Critical phenomena in (2+1)d chiral fermion systems

Lukas Janssen
Friedrich-Schiller-Universität Jena

& Jens Braun, Holger Gies, Dietrich Roscher

Workshop on Strongly-Interacting Field Theories
Jena, November 30, 2012

Research Training Group
Quantum and Gravitational Fields

seit 1558

based on: Phys. Rev. **D82** (2010) 085018
Phys. Rev. **D86** (2012) 105007

Chiral fermions: $d = 4$

- Clifford algebra:

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}, \qquad \gamma_5$$

- Chiral projector:

$$P_{\mathrm{L/R}} = \frac{1}{2}(1 \pm \gamma_5)$$

- Lagrangian:

$$\mathcal{L}_{\mathrm{kin}} = \bar{\psi}^a \mathrm{i}\partial\!\!\!/\, \psi^a \qquad = \bar{\psi}^a_{\mathrm{L}} \mathrm{i}\partial\!\!\!/\, \psi^a_{\mathrm{L}} + \bar{\psi}^a_{\mathrm{R}} \mathrm{i}\partial\!\!\!/\, \psi^a_{\mathrm{R}}$$

$$\mathcal{L}_{\mathrm{mass}} = m\bar{\psi}^a \psi^a \qquad = m\left(\bar{\psi}^a_{\mathrm{L}} \psi^a_{\mathrm{R}} + \bar{\psi}^a_{\mathrm{R}} \psi^a_{\mathrm{L}}\right)$$

- Chiral symmetry breaking (χSB) $\leftrightarrow$ dynamical mass generation:

$$\mathrm{U}(N_\mathrm{f})_\mathrm{L} \times \mathrm{U}(N_\mathrm{f})_\mathrm{R} \to \mathrm{U}(N_\mathrm{f})$$

Chiral fermions: $d = 3$

- Clifford algebra:

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu},$$

- Irreducible representation: $d_\gamma = 2^{\lfloor d/2 \rfloor} = 2$

$$\gamma_{\mu=1,2,3} \sim \sigma_{i=1,2,3}$$

$\Rightarrow$ no γ_5 ("no chirality")

Chiral fermions: $d = 3$

- Clifford algebra:

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu},$$

- Reducible representation: $d_\gamma = 4$ ... i.e. 4-component spinors

  $\Rightarrow$ 3 possible definitions of "chirality" with $\gamma_4,\ \gamma_5,\ \gamma_{45} := \mathrm{i}\gamma_4\gamma_5$:

$$P^{(4)}_{\mathrm{L/R}} = \frac{1}{2}(1 \pm \gamma_4) \quad \text{or} \quad P^{(5)}_{\mathrm{L/R}} = \frac{1}{2}(1 \pm \gamma_5) \quad \text{or} \quad P^{(45)}_{\mathrm{L/R}} = \frac{1}{2}(1 \pm \gamma_{45})$$

- Lagrangian

$$\mathcal{L} = \bar{\psi}^a \mathrm{i}\partial\!\!\!/ \psi^a + \cdots = \bar{\psi}^a_{\mathrm{L}} \mathrm{i}\partial\!\!\!/ \psi^a_{\mathrm{L}} + \bar{\psi}^a_{\mathrm{R}} \mathrm{i}\partial\!\!\!/ \psi^a_{\mathrm{R}} + \cdots$$

- Maximal "chiral" symmetry: $\mathrm{U}(2N_{\mathrm{f}})$

  ... can be dynamically/explicitly broken by masses/interactions

chiral symmetry (reducible) $\simeq$ flavor symmetry (irreducible)

Why chiral fermions in $d = 3$?

Working hypothesis:

(2+1)d fermion field theories provide an excellent playground to bridge the gap between different branches of physics.

