Inhomogeneous phases in strong-interaction physics

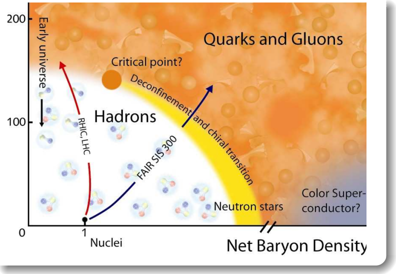
TECHNISCHE UNIVERSITÄT DARMSTADT

Michael Buballa

Workshop on Strongly-Interacting Field Theories
Jena, November 29 – December 1, 2012

Motivation

QCD phase diagram (CBM poster):



- regions of interest:
  - hadronic phase
  - quark-gluon plasma
  - critical endpoint ?
  - color superconductors ?
  - nuclear matter liquid-gas transition

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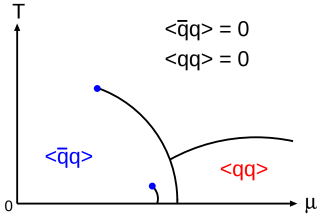

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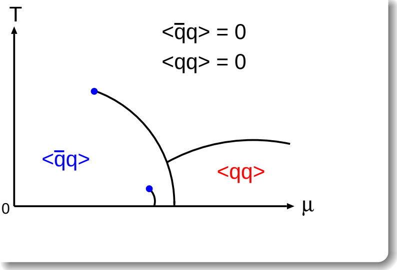

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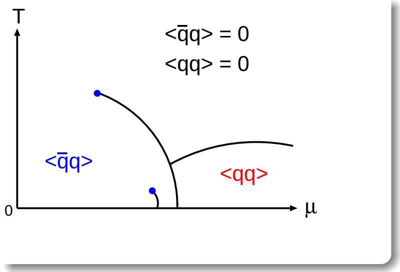

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  $\langle \bar{q}q \rangle$, $\langle qq \rangle$ constant in space
- How about non-uniform phases ?

Inhomogeneous phases: (incomplete) historical overview

- 1960s:
  - spin-density waves in nuclear matter (Overhauser)
  - crystalline superconductors (Fulde, Ferrell, Larkin, Ovchinnikov)
- 1970s – 1990s:
  - p-wave pion condensation (Migdal)
  - chiral density wave (Dautry, Nyman)
  - Skyrme crystals (Goldhaber,Manton)
- after 2000:
  - 1+1 D Gross-Neveu model (Thies et al.)
  - crystalline color superconductors (Alford, Bowers, Rajagopal)
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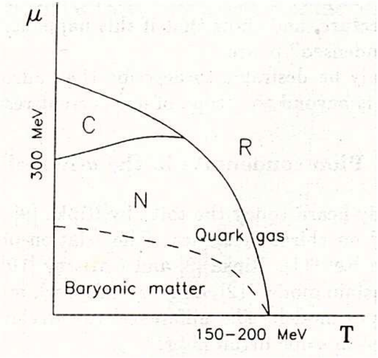

Broniowski et al. (1991)

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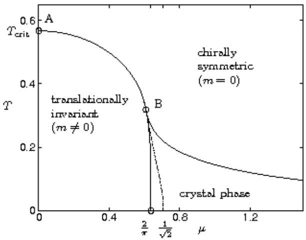

Thies, Urlichs (2003)

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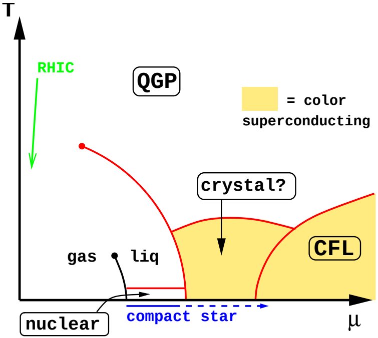

Alford (2003)

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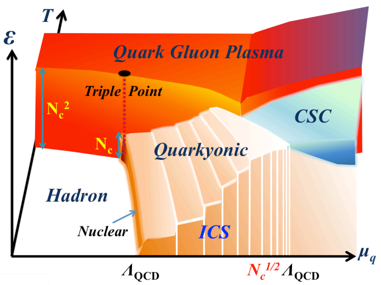

Kojo et al. (2011)

Outline

- This talk: Inhomogeneous phases in the Nambu–Jona-Lasinio model

Model

- NJL model:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G_S\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right]$$

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- bosonize: $\sigma(x) = \bar{\psi}(x)\psi(x), \quad \vec{\pi}(x) = \bar{\psi}(x)i\gamma_5\vec{\tau}\psi(x)$

$$\Rightarrow \quad \mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m + 2G_S(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\right)\psi - G_S\left(\sigma^2 + \vec{\pi}^{\,2}\right)$$

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$$\sigma(x) \to \langle\sigma(x)\rangle \equiv S(\vec{x}), \quad \pi_a(x) \to \langle\pi_a(x)\rangle \equiv P(\vec{x})\,\delta_{a3}$$

  - $S(\vec{x})$, $P(\vec{x})$ time independent classical fields
  - retain space dependence !

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- mean-field thermodynamic potential:

$$\Omega_{MF}(T,\mu) = -\frac{T}{V}\ln\int\mathcal{D}\bar{\psi}\mathcal{D}\psi\exp\left(\int_{x\in[0,\frac{1}{T}]\times V}(\mathcal{L}_{MF} + \mu\bar{\psi}\gamma^0\psi)\right)$$

Mean-field model

- mean-field Lagrangian:

$$\mathcal{L}_{MF} = \bar{\psi}(x)\,\mathcal{S}^{-1}(x)\,\psi(x) - G_S\left[S^2(\vec{x}) + P^2(\vec{x})\right]$$

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- effective Hamiltonian (in chiral representation):

$$\mathcal{H}_{MF} = \mathcal{H}_{MF}[S,P] = \begin{pmatrix} -i\vec{\sigma}\cdot\vec{\partial} & M(\vec{x}) \\ M^*(\vec{x}) & i\vec{\sigma}\cdot\vec{\partial} \end{pmatrix}$$

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  - $\mathcal{H}_{MF}$ hermitean $\Rightarrow$ can (in principle) be diagonalized (eigenvalues E_λ)
  - $\mathcal{H}_{MF}$ time-independent $\Rightarrow$ Matsubara sum as usual

Mean-field thermodynamic potential

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- thermodynamic potential:

$$\Omega_{MF}(T,\mu;S,P) = -\frac{T}{V}\,\mathbf{Tr}\ln\left(\frac{1}{T}(i\partial_0 - \mathcal{H}_{MF} + \mu)\right) + \frac{G_S}{V}\int_V d^3x\,\left(S^2(\vec{x}) + P^2(\vec{x})\right)$$

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- general case: extremely difficult!

Periodic structures

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- crystal with a unit cell spanned by vectors $\vec{a}_i\,,\;\; i = 1, 2, 3$

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- mean-field Hamiltonian in momentum space:

$$\mathcal{H}_{\vec{p}_m,\vec{p}_n} = \begin{pmatrix} -\vec{\sigma}\cdot\vec{p}_m\,\delta_{\vec{p}_m,\vec{p}_n} & \sum_{\vec{q}_k} M_{\vec{q}_k}\,\delta_{\vec{p}_m,\vec{p}_n+\vec{q}_k} \\ \sum_{\vec{q}_k} M^*_{\vec{q}_k}\,\delta_{\vec{p}_m,\vec{p}_n-\vec{q}_k} & \vec{\sigma}\cdot\vec{p}_m\,\delta_{\vec{p}_m,\vec{p}_n} \end{pmatrix}$$

  - different momenta coupled by $M_{\vec{q}_k}$ $\Rightarrow$ $\mathcal{H}$ is nondiagonal in momentum space!
  - $\vec{q}_k$ discrete $\Rightarrow$ $\mathcal{H}$ is still block diagonal

Periodic structures: minimum free energy

- general procedure:
  - choose a unit cell $\{\vec{a}_i\} \;\Rightarrow\; \{\vec{q}_k\}$
  - choose Fourier components $M_{\vec{q}_k}$
  - diagonalize $\mathcal{H}_{MF} \quad \rightarrow \quad \Omega_{MF}$
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$\to$ further simplifications necessary

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- 1 + 1D solutions known analytically: [M. Thies, J. Phys. A (2006)]

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- remaining task:
  - minimize w.r.t. 2 parameters: Δ, ν
  - (almost) as simple as CDW, but more powerful
  - $m \neq 0$: 3 parameters

Phase diagram (chiral limit)

[D. Nickel, PRD (2009)]

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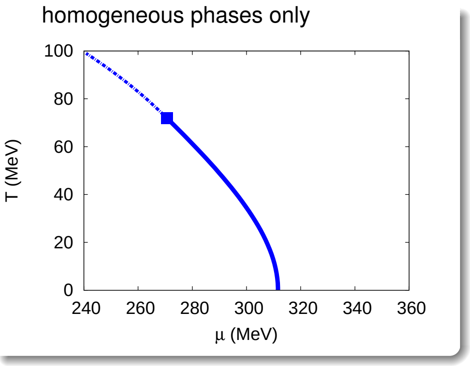

