Gauge fields far from equilibrium

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Content

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Lattice simulations of *quantum* fields

Real time:

$$\sim e^{iS}$$

non-positive definite probability measure!

→ preempts the use of standard importance sampling techniques

Nonequilibrium mapping

Quantum field dynamics is accurately described by classical-statistical evolution with MC sampling of quantum initial conditions in the large field/occupancy limit

$$\underbrace{\langle\{\Phi(x),\Phi(y)\}\rangle}_{\text{anti-commutator}} \gg \underbrace{\langle[\Phi(x),\Phi(y)]\rangle}_{\text{commutator}}$$

anti-commutator ('occupancy, field amplitudes') — commutator ('spectral function')

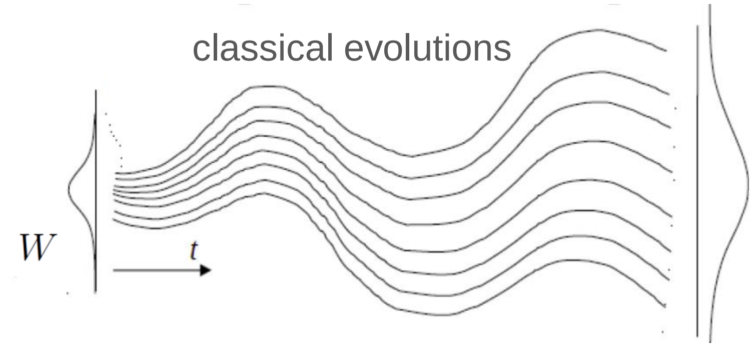
Classicality condition

Aarts, Berges, PRL 88 (2002) 041603

for bosonic fields, initial density matrix ϱ_0, $\langle\{\Phi(x),\Phi(y)\}\rangle := \mathrm{Tr}\left(\varrho_0\,\{\Phi(x),\Phi(y)\}\right)$.
In this limit, observables are obtained as ensemble averages of classical solutions:

$$\langle O\rangle_{\rm cl} = \int \mathcal{D}\phi_0\mathcal{D}\pi_0\, \underbrace{W[\phi_0,\pi_0]}_{\text{phase space density functional}}\, O_{\rm cl}[\phi_0,\pi_0]$$

phase space density functional



$O_{\rm cl}[\phi_0,\pi_0] = \int \mathcal{D}\phi\, O[\phi]\,\delta(\phi-\phi_{\rm cl}[\phi_0,\pi_0])$, initial canonical field variables $\phi_0 = \langle\Phi|_{t=0}\rangle$, π_0

Real-time simulations with fermions

Consider general class of models including lattice gauge theories

$$\mathcal{L} = \frac{1}{2}\partial\Phi^*\partial\Phi - V(\Phi) + \sum_k^{N_f} \left[i\bar{\Psi}_k \gamma^\mu \partial_\mu \Psi_k - \bar{\Psi}_k (M P_L + M^* P_R) \Psi_k \right]$$

$M \to m - g\Phi(x)$, $\quad P_L = \frac{1}{2}(1-\gamma^5)$, $\quad P_R = \frac{1}{2}(1+\gamma^5)$

$$\int \prod_k D\Psi_k^+ D\Psi_k e^{i\int \mathcal{L}(\Phi,\Psi^+,\Psi)} \quad \Longrightarrow \quad \boxed{\partial_x^2 \Phi(x) + V'(\Phi(x)) + N_f J(x) = 0}$$

$$J(x) = J^{\mathrm{S}}(x) + J^{\mathrm{PS}}(x)$$

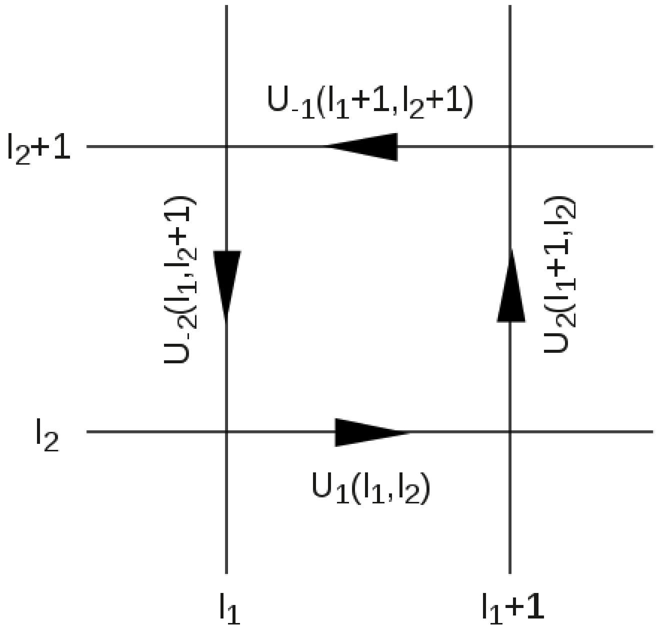
$$\begin{aligned} J^{\mathrm{S}}(x) &= -g \left\langle \bar{\Psi}(x)\Psi(x) \right\rangle = g \mathrm{Tr}\, D(x,x)\,, \\ J^{\mathrm{PS}}(x) &= -g \left\langle \bar{\Psi}(x)\gamma^5\Psi(x) \right\rangle = g \mathrm{Tr}\, D(x,x)\gamma^5 \end{aligned}$$

For classical $\Phi(x)$ the exact equation for the fermion $D(x,y)$ reads:

$$\boxed{(i\gamma^\mu \partial_{x,\mu} - m + g\mathrm{Re}\,\Phi(x) - ig\mathrm{Im}\,\Phi(x)\gamma^5) D(x,y) = 0}$$

Aarts, Smit, Nucl. Phys. B 555 (1999) 355; Borsanyi, Hindmarsh, Phys. Rev. D 79 (2009) 065010; Berges, Gelfand, Pruschke, Phys. Rev. Lett. 107 (2011) 061301; …

Example: Real-time lattice *QED*



- Dirac field:
  - $\psi(\mathbf{l},t)$ live on lattice sites
- gauge field → compact $U(1)$:
  - $U_i(\mathbf{l},t)$ live on the lattice links

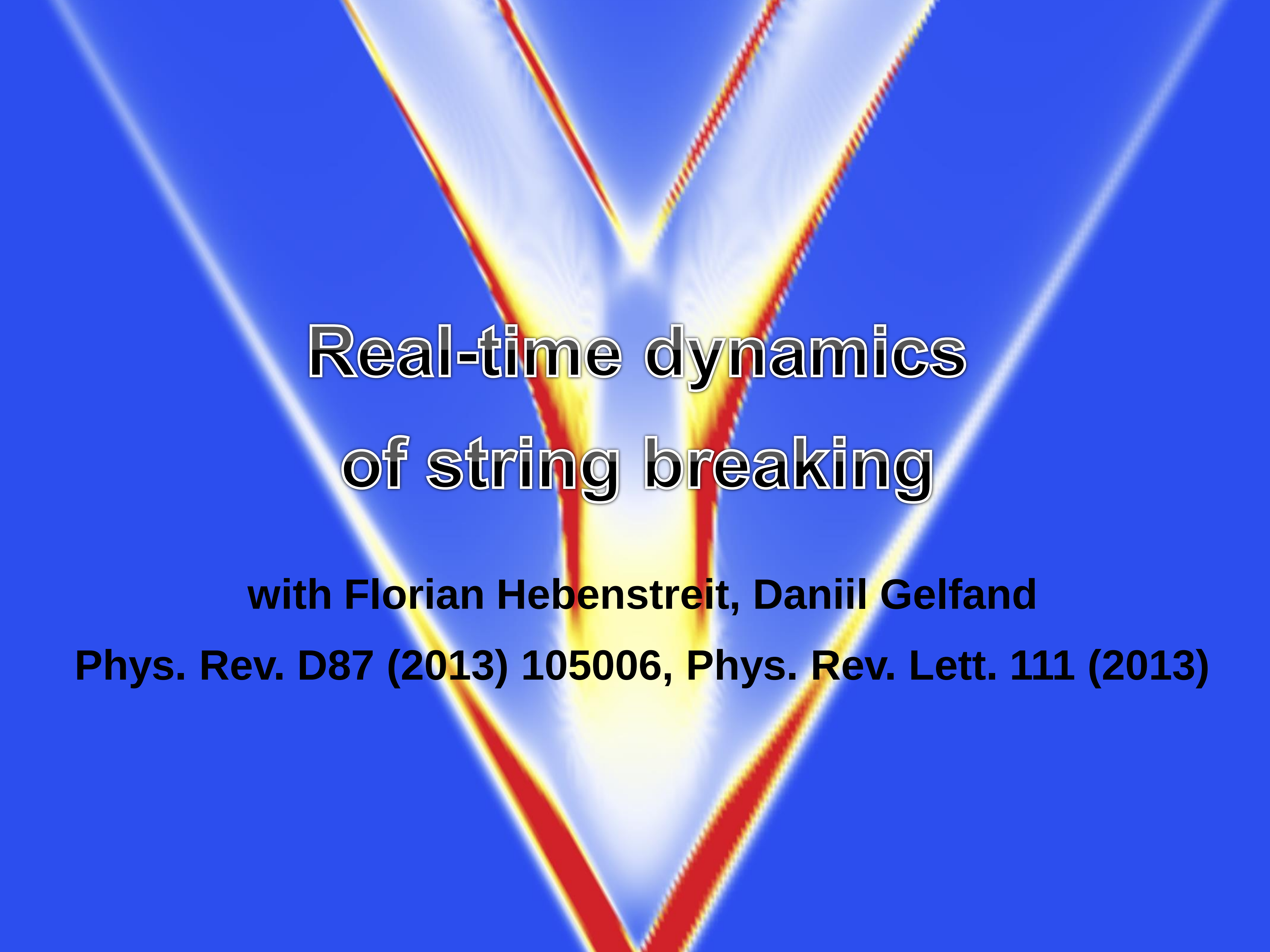
$$U_{-i}(\mathbf{l},t) = U_i^*(\mathbf{l}-\hat{i},t)$$