Particle physics:

toy models for χSB
($\sim$ QCD, EWSB)

Statistical physics:

phase transitions, critical phenomena, universal exponents (c.f. bosonic $O(N)$ models)

Condensed matter:

low-energy effective theories for (quasi) 2d materials:

- high-T_c superconductors
- graphene
- topological insulators

Quantum gravity:

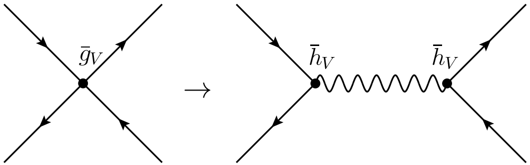
paradigm examples for nonperturbative renormalizability ("Asymptotic Safety Scenario")

Example 1: (2+1)d Thirring model

- classical action: $a = 1, \dots, N_\mathrm{f}$

$$S = \int \mathrm{d}^3 x \left[\bar{\psi}^a \mathrm{i}\partial\!\!\!/ \psi^a + \frac{\bar{g}}{2N_\mathrm{f}} \left(\bar{\psi}^a \gamma_\mu \psi^a \right)^2 \right], \qquad [\bar{g}] = 2 - d$$

- full $\mathrm{U}(2N_\mathrm{f})$ symmetry
- renormalizable in $1/N_\mathrm{f}$ expansion [Parisi '75; Hikami, Muta '77; Hands '95]
- (naive) partial bosonization involves vector boson:



$\Rightarrow$ no signature of χSB at large N_f . . . non-analyticity in N_f?

(2+1)d Thirring model: previous results

$1/N_\mathrm{f}$ expansion
[Parisi '75; Hikami & Muta '77; Hands '95]

- nonpert. renormalizability ✔
- no χSB ✗

$1/N_\mathrm{f}$ expansion & eff. pot.
[Kondo '95]

- for $N_\mathrm{f} < N_\mathrm{f}^\mathrm{cr}$: χSB ✔
- $N_\mathrm{f}^\mathrm{cr} = 2$ **?**

DSE + additional assumptions
[Gomes *et al.* '91; Hong & Park '94; Itoh *et al.* '95, ...]

- for $N_\mathrm{f} < N_\mathrm{f}^\mathrm{cr}$: χSB ✔
- $N_\mathrm{f}^\mathrm{cr} \simeq 3.24, 4.32, \infty$ **?**

Lattice MC
[Del Debbio, Hands, Lucini, ...'97, '99; Christofi, Hands, Strouthos '07; Chandrasekharan & Li '12]

- for $N_\mathrm{f} < N_\mathrm{f}^\mathrm{cr}$: χSB ✔
- $N_\mathrm{f}^\mathrm{cr} \simeq 6.6$ **?**

many open questions:

- nature of phase transition (2nd order, ∞ order?)
- phase transition mechanism?
- long-range observables (spectrum, etc.)?

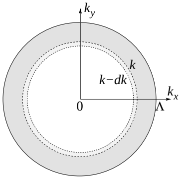
Our approach: Functional Renormalization Group (FRG)

Effective action Γ

$$\Gamma[\psi,\bar{\psi}] = \mathcal{L}\left(W[\eta,\bar{\eta}]\right), \quad W[\eta,\bar{\eta}] = \log \int_\Lambda \mathcal{D}\hat{\psi}\mathcal{D}\hat{\bar{\psi}}\, e^{-S[\hat{\psi},\hat{\bar{\psi}}]+\int \bar{\eta}\hat{\psi}+\int \hat{\bar{\psi}}\eta}$$

Idea (Wilson):

Integrate momentum shell by momentum shell!

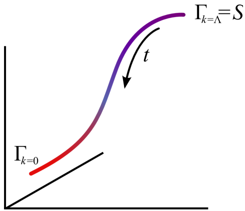


Effective average action:

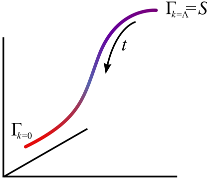
$$\Gamma[\bar{\psi},\psi] \mapsto \Gamma_k[\bar{\psi},\psi]$$

such that

$$\text{UV}: \quad \Gamma_{k=\Lambda} = S_{\text{bare}}$$
$$\text{IR}: \quad \Gamma_{k=0} = \Gamma$$



RG flow of effective average action Γ_k



Evolution for Γ_k:

[Wetterich '93]

$$\partial_k \Gamma_k[\psi, \bar{\psi}] = -\frac{1}{2} \,\mathrm{Tr}\, \frac{\partial_k R_k}{\Gamma_k^{(2)}[\psi, \bar{\psi}] + R_k}$$

with $\Gamma_k^{(2)} = \begin{pmatrix} \frac{\delta^2\Gamma}{\delta\hat{\psi}\delta\hat{\psi}} & \frac{\delta^2\Gamma}{\delta\hat{\bar{\psi}}\delta\hat{\psi}} \\ \frac{\delta^2\Gamma}{\delta\hat{\psi}\delta\hat{\bar{\psi}}} & \frac{\delta^2\Gamma}{\delta\hat{\bar{\psi}}\delta\hat{\bar{\psi}}} \end{pmatrix} \quad \Rightarrow \quad \left(\Gamma_k^{(2)} + R_k\right)^{-1}$ full propagator