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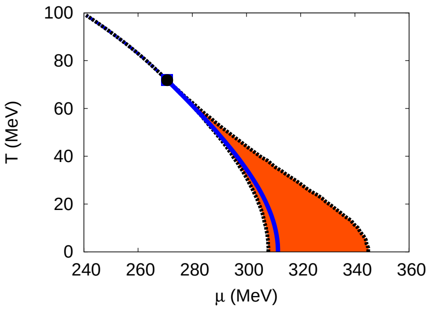
including inhomogeneous phase



- 1st-order line completely covered by the inhomogeneous phase!
- all phase boundaries 2nd order (mean-field artifact?)
- critical point coincides with Lifshitz point

Mass functions and density profiles ($T = 0$)

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- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \to \quad \begin{cases} \Delta\tanh(\Delta z) & \text{for} \quad \nu \to 1 \\ \sqrt{\nu}\Delta\sin(\Delta z) & \text{for} \quad \nu \to 0 \end{cases}$

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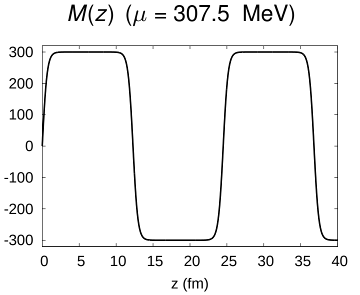

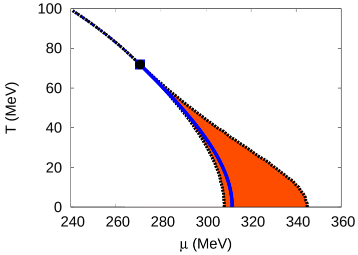

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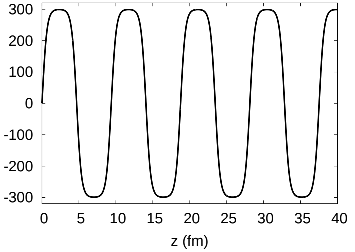
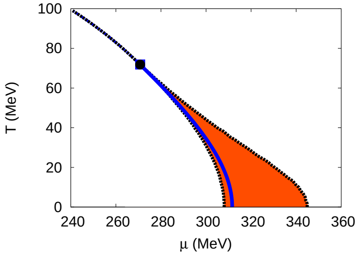
$M(z)$ ($\mu = 308$ MeV)





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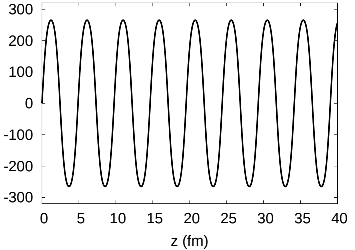
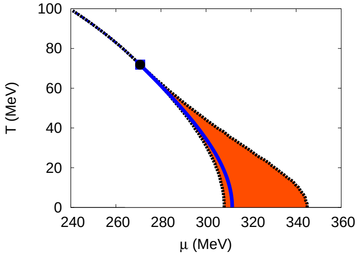
$M(z)$ ($\mu = 309$ MeV)





Mass functions and density profiles ($T = 0$)

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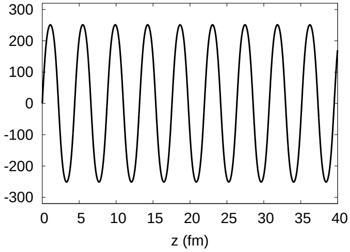

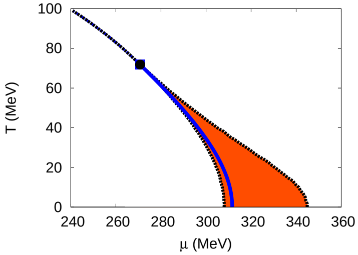

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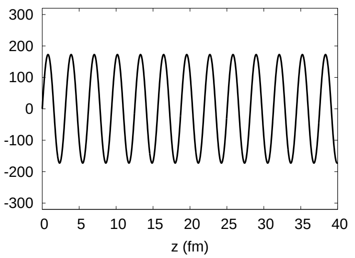

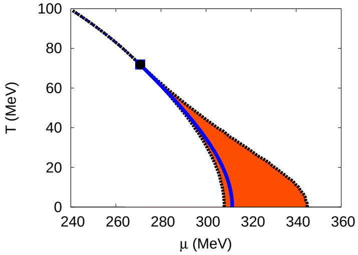

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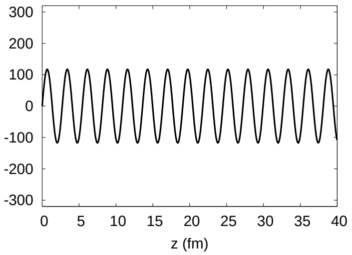

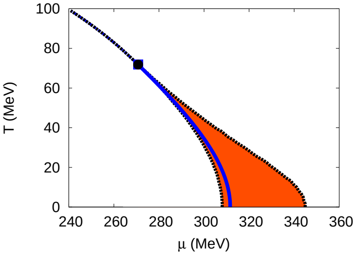

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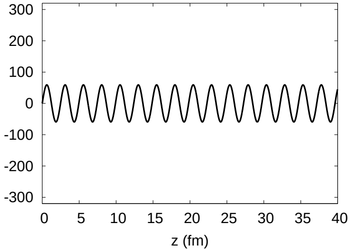
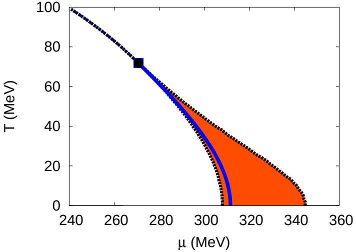
$M(z)$ (μ = 340 MeV)





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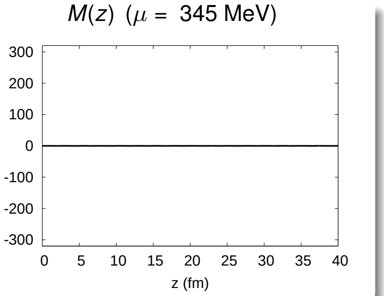

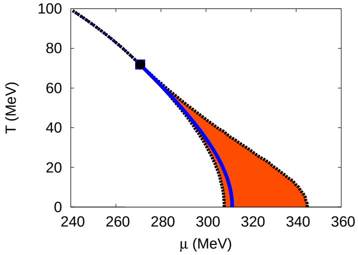

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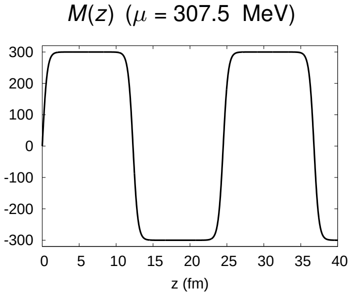

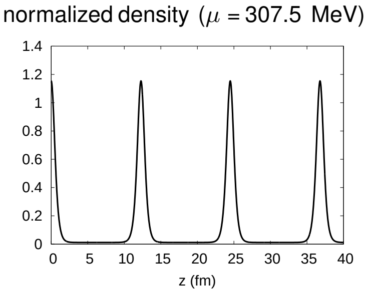

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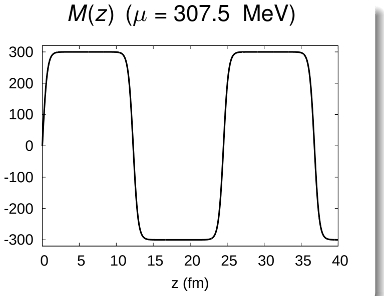

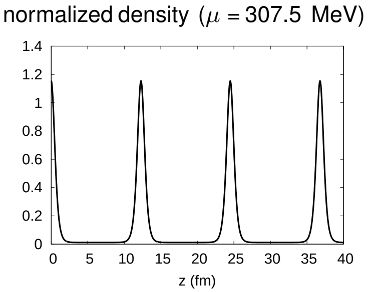

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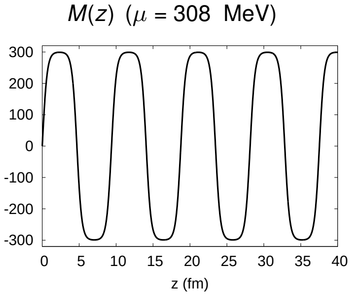

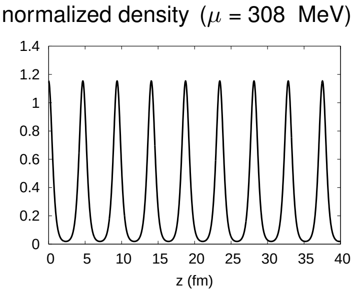

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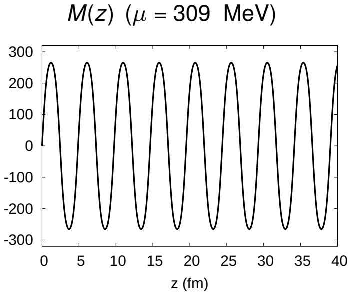