  - $U_i(\mathbf{l},t) = \exp[-iea_s\mathbf{A}_i(\mathbf{l},t)]$
  - $U_0(\mathbf{l},t) = 1$ (temporal gauge)
- gauge action:

$$S_G \sim \frac{a_s}{\Delta t}\sum_i \mathrm{Re}[U_{0i}(\mathbf{l},t)-1] - \frac{\Delta t}{a_s}\sum_{i<j}\mathrm{Re}[U_{ij}(\mathbf{l},t)-1]$$

  - temporal plaquette: $U_{0i}(\mathbf{l},t) = U_i(\mathbf{l},t+\Delta t)U_i^*(\mathbf{l},t)$
  - spatial plaquette: $U_{ij}(\mathbf{l},t) = U_i(\mathbf{l},t)U_j(\mathbf{l}+\hat{i},t)U_i^*(\mathbf{l}+\hat{j},t)U_j^*(\mathbf{l})$
- $\mathbf{E}_i(\mathbf{l},t) \sim \mathrm{Im}[U_{0i}(\mathbf{l},t)]$

  $\mathbf{B}_i(\mathbf{l},t) \sim \epsilon_{ijk}\,\mathrm{Im}[U_{jk}(\mathbf{l},t)]$



Real-time dynamics of string breaking

with Florian Hebenstreit, Daniil Gelfand

Phys. Rev. D87 (2013) 105006, Phys. Rev. Lett. 111 (2013)

Pair production in *QED*

Motivation: Schwinger effect

- QED vacuum is unstable in presence of external fields
- analytic solution for vacuum decay rate in static electric field:

$$\mathscr{P}[vac] = \frac{[eE_0]^2}{4\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n\pi m^2}{eE_0}\right)$$

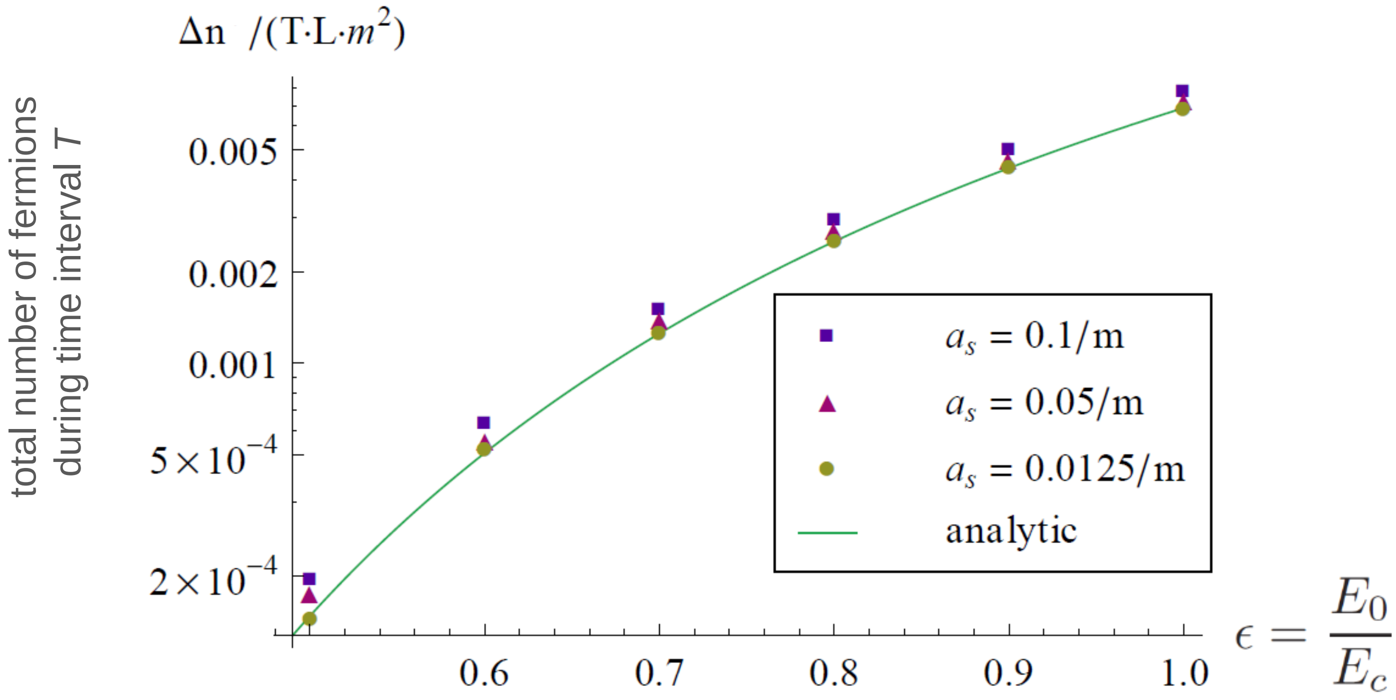
  - exponential suppression → tunneling effect → non-perturbative
  - critical field strength: $E_{cr} = \frac{m^2}{e} \sim 10^{18}\, V/m$

Schwinger, Phys. Rev. 82 (1951) 664

- create required field strength with colliding laser pulses?!
  - X-ray laser: European X-Ray Free Electron Laser
  - optical laser: Extreme Light Infrastructure

Check

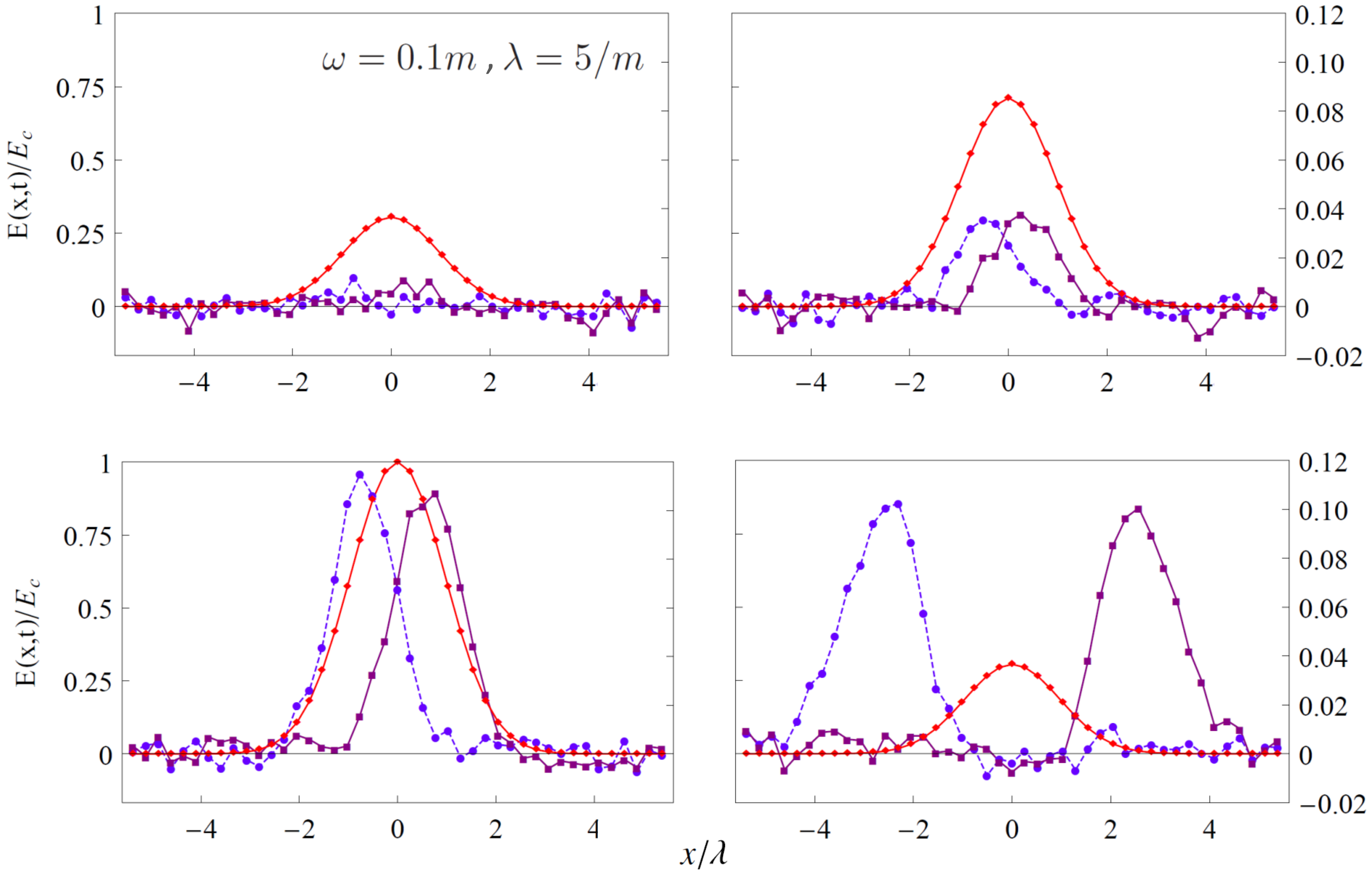
- Accurate reproduction of Schwinger formula from lattice simulations for constant E_0



1024 – 4096 spatial lattices

Real-time pair production

- spatial distribution $n(x,t)$ from initial electric pulse $E(x,t) = E_0 \, \mathrm{sech}^2(\omega t) \exp\left(-\frac{x^2}{2\lambda^2}\right)$



