Unitarity and Effective Field Theory Results in Quantum Gravity

Workshop on Strongly-Interacting Field Theories

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Introduction

Outline

- Overview of computations of amplitudes in gravity
- New toolbox for gravity
  - Use of string theory relations (Kawai-Lewellen-Tye)
  - Unitarity
  - Helicity variables
- Effective field theory computations revisited
- New unitarity one-loop gravity results and discussion

Computation of amplitudes in field theories

- Generically featuring a number of unpleasant features
  - Tedious computations with lots of contractions
  - Factorial growth in number of legs
  - Sum over Feynman diagram topologies
  - Tensor Integrations
  - .....

LHC a *motivating factor*

LHC a *motivating factor*

The LHC collider

Signals of new physics

New physics??

Supersymmetry?

Higgs!

Theory versus Experiment

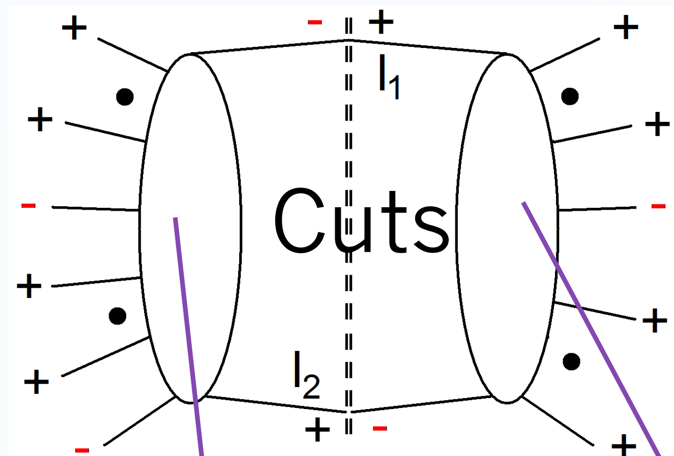
Precision calculations: QCD background at NLO

Key: Unitarity

Loop amplitudes

Simpler expressions
for amplitudes

Unitarity



Amplitudes N=4,
N=1, QCD at NLO,
Gravity..

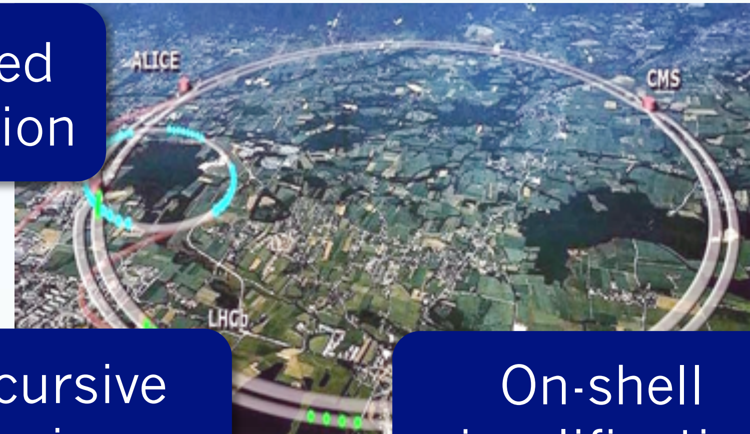
Key: Simple trees

Hidden structure!

