A functional renormalization group equation for foliated space-times

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E. Manrique, S. Rechenberger, F.S., Phys. Rev. Lett. 106 (2011) 251302

A. Contillo, S. Rechenberger, F.S., in preparation

Workshop on Strongly-Interacting Field Theories

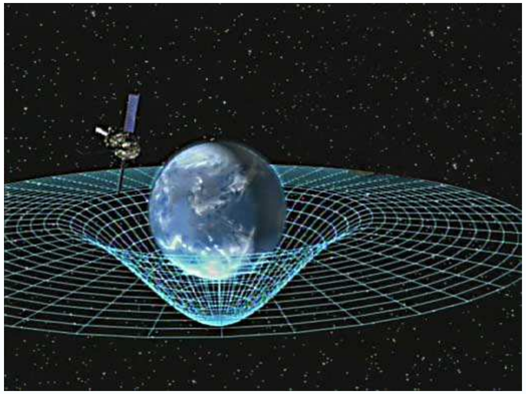
Jena, Nov. 29th, 2012

Outline

- Motivation for Quantum Gravity
- Foundations of Asymptotic Safety
- Functional Renormalization Group Equations: Part I
  - metric construction
  - Einstein-Hilbert results
- Functional Renormalization Group Equations: Part II
  - ADM construction
  - connection to Hořava-Lifshitz gravity
  - Einstein-Hilbert results
- Conclusion and perspectives

Classical General Relativity

Based on Einsteins equations



$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{space-time curvature}} = \underbrace{-\Lambda g_{\mu\nu} + 8\pi G_N T_{\mu\nu}}_{\text{matter content}}$$

- Newton's constant: $G_N = 6.67 \times 10^{-11} \, \frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$
- cosmological constant: $\Lambda \approx 10^{-35} \, \frac{\mathrm{g}}{\mathrm{m}^3}$

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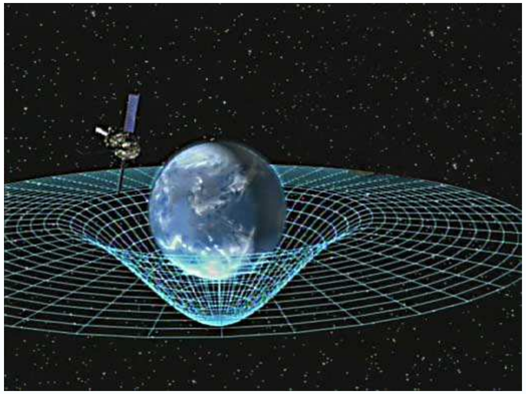

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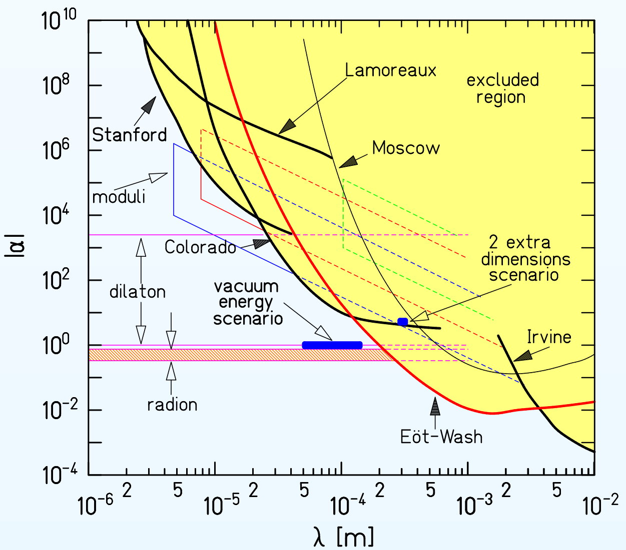
describes gravity from sub-millimeter to cosmological scales



[Adelberger, et. al., 2004]

Motivations for Quantum Gravity

1. internal consistency

$$\underbrace{R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R + \Lambda\, g_{\mu\nu}}_{\text{classical}} \quad = \quad 8\,\pi\, G_N \quad \underbrace{T_{\mu\nu}}_{\text{quantum}}$$

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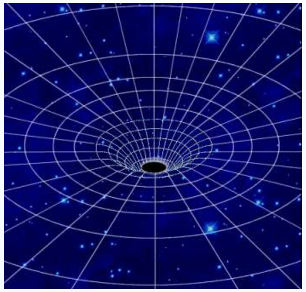

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- Big Bang singularity

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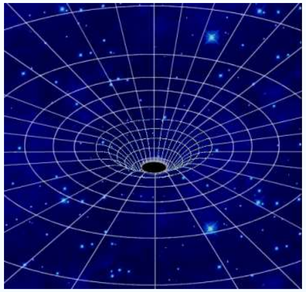

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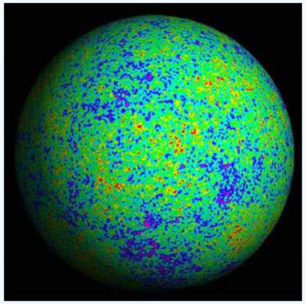

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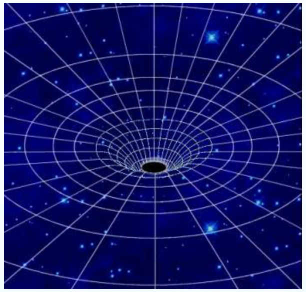

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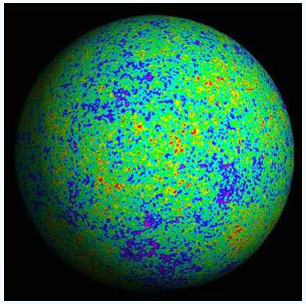

General Relativity is incomplete

Quantum Gravity may give better answers to these puzzles

General Relativity is perturbatively non-renormalizable

conclusions:

a) Treat General Relativity as effective field theory:

[J. Donoghue, gr-qc/9405057]

- compute corrections in $E^2/M_{\rm Pl}^2 \ll 1$ (independent of UV-completion)
- breaks down at $E^2 \approx M_{\rm Pl}^2$

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- loop quantum gravity:
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- perturbative UV-completion by anisotropic QFT

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basics of Asymptotic Safety

renormalizing the non-renormalizable

Renormalization from the Wilsonian perspective

- theory $\Leftarrow$ specify

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physics at different scales captured by family of effective descriptions

Fixed points of the RG flow

Central ingredient in Wilsons picture of renormalization

Definition:

- fixed point $\{g_i^*\} \Longleftrightarrow \beta$-functions vanish $\left(\beta_{g_i}(\{g_i\})|_{g_i=g_i^*} \stackrel{!}{=} 0 \right)$