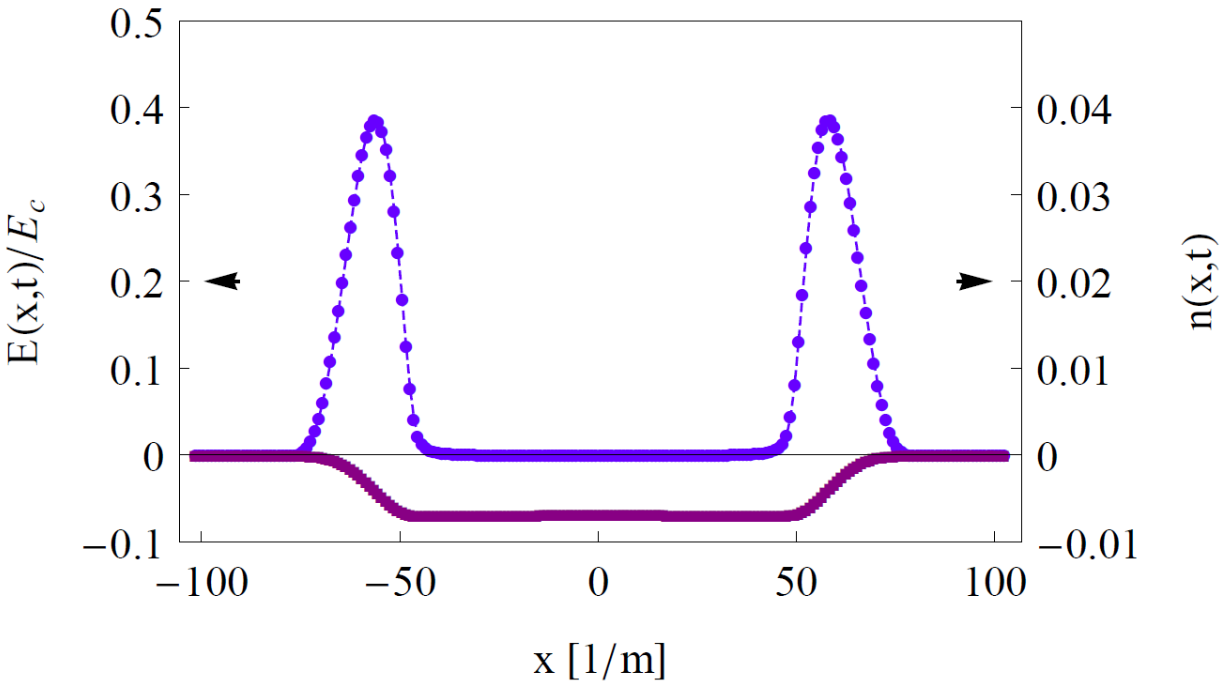
Hebenstreit, Berges, Gelfand, Phys. Rev. D87 (2013) 105006

1d capacitor

- self-consistent electric field
  - fermion bunches act as a capacitor
  - homogeneous field between bunches (charge: $\pm Q$): $E_{\rm str} = -Q$
  - linear rising potential between bunches

$$\mathscr{E}_{\rm str}(d) = \frac{1}{2}\int E_{\rm str}^2 dx = \frac{Q^2 d}{2}$$

  - secondary particle creation? $\longrightarrow$ 'string breaking'

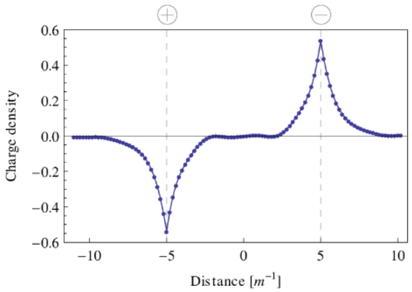


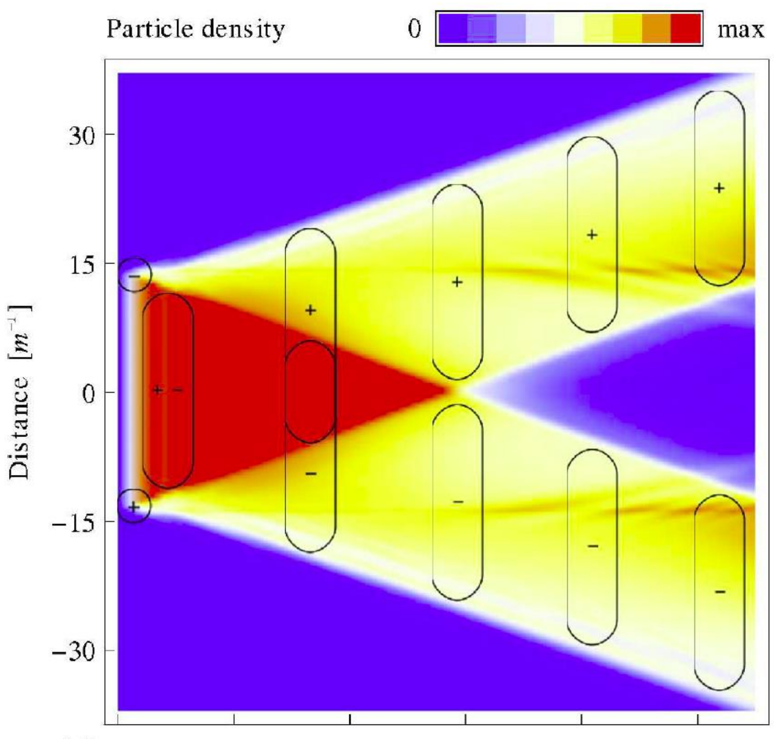
Hebenstreit, Berges, Gelfand, Phys. Rev. D87 (2013) 105006

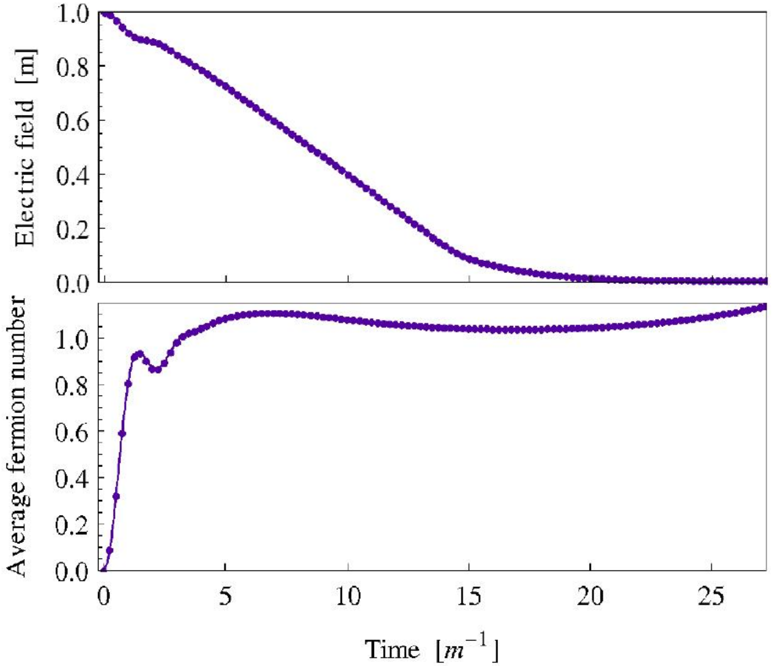
Real-time dynamics of string breaking

- static configuration: external sources with charge $\pm e$
  - string breaking: screening of sources $\longrightarrow$ electric field $\rightarrow 0$
  - two time scales: creation (short) and separation (long)
  - energy condition: $\mathcal{E}_{\text{str}} \simeq 2m + \mathcal{W}$

    $\longrightarrow$ most energy (80%) needed for charge separation!



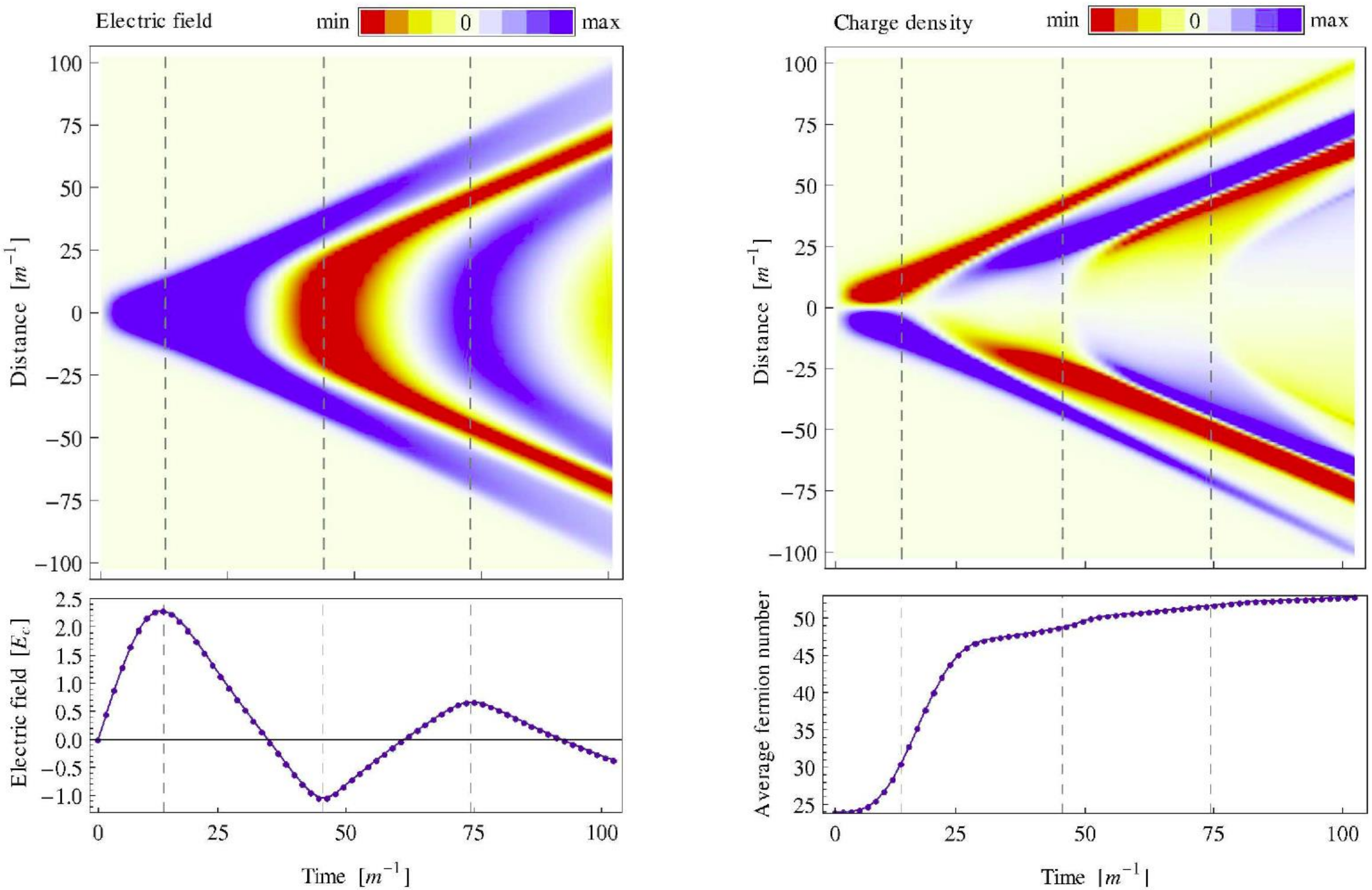




Hebenstreit, Berges, Gelfand, Phys. Rev. Lett. 111 (2013), arXiv:1307.4619

Multiple string breaking

- dynamic configuration: bunches with charge $\pm Q$
  - string breaking: screening of sources
  - additional kinetic energy available
  - multiple string breaking: zero crossings of electric field