....from compact trees to loops

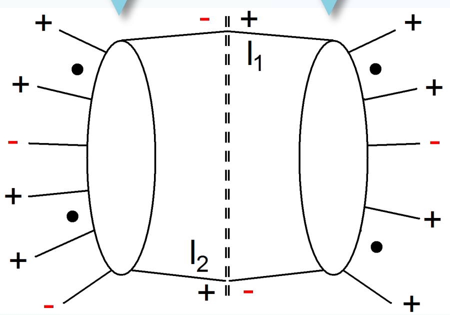
Compact, on-shell tree Amplitudes

Automated computation



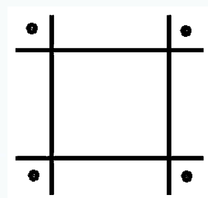
Recursive techniques

On-shell simplification

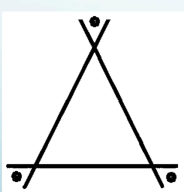


Integral basis

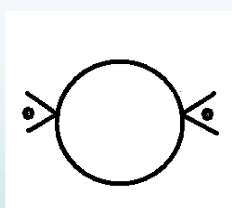
Quadruple cuts



Rational polynomials



Triple cuts



Powerful computational methods

Impossible by Feynman diagrams

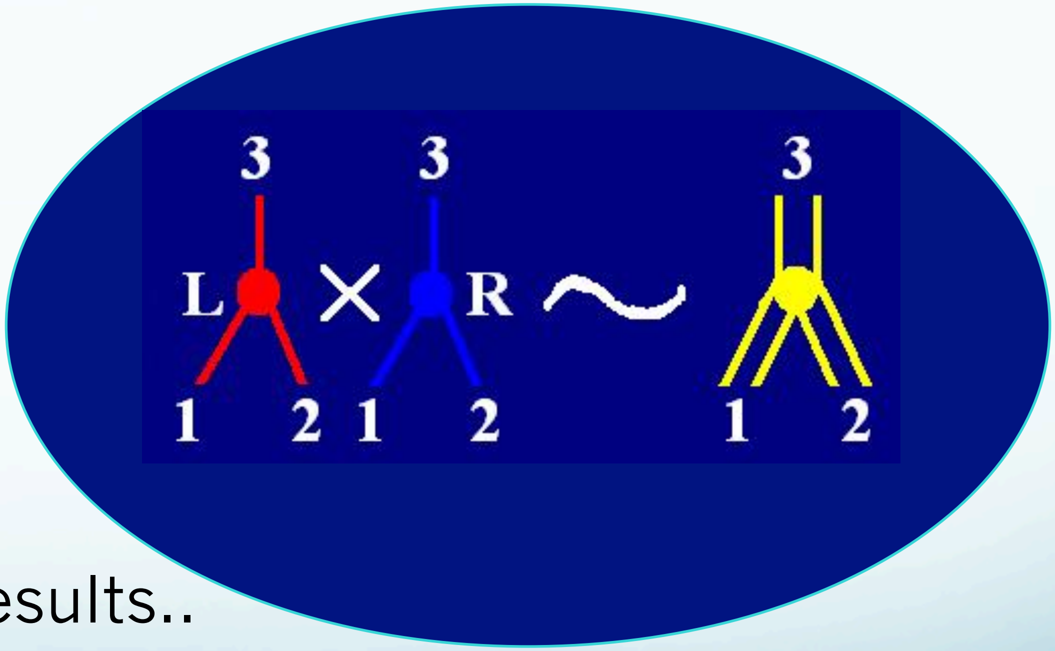
Revolution in doable computations

Squaring relation for gravity

Gravity from (Yang-Mills)^2 (Kawai, Lewellen, Tye)

Natural from the decomposition of closed strings into open.

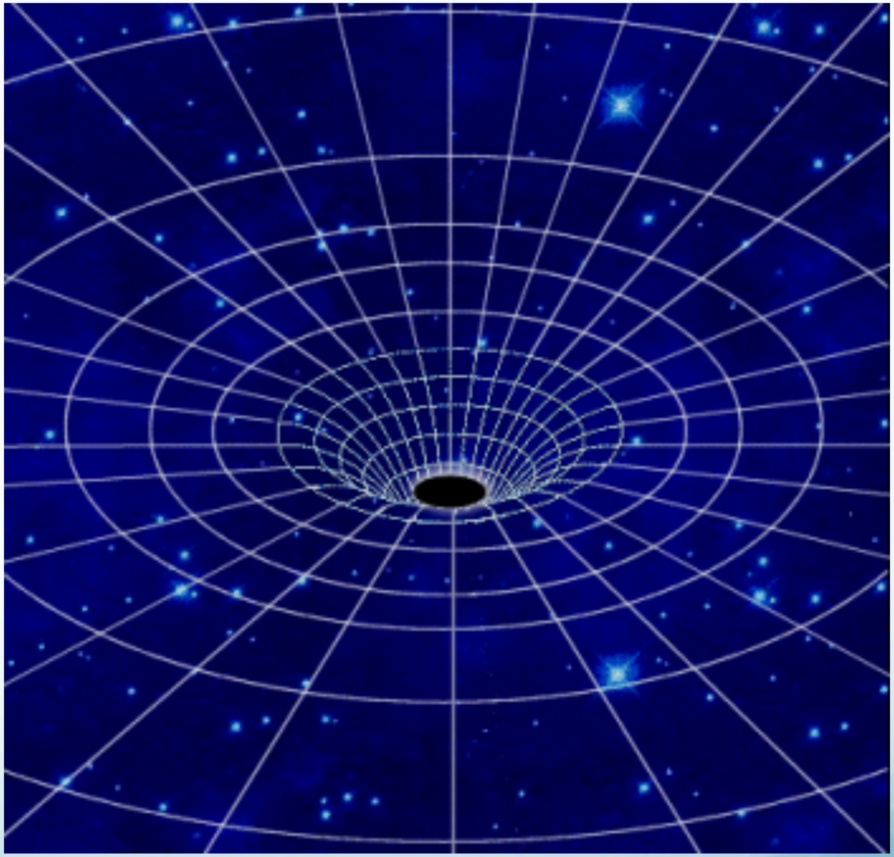
Gives a smart way to recycle Yang-Mills results into gravity results.. (Bern et. al.)



Gravity Trees

General Relativity

- Einstein's theory presents us with a beautiful theory for gravity.
- Geometrical description that does not fit well with generic (flat space) formulation of quantum mechanics.
- What could be a good quantum mechanical extension of General Relativity?



Traditional quantization of gravity

- Known since the 1960ties that a particle version of General Relativity can be derived from the Einstein Hilbert Lagrangian (Feynman, DeWitt)

- Expand Einstein-Hilbert Lagrangian :

$$\mathcal{L}_{\mathrm{E}H} = \int d^4x \left[\sqrt{-g}R\right] \qquad g_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

- Derive vertices as in a particle theory - compute amplitudes as Feynman diagrams!

Gravity Amplitudes

- Vertices: 3pt, 4pt, 5pt,..n-pt
- Complicated expressions
- Expand Lagrangian, tedious process....

45 terms + sym

$$V^{(3)}_{\mu\alpha,\nu\beta,\sigma\gamma}(k_1,k_2,k_3) = \kappa\,\mathrm{sym}\Big[-\frac{1}{2}P_3(k_1\cdot k_2\,\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \frac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma})$$
$$+\frac{1}{2}P_3(k_1\cdot k_2\,\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma}) + P_6(k_1\cdot k_2\,\eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}) + 2P_3(k_{1\nu}k_{1\gamma}\eta_{\mu\alpha}\eta_{\beta\sigma})$$
$$-P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\sigma\gamma}) + P_3(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\sigma}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta})$$
$$+2P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\sigma}) + 2P_3(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha}) - 2P_3(k_1\cdot k_2\,\eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})\Big],$$

(Sannan)

Quantum theory for gravity

- Gravity as a theory of point-like interactions
- Non-renormalisable theory!
  ('t Hooft and Veltman)

Dimensionful

$G_N = 1/M^2_{planck}$

- Traditional belief : – no known symmetry can remove all higher derivative divergences..

