QCD spectral functions and transport coefficients from functional methods

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Outline

- **Gauge dynamics far from equilibrium**

- **Spectral functions and transport coefficients**

- **Summary and outlook**

Outline

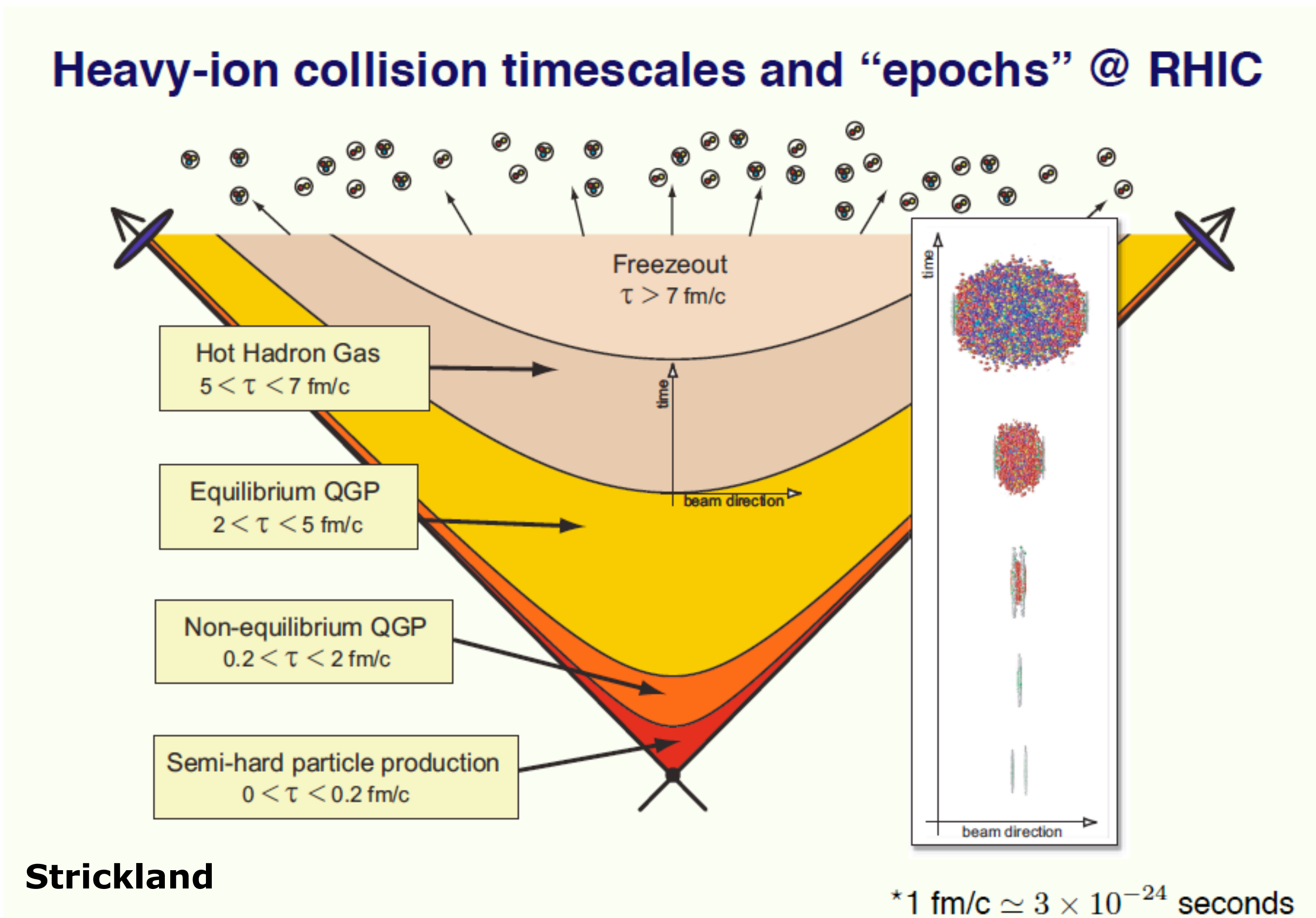
- **Gauge dynamics far from equilibrium**

- **Spectral functions and transport coefficients**

- **Summary and outlook**

Introduction and motivation: thanx to Jürgen Berges, Andreas Schäfer, ...

Gauge dynamics far from equilibrium



Gauge dynamics far from equilibrium

Abelian Higgs model in 2+1 dim

Gasenzer, McLerran, JMP, Sexty '13

Classical action:

$$S[A_\mu, \phi] = -\int_x \left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* D^\mu \phi + V(\phi)\right]$$

ϕ **Higgs**

phase $\frac{\phi}{|\phi|} = e^{i\,\varphi}$

Gauge dynamics far from equilibrium

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Classical action of Yang-Mills theory in diagonalisation gauges:

$$S_{\rm YM} \simeq \frac{1}{2}\int_x {\rm tr}F^2_{\bar\mu\bar\nu} + \frac{1}{2}\int_x {\rm tr}\,(D_{\bar\mu}A_2)^2$$

$$A_2 = A_2^c(x_0,x_1)$$

Wilson loop

$$\mathcal{W}_2 = \mathcal{P}\exp\left\{i\int_0^{L_2} dx_2\, A_2(x)\right\} = \exp\{i\,\phi\}$$

Vortex winding

$$n(\mathcal{S}) = \frac{1}{16\pi\, i}\oint_{\mathcal{S}} d^2x\,\epsilon_{ij}{\rm tr}\,\hat\phi\,\partial_i\hat\phi\,\partial_j\hat\phi$$

phase

$$\hat\phi = \frac{\phi}{\|\phi\|}$$

Quiz

Complex scalar vs Abelian Higgs

Gasenzer, McLerran, JMP, Sexty '13

phase φ of scalar field

'tachyonic' initial conditions

classical statistical lattice simulations

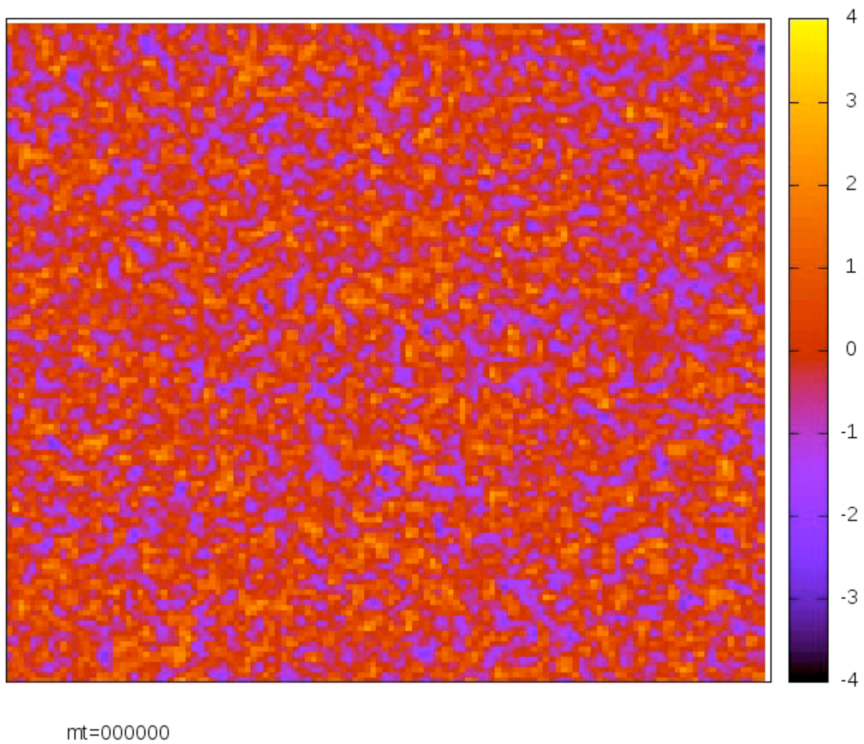
Which is which?

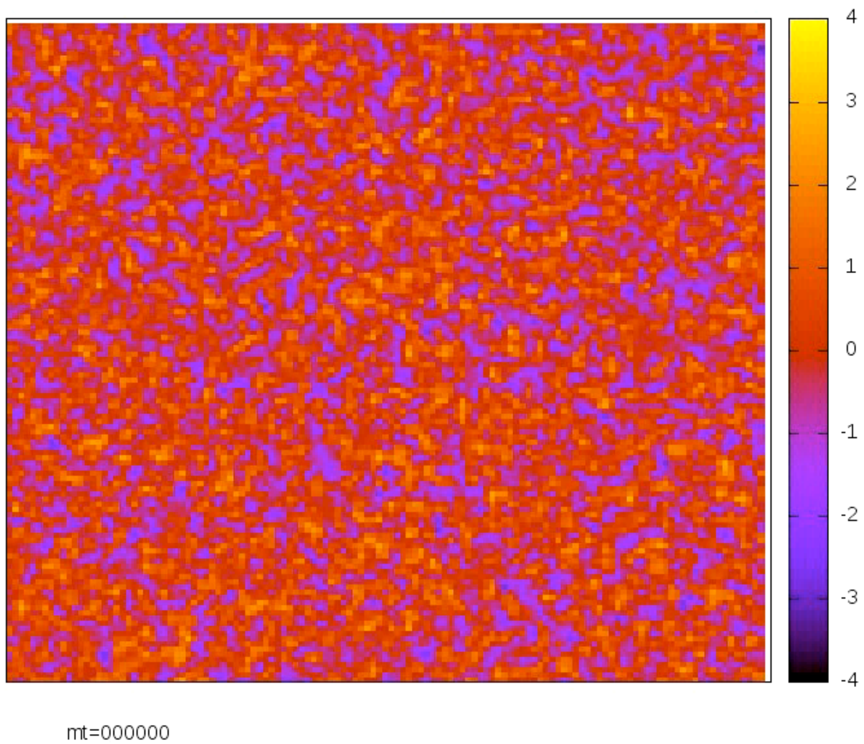
Quiz

Complex scalar vs Abelian Higgs

Gasenzer, McLerran, JMP, Sexty '13

phase φ of scalar field





'tachyonic' initial conditions

classical statistical lattice simulations

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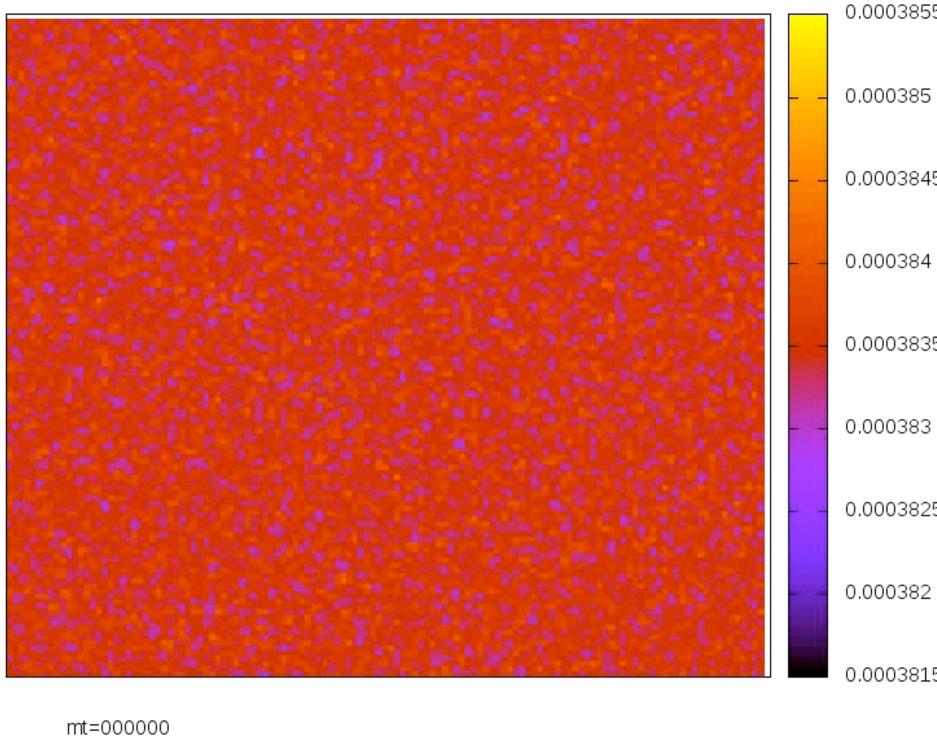
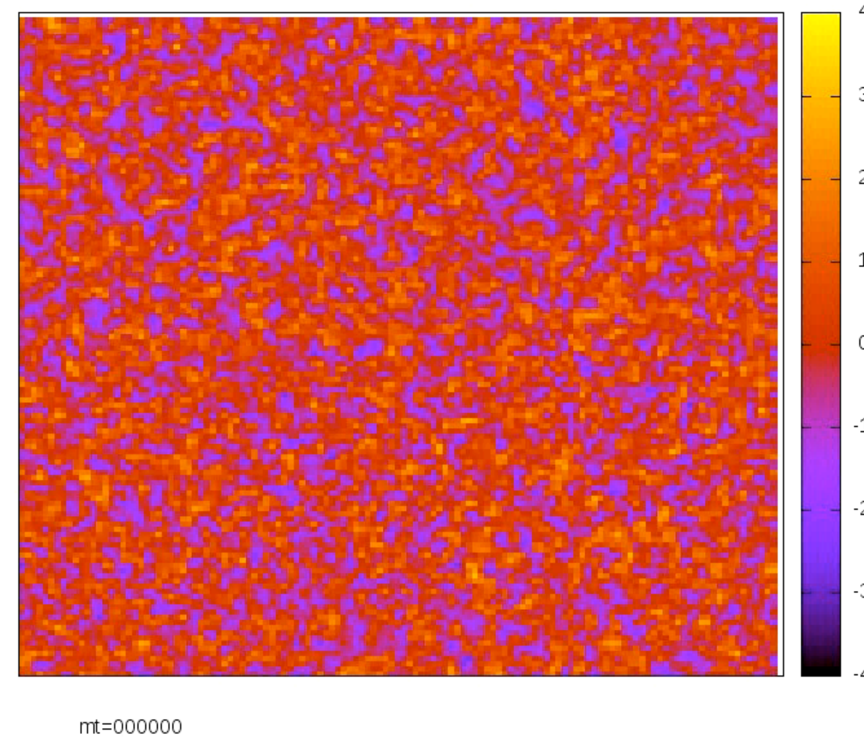
Gauge dynamics far from equilibrium