Turbulent thermalization of the Quark Gluon Plasma

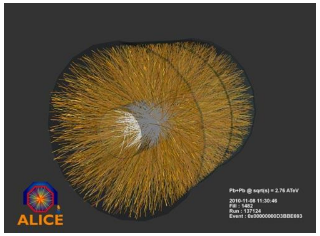
with Kirill Boguslavski, Sören Schlichting, Raju Venugopalan

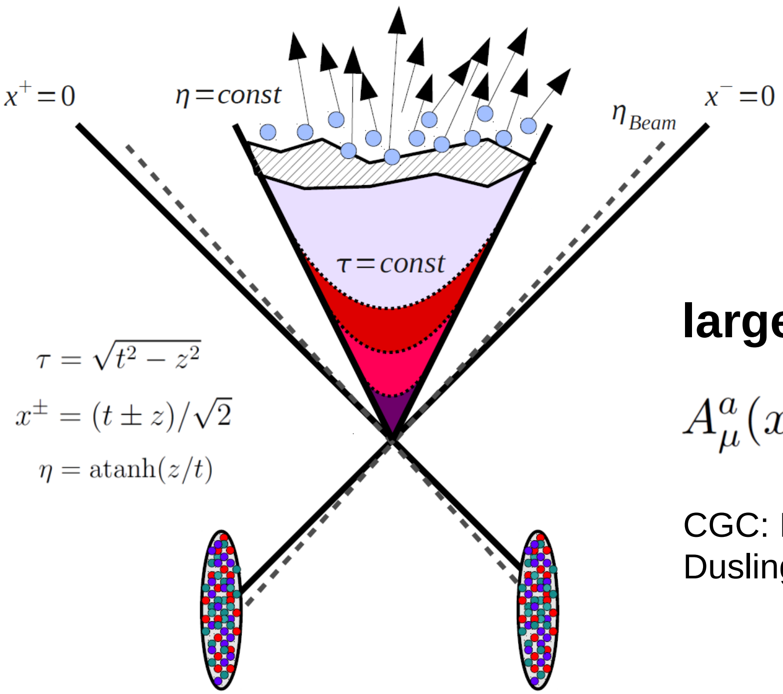
arXiv:1303.5650, arXiv:1311.3005

ALICE

Particle production in non-Abelian gauge theory

- initial stages of ultra-relativistic heavy-ion collisions





here: $g \ll 1$

large initial fields:

$$A_\mu^a(x) = \langle \hat{A}_\mu^a(x) \rangle \sim \mathcal{O}(1/g)$$

CGC: Lappi, McLerran, Dusling, Gelis, Venugopalan,…

- **small** initial (vacuum) fluctuations:

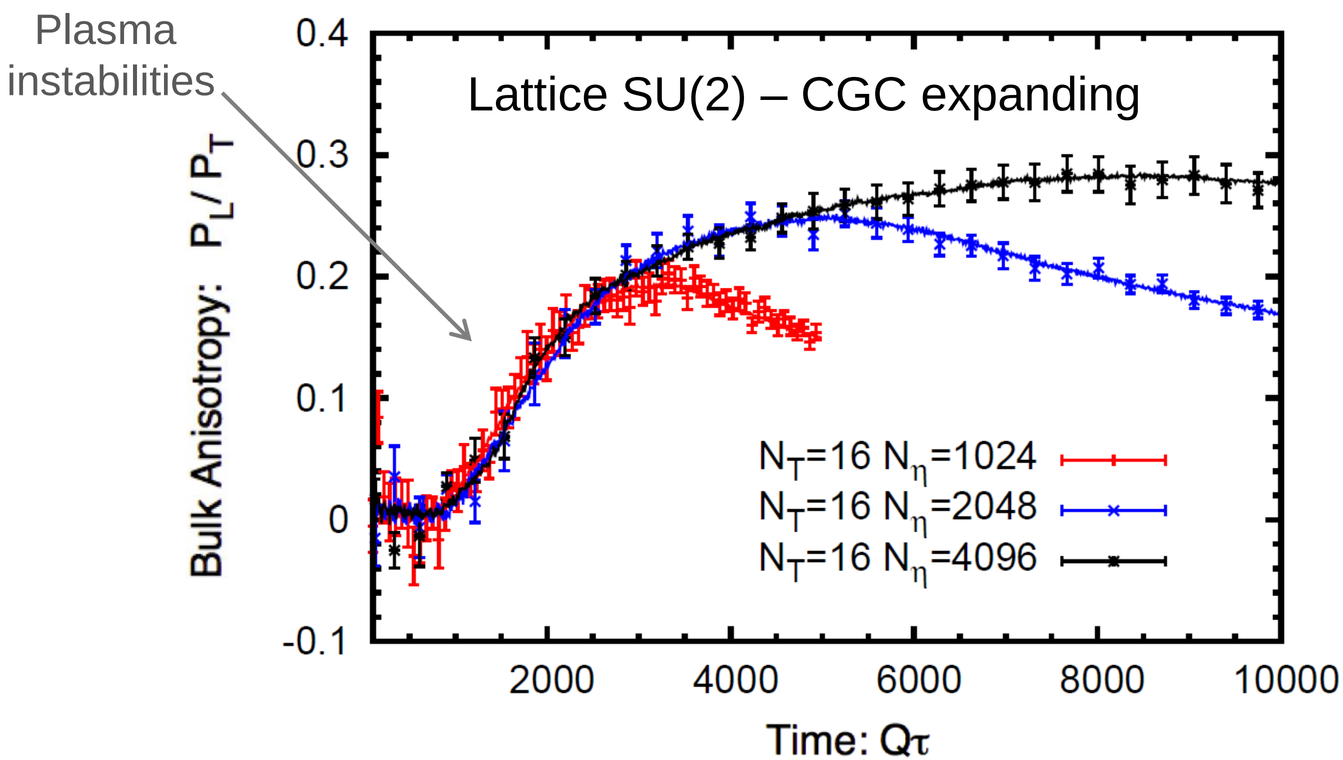
$$F_{\mu\nu}^{ab}(x,y) = \frac{1}{2}\left\langle \left\{ \hat{A}_\mu^a(x), \hat{A}_\nu^b(y) \right\} \right\rangle - A_\mu^a(x) A_\nu^b(y)$$

$$\sim \mathcal{O}(1)$$

→ ***plasma instabilities!***

Mrowczynski; Rebhan, Romatschke, Strickland; Arnold, Moore, Yaffe …

Longitudinally expanding SU(2) gauge theory



Redshift & dilution require large lattices!

Berges, Schlichting PRD 87 (2013) 014026

Which thermalization scenario is realized?

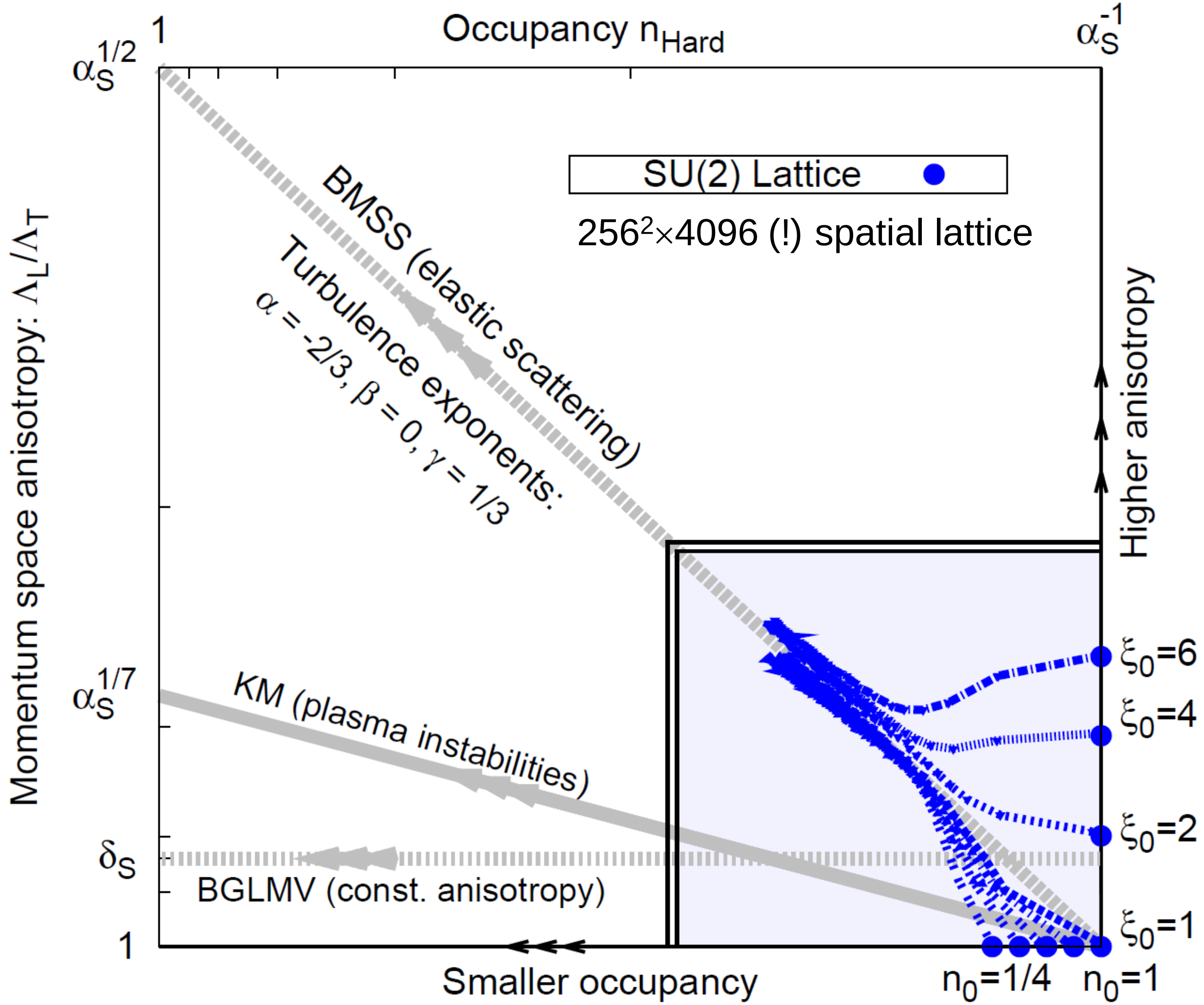
'BMSS': Baier, Mueller, Schiff, Son, PLB 502 (2001) 51; 'KM': Kurkela, Moore, JHEP 1111 (2011) 120;

