



Covariant EFT of Gravity and Dark Energy

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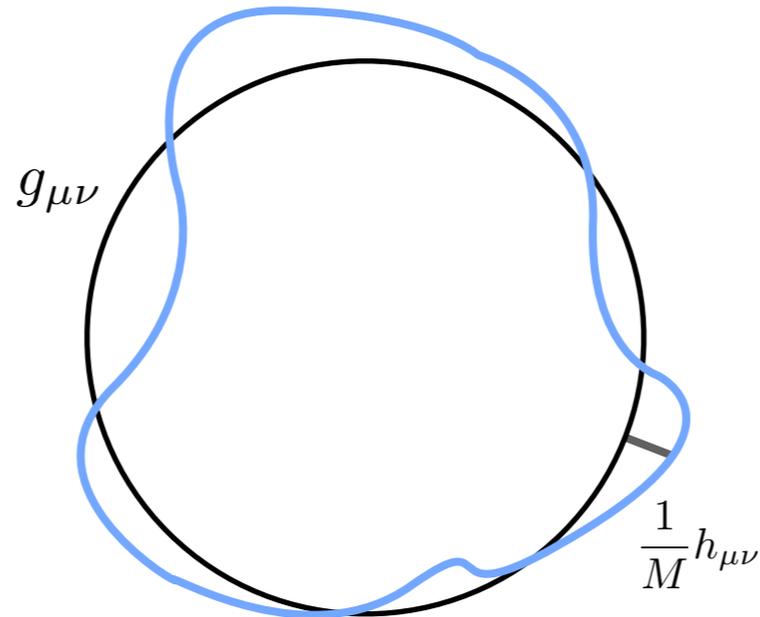
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Outline of the talk

- Covariant EFT of gravity
- LO quantum corrections
- Curvature expansion
- Cosmological effective action
- Effective Friedmann equations
- Early times: Inflation
- Late times: Dark energy

EFT of Gravity



- The theory of small fluctuations of the metric

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + \sqrt{16\pi G}h_{\mu\nu} = g_{\mu\nu} + \frac{1}{M}h_{\mu\nu}$$

- Planck's scale is the characteristic scale of gravity

$$M \equiv \frac{1}{\sqrt{16\pi G}} = \frac{M_{Planck}}{\sqrt{16\pi}}$$

$$M_{Planck} = \frac{1}{\sqrt{G}} = 1.2 \times 10^{19} \text{ GeV}$$

- Classical theory (CT) is successful over many orders of magnitude

EFT of Gravity

$$S_{eff}[g] = M^2 \left[I_1[g] + \frac{1}{M^2} I_2[g] + \frac{1}{M^4} I_3[g] + \dots \right]$$

UV action contains all couplings expressed in terms of the scale M

$$I_1[g] = \int d^4x \sqrt{g} [M^2 c_0 - c_1 R] \quad \bullet$$

$$I_2[g] = \int d^4x \sqrt{g} [c_{2,1} R^2 + c_{2,2} \text{Ric}^2 + c_{2,3} \text{Riem}^2] \quad \bullet$$

$$I_3[g] = \int d^4x \sqrt{g} [c_{3,1} R \square R + c_{3,2} R_{\mu\nu} \square R^{\mu\nu} + c_{3,3} R^3 + \dots] \quad \bullet$$

Derivative expansion of the UV action

Covariant EFT of Gravity

$$e^{-\Gamma[g]} = \int_{1PI} \mathcal{D}h_{\mu\nu} e^{-S_{eff}[g + \frac{1}{M}h]}$$

$$= \int_{1PI} \mathcal{D}h_{\mu\nu} e^{-M^2 \left\{ I_1[g + \frac{1}{M}h] + \frac{1}{M^2} I_2[g + \frac{1}{M}h] + \dots \right\}}$$

EFT: saddle point expansion in $\frac{1}{M^2}$

$$\Gamma[g] = I_1[g]$$

$$+ \frac{1}{M^2} \left\{ I_2[g] + \frac{1}{2} \text{Tr} \log I_1^{(2)}[g] \right\}$$

$$+ \frac{1}{M^4} \left\{ I_3[g] + \frac{1}{2} \text{Tr} \left[\left(I_1^{(2)}[g] \right)^{-1} I_2^{(2)}[g] \right] + 2\text{-loops with } I_1[g] \right\}$$

+ ...

Covariant EFT of Gravity

$$\begin{aligned}
 \Gamma &= && \bullet && && && && \text{CT} \\
 & && I_1 && && && && \\
 &+ \frac{1}{M^2} \left[&& \bullet &+ \frac{1}{2} \bigcirc & \right] && && && \text{LO} \\
 & && I_2 && && && && \\
 &+ \frac{1}{M^4} \left[&& \bullet &+ \frac{1}{2} \bigcirc &- \frac{1}{12} \bigcirc &+ \frac{1}{8} \bigcirc & \right] && && \text{NLO} \\
 & && I_3 && && && && \\
 &+ \dots && && && && && \text{NNLO}
 \end{aligned}$$

Covariant EFT of Gravity

LOQG: the only QG we will ever observe!

$$\begin{aligned}
 \Gamma &= \text{CT} \\
 &+ \frac{1}{M^2} \left[\text{purple dot} + \frac{1}{2} \text{circle} \right] \text{LO} \\
 &+ \frac{1}{M^4} \left[\text{pink dot} + \frac{1}{2} \text{circle with purple dot} - \frac{1}{12} \text{circle with horizontal line} + \frac{1}{8} \text{figure 8} \right] \text{NLO} \\
 &+ \dots \text{NNLO}
 \end{aligned}$$

Even if we have a fundamental theory its is generally difficult to compute phenomenological parameters...

Renormalization

Cutoff regularization

$$\bullet = \frac{1}{2} \frac{1}{(4\pi)^2} \int d^4x \sqrt{g} \left[\Lambda_{UV}^4 - 10\Lambda_{UV}^2 m^2 + 5m^4 \log \frac{\Lambda_{UV}^2}{m^2} \right. \\ \left. + \left(-\frac{23}{3} \Lambda_{UV}^2 + \frac{13}{3} m^2 \log \frac{\Lambda_{UV}^2}{m^2} \right) R \right. \\ \left. + \left(\frac{7}{20} C^2 + \frac{1}{4} R^2 + \frac{149}{180} E - \frac{19}{15} \square R \right) \log \frac{\Lambda_{UV}^2}{m^2} \right] + \text{measured phenomenological parameters}$$

Renormalization

Cutoff regularization

$$\bullet = \frac{1}{2} \frac{1}{(4\pi)^2} \int d^4x \sqrt{g} \left[\Lambda_{UV}^4 - 10\Lambda_{UV}^2 m^2 + 5m^4 \log \frac{\Lambda_{UV}^2}{m^2} \right. \\ \left. + \left(-\frac{23}{3}\Lambda_{UV}^2 + \frac{13}{3}m^2 \log \frac{\Lambda_{UV}^2}{m^2} \right) R \right. \\ \left. + \left(\frac{7}{20}C^2 + \frac{1}{4}R^2 + \frac{149}{180}E - \frac{19}{15}\square R \right) \log \frac{\Lambda_{UV}^2}{m^2} \right] \quad \begin{array}{l} \text{measured} \\ \text{+ phenomenological} \\ \text{parameters} \end{array}$$

Dimensional regularization

$$\log \Lambda_{UV}^2 \rightarrow \frac{1}{\epsilon} \quad \Lambda_{UV}^2 \rightarrow 0 \quad \Lambda_{UV}^4 \rightarrow 0$$

Renormalization

Cutoff regularization

$$m^2 = -2\Lambda$$

$$\bullet = \frac{1}{2} \frac{1}{(4\pi)^2} \int d^4x \sqrt{g} \left[\Lambda_{UV}^4 - 10\Lambda_{UV}^2 m^2 + 5m^4 \log \frac{\Lambda_{UV}^2}{m^2} \right. \\ \left. + \left(-\frac{23}{3}\Lambda_{UV}^2 + \frac{13}{3}m^2 \log \frac{\Lambda_{UV}^2}{m^2} \right) R \right. \\ \left. + \left(\frac{7}{20}C^2 + \frac{1}{4}R^2 + \frac{149}{180}E - \frac{19}{15}\square R \right) \log \frac{\Lambda_{UV}^2}{m^2} \right] \quad \begin{array}{l} \text{measured} \\ \text{+ phenomenological} \\ \text{parameters} \end{array}$$

Dimensional regularization

$$\log \Lambda_{UV}^2 \rightarrow \frac{1}{\epsilon} \quad \Lambda_{UV}^2 \rightarrow 0 \quad \Lambda_{UV}^4 \rightarrow 0$$

G runs if there is a mass scale involved also in dimensional regularization

Curvature expansion

•
$$+ \frac{1}{2} \bigcirc = -\frac{1}{2(4\pi)^{d/2}} \int d^d x \sqrt{g} \operatorname{tr} \mathcal{R} \gamma_i \left(\frac{-\square}{m^2} \right) \mathcal{R} + \dots$$

The finite physical part of the effective action is covariantly encoded in the structure functions which can be computed using the non-local heat kernel expansion

$$\gamma_i \left(\frac{X}{m^2} \right) \equiv \lim_{\Lambda_{UV} \rightarrow \infty} \int_{1/\Lambda_{UV}^2}^{\infty} \frac{ds}{s} s^{-d/2+2} [f_i(sX) - f_i(0)] e^{-sm^2}$$

Non-local heat kernel

A. O. Barvinsky and G. A. Vilkovisky, Nucl. Phys. B 282 (1987) 163

I. G. Avramidi, Lect. Notes Phys. M 64 (2000) 1

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Non-local heat kernel structure functions

Curvature expansion

$$\bullet + \frac{1}{2} \bigcirc = -\frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} \operatorname{tr} \left[\mathbf{1} R_{\mu\nu} \gamma_{Ric} \left(\frac{-\square}{m^2} \right) R^{\mu\nu} + \frac{1}{120} R \gamma_R \left(\frac{-\square}{m^2} \right) R \right. \\
 \left. - \frac{1}{6} R \gamma_{RU} \left(\frac{-\square}{m^2} \right) \mathbf{U} + \frac{1}{2} \mathbf{U} \gamma_U \left(\frac{-\square}{m^2} \right) \mathbf{U} + \frac{1}{12} \Omega_{\mu\nu} \gamma_\Omega \left(\frac{-\square}{m^2} \right) \Omega^{\mu\nu} \right]$$

Explicit form for the structure functions

$$\gamma_{Ric}(u) = \frac{1}{40} + \frac{1}{12u} - \frac{1}{2} \int_0^1 d\xi \left[\frac{1}{u} + \xi(1-\xi) \right]^2 \log [1 + u \xi(1-\xi)]$$

$$\gamma_R(u) = -\frac{23}{960} - \frac{1}{96u} + \frac{1}{32} \int_0^1 d\xi \left\{ \frac{2}{u^2} + \frac{4}{u} [1 + \xi(1-\xi)] \right. \\
 \left. - 1 + 2\xi(2-\xi)(1-\xi^2) \right\} \log [1 + u \xi(1-\xi)]$$

$$\gamma_{RU}(u) = \frac{1}{12} - \frac{1}{2} \int_0^1 d\xi \left[\frac{1}{u} - \frac{1}{2} + \xi(1-\xi) \right] \log [1 + u \xi(1-\xi)]$$

$$\gamma_U(u) = -\frac{1}{2} \int_0^1 d\xi \log [1 + u \xi(1-\xi)]$$

$$\gamma_\Omega(u) = \frac{1}{12} - \frac{1}{2} \int_0^1 d\xi \left[\frac{1}{u} + \xi(1-\xi) \right] \log [1 + u \xi(1-\xi)]$$

$$u \equiv \frac{-\square}{m^2}$$

Curvature expansion

$$\bullet + \frac{1}{2} \bigcirc = -\frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} \operatorname{tr} \left[\mathbf{1} R_{\mu\nu} \gamma_{Ric} \left(\frac{-\square}{m^2} \right) R^{\mu\nu} + \frac{1}{120} R \gamma_R \left(\frac{-\square}{m^2} \right) R \right. \\
 \left. - \frac{1}{6} R \gamma_{RU} \left(\frac{-\square}{m^2} \right) \mathbf{U} + \frac{1}{2} \mathbf{U} \gamma_U \left(\frac{-\square}{m^2} \right) \mathbf{U} + \frac{1}{12} \Omega_{\mu\nu} \gamma_{\Omega} \left(\frac{-\square}{m^2} \right) \Omega^{\mu\nu} \right]$$