String theory can by introducing new length scales

- However - as an effective field theory - one can derive a consistent point-like theory for gravity with predictions order by order in perturbation theory. Gives a 'working version' of a quantum theory for gravity below Planck scale. (Weinberg; Donoghue)

Quantum gravity as an effective field theory

- (Weinberg) proposed to view the quantization of general relativity from the viewpoint of effective field theory.

$$\mathcal{L} = \sqrt{-g}\left[\frac{2R}{\kappa^2} + \mathcal{L}_{\text{matter}}\right]$$

$$\Downarrow$$

$$\mathcal{L} = \sqrt{-g}\left\{\frac{2R}{\kappa^2} + c_1 R^2 + c_2 R^{\mu\nu} R_{\mu\nu} + \dots\right\}$$

- (Donoghue) and (NEJBB, Donoghue, Holstein) did the first one-loop concrete computation in such a framework

Amplitudes and Feynman diagrams

Diagrammatic expansion : huge permutational problem!

- Scalar field theory : constant vertex (~1 term)
- Gluons : momentum dependent vertex (~3 terms)
- Gravitons : momentum dependent vertex (~100 terms)

Naïve basic 4pt diagram count (graviton exchange) :

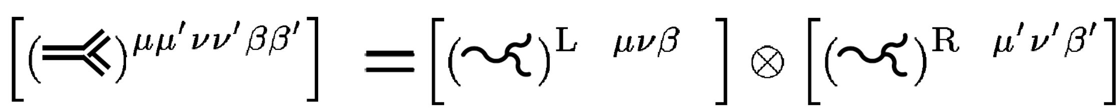
100 x 100 ~ 10^4 terms + index contractions (~ 36 pr diagram)
Number of diagrams: (~ 4 !) ~10^5 terms ~ 10^6 index contractions
n-point: (~ n !) ~ more atoms in your brain!

Too much off-shell (gauge dependent) clutter.....

Gravity Amplitudes

KLT relationship (Kawai, Lewellen and Tye) relates open and closed strings

$$A^{M}_{\text{closed}} \sim \sum_{\Pi,\tilde{\Pi}} e^{i\pi\Phi(\Pi,\tilde{\Pi})} A_M^{\text{left open}}(\Pi) A_M^{\text{right open}}(\tilde{\Pi})$$



KLT not manifestly crossing symmetric – explicit representation :

$$M_3^{\text{tree}}(1,2,3) = -iA_3^{\text{tree}}(1,2,3)A_3^{\text{tree}}(1,2,3)\,,$$

$$M_4^{\text{tree}}(1,2,3,4) = -is_{12}A_4^{\text{tree}}(1,2,3,4)A_4^{\text{tree}}(1,2,4,3)$$

$$M_5^{\text{tree}}(1,2,3,4,5) = is_{12}s_{34}\,A_5^{\text{tree}}(1,2,3,4,5)A_5^{\text{tree}}(2,1,4,3,5) + \\ +is_{13}s_{24}\,A_5^{\text{tree}}(1,3,2,4,5)A_5^{\text{tree}}(3,1,4,2,5)\,.$$

Momentum prefactors cancel double poles

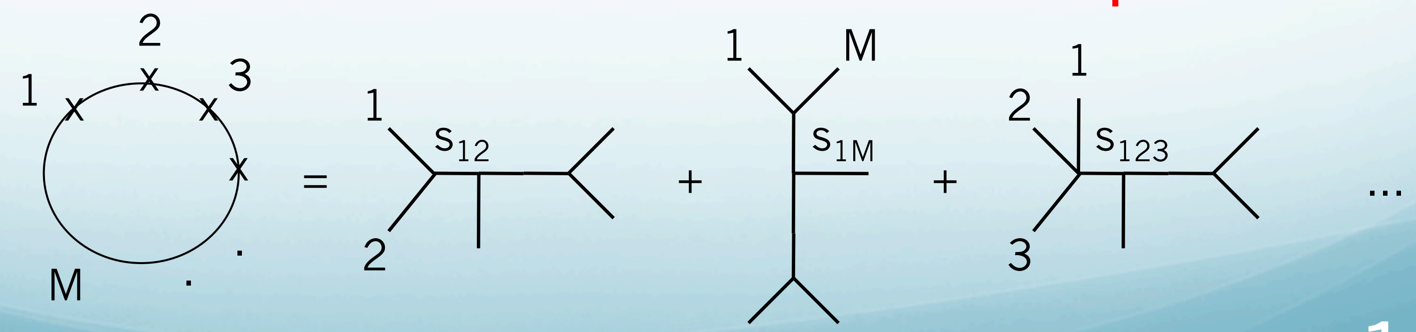
String theory

Different form for amplitude

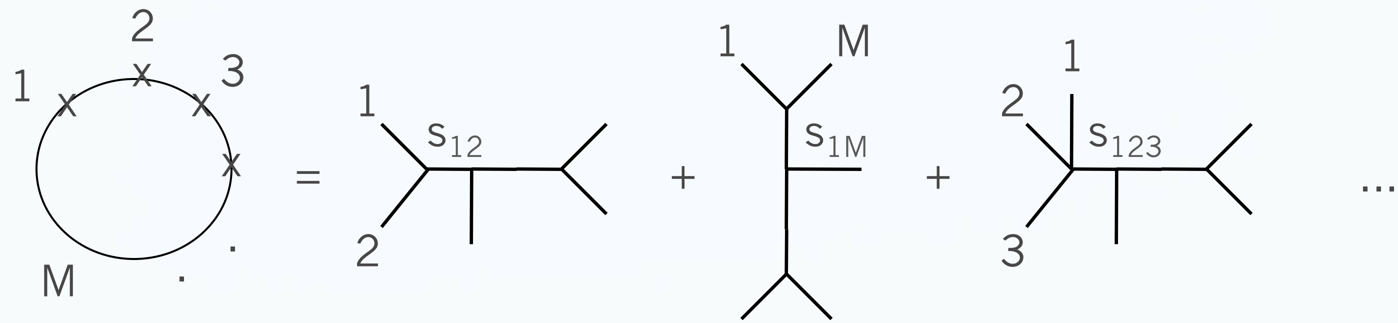
String theory adds channels up..