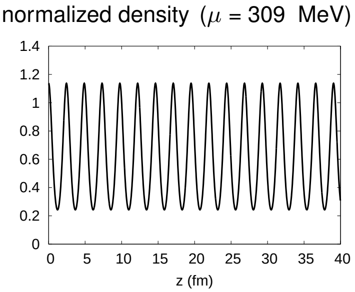

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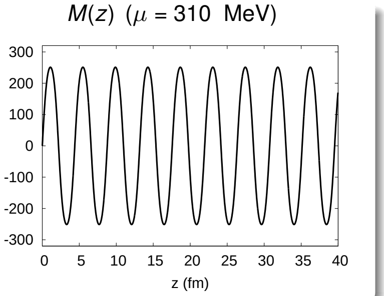

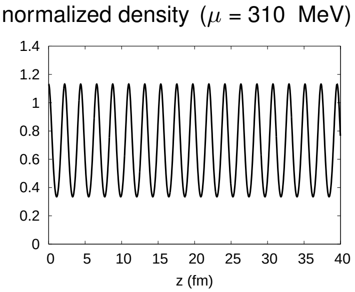

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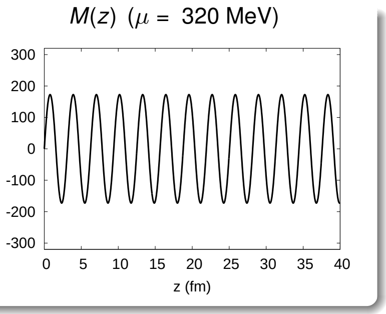

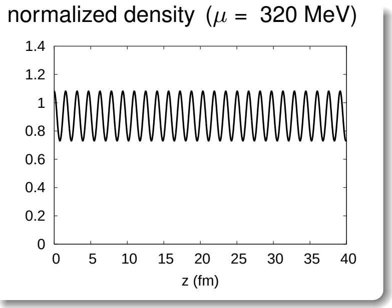

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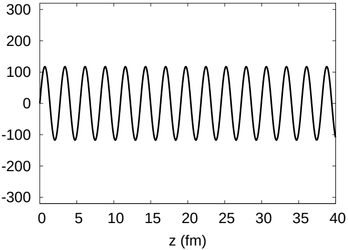
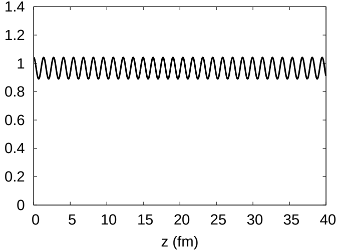


normalized density ($\mu =$ 330 MeV)



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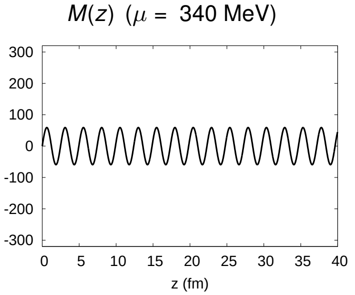

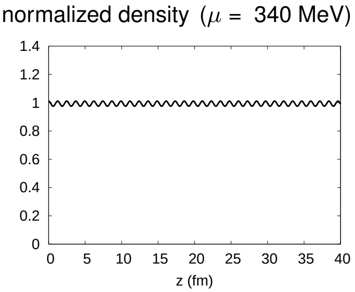

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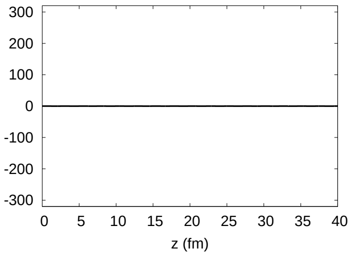

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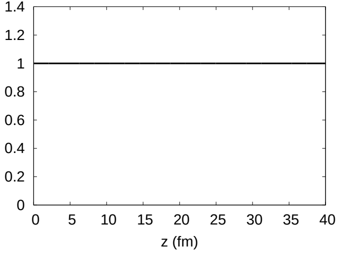

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Free energy difference

[D. Nickel, PRD (2009)]

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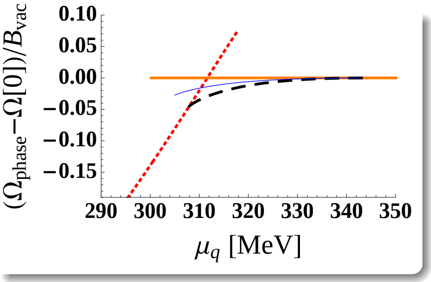

- homogeneous chirally broken
- Jacobi elliptic functions
- chiral density wave:
  $M_{CDW}(z) = M_1\, e^{iqz}$

- soliton lattice favored, when it exists
- $\delta\Omega_{Jacobi} \approx 2\delta\Omega_{CDW} \quad \Rightarrow$ CDW never favored

Including vector interactions

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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- additional vector term: $\mathcal{L}_V = -G_V(\bar{\psi}\gamma^\mu\psi)^2$

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  - $\langle\bar{\psi}\gamma^3\psi\rangle$ possible for inhomogeneous phases $\to$ will be neglected

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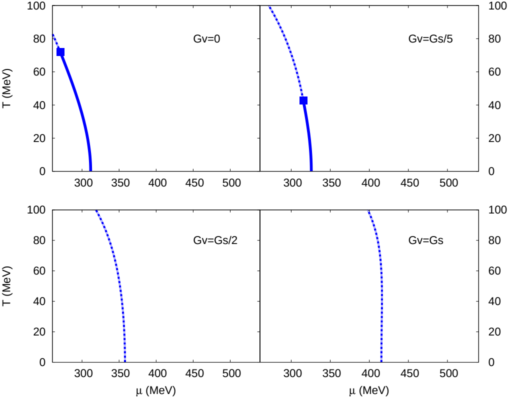
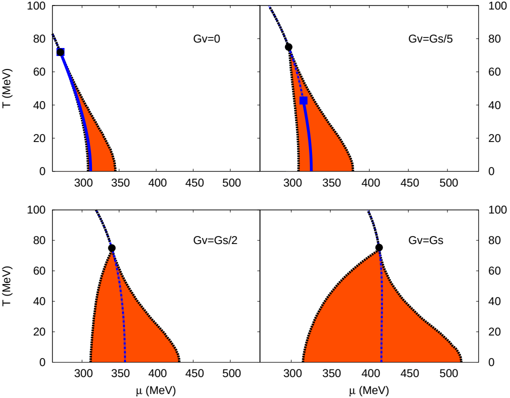
  - advantage: known analytic solutions can still be used
  - additional parameter: $\tilde\mu$, fixed by constraint $\frac{\partial\Omega_{MF}}{\partial\tilde\mu} = 0$

Phase diagram



- homogeneous phases: strong G_V-dependence of the critical point

Phase diagram



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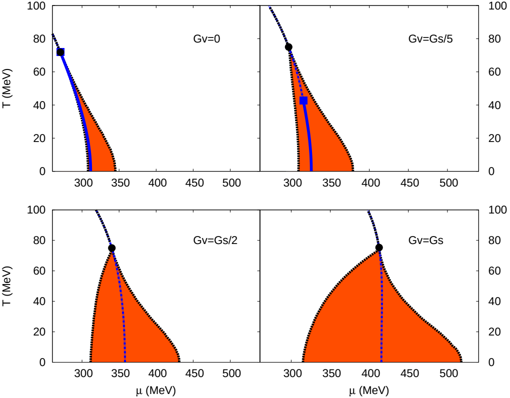
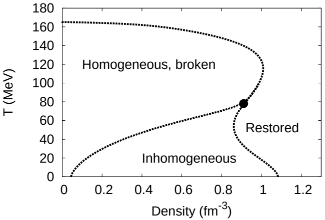
Phase diagram



T-$\langle n \rangle$ phase diagram:



- independent of G_V!

- homogeneous phases: strong G_V-dependence of the critical point
- inhomogeneous regime: stretched in μ direction, Lifshitz point at constant T

Two-dimensional modulations

[S. Carignano, M.B., PRD (2012)]

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Two-dimensional modulations

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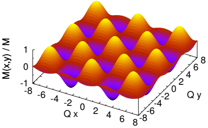
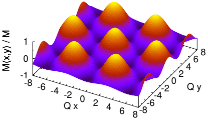
- consider two shapes:
  - square lattice ("egg carton")

    $M(x, y) = M \cos(Qx) \cos(Qy)$



  - hexagonal lattice

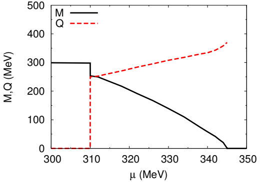
    $M(x, y) = \frac{M}{3} \left[2 \cos(Qx) \cos\left(\frac{1}{\sqrt{3}} Qy \right) + \cos\left(\frac{2}{\sqrt{3}} Qy \right) \right]$



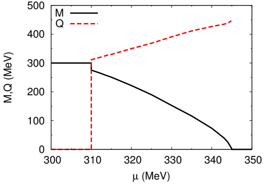
- minimize both cases numerically w.r.t. M and Q

Two-dimensional modulations: results

- amplitudes and wave numbers:
  - egg carton:



  - hexagon:



Two-dimensional modulations: results

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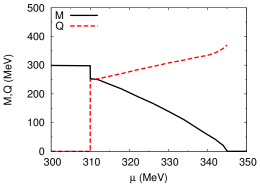

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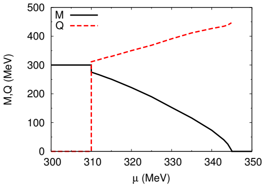

