QCD probed by strong magnetic fields

Falk Bruckmann
(Univ. Regensburg)

“Strongly interacting field theories”, Jena, Nov. 2012

with G. Bali, M. Constantinou, M. Costa, G. Endrődi, Z. Fodor,
S. Katz, T. Kovács, S. Krieg, H. Panagopoulos, A. Schäfer, K. Szabó

JHEP 1202 (2012) 044, PRD 86 (2012) 071502, 1209.6015, in prep.

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A strong field interacting with my favourite theory

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Strong Magnetic fields

early universe	$\sqrt{eB} \simeq 2$ GeV	
RHIC/LHC non-central collisions charged spectators B perp. to reaction plane	0.1.. 0.5 GeV	QCD scale!
neutron stars, magnetars	1 MeV	$B \simeq 10^{14}$ G

Strong Magnetic fields

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neutron stars, magnetars	1 MeV	$B \simeq 10^{14}$ G
cf. strongest field in lab		10^5 G (10^7 G unstable)
refrigerator magnet		100 G
earths magn. field		0.6 G

What to expect?

- quarks couple to electromagnetism: $(q_u, q_d, q_s) = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})e$

  neutral gluons do not $\leftarrow$ indirect effects

- strong electric field $\Rightarrow$ pair creation

- magn. field $\Rightarrow$ Landau orbits

  dimensional reduction to 1+1 dimensions? Mermin-Wagner?

  'chiral magnetic effect'? Kharzeev 04

  - bubble of coherent top. charge ('instanton') in collision event: prefers part. alignment of spins wrt. momenta
  - magnetic field: aligns spins

  $\Rightarrow$ alignment of momenta along B (as if E) $\Rightarrow$ part. charge correlations

  - some experimental hints

Idealized situation

- constant external magnetic field in Euclidean space

  gluons, but no photons (no QED)

- accessible to lattice simulations (no sign problem)

  magnetic fields quantized and bounded (like momenta) 't Hooft 79

  state-of-the-art: $\sqrt{eB} = 0.1 \ldots 1$ GeV

  ▸ simulation details

Free particles in magnetic fields

nonrelativistic and classical:

- motion on circle perpendicular to magn. field
- radius $r = \frac{mv}{qB} \quad (qB > 0)$
- angular velocity $\omega = \frac{qB}{m}$
- energy $E = \frac{1}{2}\,\omega L$ with angular momentum L

Bohr-Sommerfeld quantization: $L = k \in \mathbb{Z} \quad$ (in units of $\hbar = 1$)

- quantized energies: $E = k\,\frac{qB}{2m}$

  like for harmonic oscillator

… and spin

Free Dirac equation with magnetic field

gauge field for $B = B_z = (\text{curl}\, A)_z$:

$$A_y = Bx,\ A_{\text{rest}} = 0 \quad \text{(no gluons)}$$

Dirac operator $\not{D} = \gamma_\mu(\partial_\mu + qA_\mu)$, square:

$$-\not{D}^2 = -D_\mu^2 + \frac{1}{2}qF_{\alpha\beta}\sigma_{\alpha\beta} \qquad \sigma_{\alpha\beta} = [\gamma_\alpha, \gamma_\beta]/2i \qquad (1)$$

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$$= -\partial_t^2 - \partial_z^2 - \partial_x^2 - (\partial_y + qBx)^2 + qB\,\sigma_{12} \qquad \sigma_{12} = \text{diag}(1, -1, 1, -1)$$

free waves — harm. oscillator — spin

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free waves   harm. oscillator   spin

eigenvalues thereof: Landau 1930; Euler, Heisenberg 1935

$$\lambda^2 = p_t^2 + p_z^2 + |qB|(2n+1) + qB(2s)$$

$$p_t, p_z \in \mathbb{R} \quad n = 0, 1, \ldots \quad s = \pm 1/2$$

degeneracy: $|qB| \cdot$ area = |magn. flux|

eigenvalues of massive operator:

$$m^2 + \lambda^2 = m^2 + p_t^2 + p_z^2 + |qB|(2n+1) + qB(2s)$$

like class. energies plus spin

- charged quarks, spin 1/2:
  lowest Landau level (LLL) has $\lambda^2 = 0$

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  $\lambda^2 = |eB| \Rightarrow$ mass grows as: $m^2(B) = m^2(0) + |eB|$