High energy expansion $u \gg 1$

$$\gamma_{Ric}(u) = -\frac{u}{840} + \frac{u^2}{15120} - \frac{u^3}{166320} + O(u^4)$$

$$\gamma_R(u) = -\frac{u}{336} + \frac{11u^2}{30240} - \frac{19u^3}{332640} + O(u^4)$$

$$\gamma_{RU}(u) = \frac{u}{30} - \frac{u^2}{280} + \frac{u^3}{1890} + O(u^4)$$

$$\gamma_U(u) = -\frac{u}{12} + \frac{u^2}{120} - \frac{u^3}{840} + O(u^4)$$

$$\gamma_{\Omega}(u) = -\frac{u}{120} + \frac{u^2}{1680} - \frac{u^3}{15120} + O(u^4)$$

$$u \equiv \frac{-\square}{m^2}$$

Curvature expansion

$$\bullet \quad + \frac{1}{2} \bigcirc = -\frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} \operatorname{tr} \left[\mathbf{1} R_{\mu\nu} \gamma_{Ric} \left(\frac{-\square}{m^2} \right) R^{\mu\nu} + \frac{1}{120} R \gamma_R \left(\frac{-\square}{m^2} \right) R \right. \\
 \left. - \frac{1}{6} R \gamma_{RU} \left(\frac{-\square}{m^2} \right) \mathbf{U} + \frac{1}{2} \mathbf{U} \gamma_U \left(\frac{-\square}{m^2} \right) \mathbf{U} + \frac{1}{12} \Omega_{\mu\nu} \gamma_\Omega \left(\frac{-\square}{m^2} \right) \Omega^{\mu\nu} \right]$$

Low energy expansion $u \ll 1$

$$\gamma_{Ric}(u) = \frac{23}{450} - \frac{1}{60} \log u + \frac{5}{18u} - \frac{\log u}{6u} + \frac{1}{4u^2} - \frac{\log u}{2u^2} + O\left(\frac{1}{u^3}\right)$$

$$\gamma_R(u) = \frac{1}{1800} - \frac{1}{120} \log u - \frac{2}{9u} + \frac{\log u}{12u} + \frac{1}{8u^2} + \frac{\log u}{4u^2} + O\left(\frac{1}{u^3}\right)$$

$$\gamma_{RU}(u) = -\frac{5}{18} + \frac{1}{6} \log u + \frac{1}{u} - \frac{1}{2u^2} - \frac{\log u}{u^2} + O\left(\frac{1}{u^3}\right)$$

$$\gamma_U(u) = 1 - \frac{1}{2} \log u - \frac{1}{u} - \frac{\log u}{u} - \frac{1}{2u^2} + \frac{\log u}{u^2} + O\left(\frac{1}{u^3}\right)$$

$$\gamma_\Omega(u) = \frac{2}{9} - \frac{1}{12} \log u + \frac{1}{2u} - \frac{\log u}{2u} - \frac{3}{4u^2} - \frac{\log u}{2u^2} + O\left(\frac{1}{u^3}\right)$$

$$u \equiv \frac{-\square}{m^2}$$

LO effective action to \mathcal{R}^2

$$\Gamma[g] = \frac{1}{16\pi G} \int d^4x \sqrt{g} (2\Lambda - R) + \frac{1}{2\lambda} \int d^4x \sqrt{g} C^2 + \frac{1}{\xi} \int d^4x \sqrt{g} R^2$$

$$+ \int d^4x \sqrt{g} C_{\mu\nu\alpha\beta} \mathcal{G}\left(\frac{-\square}{m^2}\right) C^{\mu\nu\alpha\beta} + \int d^4x \sqrt{g} R \mathcal{F}\left(\frac{-\square}{m^2}\right) R + O(\mathcal{R}^3)$$

Graviton contributions:

$$\mathcal{G}_2(u) = -\frac{1}{2(4\pi)^2} \left(5\gamma_{Ric}(u) + 3\gamma_U(u) - 12\gamma_\Omega(u) \right. \\ \left. - 4\gamma_{Ric}(u) - \gamma_U(u) + 4\gamma_\Omega(u) \right)$$

$$\mathcal{F}_2(u) = -\frac{1}{2(4\pi)^2} \left(\frac{10}{3}\gamma_{Ric}(u) + 10\gamma_R(u) + 6\gamma_{RU}(u) + 4\gamma_U(u) - 2\gamma_\Omega(u) \right. \\ \left. - \frac{8}{3}\gamma_{Ric}(u) - 8\gamma_R(u) + 2\gamma_{RU}(u) - \frac{2}{3}\gamma_U(u) + \frac{2}{3}\gamma_\Omega(u) \right)$$

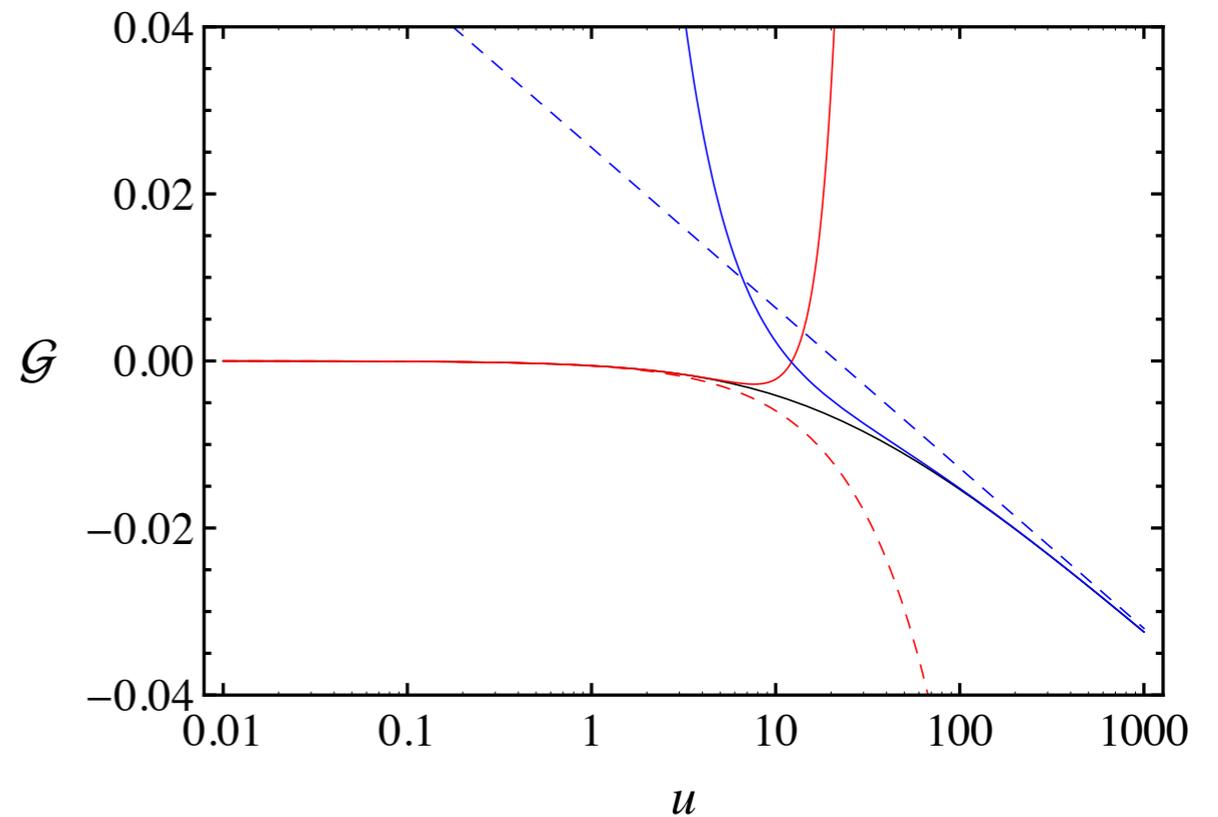
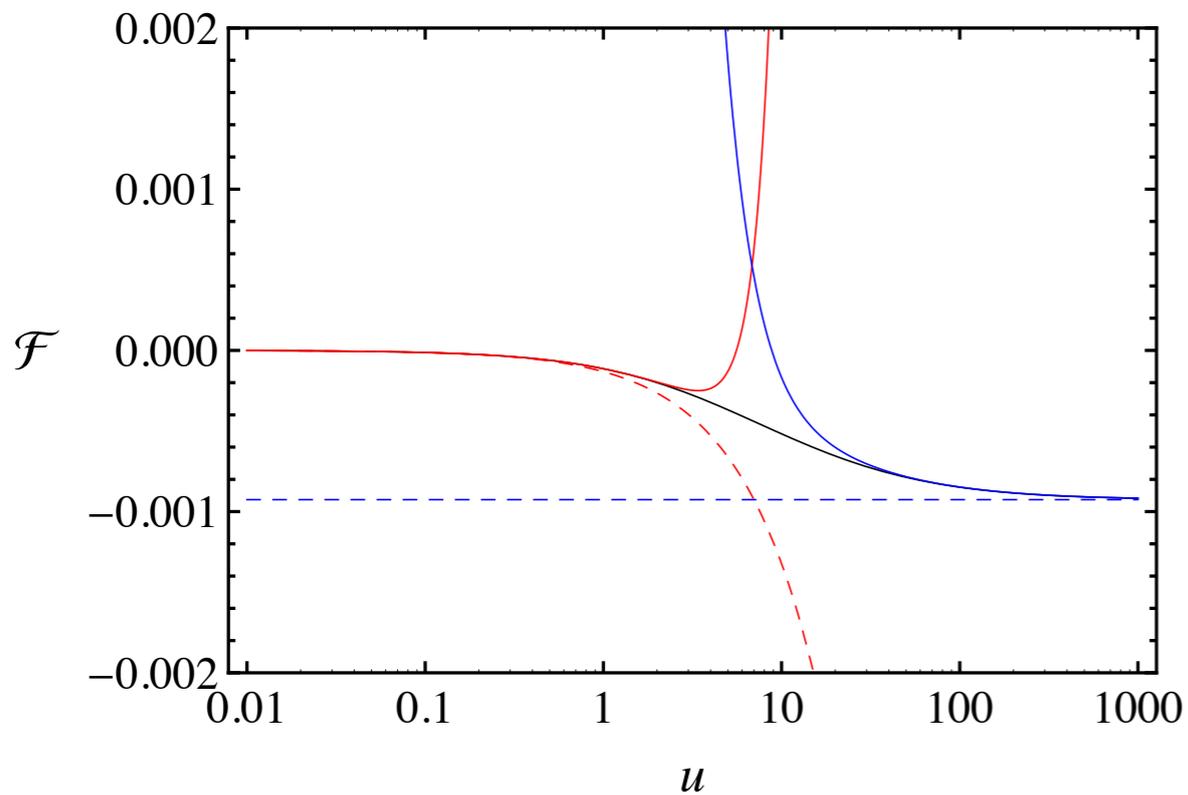
Matter contributions:

$$\mathcal{G}_0(u), \mathcal{F}_0(u), \mathcal{G}_{1/2}(u), \mathcal{F}_{1/2}(u), \mathcal{G}_1(u), \mathcal{F}_1(u)$$