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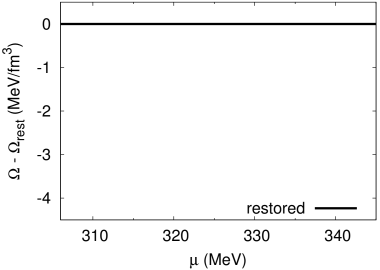

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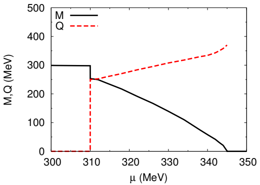

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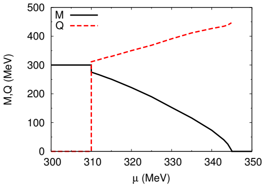

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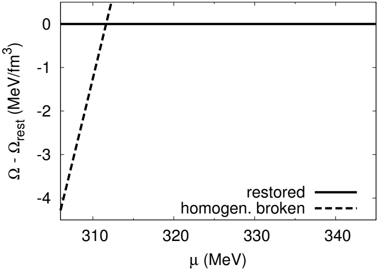

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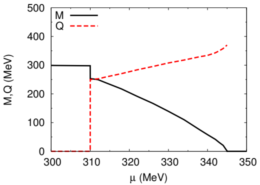

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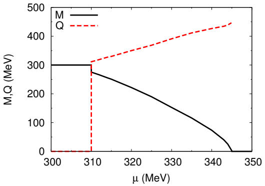

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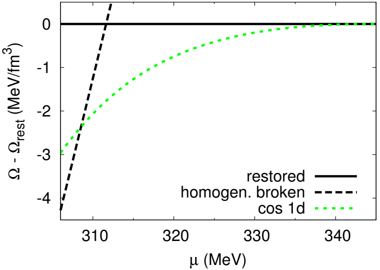

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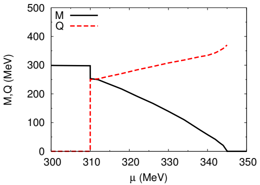

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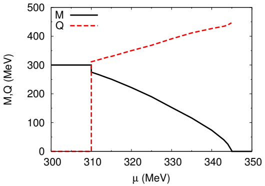

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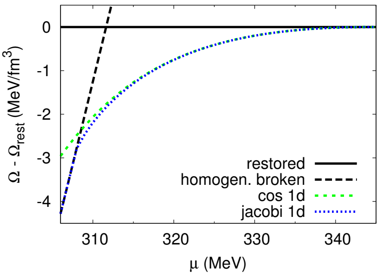

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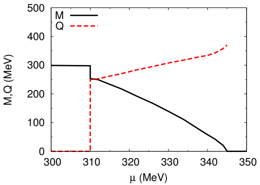

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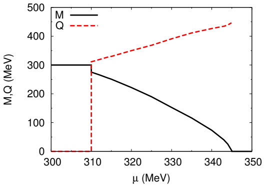

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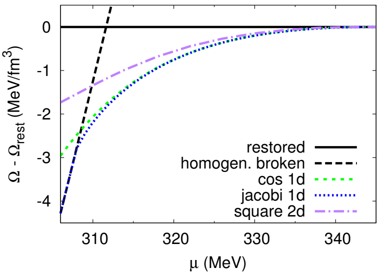

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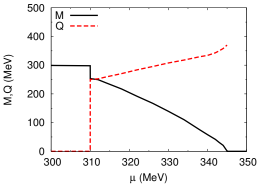

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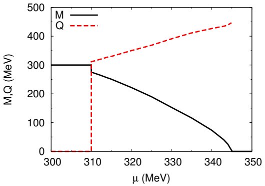

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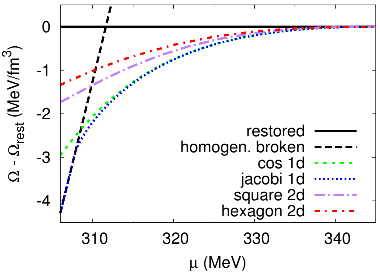

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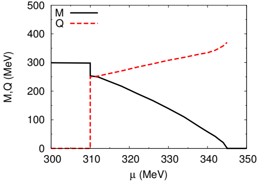

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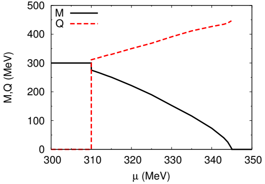

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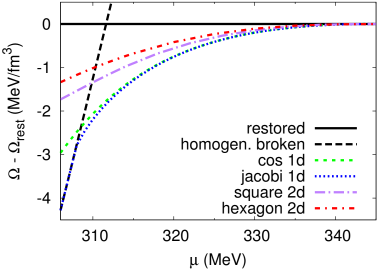

- 2d not favored over 1d in this regime

Rectangular lattice

- generalization:

  $$M(x,y) = M\cos(Q_x x)\cos(Q_y y)$$

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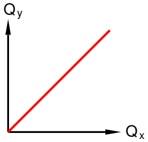

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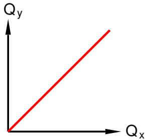

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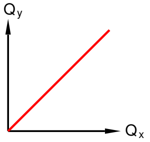
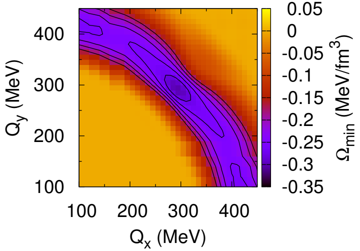


- free energy:



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Higher densities

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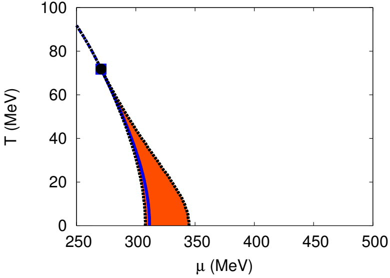

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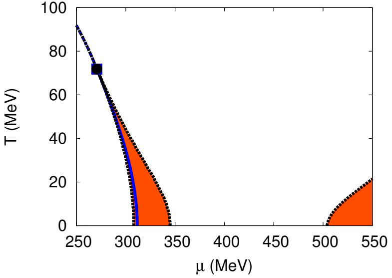

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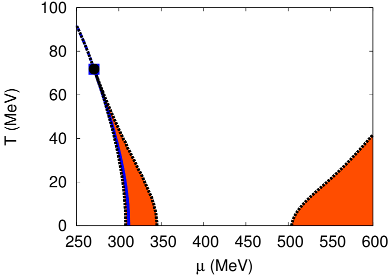

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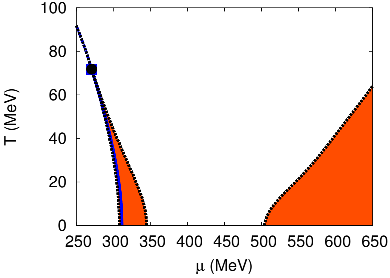

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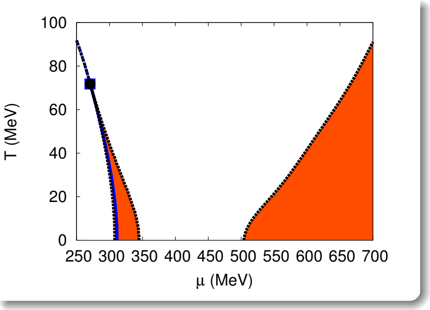

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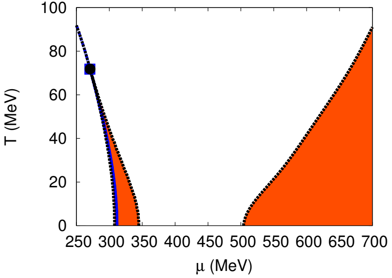

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- does not seem to end

  $\rightarrow$ "continent"
- also exists for the CDW

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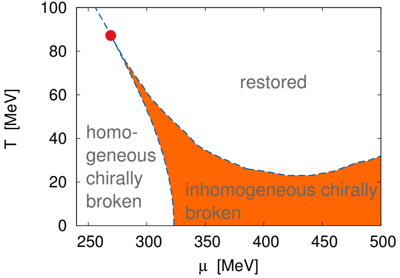

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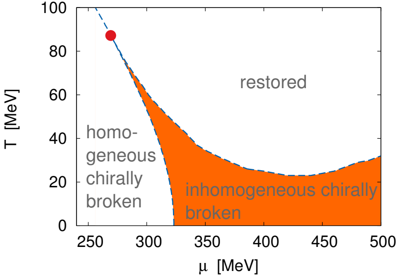

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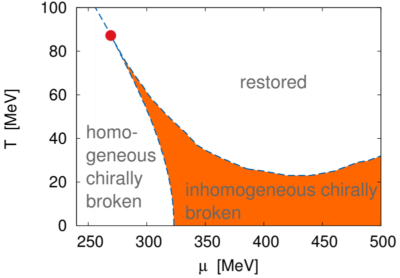

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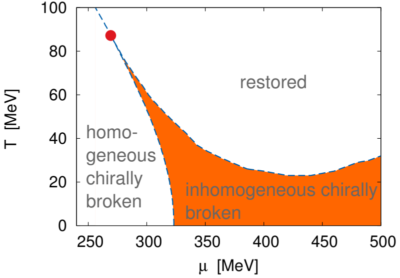

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- for now: take continent as a "laboratory" to study 2D modulations at higher μ.