Abelian Higgs model in 2+1 dim

Gasenzer, McLerran, JMP, Sexty '13

magnetic field | **phase of Higgs**





'tachyonic' initial conditions

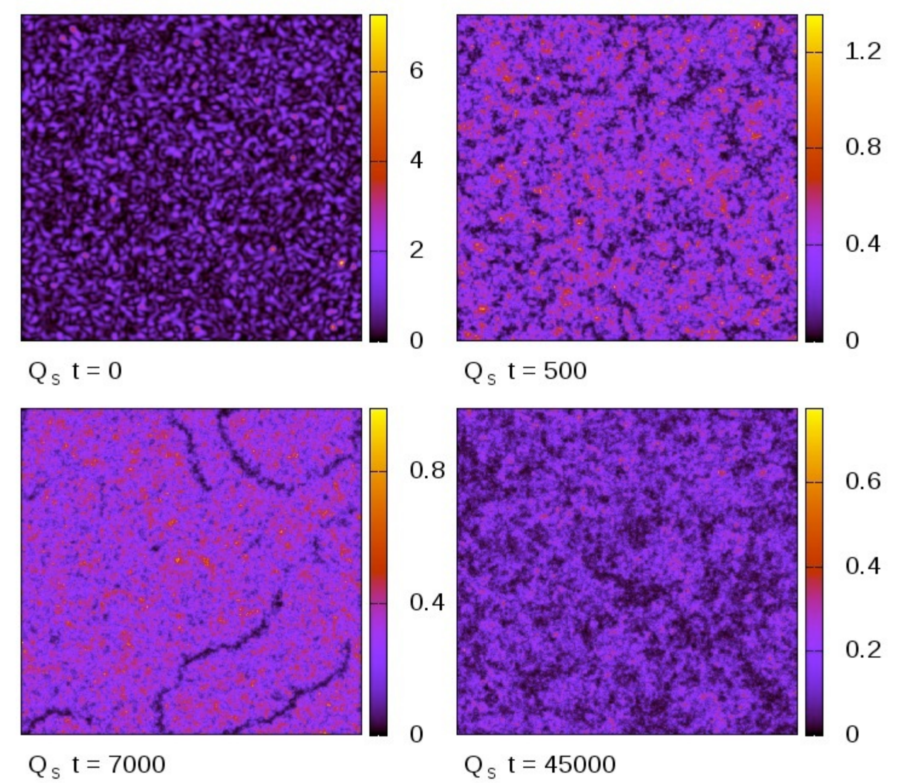
classical statistical lattice simulations

Gauge dynamics far from equilibrium

Abelian Higgs model in 2+1 dim

'overpopulation' initial conditions

modulus of Higgs

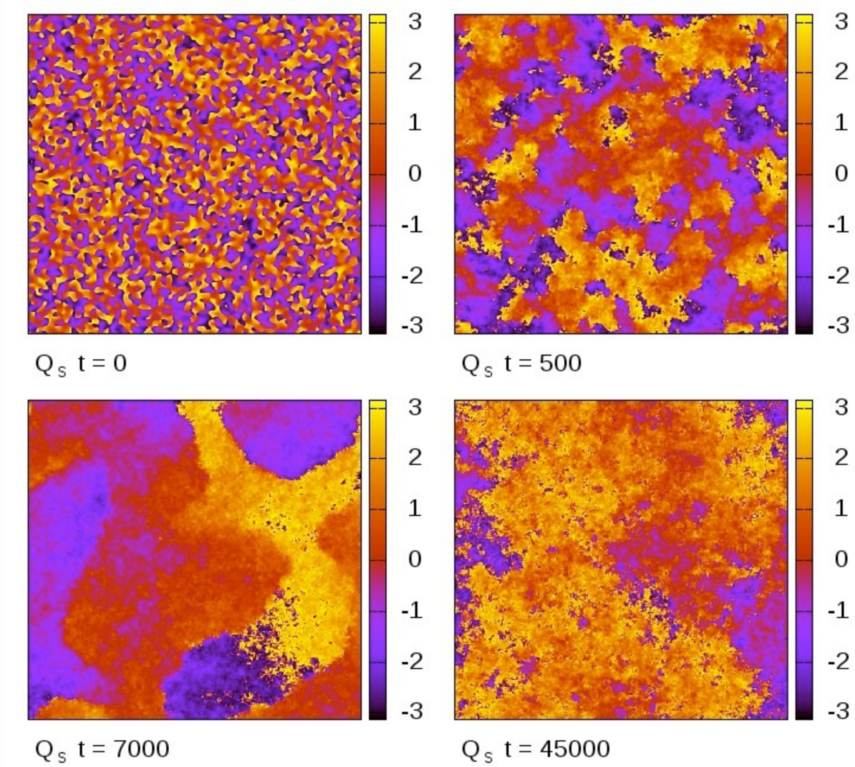


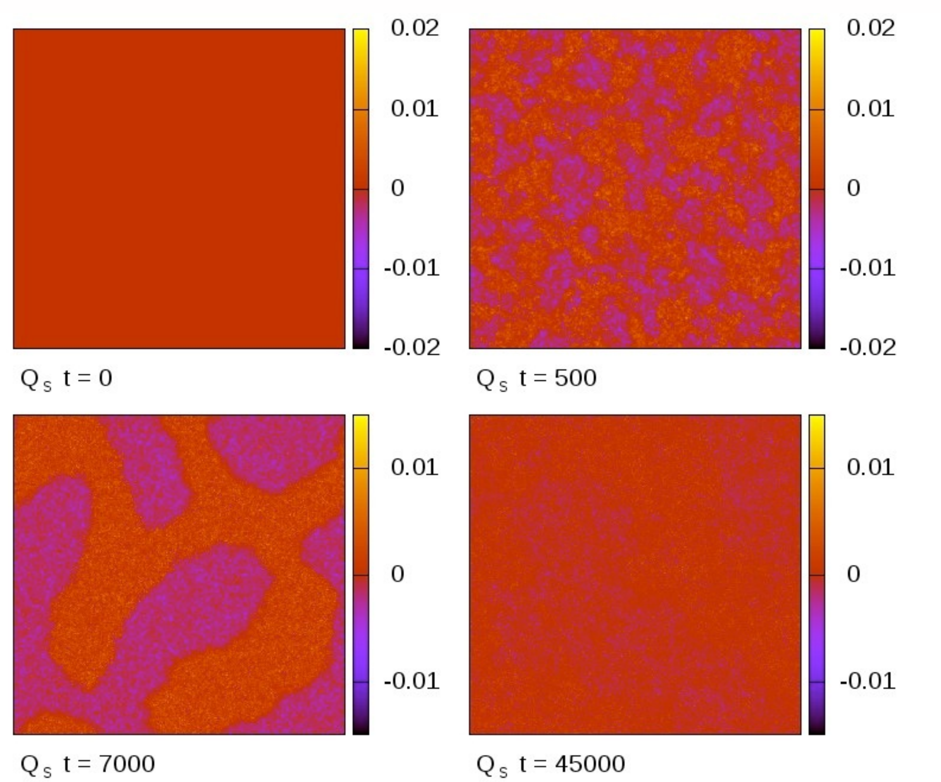
$$\xi = 0.025$$

coupling

$$\xi = \frac{6e^2}{\lambda}$$

relative phase





charge

$$\varphi^U(\vec{x},t) = \arg(G^U(\vec{0},\vec{x},t))$$

relative phase

$$G^U(\vec{x},\vec{y},t) = \langle \phi(\vec{x},t) U(\vec{x},\vec{y},t) \phi(\vec{y},t)^* \rangle_{\mathrm{cl}}$$