'BGLMV': Blaizot, Gelis, Liao, McLerran, Venugopalan, NPA 873 (2012) 68; …

Insensitivity to initial conditions: Nonthermal fixed point

Initial 'gluon distribution': (Coulomb gauge)

$$f(p_T, p_z, t_0) = \frac{n_0}{\alpha_s}\,\Theta\left(Q - \sqrt{p_T^2 + (\xi_0 p_z)^2}\right)$$



Berges, Boguslavski, Schlichting, Venugopalan, arXiv:1303.5650, arXiv:1311.3005

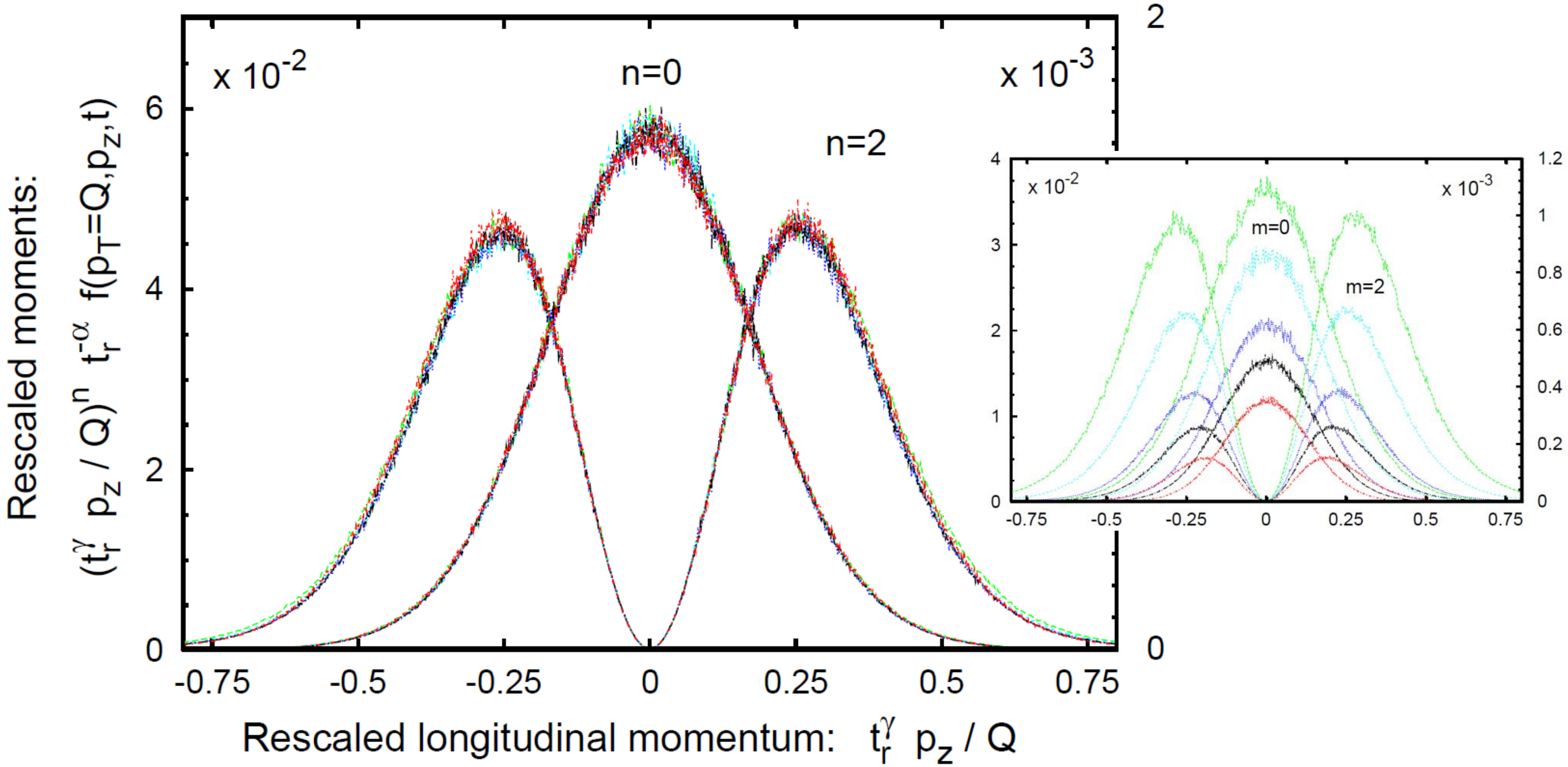
Self-similar evolution

Self-similar distribution: $f(p_T, p_z, t) = t^\alpha \underbrace{f_S(t^\beta p_T, t^\gamma p_z)}$

(Coulomb gauge)

$Qt = 750 \text{ to } 4000$

stationary fixed-point distribution



Nature of nonthermal fixed point: wave turbulence

Berges, Boguslavski, Schlichting, Venugopalan, arXiv:1303.5650, arXiv:1311.3005

Boltzmann equation with generic collision term for longitudinal expansion:

$$\left[\partial_t - \frac{p_z}{t}\partial_{p_z}\right] f(p_T, p_z, t) = C[p_T, p_z, t; f]$$

Self-similar evolution:

$$f(p_T, p_z, t) = t^\alpha f_S(t^\beta p_T, t^\gamma p_z) \;\; , \quad C[p_T, p_z, t; f] = t^\mu \, C[t^\beta p_T, t^\gamma p_z; f_S]$$

→ a) **fixed point equation for stationary distribution**:

$$\alpha f_S(p_T, p_z) + \beta p_T \partial_{p_T} f_S(p_T, p_z + (\gamma - 1) \, p_z \partial_{p_z} f_S(p_T, p_z) = C[p_T, p_z; f_S]$$

→ b) **scaling condition**:

$$\boxed{\alpha - 1 = \mu(\alpha, \beta, \gamma)}$$

Nature of nonthermal fixed point II

Interpret scaling condition with **energy/number conserving*** Fokker-Planck-type dynamics for the collisional broadening of longitudinal momentum distribution:

$$C^{(\mathrm{elast})}[p_T, p_z; f] = \hat{q}\, \partial^2_{p_z} f(p_T, p_z, t)$$

with **momentum diffusion parameter**: $\hat{q} \sim \alpha_S^2 \int \frac{d^2 p_T}{(2\pi)^2} \int \frac{dp_z}{2\pi}\, f^2(p_T, p_z, t)$

→ 1) $\mu = 3\alpha - 2\beta + \gamma$ $\xrightarrow{\alpha - 1 = \mu(\alpha,\beta,\gamma)}$ $2\alpha - 2\beta + \gamma + 1 = 0$

2) number conservation $\longrightarrow$ $\alpha - 2\beta - \gamma + 1 = 0$

3) energy conservation $\longrightarrow$ $\alpha - 3\beta - \gamma + 1 = 0$

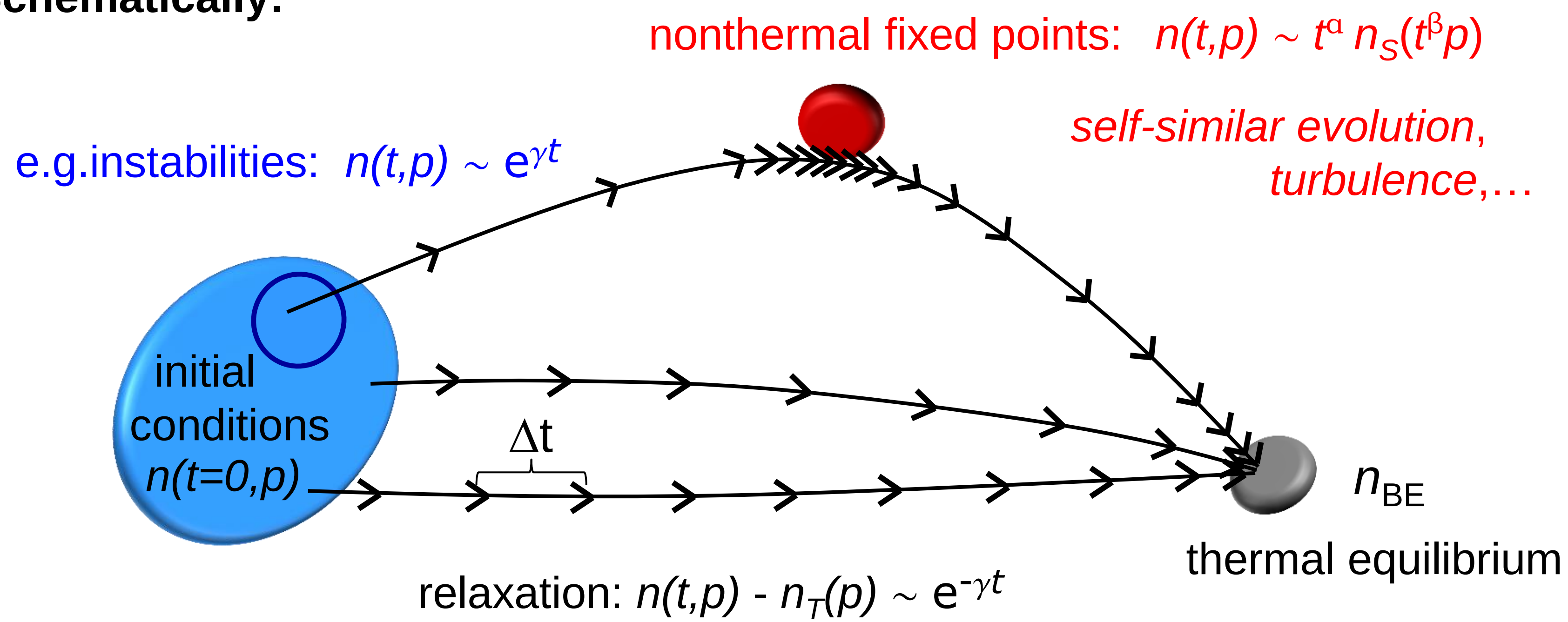
remarkable agreement with lattice data!