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- charged pions, spin 0:
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- charged rho mesons, spin 1:
  $\lambda^2 = -|eB| \Rightarrow$ mass reduces as: $m^2(B) = m^2(0) - |eB|$
  $|eB| = m_\rho^2 \Rightarrow$ condensation? Chernodub 10
  [gluons under chromomagnetic field: instability of Savvidy vacuum]

Magnetic catalysis

NJL model in 3+1 dimensions — Gusynin, Miransky, Shovkovy 96

$$\mathcal{L} = \bar{\psi} \not{D}[B] \psi + G\, O(\psi^4)$$

- chiral condensate in mean field:

$$\langle \bar{\psi}\psi \rangle \equiv \mathrm{tr} \frac{1}{\not{D} + m} \qquad \ldots \text{just } B\text{, otherwise free}$$

$$= m_0\, qB \int_{1/\Lambda^2}^{\infty} \frac{ds}{s} e^{-m_0^2 s} \coth(sqB) \qquad \ldots \text{proper time (Mellin trafo)}$$

↑ degeneracy — Schwinger 51

$$= m_0 \left[\Lambda^2 + qB \log \frac{qB}{m_0^2} \right] \qquad \ldots \text{for small } m_0\text{, large } B$$

- dynamical mass m from gap equation:

$$m = G \langle \bar{\psi}\psi \rangle = mG \left[\Lambda^2 + qB \log \frac{qB}{m^2} \right]$$

- always trivial solution $m = 0$ preserving chiral symmetry
- nontrivial solution $m > 0$ breaking chiral symmetry

$$m = \sqrt{qB}\,\exp\left(-\frac{1}{G\,qB}\right)\exp\left(\frac{\Lambda^2}{qB}\right)$$

for arbitrarily small magnetic fields & small couplings G

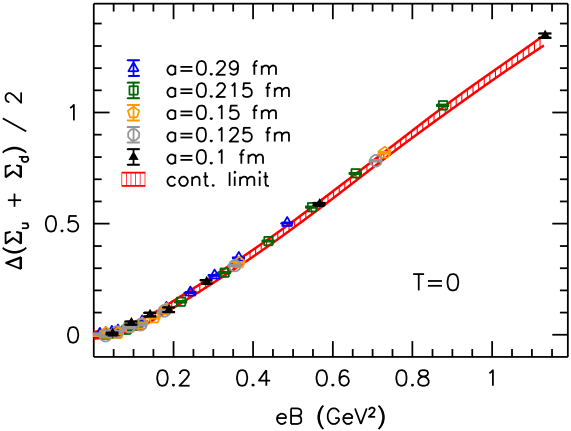
nonperturbative

magn. field catalyzes condensate and dyn. mass

- in contrast vanishing magnetic field:
  nontrivial solution only for strong couplings G

Magnetic catalysis from the lattice

- change of condensate with B: (D'Elia et al. 10) Bali, FB, Endrődi et al. 11

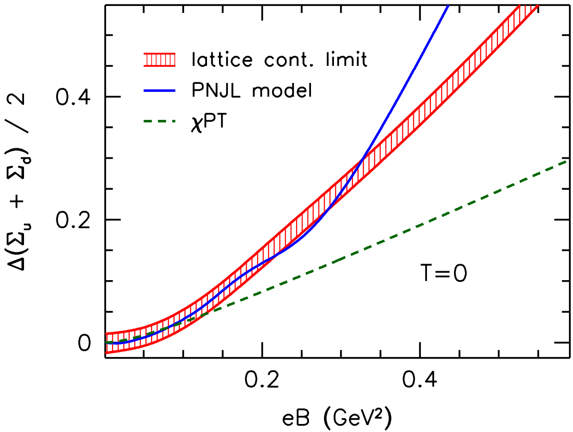


mult. and add. renorm.: $\Delta\Sigma_{u,d} \equiv \# m_{u=d} \left[\langle\bar{\psi}\psi_{u,d}\rangle(B) - \langle\bar{\psi}\psi_{u,d}\rangle(0)\right]$

continuum limit with $N_f = 2 + 1$ (staggered) quarks at phys. masses

Magnetic catalysis from the lattice: comparison

- change of condensate with B:



chiral perturbation theory — Cohen, McGady, Werbos 07, Andersen 12

& NJL model — Gatto, Ruggieri 10

$\Rightarrow$ well approximated unless $eB > {0.1 \atop 0.3}$ GeV^2 (approaches valid there?)