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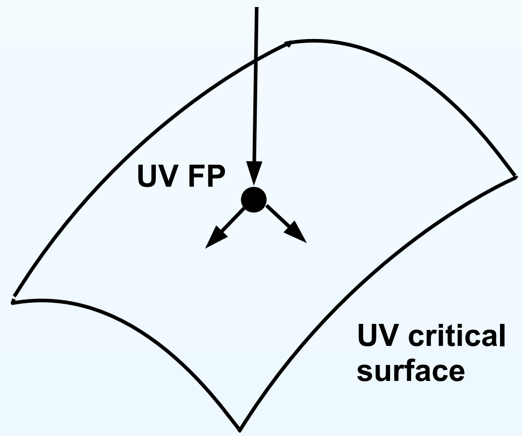
Properties:

- well-defined continuum limit
  - trajectory captured by FP in UV has no unphysical UV divergences
- 2 classes of RG trajectories:
  - relevant $=$ attracted to FP in UV
  - irrelevant $=$ repelled from FP in UV
- predictivity:
  - number of relevant directions
    $=$ free parameters (determine experimentally)



Weinbergs Asymptotic Safety scenario

Renormalization via UV fixed points $\Longrightarrow$ two classes of renormalizable QFTs

- Gaussian Fixed Point (GFP)
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  - fundamental theory: free
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  - fundamental theory: interacting
  - asymptotic safety (e.g. gravity in $d = 2 + \epsilon$)

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Gravity

> Weinberg's asymptotic safety conjecture (1979):
>
> gravity in $d = 4$ has non-Gaussian UV fixed point

Asymptotic Safety as viable theory for Quantum Gravity

Requirements:

a) non-Gaussian fixed point (NGFP)

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- ensures the absence of UV-divergences

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- RG-trajectories have part where GR is good approximation
- recover gravitational physics captured by General Relativity:
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Quantum Einstein Gravity (QEG)

Functional Renormalization Group Equations I

covariant construction

RG flows beyond perturbation theory

Functional Renormalization Group Equations:

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For the purpose of studying gravity:

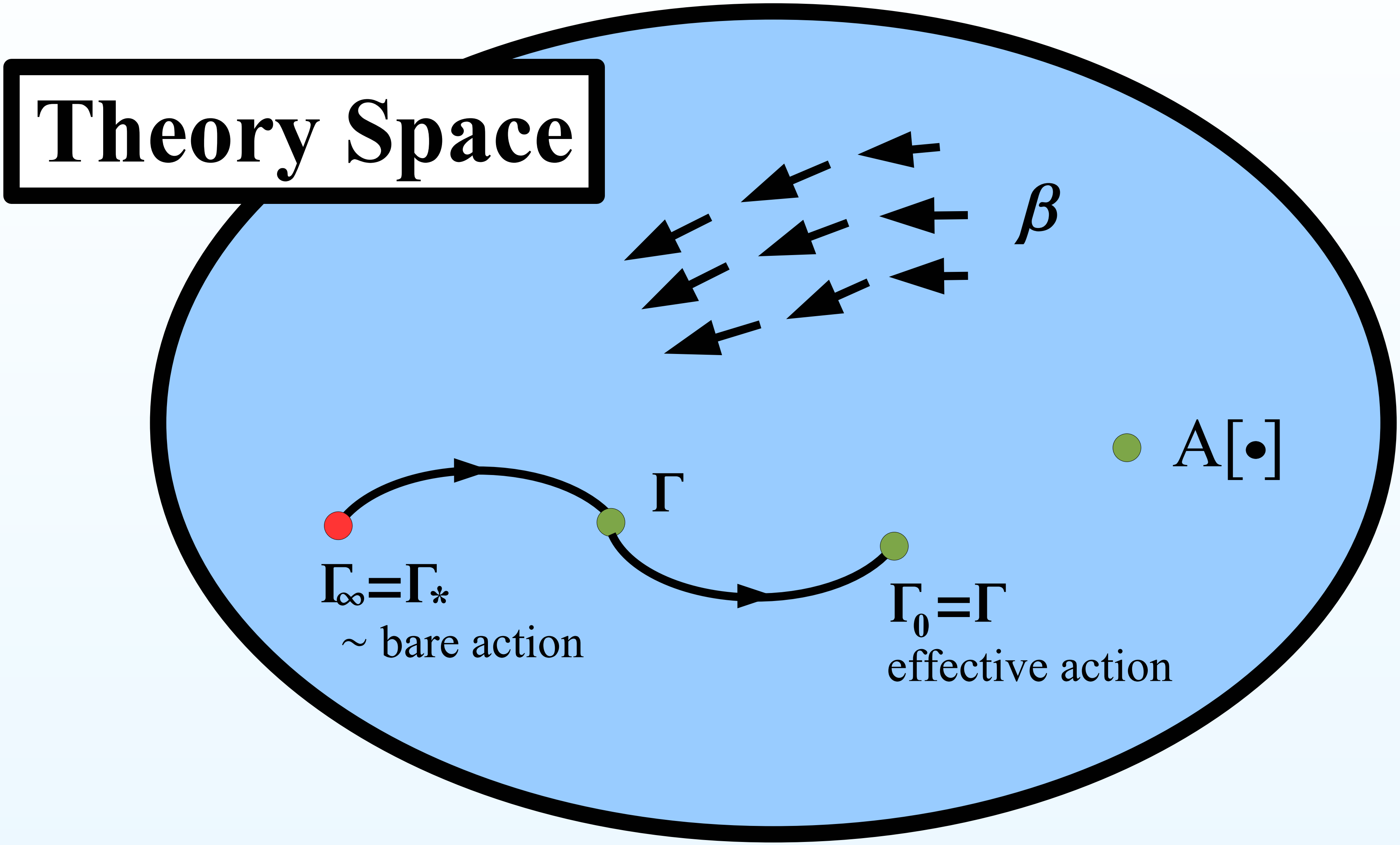
- flow equation for effective average action Γ_k

[C. Wetterich, Phys. Lett. **B301** (1993) 90]

- adapted to gravity

[M. Reuter, Phys. Rev. D **57** (1998) 971, hep-th/9605030]

Theory space underlying the Functional Renormalization Group



Effective action Γ in scalar field theory

- start: generic action $S_{\hat{k}}[\chi]$

$$S_{\hat{k}}[\chi] = \int d^4x \left\{ \tfrac{1}{2}(\partial_\mu \chi)^2 + \tfrac{1}{2} m^2 \chi^2 + \text{interactions} \right\}$$

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- effective action $\Gamma[\phi]$ gives 1PI correlation functions

$$\Gamma[\phi] = \int d^4x \, J\phi - W[J]$$

  - classical (expectation value) field

$$\phi = \langle \chi \rangle = \frac{\delta W[J]}{\delta J}$$

Effective **average** action Γ_k in scalar field theory

- start: generic action $S_{\hat{k}}[\chi]$

$$S_{\hat{k}}[\chi] = \int d^4x \left\{ \tfrac{1}{2}(\partial_\mu \chi)^2 + \tfrac{1}{2} m^2 \chi^2 + \text{interactions} \right\}$$