Two-dimensional modulations: higher densities

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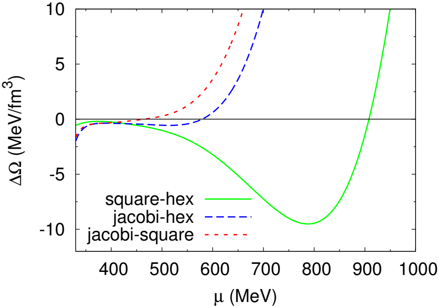
- higher chemical potentials:



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one-dim $\rightarrow$ square $\rightarrow$ hexagon

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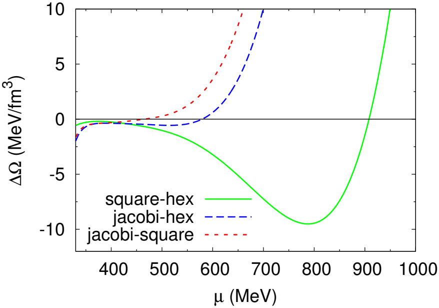
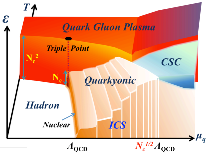
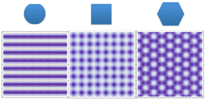


- “interweaving chiral spirals”



[Kojo et al., NPA (2012)]



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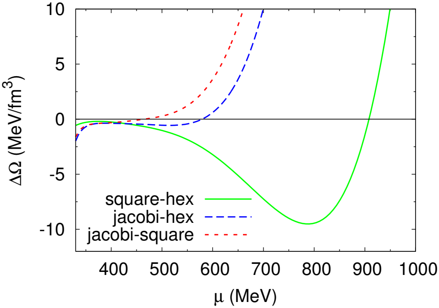

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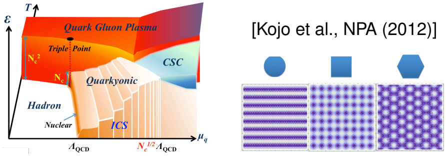

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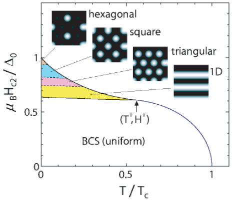

[Matsuda & Shimahara, J. Phys. Soc. Jpn. (2007)]

Self-bound quark matter

[M.B., S. Carignano, arXiv:1210.7155]

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- homogeneous NJL at $T = 0$ with strong enough attraction:
  - 1st-order phase transition from vacuum to restored quark matter

$\Rightarrow$ phase coexistence of vacuum and dense matter

$\Rightarrow$ mechanically stable quark droplets in vacuum

schematic bag-model "baryons"!

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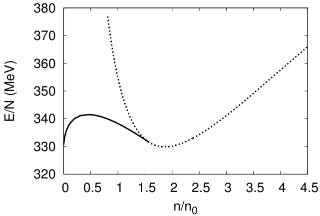

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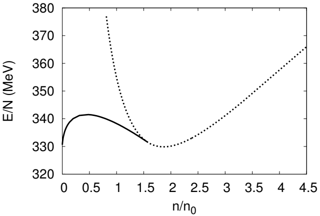

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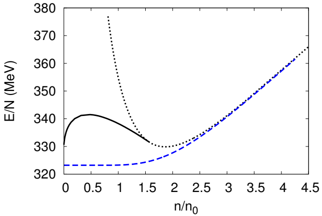

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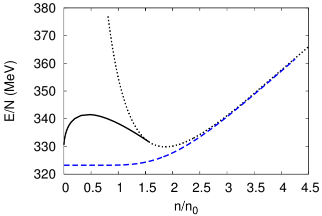

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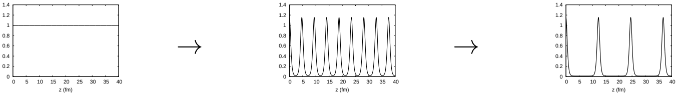

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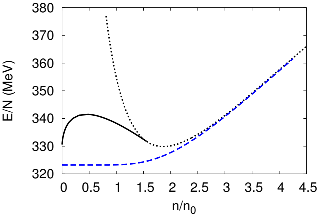

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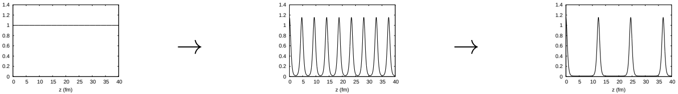

  - lowest energy at $\bar{n} = 0$ $\hat{=}$ single soliton limit

Interpretation?

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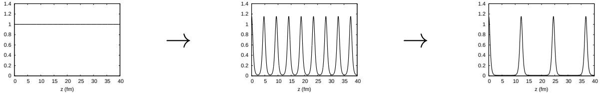

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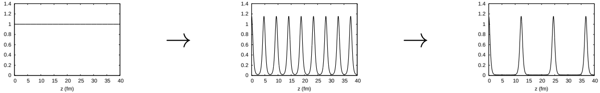

- single-soliton properties:
  - $\frac{E}{N} = \mu_{c,inh} \sim 325$ MeV $\quad \Rightarrow \quad$ "baryon" mass: $\quad M_B = 3\frac{E}{N} \sim 975$ MeV
  - central density: $\quad \rho_B = \frac{1}{4\pi} M_{vac} \mu_{c,inh}^2 \sim 2.1 \rho_0$
  - longitudinal size: $\quad \sqrt{\langle z^2 \rangle} = \frac{\pi}{\sqrt{12}} \frac{1}{M_{vac}} \sim .5$ fm

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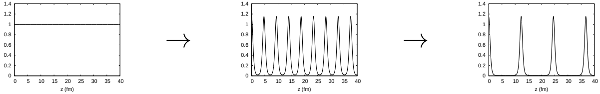

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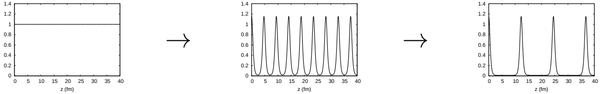

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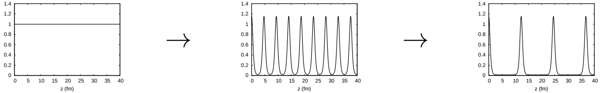

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- preformation of 1D solitons in the deconfined phase?
  - measurable effects on fireball expansions?

Color superconductivity

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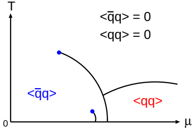
- low T, high μ:
  Cooper pairing of quarks
- diquark condensates:
  $\langle q^T \mathcal{O} q \rangle$, $\mathcal{O}$ totally antisymmetric operator



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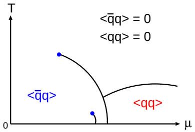

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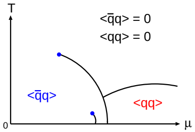

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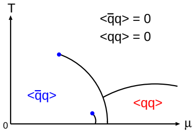

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- constraints in compact stars: beta equilibrium + electric neutrality
  - strange quarks suppressed by mass

  $\rightarrow$ more down quarks than up quarks needed

  $\rightarrow$ "stressed pairing"

BCS pairing

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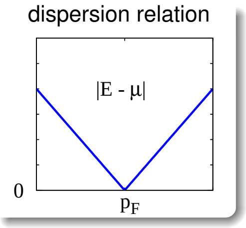
- noninteracting Fermi gas

  $\rightarrow$ no free-energy cost for particle creation at the Fermi surface



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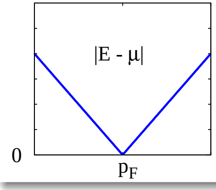

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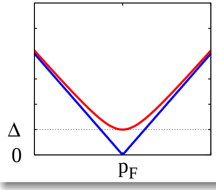

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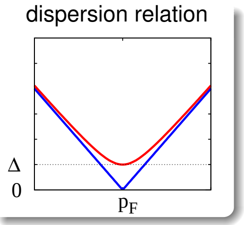

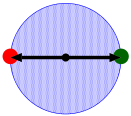

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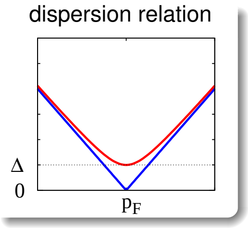

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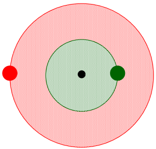

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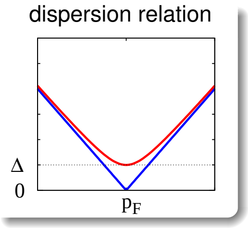

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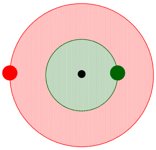

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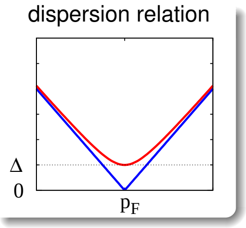

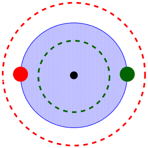

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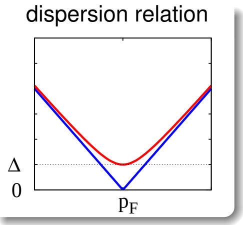