QCD phase diagram with B

- at $B = 0$: crossover — Aoki et al. 06

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- with B: catalysis seems to prefer chirally broken = low T phase, but

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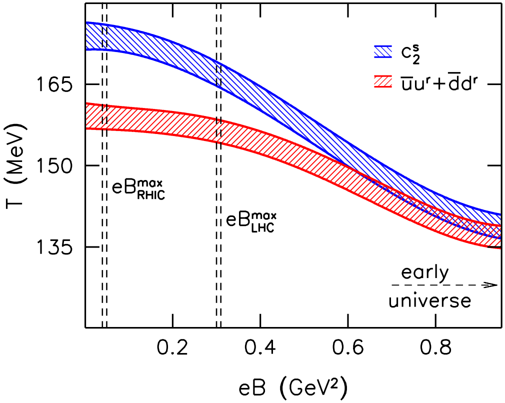
pseudo-critical temperatures $T_c(B)$: — Bali, FB, Endrődi et al. 11

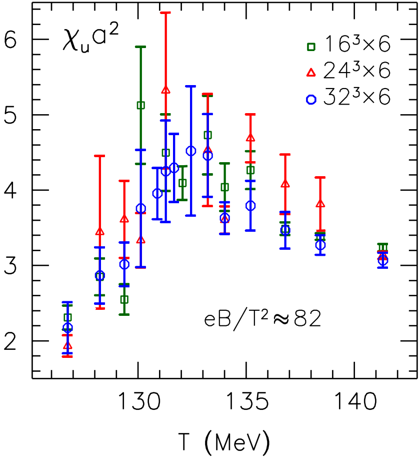


renorm. condensate of light quarks, strange number susceptibility

$\Rightarrow$ T_c decreases by O(10) MeV

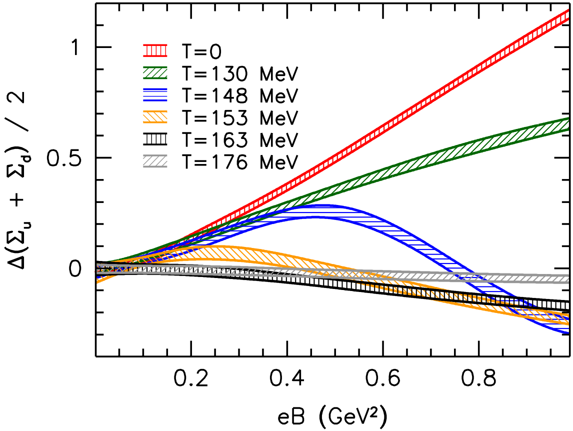
Nature of the transition

- volume dependence of light susceptibility:



no volume scaling $\Rightarrow$ remains a crossover up to $\sqrt{eB} \simeq 1$ GeV

- again change of condensate, for finite T:



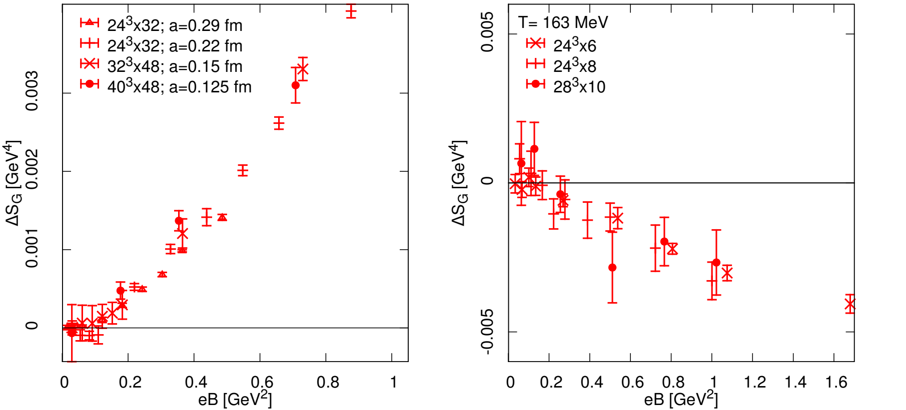
$\Rightarrow$ non-monotonic behaviour:

magn. catalysis turns into inverse magnetic catalysis around T_c

captured by models? understanding?