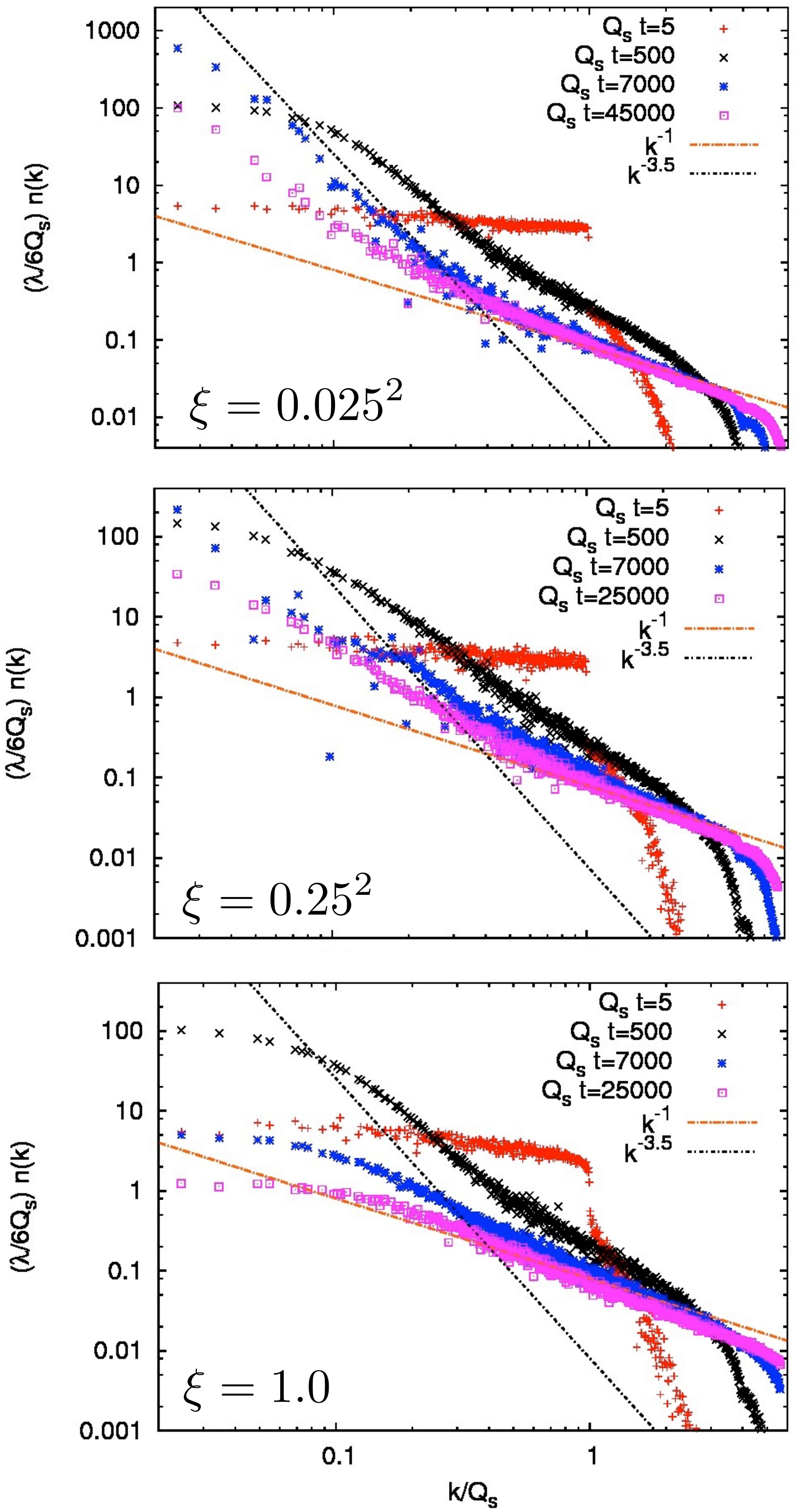
parallel transport U

Gasenzer, McLerran, JMP, Sexty '13

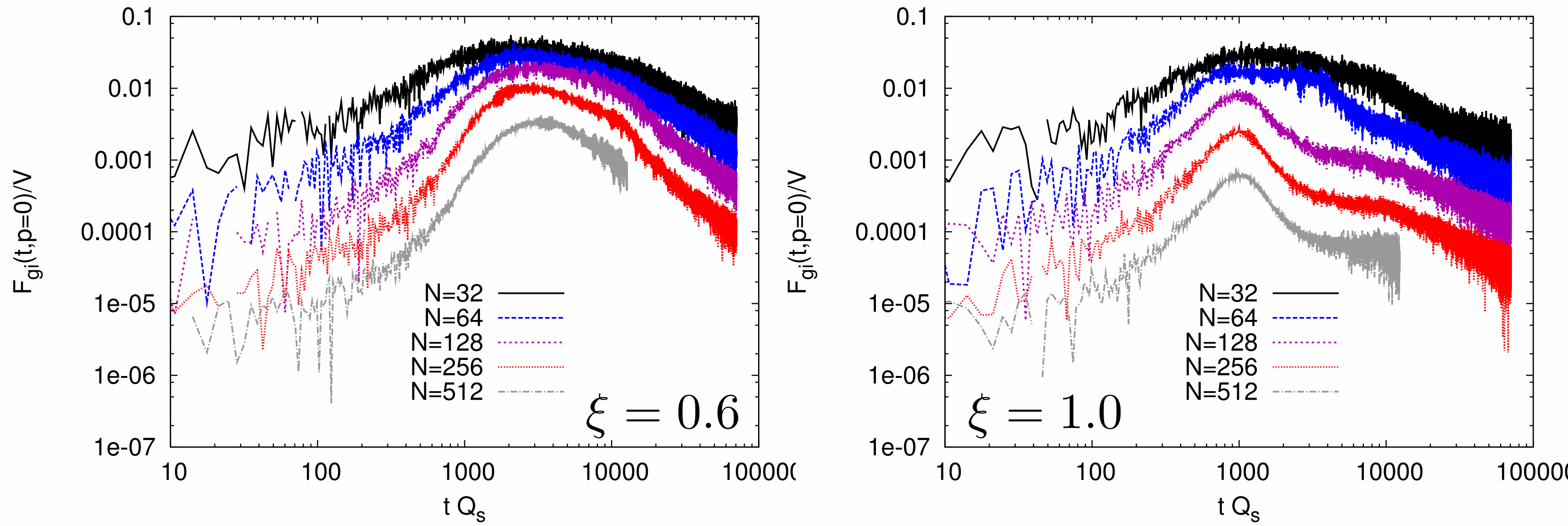
Gauge dynamics far from equilibrium

Abelian Higgs model in 2+1 dim

'overpopulation' initial conditions **Gasenzer, McLerran, JMP, Sexty '13**



$$\frac{F_{\rm gi}(p=0)}{V} = \frac{1}{V^2}\int dxdy\phi^*(x)U(x,y)\phi(y)$$



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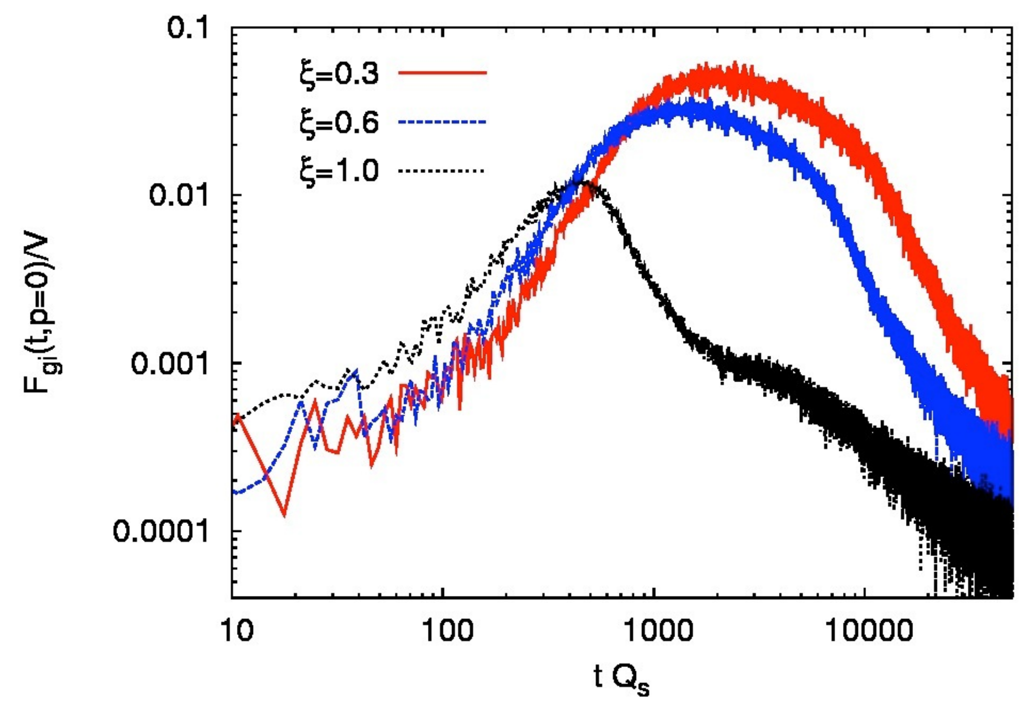
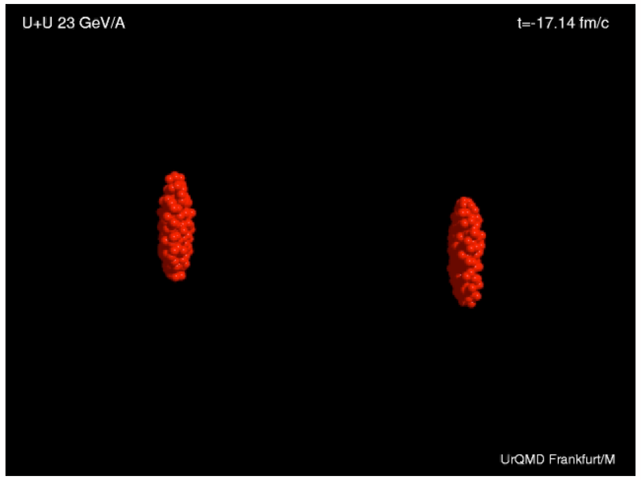


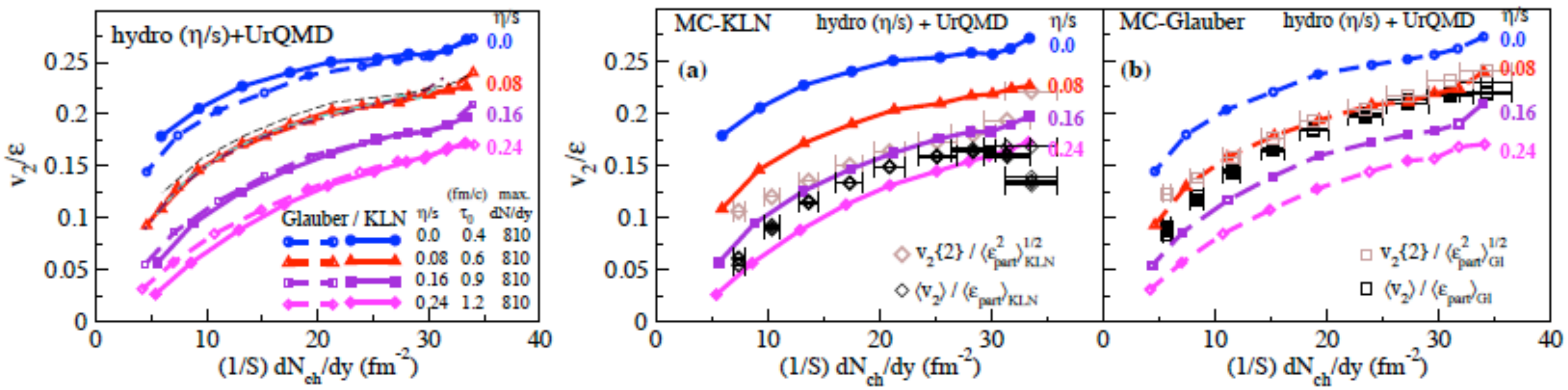
Spectral functions & transport coefficients



UrQMD Frankfurt/M

Extraction of $(\eta/s)_{\mathrm{QGP}}$ from AuAu@RHIC

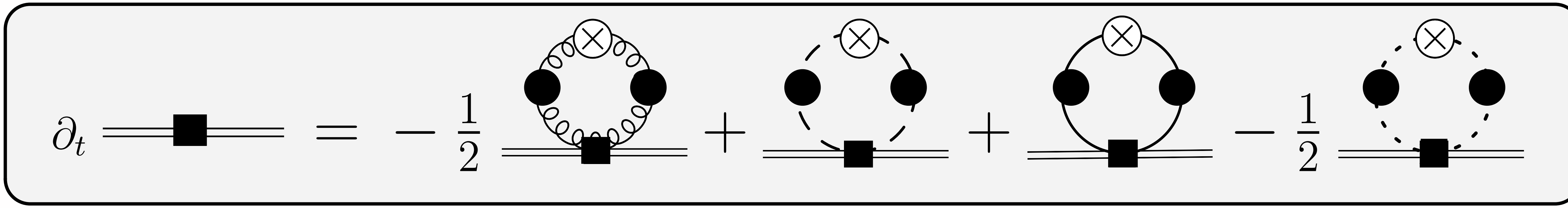
H. Song, S.A. Bass, U. Heinz, T. Hirano, C. Shen, PRL106 (2011) 192301

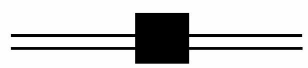


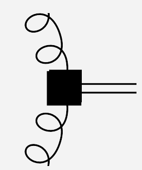
$$1 < 4\pi(\eta/s)_{\mathrm{QGP}} < 2.5$$

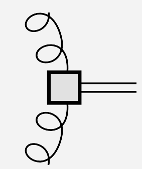
Transport in QCD

correlations of energy-momentum tensor

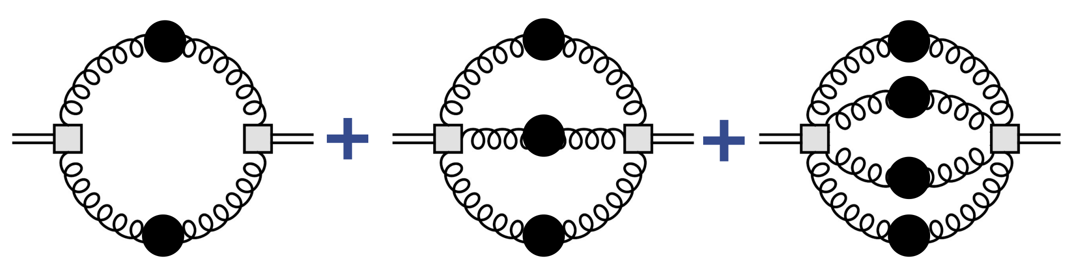


$\rho_{\pi\pi} =$ 



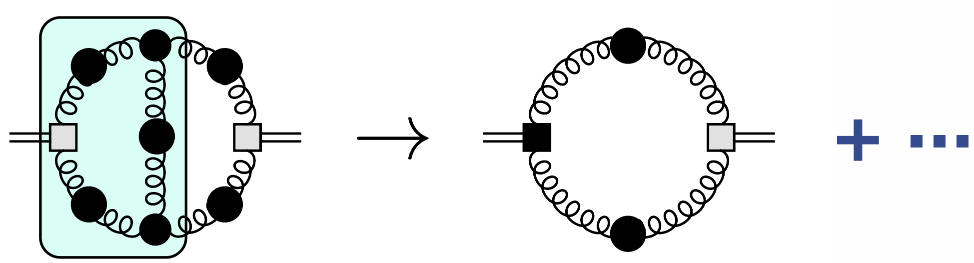


Diagrammatic representation

$\rho_{\pi\pi} =$  + ··· closed form

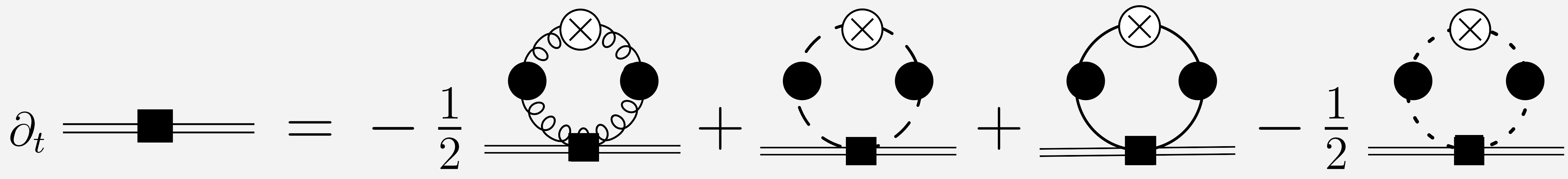
full computation Christiansen, Haas, JMP, Strodthoff, in prep.

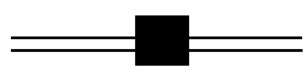
Vertex corrections



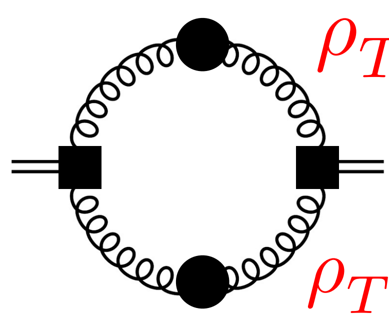
Transport in QCD

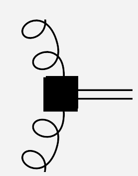
correlations of energy-momentum tensor



$\rho_{\pi\pi} =$ 

current approximation for $\mathrm{T} \approx \mathrm{T_c}$ with optimised RG-scheme from **Fister, JMP '13**

$\rho_{\pi\pi} =$ 

 **full vertex**

$\rho_{T/L}$ **with MEM**

'Those are my methods (principles), and if you don't like them...well, I have others'

Groucho Marx

direct computation

$$\rho_{\pi\pi}(p) = \frac{2d_A}{3}\int \frac{d^4k}{(2\pi)^4}\left[n(k^0) - n(k^0+p_0)\right]\left(V_{TT}\rho_T(k)\rho_T(k+p) + V_{TL}\rho_T(k)\rho_L(k+p) + V_{LL}\rho_L(k)\rho_L(k+p)\right)$$

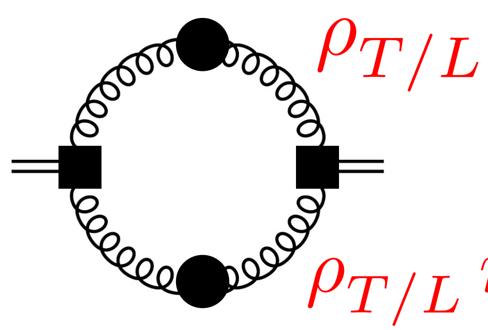
Viscosity in QCD

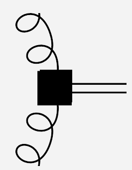
Shear viscosity

$$\eta = \frac{1}{20}\frac{d}{d\omega}\bigg|_{\omega=0}\rho_{\pi\pi}(\omega,0)$$ **Kubo relation**

current approximation for $\mathrm{T} \approx \mathrm{T_c}$ with optimised RG-scheme from **Fister, JMP '13**

$\rho_{\pi\pi} =$





full vertex

$\rho_{T/L}$ **with MEM**

MEM

- Simultaneous optimisation of the spectral function to preknowledge about its shape and to the Matsubara correlator.
- Likelihood given by:

$$L = \int d\tau \left(G_E(\tau) - G_\rho(\tau)\right)^2$$

- Extensions of Maximum Likelihood Method by adding an entropy term of Shannon-Jaynes type:

$$S = \int d\omega \left[\rho(\omega) - m(\omega) - \rho(\omega) \log \frac{\rho(\omega)}{m(\omega)}\right]$$

- Minimisation of $Q = L - \alpha S$ with a weight α

MEM Input

- Pure Yang-Mills gauge theory at finite temperature T

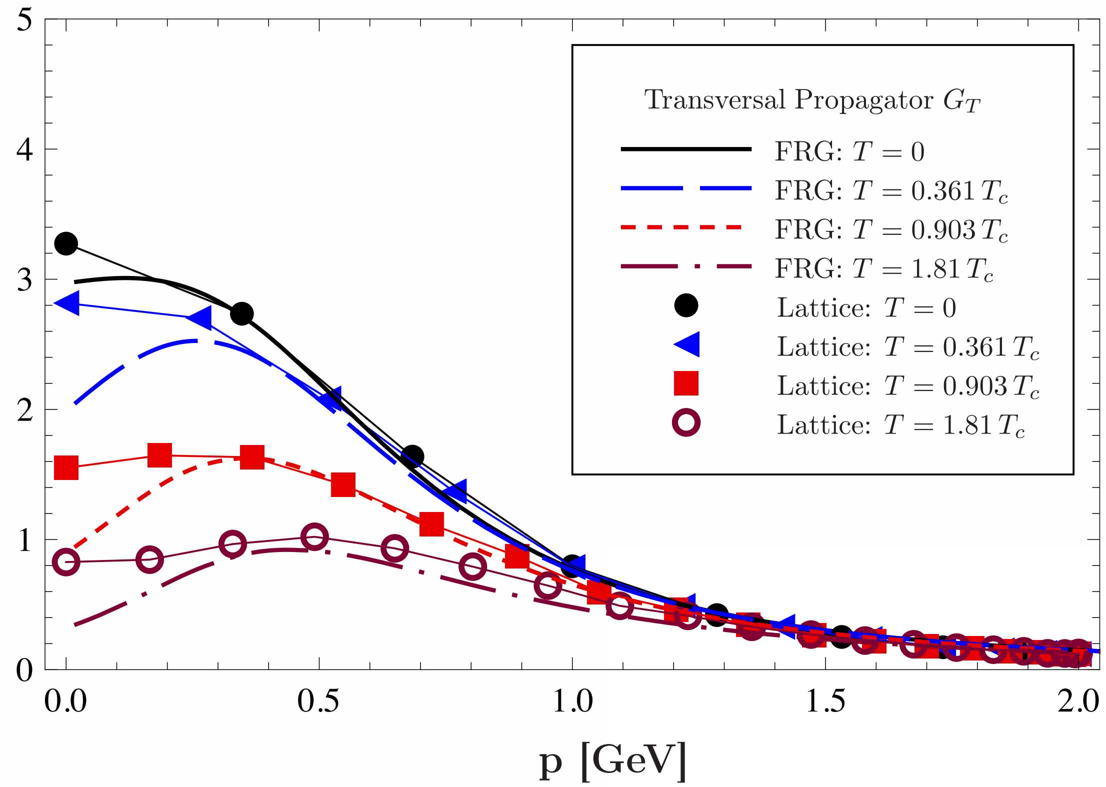
- Gluon propagator
  - Transversal part: $D_L(\omega_n = 0, q^2)$
  - Longitudinal part: $D_T(\omega_n = 0, q^2)$

- FRG results for the 0-th Matsubara mode of $D_L(0, q^2)$ and $D_T(0, q^2)$.

