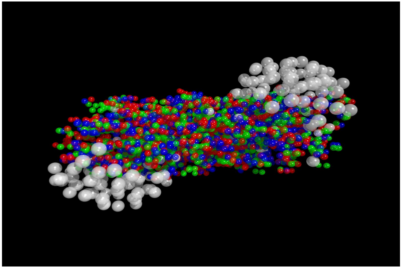
The Early Phase of High Energy Heavy Ion Collisions

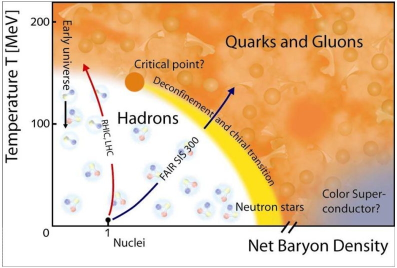
Andreas Schäfer (Regensburg)

- Thermalization and entropy production
- 1. approach: Classical Field Theory
  Kolmogorov-Sinai entropy and Husimi transformation
- 2. approach: AdS/CFT
- Conclusions

LHC: Investigating the phase diagram of QCD ?!?

p+p at 4 TeV + 4 TeV and p+Pb at 4 TeV + 1.58 A*TeV
and Pb+Pb at 1.38 A TeV + 1.38 A TeV

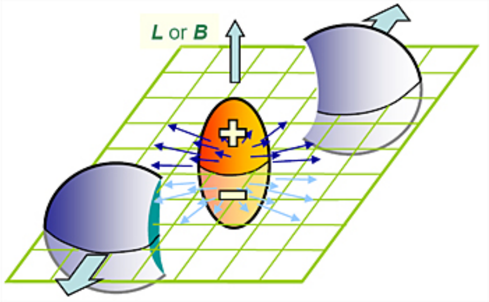


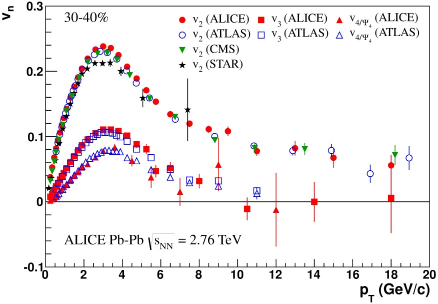


 GSI

Key question:
Does one reach thermal equilibrium fast enough to really probe the quark gluon plasma ?

Observable: Elliptic flow $v_n \sim \cos(n\phi)$



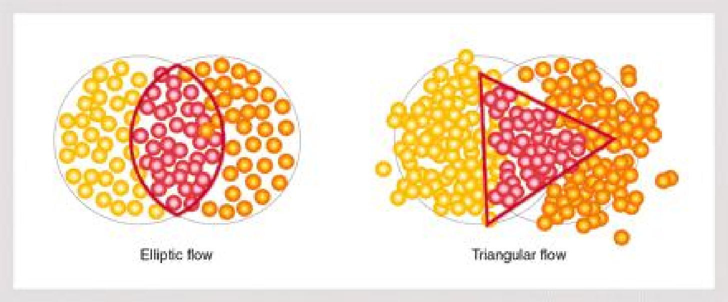


The importance of fluctuations: The system is not large

Example: v_3 and higher moments of flow, see below

$$v_n(p_T, \eta) = \langle cos[n(\phi - \Psi_n)] \rangle$$

Ψ_n: True reaction plane orientation ϕ: measured angle of hadron



The ATLAS analysis from 1305.2942

The v_n describe the observed particle distributions

The eccentricities ϵ_n describe the initial state prior to hydrodynamic expansion (r, ϕ transverse coordinates)

$$\epsilon_n = \frac{\sqrt{\langle r^n \cos n\phi \rangle^2 + \langle r^n \sin n\phi \rangle^2}}{\langle r^n \rangle}$$

generic assumption: The v_n scale linearly with the ϵ_n.

In the following: $\langle v_n \rangle$ mean; σ_{v_n} standard deviation of v_n from $\langle v_n \rangle$

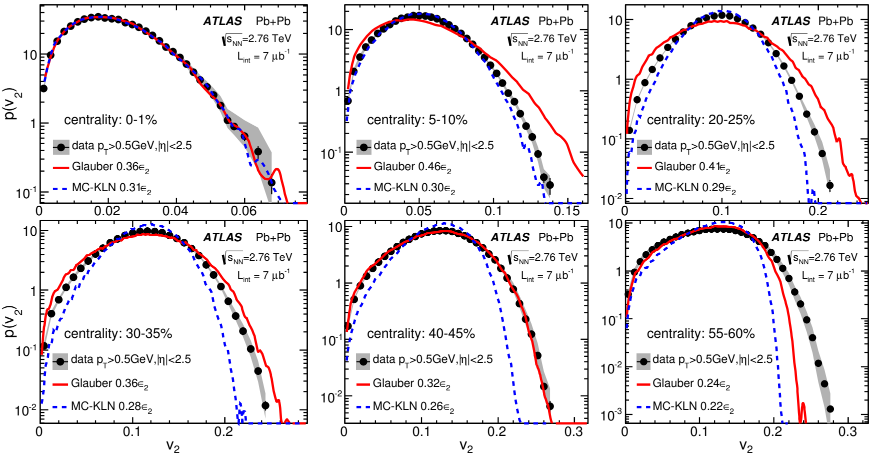
One assumes Gaussian fluctuations around a collective flow vector $\vec{v}_n = (v_n \cos(n\Phi_n), v_n \sin(n\Phi_n))$

$$p(\vec{v}_n) = \frac{1}{2\pi\delta^2_{v_n}} e^{-\left(\vec{v}_n - \vec{v}_n^{\,\mathrm{RP}}\right)^2 / \left(2\delta^2_{v_n}\right)}$$

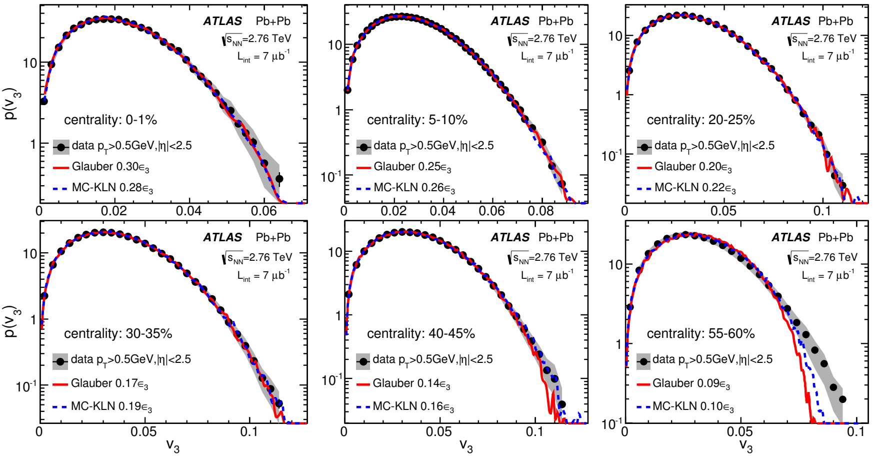
after integration over the azimuthal angle

$$p(v_n) = \frac{v_n}{\delta^2_{v_n}} e^{-\frac{(v_n)^2 + (v_n^{\mathrm{RP}})^2}{2\delta^2_{v_n}}} I_0\left(\frac{v_n^{\mathrm{RP}} v_n}{\delta^2_{v_n}}\right)$$

"fluctuations only" implies $v_n^{\mathrm{RP}} = 0$ and $\sigma_{v_n}/\langle v_n \rangle = 0.523$
Very careful systematic studies are performed
Results in comparison with a Glauber Model and a CGC model (MC-KLN)



ATLAS Pb+Pb
$\sqrt{s_{NN}}$=2.76 TeV
L_{int} = 7 μb^{-1}
centrality: 0-1%
centrality: 5-10%
centrality: 20-25%
centrality: 30-35%
centrality: 40-45%
centrality: 55-60%
data p_T>0.5GeV,|η|<2.5
Glauber 0.36ϵ_2
MC-KLN 0.31ϵ_2
Glauber 0.46ϵ_2
MC-KLN 0.30ϵ_2
Glauber 0.41ϵ_2
MC-KLN 0.29ϵ_2
Glauber 0.36ϵ_2
MC-KLN 0.28ϵ_2
Glauber 0.32ϵ_2
MC-KLN 0.26ϵ_2
Glauber 0.24ϵ_2
MC-KLN 0.22ϵ_2
p(v_2)
v_2