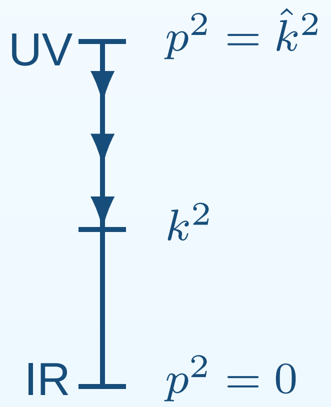
- introduce scale-dependent mass term $\Delta_k S[\chi]$ in $W[J]$

$$W_k[J] = \ln \int \mathcal{D}\chi \exp\left\{ -S_{\hat{k}}[\chi] - \Delta_k S[\chi] + \int d^4x \, J \, \chi \right\}$$

$$\Delta_k S[\chi] = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \mathcal{R}_k(p^2) \, |\widehat{\chi}_p|^2$$

  - discriminate between low/high-momentum modes

$$\mathcal{R}_k(p^2) = \begin{cases} 0 & p^2 \gg k^2 \\ k^2 & p^2 \ll k^2 \end{cases}$$



    - high momentum modes: integrated out
    - low momentum modes: suppressed by mass term

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- k-dependence governed by Functional RG Equation (FRGE)

$$k\partial_k \Gamma_k[\phi] = \tfrac{1}{2} \mathrm{Tr} \left[\left(\delta^2 \Gamma_k + \mathcal{R}_k \right)^{-1} k\partial_k \mathcal{R}_k \right]$$

  - Formally: exact equation $\Leftrightarrow$ no approximations in derivation
  - independent of "fundamental theory" $\Leftrightarrow$ $S_{\hat{k}}$ enters as initial condition
  - Limits: Γ_k interpolates continuously between:
    - $k \to \infty \quad \simeq \quad$ bare/classical action S
    - $k \to 0 \quad = \quad$ ordinary effective action Γ

Covariant functional RG equation for gravity

fundamental field: metric $\gamma_{\mu\nu}$

symmetry: general coordinate invariance

$$\delta\gamma_{\mu\nu} = \mathcal{L}_v(\gamma_{\mu\nu}) = v^\alpha \partial_\alpha \gamma_{\mu\nu} + \gamma_{\alpha\nu}\partial_\mu v^\alpha + \gamma_{\mu\alpha}\partial_\nu v^\alpha$$

starting point: generic diffeomorphism invariant action

$$S^{\mathrm{grav}}[\gamma_{\mu\nu}]$$

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- background covariance from background field formalism: $\gamma_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

$$\text{quantum trafos}: \quad \delta_Q \bar{g}_{\mu\nu} = 0\,, \quad \delta_Q h_{\mu\nu} = \mathcal{L}_v(\bar{g}_{\mu\nu} + h_{\mu\nu})$$

$$\text{background trafos}: \quad \delta_B \bar{g}_{\mu\nu} = \mathcal{L}_v(\bar{g}_{\mu\nu})\,, \quad \delta_B h_{\mu\nu} = \mathcal{L}_v(h_{\mu\nu})$$

- gauge-fixing breaks quantum gauge trafo (retains background gauge):

$$S^{\mathrm{gf}} = \frac{1}{2\alpha}\int d^4x \sqrt{\bar{g}}\, \bar{g}^{\mu\nu}\, F_\mu\, F_\nu\,, \qquad F_\mu = \bar{D}^\mu h_{\mu\nu} - \tfrac{1}{2}\bar{D}_\mu h\,,$$

- Jacobian captured by ghost term: $S^{\mathrm{gh}}[h, C, \bar{C}; \bar{g}]$

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- background covariance $\Longleftrightarrow$ background field formalism

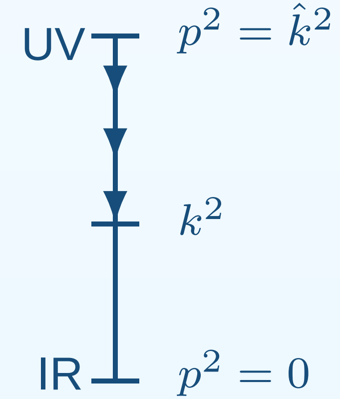
$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

- $\bar{g}$ provides structure for constructing k-dependent IR cutoff:

$$\Delta_k S[h;\bar{g}] = \int d^4x\sqrt{\bar{g}}\,\{h_{\mu\nu}\mathcal{R}_k[\bar{g}]^{\mu\nu\rho\sigma}h_{\rho\sigma} + \ldots\}$$

  - $\mathcal{R}_k[\bar{g}] \propto \mathcal{Z}_k k^2 R^{(0)}$ = k-dependent mass term
  - discriminates low/high- $\bar{D}^2$-eigenmodes

$$R^{(0)}(p^2/k^2) = \begin{cases} 0 & p^2 \gg k^2 \\ 1 & p^2 \ll k^2 \end{cases}$$



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- exact RG equation for Γ_k:

$$k\partial_k \Gamma_k[h;\bar{g}] = \tfrac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$

  - $\Gamma_k^{(2)}$ = Hessian with respect to fluctuation fields
  - “extra” $\bar{g}$-dependence necessary for formulating exact equation

Non-perturbative approximation: derivative expansion of Γ_k

- caveat: FRGE cannot be solved exactly

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- expand Γ_k in derivatives and truncate series:

$$\Gamma_k[\Phi] = \sum_{i=1}^{N} \bar{u}_i(k)\, \mathcal{O}_i[\Phi]$$

$\Longrightarrow$ substitute into FRGE

$\Longrightarrow$ projection of flow gives β-functions for running couplings

$$k\partial_k \bar{u}_i(k) = \beta_i(\bar{u}_i; k)$$

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- testing the reliability:
  - within a given truncation:
    cutoff-scheme dependence of physical quantities (= vary $\mathcal{R}_k$)
  - stability of results within extended truncations

Letting things flow

The Einstein-Hilbert truncation

The Einstein-Hilbert truncation: setup

Einstein-Hilbert truncation: two running couplings: $G(k), \Lambda(k)$

$$\Gamma_k = \frac{1}{16\pi G(k)} \int d^4x \sqrt{g}\, [-R + 2\Lambda(k)] + S^{\mathrm{gf}} + S^{\mathrm{gh}}$$

- project flow onto G-$\Lambda-$plane

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explicit β-functions for dimensionless couplings $g_k := k^2 G(k) \, , \; \lambda_k := \Lambda(k) k^{-2}$