Viscosity in pure glue

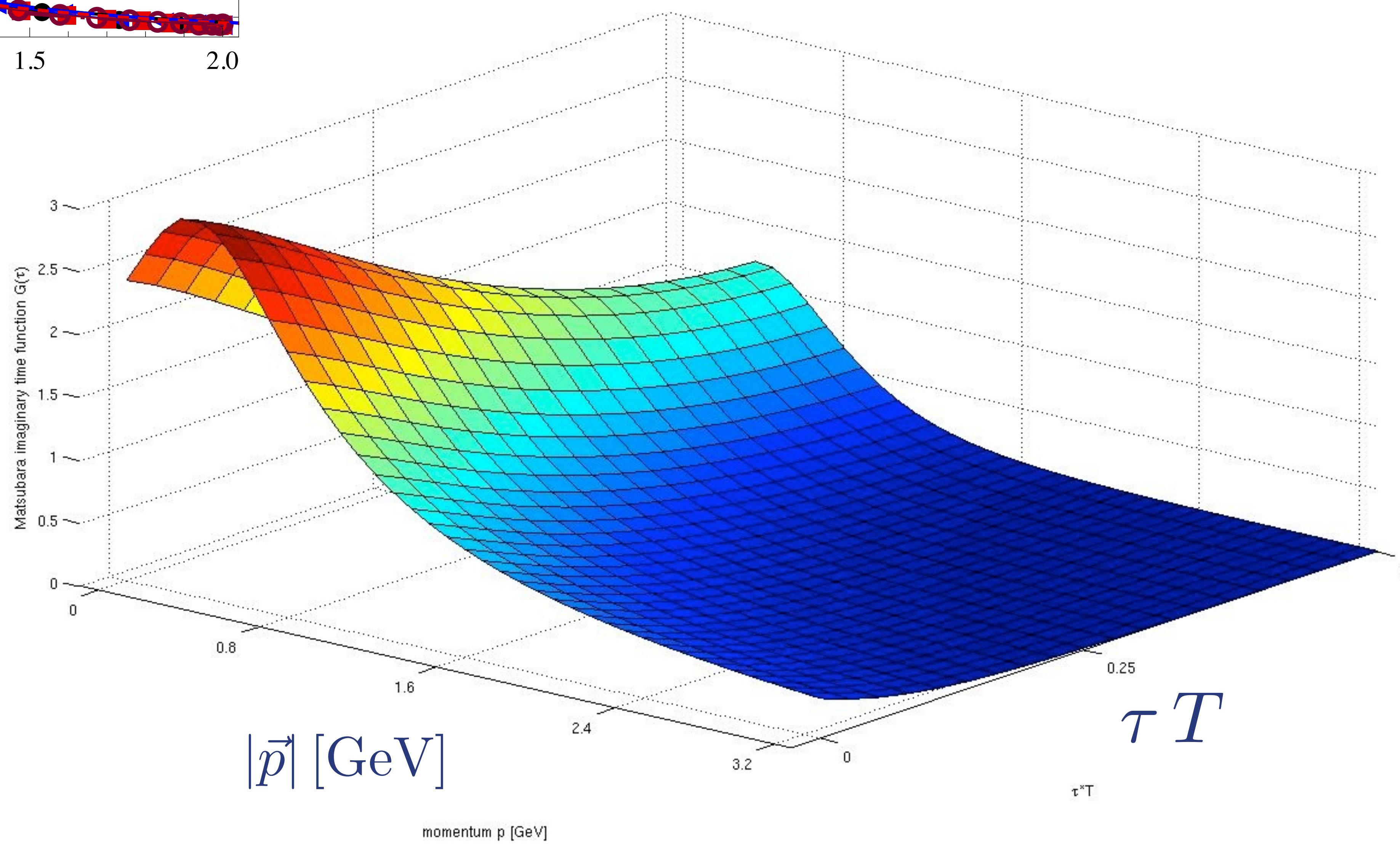
imaginary time correlations

M. Haas, Fister, JMP ´13



transversal gluon propagator

$G_T(\tau, \vec{p})$

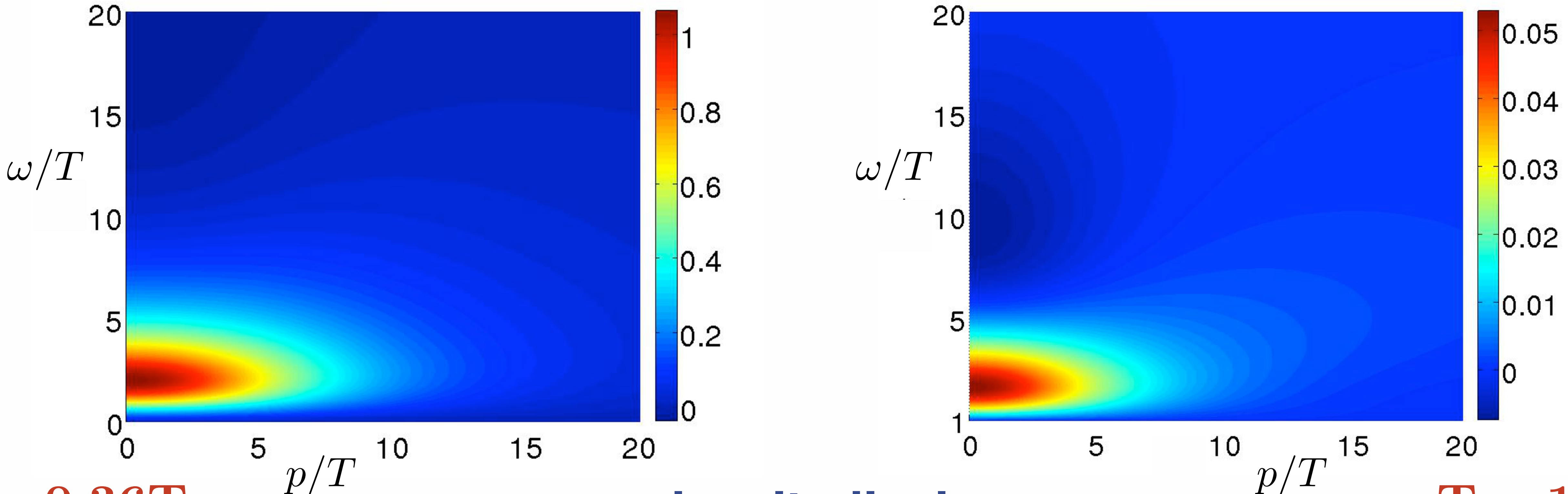


Viscosity in pure glue

spectral functions

transversal

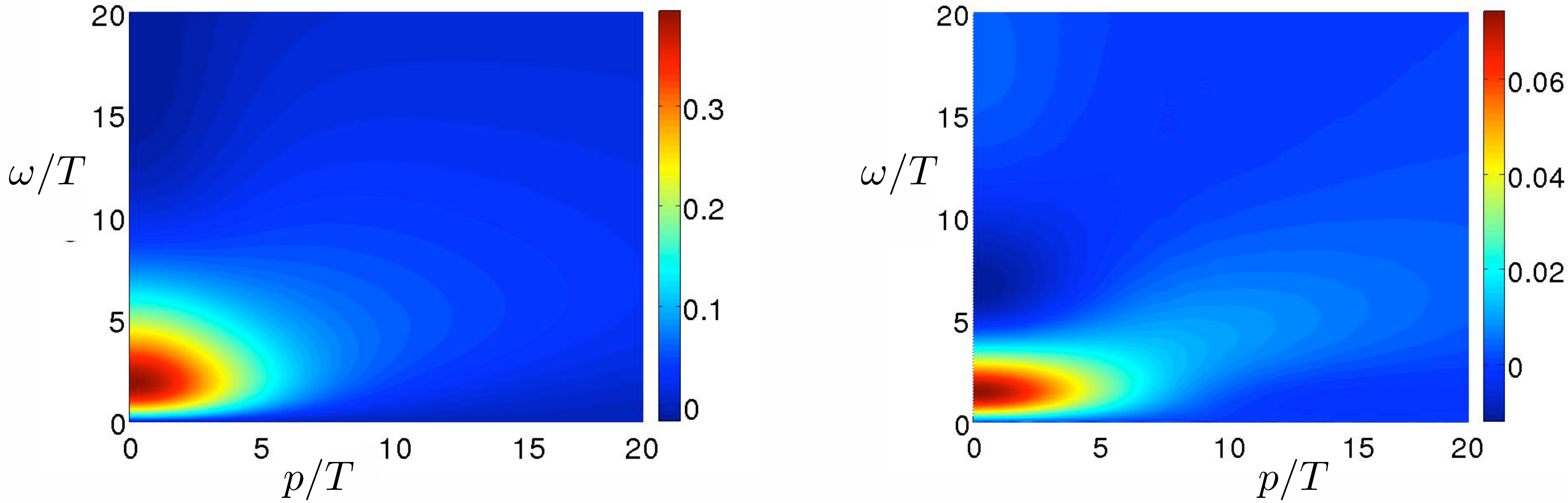
M. Haas, Fister, JMP ´13



$\mathbf{T = 0.36T_c}$

longitudinal

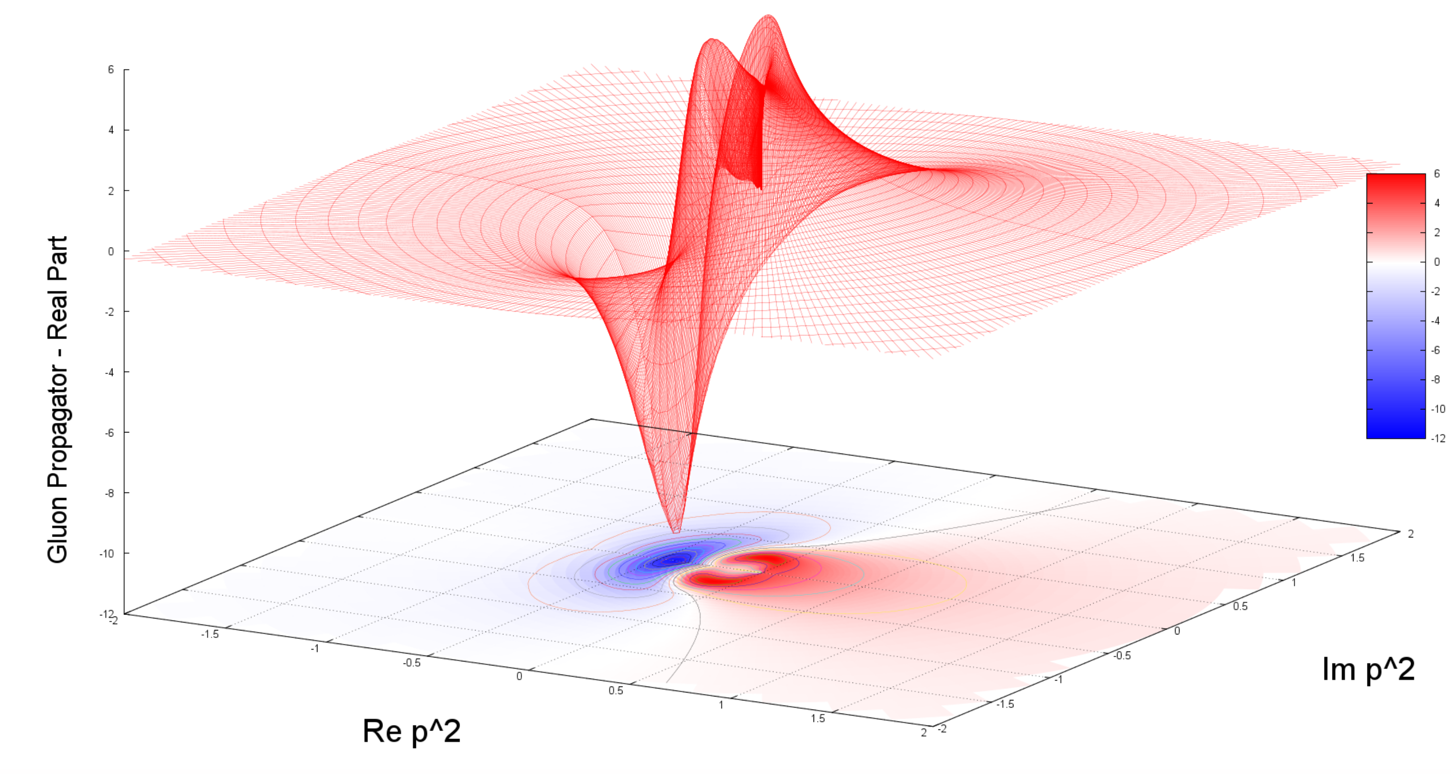
$\mathbf{T = 1.8T_c}$