Non-perturbative expansion schemes:

$$\Gamma_k = \int_x \left[V_k(\psi, \bar{\psi}) + Z_k(\psi, \bar{\psi})\, \bar{\psi}(x) \mathrm{i}\partial\!\!\!/ \psi(x) + \mathcal{O}(\partial^2) \right]$$

$$\Gamma_k = \Gamma^{(0)} + \sum_{\mathcal{O}_X} \int_{x_1,x_2} \Gamma^{(2)}(x_1, x_2)\, \bar{\psi}(x_1) \mathcal{O}_X \psi(x_2)$$

$$+ \sum_{\mathcal{O}_X,\mathcal{O}_Y} \int_{x_1,\dots,x_4} \Gamma^{(4)}(x_1, \dots, x_4)\, \bar{\psi}(x_1) \mathcal{O}_X \psi(x_2) \bar{\psi}(x_3) \mathcal{O}_Y \psi(x_4) + \mathcal{O}(\psi^6)$$

. . . all operators compatible with symmetry

$U(2N_f)$-symmetric theories: point-like limit

Simplest approximation: all local operators up to $\mathcal{O}(\psi^4)$ [Gies & LJ '10]

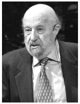
$$\Gamma_k = \int d^3x \left[\bar{\psi}^a \mathrm{i}\partial\!\!\!/ \psi^a + \bar{g}_{1,k}(V)^2 + \bar{g}_{2,k}(S)^2 + \bar{g}_{3,k}(P)^2 + \bar{g}_{4,k}(A)^2\right]$$

$$(V)^2 = \left(\bar{\psi}^a \gamma_\mu \psi^a\right)^2$$

"Thirring" interation [Thirring '58]

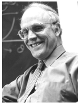
$$(S)^2 = (\bar{\psi}^a \psi^b)^2 - (\bar{\psi}^a \gamma_4 \psi^b)^2 - (\bar{\psi}^a \gamma_5 \psi^b)^2 + (\bar{\psi}^a \gamma_{45} \psi^b)^2$$

NJL-like channel [Nambu & Jona-Lasinio '61]



$$(P)^2 = \left(\bar{\psi}^a \gamma_{45} \psi^a\right)^2$$

Gross-Neveu-like channel [Gross & Neveu '74]



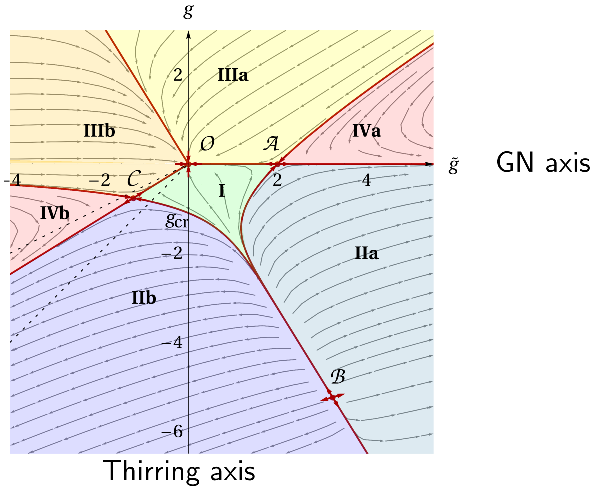
$$(A)^2 = (\bar{\psi}^a \gamma_\mu \psi^b)^2 - (\bar{\psi}^a \mathrm{i}\gamma_\mu \gamma_4 \psi^b)^2 - (\bar{\psi}^a \mathrm{i}\gamma_\mu \gamma_5 \psi^b)^2 + \tfrac{1}{2}(\bar{\psi}^a \gamma_{\mu\nu} \psi^b)^2$$

