Mass-Deformed IR Conformal Gauge Theories

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Workshop on Strongly-Interacting Field Theories

Jena, Germany, 29th November 2012

Credits

References

- L. Del Debbio *et al.*, Phys. Rev. D80 (2009) 074507 [arXiv:0907.3896]
- L. Del Debbio *et al.*, Phys. Rev. D82 (2010) 014509 [arXiv:1004.3197]
- L. Del Debbio *et al.*, Phys. Rev. D82 (2010) 014510 [arXiv:1004.3206]
- L. Del Debbio *et al.*, Phys. Rev. D84 (2011) 034506 [arXiv:1104.4301]
- A. Patella *et al.*, PoS LATTICE2011 (2011) 084 [arXiv:1111.4672]

Collaborators: E. Bennett, F. Bursa, L. Del Debbio, D. Henty, E. Kerrane, A. Patella, T. Pickup, C. Pica, A. Rago

Standard Model and beyond

- The strong and electroweak interactions successfully described by the standard model (QCD for the strong sector, $SU(2)_L \otimes U(1)_Y$ with Higgs mechanism for the electroweak sector)
- The strong sector is believed to be valid at all energies, while the weak sector has a natural cut-off at the scale of the TeV
- Among the various models formulated to extend the electroweak sector of the SM above the TeV, strongly interacting BSM dynamics is based on the existence of a new strong interaction
- The lattice provide a natural framework to perform calculations in strongly coupled gauge theories from first principles

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DEWSB in QCD

As a result of chiral symmetry breaking, in QCD there is a quark condensate

$$\langle \bar{u}u + \bar{d}d \rangle \approx (200 \text{ MeV})^3$$

that is not invariant under $SU(2)_L \otimes U(1)_Y$

Not enough for accounting for the symmetry breaking of the Standard Model:

$$\langle \phi \rangle = 246 \text{ GeV}$$

Can a scaled-up version of QCD work?

From Colour to Technicolour

- New strongly interacting gauge theory with N_{TC} colours and N_{TCf} fermions
- The (bilinear) fermionic condensate replaces the Higgs condensate
- Some of the Goldstone bosons of the techni-chiral symmetry are absorbed by three gauge bosons, which become the massive $W^{\pm}$, Z, while others acquire a mass of the order of the TeV
- The chiral condensate provide standard model quarks with a non-zero mass (Extended Technicolour)

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From QCD-like to walking dynamics

Problems:

- Large flavour changing neutral currents
- Quark mass hierarchy
- Prediction for the S parameter

The problems of the technicolour models can be traced back to the logarithmic running of the coupling in QCD

Ultimately, QCD-like dynamics will dominate in the infrared (confinement) and in the ultraviolet (asymptotic freedom)

A very slowly running (*walking*) coupling in an intermediate energy domain could determine a natural mass hierarchy, suppress flavour changing neutral currents and give a small contribution to the S parameter, but needs a large (order 1) anomalous dimension of the chiral condensate

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Perturbative IR fixed point

The running of the coupling in SU(N) gauge theories with N_f fermion flavours transforming in the representation R is determined by the β-function

$$\mu \frac{dg}{d\mu} = -b_0 g^3 - b_1 g^5 + \dots \ ,$$

with

$$b_0 = \frac{1}{(4\pi^2)} \left(\frac{11}{3} N - \frac{4}{3} T_R N_f \right) \ , \qquad b_1 = \frac{1}{(4\pi)^4} \left[\frac{34}{3} N^2 - \frac{20}{3} N T_R N_f - 4 \frac{N^2 - 1}{d_R} N_f \right]$$

Banks-Zaks (perturbative) fixed point (two-loops):

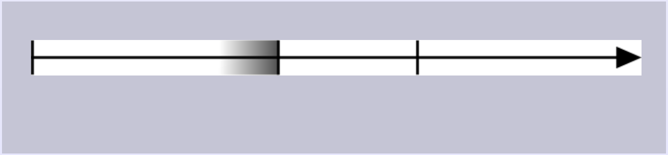
$$\left. \frac{dg}{d\mu} \right|_{2-L} = 0 \qquad \Rightarrow \qquad g^\star \simeq - \frac{b_0}{b_1} \ll 1$$

Starting from $g < g^\star$ in the ultraviolet, in the infrared $g \to g^\star$ (IR fixed point)

