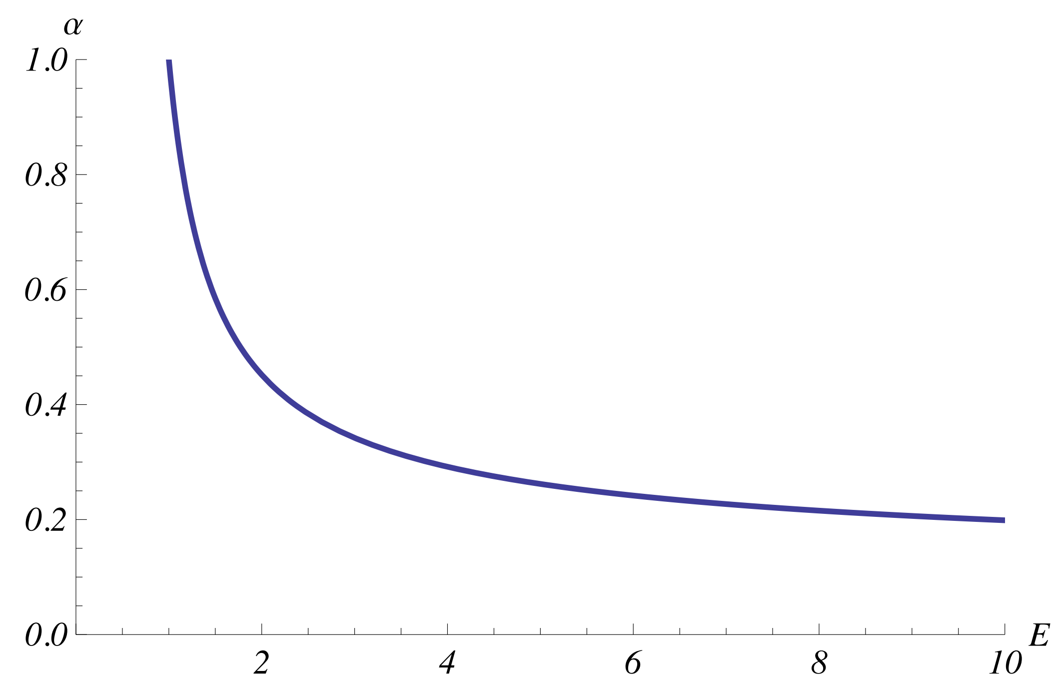

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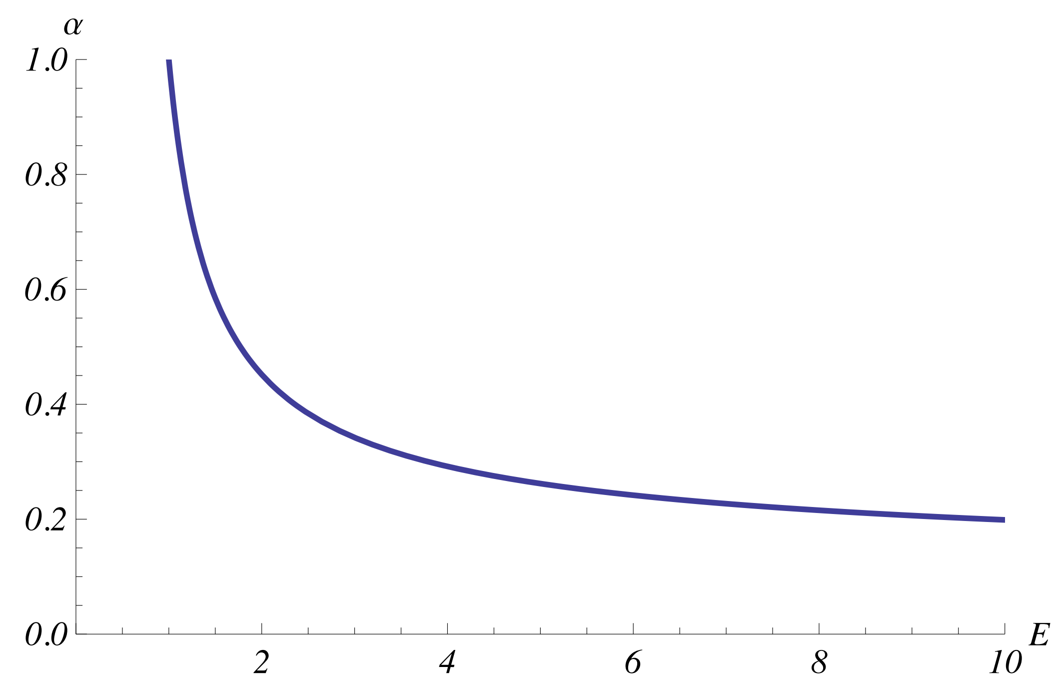

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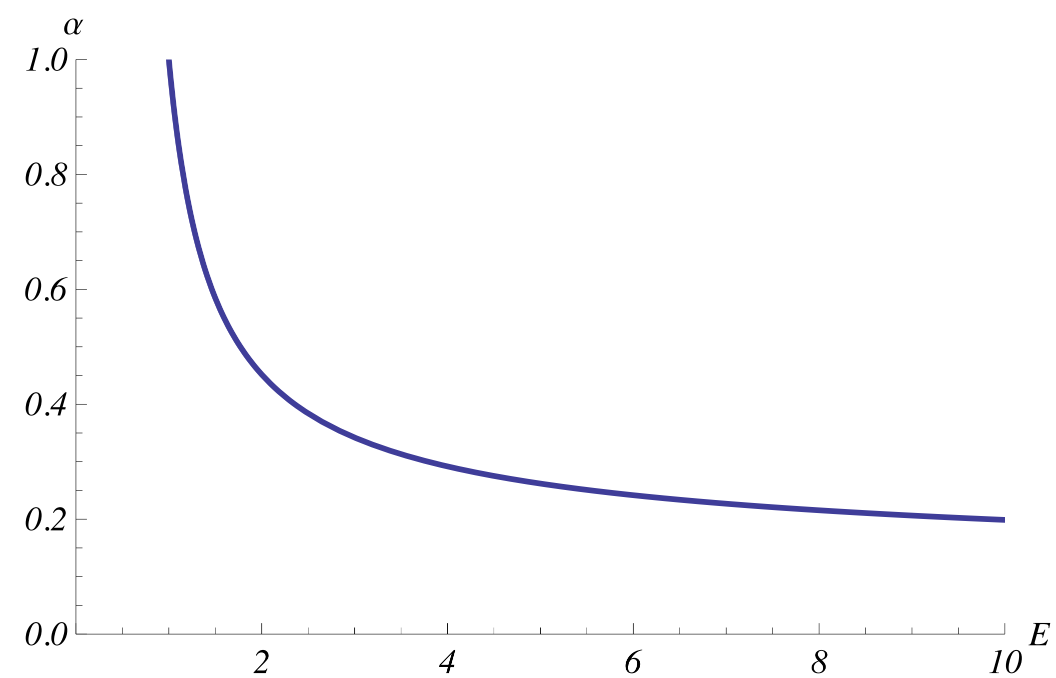