- Particular choice of $\mathcal{R}_k$ (optimized cutoff)

$$k\partial_k g_k = (\eta_N + 2) g_k \, ,$$

$$k\partial_k \lambda_k = -(2 - \eta_N)\, \lambda_k - \frac{g_k}{2\pi} \left[5 \frac{1}{1-2\,\lambda_k} - 4 - \frac{5}{6} \frac{1}{1-2\lambda_k} \eta_N \right]$$

- anomalous dimension of Newton's constant:

$$\eta_N = \frac{g B_1}{1 - g B_2}$$

$$B_1 = \frac{1}{3\pi} \left[5 \frac{1}{1-2\lambda} - 9 \frac{1}{(1-2\lambda)^2} - 7 \right] \, , \; B_2 = -\frac{1}{12\pi} \left[5 \frac{1}{1-2\lambda} + 6 \frac{1}{(1-2\lambda)^2} \right]$$

Einstein-Hilbert truncation: Fixed Point structure

β-functions for $g_k := k^2 G(k)\,,\ \lambda_k := \Lambda(k) k^{-2}$

$$k\partial_k g_k = (\eta_N + 2) g_k\,,$$

$$k\partial_k \lambda_k = -(2-\eta_N)\,\lambda_k - \frac{g_k}{2\pi}\left[5\,\frac{1}{1-2\,\lambda_k} - 4 - \frac{5}{6}\,\frac{1}{1-2\lambda_k}\,\eta_N\right]$$

microscopic theory $\Longleftrightarrow$ fixed points of the β-functions

$$\beta_g(g^*, \lambda^*) = 0\,, \qquad \beta_\lambda(g^*, \lambda^*) = 0$$

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  - UV attractive in g_k, λ_k

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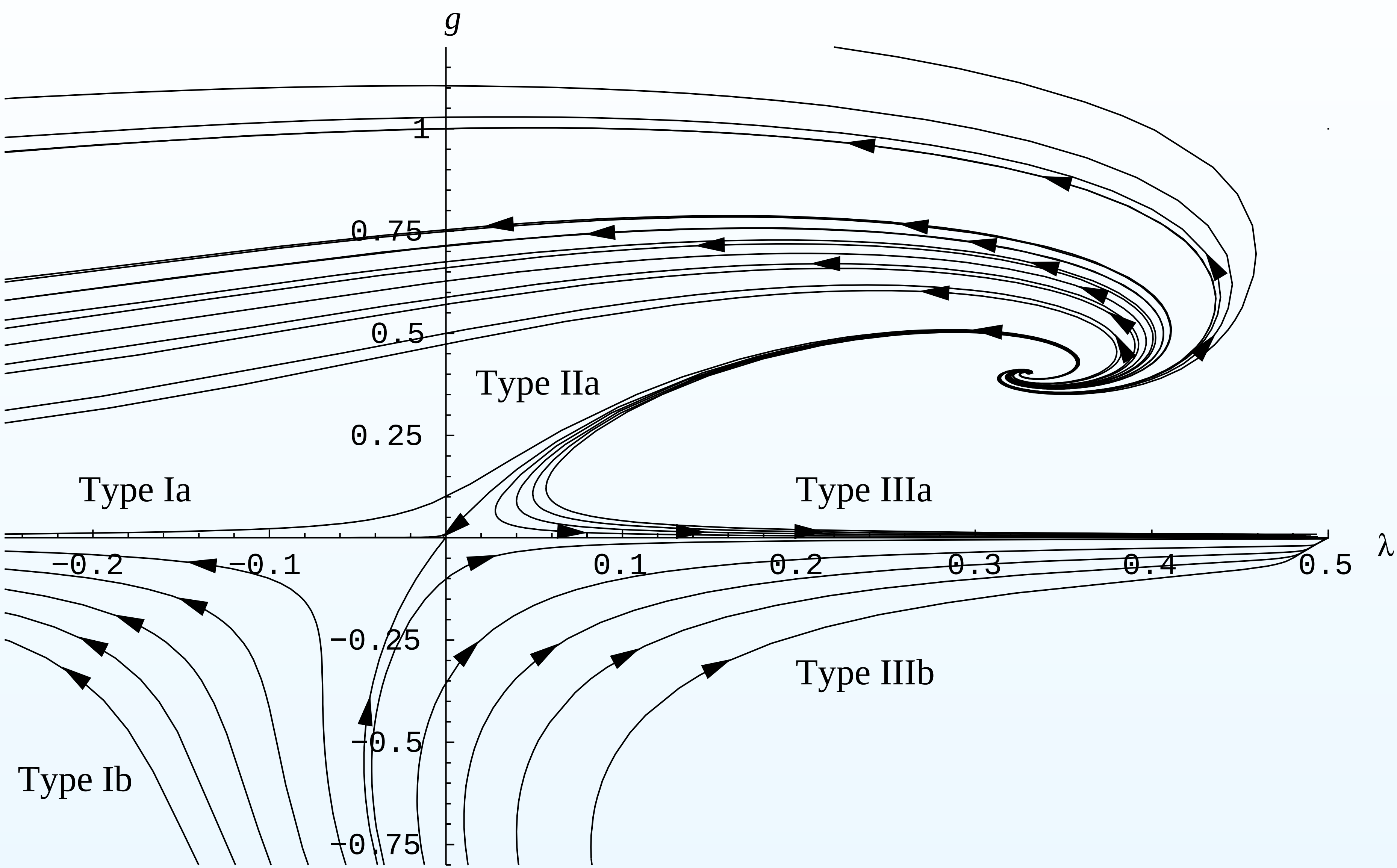
Asymptotic safety: non-Gaussian Fixed Point is UV completion for gravity

Einstein-Hilbert-truncation: the phase diagram



Einstein-Hilbert truncation: Stability properties

Ref.	g^*	λ^*	$g^*\lambda^*$	$\theta' \pm i\theta''$	gauge	$\mathcal{R}_k$
BMS	0.902	0.109	0.099	$2.52 \pm 1.78i$	geometric	II, opt
RS	0.403	0.330	0.133	$1.94 \pm 3.15i$	harmonic	I, sharp
LR	0.272	0.348	0.095	$1.55 \pm 3.84i$	harmonic	I, exp
	0.344	0.339	0.117	$1.86 \pm 4.08i$	Landau	I, exp
L	1.17	0.25	0.295	$1.67 \pm 4.31i$	Landau	I, opt
CPR	0.707	0.193	0.137	$1.48 \pm 3.04i$	harmonic	I, opt
	0.556	0.092	0.051	$2.43 \pm 1.27i$	harmonic	II, opt
	0.332	0.274	0.091	$1.75 \pm 2.07i$	harmonic	III, opt

BMS: Benedetti, Machado, Saueressig, 2009.

RS: Reuter, Saueressig, 2002.