Phases of a gauge theory

At fixed N a critical number of flavours N_f^{cu} exists above which asymptotic freedom is lost

Banks and Zaks conjectured that an N_f^{lu} exists such that a non-trivial infrared fixed point appears for $N_f^{lu} \leq N_f \leq N_f^{cu}$ (conformal window)



At fixed fermion representation N_f^{lu} depends on the number of flavours

Near the BZ point naive scaling arguments can not be applied and walking can arise

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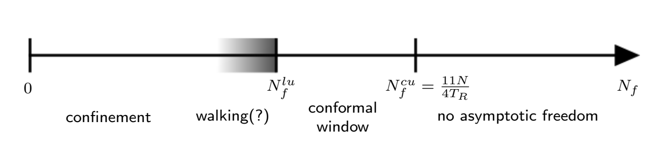

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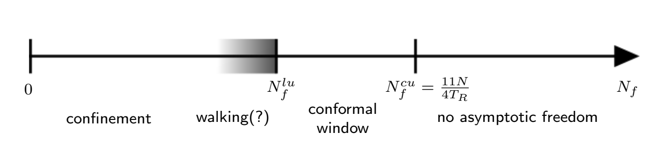

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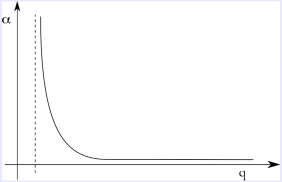
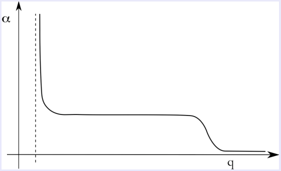
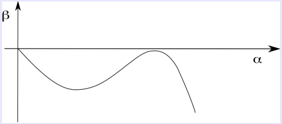
Near-conformality and β-function

QCD-like vs. Walking







Walking needs two separate scales Λ_χ (onset of the plateau at high energy) and Λ_{QCD} (start of QCD-like low-energy running)

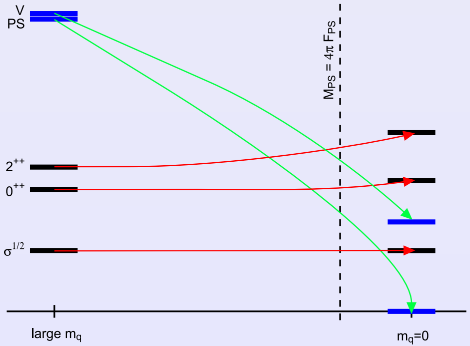
Open problems

- Can we determine the extent of the conformal window?
- Can we see a walking behaviour just below the conformal window?
- Can we measure the anomalous dimension?
- How does the spectrum of an IR conformal theory differs from that of a QCD-like theory?
- In particular, do (near-)conformal gauge theories have light scalars and if yes under which assumptions?

The inherently non-perturbative nature of the problem requires an approach from first principles like lattice calculations

Outline

The spectrum for a QCD-like theory



- At high fermion masses the theory is nearly-quenched
- At low fermion masses the relevant degrees of freedoms are the pseudoscalar mesons

IR Conformality, Walking and Confinement

1. Confinement (and χSB) characterised by a string tension (and a chiral condensate) of the order of the dynamically generated scale
2. Conformality characterised by power-law behaviour of correlators (unparticles)
3. A small mass term m in a conformal theory generates dynamical scales, a meson spectrum scaling as m^{ρ} and a non-trivial running of the coupling
4. A walking theory is confining in the infrared, but presents features of a IR-conformal theory in an intermediate energy range

A study in the chiral limit is mandatory

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Mass-deformation of the IR-conformal theory

A mass term drive the system outside the IR conformal point

Running of the mass

$$m(\mu) = m(\mu_0) \exp\left\{ - \int_{g(\mu_0)}^{g(\mu)} \frac{\gamma(z)}{\beta(z)} dz \right\} \equiv Z(\mu, \mu_0, \Lambda) m(\mu_0)$$

Close to the IR fixed point we assume a regular behaviour for the RG functions:

$$g \to g_* : \begin{cases} \beta(g) \simeq \beta_*(g - g_*) \\ \gamma(g) \simeq \gamma_* \end{cases}$$