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- Too many flavors destroy asymptotic freedom

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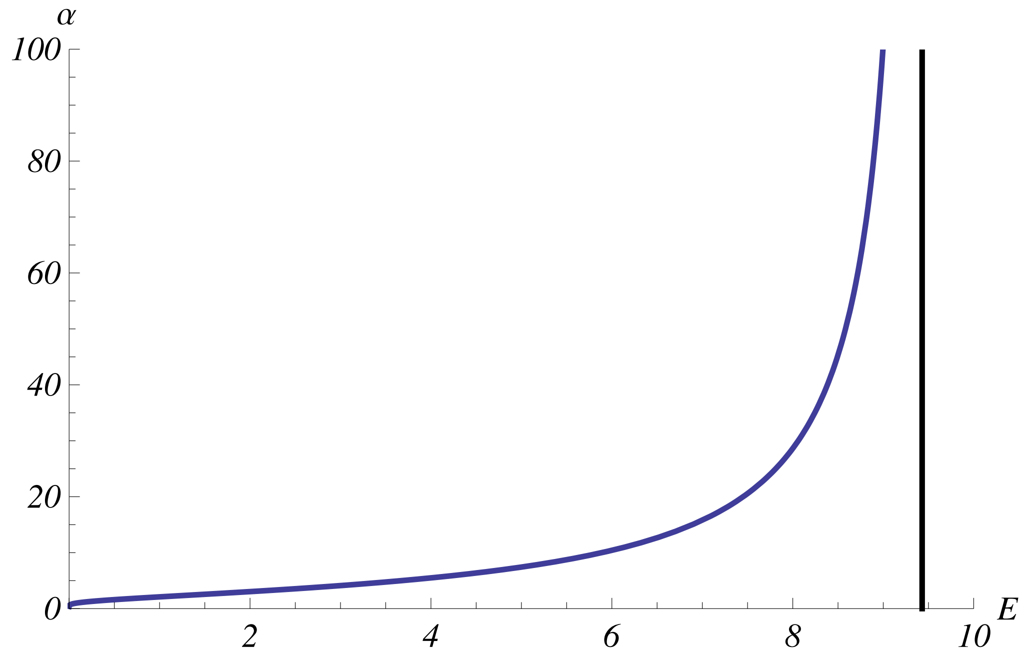

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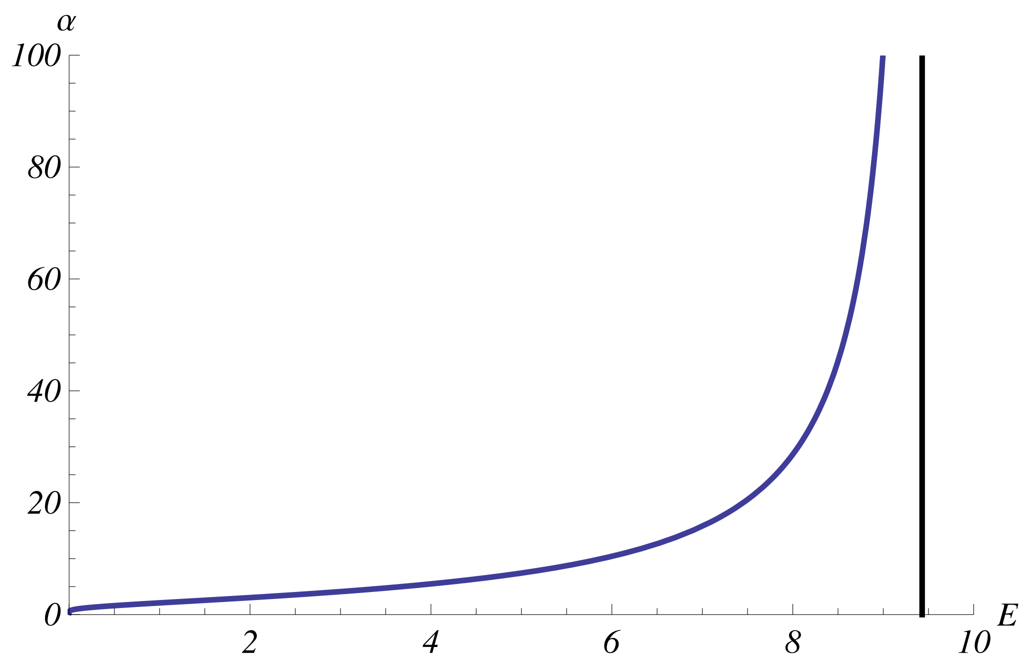

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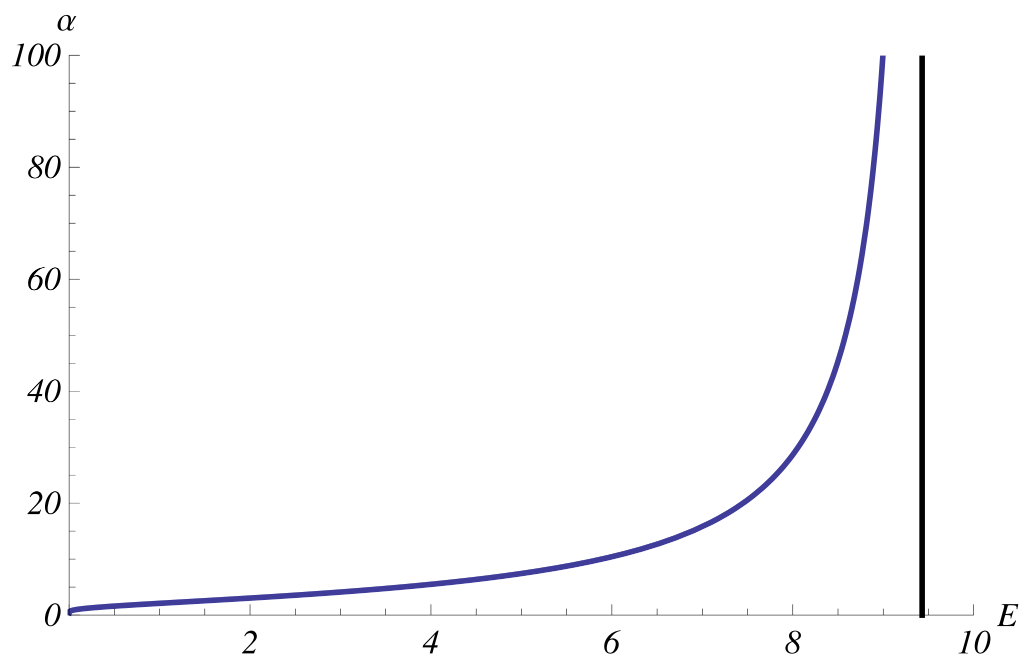