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data p_T>0.5GeV,|η|<2.5
Glauber 0.30ϵ_3
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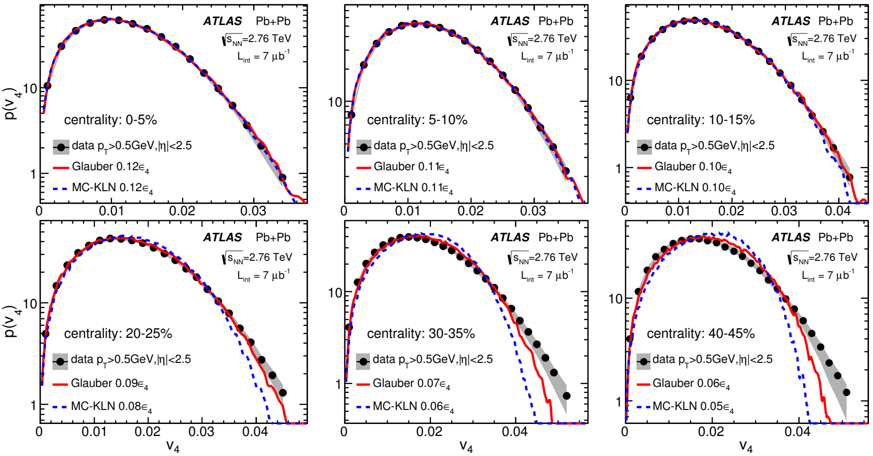
Glauber 0.25ϵ_3
MC-KLN 0.26ϵ_3
Glauber 0.20ϵ_3
MC-KLN 0.22ϵ_3
Glauber 0.17ϵ_3
MC-KLN 0.19ϵ_3
Glauber 0.14ϵ_3
MC-KLN 0.16ϵ_3
Glauber 0.09ϵ_3
MC-KLN 0.10ϵ_3
$p(v_3)$
v_3

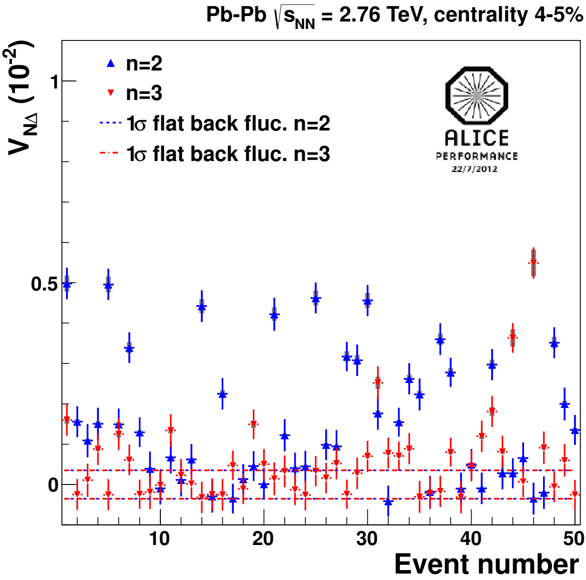


ATLAS Pb+Pb
$\sqrt{s_{NN}}$=2.76 TeV
L_{int} = 7 μb^{-1}
centrality: 0-5%
centrality: 5-10%
centrality: 10-15%
centrality: 20-25%
centrality: 30-35%
centrality: 40-45%
data p_T>0.5GeV,|η|<2.5
Glauber 0.12ϵ_4
MC-KLN 0.12ϵ_4
Glauber 0.11ϵ_4
MC-KLN 0.11ϵ_4
Glauber 0.10ϵ_4
MC-KLN 0.10ϵ_4
Glauber 0.09ϵ_4
MC-KLN 0.08ϵ_4
Glauber 0.07ϵ_4
MC-KLN 0.06ϵ_4
Glauber 0.06ϵ_4
MC-KLN 0.05ϵ_4
$p(v_4)$
v_4

ATLAS:

Both models fail to describe the experimental data consistently over most of the measured centrality range

The ALICE analysis from 1301.6084

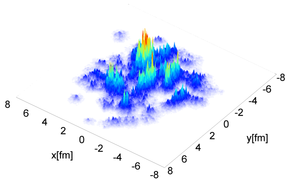


On an event-by-event basis one observes far larger fluctuations than expected

The two possible interpretations are:

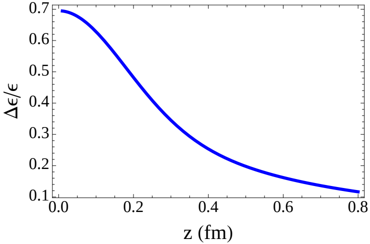
- different initial distributions, i.e. different ϵ_n
- or strongly non-linear dynamics, i.e. no proportionality between v_n and ϵ_n

Theory actually predicts large fluctuations



B. Schenke, Tribedy, Venugopalan 1206.6805

energy density at 0.2 fm/c



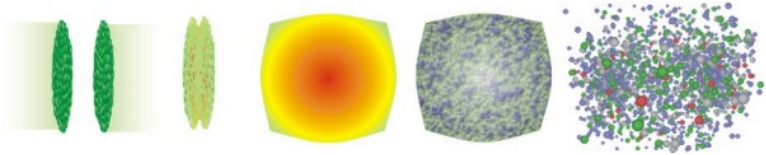
B.M. & A.S. 1111.3347

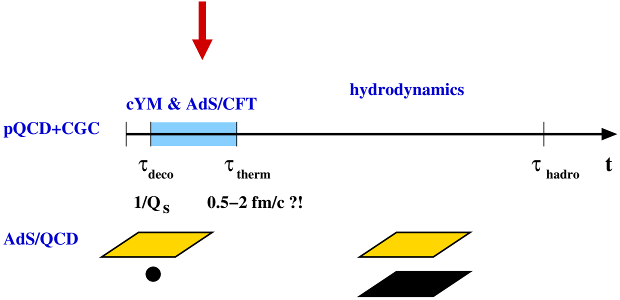
How is thermalization at all possible ?
Thermalization requires massive entropy production at early stages ($<$ 1 fm/c)

But QCD is time-reversal invariant. Entropy is typically produced by measurements $\Rightarrow$ Coarse-graining.

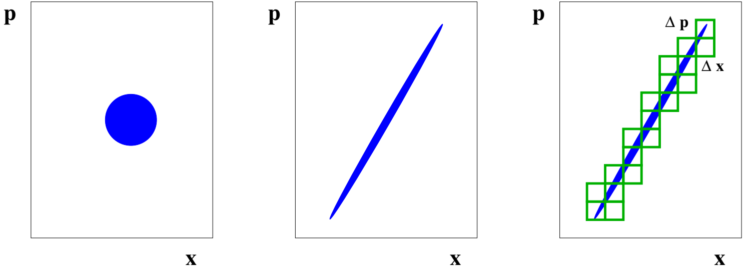
Q1: How can entropy be produced at all before any measurement takes place **?**
Q2: How can it be produced so fast **?**

Different stages of entropy production in a HIC



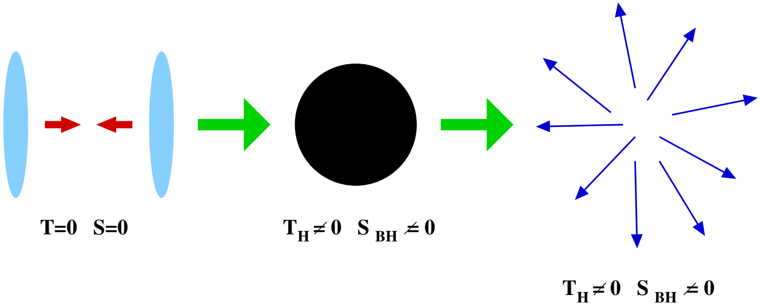


Coarse graining in non-linear mechanics



- Under time evolution the phase space volume is conserved
- The finite resolution of any measurement implies an increase in phase space volume

BH formation at LHC ?!?