Define a renormalised mass M from the condition $m(M) = M$

A large M destroys conformality and the theory looks like Yang-Mills with heavy sources

$$m_{mes} = 2M$$
$$m_{glue} = B_{glue}\Lambda$$

We are interested in the opposite regime $M \ll \Lambda$

Mass-deformation near the BZ fixed point

V. A. Miransky. Dynamics in the Conformal Window in QCD like theories. hep-ph/9812350.

$$\Lambda_{YM} = M\, e^{-\frac{1}{2b_0^{YM} g_*^2}} \ll M \ll \Lambda$$

- At energies much lower than M, the original theory is effectively described by a pure Yang-Mills theory with scale Λ_{YM}.
- Glueballs are lighter than mesons.
- A deconfinement transition occurs at a temperature $T_c \simeq \Lambda_{YM}$.
- Mesons are effectively quenched. The mesons are bound states of the quark-antiquark pair interacting via the YM static potential, the bound energy is small with respect to the mass of the fermions, and the correction to the potential due to quark-antiquark pair creation are negligible.
- As the mass M is reduced, the IR physics is always the same, provided that all the masses are rescaled with M.

Locking

For a physical quantity

$$m_X = A_X \mu^{\frac{\gamma_*}{1+\gamma_*}} m(\mu)^{\frac{1}{1+\gamma_*}} \ .$$

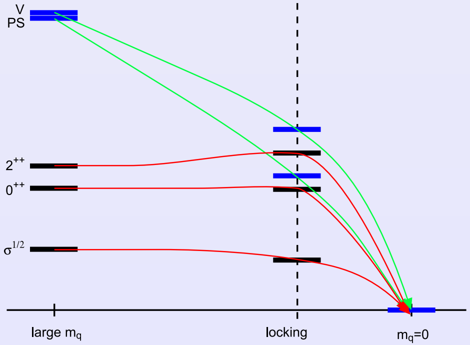
On the lattice, choosing $\mu = a^{-1}$ gives

$$a m_X = A_X (a m_0)^{\frac{1}{1+\gamma_*}}$$

Consequences

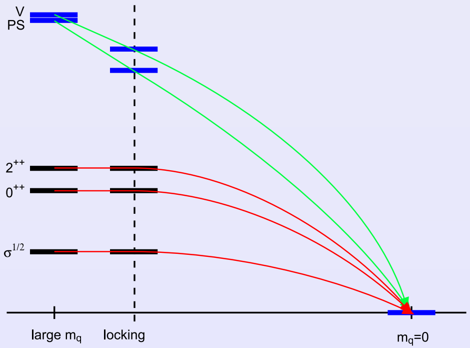
- Ratios of physical quantities with the same mass dimension are independent of the fermion mass if the latter is sufficiently small
- γ_* can be determined by looking at the small-mass scaling of a physical observable

Locking at intermediate M_{lock}



All spectral mass ratios depend very mildly on m below the locking scale

Locking at large M_{lock}



If this scenario is valid beyond BZ, at all scales $\ll \Lambda$

$$m_V/m_{PS} \simeq 1 + \epsilon$$
$$m_{PS} \gg \sigma^{1/2}$$
$$m_G/\sigma^{1/2} \simeq \left[m_G/\sigma^{1/2}\right]^{(YM)}$$

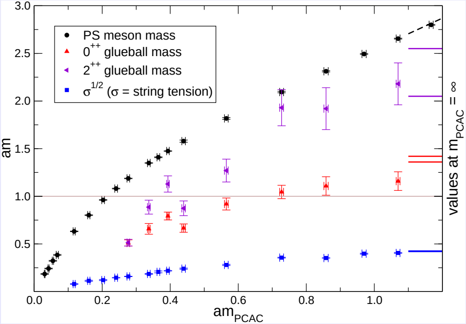
Outline

Details of the Simulations

- Study SU(2) with two adjoint Dirac flavour (suggested to be walking by Sannino-Dietricht) on the lattice
- SU(2) 1x1 plaquette action in the fundamental representation
- 2 Dirac Wilson fermions in the adjoint representation
- fixed lattice spacing: $\beta = 2.25$
- various volumes to account for finite size effects

Spectrum hierarchy

(L. Del Debbio *et al.*, arXiv:0907.3896)



