Spontaneous supersymmetry breaking on the lattice

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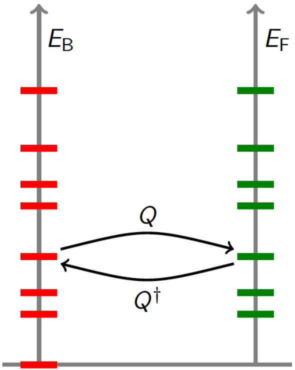
Outline

- Spontaneous supersymmetry breaking:
  - Witten index and sign problem
- New approach for simulating fermions on the lattice:
  - Loop formulation for Majorana Wilson fermions
  - Solution of the sign problem
- Two examples:
  - $\mathcal{N} = 2$ supersymmetric QM
  - $\mathcal{N} = 1$ Wess-Zumino model in $d = 2$

Supersymmetry and its breaking

- Unbroken supersymmetry:
  - Vanishing ground state energy
  - Degenerate mass spectrum
- Broken supersymmetry:
  - No supersymmetric ground state
  - Particle masses not degenerate
  - Emergence of Goldstino mode



Spontaneous SUSY breaking (SSB) and the Witten index

- Witten index provides a necessary but not sufficient condition for SSB:

$$W \equiv \lim_{\beta\to\infty} \mathrm{Tr}(-1)^F \exp(-\beta H) \quad \Rightarrow \begin{cases} = 0 & \text{SSB may occur} \\ \neq 0 & \text{no SSB} \end{cases}$$

- Index counts the difference between the number of bosonic and fermionic zero energy states:

$$W \equiv \lim_{\beta\to\infty} \left[\mathrm{Tr}_B \exp(-\beta H) - \mathrm{Tr}_F \exp(-\beta H)\right] = n_B - n_F$$

- Index is equivalent to partition function with periodic b.c.:

$$W = \int_{-\infty}^{\infty} \mathcal{D}\phi \ \det\left[\not{D}(\phi)\right] \, e^{-S_B[\phi]} = Z_\mathrm{p}$$

$\Rightarrow$ Determinant (or Pfaffian) must be indefinite for SSB.

Example: $\mathcal{N}=2$ SUSY QM

- Consider the Lagrangian for $\mathcal{N}=2$ supersymmetric quantum mechanics

$$\mathcal{L} = \frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 + \frac{1}{2}P'(\phi)^2 + \overline{\psi}\left(\frac{d}{dt} + P''(\phi)\right)\psi\,,$$

  - real commuting bosonic 'coordinate' ϕ,
  - complex anticommuting fermionic 'coordinate' ψ,
  - superpotential $P(\phi)$

- Two supersymmetries in terms of Majorana fields $\psi_{1,2}$:

$$\begin{aligned}
\delta_A\phi &= \psi_1\varepsilon_A, & \delta_B\phi &= \psi_2\varepsilon_B,\\
\delta_A\psi_1 &= \tfrac{d\phi}{dt}\varepsilon_A, & \delta_B\psi_1 &= -iP'\varepsilon_B,\\
\delta_A\psi_2 &= iP'\varepsilon_A, & \delta_B\psi_2 &= \tfrac{d\phi}{dt}\varepsilon_B.
\end{aligned}$$

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Example: $\mathcal{N}=2$ SUSY QM

- Integrating out the fermion fields yields (indefinite) determinant

$$\int \mathcal{D}\overline{\psi}\,\mathcal{D}\psi \exp\left(-\overline{\psi}D(\phi)\psi\right) = \det D(\phi)$$

- The (regulated) fermion determinant can be calculated exactly:

$$\det\left[\frac{\partial_t + P''(\phi)}{\partial_t + m}\right] = \sinh\int_0^T \frac{P''(\phi)}{2}dt \quad\Longrightarrow\quad Z_0 - Z_1$$

- If under some symmetry $\phi \leftrightarrow \tilde{\phi}$ of $S_B(\phi)$ we have

$$\int_0^T \frac{P''(\tilde{\phi})}{2}dt = -\int_0^T \frac{P''(\phi)}{2}dt \quad\Longrightarrow\quad Z_0 = Z_1 \text{ and SSB}$$

Spontaneous SUSY breaking (SSB) and the sign problem

- On the lattice we find with Wilson type fermions

$$\det\left[\nabla^* + P''(\phi)\right] = \prod_t \left[1 + P''(\phi_t)\right] - 1\,.$$

- For odd potentials, e.g. $P(\phi) = \frac{m^2}{2\lambda}\,\phi + \frac{1}{3}\lambda\phi^3$ we have

$$\det\left[\nabla^* + P''\right] = \prod_t \left[1 + 2\lambda\phi_t\right] - 1$$

no longer positive... $\Rightarrow$ sign problem!

- Every supersymmetric model which allows SSB must have a sign problem:
  - SUSY QM with odd potential,
  - $\mathcal{N}=16$ Yang-Mills quantum mechanics [Catterall, Wiseman '07],
  - $\mathcal{N}=1$ Wess-Zumino model in 2D [Catterall '03; Wipf, Wozar '11],
  - ...

Solution of the sign problem

- We propose a (novel) approach circumventing these problems:

  $\Rightarrow$ fermion loop formulation.

- Alternative way of simulating fermions on the lattice:
  - based on the exact hopping expansion of the fermion action,
  - eliminates critical slowing down,
  - allows simulations directly in the massless limit,

  $\Rightarrow$ solves the fermion sign problem.

- Applicable to Wilson fermions in the
  - $O(N)$ Gross-Neveu model in $d = 2$ dimensions,
  - Schwinger model in the strong coupling limit in $d = 2$ and 3,
  - SUSY QM,
  - $\mathcal{N} = 1$ and 2 supersymmetric Wess-Zumino model,
  - supersymmetric matrix QM.

Exact hopping expansion for Wilson fermions

- Fermionic part of $\mathcal{N} = 2$ SUSY QM,

$$\mathcal{L} = \overline{\psi}(\partial_t + P''(\phi))\psi \,.$$

- Using Wilson lattice discretisation yields backward derivative

$$\nabla^* \psi(t) = \psi(t) - \psi(t-a)$$

  and eliminates fermion doubling in 1D.

- Using the nilpotency of Grassmann elements we expand the Boltzmann factor

$$\int \mathcal{D}\overline{\psi}\,\mathcal{D}\psi \prod_t \left(1 - M(t)\overline{\psi}(t)\psi(t)\right) \prod_t \left(1 + \overline{\psi}(t)\psi(t-a)\right)$$

  where $M(t) = 1 + P''(\phi(t))$.