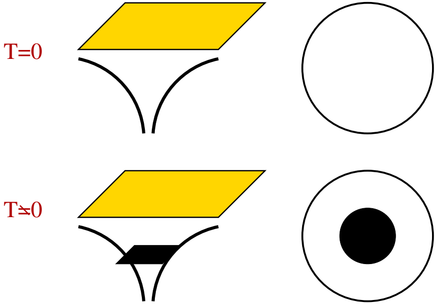


If the S-Matrix is unitary $S_{inital} = S_{final}$.

Entropy production corresponds to information loss.
Q1: Polchinski et al.: Unmeasurable information is lost information, Hawking radiation is not "really" a thermal but an entangled state

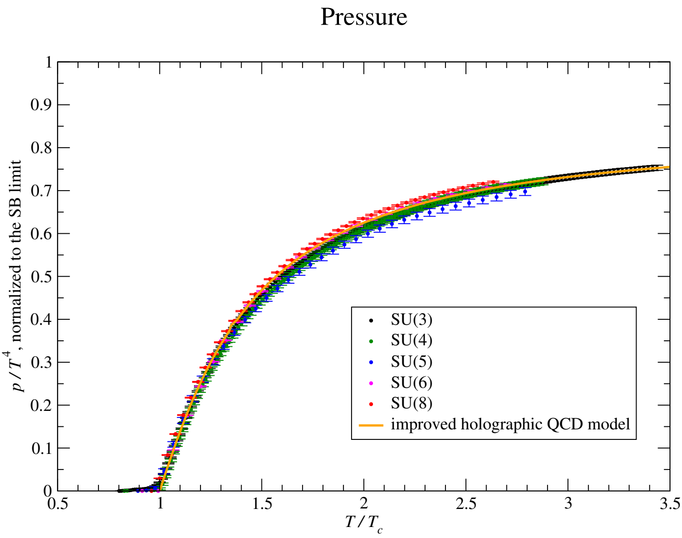
Holography: The Bekenstein entropy $S_B = k_B A/(4\ell_P^2)$ is proportional to the surface not the volume of a BH

The dictionary



Hawking temperature = boundary temperature
(endless debates on hep-th for the non-equilibrium situation)

BH entropy problem = QCD entropy problem

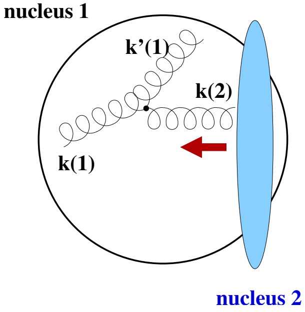


AdS/CFT is only valid for large N $\Rightarrow$ large N lattice simulations

M. Panero, 0907.3719

I shall concentrate on the second phase but as a reminder:

1. Phase: Interaction between gluons from different nuclei; pQCD evolution of the density matrix D.
decay time of the ratio $\mathrm{Tr}D^2/(\mathrm{Tr}D)^2$
generated entropy from $S(\tau_f) = \mathrm{Tr}\,\{D(\tau_f)\log D(\tau_f)\}$



This turned out to be a very hard calculation giving a finite result only for (Λ IR cutoff of CGC model)

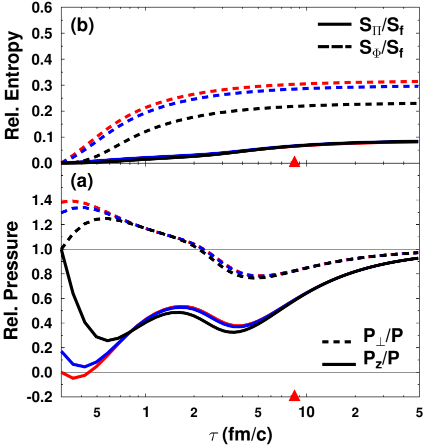
$$g^4 \to g^2(\Lambda^2)\, g^2(1/x^2)$$

which is exactly what was found by Kovchegov & Weigert and Balitsky.

Q1: Entropy is generated (at mid-rapidity) by mixing gluons from the two nuclei

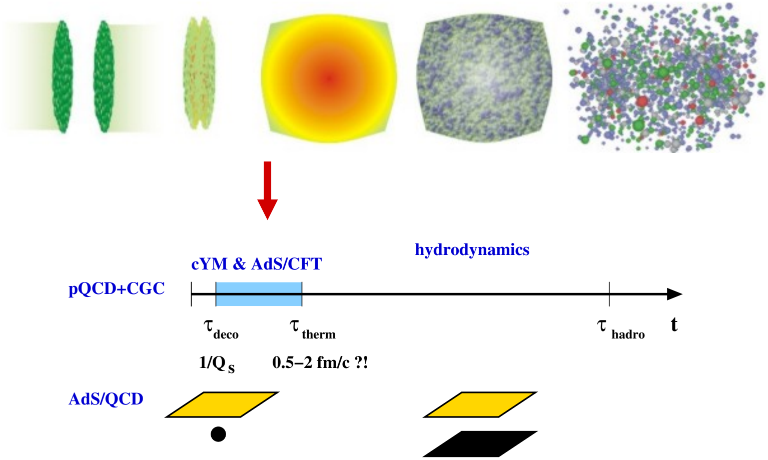
Q2: This process is very fast (0.3 fm/c) but only $\sim$ 30-40 % of the needed entropy is generated

3. Phase: Viscous hydrodynamics



Conclusion: Nearly ideal hydrodynamics. Only little enropy is produced. Entropy production needs 1-2 fm/c)

Different stages of entropy production in a HIC



1. approach: Classical Field Theory

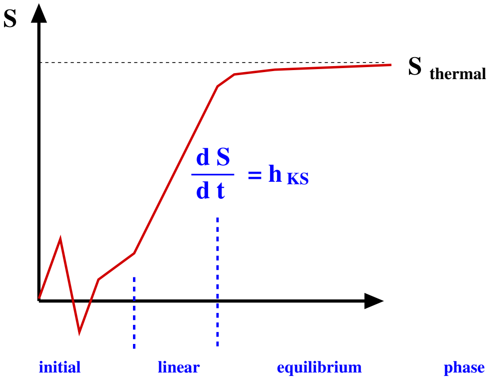
Non-linear Dynamics and Quantum Decoherence
Q1: Husimi coarse graining

The Lyapunov exponents of a classical theory are determine numerically. The Kolmogorov-Sinai entropy is defined as

$$h_{KS} = \sum_{i, \lambda_i > 0} \lambda_i$$

The "Kolmogorov-Sinai entropy" is no entropy, but an entropy growth rate.

a generic picture



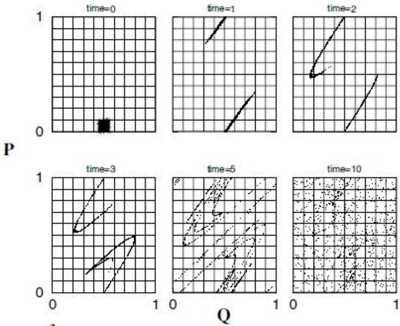
In the linear phase: $\frac{dS}{dt} = h_{KS}$

h_{KS} seems to play the same role in quantum theories !

Example: Standard map

M. Baranger, Chaos, Solitons and Fractals 13(2002)471

$$q, p \to Q, P: \qquad P = p + \frac{q\sin(2\pi q)}{2\pi}\ [\mathrm{mod}\ 1]; \qquad Q = q + P\ [\mathrm{mod}\ 1]$$



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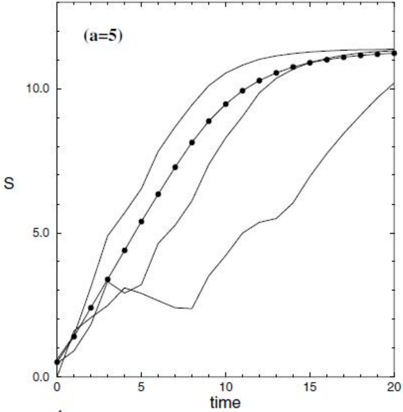

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Husimi function and Wehrl entropy

The Husimi function

$$H_\Delta(p,x;t) \;=\; \int \frac{dp'\,dx'}{\pi\hbar}\exp\left(-\frac{1}{\hbar\Delta}(p-p')^2-\frac{\Delta}{\hbar}(x-x')^2\right)W(p',x';t)$$

The Wehrl entropy

$$S_{H,\Delta}(t) \;=\; -\int \frac{dp\,dx}{2\pi\hbar}H_\Delta(p,x;t)\ln H_\Delta(p,x;t); \qquad \lim_{t\to\infty}\frac{dS_{H,\Delta}}{dt} \;=\; h_{KS}$$

Crucial assumption: Classical and quantum system have similar entropy growth rate

The Husimi function takes into account that the uncertainty principle implies coarse graining independent of the actual measurement.