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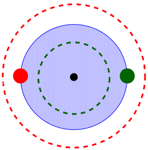

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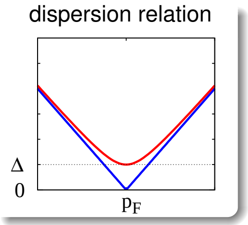

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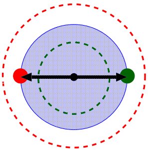

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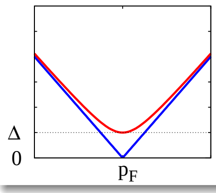

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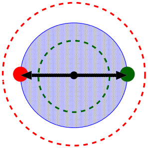

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  - not favored for $\delta p_F \gtrsim \frac{\Delta}{\sqrt{2}}$ (Chandrasekhar, Clogston (1962))

Inhomogeneous pairing

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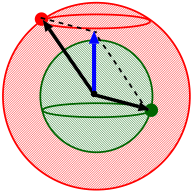
  - disfavored by phase space



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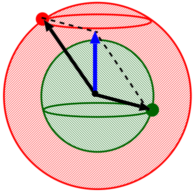

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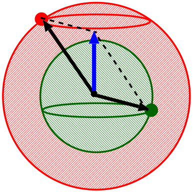

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- inhomogeneous (“crystalline”) color superconductivity:
  - so far mostly FF ansatz [Alford, Bowers, Rajagopal (2001); Sedrakian, Rischke (2009); ...]
  - or Ginzburg-Landau studies of certain LO patterns
    [Bowers, Rajagopal (2002); Casalbuoni et al. (2006), Rajagopal, Sharma (2006); ...]

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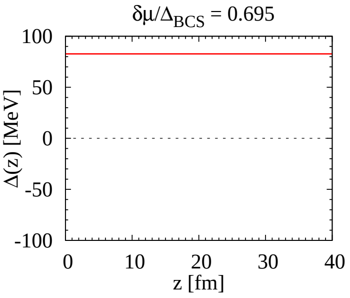

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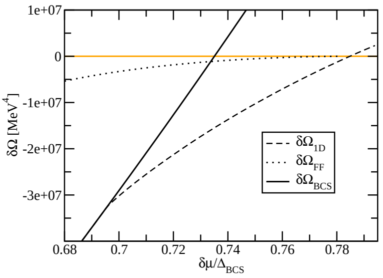

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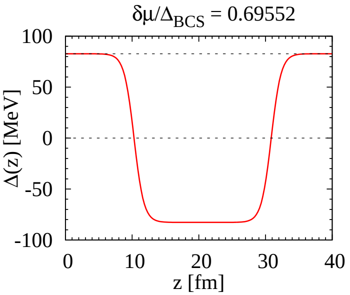

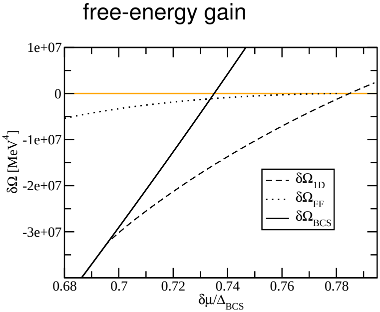

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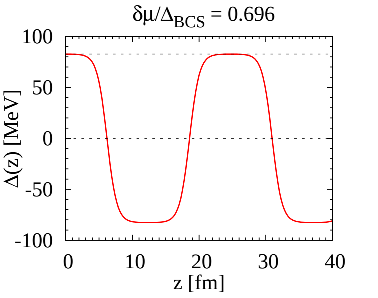

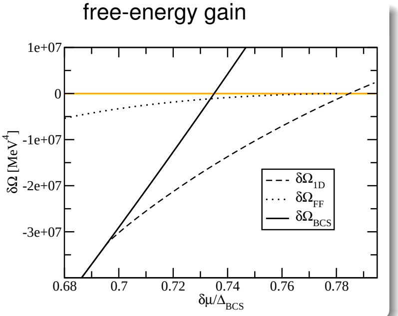

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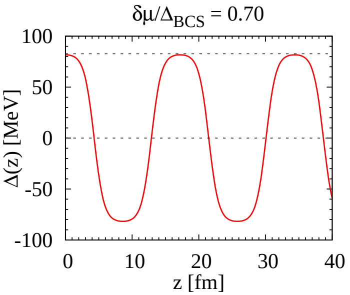

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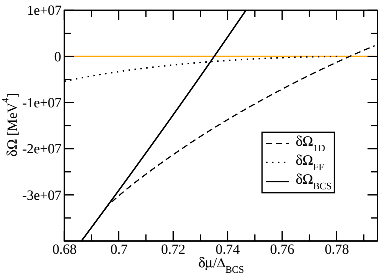

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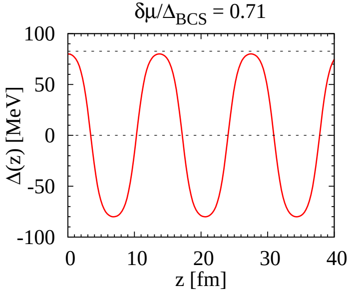

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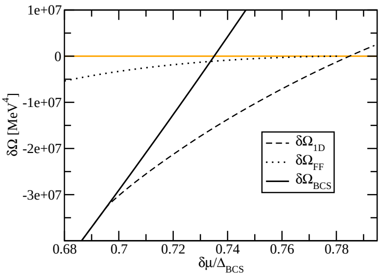

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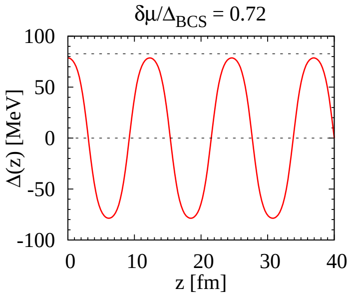

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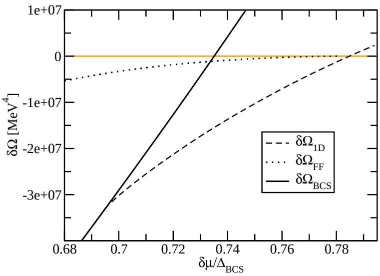

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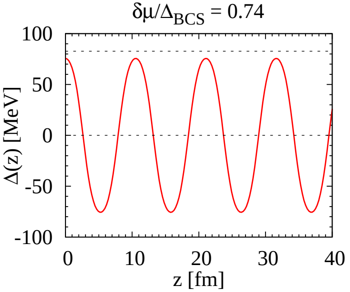

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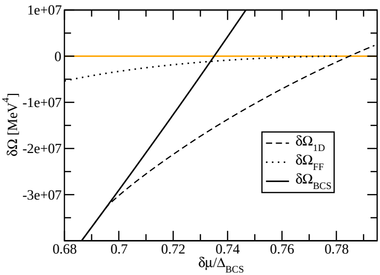

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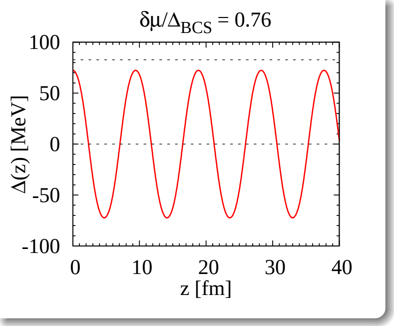

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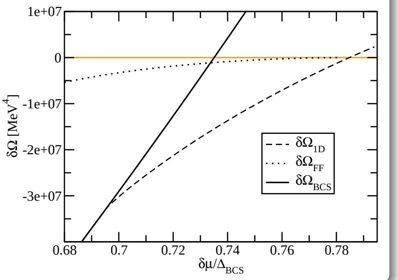

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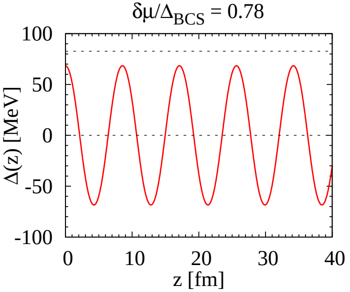

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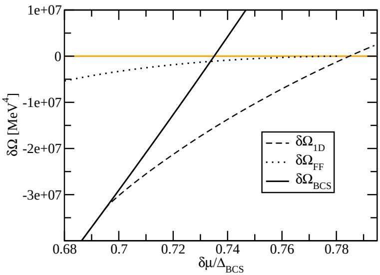

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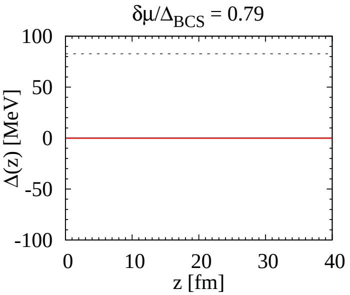

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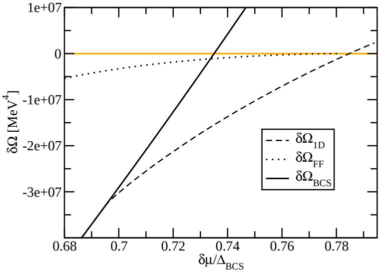

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Pion condensation

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- finite isospin chemical potential:

  $\mu_u = +\delta\mu, \quad \mu_d = -\delta\mu$

- charged pion condensation:

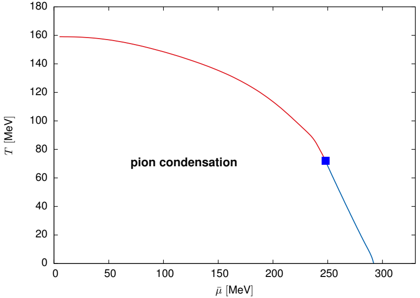
  $\delta\mu > m_\pi/2 \quad \to \quad \langle \bar{u}\, i\gamma_5\, d\rangle \neq 0$

- + quark chemical potential:

  $\mu_u = \delta\mu + \bar{\mu}, \quad \mu_d = -\delta\mu + \bar{\mu}$

  $\to$ stressed pion condensation

homogeneous phases only



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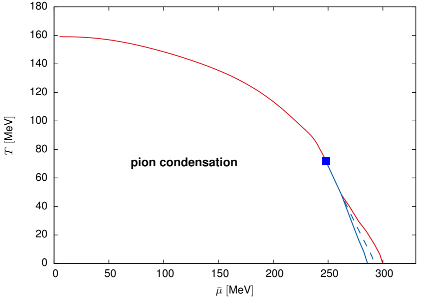
- allowing for inhomogeneous condensates:

  analogous phase structure as before

including inhomogeneous phase



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Comparison

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condensate	favored by	stressed by
$\langle \bar{q}q \rangle$	(vacuum)	$\bar{\mu}$
$\langle q^T \mathcal{O} q \rangle$	$\bar{\mu}$	$\delta\mu$
$\langle \bar{u}\, i\gamma_5\, d \rangle$	$\delta\mu$	$\bar{\mu}$

- similar structures in the $T - \mu_{stress}$ phase diagram!

Conclusions

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Collaborators

Stefano Carignano

Daniel Nowakowski

Backup slides

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Susceptibilities

- signature of the critical point: divergent susceptibilities
- e.g., quark number susceptibility:

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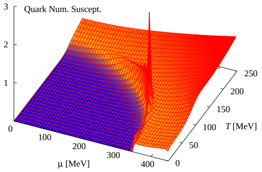
homogeneous phases only:



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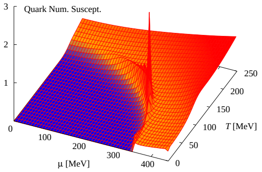

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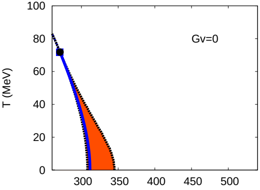

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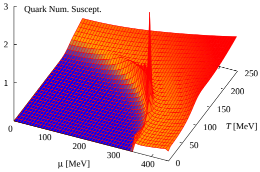

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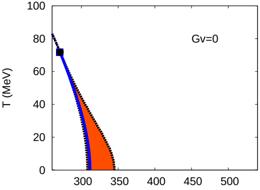

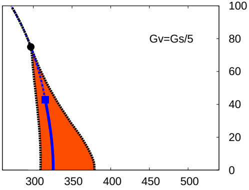

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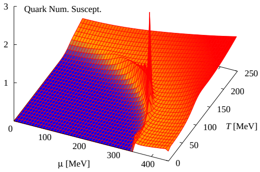

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- $\underline{G_V = 0:}$
  CP = Lifshitz point
  $\rightarrow$ no qualitative change
- $\underline{G_V > 0:}$
  no CP $\rightarrow$ no divergence

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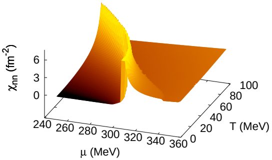

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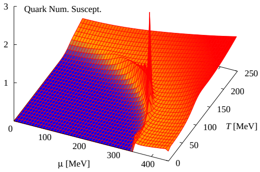

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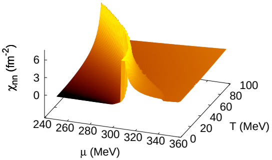

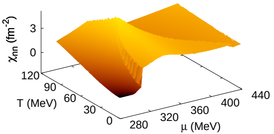

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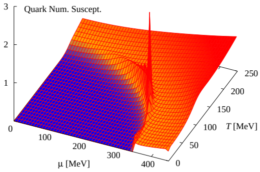

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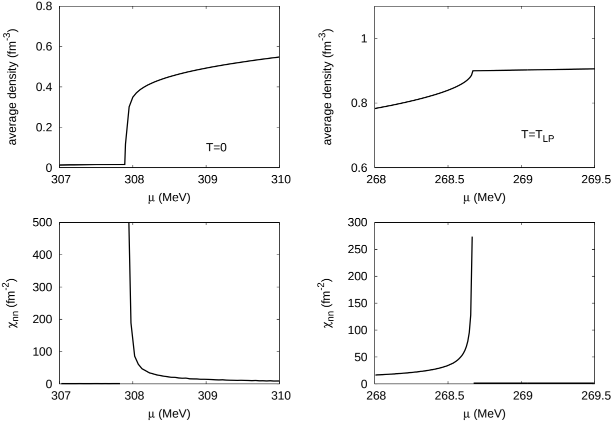
- densities and quark number susceptibilities for $G_V = 0$:



- $\underline{T = T_{CP},\ \mu < \mu_c}$:

  $\chi_{nn} \propto \frac{1}{\sqrt{\mu_c - \mu}}$

- $\underline{T = 0,\ \mu > \mu_{cr}}$:

  $\chi_{nn} \propto \frac{1}{(\mu - \mu_{cr}) \log^2(\mu - \mu_{cr})}$

- $\underline{G_V > 0}$:

  $\delta\chi_{nn}\big|_{T=0,\mu=\mu_{cr}} \approx \frac{1}{2G_V}$

Technical origin of the "continent" (chiral density wave, $T = 0$)

- $\Omega(\mu) - \Omega_{rest.}(\mu) =$

  $$-2N_c \int_0^\infty dE\, [f_{vac}(E,\Lambda) + f_{med}(E,\mu)]\,[\rho(E,M,q) - \rho_{rest.}(E)] + \frac{M^2}{4G}$$

  $f_{vac}(E,\Lambda) = E$ + regulator terms

  $f_{med}(E,\mu) = \theta(\mu - E)(\mu - E)$

  ρ = density of states

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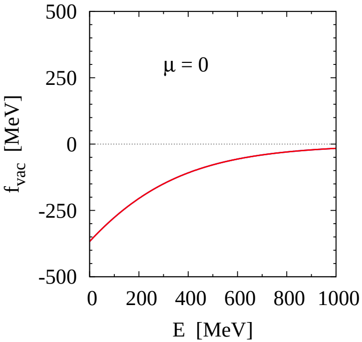

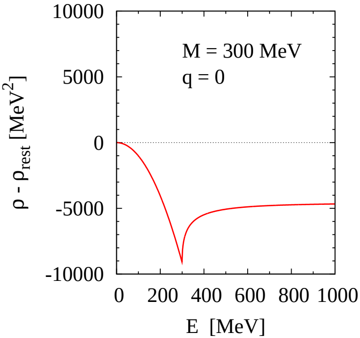

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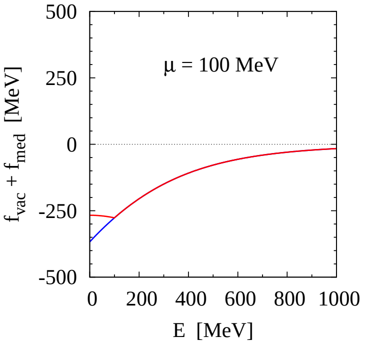

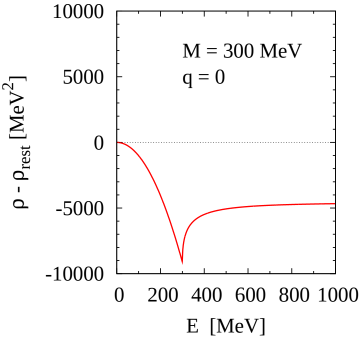

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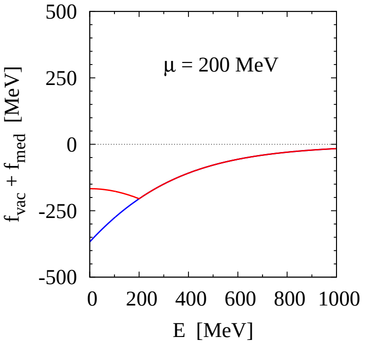

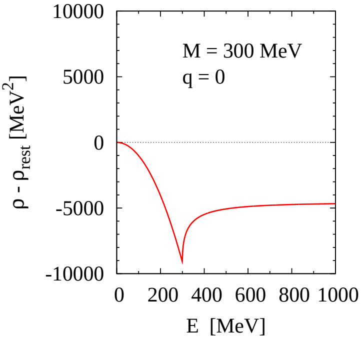

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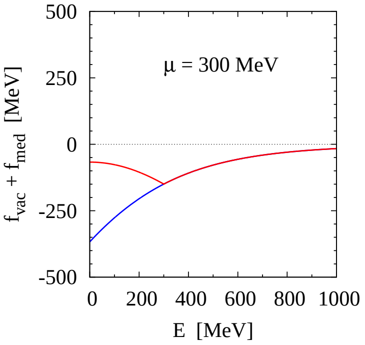