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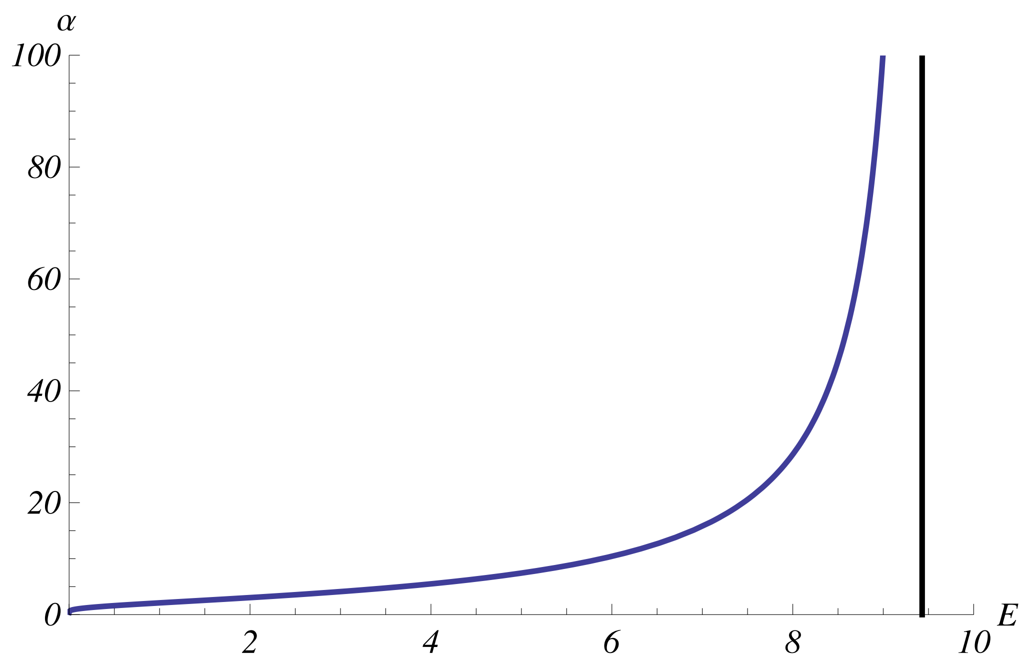

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- At high energies α diverges at a Landau pole

Questions

- Is the theory asymptotically free?
- Does it reproduce the correct phenomenology?
- Does it resolve previous issues?

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COVARIANT EFFECTIVE ACTIONS IN HL GRAVITY

Anisotropic scalar field

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- Horava-Lifshitz gravity minimally coupled to an anisotropic scalar

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- Gravitational sector

$$\Gamma_k^{\mathrm{HL}} = \frac{1}{16\pi G_k} \int dt d^d x N \sqrt{\sigma} \left[K_{ij} K^{ij} - \lambda_k K^2 + V_k \right]$$

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$$\Gamma_k[N, N_i, \sigma, \phi] = \Gamma_k^{\mathrm{HL}}[N, N_i, \sigma] + S^{\mathrm{LS}}[N, N_i, \sigma, \phi]$$

- Gravitational sector

$$\Gamma_k^{\mathrm{HL}} = \frac{1}{16\pi G_k} \int dt d^d x N \sqrt{\sigma} \left[K_{ij} K^{ij} - \lambda_k K^2 + V_k \right]$$

Potential V is a function of the intrinsic curvatures

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- Projectable version

$$V_k^{(d=2)} = g_0 + g_1 R + g_2 R^2$$

$$V_k^{(d=3)} = g_0 + g_1 R + g_2 R^2 + g_3 R_{ij} R^{ij} - g_4 R \Delta_x R - g_5 R_{ij} \Delta_x R^{ij} + g_6 R^3 + g_7 R R_{ij} R^{ij} + g_8 R^i_j R^j_k R^k_i$$

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$$\begin{aligned} \Delta \mathcal{V} \quad = \quad & u_1 \, a_i \, a^i + u_{2,1} \, R \, \nabla_i \, a^i + u_{2,2} \, a_i \, \Delta_x \, a^i + \\ & u_{3,1} (\Delta_x R) \, \nabla_i \, a^i + u_{3,2} \, a_i \, (\Delta_x)^2 \, a^i + \ldots \end{aligned}$$

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- Divergences are encoded in the Seeley-deWitt expansion

ANISOTROPIC HEAT-KERNELS

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$$\mathrm{Tr}\, e^{-sD^2} = (4\pi)^{-\frac{d+1}{2}}\, s^{-\frac{1+d/z}{2}} \int dt d^d x\, N\sqrt{\sigma} \sum_{l,m,n\geq 0} s^{\frac{l(1-z)}{2z}+\frac{m}{2}+\frac{n}{2z}}\, \mathrm{tr}\, \mathbf{b}_{l,m,n}$$

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$$\underbrace{e_2\, K_{ij}K^{ij}}_{\mathbf{b}_{0,2,0}} \qquad \underbrace{a_2\, R^{(3)}}_{\mathbf{b}_{0,0,2}} \qquad \underbrace{c_1\, a_i a^i}_{\mathbf{b}_{0,0,2}}$$

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  - This gives

$$\mathrm{Tr}\, e^{-sD^2} = \mathrm{Tr}\left[e^{-s\Delta_t} e^{-s(\Delta_x)^z} \mathcal{C}(\Delta_t, \Delta_x) \right]$$

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  - *Step 3*: Rescale the metric

$$\tilde{\sigma}_{ij} = \frac{s}{v}\,\sigma_{ij} \quad \longrightarrow \quad \tilde{\Delta}_x = \frac{v}{s}\,\Delta_x$$

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  - The result is of the off-diagonal heat-kernel type

$$\mathrm{Tr}\, e^{-sD^2} = \mathrm{Tr}\left[\mathcal{O}\, e^{-s\Delta^{(D)}} \right]$$

  - in terms of the “fake” Laplacian

$$\Delta^{(D)} = \Delta_t + \tilde{\Delta}_x$$

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  - *Step 5*: Use the off-diagonal Heat-Kernel !