$$\alpha = -2/3\,, \qquad \beta = 0\,, \qquad \gamma = 1/3$$

*cf. early stages of Baier, Mueller, Schiff, Son, PLB 502 (2001) 51 (‘BMSS‘)

Nonthermal fixed points in quantum many-body systems

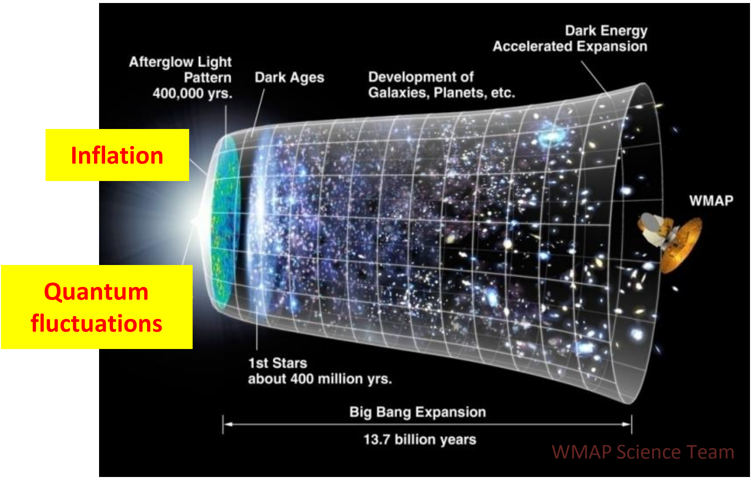
Schematically:

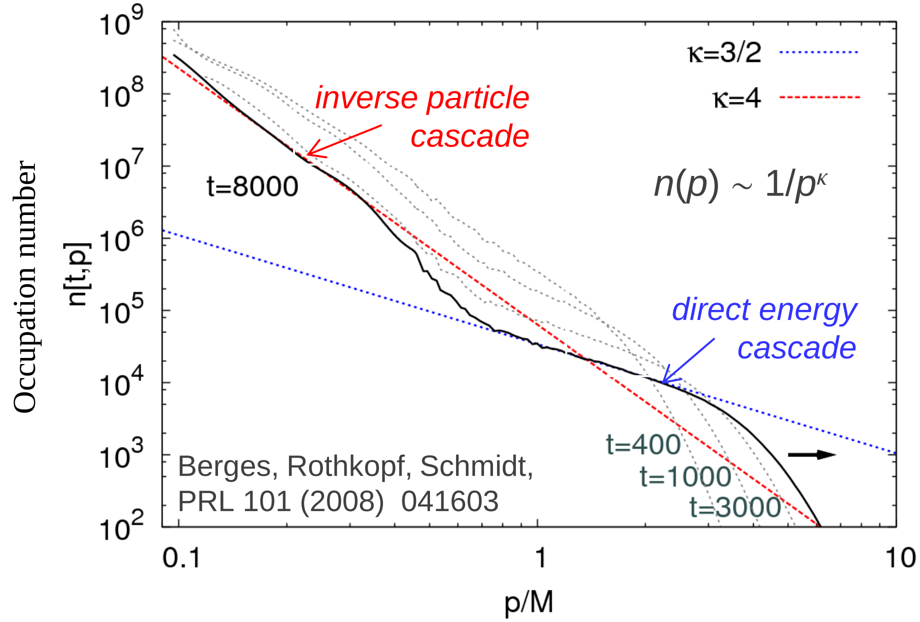


Universality far from equilibrium

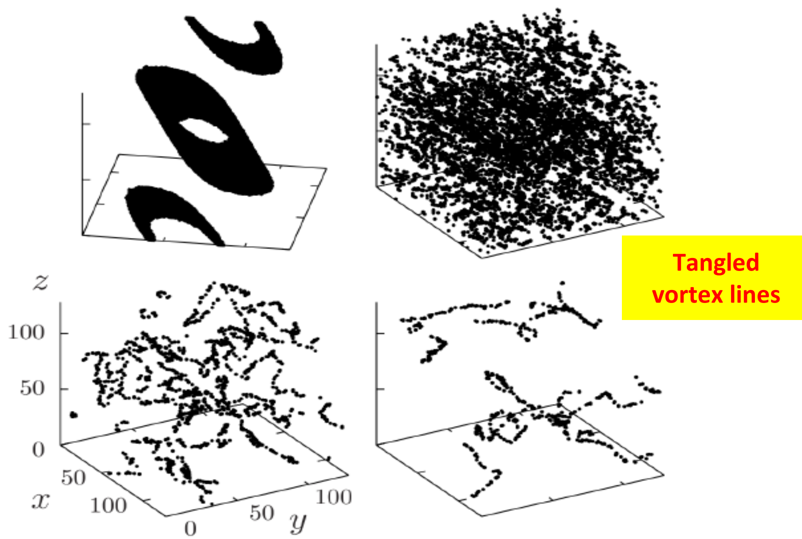
Universality far from equilibrium

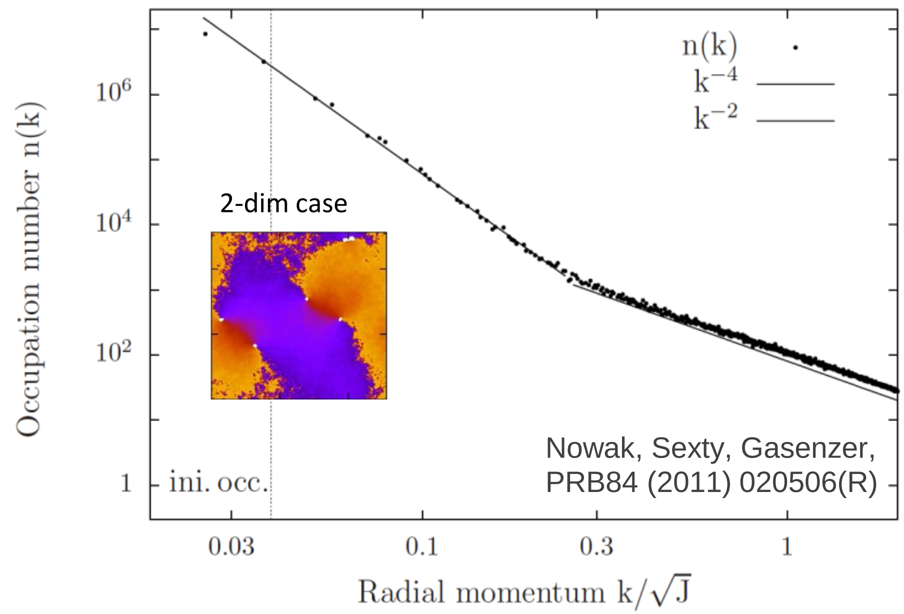
- Reheating dynamics after chaotic inflation





- Superfluid turbulence in a cold Bose gas





Conclusions

Real-time lattice gauge theory techniques provide powerful tool for ab initio calculations in a large range of **nonequilibrium problems**

Nonequilibrium particle production:

→ QED_{1+1}: real-time dynamics of **string breaking**

Thermalization dynamics:

→ QCD_{3+1} with longitudinal expansion: Turbulent thermalization/ Universality far from equilibrium near **nonthermal fixed points**