LR: Lauscher, Reuter, 2002.

L: Litim, 2004.

CPR: Codello, Percacci, Rahmede, 2009.

Exploring the gravitational theory space

Some key results . . .

- evidence for asymptotic safety

  $\Rightarrow$ non-Gaussian fixed point provides UV completion of gravity

- finite dimensional UV-critical surface

  $\Rightarrow$ possibly: 3 relevant parameters

- perturbative counterterms:
  - gravity + matter: asymptotic safety survives 1-loop counterterm

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- construction of fixed functionals in $f(R)$-gravity?

[D. Benedetti, F. Caravelli, JHEP 06 (2012) 017]
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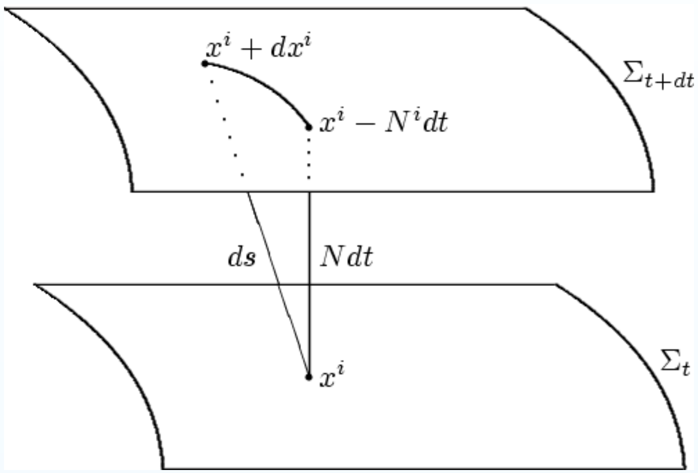
How does a “time”-direction affect Asymptotic Safety?

Functional Renormalization Group Equations II

foliated construction

Foliation structure via ADM-decomposition

Preferred “time”-direction via foliation of space-time



- foliation structure $\mathcal{M}^{d+1} = S^1 \times \mathcal{M}^d$ with $y^\mu \mapsto (\tau, x^a)$:

$$ds^2 = N^2 dt^2 + \sigma_{ij} \left(dx^i + N^i dt\right) \left(dx^j + N^j dt\right)$$

- fundamental fields: $g_{\mu\nu} \mapsto (N, N_i, \sigma_{ij})$

$$g_{\mu\nu} = \begin{pmatrix} N^2 + N_i N^i & N_j \\ N_i & \sigma_{ij} \end{pmatrix}$$

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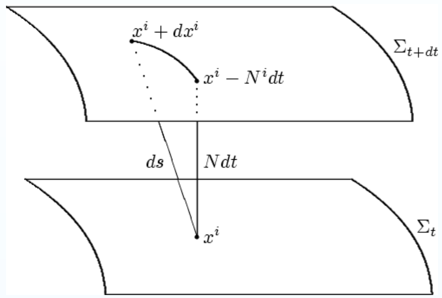

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Allows to include signature parameter $\epsilon = \pm 1$

Foliated FRGE: full diffeomorphism symmetry

fundamental fields: $\{\tilde{N}(\tau,x), \tilde{N}_i(\tau,x), \tilde{\sigma}_{ij}(\tau,x)\}$

symmetry: general coordinate invariance inherited from $\gamma_{\mu\nu}$:

$$\delta\gamma_{\mu\nu} = \mathcal{L}_v(\gamma_{\mu\nu})\,, \qquad v^\alpha = (f(\tau,x)\,,\, \zeta^a(\tau,x))$$

induces

$$\delta\tilde{N} = f\partial_\tau\tilde{N} + \zeta^k\partial_k\tilde{N} + \tilde{N}\partial_\tau f - \tilde{N}\tilde{N}^i\partial_i f\,,$$

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- symmetry realized non-linearly

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- symmetry realized non-linearly
- impossible to construct regulator $\Delta_k S$
  - quadratic in fluctuation fields
  - perserving Diff($\mathcal{M}$) as background symmetry

Foliated FRGE: projectable Hořava-Lifshitz

fundamental fields: $\{\tilde{N}(\tau), \tilde{N}_i(\tau,x), \tilde{\sigma}_{ij}(\tau,x)\}$

symmetry: foliation-preserving diffeomorphisms $\subset$ Diff($\mathcal{M}$):

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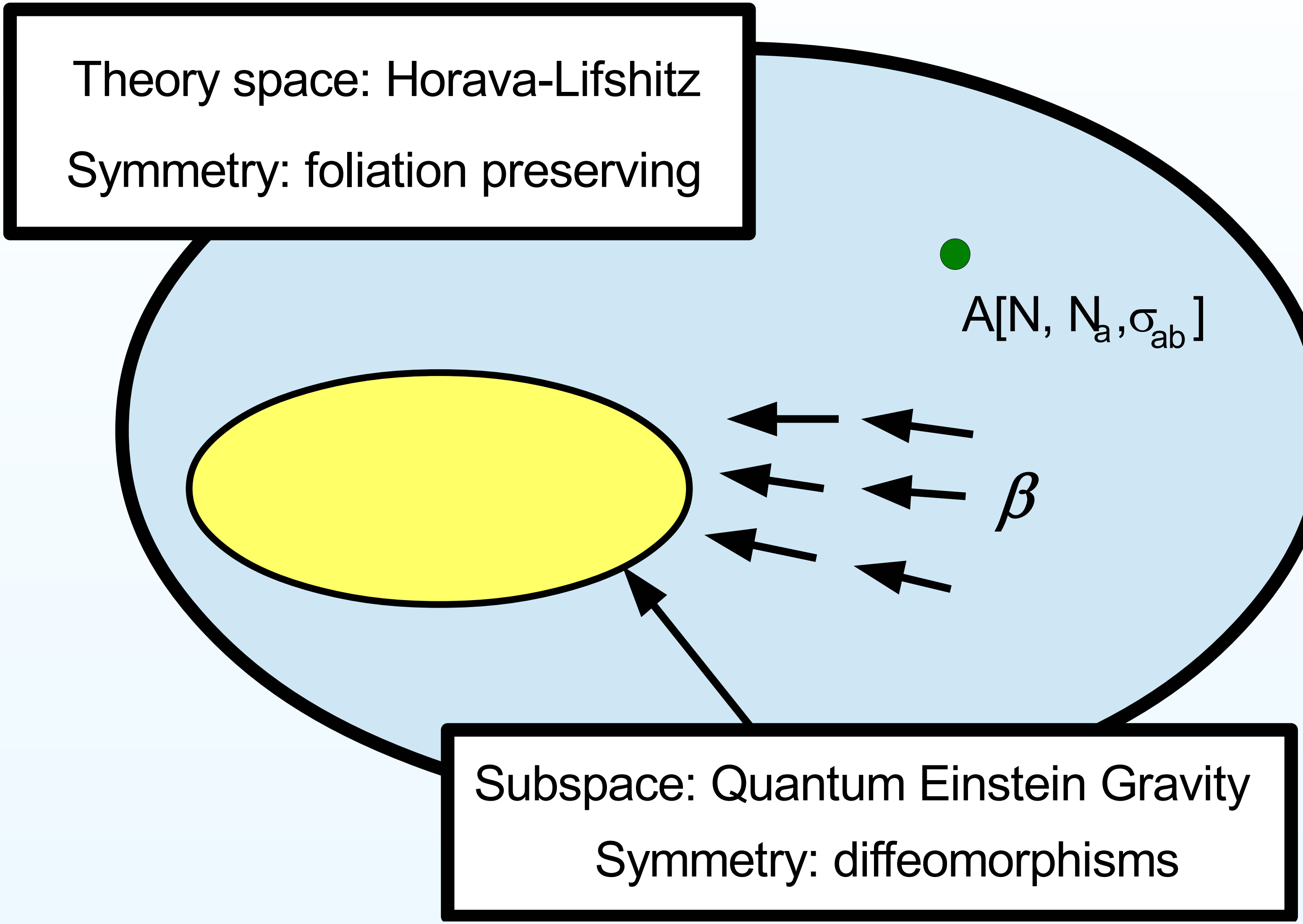
- symmetry realized linearly
- construct regulator $\Delta_k S$:
  - quadratic in fluctuation fields
  - allows foliation-preserving diffeomorphisms as background symmetry