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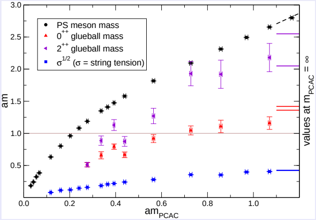
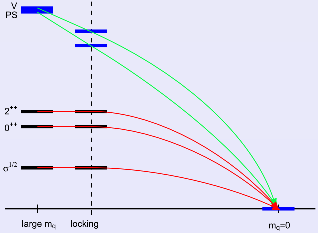




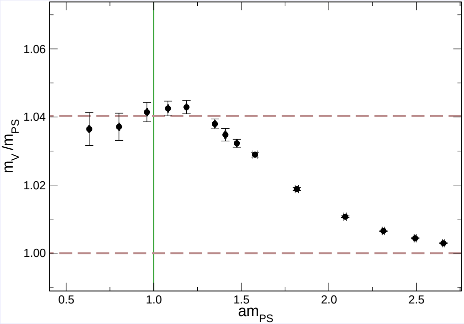
The pseudoscalar is always higher in mass than the 0^{++} glueball

$\Downarrow$

The locking scenario at high fermion mass looks plausible

m_V/m_{PS} vs. am_{ps}

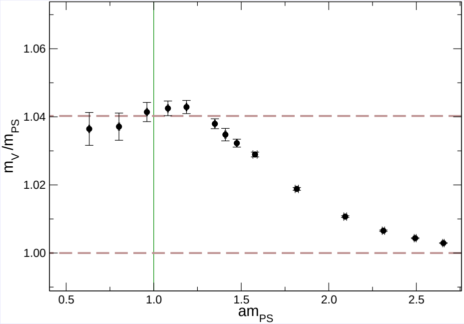
(L. Del Debbio *et al.*, arXiv:0907.3896)



Locking at $am_{ps} \simeq 1.25$

m_V/m_{PS} vs. am_{ps}

(L. Del Debbio *et al.*, arXiv:0907.3896)



Is it dynamical quenching?
How do we go to lower masses of the fermions?

Comparing with quenched data

Note that the spirit of the comparison is different than for the case of heavy quarks in QCD: here we are tuning β quenched to define the string tension in units of the cut-off and assuming that the latter is fixed while the other changes

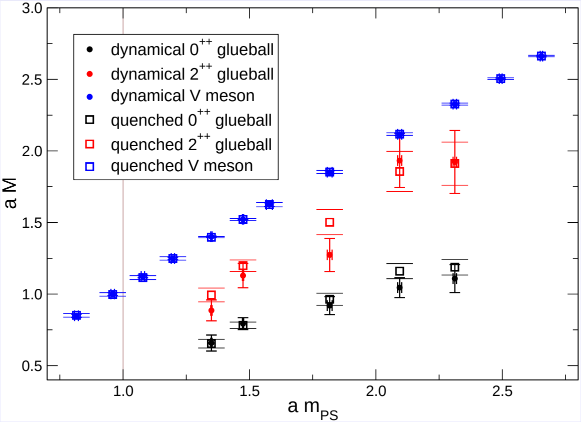
To compare with quenched data, we need to match the bare parameters of the quenched and the dynamical calculation

- β is matched demanding that $\sqrt{\sigma}$ is the same
- κ is matched demanding that m_{PS} is the same

The other observables can then be compared

IR dynamics - The spectrum

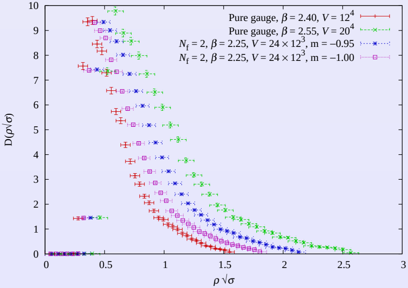
(L. Del Debbio *et al.*, arXiv:1004.3206)



The large-distance dynamics is nearly quenched

IR dynamics - Instantons

(E. Bennett and B. Lucini, arXiv:1209.5579)



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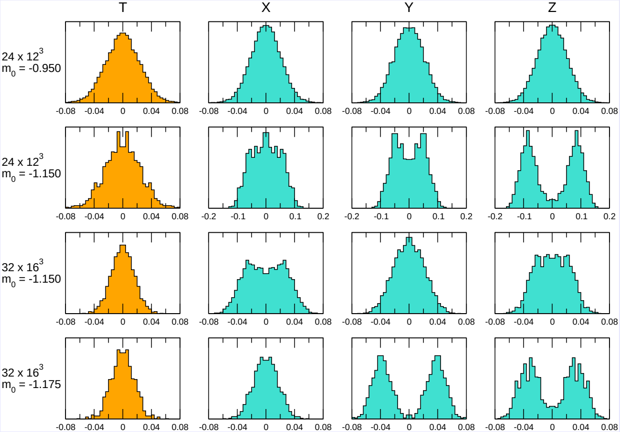
Polyakov loop distribution