[Anselmi, Benini JHEP 10 (2007) 099]
[Benedetti, Groh, Machado, Saueressig JHEP 06 (2011) 079]
[Groh, Saueressig, Zanusso arXiv:1112.4856]

Numerical Coefficients

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- **Extrinsic curvature (kinetic) terms**

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basis of curvature monomials with 2n spatial derivatives

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- Power-counting irrelevant terms $\qquad k = n + 1 - \lfloor d/2 \rfloor$

$$b_n(d,z) = \frac{(-1)^k}{\Gamma(d/2-n+k)} \int_0^\infty dx\, x^{d/2-n+k-1}\, (\partial_x)^k\, e^{-x^z}\,, \qquad n > \lfloor d/2 \rfloor$$

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- Note: determining the c's is computationally very intensive !

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		$d=2$			$d=3$			
	dim	a_{2n}	$z=2$	$z=3$	a_{2n}	$z=2$	$z=3$	$z=4$
K^2	$\frac{d-z}{2z}$	$\frac{1}{6}$	$\frac{\sqrt{\pi}}{16}$	$\frac{1}{36}\Gamma\left(\frac{1}{3}\right)$	$\frac{1}{6}$	$\frac{2\Gamma\left(\frac{3}{4}\right)}{15\sqrt{\pi}}$	$\frac{1}{15}$	$\frac{\Gamma\left(\frac{3}{8}\right)}{30\sqrt{\pi}}$
$K_{ij}K^{ij}$	$\frac{d-z}{2z}$	$-\frac{1}{6}$	$-\frac{\sqrt{\pi}}{8}$	$-\frac{1}{9}\Gamma\left(\frac{1}{3}\right)$	$-\frac{1}{6}$	$-\frac{7\Gamma\left(\frac{3}{4}\right)}{30\sqrt{\pi}}$	$-\frac{1}{5}$	$-\frac{11\Gamma\left(\frac{3}{8}\right)}{60\sqrt{\pi}}$
1	$\frac{d+z}{2z}$	1	$\frac{\sqrt{\pi}}{2}$	$\Gamma\left(\frac{4}{3}\right)$	1	$\frac{4\Gamma\left(\frac{7}{4}\right)}{3\sqrt{\pi}}$	$\frac{2}{3}$	$\frac{4\Gamma\left(\frac{11}{8}\right)}{3\sqrt{\pi}}$
R	$\frac{d-2+z}{2z}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{\Gamma\left(\frac{5}{4}\right)}{3\sqrt{\pi}}$	$\frac{\Gamma\left(\frac{7}{6}\right)}{3\sqrt{\pi}}$	$\frac{\Gamma\left(\frac{9}{8}\right)}{3\sqrt{\pi}}$
R^2	$\frac{d-4+z}{2z}$	$\frac{1}{60}$	0	0	$\frac{1}{120}$	$\frac{\Gamma\left(\frac{5}{4}\right)}{120\sqrt{\pi}}$	$\frac{\Gamma\left(\frac{7}{6}\right)}{120\sqrt{\pi}}$	$\frac{\Gamma\left(\frac{9}{8}\right)}{120\sqrt{\pi}}$
$R_{ij}R^{ij}$	$\frac{d-4+z}{2z}$	–	–	–	$\frac{1}{60}$	$\frac{\Gamma\left(\frac{5}{4}\right)}{60\sqrt{\pi}}$	$\frac{\Gamma\left(\frac{7}{6}\right)}{60\sqrt{\pi}}$	$\frac{\Gamma\left(\frac{9}{8}\right)}{60\sqrt{\pi}}$
$-R\Delta_x R$	$\frac{d-6+z}{2z}$	–	–	–	$\frac{1}{336}$	$-\frac{\Gamma\left(\frac{5}{4}\right)}{168\sqrt{\pi}}$	$-\frac{1}{672}$	$-\frac{\Gamma\left(\frac{5}{8}\right)}{672\sqrt{\pi}}$
$-R_{ij}\Delta_x R^{ij}$	$\frac{d-6+z}{2z}$	–	–	–	$\frac{1}{840}$	$-\frac{\Gamma\left(\frac{5}{4}\right)}{420\sqrt{\pi}}$	$-\frac{1}{1680}$	$-\frac{\Gamma\left(\frac{5}{8}\right)}{1680\sqrt{\pi}}$
R^3	$\frac{d-6+z}{2z}$	$\frac{1}{756}$	-2	0	$-\frac{1}{560}$	$\frac{\Gamma\left(\frac{5}{4}\right)}{280\sqrt{\pi}}$	$\frac{1}{1120}$	$\frac{\Gamma\left(\frac{5}{8}\right)}{1120\sqrt{\pi}}$
$RR_{ij}R^{ij}$	$\frac{d-6+z}{2z}$	–	–	–	$\frac{1}{105}$	$-\frac{2\Gamma\left(\frac{5}{4}\right)}{105\sqrt{\pi}}$	$-\frac{1}{210}$	$-\frac{\Gamma\left(\frac{5}{8}\right)}{210\sqrt{\pi}}$
$R^i_j R^j_k R^k_i$	$\frac{d-6+z}{2z}$	–	–	–	$-\frac{1}{180}$	$\frac{\Gamma\left(\frac{5}{4}\right)}{90\sqrt{\pi}}$	$\frac{1}{360}$	$\frac{\Gamma\left(\frac{5}{8}\right)}{360\sqrt{\pi}}$
a^2	$\frac{d-2+z}{2z}$	0	$-\frac{13}{12}$	$-\frac{13}{6}$	0	$-\frac{13\Gamma\left(\frac{5}{4}\right)}{9\sqrt{\pi}}$	$-\frac{26\Gamma\left(\frac{7}{6}\right)}{9\sqrt{\pi}}$	$-\frac{13\Gamma\left(\frac{9}{8}\right)}{3\sqrt{\pi}}$

SCALAR-DRIVEN RG FLOWS IN HL GRAVITY

One-loop Effective Action

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- The one-loop part of the affective action is

$$\Gamma_1 = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\, e^{-sm^2}\, \mathrm{Tr}\left[e^{-sD^2}\right]$$

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- From this we can read the matter-induced beta functions

Beta functions for d=z=3

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$$\beta_g = \frac{n_s}{5\pi}\, g^2 \,, \qquad \beta_\lambda = \frac{n_s}{15\pi}\, (3\lambda - 1)\; g$$