LO effective action to \mathcal{R}^2

$$\Gamma[g] = \frac{1}{16\pi G} \int d^4x \sqrt{g} (2\Lambda - R) + \frac{1}{2\lambda} \int d^4x \sqrt{g} C^2 + \frac{1}{\xi} \int d^4x \sqrt{g} R^2$$

$$+ \int d^4x \sqrt{g} C_{\mu\nu\alpha\beta} \mathcal{G}\left(\frac{-\square}{m^2}\right) C^{\mu\nu\alpha\beta} + \int d^4x \sqrt{g} R \mathcal{F}\left(\frac{-\square}{m^2}\right) R + O(\mathcal{R}^3)$$



$$u \equiv \frac{-\square}{m^2}$$

Adding matter

$$\Gamma = \begin{array}{l} \bullet \\ + \frac{1}{M^2} \left[\begin{array}{l} \bullet + \bullet + \frac{1}{2} \bigcirc + \frac{1}{2} \bigcirc \end{array} \right] \\ + \dots \end{array} \quad \begin{array}{l} \text{CT} \\ \text{LO} \\ \text{NLO} \end{array}$$

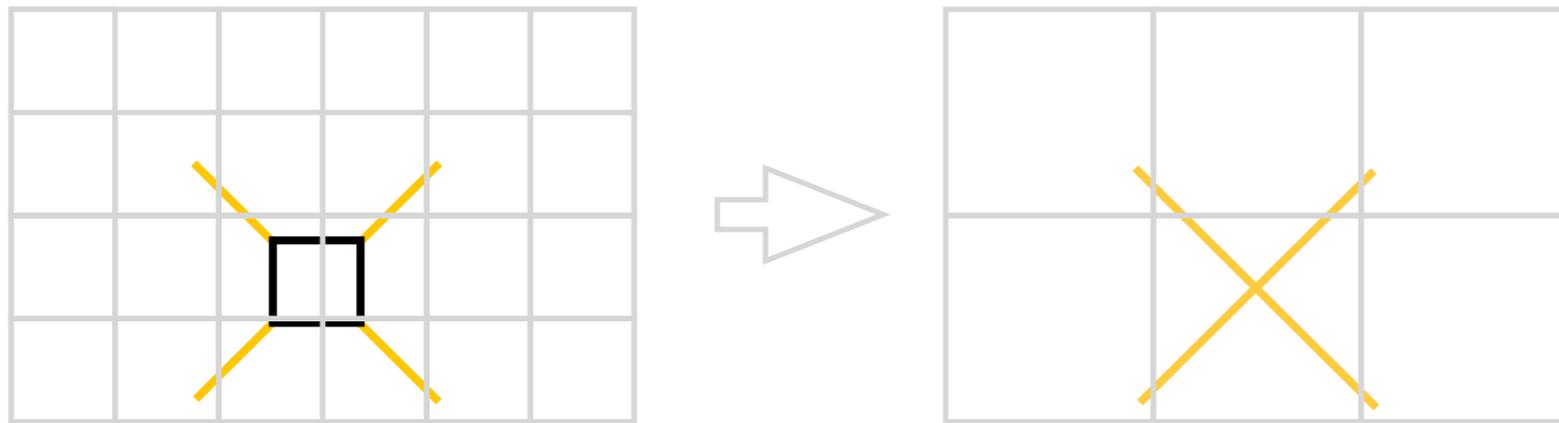
Integrating out matter

$$\frac{1}{2} \bigcirc = -\frac{1}{2} \frac{1}{(4\pi)^2} \sum_{n=\frac{d}{2}+1} \frac{1}{m^{2n-d}} B_{2n}$$

Matter induced effective action:
expandable in inverse powers of the (lightest) particle mass

Integrating out matter

$$\frac{1}{2} \bigcirc = -\frac{1}{2} \frac{1}{(4\pi)^2} \frac{1}{m^2} B_6 + \dots$$



As in QED when we integrate out electrons

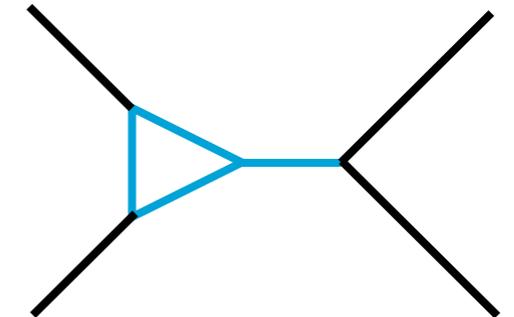
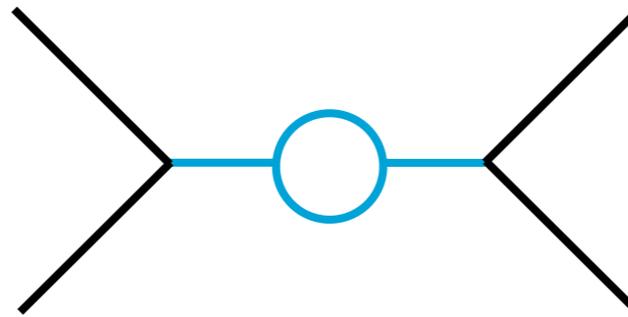
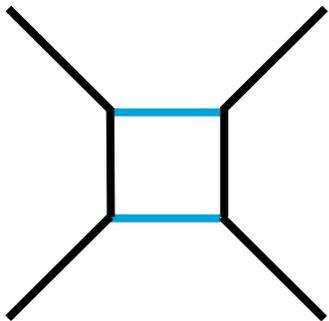
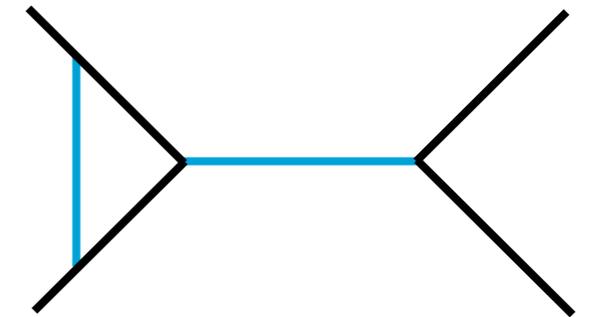
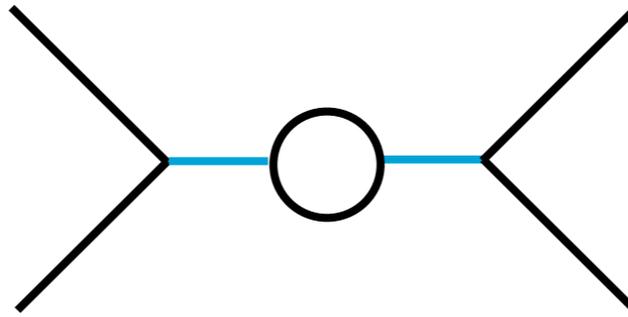
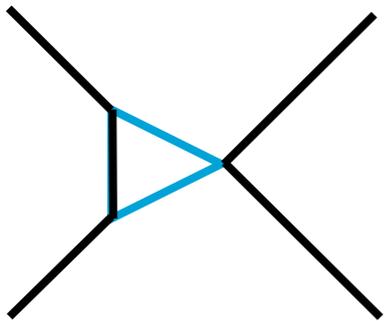
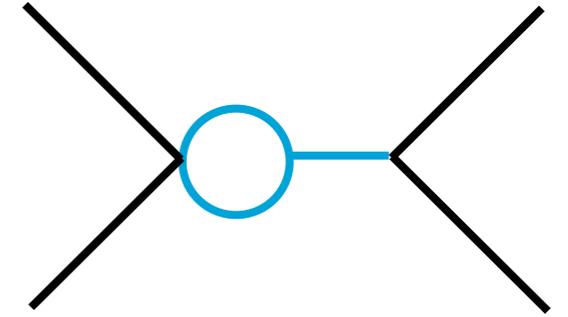
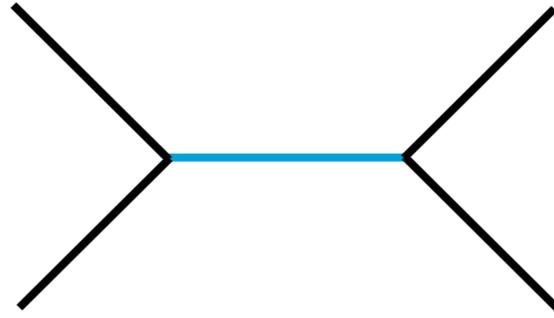
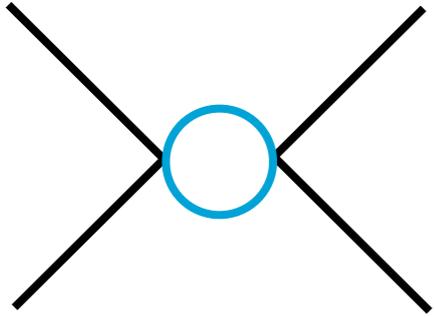
Integrating out matter

$$\begin{aligned}
 \frac{1}{2} \bigcirc &= -\frac{1}{2} \frac{1}{(4\pi)^2} \frac{1}{m^2} \int d^4x \sqrt{g} \left[\frac{1}{336} R \square R + \frac{1}{840} R_{\mu\nu} \square R^{\mu\nu} \right. \\
 &+ \frac{1}{1296} R^3 - \frac{1}{1080} R R_{\mu\nu} R^{\mu\nu} - \frac{4}{2835} R_{\mu}^{\nu} R_{\nu}^{\alpha} R_{\alpha}^{\mu} \\
 &+ \frac{1}{945} R_{\mu\nu} R_{\alpha\beta} R^{\mu\alpha\nu\beta} + \frac{1}{1080} R R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} + \frac{1}{7560} R_{\mu\nu} R^{\mu\alpha\beta\gamma} R^{\nu}_{\alpha\beta\gamma} \\
 &\left. + \frac{17}{45360} R_{\mu\nu}^{\alpha\beta} R_{\alpha\beta}^{\gamma\delta} R_{\gamma\delta}^{\mu\nu} - \frac{1}{1620} R_{\mu}^{\alpha\beta} R_{\nu}^{\mu\nu} R_{\gamma\delta}^{\gamma\delta} R_{\alpha\beta}^{\alpha\beta} \right] + \dots
 \end{aligned}$$