Exact hopping expansion for Wilson fermions

- At each site t, the fields $\overline{\psi}$ and ψ must be exactly paired to give a contribution to the path integral:

$$\int \mathcal{D}\overline{\psi}\,\mathcal{D}\psi \prod_t \left(-M(t)\overline{\psi}(t)\psi(t)\right)^{m(t)} \prod_t \left(\overline{\psi}(t)\psi(t-a)\right)^{n_f(t)}$$

  with occupation numbers
  - $m(t) = 0, 1$ for monomers,
  - $n_f(t) = 0, 1$ for fermion bonds (or dimers),

  satisfying the constraint

$$m(t) + \frac{1}{2}\left(n_f(t) + n_f(t-a)\right) = 1.$$

- Only closed paths survive the integration.

Exact hopping expansion for scalar fields

- Analogous treatment for the bosonic field [Prokof'ev, Svistunov '01]:
  - $(\partial_t \phi)^2 \quad \rightarrow \quad \phi_{t+a}\phi_t,$
  - expand hopping term $\exp\{-\phi_{t+a}\phi_t\}$ to all orders:

$$\int \mathcal{D}\phi \prod_t \sum_{n_b(t)} \frac{1}{n_b(t)!} \left(\phi_t \phi_{t+a}\right)^{n_b(t)} \exp\left(-V(\phi_t)\right) M(\phi_t)^{m(t)}$$

with bosonic bond occupation numbers $n_b(t) = 0, 1, 2, \ldots$

- Integrating out $\phi(t)$ yields bosonic site weights

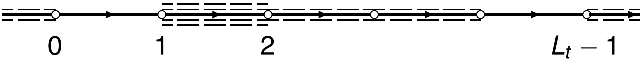
$$Q(N) = \int d\phi \; \phi^N \exp\left(-V(\phi)\right)$$

where N includes powers from $M(\phi)$.

Loop formulation of SUSY QM

- Loop representation in terms of fermionic monomers and dimers and bosonic bonds.
- Especially simple for supersymmetric QM:

$$\begin{aligned} \{n_f = 0, m = 1\} &\quad\Rightarrow\quad \text{no fermion, bosonic vacuum} \\ \{n_f = 1, m = 0\} &\quad\Rightarrow\quad \text{fermion present, fermionic vacuum} \end{aligned}$$

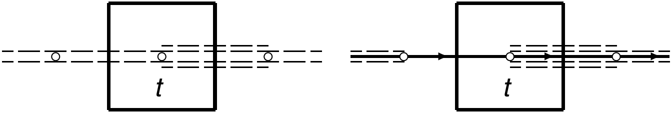


- Partition functions sum over all bond configurations:

$$\begin{aligned} Z_p &= Z_0 - Z_1 \quad\Rightarrow\text{Witten index} \\ Z_a &= Z_0 + Z_1 \quad\Rightarrow\text{finite temperature} \end{aligned}$$

Exact calculation using a transfer matrix

- Loop representation allows the construction of a transfer matrix.
- Each state on the link is characterised by the bond occupation numbers, i.e. $|n_f, n_b\rangle$,



- $T_{n';n}$ takes the system from state $|n_f, n_b\rangle$ to state $|n'_f, n'_b\rangle$.
- $T_{n';n}$ block diagonalises into $T^{n_f=0}_{n'_b;n_b}$ and $T^{n_f=1}_{n'_b;n_b}$.

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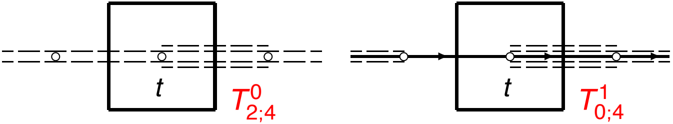

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Exact calculation using a transfer matrix

- Specifically,

$$T^1_{n',n} = \frac{Q(n'+n)}{\sqrt{n'!\,n!}} \quad \text{with} \quad Q(N) = \int d\phi\ \phi^N \exp\left(-V(\phi)\right)$$

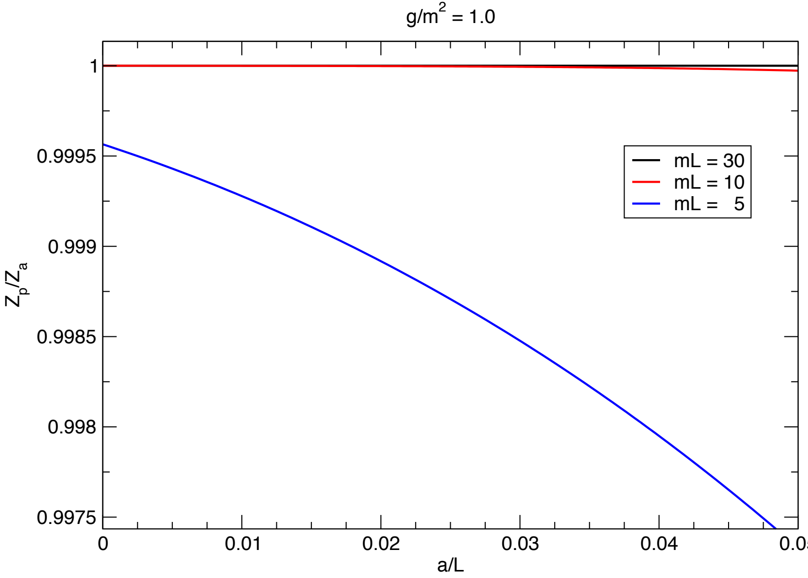
  and T^0 analogously.

- Exact lattice partition functions given by $Z_{n_f} = \mathrm{Tr}\left[\left(T^{n_f}\right)^{L_t}\right]$, Witten index by $W \equiv Z_p = Z_0 - Z_1$.

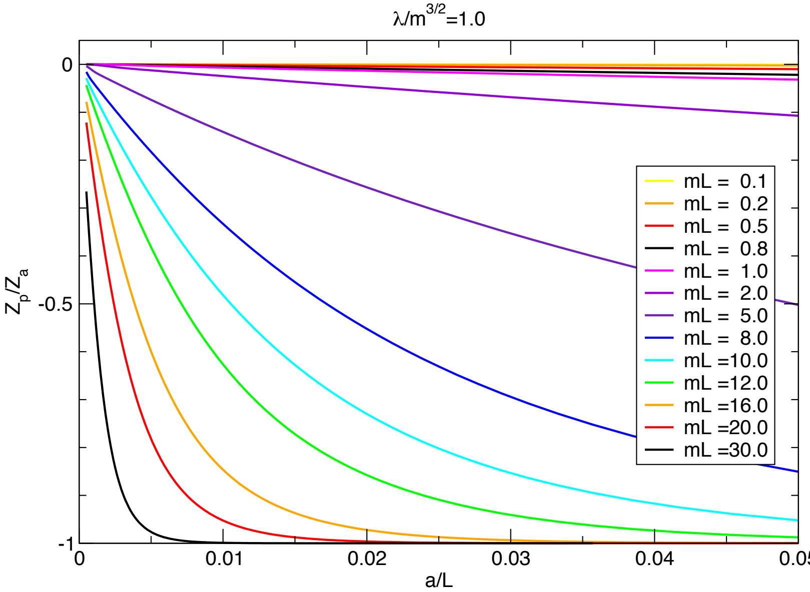
- Mass gaps from ratios of eigenvalues of T^0 and T^1.