"axial" channel

Fierz identities: only 2 interactions independent in point-like limit

U($2N_\mathrm{f}$)-symmetric theories: fermionic RG flow

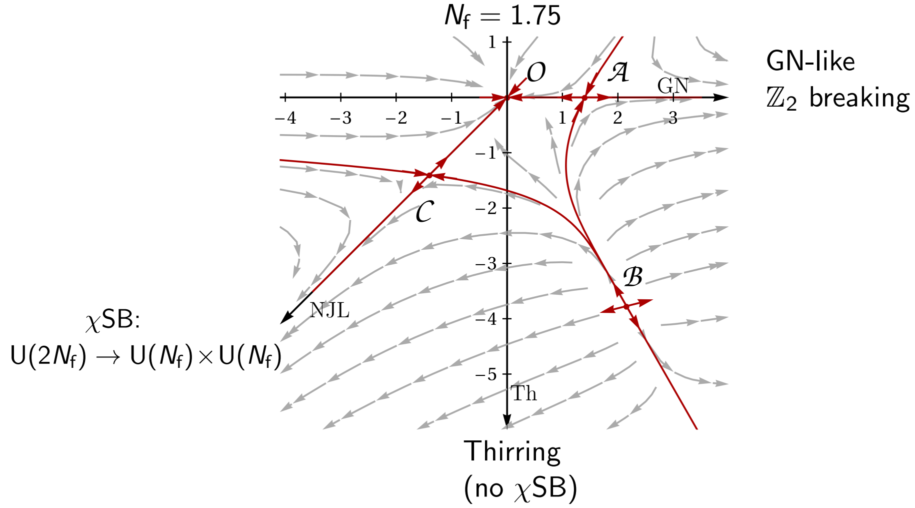
Phase diagram for $N_\mathrm{f} = 1$: [Gies & LJ '10]



⇒ 3 non-Gaußian FPs, but no "pure Thirring" FP

⇒ $\mathcal{C}$: "Thirring FP"

(2+1)d Thirring universality class: N_f dependence

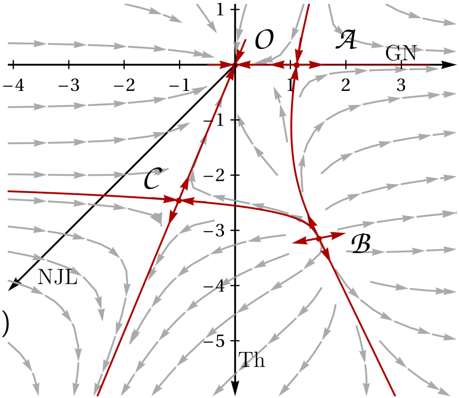


Phase transition mechanism:

competition between chiral-scalar and vector channel

(2+1)d Thirring universality class: N_f dependence

$N_\mathrm{f} \simeq 5.1$



GN-like
$\mathbb{Z}_2$ breaking

χSB:
$\mathrm{U}(2N_\mathrm{f}) \to \mathrm{U}(N_\mathrm{f}) \times \mathrm{U}(N_\mathrm{f})$

Thirring
(no χSB)

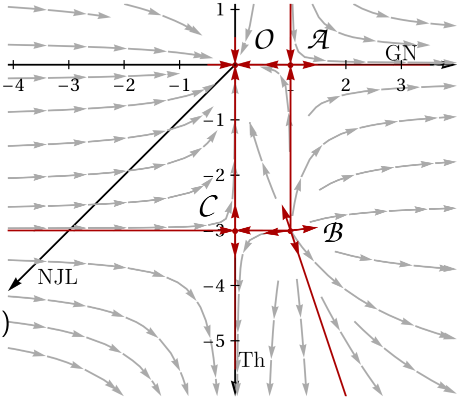
Phase transition mechanism:

competition between chiral-scalar and vector channel

(2+1)d Thirring universality class: N_f dependence

$N_\mathrm{f} = \infty$

GN-like
$\mathbb{Z}_2$ breaking



χSB:
$\mathrm{U}(2N_\mathrm{f}) \to \mathrm{U}(N_\mathrm{f}) \times \mathrm{U}(N_\mathrm{f})$

Thirring
(no χSB)

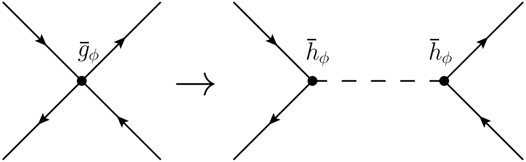
Phase transition mechanism:

competition between chiral-scalar and vector channel

Partial bosonization: competing channels

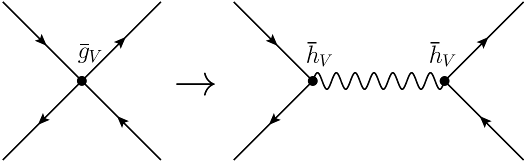
chiral scalar channel:

$(S)^2 = (\bar{\psi}^a \psi^b)^2 - (\bar{\psi}^a \gamma_4 \psi^b)^2 - (\bar{\psi}^a \gamma_5 \psi^a)^2 + (\bar{\psi}^a \gamma_{45} \psi^b)^2 \rightarrow h_\phi \phi^{ab} \bar{\psi}^a \psi^b$

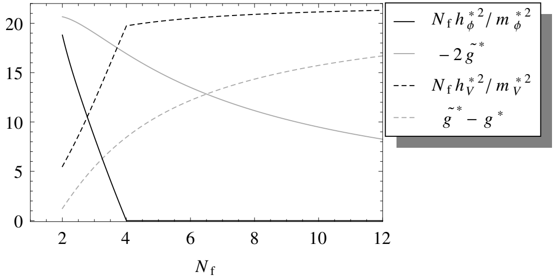


versus

vector channel: $(V)^2 = g_V (\bar{\psi}^a \gamma_\mu \psi^a)^2 \rightarrow h_V V_\mu \bar{\psi}^a \gamma_\mu \psi^a$



Partial bosonization: fixed-point structure



black: part. bosonized RG   gray: fermionic RG

part. bosonized RG:

+ allows momentum dependence
– assumes distrib. onto channels, "Fierz ambiguity"

fermionic RG:

– no momentum dependence ... not so important in UV?
+ resolves Fierz ambiguity ... essential for competing-channel interplay?

⇒ solution: Dynamical Bosonization

Dynamical Bosonization

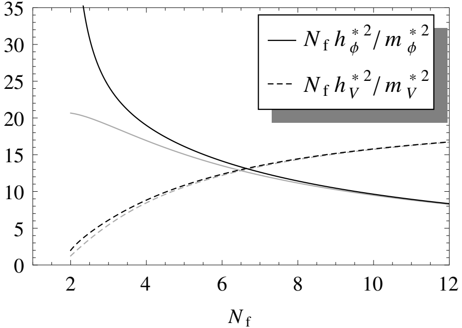
[Gies, Wetterich '01, '04; Pawlowski '07; Flörchinger, Wetterich '09]

Idea: perform Hubbard-Stratonovich transformation at each RG step

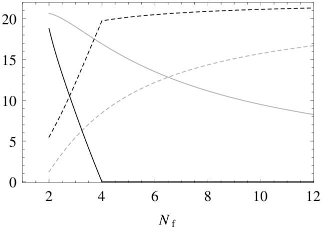
⇒ scale-dependent field transformation:

$$\phi \to \phi_k[\phi, V_\mu, \psi, \dots]$$
$$V_\mu \to V_{\mu,k}[\phi, V_\mu, \psi, \dots]$$

Result, e.g. FP structure:



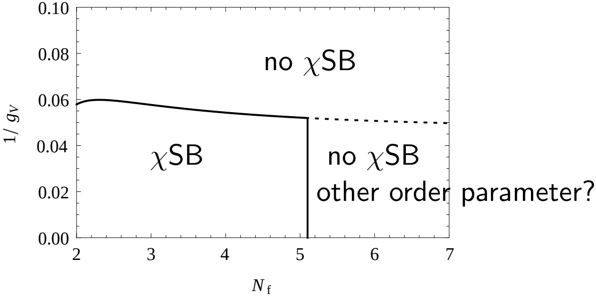
c.f. part. bos.



⇒ crucial for quantitative description of competing-channel interplay

Phase diagram of (2+1)d Thirring model

[LJ, Gies '12]



critical flavor number:

$$N_f^{cr} \simeq 5.1(7)$$

c.f. MC results: $N_f^{cr} = 6.6(1)$ [Christofi, Hands, Strouthos '07]

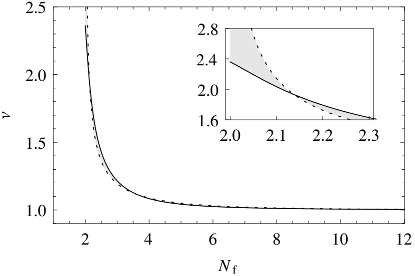
Phase transition as function of g: critical behavior