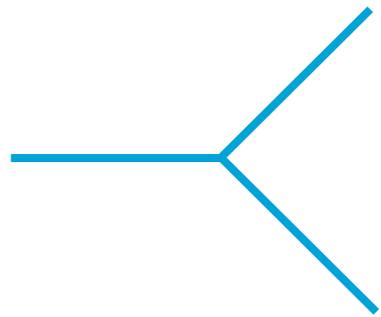
Changes how cubic terms are suppressed

$$\frac{1}{M^4} \rightarrow \frac{1}{m^2 M^2}$$

Corrections to Newton's potential



Corrections to Newton's interaction

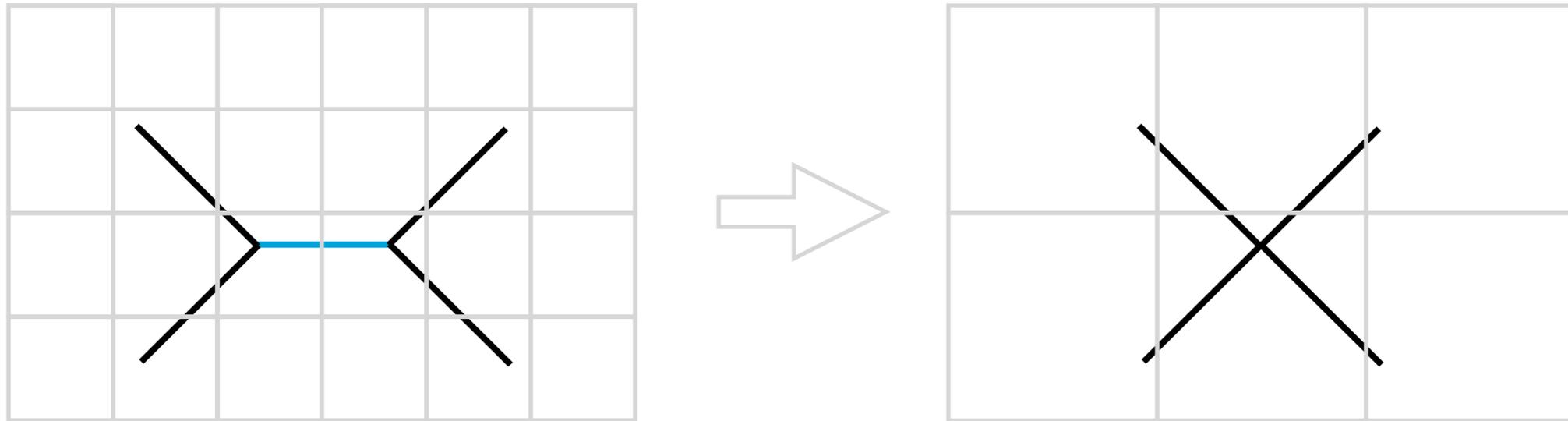


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$$\begin{aligned}
 & \frac{i\kappa}{2} \left(P_{\alpha\beta,\gamma\delta} \left[k^\mu k^\nu + (k-q)^\mu (k-q)^\nu + q^\mu q^\nu - \frac{3}{2} \eta^{\mu\nu} q^2 \right] \right. \\
 & + 2q_\lambda q_\sigma \left[I^{\lambda\sigma, \alpha\beta} I^{\mu\nu, \gamma\delta} + I^{\lambda\sigma, \gamma\delta} I^{\mu\nu, \alpha\beta} - I^{\lambda\mu, \alpha\beta} I^{\sigma\nu, \gamma\delta} - I^{\sigma\nu, \alpha\beta} I^{\lambda\mu, \gamma\delta} \right] \\
 & + \left[q_\lambda q^\mu \left(\eta_{\alpha\beta} I^{\lambda\nu, \gamma\delta} + \eta_{\gamma\delta} I^{\lambda\nu, \alpha\beta} \right) + q_\lambda q^\nu \left(\eta_{\alpha\beta} I^{\lambda\mu, \gamma\delta} + \eta_{\gamma\delta} I^{\lambda\mu, \alpha\beta} \right) \right. \\
 & \left. - q^2 \left(\eta_{\alpha\beta} I^{\mu\nu, \gamma\delta} + \eta_{\gamma\delta} I^{\mu\nu, \alpha\beta} \right) - \eta^{\mu\nu} q^\lambda q^\sigma \left(\eta_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} + \eta_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} \right) \right] \\
 & + \left[2q^\lambda \left(I^{\sigma\nu, \alpha\beta} I_{\gamma\delta,\lambda\sigma} (k-q)^\mu + I^{\sigma\mu, \alpha\beta} I_{\gamma\delta,\lambda\sigma} (k-q)^\nu \right. \right. \\
 & \quad \left. \left. - I^{\sigma\nu, \gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\mu - I^{\sigma\mu, \gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\nu \right) \right. \\
 & \left. + q^2 \left(I^{\sigma\mu, \alpha\beta} I_{\gamma\delta,\sigma}{}^\nu + I_{\alpha\beta,\sigma}{}^\nu I^{\sigma\mu, \alpha\delta} \right) + \eta^{\mu\nu} q^\lambda q_\sigma \left(I_{\alpha\beta,\lambda\rho} I^{\rho\sigma, \gamma\delta} + I_{\gamma\delta,\lambda\rho} I^{\rho\sigma, \alpha\beta} \right) \right] \\
 & + \left\{ (k^2 + (k-q)^2) \left(I^{\sigma\mu, \alpha\beta} I_{\gamma\delta,\sigma}{}^\nu + I^{\sigma\nu, \alpha\beta} I_{\gamma\delta,\sigma}{}^\mu - \frac{1}{2} \eta^{\mu\nu} P_{\alpha\beta,\gamma\delta} \right) \right. \\
 & \quad \left. - \left(k^2 \eta_{\gamma\delta} I^{\mu\nu, \alpha\beta} + (k-q)^2 \eta_{\alpha\beta} I^{\mu\nu, \gamma\delta} \right) \right\}
 \end{aligned}$$

the truth behind Feynman diagrams...

Corrections to Newton's interaction

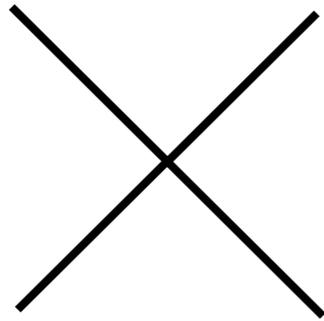


$$V = -\frac{GMm}{r} \left[1 + 3 \frac{G(M+m)}{c^2 r} + \frac{41}{10\pi} \frac{G\hbar}{c^3 r^2} + \dots \right]$$

Leading quantum corrections to Newton's potential

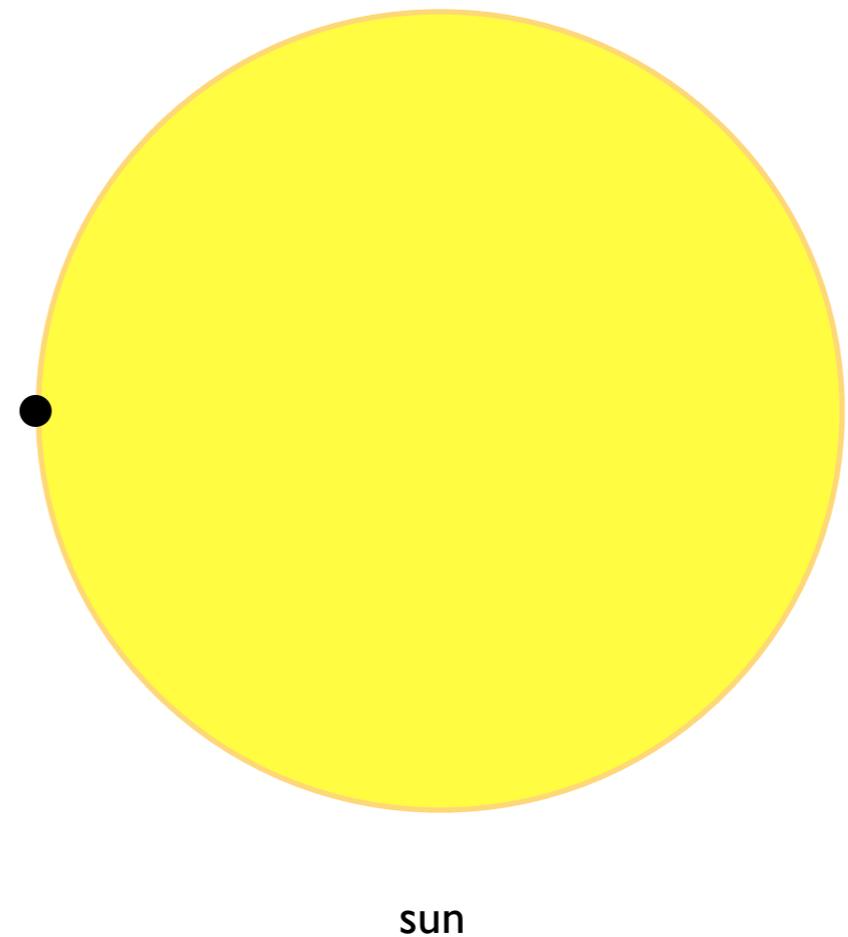
J.F. Donoghue, Phys. Rev. Lett. 72, 2996 (1994)

Corrections to Newton's interaction



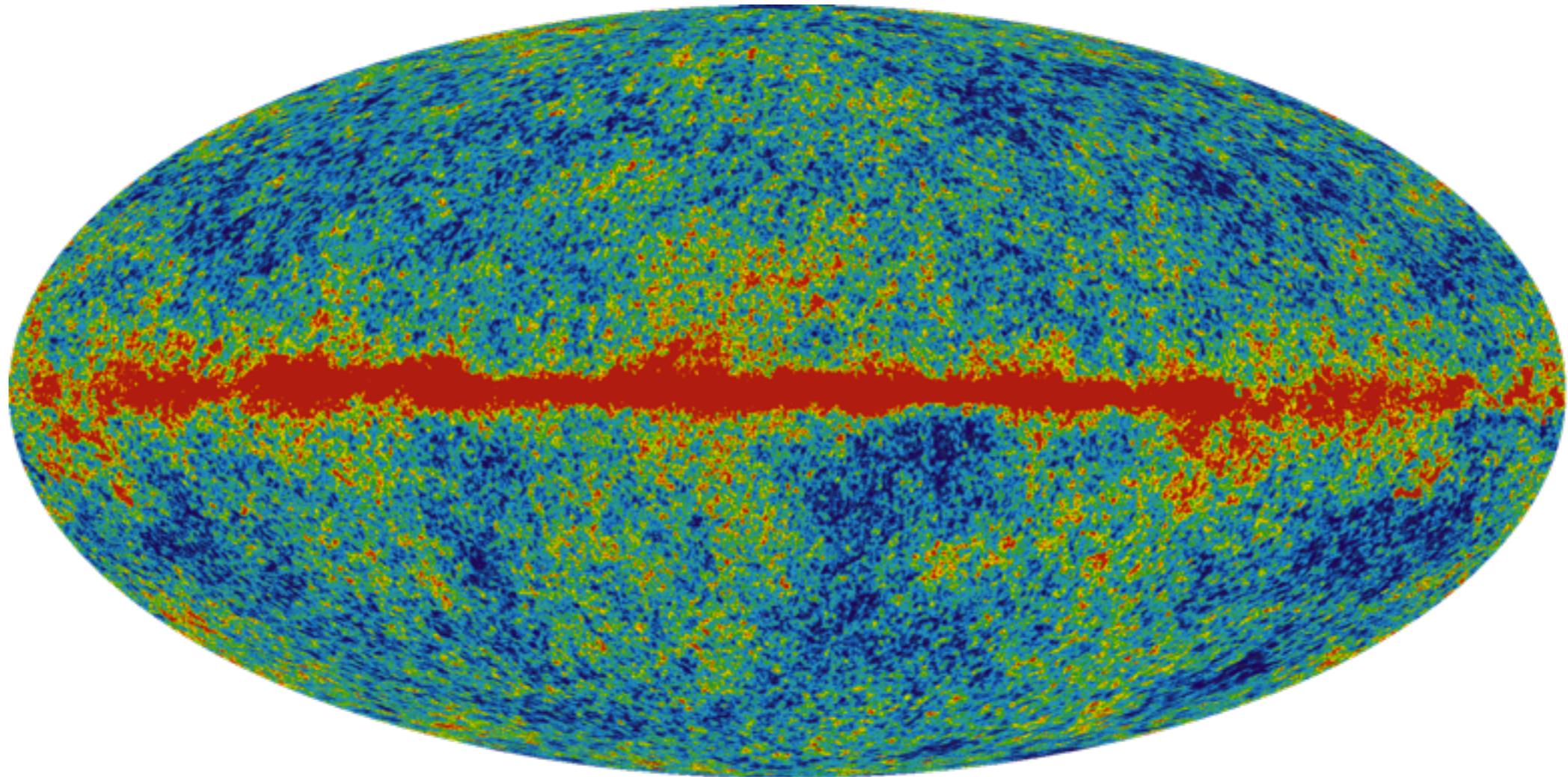
$$\frac{GM_{\odot}}{c^2 r_{\odot}} \sim 10^{-6}$$

$$\frac{G\hbar}{c^3 r_{\odot}^2} \sim 10^{-88}$$



Leading quantum corrections to Newton's law are incredibly small!