- Exact results for other observables (n-point functions, etc.)

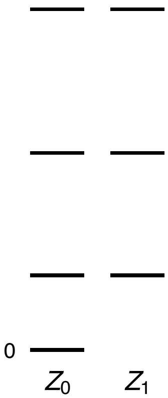
Witten index for unbroken supersymmetry, $P = \frac{1}{2}m\phi^2 + \frac{1}{4}g\phi^4$

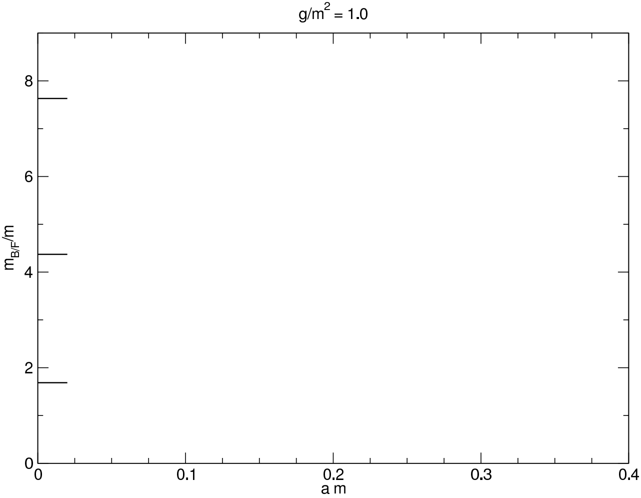


Witten index for broken supersymmetry, $P=-\frac{1}{2\lambda}m^2\phi+\frac{1}{3}\lambda\phi^3$



Mass gaps for unbroken SUSY





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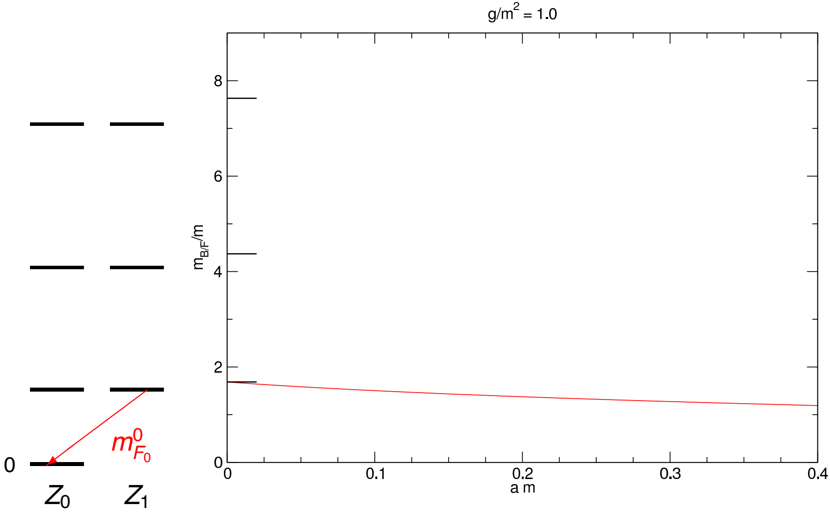

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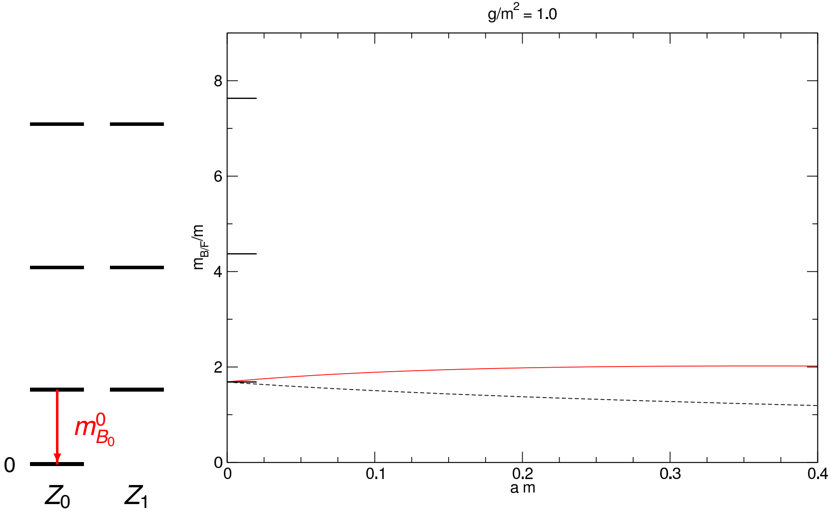

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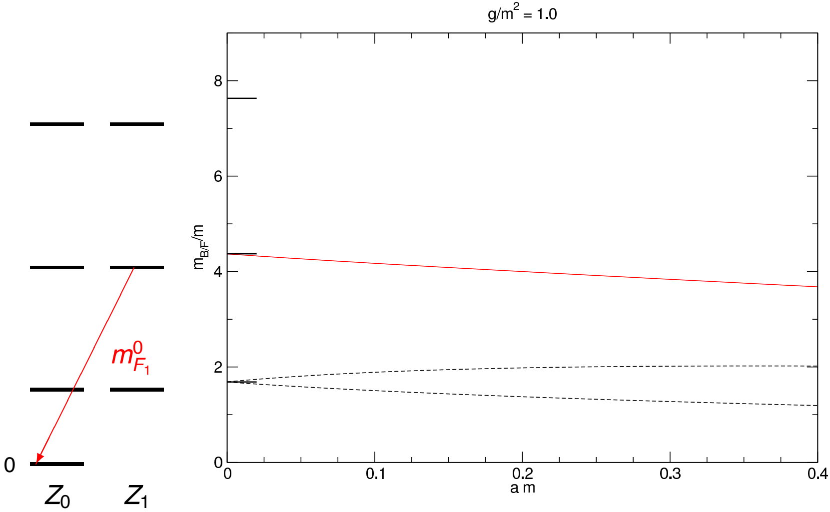

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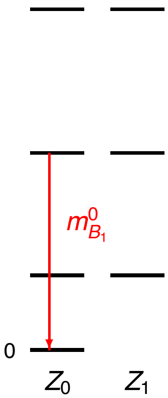