Embedding of QEG in Hořava-Lifshitz gravity



Foliated FRGE: construction

fundamental fields: $\{\tilde{N}(\tau), \tilde{N}_i(\tau, x), \tilde{\sigma}_{ij}(\tau, x)\}$

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- starting point: action invariant under foliation-preserving diffeomorphisms

$$S^{\mathrm{grav}}[\tilde{N}, \tilde{N}_i, \tilde{\sigma}_{ij}]$$

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- gauge-fixing: background field formalism for $\left\{\tilde{N}, \tilde{N}_i, \tilde{\sigma}_{ij}\right\}$:

$$\tilde{N} = \bar{N} + h\,, \qquad \tilde{N}_i = \bar{N}_i + h_i\,, \qquad \tilde{\sigma}_{ij} = \bar{\sigma}_{ij} + h_{ij}$$

  - impose temporal gauge: $h = 0, h_i = 0$

$$S^{\rm gf} = \tfrac{1}{2\alpha}\sqrt{\epsilon}\int d\tau \int d^3x \sqrt{\bar{\sigma}}\bar{N}^{-1}\,\left\{h^2 + \bar{\sigma}^{ij}h_i h_j\right\}$$

- ghost action:

$$S^{\rm gh} = \sqrt{\epsilon}\int d\tau \int d^3x \sqrt{\bar{\sigma}}\left\{\bar{C}\partial_\tau C + \bar{C}_i\partial_\tau C^i\right\}$$

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- k-dependent IR-cutoff $\Delta_k S$

$$\Delta_k S[h; \bar{\sigma}] = \sqrt{\epsilon} \int d\tau \int d^3x \sqrt{\bar{\sigma}}\ \{h_{ij} \mathcal{R}_k[\bar{\sigma}] h^{ij} + \ldots\}$$

  - depends on spatial Laplacian: $\Delta = -\bar{\sigma}^{ij} \bar{D}_i \bar{D}_j$ is positive definite
  - respects foliation-preserving diffeomorphisms

  $\Rightarrow$ explore RG-flows in Hořava-Lifshitz gravity

Foliated functional renormalization group equation

Flow equation: formally the same as in covariant construction

$$k\partial_k \Gamma_k[h, h_i, h_{ij}; \bar{\sigma}_{ij}] = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$

- covariant: $\mathcal{M}^4$

$$\mathrm{STr} \approx \sum_{\mathrm{fields}} \int d^4y \sqrt{\bar{g}}$$

- foliated: $S^1 \times \mathcal{M}^3$

$$\mathrm{STr} \approx \sqrt{\epsilon} \sum_{\mathrm{component\ fields}} \sum_{\mathrm{KK-modes}} \int d^3x \sqrt{\bar{\sigma}}$$

  - structure resembles: quantum field theory at finite temperature!

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Advantages of the foliated flow equation:

- limits: same as covariant equation
- ϵ-dependence: keep track of signature effects
- structure: same as Monte-Carlo simulations following CDT

signature-dependent renormalization group flows

Einstein-Hilbert truncation

ADM-decomposed Einstein-Hilbert truncation: setup

ADM-decomposed Einstein-Hilbert truncation: running couplings: G_k, Λ_k

$$\Gamma_k^{\mathrm{ADM}} = \frac{\sqrt{\epsilon}}{16\pi G_k} \int d\tau d^3x N \sqrt{\sigma} \left\{ \epsilon^{-1} K_{ij} \left[\sigma^{ik}\sigma^{jl} - \sigma^{ij}\sigma^{kl} \right] K_{kl} - R^{(3)} + 2\Lambda_k \right\} + S^{\mathrm{gf}} + S^{\mathrm{gh}}$$

- K_{ij}: extrinsic curvature
- $R^{(d)}$: intrinsic curvature
- $\epsilon = \pm 1$: signature parameter

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Structure of the flow equation

$$k\partial_k \Gamma_k = T^{\rm TT} + T^0$$

$$T^{\rm TT} = \frac{\sqrt{\epsilon} k^3 d_{\rm 2T}}{(4\pi)^{3/2}} \sum_n \int d^3x \sqrt{\bar{\sigma}} \left[q^{1,0}_{3/2}(w_{\rm 2T}) + \frac{\bar{R}}{k^2} \left(\frac{1}{6} q^{1,0}_{1/2}(w_{\rm 2T}) - \frac{2}{3} q^{2,0}_{3/2}(w_{\rm 2T}) \right) \right]$$

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β-functions depend parametrically on $m = \frac{2\pi}{Tk}$:

$$k\partial_k g_k = \beta_g(g, \lambda; m), \qquad k\partial_k \lambda_k = \beta_\lambda(g, \lambda; m)$$

Analyticity properties of β-functions

Kaluza-Klein sums: carry out analytically:

$$\sum_n q_{d/2}^{1,0}(w_{2\mathrm{T}}) \propto \sum_n \frac{1}{1 + \frac{1}{2\epsilon} m^2 n^2 - 2\lambda_k}$$

Summation: depends on signature ϵ:

$$\sum_n \frac{1}{n^2 + x^2} = \frac{\pi}{x \tanh(\pi x)}, \quad x^2 > 0 \quad \text{(hyperbolic functions)}$$

$$\sum_n \frac{1}{n^2 + x^2} = \frac{\pi}{i\, x \tan(\pi x)}, \quad x^2 < 0 \quad \text{(trigonometric functions)}$$