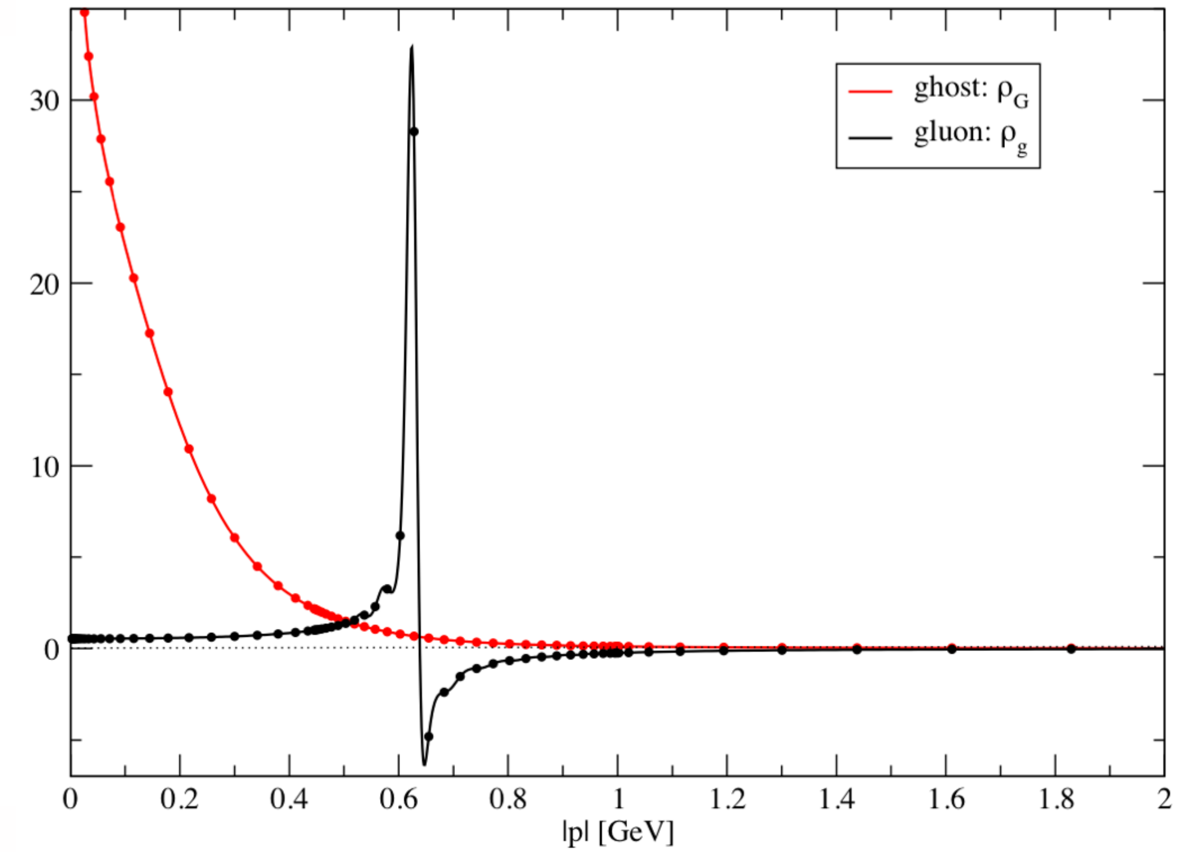


Viscosity in pure glue

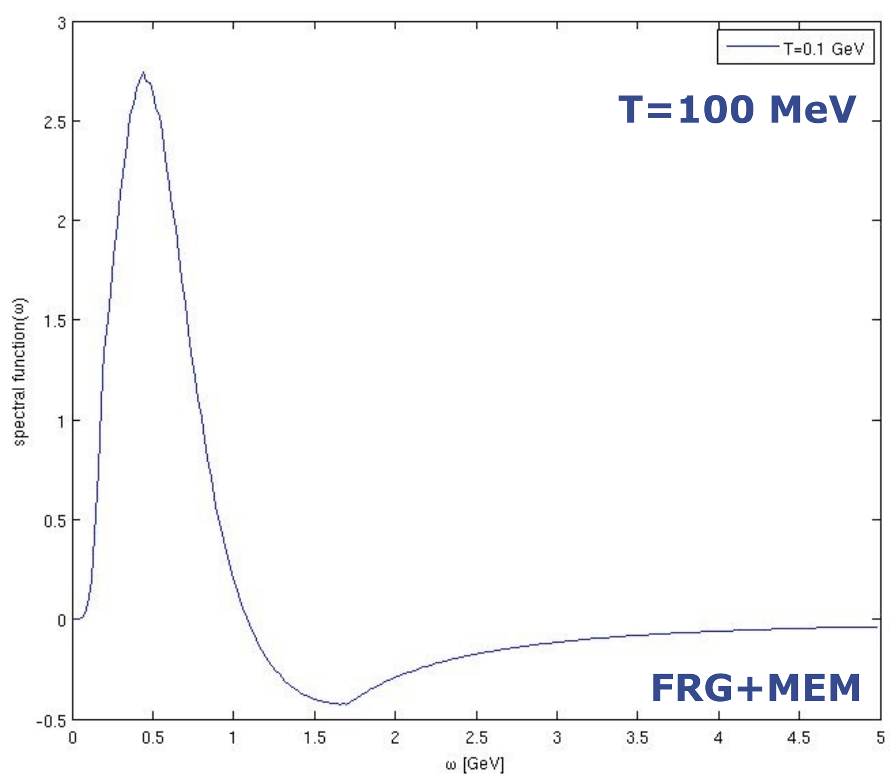
spectral functions

Complex DSEs





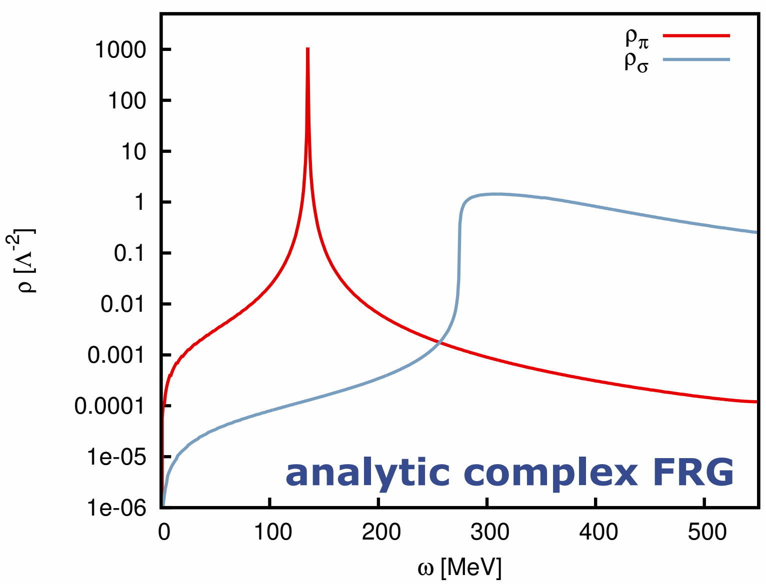
Strauss, Fischer, Kellermann ’12



transversal spectral function

M. Haas, Fister, JMP ´13

pion and sigma spectral functions

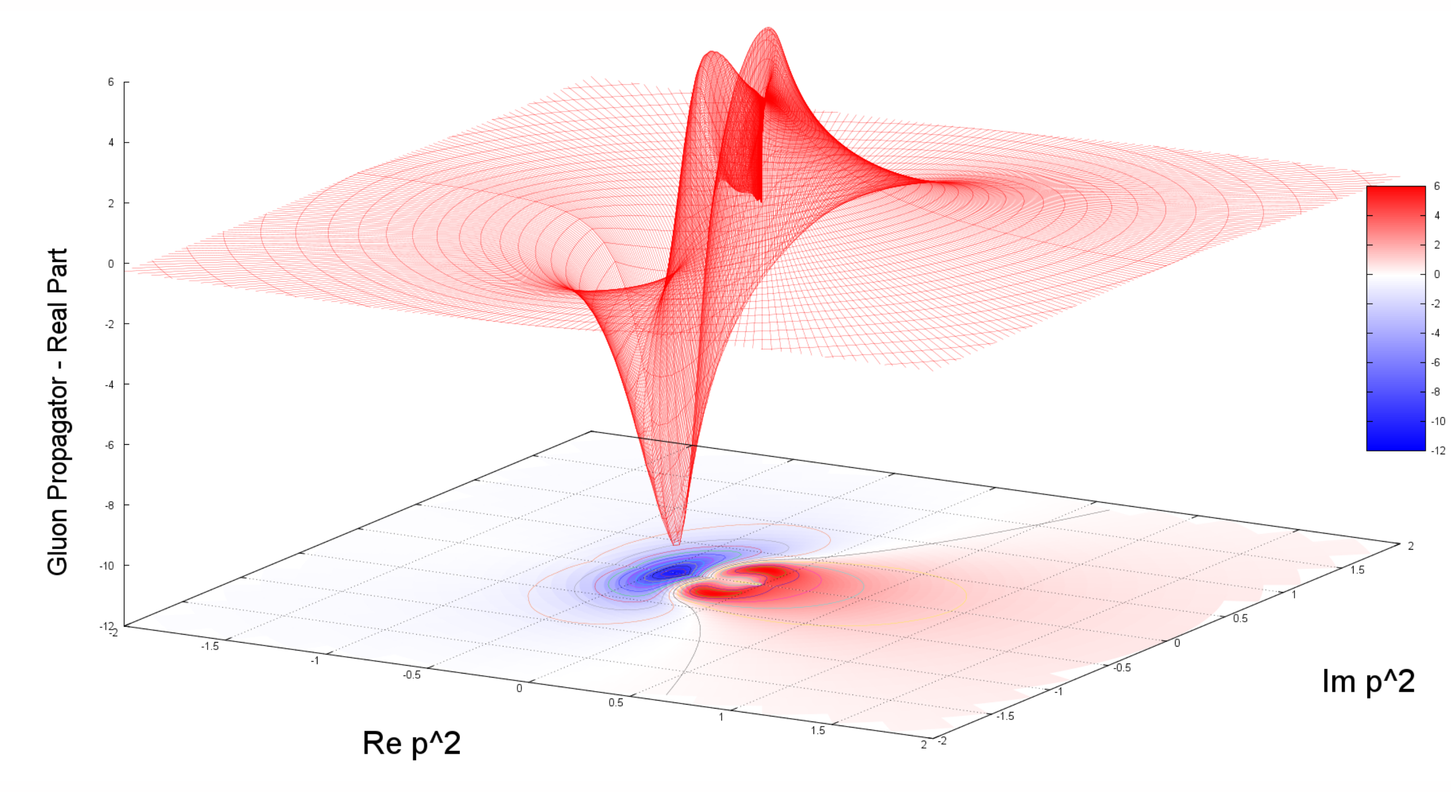


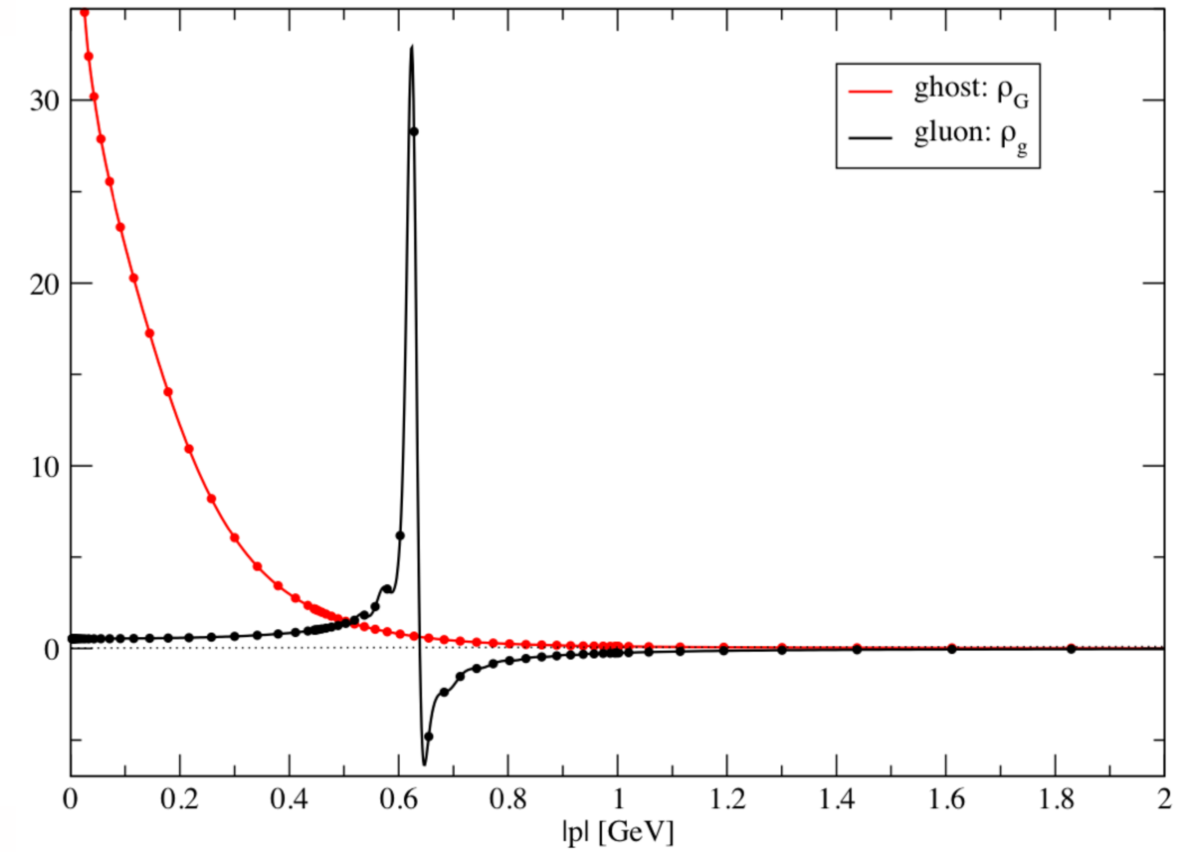
Kamikado, Strodthoff, von Smekal, Wambach ’13