Can we ever observe
quantum gravity effects?



Look for physical situations where
LO corrections are enhanced

Cosmological effective action

FRW metric

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

Weyl tensor vanishes

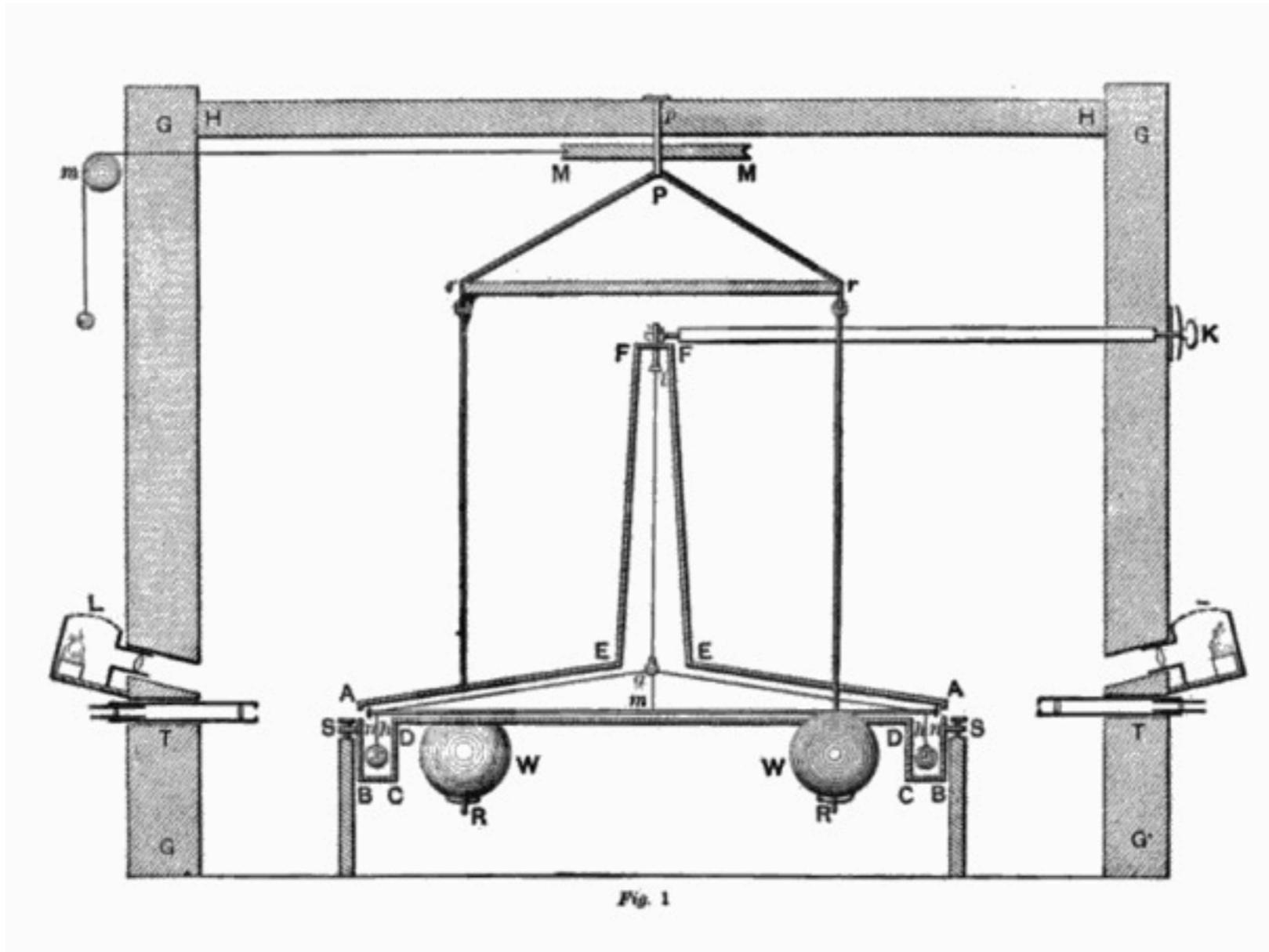
$$C_{\alpha\beta\gamma\delta} = 0$$

Euclidean to Lorentzian

$$\Gamma[g] = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\Lambda) + \xi \int d^4x \sqrt{-g} R^2 + \int d^4x \sqrt{-g} RF(\square)R$$

A predictive framework for cosmology

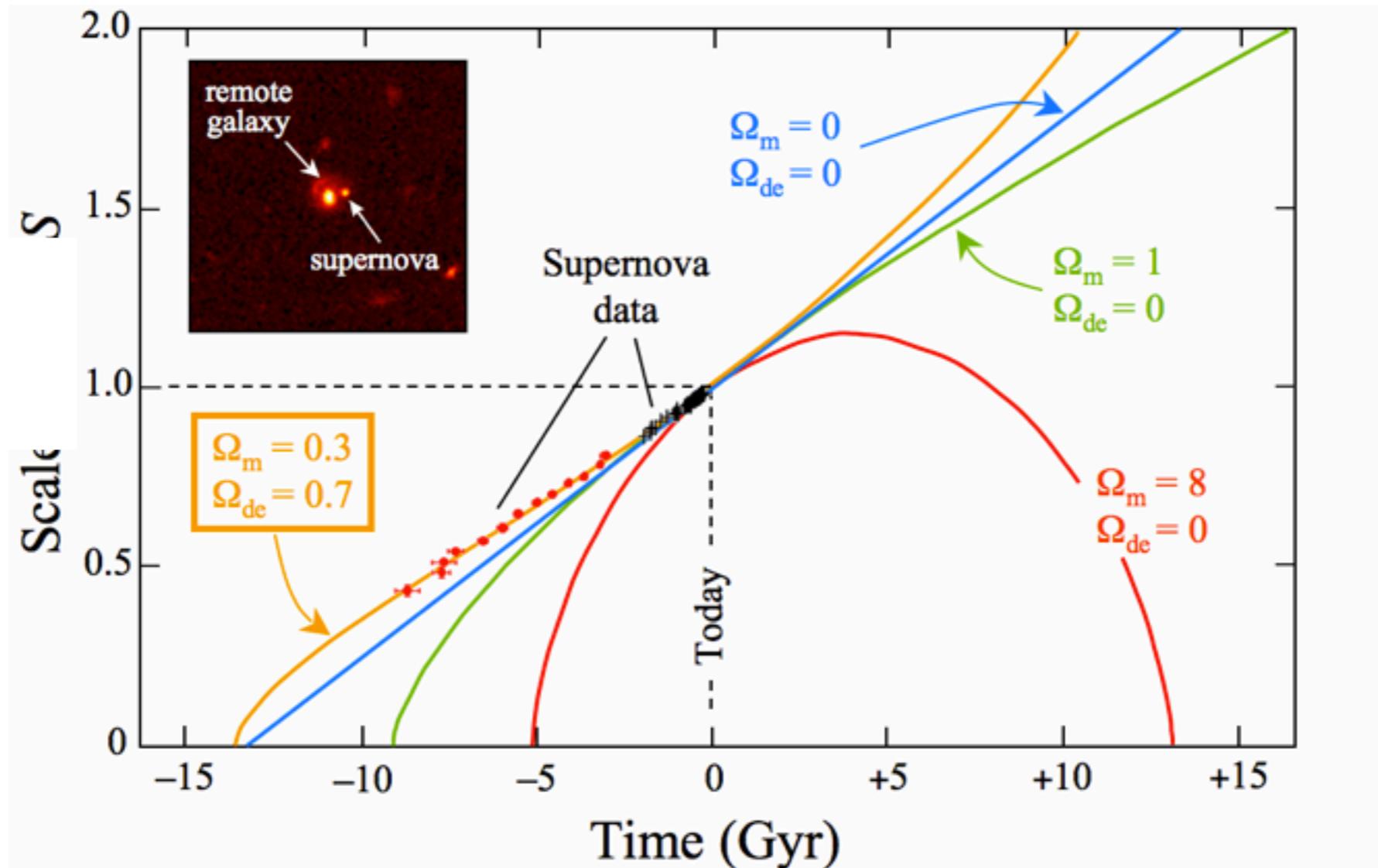
Phenomenological parameters



$$G = 6.67428 \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$$

Cavendish 1797 (1% off best value!)

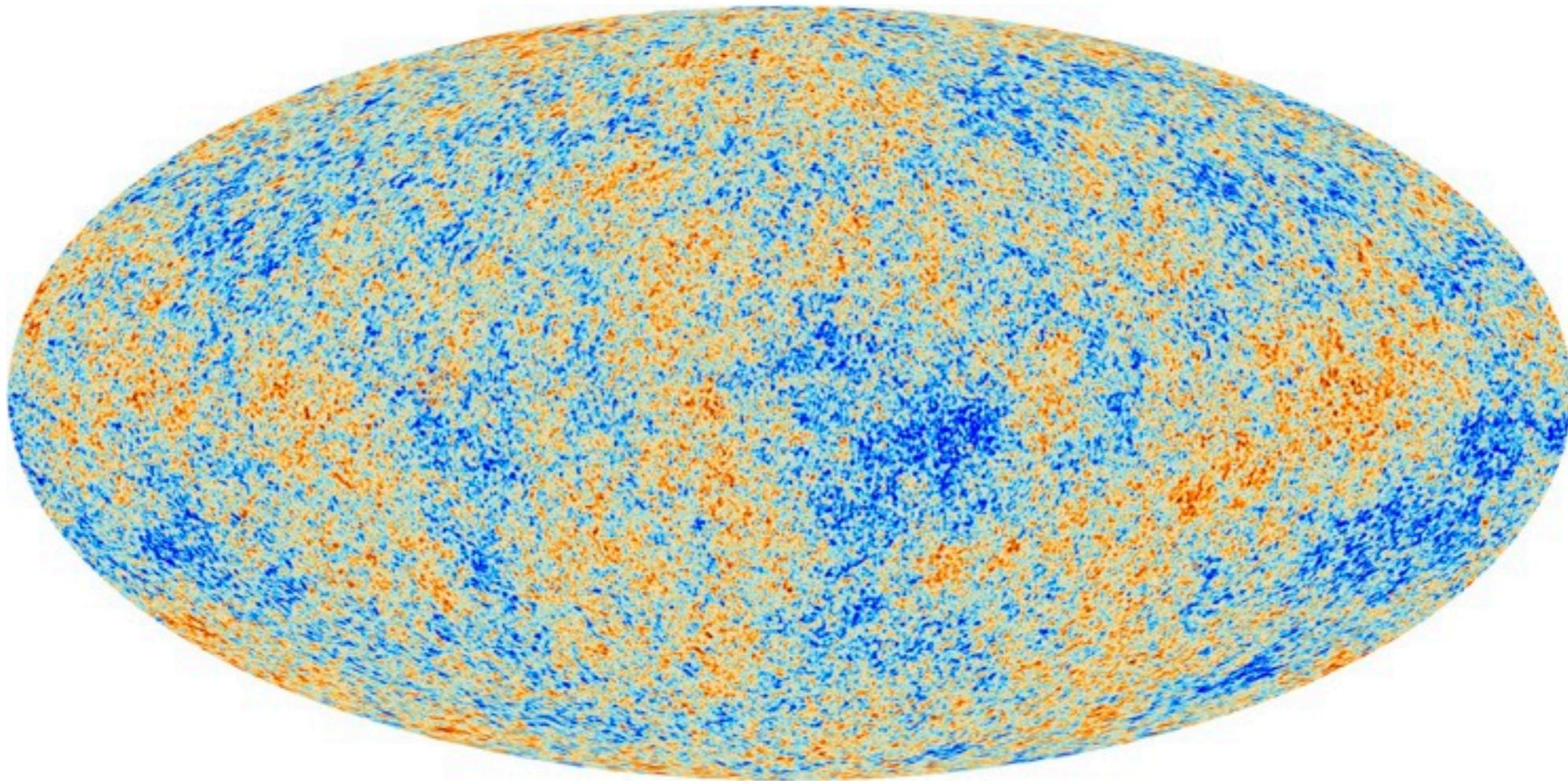
Phenomenological parameters



$$\Lambda = 10^{-47} \text{ GeV}^4$$

Supernova Cosmology Project

Phenomenological parameters



$$\xi \equiv c_{R^2}$$

$$\xi(k_*) \sim 10^9$$

$$k_* \equiv 0.05 \text{ Mpc}^{-1} \sim 10^{-40} \text{ GeV}$$

Planck mission

$$C_{\alpha\beta\gamma\delta} = 0$$

Cosmological effective action

$$\Gamma[g] = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\Lambda) + \xi \int d^4x \sqrt{-g} R^2 + \int d^4x \sqrt{-g} R F(\square) R$$