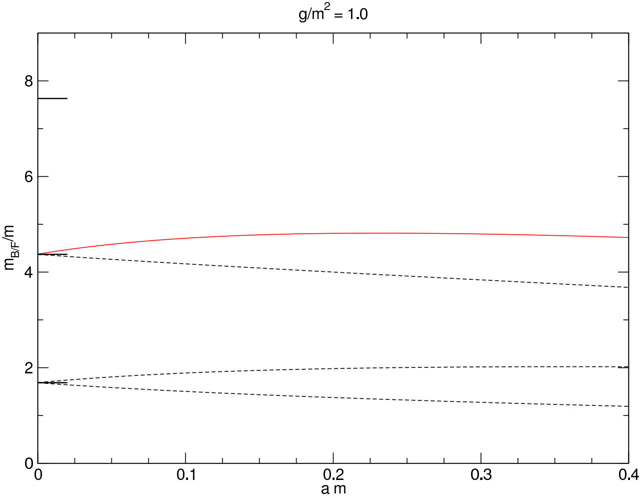

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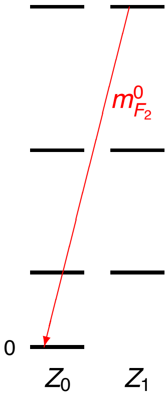

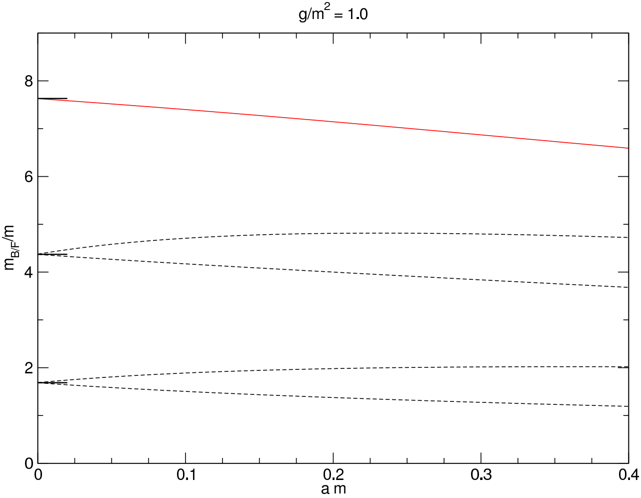

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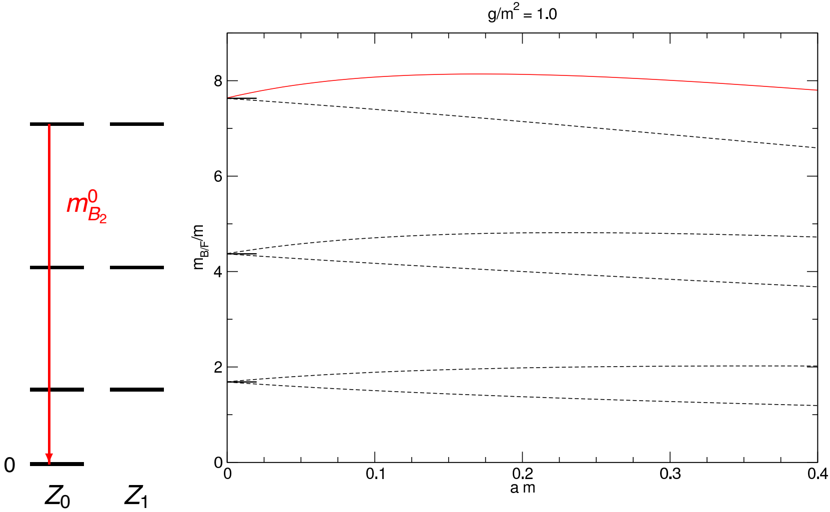

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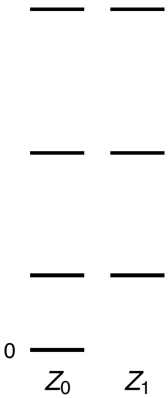

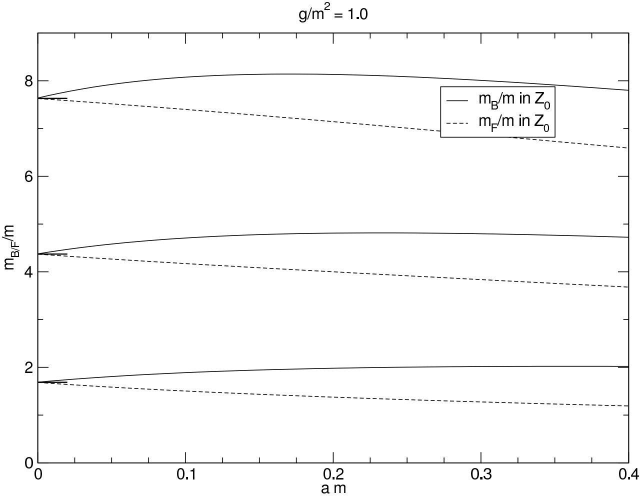

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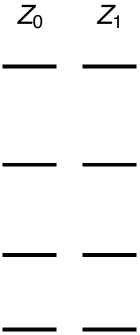
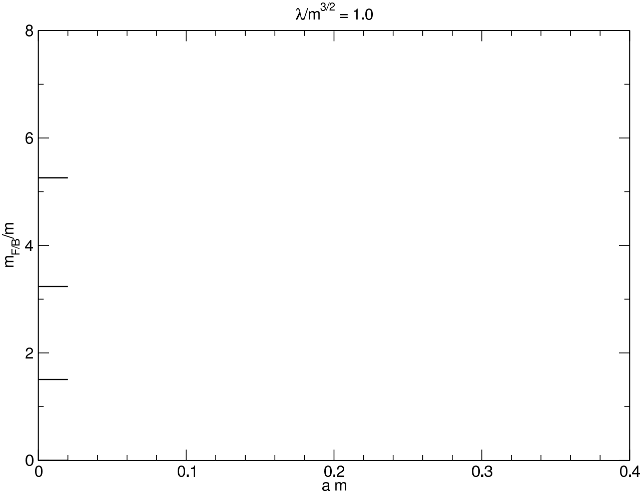


$E_0 = 0.1685$



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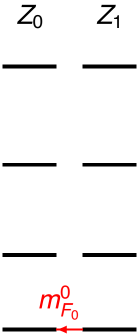

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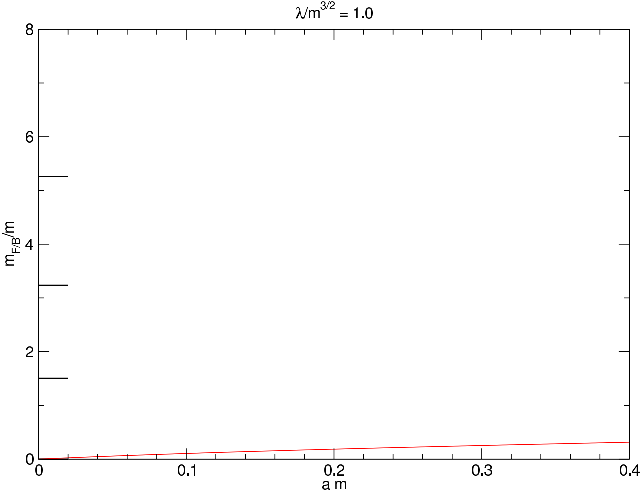