Viscosity in pure glue

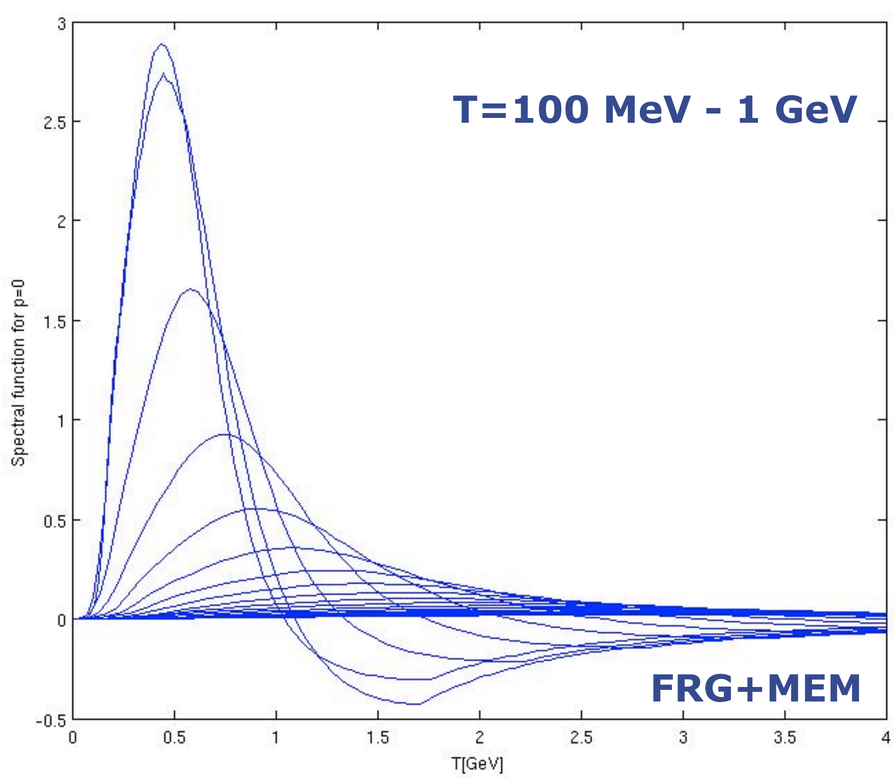
spectral functions

Complex DSEs





Strauss, Fischer, Kellermann '12



transversal spectral function

M. Haas, Fister, JMP ´13

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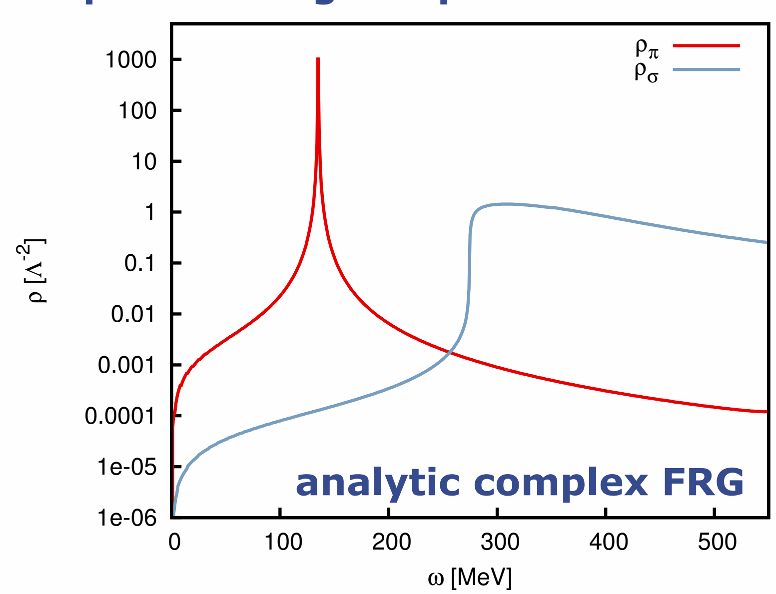
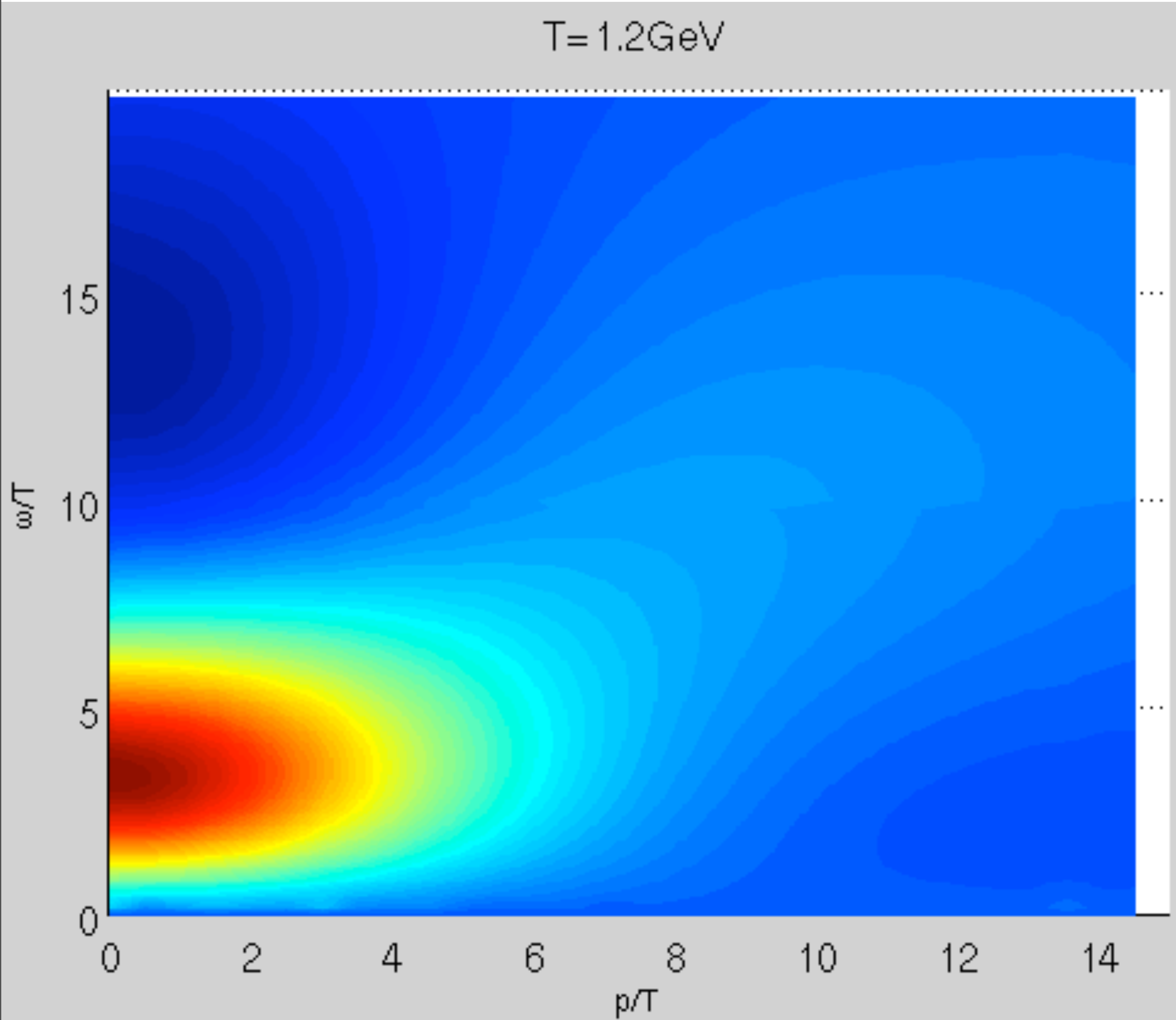
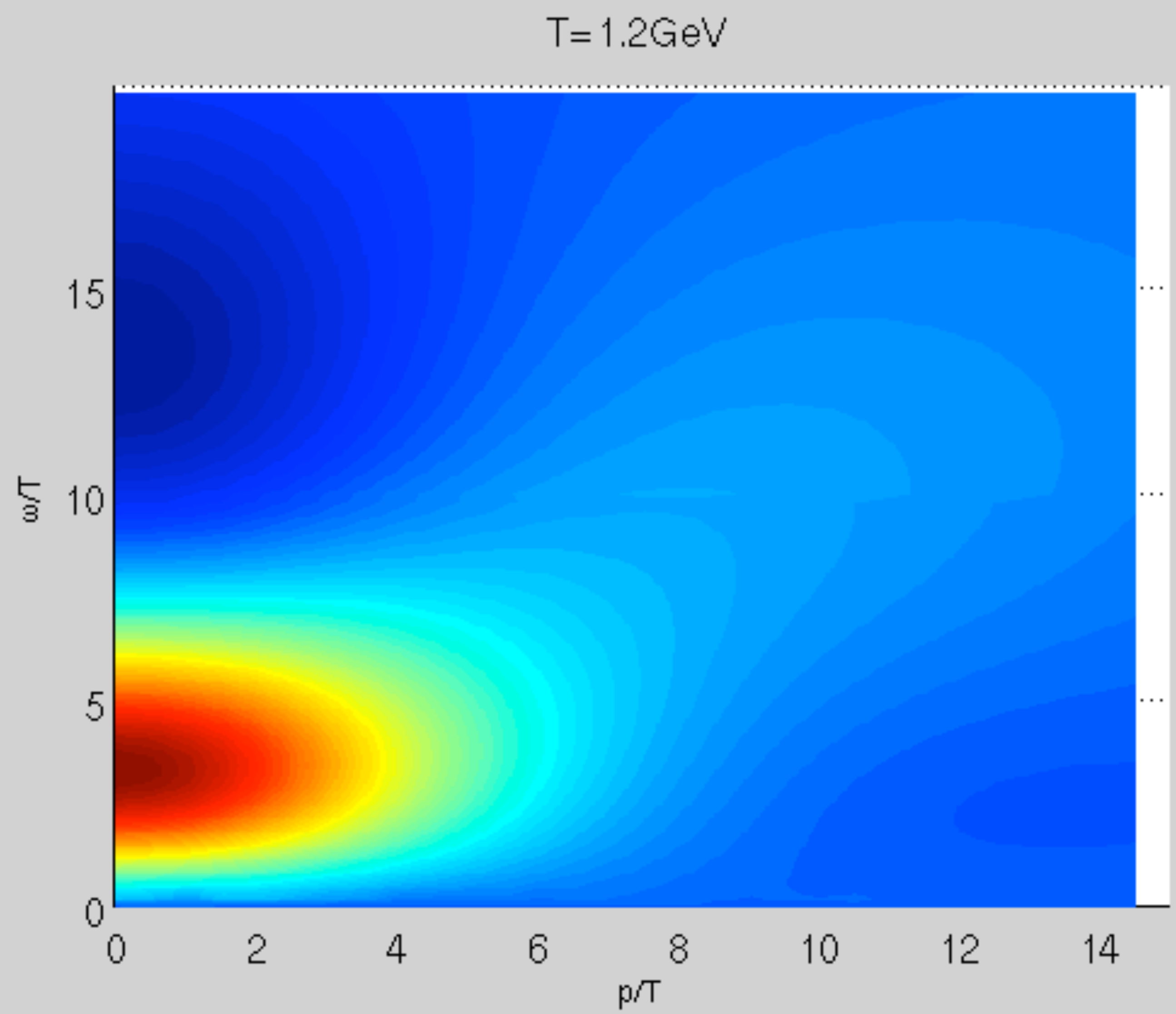


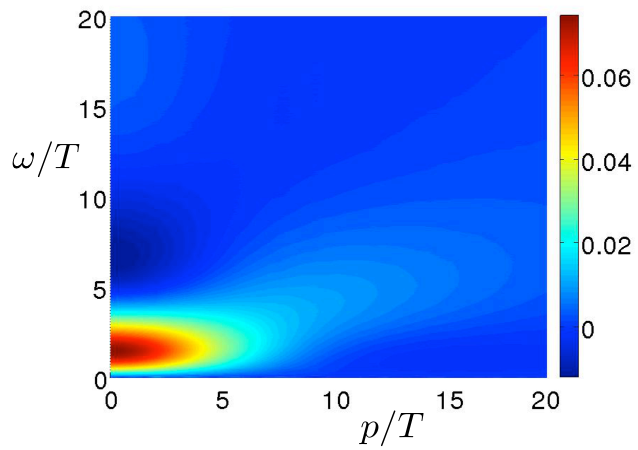
Kamikado, Strodthoff, von Smekal, Wambach '13