It is easiest explained for a simple example: The inverse 1-dim oscillator.

$$\hat{\mathcal{H}} = \frac{1}{2}\hat{p}^2 - \frac{1}{2}\lambda^2\hat{x}^2$$

Initial state: Gaussian wave packet of width $\sqrt{\hbar/\omega}$:

$$\langle x|\psi_0\rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/4} e^{-\omega x^2/2\hbar}$$

The Wigner function associated with the density matrix $\hat{\rho}$ is defined as

$$W(p,x;t) = \int du\; e^{\frac{i}{\hbar}pu}\langle x - \frac{u}{2}|\;\hat{\rho}(t)\;|x + \frac{u}{2}\rangle$$

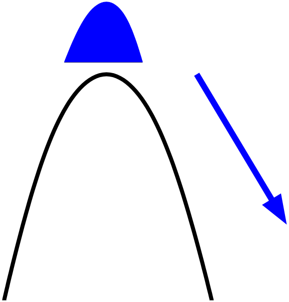
$$\int \frac{dp\,dx}{2\pi\hbar} W(p,x;t) = \mathrm{Tr}\,[\hat{\rho}] = 1$$

$$\int \frac{dp\,dx}{2\pi\hbar} [W(p,x;t)]^2 = \mathrm{Tr}\,[\hat{\rho}^2] \leq 1$$

The Wigner function is constant along the classical path.

$$x = x_0 \cosh \lambda t + \frac{p_0}{\lambda} \sinh \lambda t, \qquad p = \lambda x_0 \sinh \lambda t + p_0 \cosh \lambda t,$$

Thus there is only one positive Lyapunov exponent, namely λ and $h_{KS} = \lambda$



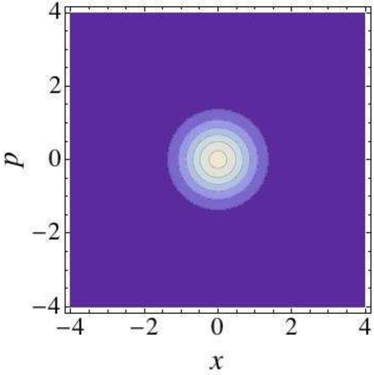
The Husimi function

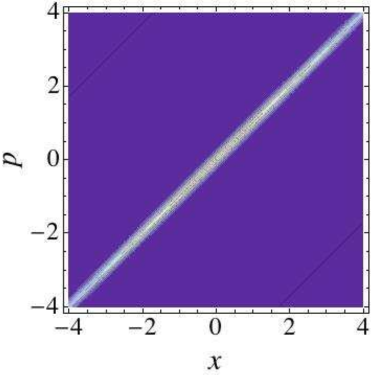
$$H_\Delta(p,x;t) \;=\; \int \frac{dp'\,dx'}{\pi\hbar}\,\exp\left(-\frac{1}{\hbar\Delta}(p-p')^2-\frac{\Delta}{\hbar}(x-x')^2\right)W(p',x';t)$$

is non-negative, in contrast to the Wigner function. Δ is an arbitrary parameter corresponding to the actual measurement.

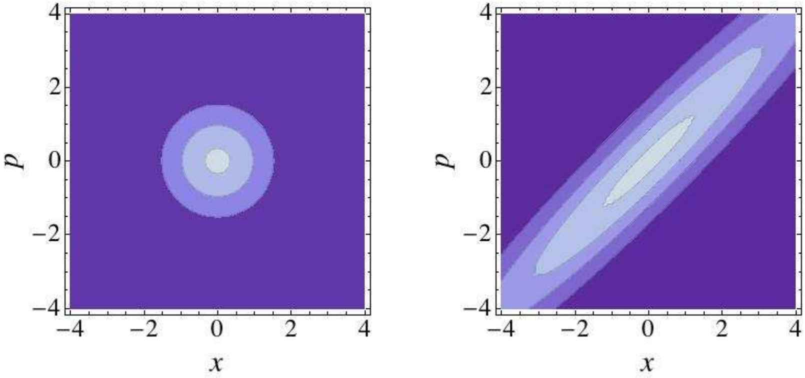
For this simple case the differential equation for W and the integration can be done analytically

The Wigner function for $t = 0$ and $t = 2/\lambda$





The Husimi function for $t = 0$ and $t = 2/\lambda$ for $\Delta = \lambda$. The phase space volume increases due to smearing.



For the Wehrl entropy

$$S_{H,\Delta}(t) \;=\; -\int \frac{dp\,dx}{2\pi\hbar} H_\Delta(p,x;t) \ln H_\Delta(p,x;t)$$

one finds

$$\lim_{t\to\infty} \frac{dS_{H,\Delta}}{dt} \;=\; \lambda \;=\; h_{KS}$$

independent of Δ **!**

Relation to Other Definitions of Entropy

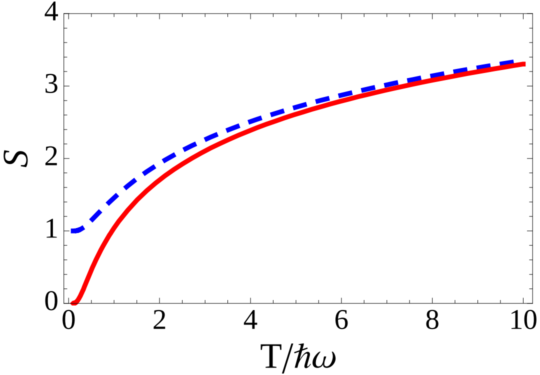
one harmonic oscillator; occupation probability of eigenstate $|n\rangle$:

$$\begin{aligned} w_n &= e^{-n\beta\hbar\omega}/\mathcal{Z}_\beta \\ \mathcal{Z}_\beta &= \sum_{n=0}^{\infty} e^{-n\beta\hbar\omega} = (1 - e^{-\beta\hbar\omega})^{-1} \end{aligned}$$

von Neumann entropy

$$\begin{aligned} S_{\mathrm{vN}} &\equiv -\sum_{n=0}^{\infty} w_n \ln w_n = \frac{\beta\hbar\omega}{e^{\beta\hbar\omega} - 1} - \ln(1 - e^{-\beta\hbar\omega}) \\ &= -\bar{n}\ln\bar{n} + (\bar{n}+1)\ln(\bar{n}+1) \end{aligned}$$

S_{vN} (solid) versus S_H (dashed)



Classical YM theory

$$
\begin{aligned}
H &= \frac{1}{2}\sum_{x,a,i} E_i^a(x)^2 + \frac{1}{4}\sum_{x,a,i,j} F_{ij}^a(x)^2 \\
F_{ij}^a(x) &= \partial_i A_j^a(x) - \partial_j A_i^a(x) + \sum_{b,c} f^{abc} A_i^b(x) A_j^c(x)
\end{aligned}
$$

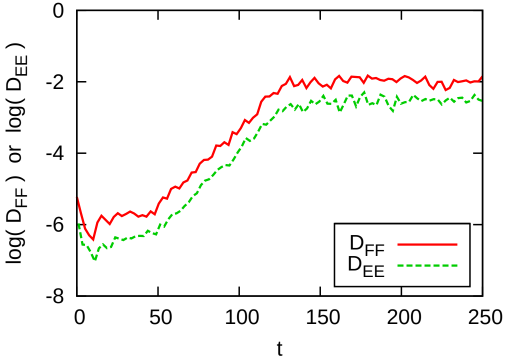
$$
\delta\dot{X}(t) = \mathcal{H}\delta X(t)
$$

$$
\begin{aligned}
\dot{A}_i^a(x) &= E_i^a(x) \\
\dot{E}_i^a(x) &= \sum_j \partial_j F_{ji}^a(x) + \sum_{b,c,j} f^{abc} A_j^b(x) F_{ji}^c(x)
\end{aligned}
$$