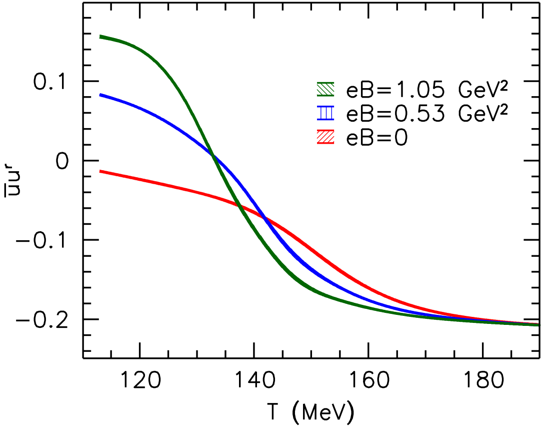
Inverse magnetic catalysis in the gluonic sector

- change of gluonic action at $T = 0$ and $T \gtrsim T_c$: Bali, FB, Endrődi in prep.



similar behaviour: magnetic catalysis $\rightarrow$ inverse magnetic catalysis mediated by quarks

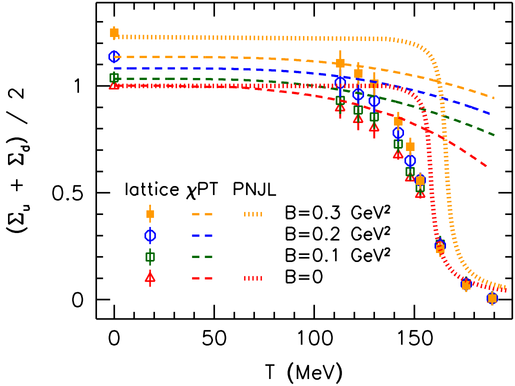
- again condensate, now as function of T at fixed B:



magn. catalysis and inverse magnetic catalysis (vertically) ✓

T_c decreases ✓

- again to χPT and PNJL model: Andersen 12; Gatto, Ruggieri 10



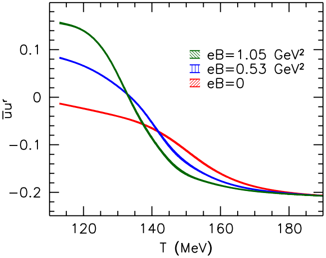
⇒ χPT ok unless $T > 100$ MeV (valid there?!)

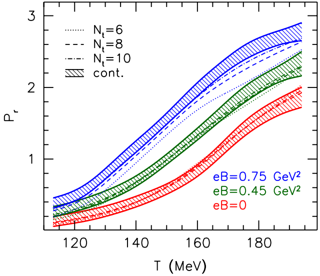
PNJL? this one uses Polyakov loop potential from $N_f = 2$ lattice data

⇒ no inverse magn. catalysis (= crossing of curves)

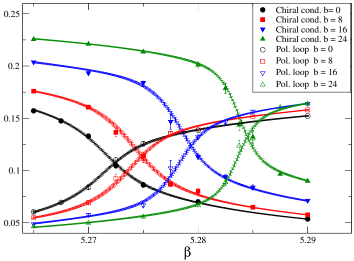
Inverse magnetic catalysis: previous lattice simulations

$1+1+1$ (staggered, smeared) us
at phys. masses





$1+1$ (staggered) D'Elia et al. 10
at higher-than-phys. masses



similar in $SU(2)$ Ilgenfritz et al. 12

! $T_c(B)$ different

! monotonicities different

rationale: continuum limit,

light quark masses!