Viscosity in pure glue

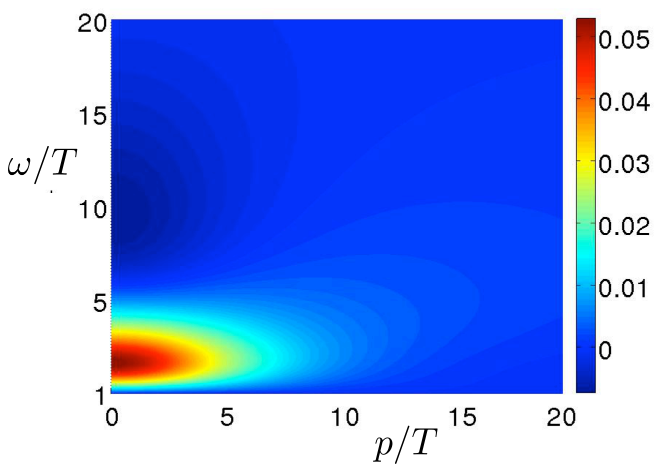
spectral functions

longitudinal	transversal
	

$\mathbf{T = 1.8T_c}$



$\rho_{T/L}$ **with MEM**

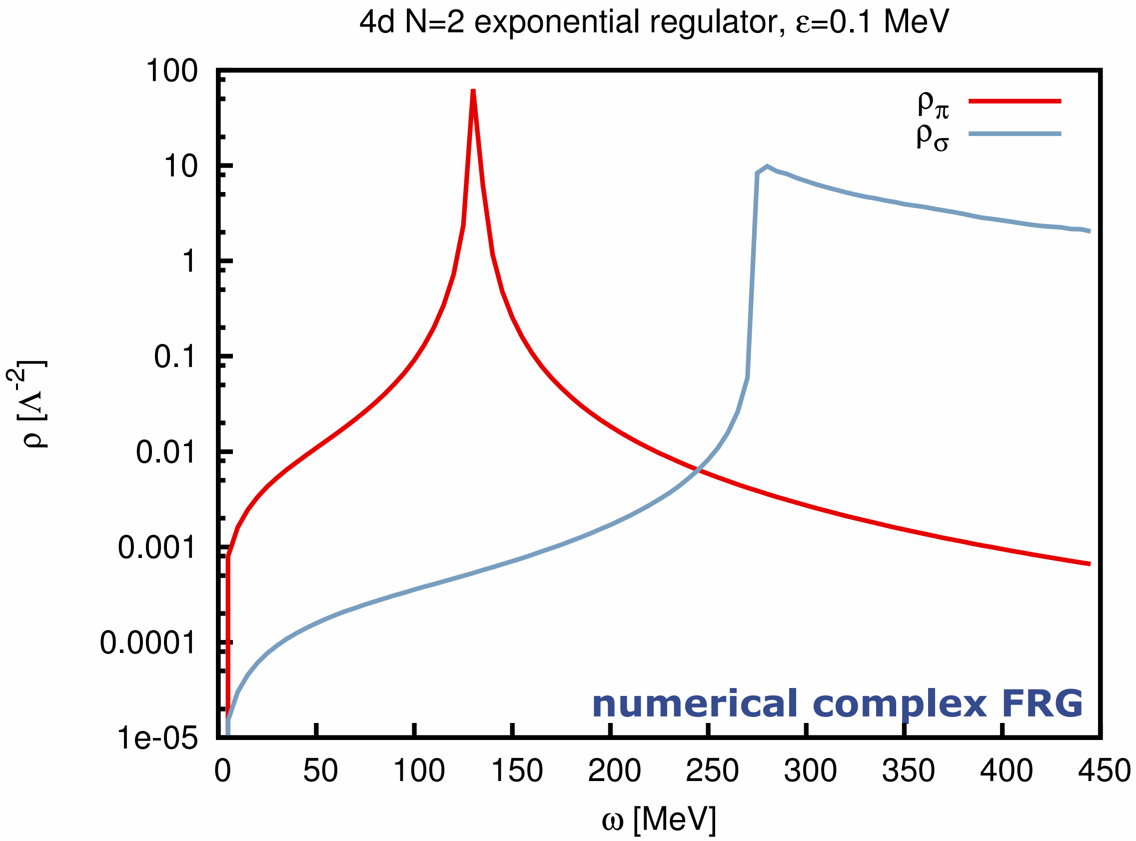


$\mathbf{T = 1.8T_c}$

Viscosity in pure glue

spectral functions

pion and sigma spectral functions



JMP, Strodthoff, in preparation

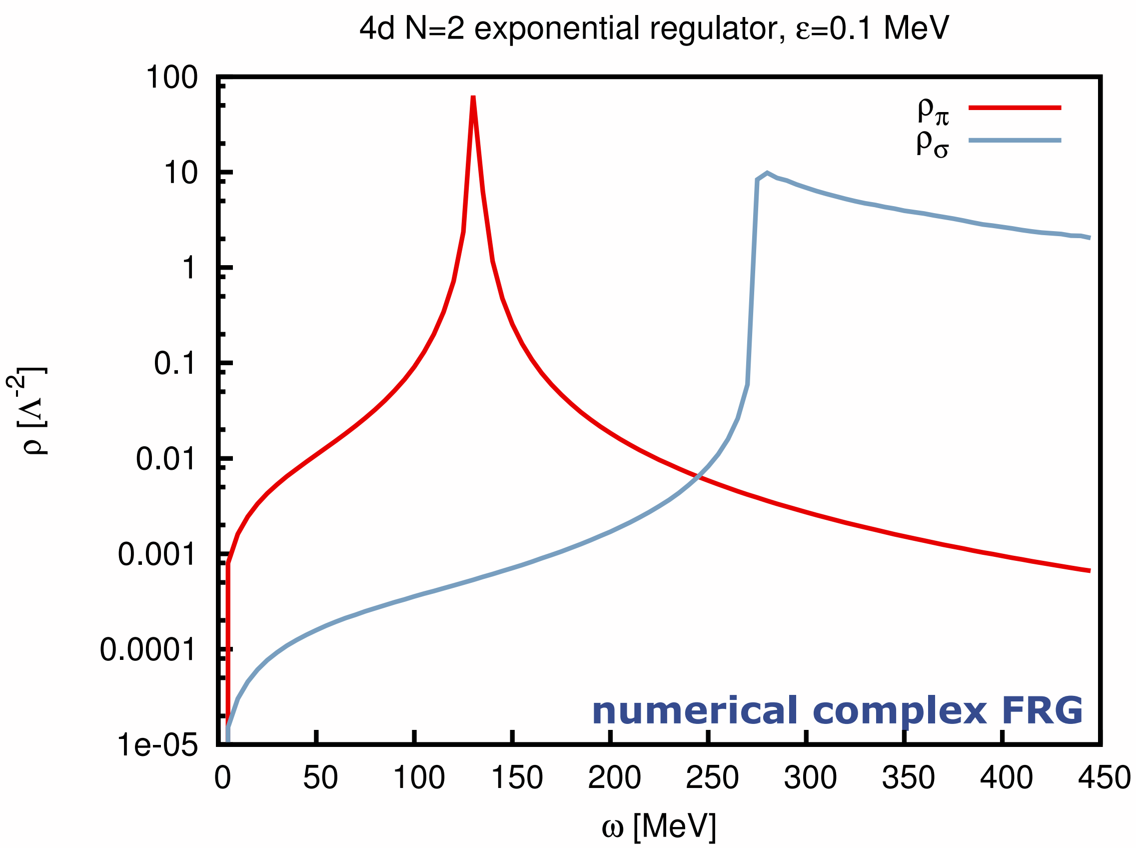
‘Those are my methods (principles), and if you don’t like them...well, I have others’

direct computation **Groucho Marx**

Viscosity in pure glue

spectral functions

pion and sigma spectral functions



JMP, Strodthoff, in preparation

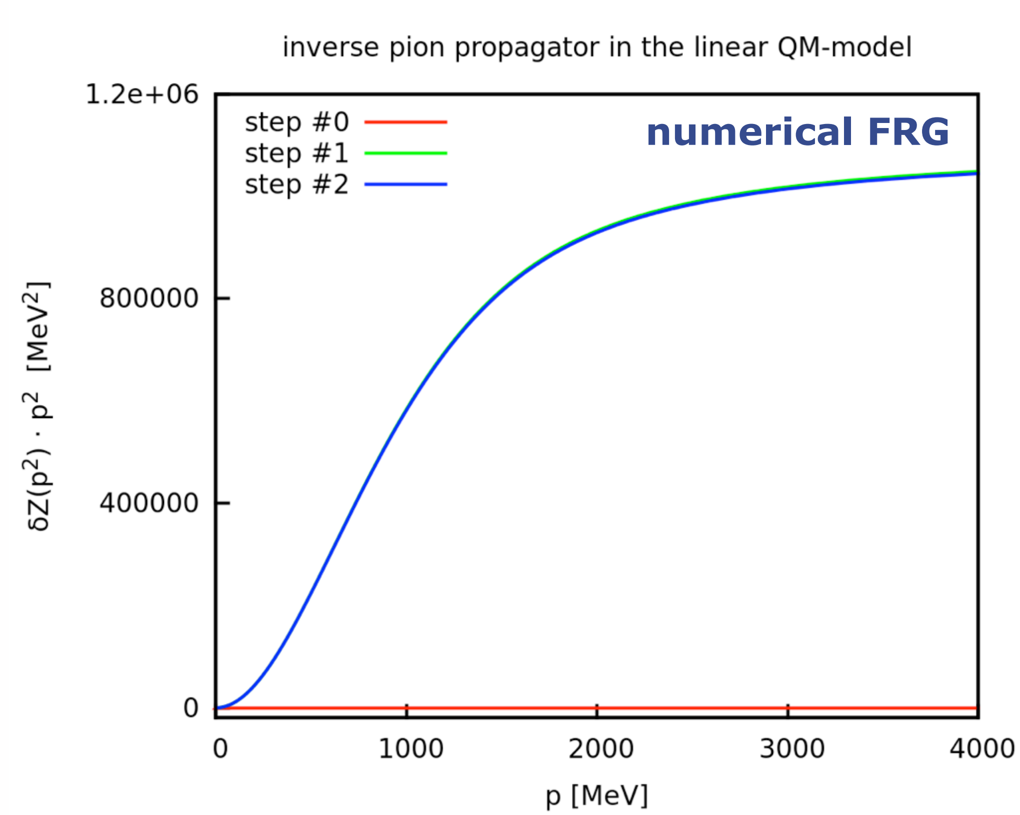
O(N)-model

iteration step	σ_0 [MeV]	δ_ρ [%]	m_{pole} [MeV]	m_{screen} [MeV]	δ_m [%]
0	93.55	0.0043	130.3113	136.7593	4.9
1	100.05	0.0028	126.6390	126.4590	0.14
5	99.38	0.0043	127.0347	127.0110	0.019

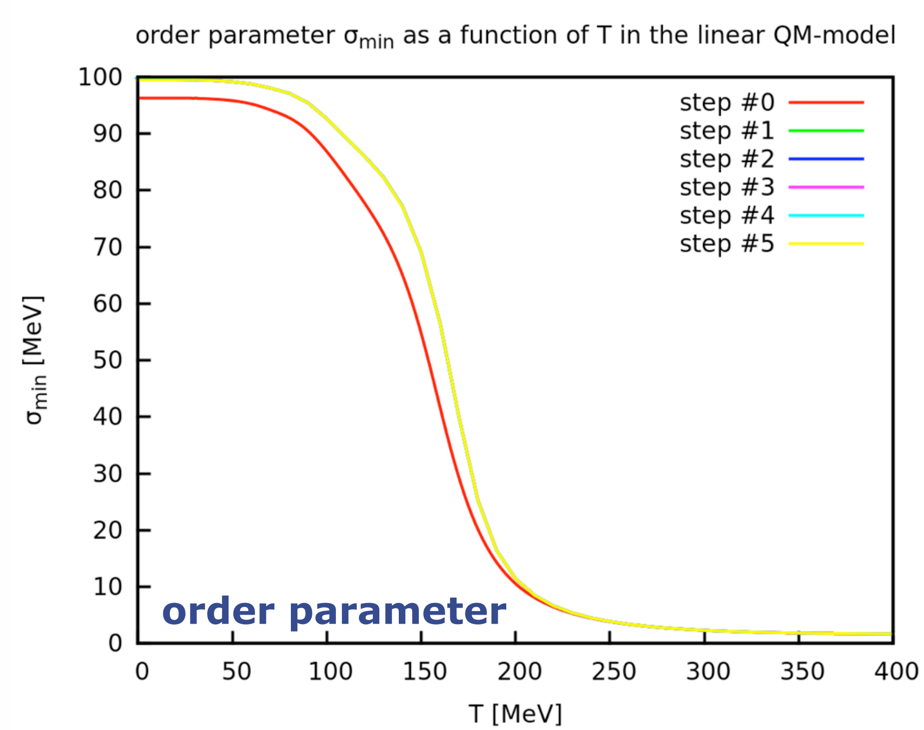
iteration step	σ_0 [MeV]	δ_ρ [%]	m_{pole} [MeV]	m_{screen} [MeV]	δ_m [%]
0	96.25	0.0052	91.4911	134.8281	47
1	99.56	0.0044	90.8841	91.1611	0.30
5	99.56	0.0073	90.9244	91.1551	0.25