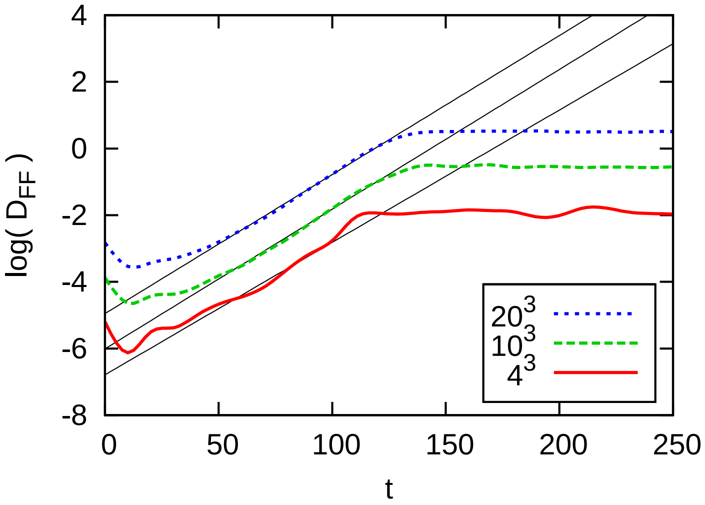
Different distance measures give the same result.

$$D_{EE} = \sqrt{\sum_x \{ \sum_{a,i} E_i^a(x)^2 - \sum_{a,i} E'^a_i(x)^2 \}^2}$$

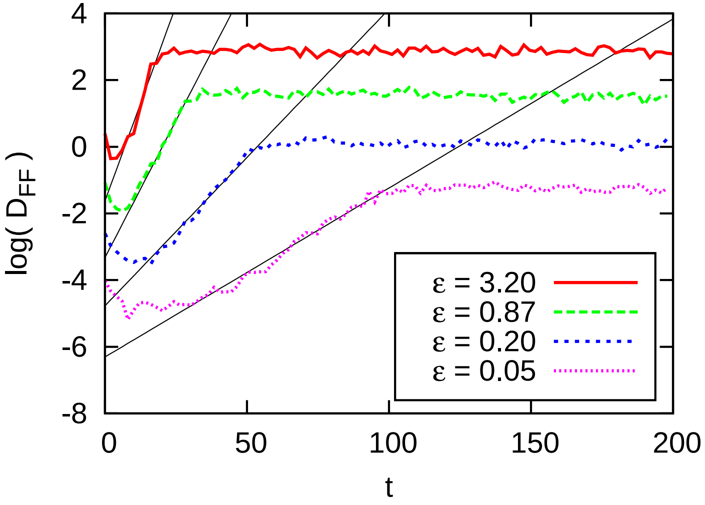
$$D_{FF} = \sqrt{\sum_x \{ \sum_{a,i,j} F_{ij}^a(x)^2 - \sum_{a,i,j} F'^a_{ij}(x)^2 \}^2}$$



Time evolution in SU(2) simulation on 4^3, 10^3, and 20^3 lattices with the same energy density



Time evolution in SU(3) simulation on a 4^3 lattice



Classical YM theories are UV divergent
$\Rightarrow$ The lattice constant has the physical meaning of a screening length

Scale setting can be done by different means:
equation the energy densities on the lattice

$$
\begin{aligned}
\epsilon_{cl}(T) &= \frac{\epsilon^{\mathrm{L}}}{a^4 g^2} \\
\epsilon^{\mathrm{L}} &= 2(N_c^2 - 1)\frac{1}{L^3}\sum_{\mathbf{k}} |\mathbf{k}| \frac{T^{\mathrm{L}}}{|\mathbf{k}|} \\
\epsilon(T) &= 2(N_c^2 - 1)\int \frac{d^3 k}{(2\pi)^3} \frac{|\mathbf{k}|}{e^{|\mathbf{k}|/T} - 1}
\end{aligned}
$$

or choosing the damping length of SU(N) gauge theories

All approaches give

$$a \;=\; \frac{\theta}{T} \;\sim\; \epsilon^{-1/4}$$

with θ of order unity

The thermalization time is estimated from

$$\tau_{\rm eq} \;=\; \frac{\Delta s}{s_{KS}} \;+\; \tau_{delay}$$

The delay time is found to be of the order

$$\frac{1}{6(N_c^2-1)L^3}\, e^{\lambda_{\rm max}\tau_{\rm delay}} \;\approx\; 1$$

Q2: numerically we find

$$\tau_{\text{eq}} \approx 2 \text{ fm/c}$$

with substantial theoretical uncertainties

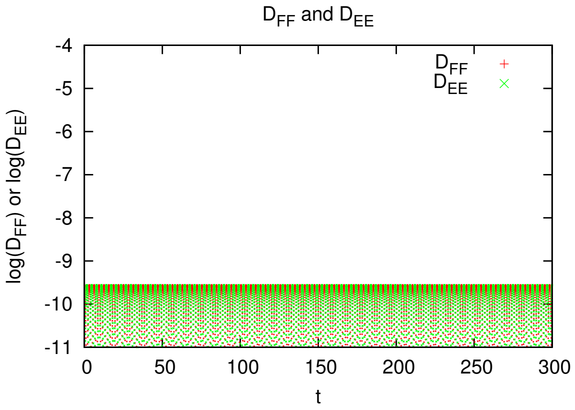
$\Rightarrow$ This would suggest substantial corrections due to incomplete thermalization for the LHC experiments.

Initial fluctuations might make $\leq$ 1 fm/c possible

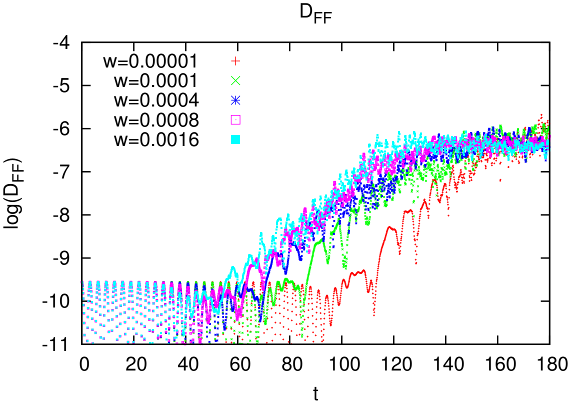
New results: arXiv:1304.1807

We tested whether more realistic initial gauge field configurations lead to smaller τ_{eq}.

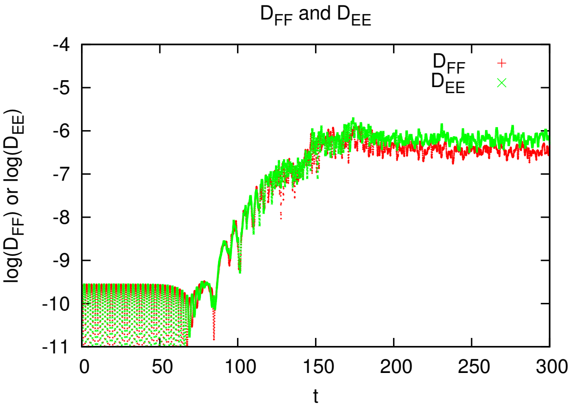
$$
\begin{aligned}
A_i^a(\vec{r}) &= \delta_{i2}\left[\epsilon_1 \sin\left(\frac{2x\pi}{N_x}\right) + \epsilon_2 \sin\left(\frac{2nx\pi}{N_x}\right)\sin\left(\frac{2mz\pi}{N_z}\right)\right] + \eta_i^a(\vec{r}) \\
E_i^a(\vec{r}) &= 0
\end{aligned}
$$

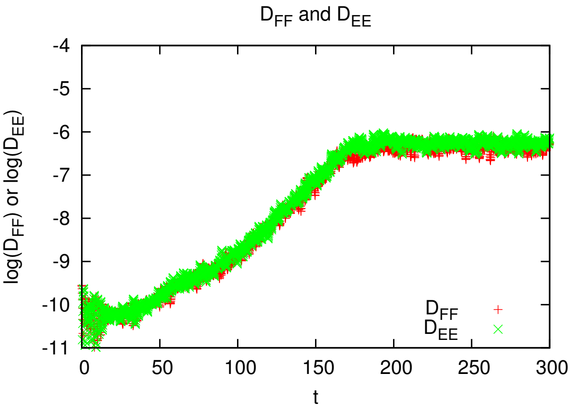


CGC-type field without initial fluctuations ⇒ purely numerical fluctuations are under control.