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analytic structure of β-functions: determined by ϵ, λ:

ϵ	$\lambda < \lambda^{(1)} < 0$	$\lambda^{(1)} < \lambda < 1/2$	$1/2 < \lambda$
$+1$	hyperbolic	mixture	trigonometric
-1	trigonometric	mixture	hyperbolic

results: compactification limit $T \to 0, m \to \infty$

- "time"-circle collapses $\Longrightarrow$ β-functions independent of signature ϵ
- dimensionless couplings

$$\lambda_k \ , \quad g_k^{(d)} = g_k^{(D)} \frac{m}{2\pi} \ , \quad \tau_* = \lambda_* (g_*^{(d)})^{2/(d-2)}$$

- β-functions give non-Gaussian fixed point:

d	$g_*^{(d)}$	λ_*	τ_*	$\theta_{1,2}$	τ_*^{LF}	$\theta_{1,2}^{\mathrm{LF}}$
3	0.24	0.30	0.02	$0.89 \pm 3.22i$	–	–
4	0.77	0.28	0.22	$2.69 \pm 4.63i$	0.14	$1.48 \pm 3.04i$
5	3.17	0.29	0.62	$4.55 \pm 6.26i$	0.48	$2.69 \pm 5.15i$
6	15.3	0.29	1.14	$6.64 \pm 7.86i$	0.96	$4.33 \pm 7.14i$
7	84.0	0.30	1.75	$8.97 \pm 9.41i$	1.55	$6.27 \pm 9.05i$

[comparison: P. Fischer, D. Litim, hep-th/0602203]

results: decompactification limit $T \to \infty, m \to 0$

- trigonometric terms in β-functions diverge:

ϵ	$\lambda < \lambda^{(1)}$	$\lambda^{(1)} < \lambda < 1/2$	$1/2 < \lambda$
$+1$	hyperbolic	~~mixture~~	~~trigonometric~~
-1	~~trigonometric~~	~~mixture~~	hyperbolic

no Non-Gaussian fixed point!

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$+1$	hyperbolic	~~mixture~~	~~trigonometric~~
-1	~~trigonometric~~	~~mixture~~	hyperbolic

no Non-Gaussian fixed point!

- can be traced to the conformal factor problem
- artifact of the truncation

results: NGFP for Kaluza-Klein mass

$$T \propto k^{-1} \quad \Longleftrightarrow \quad \lim_{k\to\infty} m_k = m^* \neq 0 \qquad (\text{sequel}: m = 2\pi)$$

β-functions well-defined in all regions:

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ϵ	$\lambda < \lambda^{(1)}$	$\lambda^{(1)} < \lambda < 1/2$	$1/2 < \lambda$
$+1$	hyperbolic	mixture	trigonometric
-1	trigonometric	mixture	hyperbolic

Obtain: NGFP for both signatures:

ϵ	g_*	λ_*	$g_*\lambda_*$	$\theta_{1,2}$
$+1$	0.23	0.30	0.07	$0.92 \pm 3.27i$
-1	0.24	0.30	0.07	$0.87 \pm 3.16i$

results: NGFP for Kaluza-Klein mass

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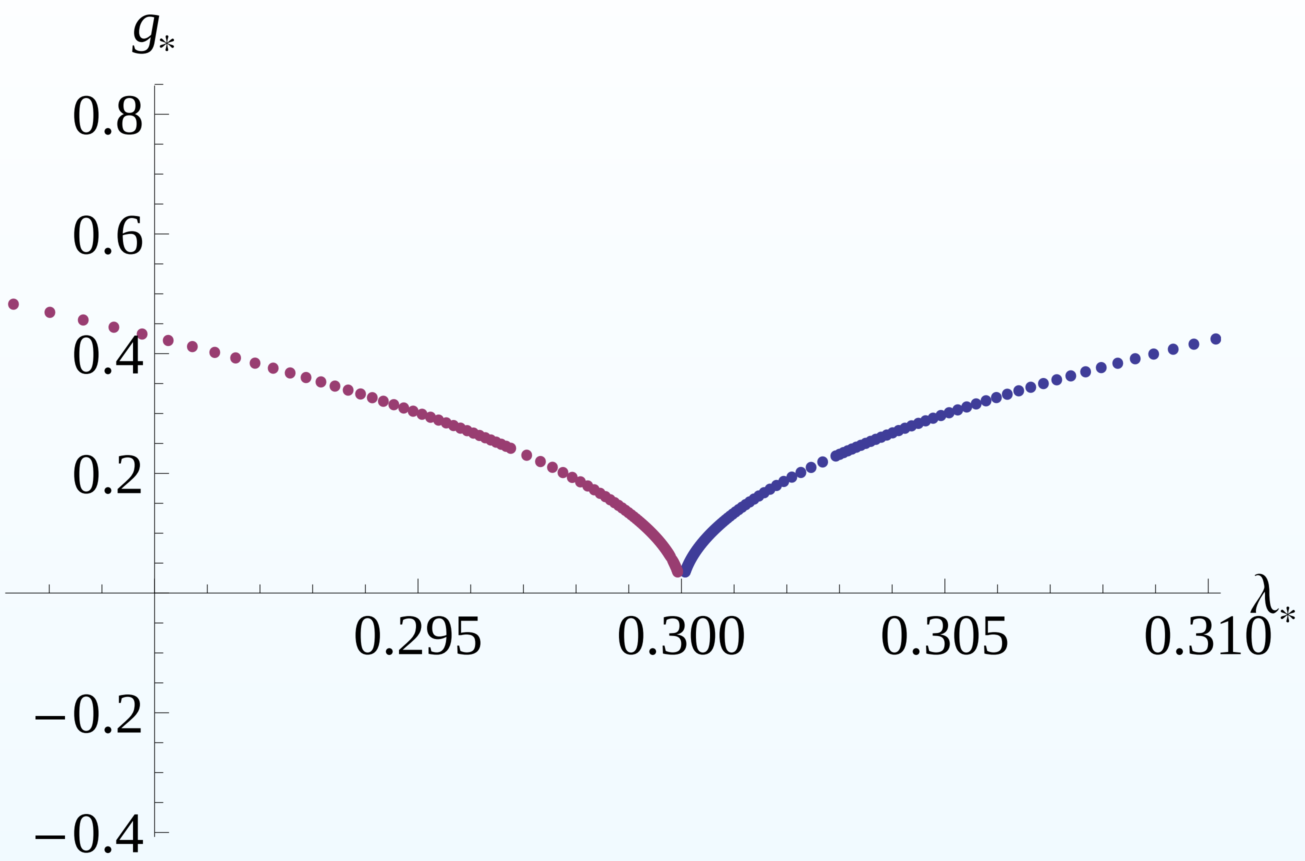
stability coefficients: almost the same!