<·>

Feynman diagrams sums separate kinematic poles

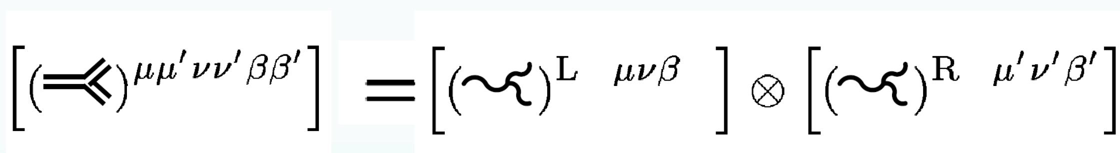


Gravity Amplitudes



(Link to individual Feynman diagrams lost..)

Certain vertex relations possible



Concrete Lagrangian formulation possible?

(Bern and Grant;
Ananth and Theisen;
Hohm)

Yang-Mills Trees

Helicity states formalism

Spinor products :

$$\langle i\ j\rangle = \epsilon^{mn}\lambda^i_m\lambda^j_n \qquad [i\ j] = \epsilon^{\dot m\dot n}\tilde\lambda^i_{\dot m}\tilde\lambda^j_{\dot n}$$

Different representations of the Lorentz group

$$p_{a\dot a} = \sigma^\mu_{a\dot a}p_\mu$$

$$p^\mu p_\mu = 0 \qquad p_{a\dot a} = \lambda_a\tilde\lambda_{\dot a}$$

Momentum parts of amplitudes:

$$q_{a\dot a}=\mu_a\tilde\mu_{\dot a} \quad p_{a\dot a} = \lambda_a\tilde\lambda_{\dot a} \quad 2(p\cdot q) = s_{ij} = -\langle\lambda,\mu\rangle[\tilde\lambda,\tilde\mu]$$

Spin-2 polarisation tensors in terms of helicities, (squares of those of YM):

(Xu, Zhang, Chang)

$$\varepsilon^-_{a\dot a} = \frac{\lambda_a\tilde\mu_{\dot a}}{[\tilde\lambda,\tilde\mu]} \qquad \tilde\varepsilon^+_{a\dot a} = \frac{\mu_a\tilde\lambda_{\dot a}}{\langle\mu,\lambda\rangle} \qquad \begin{matrix}\varepsilon^- & \varepsilon^- \\ \tilde\varepsilon^+ & \tilde\varepsilon^+\end{matrix}$$

Yang-Mills MHV-amplitudes

(n) same helicities vanishes

$A^{tree}(1^+,2^+,3^+,4^+,..) = 0$

(n-1) same helicities vanishes

$A^{tree}(1^+,2^+,...,j^-,..) = 0$

(n-2) same helicities:

$A^{tree}(1^+,2^+,...,j^-,...,k^-,..)$

First non-trivial example,

(M)aximally

(H)elicity (V)iolating

(MHV) amplitudes

One single term!!

$$i \frac{\langle j\, k\rangle^4}{\langle 1\, 2\rangle\langle 2\, 3\rangle \cdots \langle n\, 1\rangle}$$

$A^{tree\ MHV}$ Given by the formula (Parke and Taylor) and proven by (Berends and Giele)

Gravity MHV amplitudes

Can be generated from KLT via YM MHV amplitudes.

$$M_4^{\rm tree}(1^-,2^-,3^+,4^+) = i\,\langle 1\,2\rangle^8\,\frac{[1\,2]}{\langle 3\,4\rangle\;N(4)}$$

$$M_5^{\rm tree}(1^-,2^-,3^+,4^+,5^+) = i\,\langle 1\,2\rangle^8\,\frac{\varepsilon(1,2,3,4)}{N(5)}$$

Anti holomorphic Contributions – feature in gravity

(Berends-Giele-Kuijf) recursion formula

$$M_n^{\rm tree}(1^-,2^-,3^+,\cdots,n^+) = -i\,\langle 1\,2\rangle^8 \times \left[\frac{[1\,2]\,[n-2\,n-1]}{\langle 1\,n-1\rangle\,N(n)}\left(\prod_{i=1}^{n-3}\prod_{j=i+2}^{n-1}\langle i\,j\rangle\right)\prod_{l=3}^{n-3}(-[n|K_{l+1,n-1}|l\rangle) + \mathcal{P}(2,3,\cdots,n-2)\right]$$

Simplifications from Spinor-Helicity

$$s_{ij} = -\langle\lambda, \mu\rangle[\tilde{\lambda}, \tilde{\mu}]$$

Huge simplifications

$$V^{(3)}_{\mu\alpha,\nu\beta,\sigma\gamma}(k_1,k_2,k_3) = \kappa\,\mathrm{sym}\Big[-\frac{1}{2}P_3(k_1\cdot k_2\,\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \frac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma})$$
$$+\frac{1}{2}P_3(k_1\cdot k_2\,\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma}) + P_6(k_1\cdot k_2\,\eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}) + 2P_3(k_{1\nu}k_{1\gamma}\eta_{\mu\alpha}\eta_{\beta\sigma})$$
$$-P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\sigma\gamma}) + P_3(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\sigma}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta})$$
$$+2P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\sigma}) + 2P_3(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha}) - 2P_3(k_1\cdot k_2\,\eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})\Big],$$

45 terms + sym

Vanish in spinor helicity formalism

Contractions

$$\varepsilon^-_{a\dot{a}} = \frac{\lambda_a\tilde{\mu}_{\dot{a}}}{[\tilde{\lambda},\tilde{\mu}]} \qquad \tilde{\varepsilon}^+_{a\dot{a}} = \frac{\mu_a\tilde{\lambda}_{\dot{a}}}{\langle\mu,\lambda\rangle}$$

Gravity:

$\varepsilon^-\ \varepsilon^-$

$\tilde{\varepsilon}^+\ \tilde{\varepsilon}^+$

$$A_3(1^-,2^-,3^+) = -i\frac{\langle 1\,2\rangle^6}{\langle 2\,3\rangle\,\langle 3\,1\rangle}$$