[LJ, Gies '12]

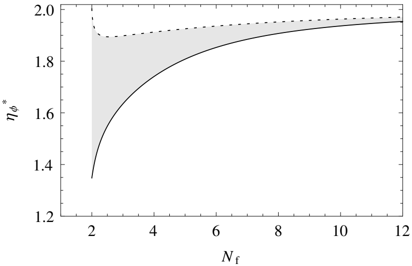
$$\langle \phi \rangle \sim (g - g_{\mathrm{cr}})^{\beta}$$

$\Rightarrow$ 2nd order chiral phase transition for all $N_{\mathrm{f}} < N_{\mathrm{f}}^{\mathrm{cr}}$

correlation length exponent



anomalous dimension

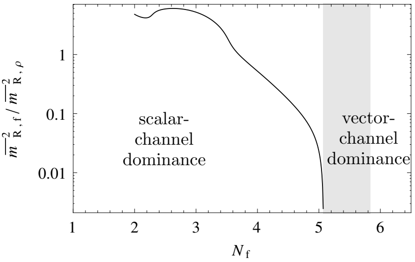


c.f. RG results for $N_{\mathrm{f}} = 1$: $\nu \simeq 1.9$, $\eta_{\phi}^{*} \simeq 1.0$ [Mesterházy, Berges, von Smekal '12]

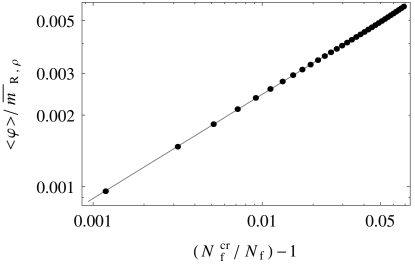
Phase transition as function of N_f: critical behavior

[LJ, Gies '12]

fermion mass



scaling of chiral condensate



$$\langle \phi \rangle \sim (N_f^{cr} - N_f)^b, \ b \simeq 0.44$$

$\Rightarrow$ 2nd order phase transition

c.f. MC results: $b \simeq 0.37$ [Christofi, Hands, Strouthos '07]

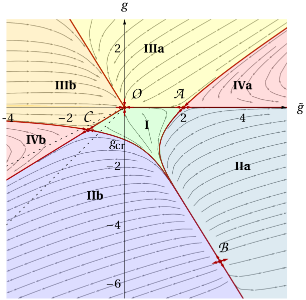
Example 2: QED_3 (preliminary results)

- classical action:

$$S = \int \mathrm{d}^3 x \left[\bar{\psi}^a \left(\mathrm{i}\partial_\mu + \bar{e} A_\mu \right) \gamma^\mu \psi^a + F_{\mu\nu} F^{\mu\nu} \right], \qquad [\bar{e}] = \frac{4-d}{2}$$

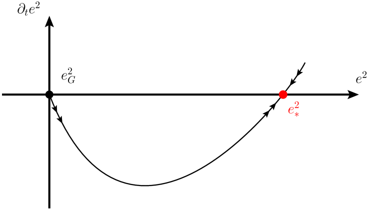
$\Rightarrow$ QED_3 has a Gaußian UV fixed point ...i.e., is asymptotically free

- maximal $\mathrm{U}(2N_\mathrm{f})$ symmetry $\Rightarrow$ fermionic theory space $\simeq$ Thirring



Charge renormalization

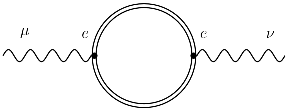
- Ward-Takahashi identity: $Z_e = Z_\psi$ ... can be modified by regulator
- Running gauge coupling:



$$\partial_t e^2 = (d - 4 + \eta_A) e^2, \qquad \eta_A > 0$$

with anomalous dimension:

$$\eta_A = \eta_A(e^2, N_\mathrm{f}) \qquad \sim$$

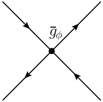


$$\sim N_\mathrm{f} e^2$$

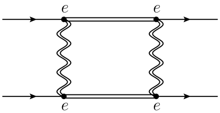
$\Rightarrow e^2_* \sim 1/N_\mathrm{f}$

- finite e^2 can generate 4-fermi interactions:

∂_t

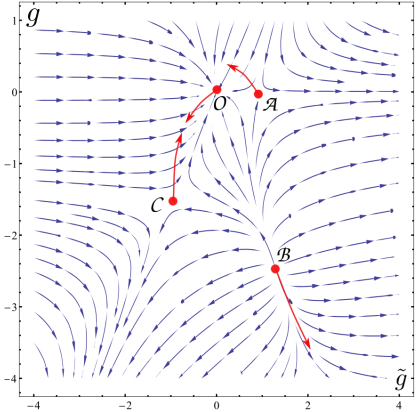


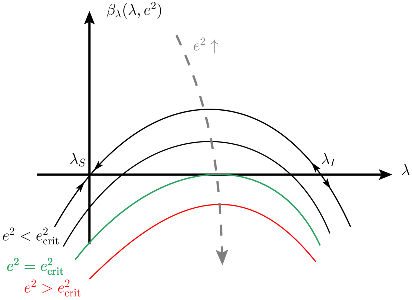
$\sim$



QED_3: fermionic fixed-point structure

- for increasing e^2: [Braun, Gies, LJ, Roscher 'prelim.]





- Gaußian and Thirring fixed point annihilate for

$$e^2 > e^2_{\text{cr}}$$

$\Rightarrow$ prerequisite for χSB: $e^2_{\text{cr}} < e^2_* = \mathcal{O}(1/N_\text{f})$

$\Rightarrow$ no χSB at large N_f [Kubota & Terao '01; Fischer & Alkofer *et al.* '04; Kaveh & Herbut '05; ...]

QED$_3$: pseudo-critical flavor number $N_f^{\text{p-cr}}$

- for $N_f > N_f^{\text{p-cr}}$:
  - effective charge: $e^2 \xrightarrow{k\to 0} e_*^2 < e_{\text{cr}}^2$
  - no FP annihilation; all 4-fermi couplings remain finite
  - "pseudo-conformal" phase; no χSB [Miransky & Yamawaki '97]

... "pseudo", since $\bar{e}^2$ is dimensionful

- RG estimate for $N_f^{\text{p-cr}}$: [Braun, Gies, LJ, Roscher 'prelim.]

(fermionic NLO trunc., momentum-dep. photon propagator, WTI)

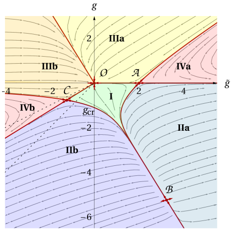
regulator	linear	exponential	Callan-Symanzik	sharp
$N_f^{\text{p-cr}}$	10.0	8.1	7.5	11.0

- true critical flavor number: $N_f^{\text{cr}} \leq N_f^{\text{p-cr}} \simeq 10.0 \pm 2.5$
- compatible with literature:

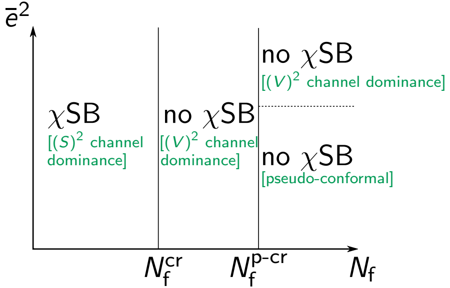
	thermodynamic argument [Appelquist *et al.* '99]	MC (staggered) [Strouthos & Kogut '08]	DSE [Fischer, Alkofer *et al.* '04]	RG [Kubota & Terao '01]
N_f^{cr}	$\leq \frac{3}{2}$	≈ 1.5	≈ 4	≈ 3.1

QED$_3$ vs. Thirring conjecture

If long-range dynamics is dominantly driven by fermion dynamics:



Schematic phase diagram:



critical flavor number:

$$N_f^{cr}(\mathrm{QED}_3) \simeq N_f^{cr}(\mathrm{Thirring})$$

with similar critical behavior.

Conclusions

Low-dimensional chiral fermion systems:

- plethora of theories

  ... Gross-Neveu, Thirring, NJL, axial, ...

- "perfect" quantum field theories

  ... non-perturbatively renormalizable, asymptotically safe

- wide variety of universality classes

  ... & variety of symmetry breaking patterns

- plenty of critical exponents to be computed

  ... perfect testing ground for non-perturbative methods

- relevant as low-energy effective theories of 2d cond-mat systems

  ... excellent playground to bridge the gap between cond-mat & hep-th