▸ cross-checked

Inverse magnetic catalysis: mechanism

$$\bar{\psi}\psi^{\text{full}} = \frac{\int DA_\mu e^{-S_g[A_\mu]} \det(\not{D}[A_\mu; B] + m) \operatorname{tr}(\not{D}[A_\mu; B] + m)^{-1}}{\int DA_\mu e^{-S_g[A_\mu]} \det(\not{D}[A_\mu; B] + m)}$$

Inverse magnetic catalysis: mechanism

$$\bar{\psi}\psi^{\text{val}} = \frac{\int DA_\mu e^{-S_g[A_\mu]} \det(\not{D}[A_\mu; 0\,] + m) \operatorname{tr}(\not{D}[A_\mu; B] + m)^{-1}}{\int DA_\mu e^{-S_g[A_\mu]} \det(\not{D}[A_\mu; 0\,] + m)}$$

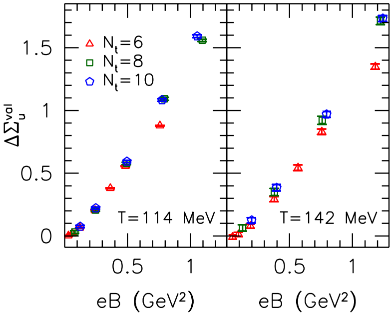
Inverse magnetic catalysis: mechanism

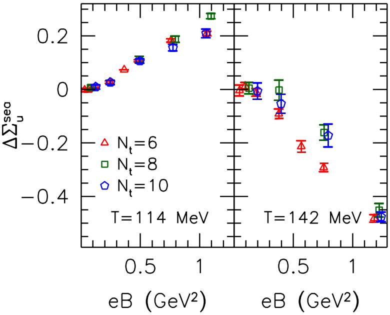
$$\bar{\psi}\psi^{\text{sea}} = \frac{\int DA_\mu e^{-S_g[A_\mu]} \det(\not{D}[A_\mu; B] + m)\, \text{tr}(\not{D}[A_\mu; 0\,] + m)^{-1}}{\int DA_\mu e^{-S_g[A_\mu]} \det(\not{D}[A_\mu; B] + m)}$$

$$\bar\psi\psi^{\text{full}} = \frac{\int DA_\mu e^{-S_g[A_\mu]} \det(\not{D}[A_\mu; B] + m)\, \text{tr}(\not{D}[A_\mu; B] + m)^{-1}}{\int DA_\mu e^{-S_g[A_\mu]} \det(\not{D}[A_\mu; B] + m)}$$

approx. summed up: $\bar\psi\psi^{\text{full}} \simeq \bar\psi\psi^{\text{val}} + \bar\psi\psi^{\text{sea}}$ D'Elia, Negro 11

- at low T and around T_c: FB, Endrődi, Kovács (in prep.)





valence: *B* generates condensates via low modes (catalysis)

sea: low modes disfavored, det = weight ! (quarks feed back)

Summary I

- magnetic catalysis: $\langle\bar{\psi}\psi\rangle(B) \nearrow$ at $T = 0$
- inv. magnetic catalysis: $\langle\bar{\psi}\psi\rangle(B) \searrow$ at $T \simeq T_c$
  - sea quark effect
  - only for light (phys.) quark masses
- QCD crossover: $T_c(B)$ decreases slightly

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---

second part: Is the QCD vacuum diamagnetic or paramagnetic?

Does the free energy increase/decrease with B?

$dF = -M\,dB$: do spins align antiparallel or parallel to B

from partition function:

$$e^{-F/T} = Z = \int DA\, e^{-S_{\text{gauge}}} \det(\not{D} + m)$$

reminder m-derivative and condensate:

$$\frac{\partial \log Z}{\partial m} \overset{\det = \exp \operatorname{tr} \log}{=} \left\langle \operatorname{tr} \frac{1}{\not{D} + m} 1 \right\rangle = \langle \bar{\psi}\psi \rangle$$