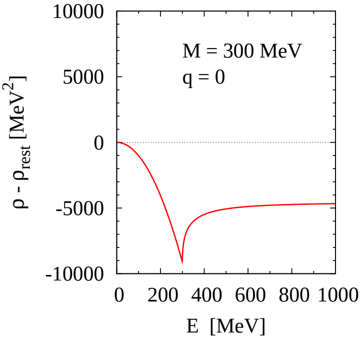

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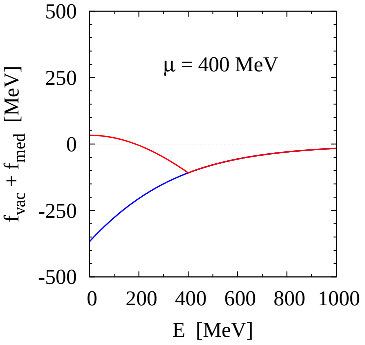

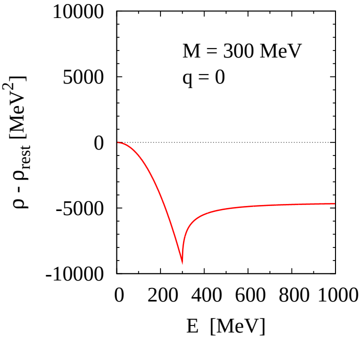

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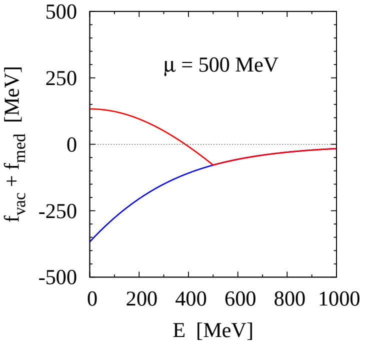

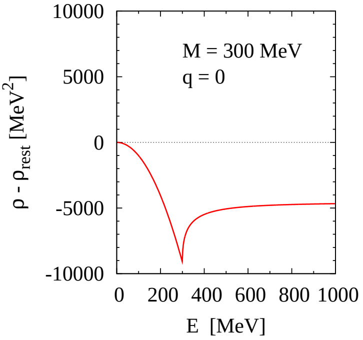

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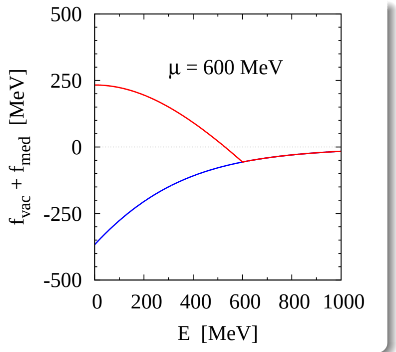

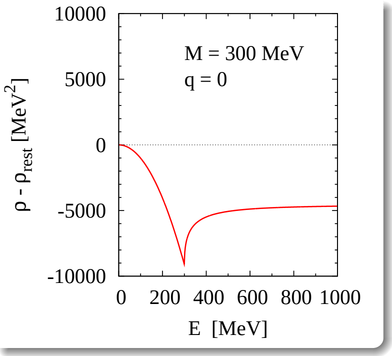

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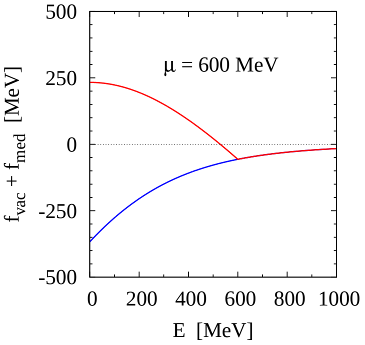
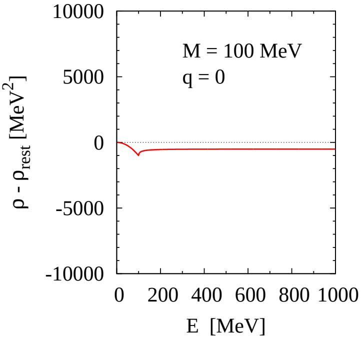
TECHNISCHE UNIVERSITÄT DARMSTADT

- $\Omega(\mu) - \Omega_{rest.}(\mu) =$

$$-2N_c \int_0^\infty dE \, [f_{vac}(E, \Lambda) + f_{med}(E, \mu)] \, [\rho(E, M, q) - \rho_{rest.}(E)] \; + \; \frac{M^2}{4G}$$





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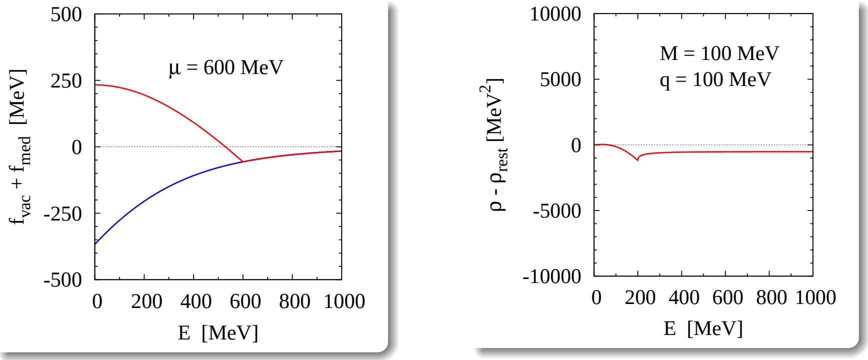

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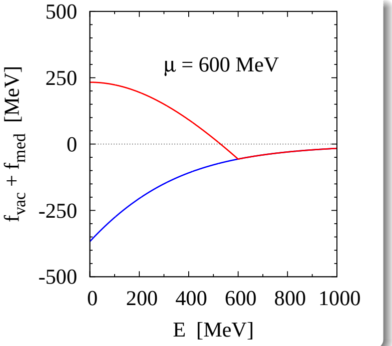

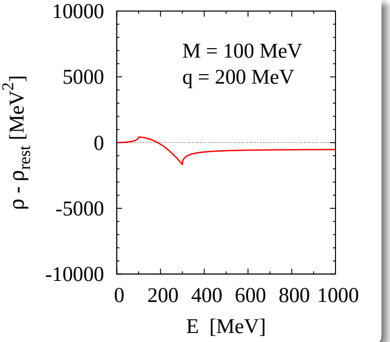

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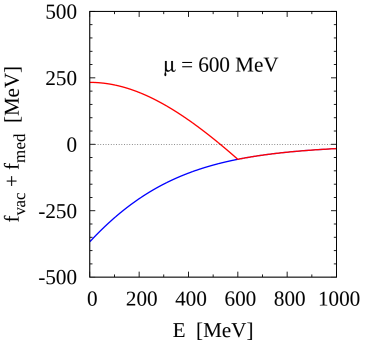

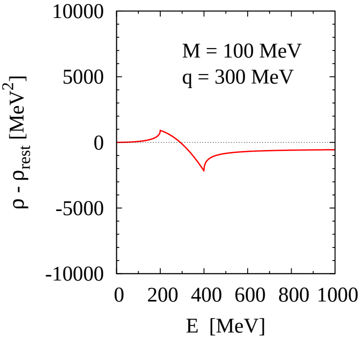

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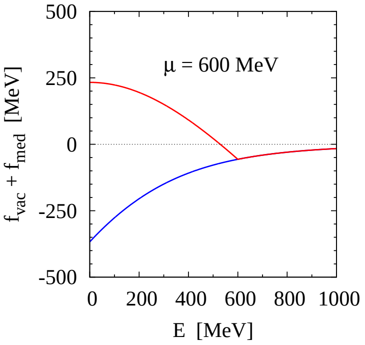

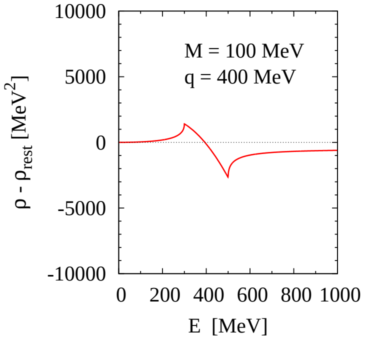

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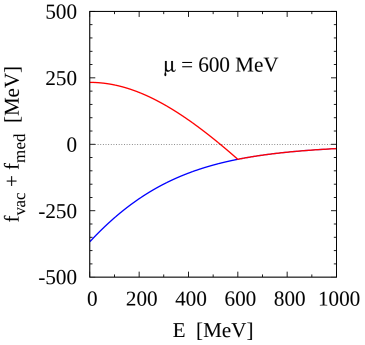

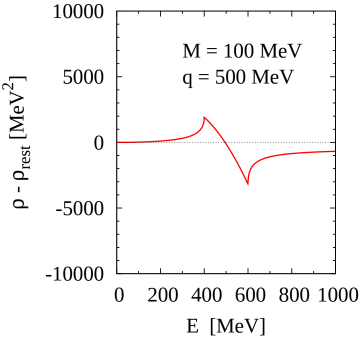

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