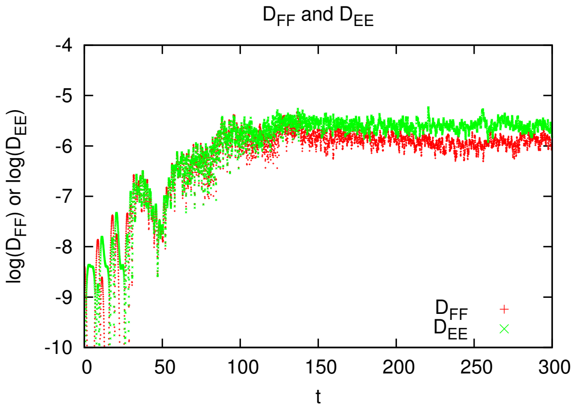


effect of increasing fluctuations


D_{FF} and D_{EE}
D_{FF}
D_{EE}
log(D_{FF}) or log(D_{EE})
-4
-5
-6
-7
-8
-9
-10
-11
0
50
100
150
200
250
300
t



For comparison: Behavior for fluctuations only

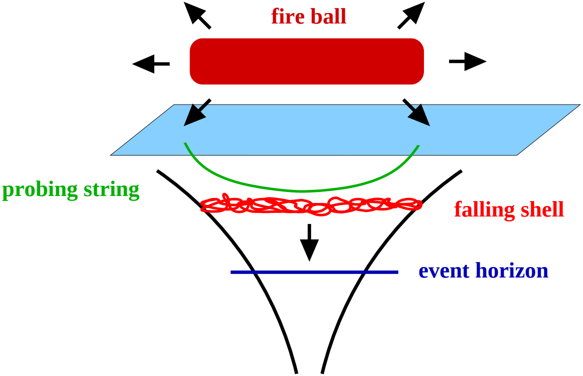


Constant background plus fluctuations

Upshot: The behavior is indeed generic

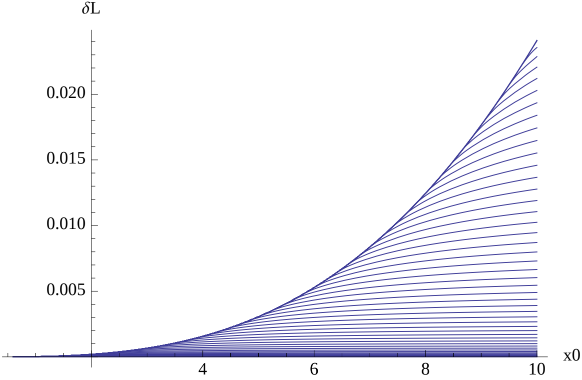
τ_{delay} depends on details

Idea: Probe black brane formation with a string or membrane.
PRL: 1012.4753 PRD: 1103.2683

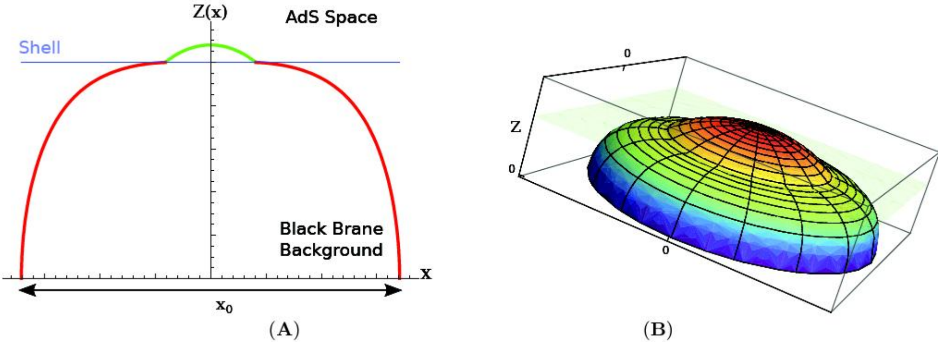


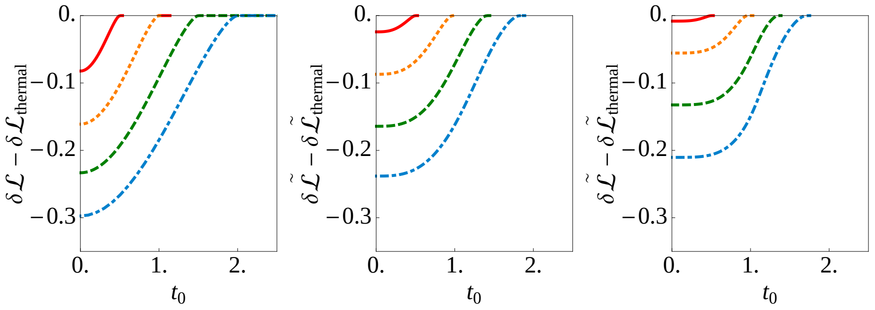
The change in geodetic length is sensitive to equal time correlators of high dimension gluonic operators.

$$\frac{\langle \mathcal{O}(t_{shell}, x)\mathcal{O}(t_{shell}, 0)\rangle_{\text{shell}}}{\langle \mathcal{O}(t_{shell}, x)\mathcal{O}(t_{shell}, 0)\rangle_{\text{AdS}}} \approx e^{-\Delta \delta L(t_{shell}, x)}$$

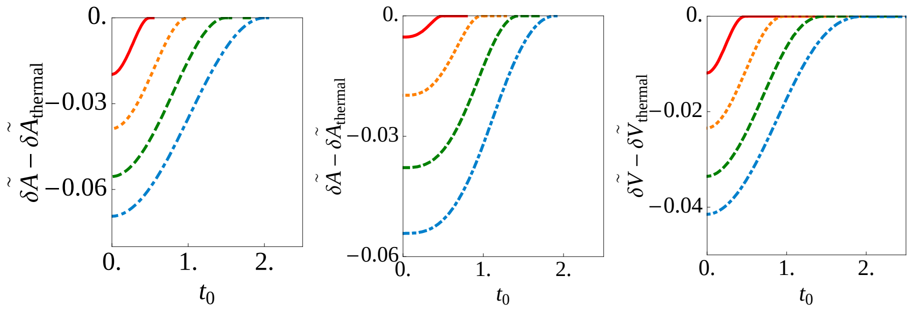


We solved analytically and numerically different cases: $AdS_3 \sim CFT(1+1)$, $AdS_4 \sim CFT(1+2)$, $AdS_5 \sim CFT(1+3)$ and analysed how the length of the geodesic/the area of the surface approaches its thermal value, as a function of ℓ and t_0.





$\delta\tilde{\mathcal{L}} - \delta\tilde{\mathcal{L}}_{\text{thermal}}$ ($\tilde{\mathcal{L}} \equiv \mathcal{L}/\ell$) for $d = 2, 3, 4$ (left,right, middle) and $\ell = 1, 2, 3, 4$ (top to bottom curve).



$\delta\tilde{\mathcal{A}} - \delta\tilde{\mathcal{A}}_{\mathrm{thermal}}$ ($\tilde{\mathcal{A}} \equiv \mathcal{A}/\pi R^2$ and $\delta\tilde{V} - \delta\tilde{V}_{\mathrm{thermal}}$ ($\tilde{V} \equiv V/(4\pi R^3/3)$) as a function of t_0 for radii $R = 0.5, 1, 1.5, 2$ (top to bottom) in $d = 3$ (left panel) and $d = 4$ (middle and right panel) CFT

Observations

- Thermalization is approached as fast as compatible with causality.
  Note: speed of light (gluons), not speed of sound (density fluctuations)
  For heavy ion collisions this implies
  $\tau \sim 1/(2Q_s) \sim 0.1 fm/c$
  Clear contradiction to YM result ?
- Short distances thermalize first, top-down rather than bottom-up thermalization
  Unavoidable in the AdS dual theory. A fundamental difference between strong and weak coupling ???