$$C_{\alpha\beta\gamma\delta} = 0$$

Cosmological effective action

$$\Gamma[g] = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\Lambda) + \xi \int d^4x \sqrt{-g} R^2 + \int d^4x \sqrt{-g} R F(\square) R$$

$$F(\square) = \alpha \log \frac{-\square}{m^2}$$

$$+ \beta \frac{m^2}{-\square}$$

$\alpha, \beta, \gamma, \delta$

$$+ \gamma \frac{m^2}{-\square} \log \frac{-\square}{m^2}$$

$$+ \delta \frac{m^4}{(-\square)^2}$$

+ ...

are calculable
constants
depending on
effective gravitons
and matter content

$$C_{\alpha\beta\gamma\delta} = 0$$

Cosmological effective action

$$\Gamma[g] = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\Lambda) + \xi \int d^4x \sqrt{-g} R^2 + \int d^4x \sqrt{-g} R F(\square) R$$

$$F(\square) = \alpha \log \frac{-\square}{m^2}$$

Leading logs

J. F. Donoghue and B. K. El-Menoufi, Phys. Rev. D 89, 104062 (2014)

$$+ \beta \frac{m^2}{-\square}$$

Non-local cosmology

S. Deser and R. P. Woodard, Phys. Rev. Lett. 99, 111301 (2007)

$\alpha, \beta, \gamma, \delta$

$$+ \gamma \frac{m^2}{-\square} \log \frac{-\square}{m^2}$$

are calculable
constants
depending on
effective gravitons
and matter content

$$+ \delta \frac{m^4}{(-\square)^2}$$

Non-local gravity and dark energy

M. Maggiore and M. Mancarella, Phys. Rev. D 90, 023005 (2014).

+ ...

Effective non-local cosmology

A. C. and K. J. Jain

Effective Friedmann equations (local)

Modified Einstein's equations

$$G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Local correction (covariant form)

$$-\frac{\xi}{16\pi G} \Delta G_{\mu\nu} R^2 = 2 \left(R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R \right) R - 2 (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) R$$

Local correction (FRW form)

$$H^2 + 96\pi\xi G \left[2H\ddot{H} + 6H^2\dot{H} - \dot{H}^2 \right] = \frac{1}{3}\Lambda + \frac{8\pi G}{3}\rho$$

Effective Friedmann equations (local)

Jordan frame Starobinsky gravity

$$R \quad R^2 \quad \Lambda \quad \rho$$
$$H^2 + 96\pi\xi G \left[2H\ddot{H} + 6H^2\dot{H} - \dot{H}^2 \right] = \frac{1}{3}\Lambda + \frac{8\pi G}{3}\rho$$

Pure Starobinsky gravity is exactly solvable

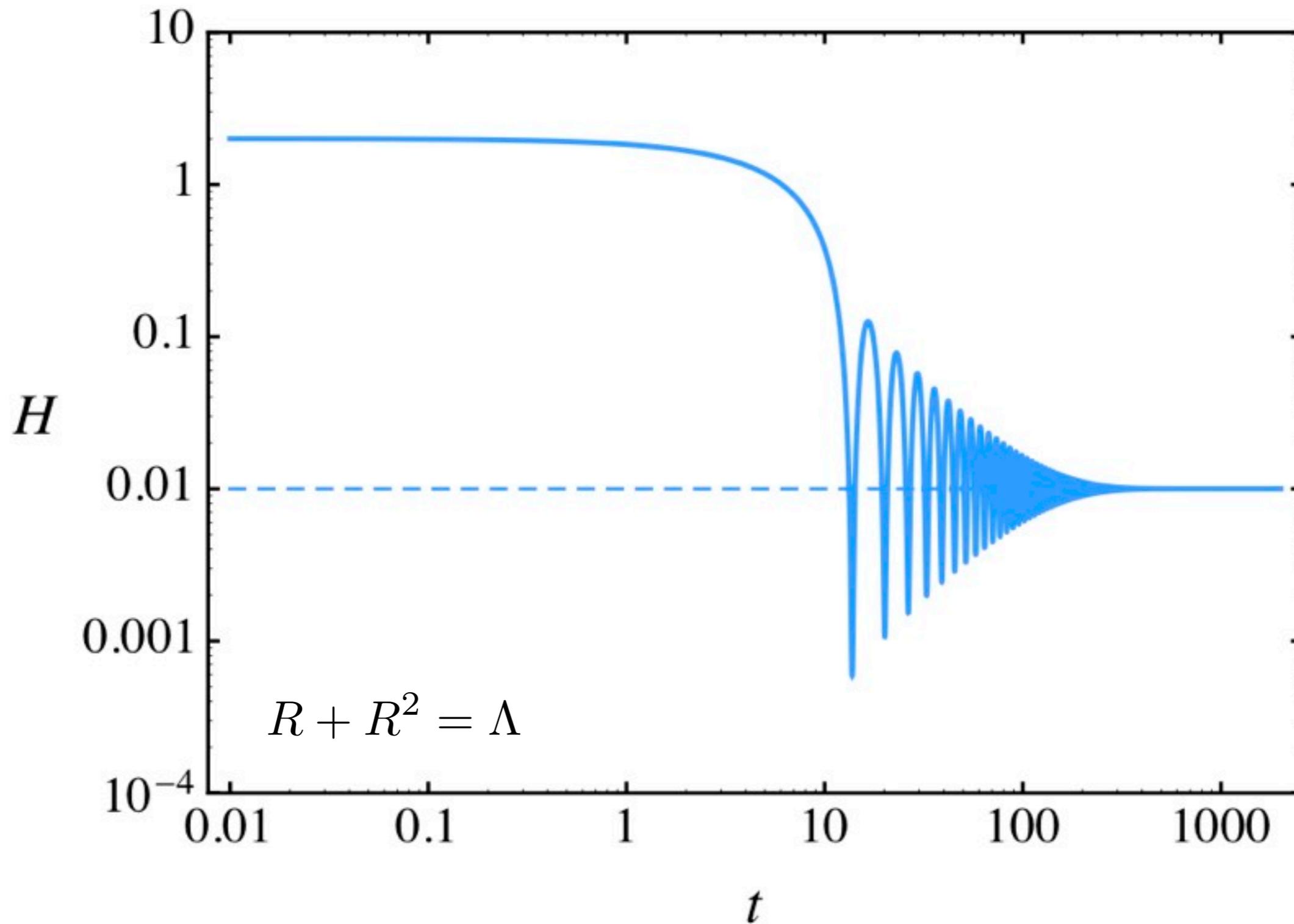
$$2H\ddot{H} + 6H^2\dot{H} - \dot{H}^2 = 0$$

$$H = 0 \quad \text{inflation}$$

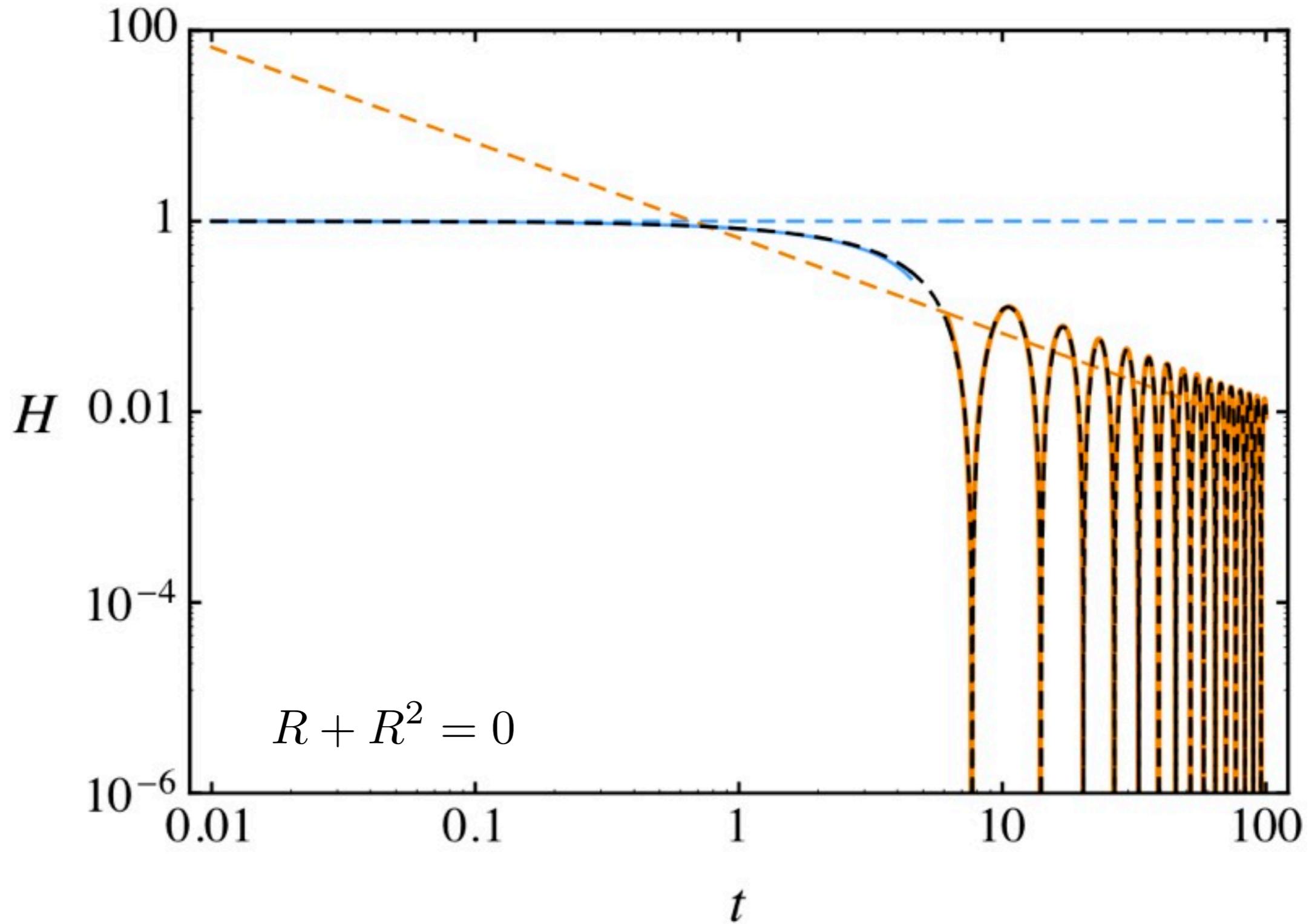
$$t + C_2 = \int \frac{dH}{C_1\sqrt{H} - 2H^2}$$