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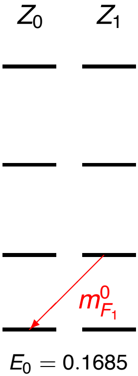

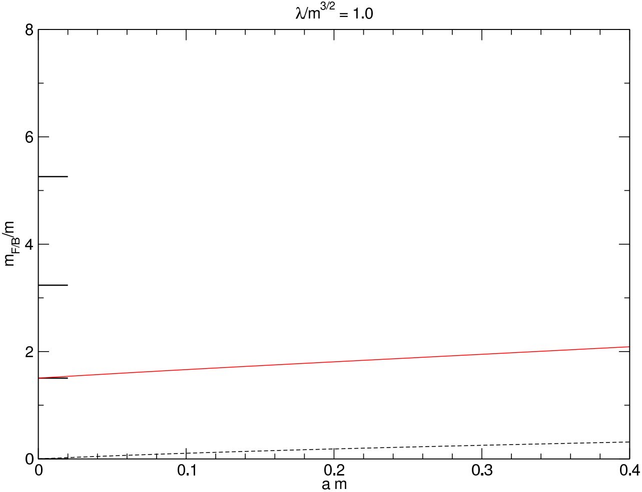

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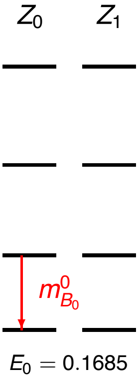

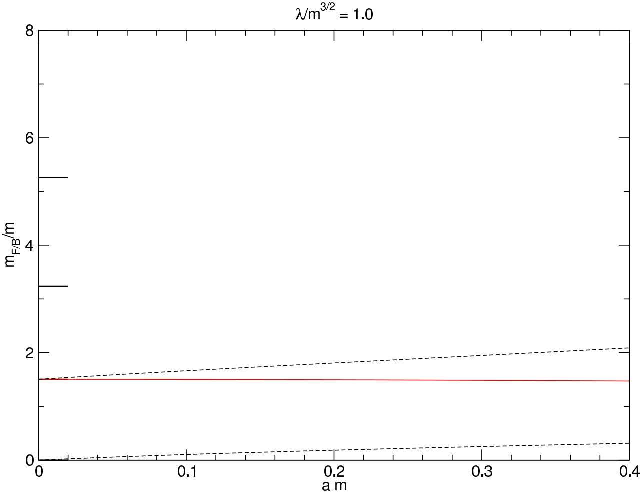

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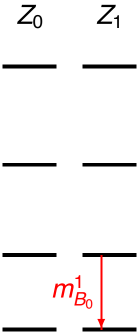

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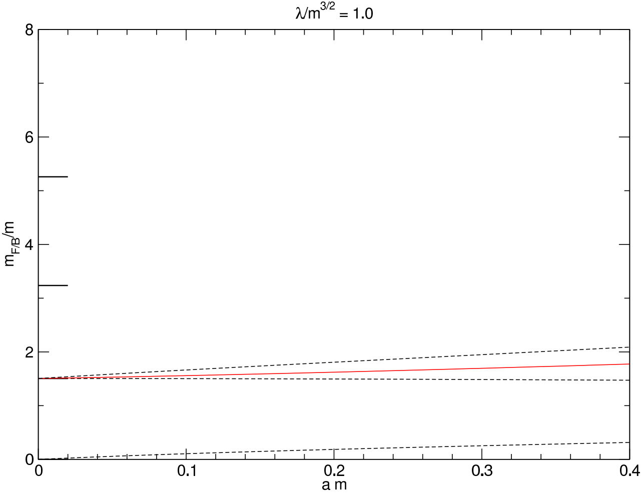

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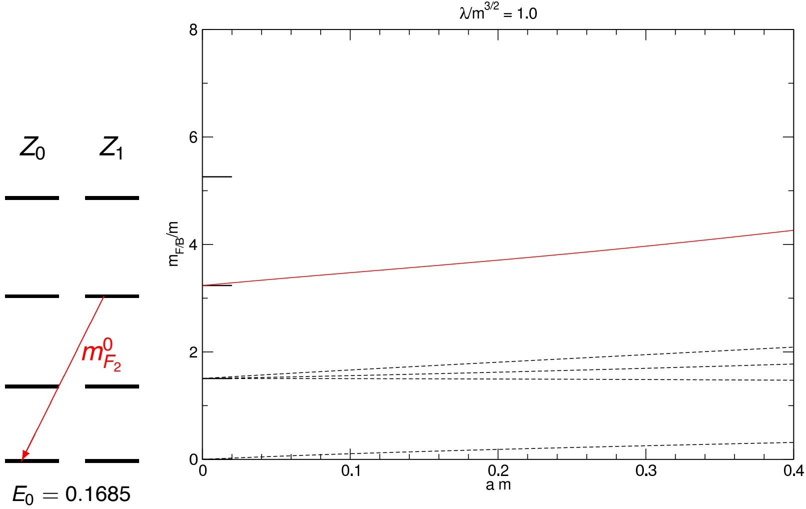

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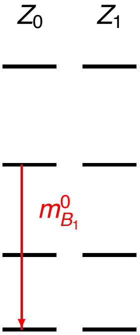

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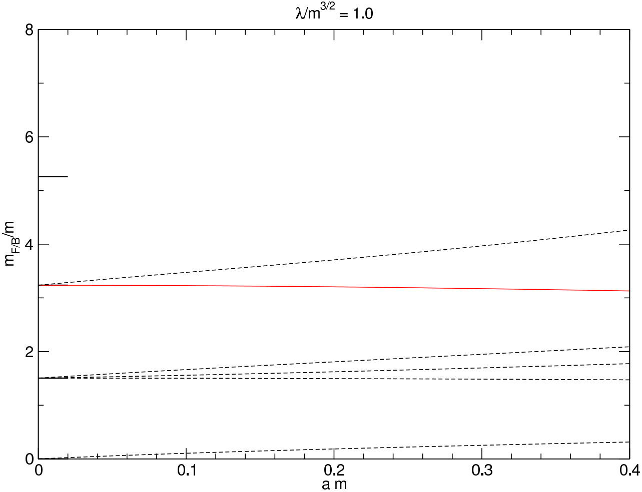

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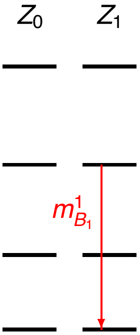