New papers: PRL 1307.1487; JHEP 1307.7086

We included fluctuations in one spatial dimension of a scalar field on a 1+2 dimensional boundary and calculated in AdS_4 using perturbative expansions.

$$\phi(t,x) \; = \; \epsilon \left(1 + e^{-\mu^2 x^2}\right) \, e^{-(t-\nu)^2/\sigma^2}$$

We compared with

- free streaming
- second order viscous hydrodynamics

$\epsilon(x,y) \; \Rightarrow$ "local temperature" $\epsilon = \frac{1}{8\pi G_N}\left(\frac{4}{3}\pi T\right)^3$

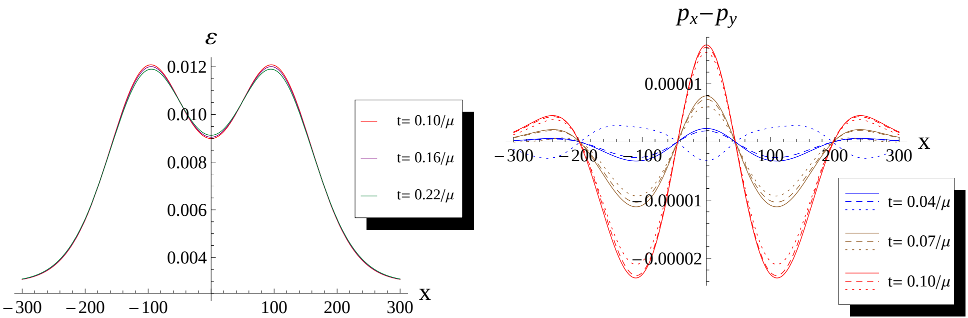
shear and bulk viscosity $\eta = \frac{1}{16\pi G_N}\left(\frac{4}{3}\pi T\right)^2$ and $\zeta = 0$

$\Rightarrow$ [Hubeny and Rangamani, 1006.3675] shear stress relaxation time $\tau_\Pi = \frac{3}{4\pi T}\left(1 + \frac{\gamma}{3} + \frac{\Gamma'(-1/3)}{3\Gamma(-1/3)}\right)$

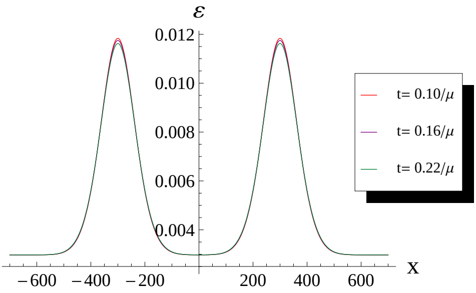
"We" (i.e. primarily A. Bernamonti and F. Galli) solve

$$
\begin{aligned}
D &= -u_t\partial_t + u_x\partial_x \\
M^{\alpha\beta} &= \frac{\partial^{\langle\alpha}u^{\beta\rangle}}{\theta} \\
\theta &= \partial_\rho u^\rho \\
\sigma^{\alpha\beta} &= P^{\alpha\rho}P^{\beta\sigma}\left(\partial_{\langle\rho}u_{\sigma\rangle} - \frac{1}{2}P_{\rho\sigma}\theta\right) \qquad \text{shear tensor} \\
D\sigma^{\alpha\beta} &= 2(D\theta)M^{\alpha\beta\rangle} + 2\theta DM^{\alpha\beta} \\
\Pi^{\alpha\beta}_{(2)} &= \eta\tau_\Pi\left[\,^{\langle}D\sigma^{\alpha\beta} + \frac{1}{2}\sigma^{\alpha\beta}\theta\right] + \dots
\end{aligned}
$$

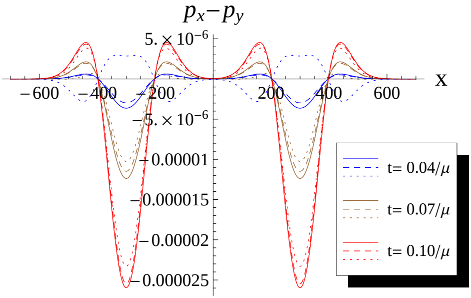
From the stress tensor Π one can read of $p_x - p_y$



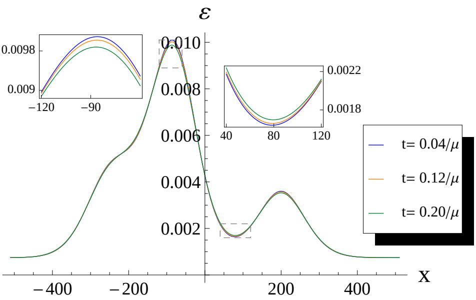
Conclusion: hydroization is reached long before equilibration



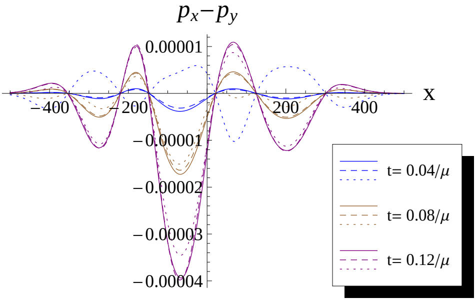
ε
0.012
0.010
0.008
0.006
0.004
−600
−400
−200
200
400
600
x
t= 0.10/μ
t= 0.16/μ
t= 0.22/μ



p_x−p_y
5. × 10^−6
−5. × 10^−6
−0.00001
−0.000015
−0.00002
−0.000025
−600
−400
−200
200
400
600
x
t= 0.04/μ
t= 0.07/μ
t= 0.10/μ



ε
0.010
0.008
0.006
0.004
0.002
−400
−200
200
400
x
0.0098
0.009
−120
−90
0.0022
0.0018
40
80
120
t= 0.04/μ
t= 0.12/μ
t= 0.20/μ



$p_x - p_y$
0.00001
−0.00001
−0.00002
−0.00003
−0.00004
−400
−200
200
400
x
t= 0.04/μ
t= 0.08/μ
t= 0.12/μ

phase	Q1:mechanism	Q2: τ_{therm}	ΔS
1.) pQCD	mixing	0.3 fm/c	30-40 %
2.) cYM	Husimi trafo	2 fm/c	40-50%
2.) AdS/CFT	Bekenstein	0.1 $\Rightarrow$? fm/c	40-50 %
3.) hydrodynamics	viscosity	1-2 fm/c	10-20 %
4.) hadron gas	fragmentation	1 fm/c	10 %

Interpretation: “hydroization” time $<<$1 fm/c, equilibration time 2 fm/c

The same picture emerges from detailled numerical simulations which are the way to go
$\Rightarrow$ Michal Heller's talk.

Michal gave us a great hands-on introduction into numerical AdS calculations in Regensburg Thank you again !!

Our string collaboration will now also start with numerical simulations van der Schee, Romatschke, Pratt, Hogg 1307.2539 and 1301.2635

In many aspects this is close to what one would like to do, but no fluctuations $\Rightarrow$ no chance to describe data ?!?

- AdS/CFT analytic: short time expansion $\Rightarrow$ starting condition in local rest frame

$$T^{\mu\nu} = \mathrm{diag}(-\epsilon, P_T, P_T, -1.5P_T)$$

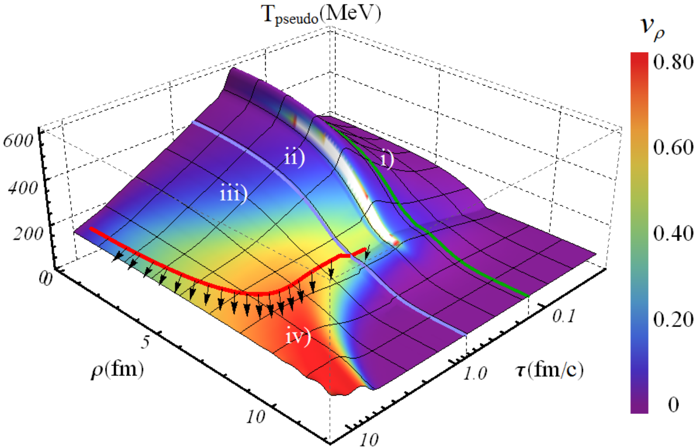
- AdS/CFT numerical solution of Einstein equations, enforcing convergence

$$\begin{aligned} ds^2 &= -Ad\tau^2 + \Sigma^2\left(e^{-B-C}d\xi^2 + e^B d\rho^2 + e^C d\theta^2\right) \\ &\quad +2drd\tau + 2Fd\rho d\tau \\ B(r,\tau,\rho) &\to B_0(r,\tau,\rho) + \sum_{i=0}^{6} \frac{b_i(\tau,\rho)r^{-i}}{1+\sigma^7 r^{-7}} \end{aligned}$$