One Loop

One-loop result for gravity

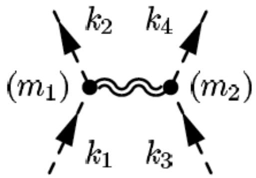
- 4 pt Amplitude can be deduced to take the form

$$\mathcal{M} \sim \left(A + Bq^2 + \ldots + \alpha\kappa^4 \frac{1}{q^2} + \beta_1 \kappa^4 \ln(-q^2) + \beta_2 \kappa^4 \frac{m}{\sqrt{-q^2}} + \ldots \right)$$

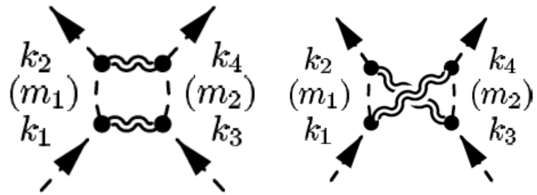
Focus on deriving these ~>
Long-range behavior

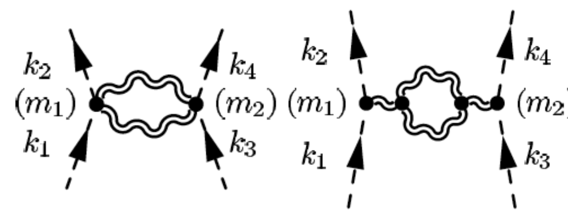
Short range behaviour

Off-shell Computation

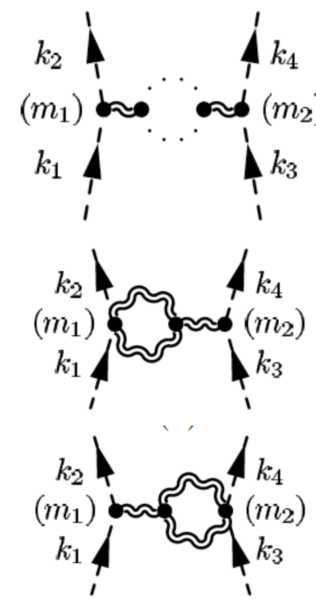


Boxes



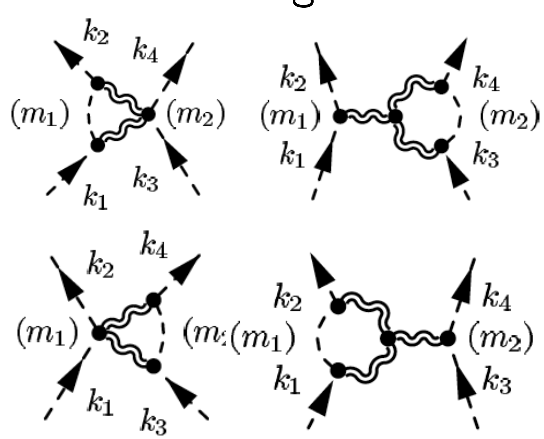


Bubbles



Tree

Triangles



Result for the one-loop amplitude

The result for the amplitude (in coordinate space) after summing all diagrams (more than 10.000 terms) is

$$-\frac{Gm_1m_2}{r}\left[1+3\frac{G(m_1+m_2)}{r}+\frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

As expected from Newtonian gravity

As expected from General relativity

Novel quantum result

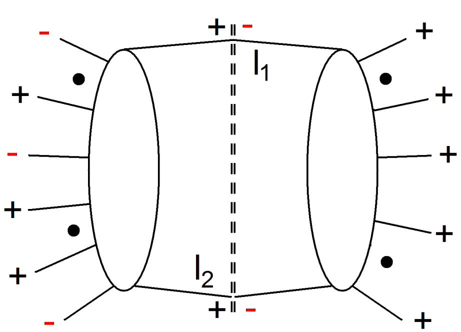
Very long an tedious computation, hard to extend to more legs... and more loops... Search for a better way

Unitarity cuts

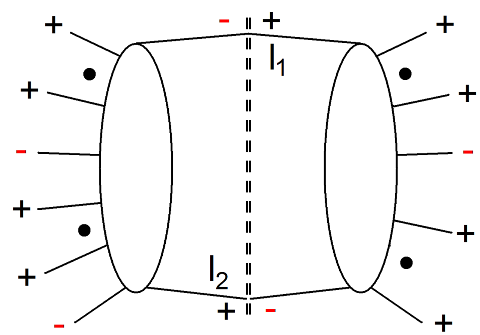
Helicity formalism require unitarity methods

$$C_{i,\ldots,j} = \mathrm{Im}_{K_{i,\ldots,j}>0}\, M^{\text{1-loop}}$$

Singlet



Non-Singlet



$$C_{i,\ldots,j} \equiv \frac{i}{2}\int d\mathrm{LIPS}\Big[M^{\text{tree}}(\ell_1, i, i+1, \ldots, j, \ell_2) \times$$
$$\times\, M^{\text{tree}}(-\ell_2, j+1, j+2, \ldots, i-1, -\ell_1)\Big]$$

Use of on-shell methods to derive such results

- The starting point for a unitarity computation is compact trees.
- Trees in gravity can be derived using Yang-Mills results and the KLT relations.
- Next the necessary cuts is constructed.
- On the cut it is helpful to fix one-loop amplitude employing a basis of integral functions, and determining, using that, where the different singularities in the cut go. (Bern, Dixon, Perelstein, Rozowsky); Dunbar and Norridge)