B-derivative:

$$\frac{\partial \log Z}{\partial (qB)} = \left\langle \operatorname{tr} \frac{1}{\not{D} + m} \frac{\partial \not{D}}{\partial (qB)} \right\rangle = -\frac{1}{2m} \left\langle \operatorname{tr} \frac{1}{\not{D} + m} \frac{\partial \not{D}^2}{\partial (qB)} \right\rangle$$

$$\Rightarrow -\frac{\partial F}{\partial (qB)} \overset{(1)}{\propto} \left\langle \operatorname{tr} \frac{1}{\not{D} + m} L_{12} \right\rangle + \left\langle \operatorname{tr} \frac{1}{\not{D} + m} \sigma_{12} \right\rangle \propto \ldots + \langle \bar{\psi} \sigma_{12} \psi \rangle$$

angular momentum + spin

$.._{12}$ allowed because $B = F_{12}$

Free quarks

$$\langle\bar{\psi}\sigma_{12}\psi\rangle = \frac{T}{V}\langle \mathrm{tr}\,\frac{\sigma_{12}}{\not{D}+m}\rangle = \frac{T}{V}m\,\langle \mathrm{tr}\,\frac{\sigma_{12}}{-\not{D}^2+m^2}\rangle$$

$$= \frac{m\,qB}{\pi}\int\frac{d^2p}{(2\pi)^2}\sum_{\substack{n=0,1,\dots\\2s=\pm1}}\frac{2s}{p^2+qB(2n+1)+qB2s+m^2}$$

$$= \frac{m\,qB}{\pi}\int\frac{d^2p}{(2\pi)^2}\frac{-1}{p^2+m^2} \qquad \text{only LLL survived!}$$

- negative
- linear in qB: degeneracy

  for LLL: 2d index predicts magn. flux ✓
- log. divergence $m\log(m/\Lambda) = m\log(ma)$

  not present in chiral limit

  removed through $1 - m\frac{\partial}{\partial m}$ and renorm. factor of tensor operators

  connected to charge renormalisation

Magnetic susceptibility

- χ and tensor coefficient τ from leading behavior: Ioffe, Smilga 84

$$\langle\bar{\psi}\sigma_{12}\psi\rangle \equiv qB\,\underbrace{\langle\bar{\psi}\psi\rangle\cdot\chi}_{\tau} + O((qB)^3)$$

  for each quark flavor

- χ relevant for radiative D_s meson transitions, anomalous magn. moment of the muon, chiral-odd photon distribution amplitudes
- in free energy:

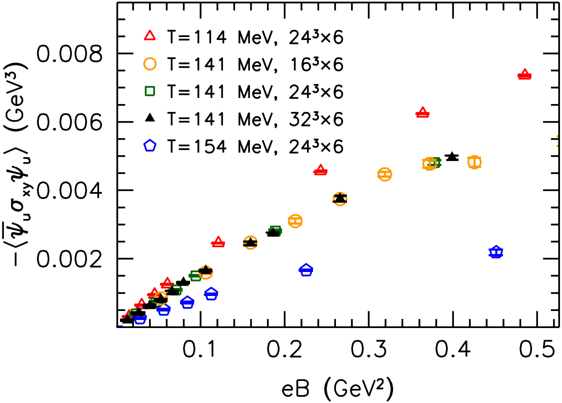
$$F \propto -\tau(qB)^2 + O((qB)^4)$$

  negative τ as we will find $\Rightarrow$ free energy increases with $B \Rightarrow$ diamagnetic contribution of spin

- technically: $\langle\bar{\psi}\sigma_{12}\psi\rangle$ is easily available in staggered formalism (the angular momentum term seems not)

Magnetic susceptibility from the lattice

- linear behaviour of $-\langle\bar{\psi}\sigma_{12}\psi\rangle$ in B: Bali, FB, Constantinou et al. 12

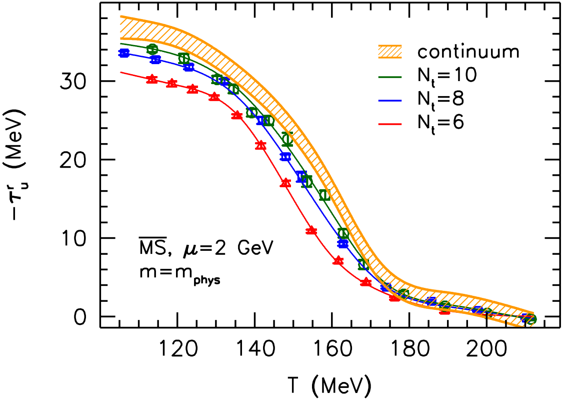


$\Rightarrow$ negative

$\Rightarrow$ decreasing slope for higher temperatures

Order parameter

- behaviour of tensor coefficient $\tau = \frac{\langle \bar{\psi}\sigma_{12}\psi \rangle}{qB}$ at different T's:



$\Rightarrow$ inflection point: $T_c = 162(3)(3)$ MeV

compatible with $T_c^{\langle \bar{\psi}\psi \rangle} = 159(3)(3)$ MeV at $B = 0$

Numbers

- at $T = 0$ and $\overline{\text{MS}}$ scheme at 2 GeV:

$$\tau_{\text{up}} = -(40.7 \pm 1.3)\ \text{MeV}$$
$$\tau_{\text{down}} = -(39.4 \pm 1.4)\ \text{MeV}$$
$$\tau_{\text{strange}} = -(53.0 \pm 7.2)\ \text{MeV}$$
$$\chi_{\text{up}} = -(2.08 \pm 0.08)\ \text{GeV}^{-2}$$
$$\chi_{\text{down}} = -(2.02 \pm 0.09)\ \text{GeV}^{-2}$$

quenched unrenorm.: $\tau_{\text{up/down}} = -52$ MeV   Braguta, Buividovich et al. 2010

QCD sum rules: $\chi_{\text{light}} = -(2.11 \pm 0.23)\ \text{GeV}^{-2}$ Ball, Braun, Kivel 2003

vector dominance: $\chi_{\text{light}} = -\frac{2}{m_\rho^2} \approx -3.3\ \text{GeV}^{-2}$

Summary II

spin polarization $\langle\bar{\psi}\sigma_{12}\psi\rangle$:

- exists due to anisotropy $F_{12}^{\text{ext}} = B$
- $\propto qB$ already for free quarks

  divergence: removed
- coefficient τ is an order parameter
- contributes to free energy: spin diamagnetism

  condensed matter, ideal Fermi gas:

  spin paramagn. & angular diamagn., 3 : -1 — Landau 30

  here we discuss the (QCD) vacuum:

  related to asymptotic freedom — Nielsen 81

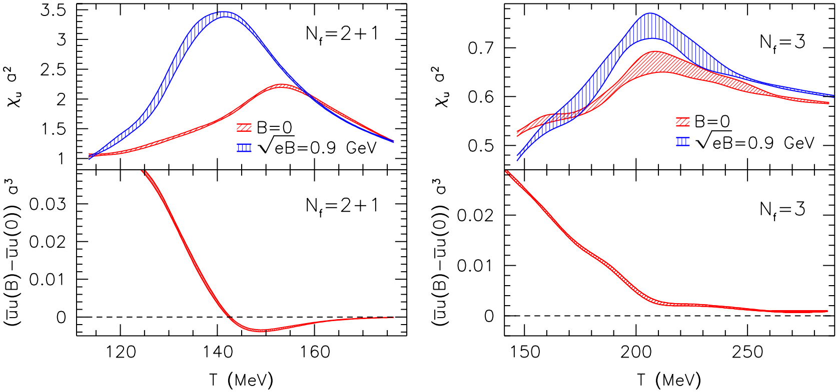
Backup: Simulation details

as for transition studies at $B = 0$ — Budapest-Wuppertal

- tree-level improved gauge action
- stout smeared staggered fermions (rooting trick)
- 2 light quarks + strange quark, charges $(q_u, q_d, q_s) = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})e$
- lattice spacing set at $T = 0, B = 0$
  physical pion masses
  set by f_K, f_K/m_π and f_K/m_K
- $T = 0$: $24^3 \times 32$, $32^3 \times 48$ and $40^3 \times 48$ lattices
- $T > 0$: $N_t = 6, 8, 10$ meaning $a = 0.2, 0.15, 0.12$ fm
  $N_s = 16, 24, 32$ for finite volumes
- condensates from stochastic estimator method with 40 vectors
- magn. flux quanta: $N_B \leq 70 < \frac{N_x N_y}{4} = 144$

Backup: Mass sensitivity

- what if we put $(m_{\text{light}}, m_{\text{light}}, m_{\text{strange}}) \to (m_{\text{strange}}, m_{\text{strange}}, m_{\text{strange}})$?



T-dep. of *u*-susceptibility (top) and change of *u*-condensate (bottom)

$\Rightarrow$ effects of decreasing T_c & inverse magn. catalysis disappear

light quark masses are important