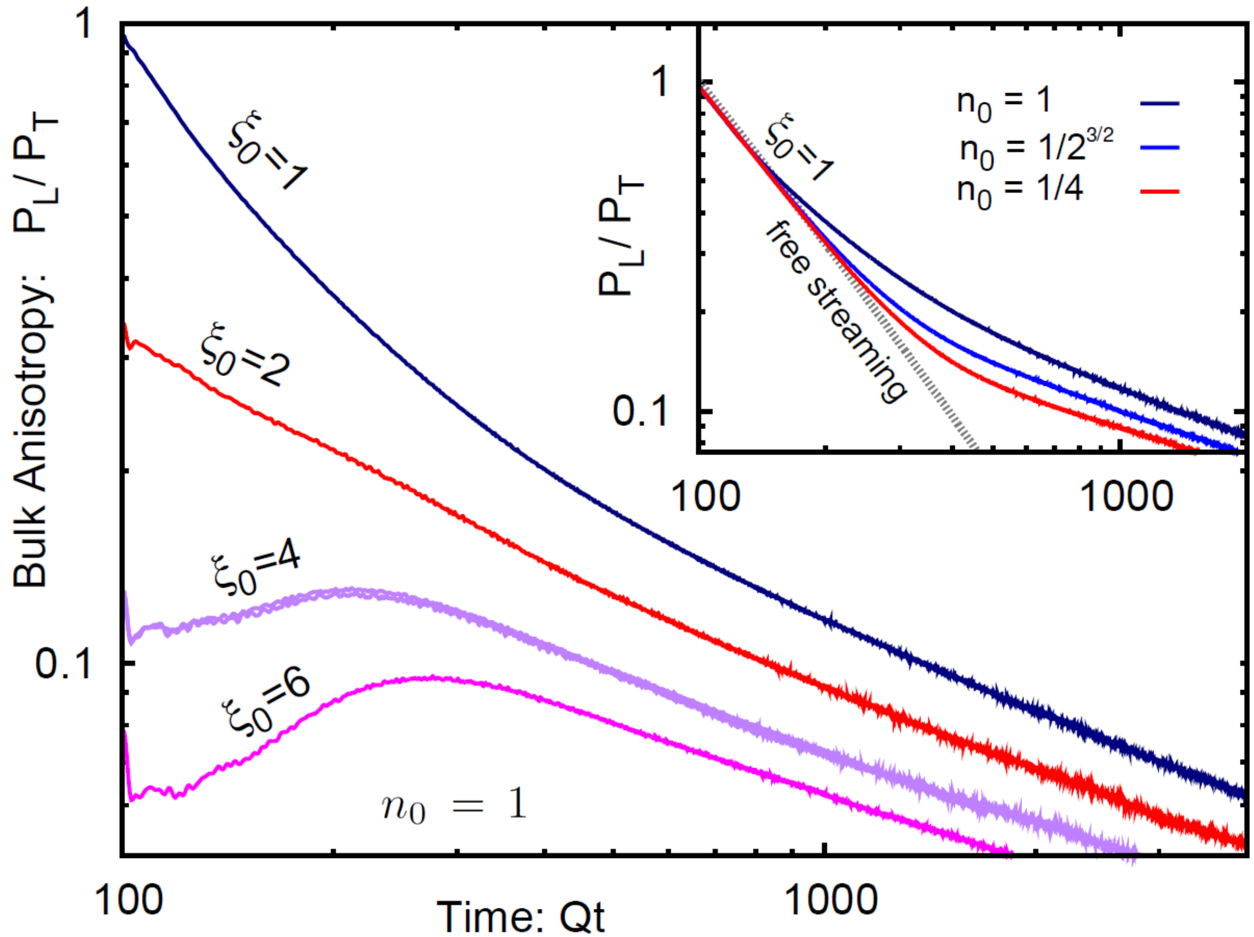
Insensitivity to initial condition details

occupancy parameter

Initial gluon distribution: $$f(p_T, p_z, t_0) = \frac{n_0}{\alpha_s}\Theta\left(Q - \sqrt{p_T^2 + (\xi_0 p_z)^2}\right)$$

anisotropy parameter

Berges, Boguslavski, Schlichting, Venugopalan, arXiv:1303.5650



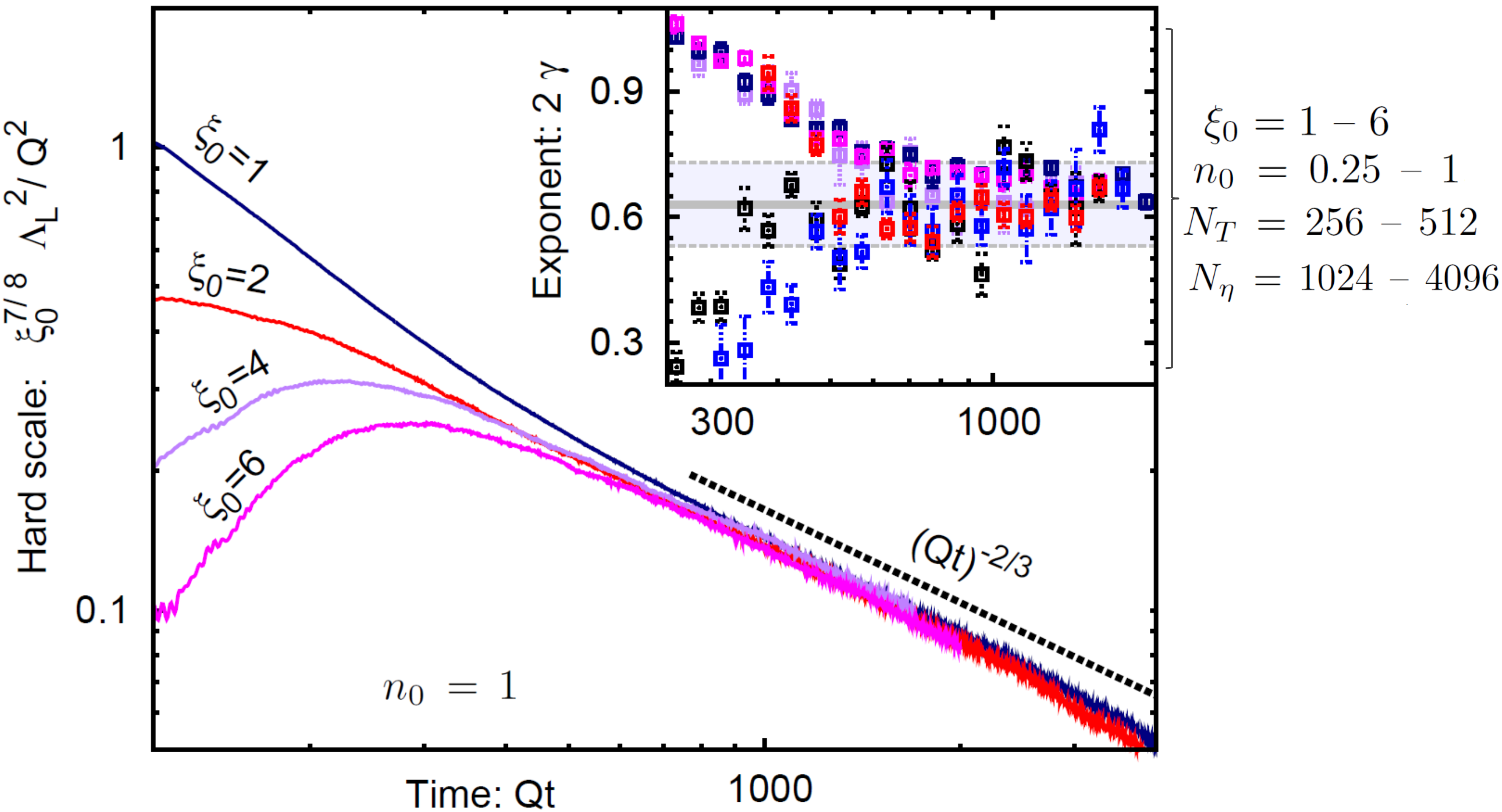
$256^2 \times 4096$

spatial lattice

Universal scaling exponents

Typical ('hard') momenta: $\Lambda_L^2(t) \sim (Qt)^{-2\gamma}\,, \quad \Lambda_T^2(t) \sim (Qt)^{-2\beta}$

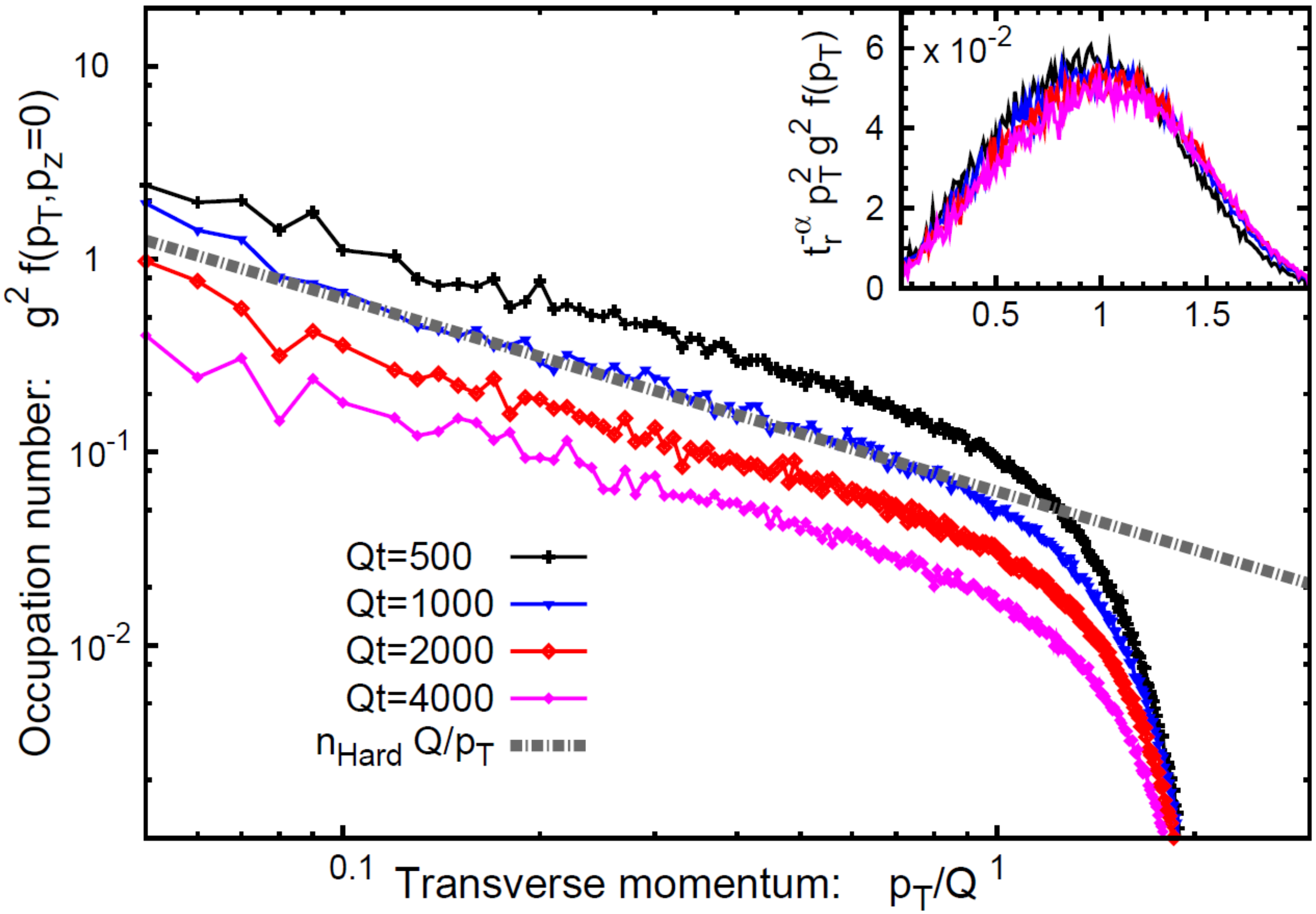
$2\gamma \;=\; 0.67 \pm 0.07$, monoton. decr. $|\beta| < 0.04$ for $Qt \gtrsim 650$