Unitarity method trees

- Starting from Yang-Mills trees we have $\kappa_{(4)}^2 = 32\pi G_N$

$$iM_s^{\text{tree}}(p_1,p_2,k_1,k_2) = \kappa_{(4)}^2\,(p_1\cdot k_1)\,A_s^{\text{tree}}(p_1,p_2,k_2,k_1)A_0^{\text{tree}}(p_1,k_2,p_2,k_1)$$

s = 0, ½, 1,

- The color striped YM amplitude satisfies

$$A_s^{\text{tree}}(p_1,p_2,k_2,k_1) = \frac{p_1\cdot k_2}{k_1\cdot k_2}\,A_s^{\text{tree}}(p_1,k_2,p_2,k_1)$$

$$iM_s^{\text{tree}}(p_1,p_2,k_1,k_2) = \frac{\kappa_{(4)}^2}{e^2}\frac{(p_1\cdot k_1)\,p_1\cdot k_2}{k_1\cdot k_2}\,A_s^{\text{tree}}(p_1,k_2,p_2,k_1)A_0^{\text{tree}}(p_1,k_2,p_2,k_1)$$

(NEJBB, Donoghue, Vanhove)

KLT squaring and traces

In all generality we have

$$iM^{\text{tree}} = \sum_{\sigma,\gamma \in \mathfrak{S}_{n-3}} \mathcal{S}[\sigma(2,\cdots,n-2)|\gamma(2,\cdots,n-2)]|_{k_1} \times$$
$$\times A^{\text{tree}}(1,\sigma(2,\cdots,n-2),n-1,n) A^{\text{tree}}(n,n-1,\gamma(2,\cdots,n-2),1)$$

Where

$$S[i_1,\ldots,i_r|j_1,\ldots,j_r]|_p = \prod_{t=1}^{r}(p\cdot k_{i_r} + \sum_{s>t}^{r}\theta(i_r,i_s)\,k_{i_r}\cdot k_{i_s})$$

(NEJBB, Damgaard, Feng, Søndergaard; NEJBB, Damgaard, Sondergaard, Vanhove)

Unitary cut

- Now one sees that

$$A_0^{\text{tree}}(p_1, k_2^+, p_2, k_1^+) = -\frac{m^2\,[k_1\,k_2]^2}{4(p_1\cdot k_1)\,(p_1\cdot k_2)}$$

$$A_0^{\text{tree}}(p_1, k_2^-, p_2, k_1^+) = \frac{\langle k_2|p_1|k_1]^2}{4\,(k_1\cdot p_1)(p_1\cdot k_2)}$$

- This yields

$$iM_0^{\text{tree}}(p_1, p_2, k_1^+, k_2^+) = \frac{\kappa_{(4)}^2}{16}\,\frac{1}{(k_1\cdot k_2)}\,\frac{m^4\,[k_1\,k_2]^4}{(k_1\cdot p_1)(k_1\cdot p_2)}$$

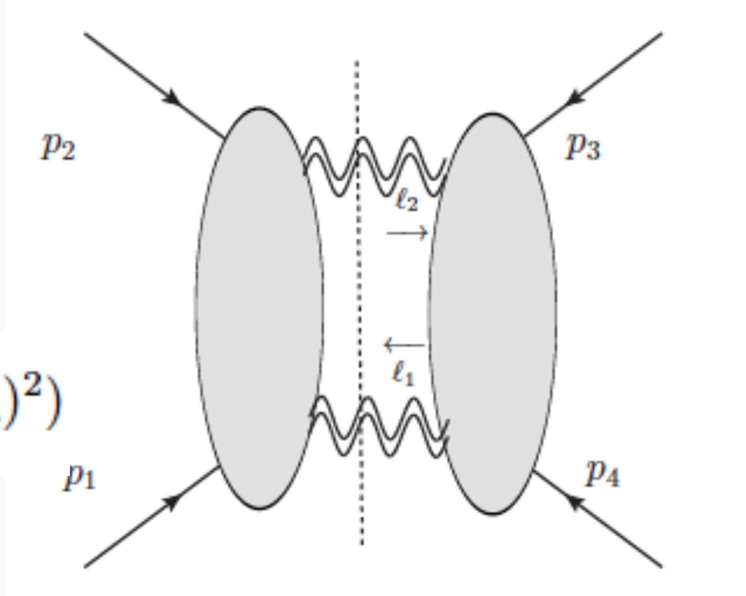
$$iM_0^{\text{tree}}(p_1, p_2, k_1^-, k_2^+) = \frac{\kappa_{(4)}^2}{16}\,\frac{1}{(k_1\cdot k_2)}\,\frac{\langle k_1|p_1|k_2]^2\,\langle k_1|p_2|k_2]^2}{(k_1\cdot p_1)(k_1\cdot p_2)}$$

Simplifications and singularities

- In the cut we have

$$\mathcal{N}^{\text{singlet}} = m_1^2 m_2^2 s^2$$

$$\mathcal{N}^{\text{non-singlet}} = \frac{1}{2}(\text{tr}_-(\ell_2 p_1 \ell_1 p_3)^2 + \text{tr}_+(\ell_2 p_1 \ell_1 p_3)^2)$$



$$iM^{1-\text{loop}}\Big|_{disc} = \int \frac{d^D\ell}{(2\pi)^D} \frac{\sum_{\lambda_1,\lambda_2} M^{\text{tree}}_{\lambda_1\lambda_2}(p_1, p_2, -\ell_2^{\lambda_2}, \ell_1^{\lambda_1})(M^{\text{tree}}_{\lambda_1\lambda_2}(p_3, p_4, \ell_2^{\lambda_2}, -\ell_1^{\lambda_1}))^*}{\ell_1^2 \ell_2^2}\Big|_{cut}$$

$$iM^{1-\text{loop}}\Big|_{disc} = \frac{e^4}{16} \int \frac{d^D\ell}{(2\pi)^D} \frac{\mathcal{N}}{\ell_1^2 \ell_2^2 \prod_{i=1}^4 (p_i \cdot \ell_1)}$$