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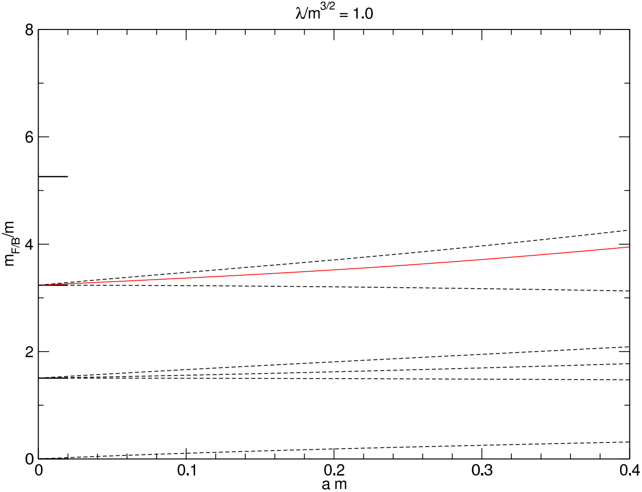

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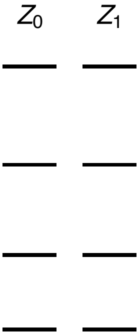

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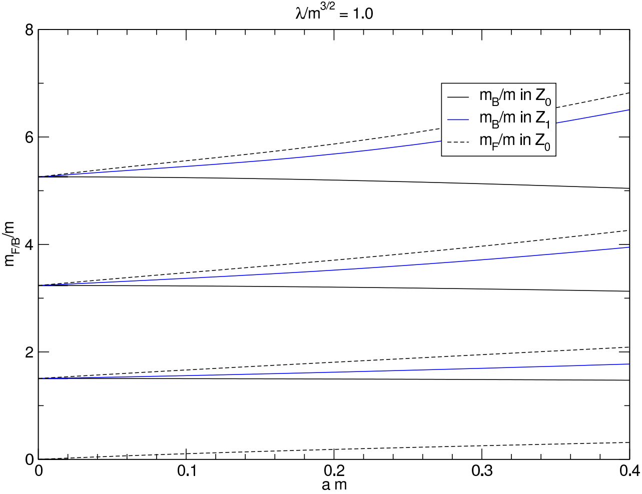
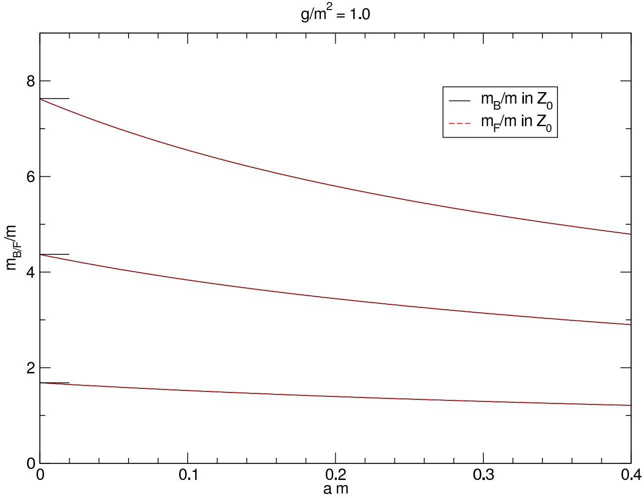


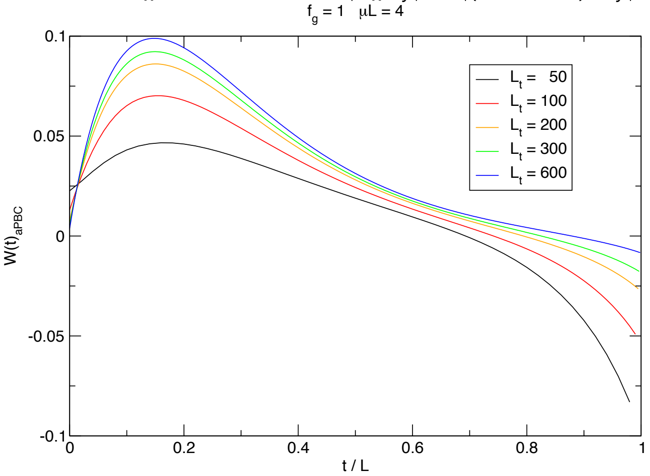
Mass gaps for unbroken SUSY, Q_1-exact action (twisted SUSY)



Ward identity for unbroken SUSY

Ward identity $\langle \delta\mathcal{O} \rangle = \langle \mathcal{O}\delta S \rangle$:

$$\mathcal{O} = \overline{\psi}_x \phi_x \quad \rightarrow \quad W = \langle \overline{\psi}_x \psi_y \rangle + \langle (\Delta^- + P')_x \phi_y \rangle$$



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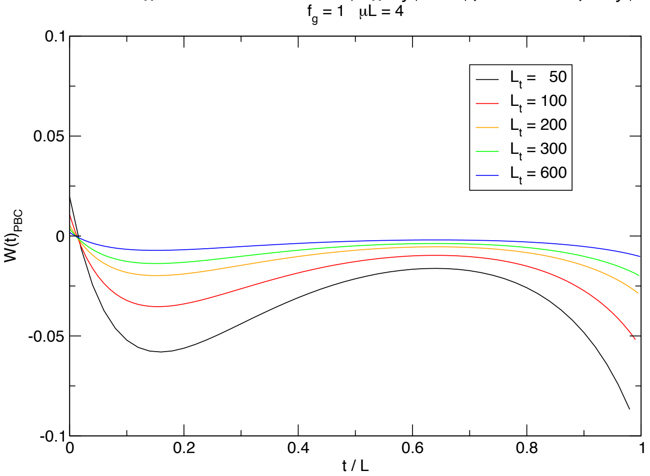

Ward identity for unbroken SUSY, continuum limit

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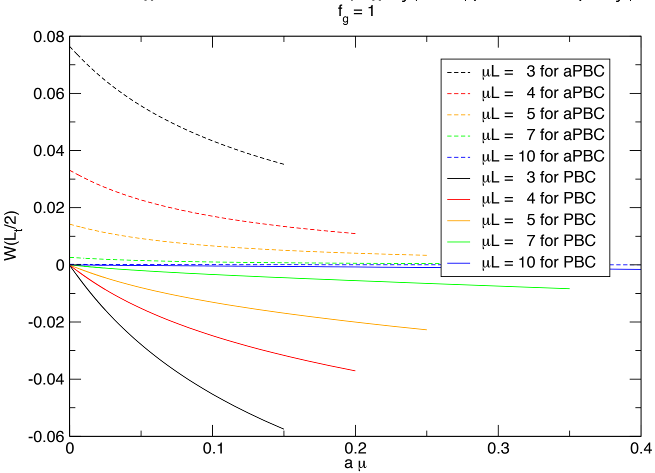