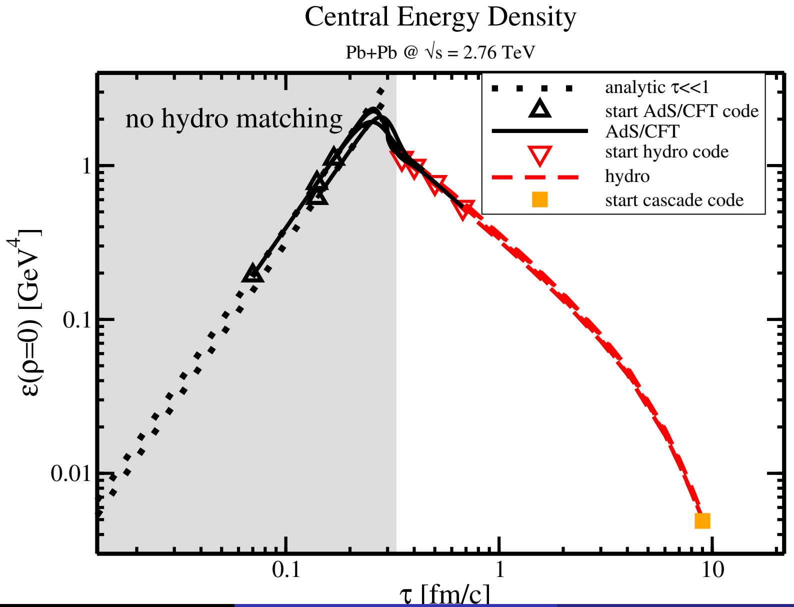
- change at some time to a hydro code
- hadronization by MC code describing particle scattering

The general scheme

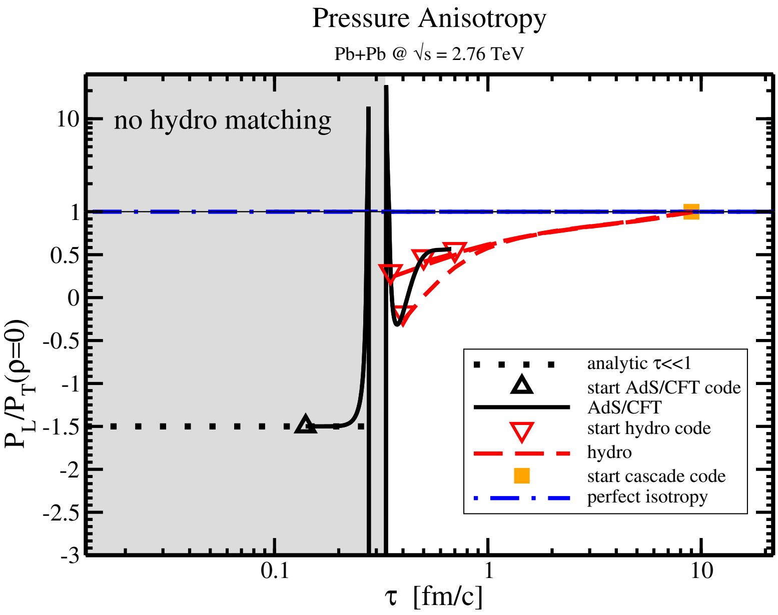
i)early time expansion   ii) numerical AdS   iii) hydro   iv) kinetic theory



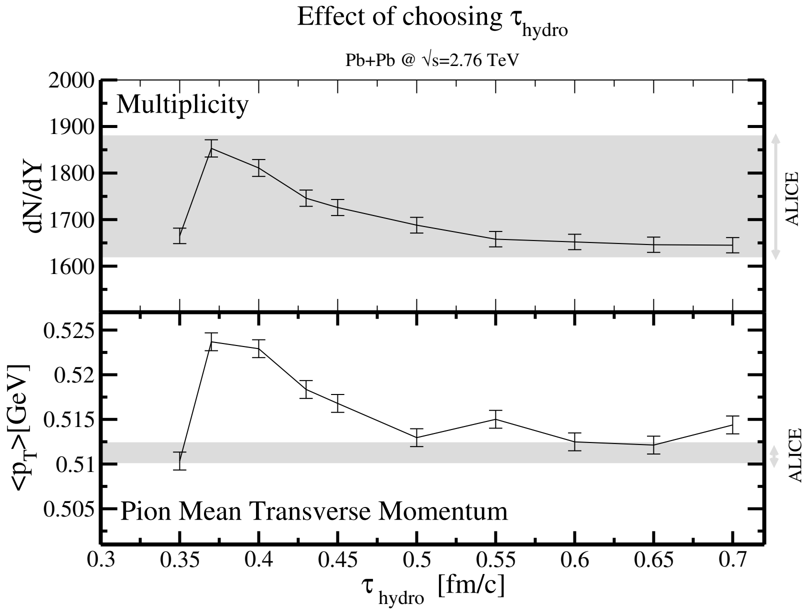
Time evolution of energy density at center of fireball for different values of σ , τ_{init} and τ_{hydro}.



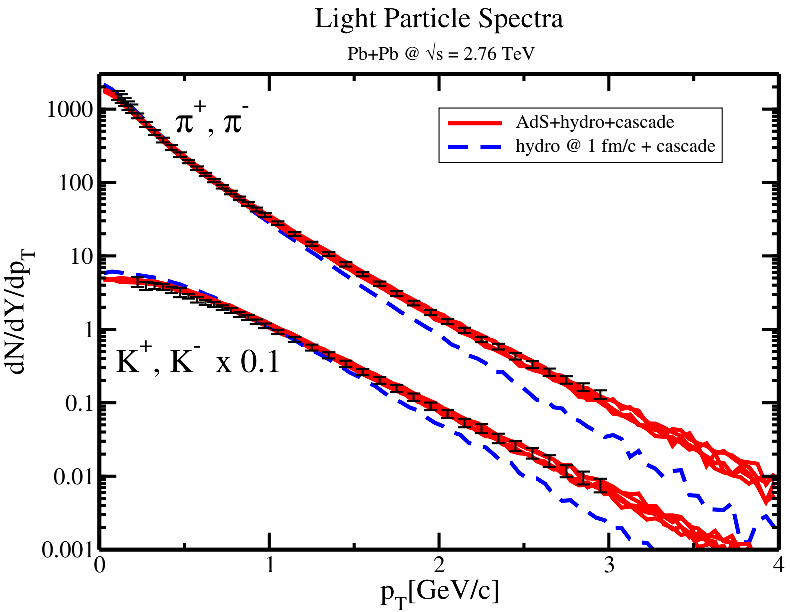
P_L/P_T at center of fireball for different values of $\tau_{\rm hydro}$



Comparison with ALICE data



Comparison with ALICE data



Comments:

The results look a bit too good to be true to me

- It is hard to believe that the instability induced by $P_L/P_T = -1.5$ is free of uncertainties, e.g. , $T_{tt} = \delta(t+z)\epsilon_0 \int_{-\infty}^{\infty} dz \left[1 + e^{(\sqrt{\rho^2+z^2}-R)/a}\right]^{-1}$ is schematic
- Exact agreement with data without including fluctuations is very surprising

Note: During the oscillatory phase free-streaming is not a valid description in this calculation

 This is something we might be able to improve upon

Conclusions

- Understanding entropy production, hydroization and thermalization in HICs is a problem of fundamental importance.
- Thermalization via non-linear dynamics and coarse graining with $\hbar$, including initial fluctuations, needs $\tau \approx 1-2\ \mathrm{fm/c}$.
- Thermalization for strong coupling as described by AdS/CFT is top-down and very fast $\tau \approx 0.1\mathrm{fm/c}$. Initial fluctuations change this conclusion.
- Perhaps the initial magentic field helps, see Gergo's talk
- The practical aim: Obtain precise results for e.g. η

Many Thanks to

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