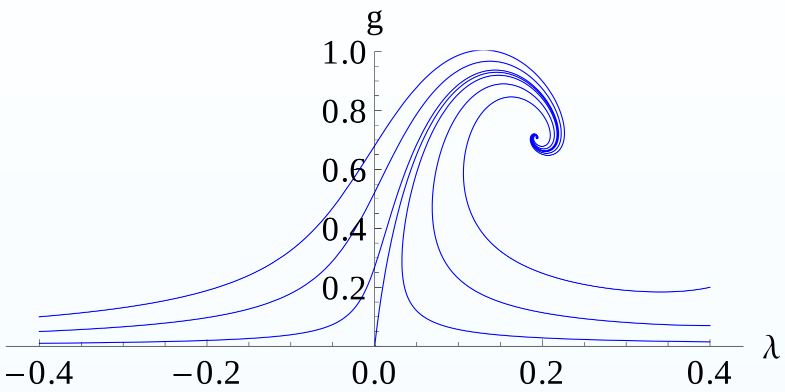
result: signature dependence of NGFP



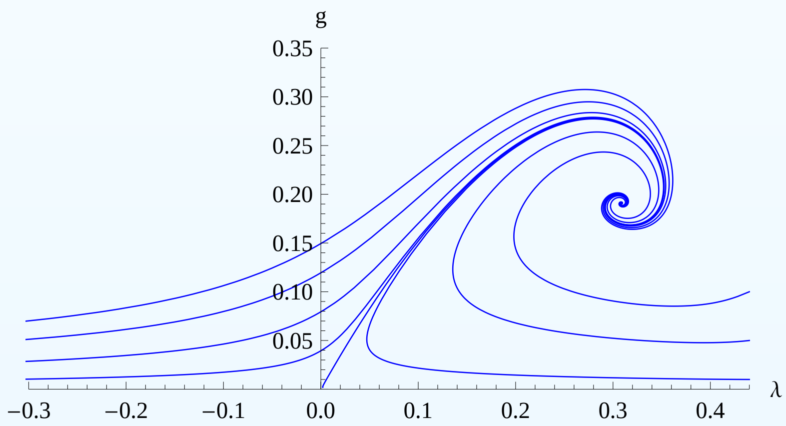
for m finite NGFPs separate:

- $\epsilon = +1$: Euclidean signature (blue)
- $\epsilon = -1$: Lorentzian signature (magenta)

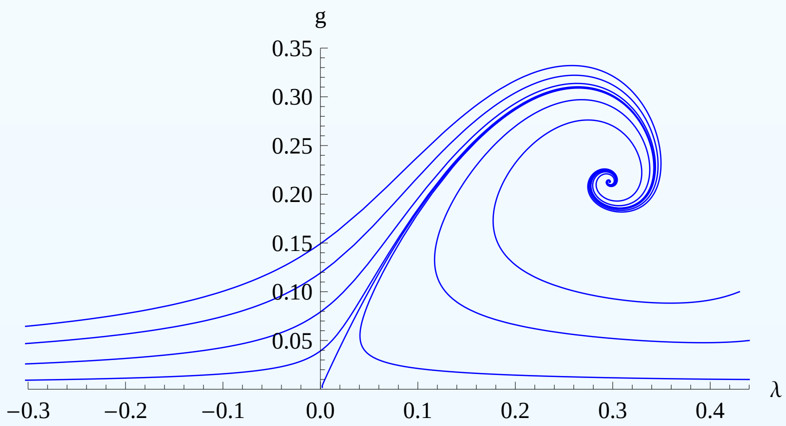
result: phase diagrams



covariant computation



Euclidean



Lorentzian

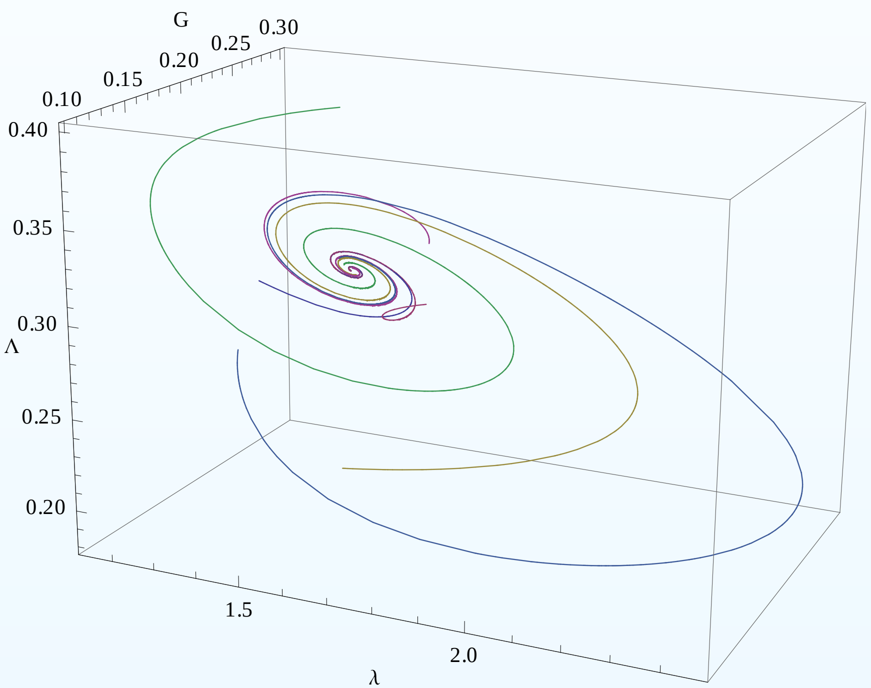
including anisotropy

preliminary results

Probing the RG-flow of Hořava-Lifshitz gravity

include anisotropy parameter λ:

$$\Gamma_k^{\rm HL} = \frac{1}{16\pi G_k} \int d\tau d^3x N \sqrt{\sigma} \left\{ K_{ij} \left[\sigma^{ik}\sigma^{jl} - \lambda_k \sigma^{ij}\sigma^{kl} \right] K_{kl} - R^{(3)} + 2\Lambda_k \right\}$$



- NGFP known from Einstein-Hilbert still present
- no evidence for diffeomorphism invariant surface as IR-attractor

Conclusions

Conclusion and Perspectives

novel causal functional renormalization group equation

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- applications:
  - RG flows of Euclidean and Lorentzian signature metrics
  - analytic complement to causal dynamical triangulations
  - Hořava-type gravitational theories

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- ADM-decomposed Einstein-Hilbert action:
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gravity in UV

signature does not affect asymptotic safety