General 1-loop amplitudes

n-pt amplitude

Vertices carry factors of loop momentum

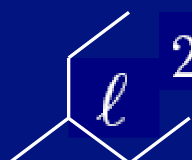
$$\int d^4\ell \frac{F^p(\ell, k, \epsilon)}{\Pi_i p_i^2}$$

p = 2n for gravity

p=n for YM

Propagators

(Passarino-Veltman) reduction

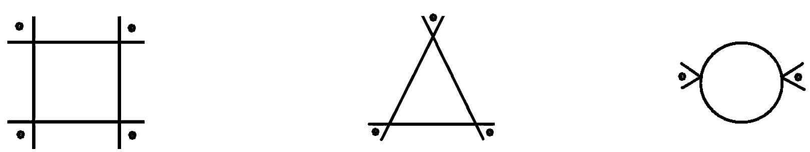


$$2(k\cdot\ell) = (k-\ell)^2 - \ell^2$$

Collapse of a propagator

$$I_r[P^m(l)] \longrightarrow \sum_i I^i_{r-1}[P^{m-1}(l)]$$

$$I^i_4[P^{m'}(l)] \longrightarrow c_i\, I^i_4[1] + \sum_j I^j_3[P^{m'-1}(l)]$$



$$M^{1-\text{loop}} = \sum_a c_a I^a_4 + \sum_a d_a I^a_3 + \sum_a e_a I^a_2 + R$$

Result from unitarity for the one-loop amplitude

- The (off-shell) result for the amplitude (in coordinate space) after summing all diagrams is confirmed:

$$-\frac{Gm_1m_2}{r}\left[1+3\frac{G(m_1+m_2)}{r}+\frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

New features:

- Much simpler way to derive result.
- Gives directly argument for universality of the amplitude for different external matter.

Helicity method vs. covariant

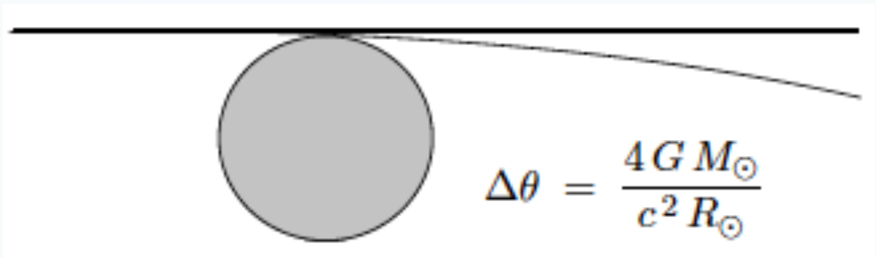
- The cut is here written down in terms of helicity variables (*i.e.* a physical transverse polarisations), this has the advantage that 'ghost' contributions are avoided.
- For a covariant cut which is also possible, 'ghosts' would have to be taken into account.
- All symmetry factors plus the various Feynman channels that would normally have be mapped out before the computation are automatically included when calculating the loop amplitude from the cut.

New possibilities and matter fields

- Unitarity offers make other advantages
  - On-shell tree, recursive methods can be used to compute such trees.
  - It is easy to consider other types of matter fields just by making the cut with other external particles.
  - Immediate extension to higher loop cases once trees are known.
    - Extensions to any loop order in principle possible (or less impossible than off-shell approach)

Photons and massless scalars

- Next we will turn to the scattering of mass-less matter



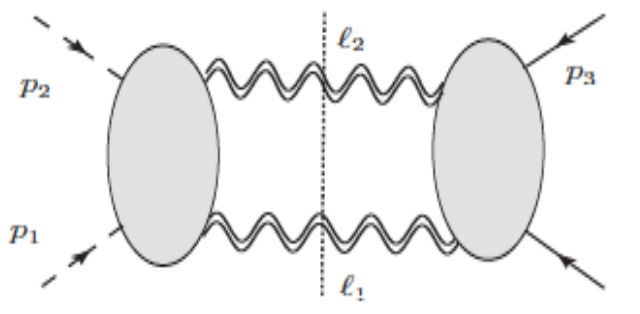
- Bending of light/massless scalars around the Sun
- New features: mass-less external fields ~> IR singularities
- New test of universality of matter

Trees and the cut

- We have the Lagrangian

$$\mathcal{S} = \int d^4x \sqrt{-g} \Big[\Big(\frac{2}{\kappa^2} \mathcal{R} - \frac{1}{4} (\nabla_\mu A_\nu - \nabla_\nu A_\mu)^2 \Big) + \Big(-\frac{1}{2} (\partial_\mu \varphi)^2 - \frac{1}{2} ((\partial_\mu \phi)^2 - M^2 \phi^2) \Big) + S_{\text{EF}} \Big]$$

- We want to compute the cut



$$i\overset{1}{\mathcal{M}}{}^{[\eta(p_1)\eta(p_2)]}_{[\phi(p_3)\phi(p_4)]}\Big|_{\text{disc}} = \int \frac{d^D\ell}{(2\pi)^4} \frac{\sum_{h_1,h_2} \overset{0}{\mathcal{M}}{}^{[h^{h_1}(\ell_1)h^{h_2}(-\ell_2)]}_{[\eta(p_1)\eta(p_2)]} \, \overset{0}{\mathcal{M}}{}^{[h^{h_1}(-\ell_1)h^{h_2}(\ell_2)]\star}_{[\phi(p_3)\phi(p_4)]}}{4\ell_1^2\ell_2^2}$$

Photons and scalars

For photons we have

$$i\overset{0}{\mathcal{M}}{}^{[h^+(k_1)h^-(k_2)]}_{[\gamma^+(p_1)\gamma^-(p_2)]} = \frac{\kappa^2}{4}\frac{[p_1\,k_1]^2\,\langle p_2\,k_2\rangle^2\,\langle k_2|p_1|k_1]^2}{(p_1\cdot p_2)(p_1\cdot k_1)(p_1\cdot k_2)}$$

While for scalars

$$i\overset{0}{\mathcal{M}}{}^{[h^+(k_1)h^+(k_2)]}_{[\phi(p_1)\phi(p_2)]} = \frac{\kappa^2}{4}\frac{M^4\,[k_1\,k_2]^4}{(k_1\cdot k_2)(k_1\cdot p_1)(k_1\cdot p_2)}$$

$$i\overset{0}{\mathcal{M}}{}^{[h^-(k_1)h^+(k_2)]}_{[\phi(p_1)\phi(p_2)]} = \frac{\kappa^2}{4}\frac{\langle k_1|p_1|k_2]^2\,\langle k_1|p_2|k_2]^2}{(k_1\cdot k_2)(k_1\cdot p_1)(k_1\cdot p_2)}$$