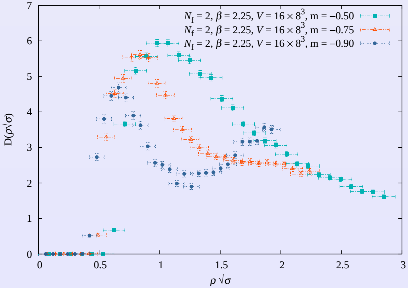
(L. Del Debbio *et al.*, arXiv:1004.3206)



We need larger lattices closer to the chiral limit

Instanton size distribution

(E. Bennett and B. Lucini, arXiv:1209.5579)



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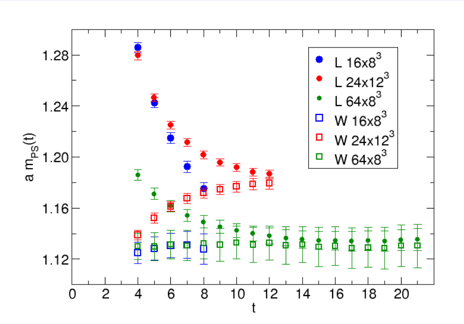
Temporal size effect

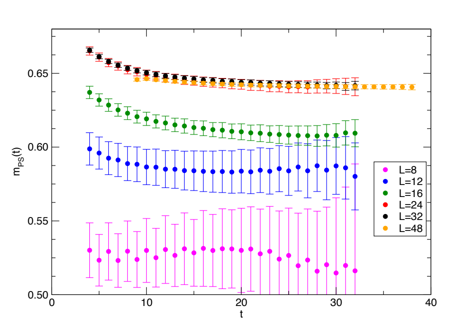
(F. Bursa *et al.*, arXiv:1104.4301)



A short temporal direction increases masses

Spatial size effect

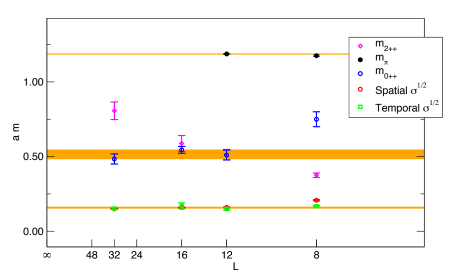
(F. Bursa *et al.*, arXiv:1104.4301)



A small spation volume decreases masses

Spectrum vs. Size ($M_{PS} = 1.187(2)$)

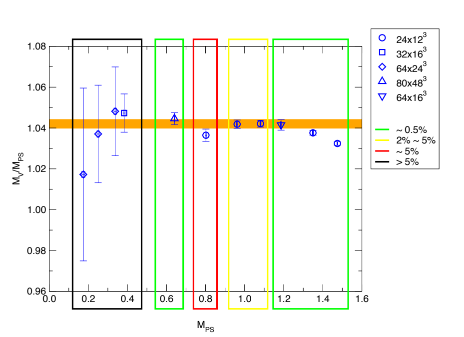
(F. Bursa *et al.*, in preparation)



Lighter states are more difficult to keep under control, very large lattices are needed

Estimate of Finite Size Effects

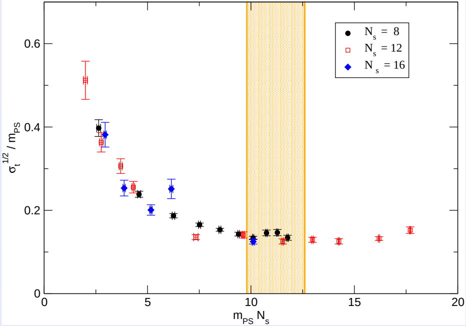
(A. Patella *et al.*, arXiv:1111.4672)