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$$\begin{aligned}
\beta_{\tilde{g}_0} &= -\,2\,\tilde{g}_0 + n_s\,\tfrac{g}{\pi}\,\left(b_0 + \tfrac{1}{5}\,\tilde{g}_0\right)\\
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\end{aligned}$$

- Non-projectable potential terms

$$\begin{aligned}
\beta_{\tilde{u}_1} &= -\tfrac{4}{3}\,\tilde{u}_1 + n_s \tfrac{g}{\pi} \left(c_1 + \tfrac{1}{5}\,\tilde{u}t_1 \right) , \\
\beta_{\tilde{u}_{2,i}} &= -\tfrac{2}{3}\,\tilde{u}_{2,i} + n_s \tfrac{g}{\pi} \left(c_{2,i} + \tfrac{1}{5}\,\tilde{u}_{2,i} \right) , \\
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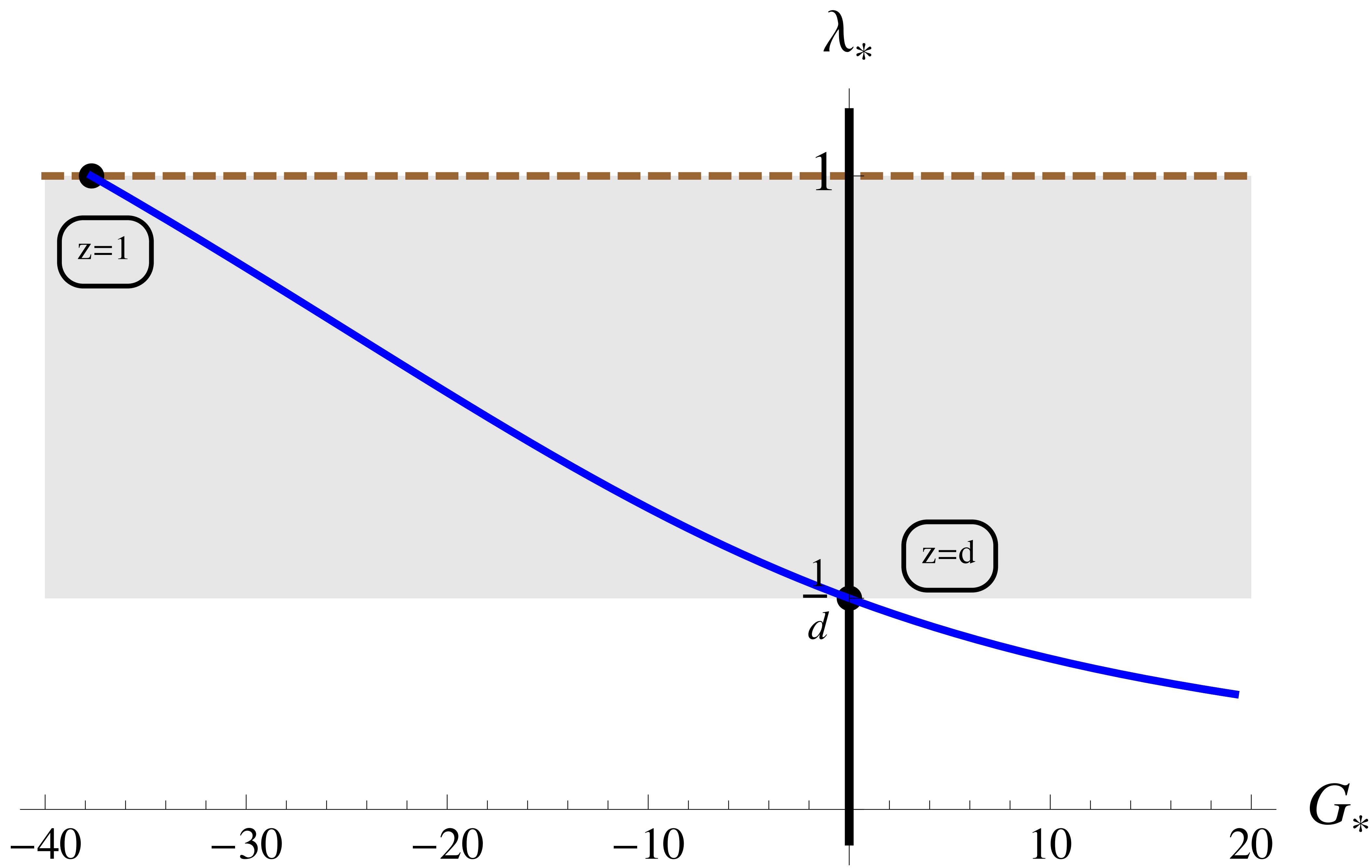
\end{aligned}$$

A parametric view of theory space



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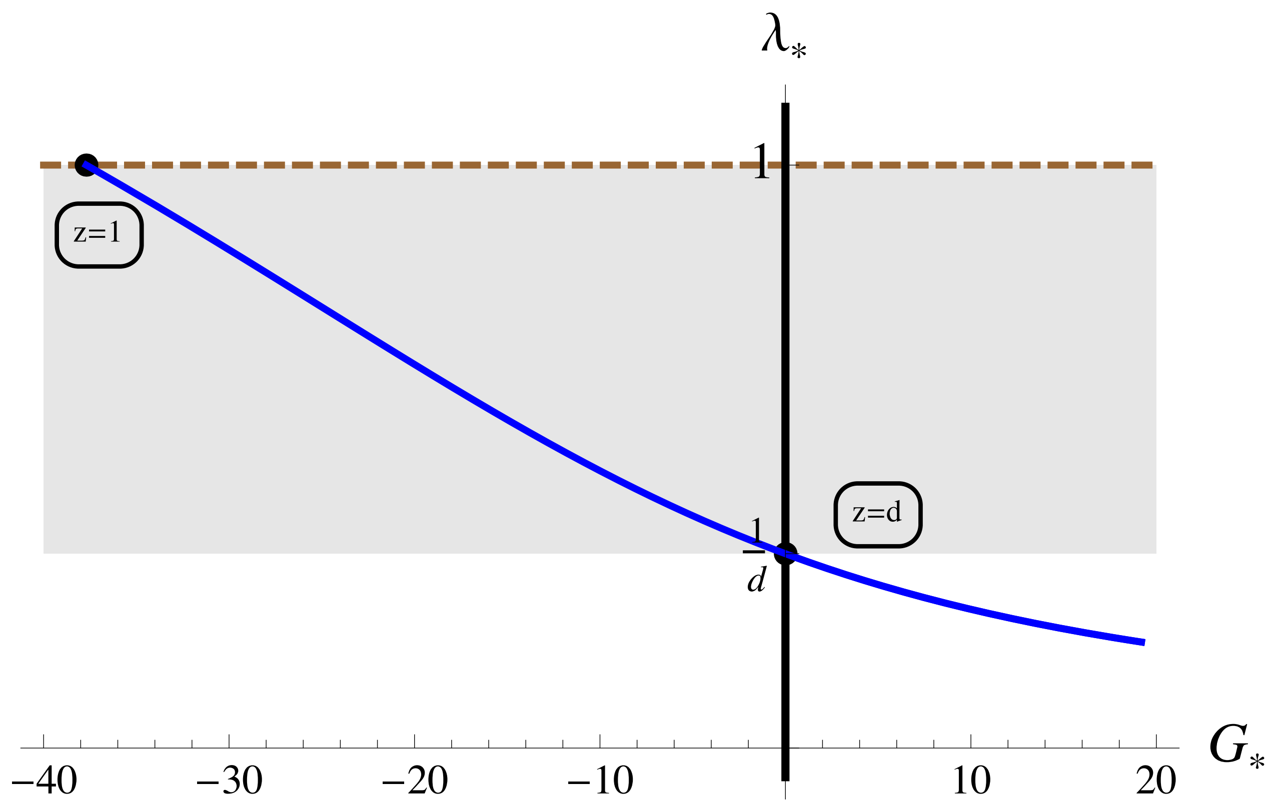