$$H = \frac{1}{2t} \quad \text{radiation}$$

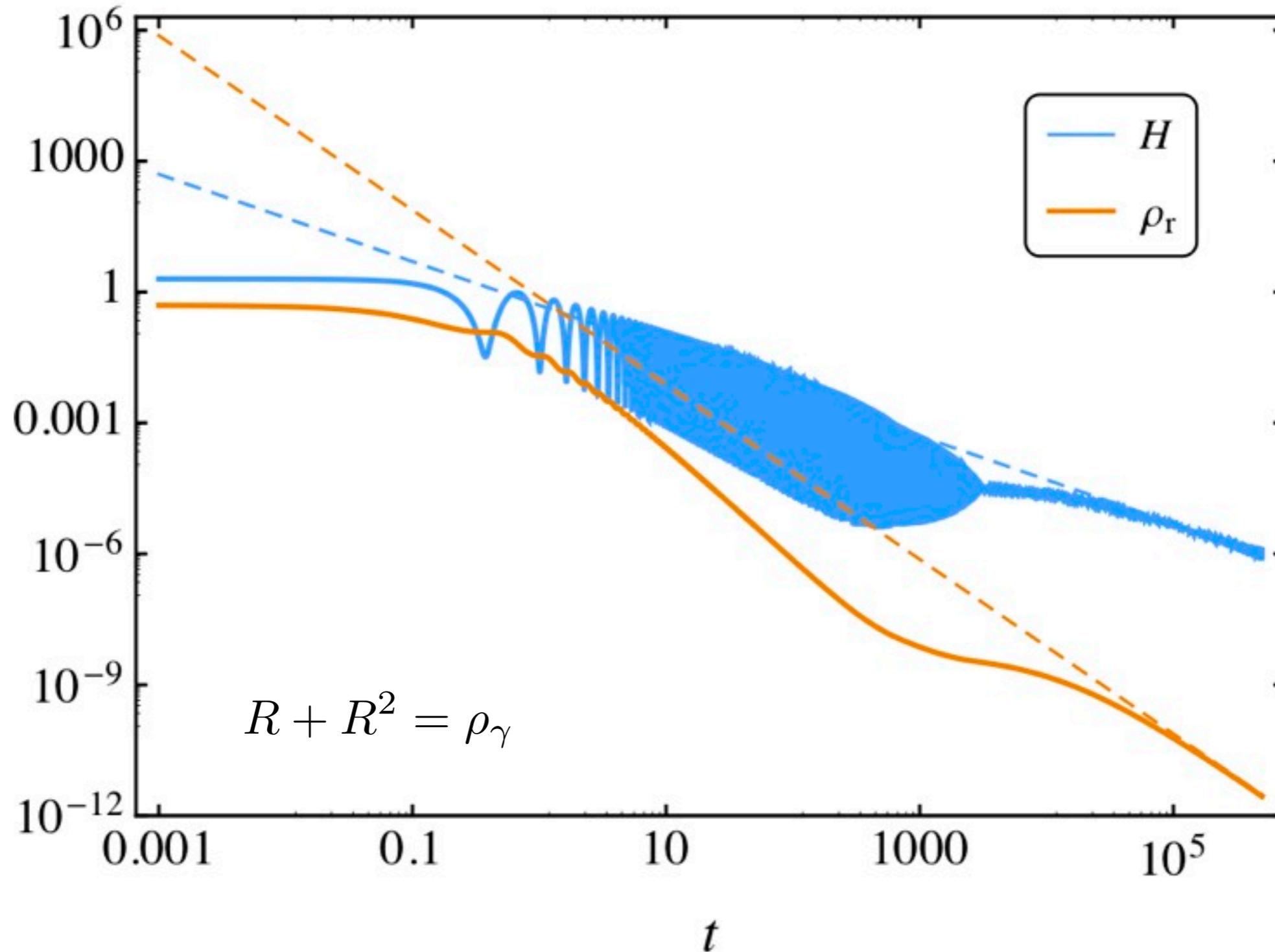
Early times: Inflation



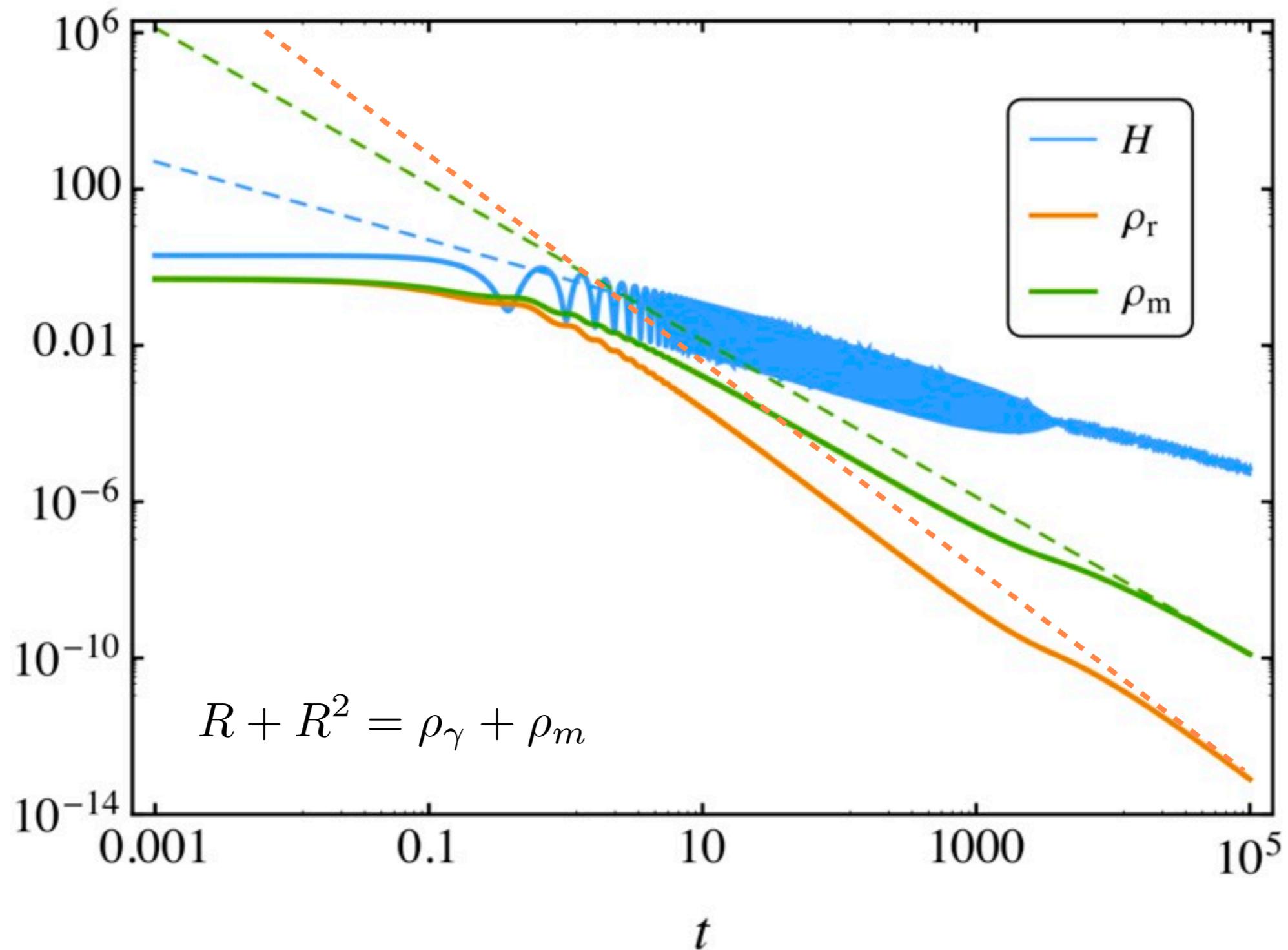
Early times: Inflation



Early times: Inflation



Early times: Inflation



Effective Friedmann equations (non-local)

Modified Einstein's equations

$$G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Non local correction (covariant form)

$$\begin{aligned} \frac{1}{16\pi G\beta m^2} \Delta G_{\mu\nu}^{R-\frac{1}{\square}R} &= -2G_{\mu\nu}U + 2g_{\mu\nu}R + 2\nabla_\mu\nabla_\nu U + \nabla_\mu U\nabla_\nu U \\ &\quad - \frac{1}{2}g_{\mu\nu}\nabla^\alpha U\nabla_\alpha U \\ -\square U &= R \end{aligned}$$

Non local correction (FRW form)

$$\begin{aligned} H^2 - 16\pi G\beta m^2 \left(\frac{1}{6}\dot{U}^2 - 2H\dot{U} - 2H^2U \right) &= \frac{8\pi G}{3}\rho \\ \ddot{U} + 3H\dot{U} &= 6 \left(2H^2 + \dot{H} \right) \end{aligned}$$

Effective Friedmann equations (non-local)

$$\begin{aligned} \frac{1}{16\pi G\delta m^4} \Delta G_{\mu\nu}^{R\frac{1}{2}R} &= -2G_{\mu\nu}S + 2g_{\mu\nu}U + 2\nabla_\mu\nabla_\nu S \\ &+ 2\nabla_\mu U\nabla_\nu S - g_{\mu\nu}\nabla^\alpha U\nabla_\alpha S + \frac{1}{2}g_{\mu\nu}U^2 \\ -\square U &= R \\ -\square S &= U \end{aligned}$$

$$\begin{aligned} H^2 - 32\pi G\delta m^4 \left(2H^2S + H\dot{S} + \frac{1}{2}\dot{H}S - \frac{1}{6}\dot{U}\dot{S} \right) &= \frac{8\pi G}{3}\rho \\ \ddot{U} + 3H\dot{U} &= 6 \left(2H^2 + \dot{H} \right) \\ \ddot{S} + 3H\dot{S} &= U \end{aligned}$$