Ward identity for unbroken SUSY, Q-exact action

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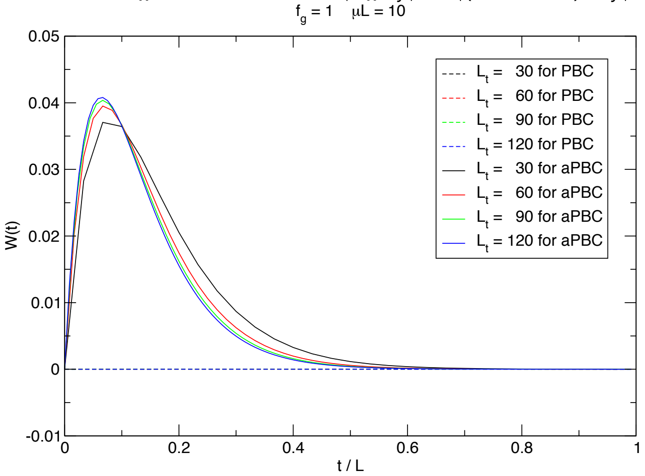

Ward identity for unbroken SUSY, *Q*-exact action, continuum limit

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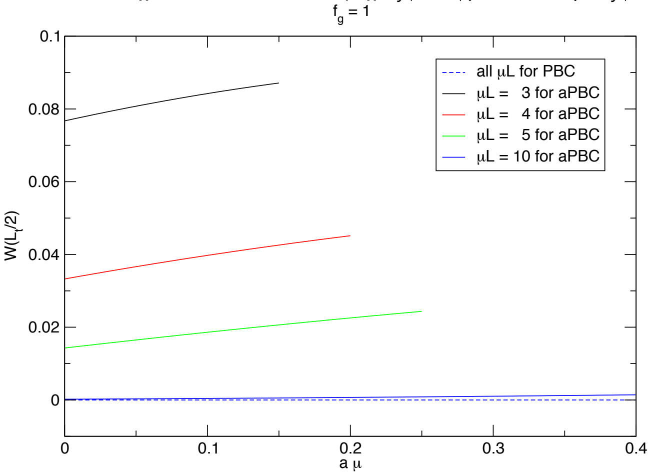
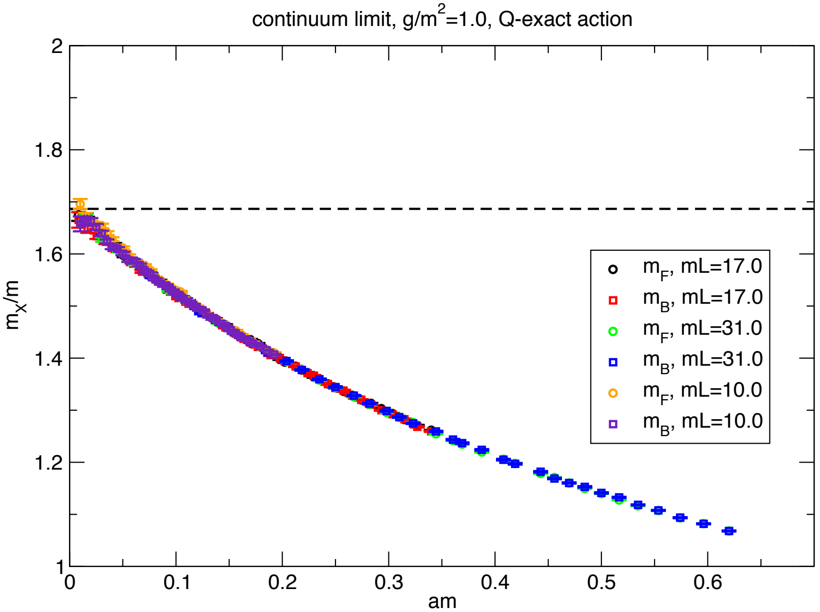


MC simulation of fermion loops: proof of concept



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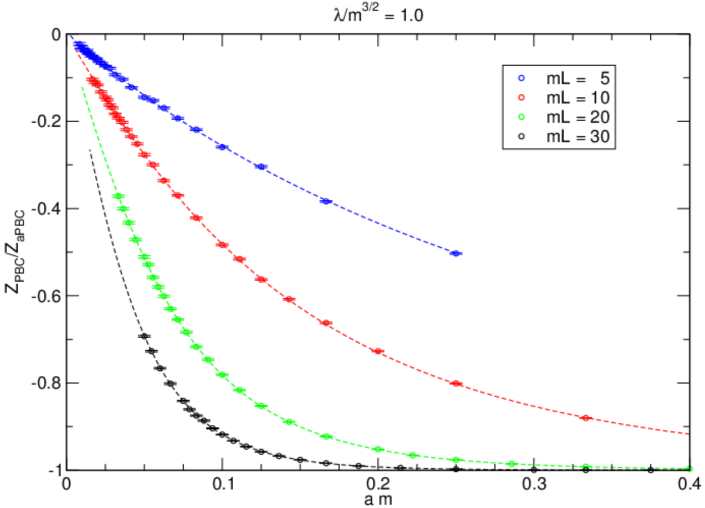


$\mathcal{N}=1$ Wess-Zumino model

- The $\mathcal{N}=1$ Wess-Zumino model in 2D:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}P'(\phi)^2 + \frac{1}{2}\overline{\psi}\left(\partial\!\!\!/ + P''(\phi)\right)\psi$$

  - with ψ a two component Majorana field,
  - and ϕ real bosonic field,
  - superpotential, e.g. $P(\phi) = -\frac{m^2}{4g}\phi + \frac{1}{3}g\phi^3$.

- Symmetries:
  - Supersymmetry

$$\begin{aligned} \delta\phi &= \bar{\epsilon}\psi \\ \delta\psi &= (\partial\!\!\!/\phi - P')\epsilon \\ \delta\overline{\psi} &= 0 \end{aligned}$$

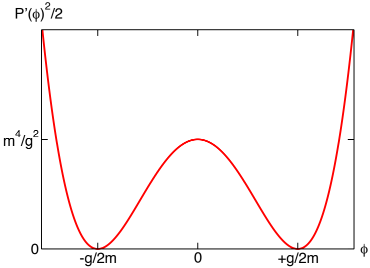
  - $\mathbb{Z}(2)$ chiral symmetry

$$\begin{aligned} \phi &\to -\phi \\ \psi &\to \gamma_5\psi \\ \overline{\psi} &\to -\overline{\psi}\gamma_5 \end{aligned}$$

Classical potential

The scalar potential is a standard ϕ^4 theory:

$$\frac{1}{2}P'(\phi)^2 = -\frac{1}{2}\frac{m^2}{2}\phi^2 + \frac{1}{2}g^2\phi^4 + \text{const}$$



- $\mathbb{Z}(2)$ symmetry may be broken.