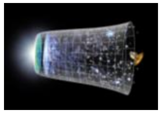


‘Thermal‘ $1/p_T$ spectrum far from equilibrium

Self-similar evolution with decreasing amplitude:

$$n_{\rm Hard}(t) = f(p_T = Q, p_z = 0, t)$$

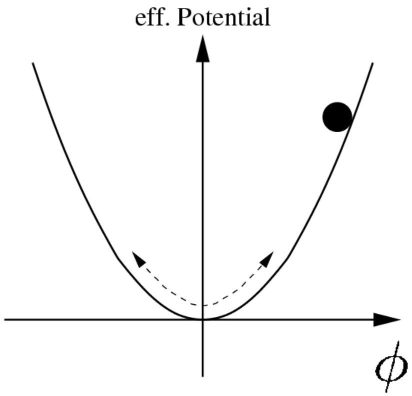




A well understood *quantum* example

Early universe preheating:

Kofman, Linde, Starobinsky, PRL 73 (1994) 3195



Scalar $\lambda\Phi^4$ inflaton: $\lambda \ll 1$

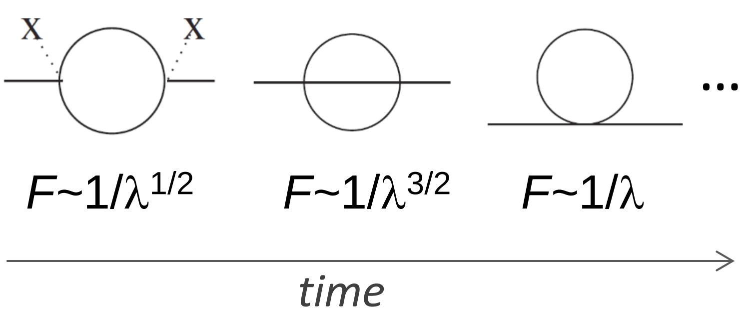
- **large** initial field $\phi = \langle\Phi\rangle \sim 1/\lambda^{1/2}$
- **small** fluctuation $F \sim \langle\{\Phi,\Phi\}\rangle - \phi\phi \sim 1$

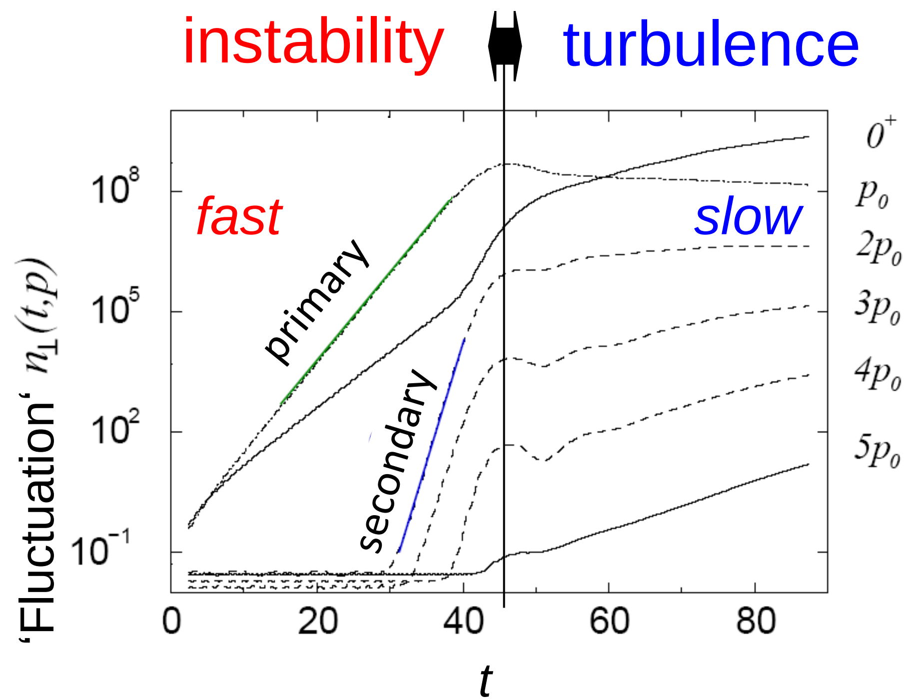
Instability: $F(t) \sim e^{\gamma t}$ $(\gamma > 0)$

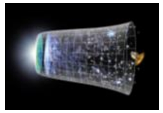
Quantum field theory:

Berges, Serreau, PRL 91 (2003) 111601

Dynamical power counting (2PI):

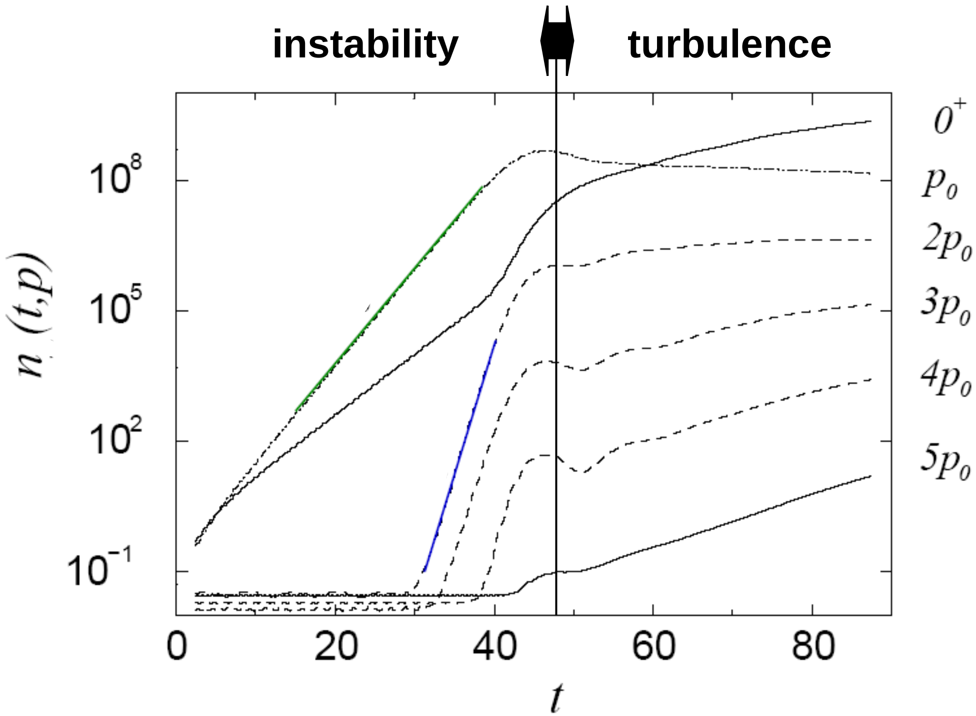




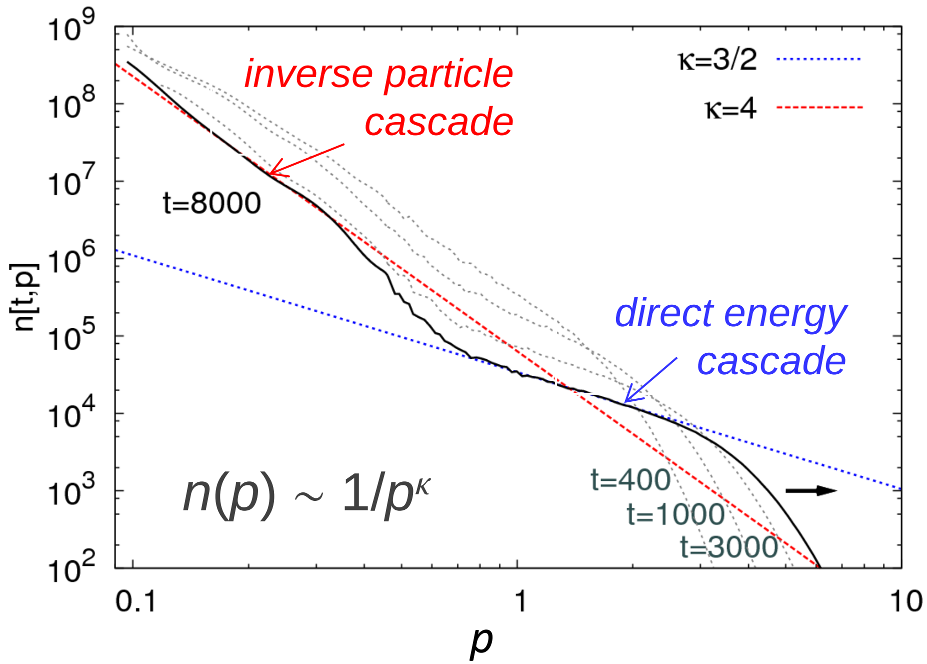


Turbulent thermalization

instability | **turbulence**



dual cascade



Direct cascade: Micha, Tkachev, PRL 90 (2003) 121301, …

Inverse cascade: Berges, Rothkopf, Schmidt, PRL 101 (2008) 041603, … ($\kappa = d+z-\eta$)

Bose condensation: Berges, Sexty, PRL 108 (2012) 161601, …

Cascade dynamics described by universal power laws!

self-similar evolution, same universality class as classical-statistical field theory