Effective Friedmann equations (non-local)

$$\frac{1}{16\pi G\delta m^4} \Delta G_{\mu\nu}^{R\frac{1}{\square^2}R} = -2G_{\mu\nu}S + 2g_{\mu\nu}U + 2\nabla_\mu\nabla_\nu S$$

$$+ 2\nabla_\mu U\nabla_\nu S - g_{\mu\nu}\nabla^\alpha U\nabla_\alpha S + \frac{1}{2}g_{\mu\nu}U^2$$

$$-\square U = R$$

$$-\square S = U$$

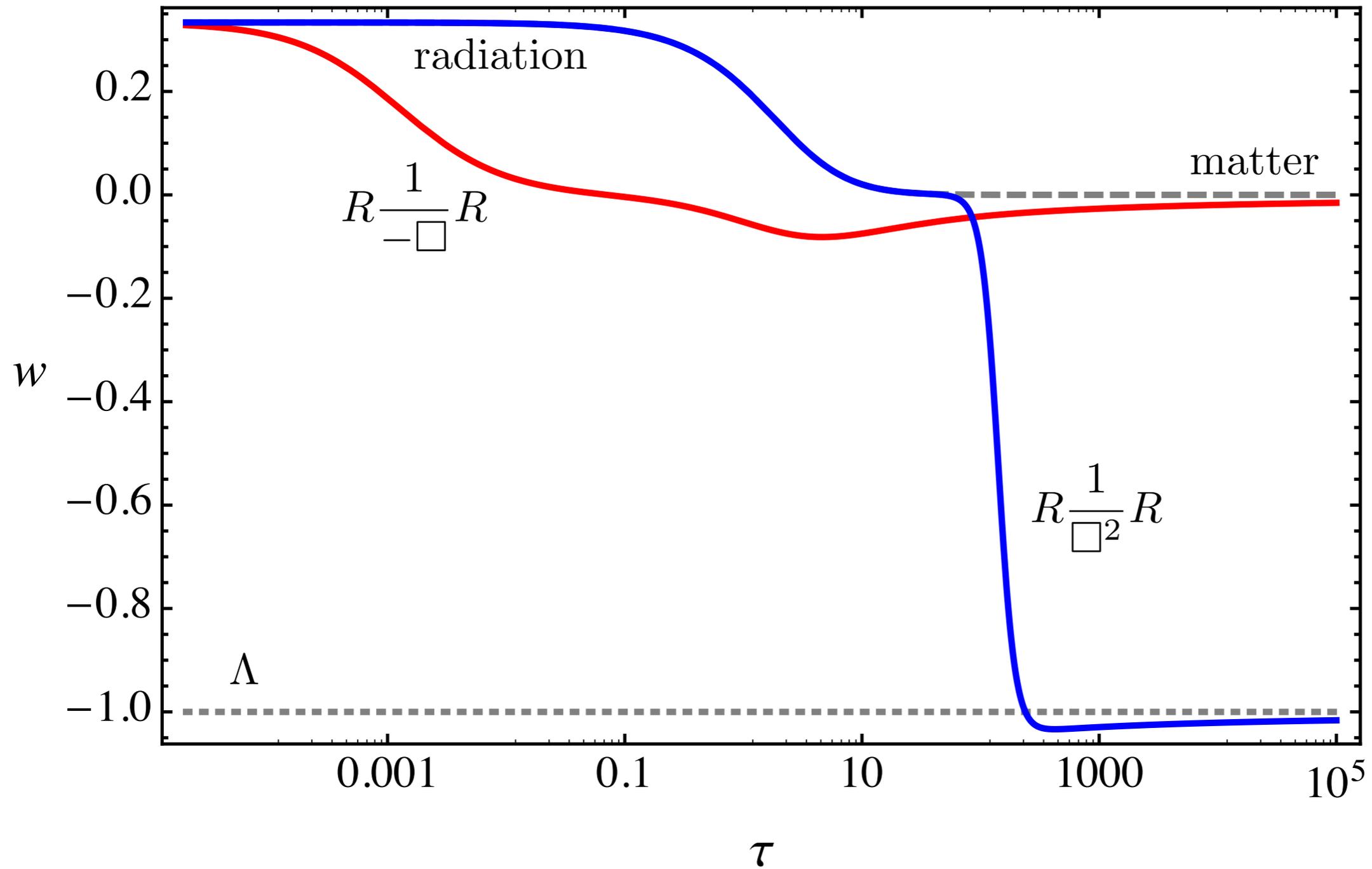
$$H^2 - 32\pi G\delta m^4 \left(2H^2S + H\dot{S} + \frac{1}{2}\dot{H}S - \frac{1}{6}\dot{U}\dot{S} \right) = \frac{8\pi G}{3}\rho$$

$$\ddot{U} + 3H\dot{U} = 6 \left(2H^2 + \dot{H} \right)$$

$$\ddot{S} + 3H\dot{S} = U$$

$$H^2 = \frac{8\pi G}{3}(\rho + \rho_{DE})$$

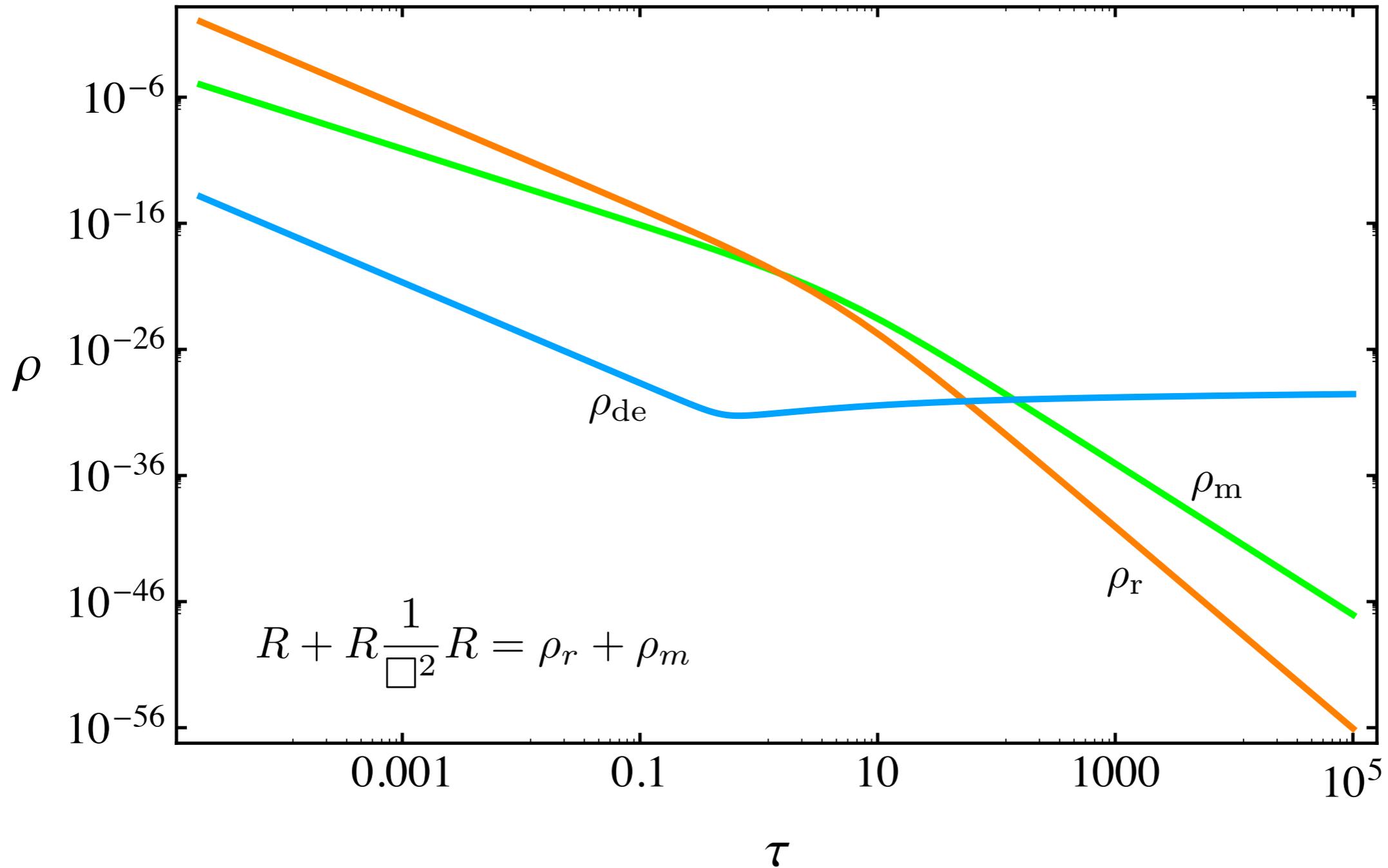
Late times: Dark energy



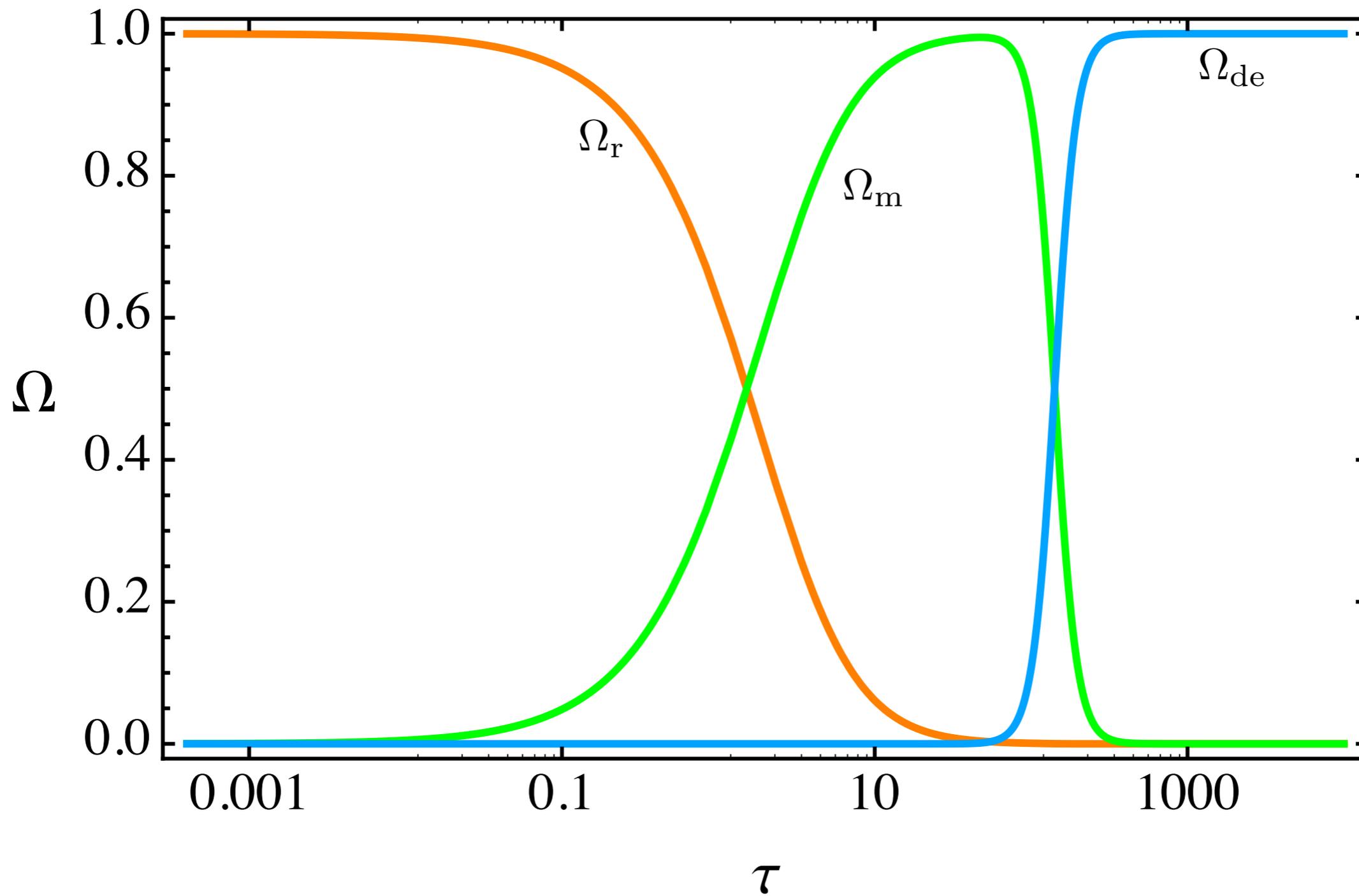
$$\dot{\rho}_{\text{tot}} + 3H(1 + w)\rho_{\text{tot}} = 0$$

$$w = -1 - \frac{1}{3H} \frac{\dot{\rho}_{\text{tot}}}{\rho_{\text{tot}}}$$

Late times: Dark energy



Late times: Dark energy



Conclusions and Outlook

- Consistent cosmological history
- Compute all LO terms
- The role of the Conformal anomaly
- Apply to stars/black holes
- Connection with high energy quantum gravity
- Falsify!

Thank you