We can assume scaling with $M_{PS}L$

Outline

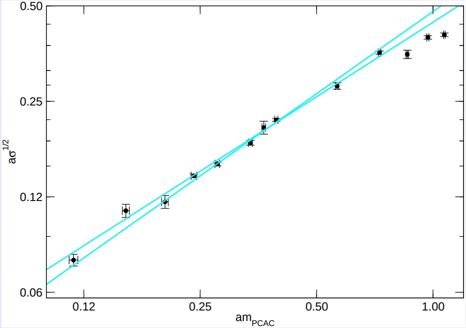
Finite size scaling



Ratios of spectral quantities are universal functions of $x = N_s(am_q)^{\frac{1}{1+\gamma}}$ or equivalently $y = N_s m_{PS}$ (B. Lucini, arXiv:0911.0020)

Condensate anomalous dimension - I

(Del Debbio et al., arXiv:1004.3206)

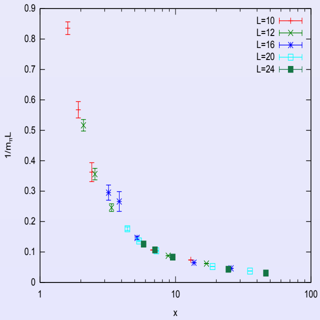


$$a\sigma^{1/2} = A_\sigma (am_q)^{\frac{1}{1+\gamma}} \qquad \gamma = 0.22(6)$$

γ too small for phenomenology?

Condensate anomalous dimension - II

(Giedt and Weinberg, arXiv:1201.6262)

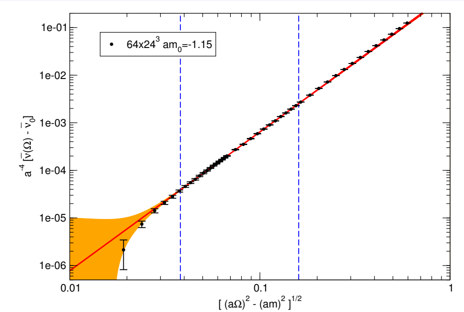


$$\gamma = 0.51(16)$$

γ too small for phenomenology?

Condensate anomalous dimension - III

(Patella, arXiv:1204.4432)



$$\gamma = 0.371(20)$$

γ too small for phenomenology?

Condensate anomalous dimension - Summary

Method	γ
FSS (Lucini:2009)	$0.05 < \gamma < 0.25$
SF (Bursa:2009)	$0.05 < \gamma < 0.56$
FSS (DelDebbio:2010)	$0.05 < \gamma < 0.20$
FSS (DelDebbio:2010)	0.22 ± 0.06
MCRG (Catterall:2011)	$-0.6 < \gamma < 0.6$
SF (DeGrand:2011)	0.31 ± 0.06
FSS (Giedt:2012)	0.51 ± 0.16
MNS (Patella:2012)	0.371 ± 0.020
Perturbative 4-loop (Pica:2010)	0.500
Schwinger-Dyson (Ryttov:2010)	0.653
All-orders hypothesis (Pica:2010)	0.46

All estimates are well below one

Discrepancies accounted for by finite size effects?

Can we quantify better systematic effects?

Outline

The Emerging Picture

1. The spectrum of SU(2) gauge theory shows the following features:
   - Well-defined mass hierarchy $m_{PS,V} > m_G > \sqrt{\sigma}$
   - Dynamical quenching
   - Dynamically-generated scale sliding with m_{PCAC}
   - Anomalous dimension $\gamma \simeq 0.37(2)$ preliminary result!
2. These results are compatible with the idea that SU(2) with $N_f = 2$ flavours of Dirac fermions is in a (near-)conformal phase
3. However
   - The anomalous dimension is too small for fitting current models of DEWSB
   - A walking scenario can not be completely excluded at this stage

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Perspectives

- Improvement of the finite size analysis
- Investigation of other observables
- Better control over the chiral limit
- Extrapolation to the continuum limit
- Finite temperature studies
- Extension to other gauge groups/representations

Conclusions

- We have developed tools and techniques for looking at IR conformality signatures in the spectrum
- In particular, FSS and (mostly) Dirac MNS allow precise measurements of the condensate anomalous dimension (relevant for phenomenology)
- These techniques played a crucial role in establishing IR conformality in SU(2) gauge theory with $N_f = 2$ Dirac adjoint fermions
- With this result resting on firm ground, the latter theory is an ideal play ground for testing methods of numerical investigations of IR conformal behaviour