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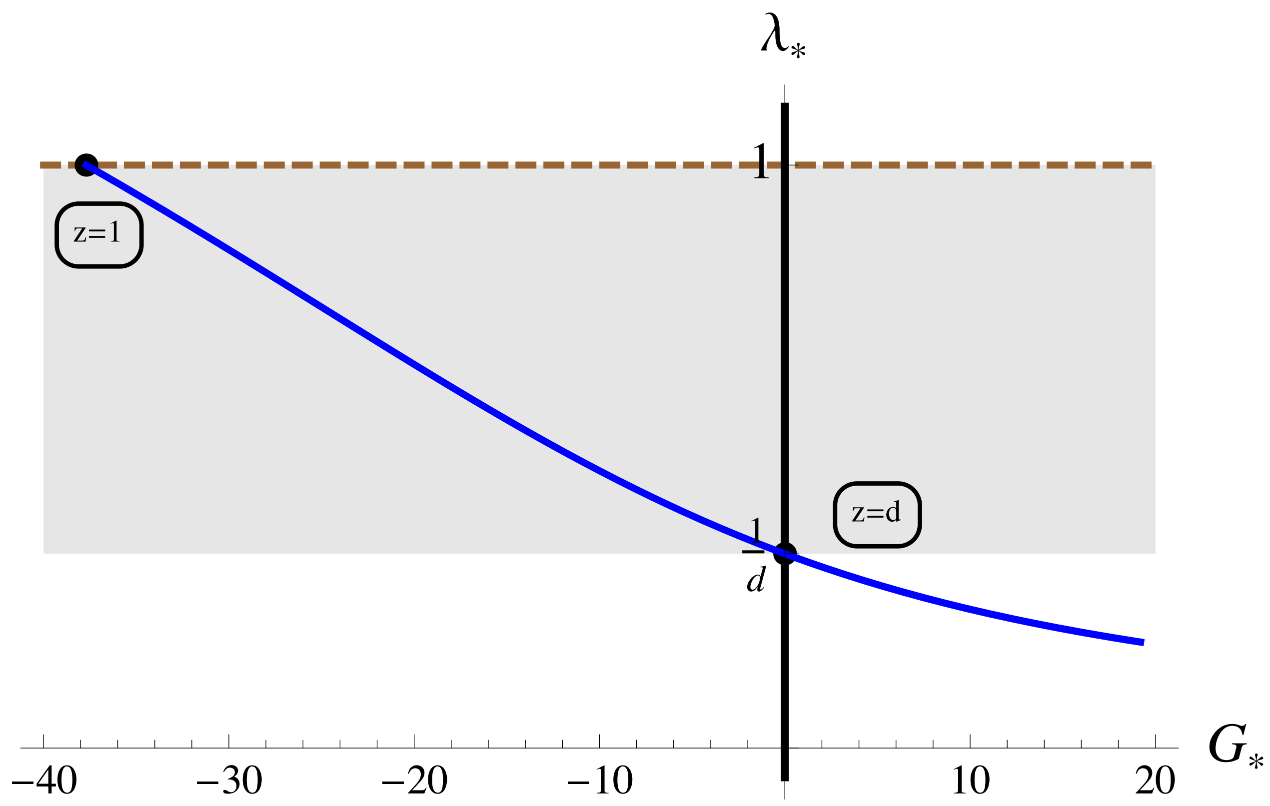

- z = 1: isotropic non-Gaussian fixed point (Asymptotic Safety)

$$g_* = -\frac{3(d-1)}{2}\,(4\pi)^{(d-1)/2}\,\Gamma\left(\frac{d+1}{2}\right) \quad , \quad \lambda_* = 1$$