QM-model

QM-model



Helmboldt, JMP, Strodthoff, in preparation



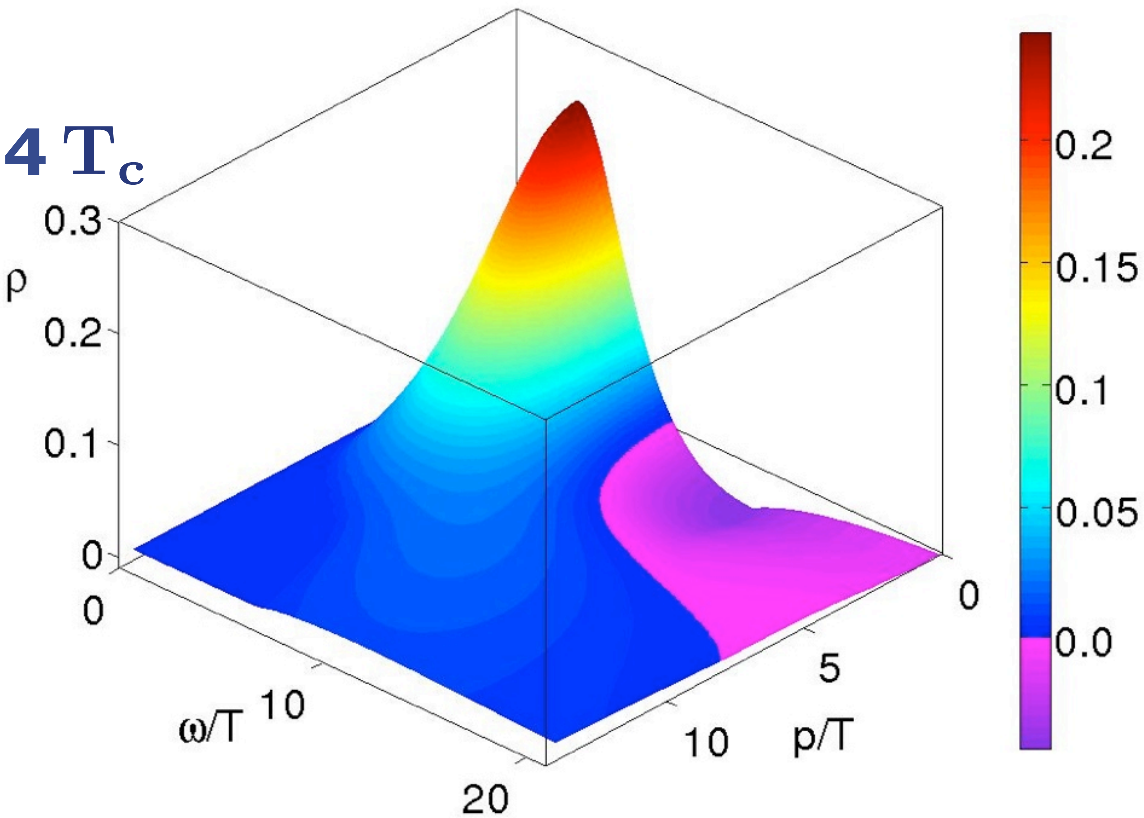
Viscosity in pure glue

spectral functions

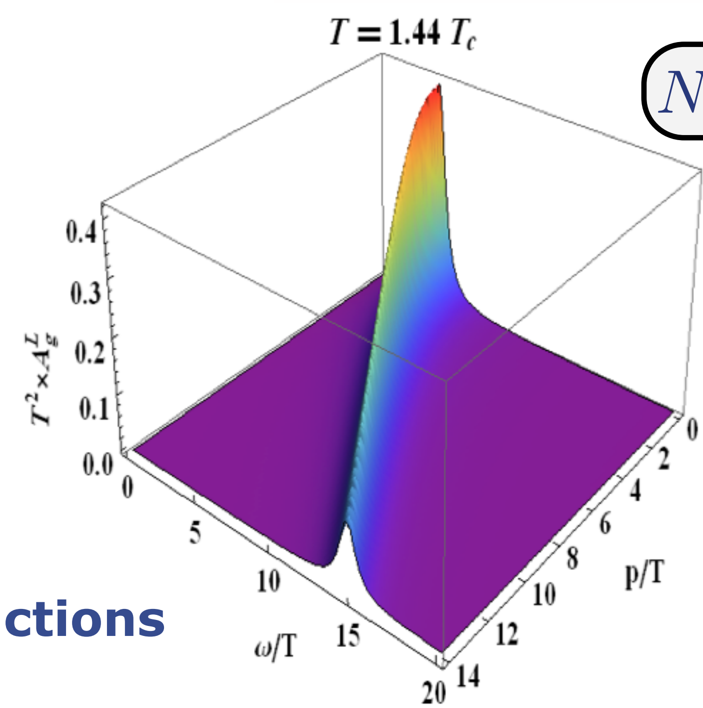
M. Haas, Fister, JMP ´13

transversal

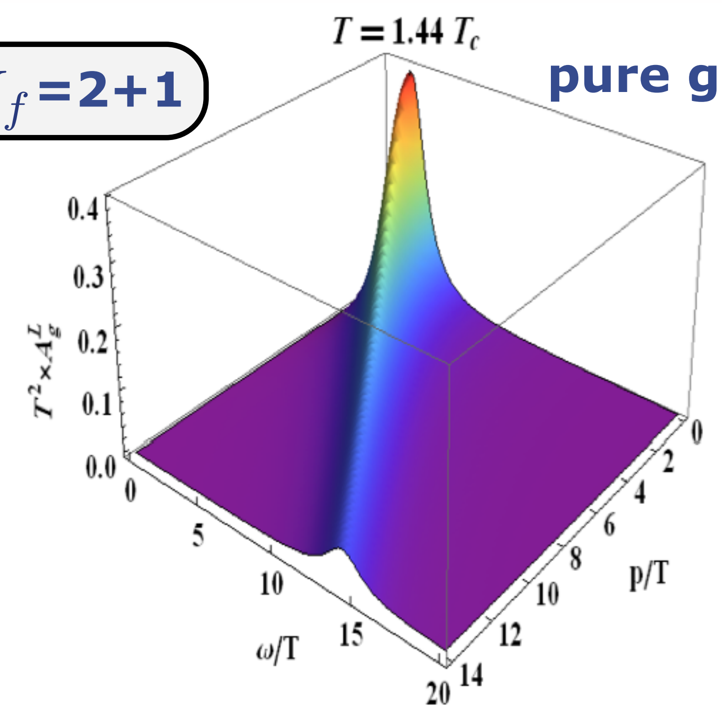
T=1.44 $\mathrm{T_c}$



T=1.44 $\mathrm{T_c}$



N_f=2+1



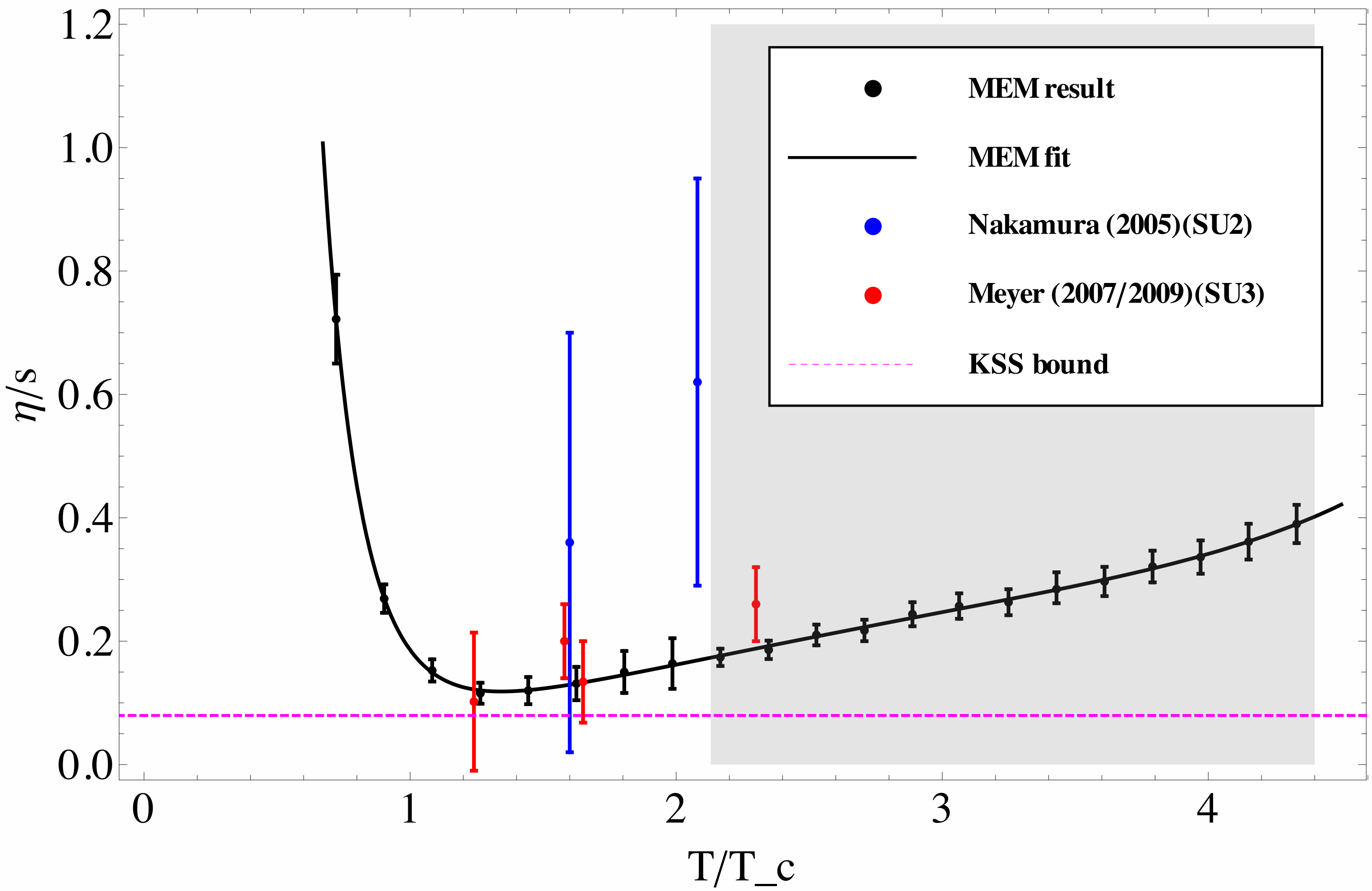
pure glue

PHSD spectral functions

Viscosity in pure glue

shear viscosity

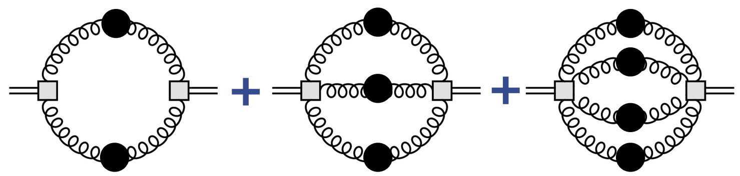
M. Haas, Fister, JMP ´13



Kubo relation

$$\eta = \frac{1}{20}\frac{d}{d\omega}\bigg|_{\omega=0}\rho_{\pi\pi}(\omega,0)$$

Diagrammatic representation

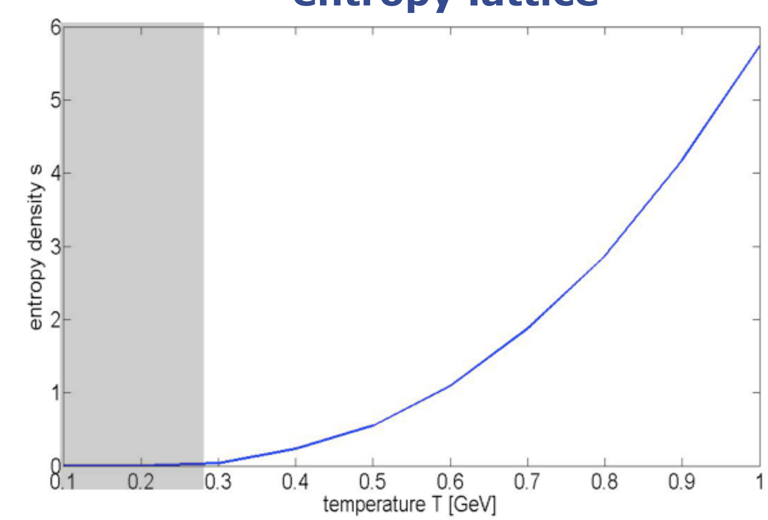
$\rho_{\pi\pi} =$ 

+ ··· closed form

$T \lesssim 2T_c$ **: MEM+optimised RG-scheme systematic error estimates**

Shaded area: MEM error estimates

entropy lattice



H. Meyer '09

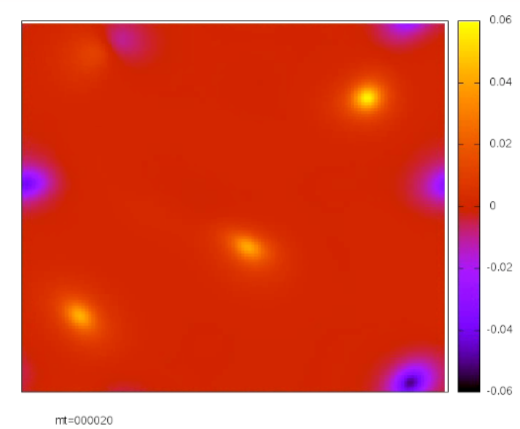
Boyd, Engels, Karsch '95

Summary & Outlook

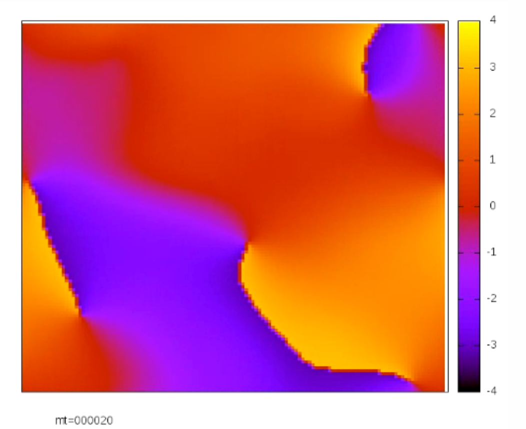
Summary & outlook

- **Gauge dynamics far from equilibrium**

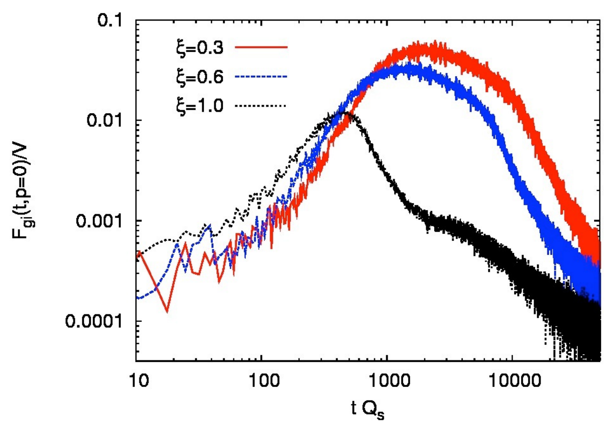
Abelian Higgs



magnetic field



phase of Higgs



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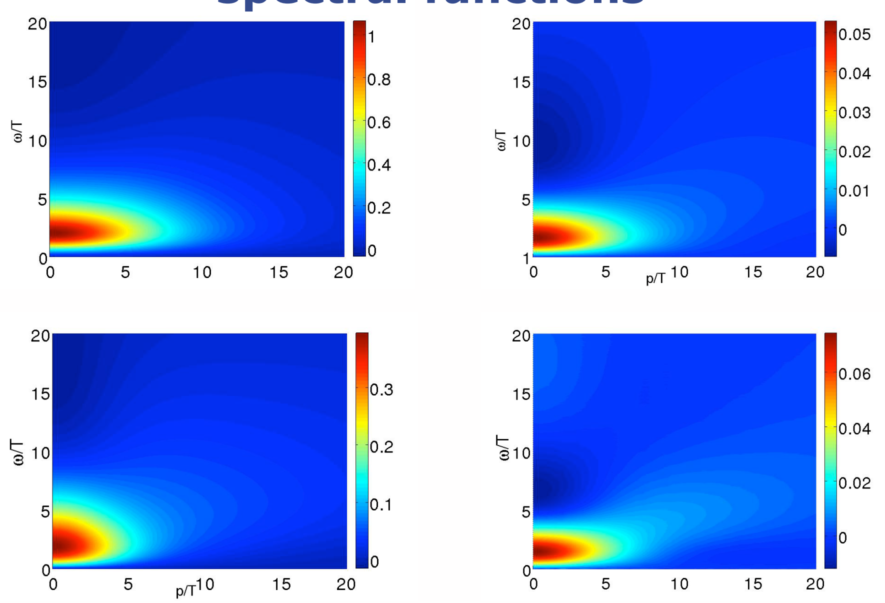
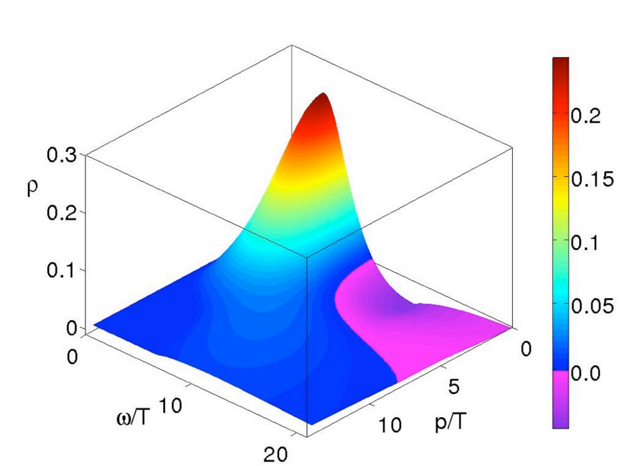
- **Spectral functions and transport coefficients**

spectral functions





viscosity over entropy ratio