Result for cut

So that

$$i\overset{1}{\mathcal{M}}{}^{[\eta(p_1)\eta(p_2)]}_{[\phi(p_3)\phi(p_4)]}\Big|_{\text{disc}} = -\frac{\kappa^4}{4t^4}\sum_{h_1,h_2}\sum_{i=1}^{2}\sum_{j=3}^{4}\int\frac{d^D\ell}{(2\pi)^4}\,\frac{\mathcal{N}^{h_1\,h_2}}{\ell_1^2\ell_2^2(p_i\cdot\ell_1)(p_j\cdot\ell_1)}$$

where

$$\mathcal{N}^{+-}+\mathcal{N}^{-+}=\Re\mathrm{e}\left[\left(\mathrm{tr}_-(\not\ell_1\not p_1\not\ell_2\not p_3)\right)^4\right]$$

Scalar case

$$\mathcal{N}^{+-}+\mathcal{N}^{-+}=\Re\mathrm{e}\left[\frac{\left(\mathrm{tr}_-(\not\ell_2\not p_2\not\ell_1\not p_3)\,\mathrm{tr}_+(\not\ell_2\not p_3\not\ell_1\not p_1\not p_3\not p_2)\right)^2}{\langle p_1|p_3|p_2]^2}\right]$$

Photon case

Amplitude result

- The result for the amplitude is of the form

$$i\mathcal{M}^{[\eta(p_1)\eta(p_2)]}_{[\phi(p_3)\phi(p_4)]} \simeq \frac{\mathcal{N}^\eta}{\hbar}(M\omega)^2$$
$$\times\Big[\frac{\kappa^2}{t} + \kappa^4 \frac{15}{512}\frac{M}{\sqrt{-t}}$$
$$+\hbar\kappa^4\frac{15}{512\pi^2}\log\left(\frac{-t}{M^2}\right) - \hbar\kappa^4\frac{bu^\eta}{(8\pi)^2}\log\left(\frac{-t}{\mu^2}\right)$$
$$+\hbar\kappa^4\frac{3}{128\pi^2}\log^2\left(\frac{-t}{\mu^2}\right) + \kappa^4\frac{M\omega}{8\pi}\frac{i}{t}\log\left(\frac{-t}{M^2}\right)\Big]$$

Taking the Non-Relativistic low energy limit

$$\simeq \quad -\frac{2GM\omega}{r} + \frac{15}{4}\frac{(GM)^2\omega}{r^2}$$
$$+ \frac{8bu^\eta - 15}{4\pi}\frac{G^2 M\omega\hbar}{r^3} + \frac{12G^2 M\omega\hbar}{\pi}\frac{\log\frac{r}{r_o}}{r^3}$$

(NEJBB, Donoghue, Holstein, Plante, Vanhove)

Bending of light

Interpreted as a bending angle (eikonal approximation) we have:

$$\theta_\eta \simeq \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2}$$

plus a quantum effect of the order of magnitude:

$$+\frac{8bu^\eta + 9 + 48\log\frac{b}{2r_o}}{\pi}\frac{G^2\hbar M}{b^3}$$

We see that we have universality between scalars and photons only for the 'Newton' and 'post-Newtonian' contributions

- Quantum part seems to violate universality (can been seen as a tidal effect).

Bending angle for quantum effects is too naïve!

- Should really be treated by quantum means like in QCD… likely to give a diffraction effect as a wave packet treatment.

Conclusions

Discussion / Conclusion

- Treating general relativity as an effective field is a smart way to avoid the usual complications and confusions in quantizing gravity.
- The results are unique consequences of an underlying more fundamental theory.
  - Effects are tiny but this is a consequence of gravity being a very weak force.
- Show that classical GR has a huge validity for normal energies, but GR-EFT provides a natural alternative that takes into account quantum corrections.

New possibilities

- As interesting projects one could consider:
  - Scattering of other types of matter.
  - Higher loop computations (much harder than one-loop)
  - As an application: inclusion of supersymmetry.
    - *E.g.* can universality be restored from SUGRA?
  - Full quantum mechanical treatment (realistic wave packet)
- The on-shell unitarity toolbox for computations is crucial to make further progress in this field.

Conclusion

- Effective Field theory for Gravity 'good theory' at normal energy scales, (for another 16 orders of magnitude).
- Experimentally: interesting to think about where effects could be possible to observe.
  - Right now foremost a new theoretical tool for computation.
  - Could envision more phenomenological applications in the future (esp. with more automatic routines for computations).

Conclusion

- New prospects for further theoretical breakthroughs
  - On-shell and helicity methods has progressed much in short time.
  - New multi-loop (automatic?) toolboxes might yet again alter the landscape of doable computations.
  - On-shell methods might develop into whole new ways of doing perturbative computations.
  - Such 'new' applications will also have implications for gravity computations.

Other new tools…

- Scattering equations: (Cachazo, He, Yuan)
  - Formulas for scalars, gauge theories and gravity.
  - Tree formula

$$\mathcal{A}_n = \int \frac{d^n\sigma}{\mathrm{volSL}(2,\mathrm{c})} \prod_a{}' \delta\left(\sum_{a\neq b} \frac{k_a \cdot k_b}{z_a - z_b}\right)$$
$$\left(\frac{\mathrm{Tr}(T^{a_1}T^{a_2}T^{a_3}\cdots T^{a_n})}{(z_1-z_2)(z_2-z_3)\cdots(z_n-z_1)} + \cdots\right)^{2-s} (\mathrm{Pf}'\Psi)^s$$

  - Also possible extensions to loops (Geyer, Mason, Monteiro, Tourkine).

Exciting times!!