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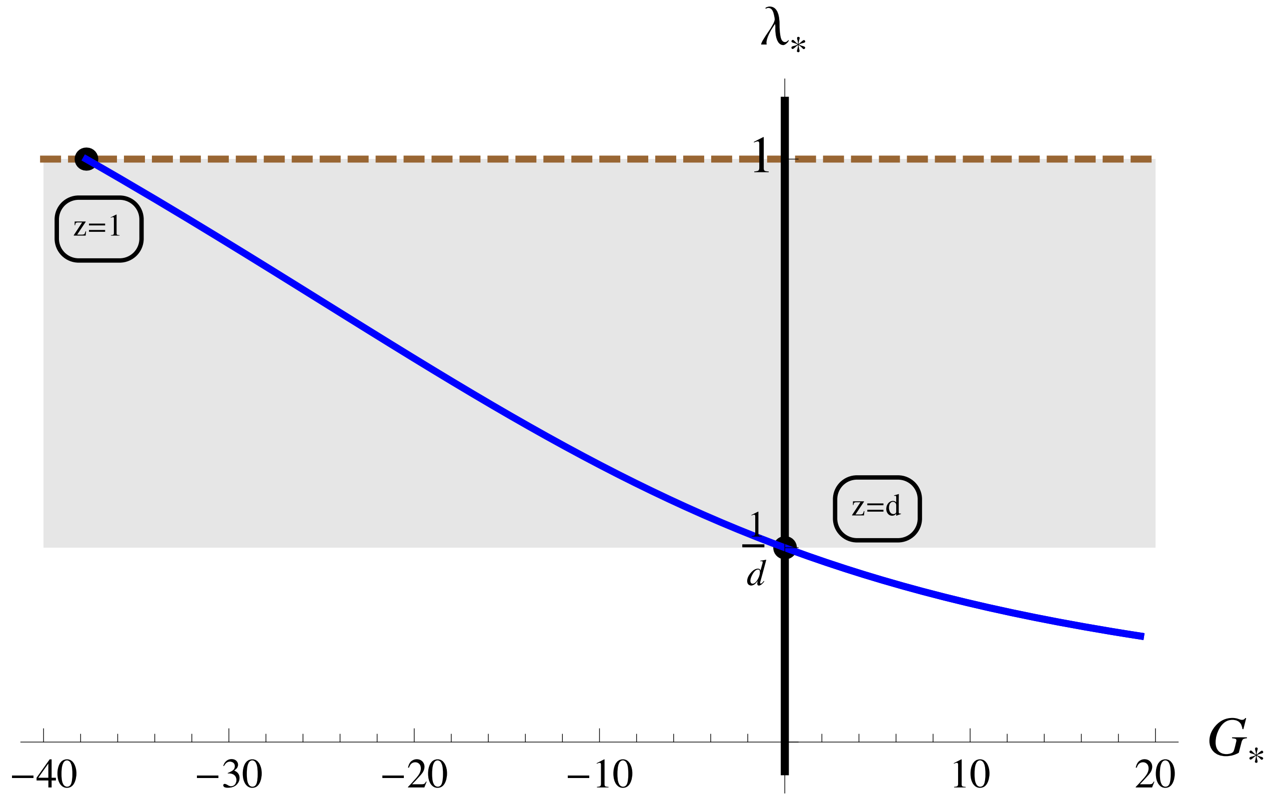

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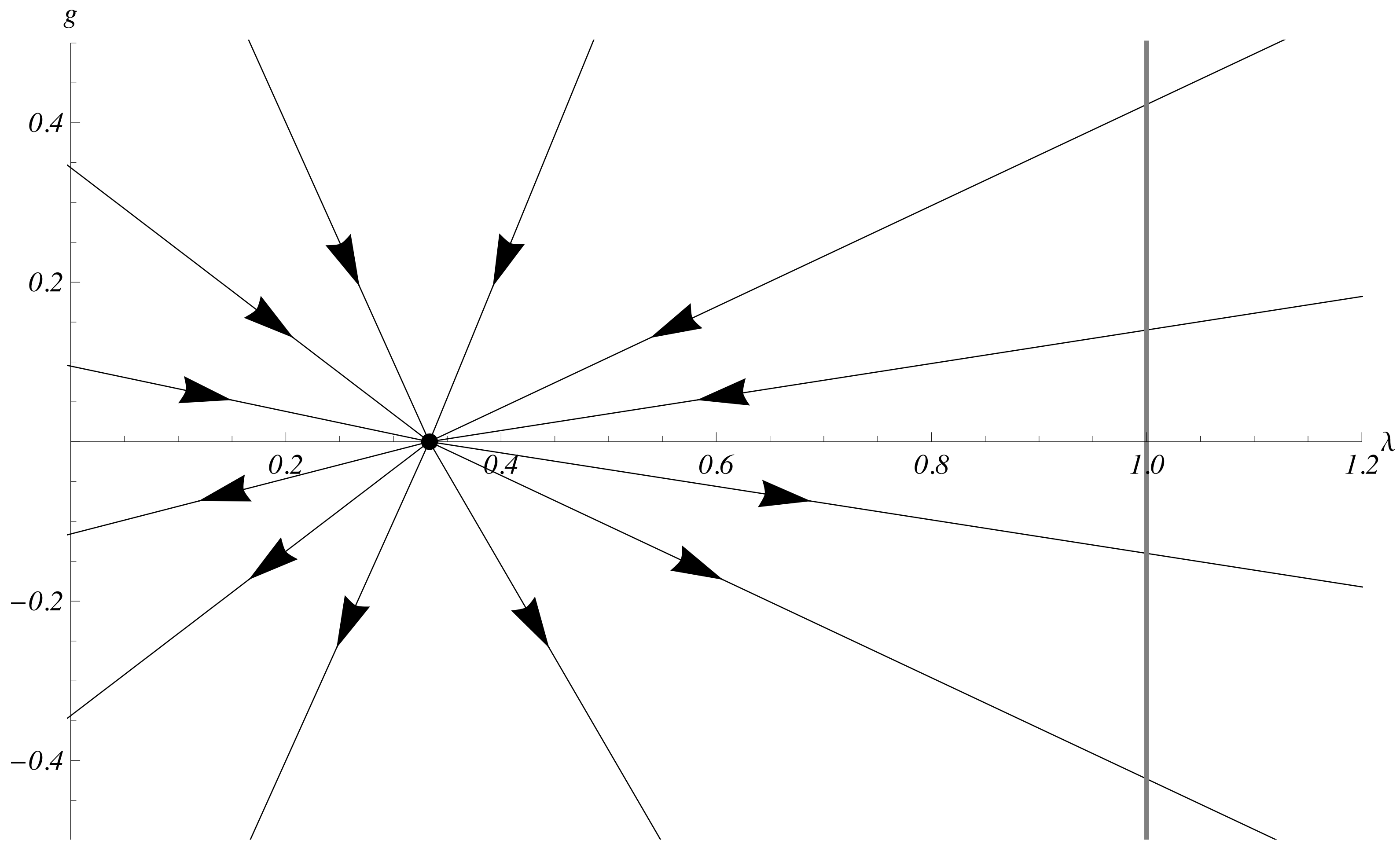
- z = d: anisotropic Gaussian fixed point (Horava-Lifshitz)

$$g_* = 0 \quad , \quad \lambda_* = \frac{1}{d}$$

Massless flows at criticality (d=z=3)



- Arrows point towards the infrared
- The matter-induced anisotropic Gaussian fixed point is an infrared attractor!
- Isotropic plane: no special properties

CONCLUSIONS

Conclusions

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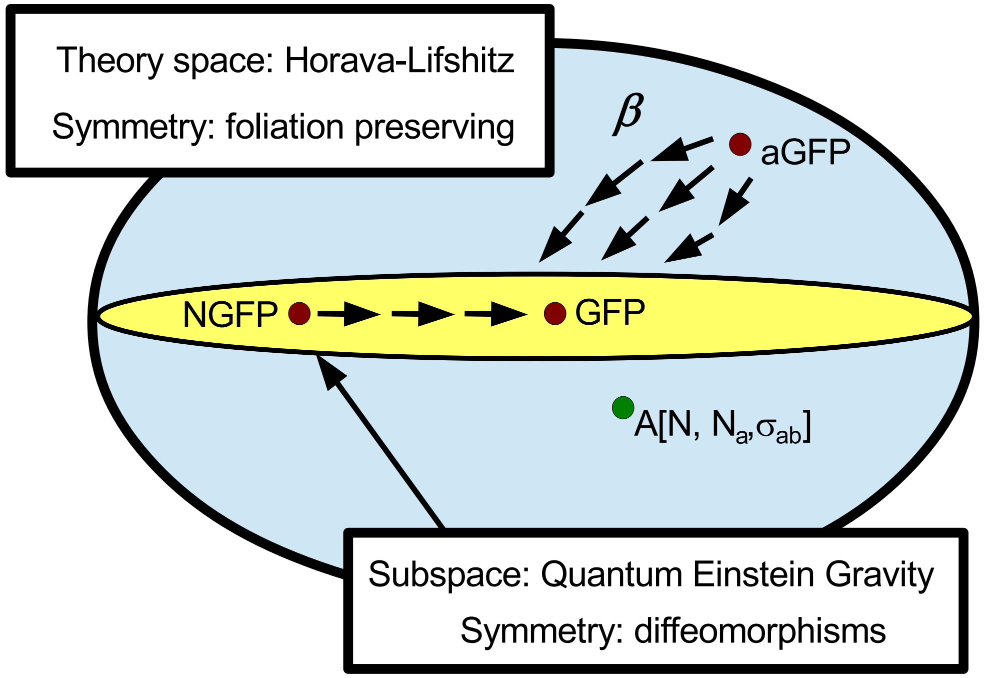
- Two different universality classes for quantum gravity



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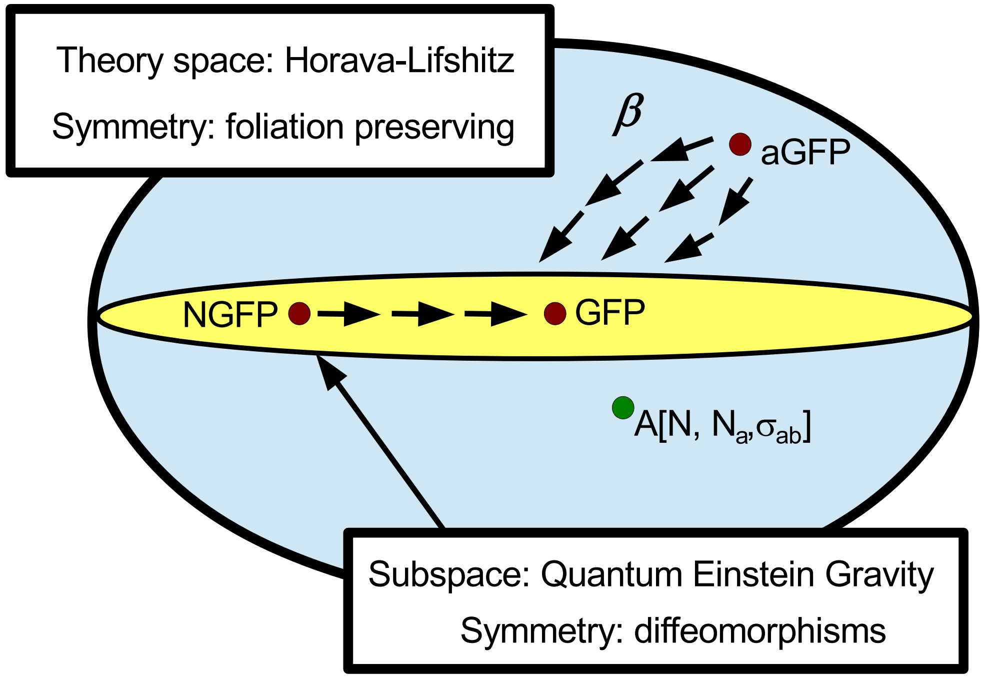

- Scalar induced beta function at criticality similar to QCD

$$\beta(G) = -\,(c - n_s)\,\frac{G^2}{5\pi}$$

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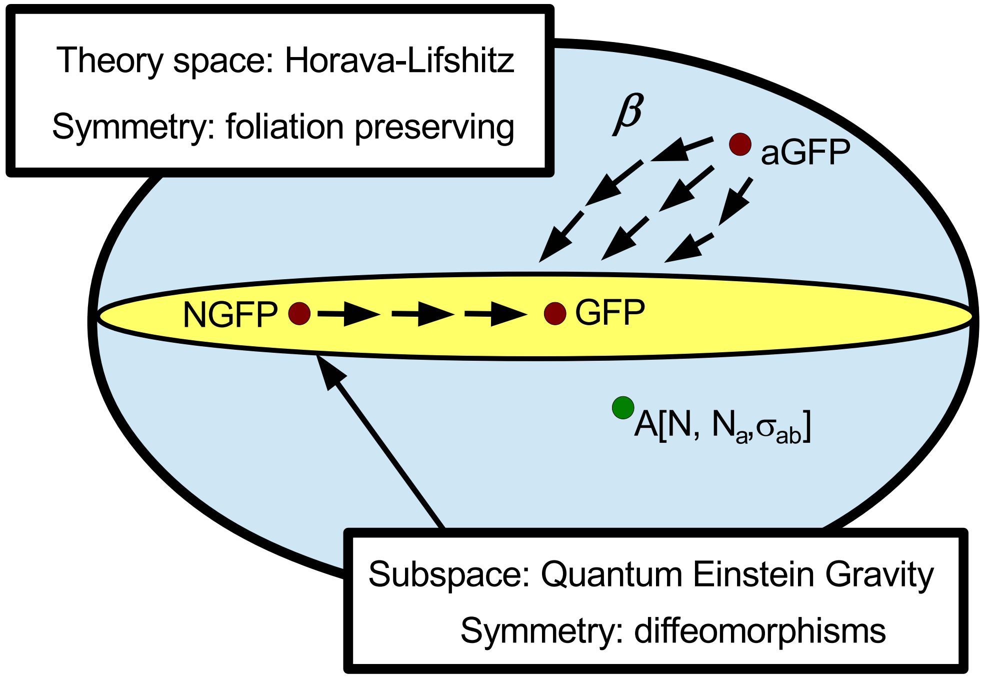

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- Asymptotic freedom requires $c > 0$

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THANK YOU