- Witten index is vanishing ($W = 0$) with one bosonic and one fermionic ground state.

- Integrating out Majorana fermions yields indefinite Pfaffian.

$\mathbb{Z}(2)$ and supersymmetry breaking

- For large $\frac{m}{g}$ the $\mathbb{Z}(2)$ symmetry is spontaneously broken, $\overline{\phi} = \pm m/2g$ selects a definite ground state:

$$\begin{aligned} \overline{\phi} = +m/2g \quad &\Longrightarrow \quad \text{bosonic} \\ \overline{\phi} = -m/2g \quad &\Longrightarrow \quad \text{fermionic} \end{aligned}$$

$\Rightarrow$ SUSY unbroken

- For small $\frac{m}{g}$ the $\mathbb{Z}(2)$ symmetry is unbroken, $\overline{\phi} = 0$ allows both bosonic and fermionic ground state:

$\Rightarrow$ SUSY broken

Tunneling between the two ground states:

$\Rightarrow$ Goldstino mode, Pf $M_{pp} = 0$

$\mathcal{N}=1$ Wess-Zumino model on the lattice

- Using Wilson lattice discretisation for the fermionic part

$$\mathcal{L} = \frac{1}{2}\xi^T \mathcal{C}(\gamma_\mu \tilde{\partial}_\mu - \frac{1}{2}\partial^*\partial + P''(\phi))\xi\,,$$

and expanding the Boltzmann factor,

$$\int \mathcal{D}\xi \prod_x \left(-M(x)\xi^T(x)\mathcal{C}\xi(x)\right)^{m(x)} \prod_{x,\mu} \left(\xi^T(x)\mathcal{C}\Gamma(\mu)\xi(x+\hat{\mu})\right)^{b_\mu(x)}$$

where $M(x) = 2 + P''(\phi)$.

- $m(x) = 0, 1$ and $b_\mu(x) = 0, 1$ satisfy the constraint

$$m(x) + \frac{1}{2}\sum_\mu b_\mu(x) = 1.$$

- Only closed, non-intersecting paths survive the integration.

Fermion loop formulation

- Loop representation in terms of fermionic monomers and dimers and bosonic fields ϕ.
- Partition function becomes a sum over all non-oriented, self-avoiding fermion loops ℓ

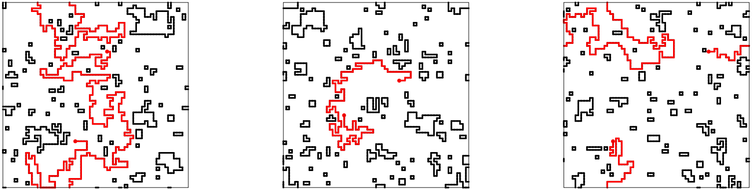
$$Z_{\mathcal{L}} = \int \mathcal{D}\phi \sum_{\{\ell\}\in\mathcal{L}} \omega[\ell,\phi], \quad \mathcal{L} \in \mathcal{L}_{00} \cup \mathcal{L}_{10} \cup \mathcal{L}_{01} \cup \mathcal{L}_{11}$$
$$= Z_{\mathcal{L}_{00}} + Z_{\mathcal{L}_{10}} + Z_{\mathcal{L}_{01}} + Z_{\mathcal{L}_{11}}$$

  Represents system with unspecified fermionic b.c. [Wolff '07].

- Simulate fermions by enlarging the configuration space by one open fermionic string [Wenger '08].

Reconstructing the fermionic boundary conditions

- The open fermionic string corresponds to the insertion of a Majorana fermion pair $\{\xi^T(x)\mathcal{C}, \xi(y)\}$ at position x and y:



- It samples the relative weights between $Z_{\mathcal{L}_{00}}, Z_{\mathcal{L}_{10}}, Z_{\mathcal{L}_{01}}, Z_{\mathcal{L}_{11}}$.
- Reconstruct the Witten index a posteriori

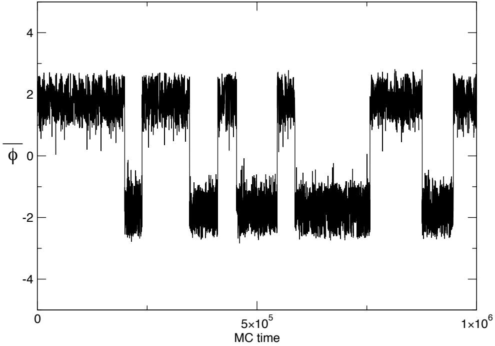
$$W \equiv Z^{pp} = Z_{\mathcal{L}_{00}} - Z_{\mathcal{L}_{10}} - Z_{\mathcal{L}_{01}} - Z_{\mathcal{L}_{11}},$$

or a system at finite temperature

$$Z^{pa} = Z_{\mathcal{L}_{00}} - Z_{\mathcal{L}_{10}} + Z_{\mathcal{L}_{01}} + Z_{\mathcal{L}_{11}}.$$

$\mathbb{Z}(2)$ broken phase

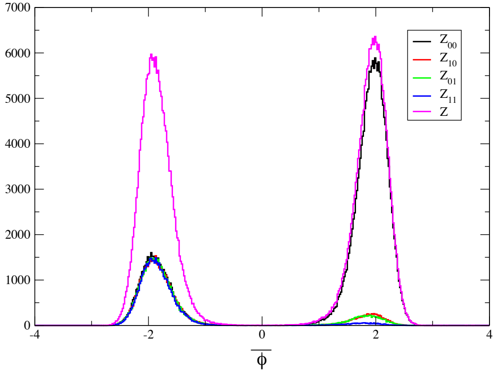
$\mathbb{Z}(2)$ broken phase with $\langle|\overline{\phi}|\rangle \simeq 2$ at $m/g = 4$



$\Rightarrow$ tunneling suppressed for $L \to \infty$ or $m/g \to \infty$

$\mathbb{Z}(2)$ broken phase

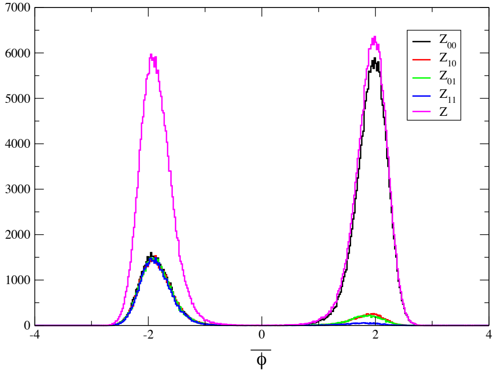
$\mathbb{Z}(2)$ broken phase with $\langle|\overline{\phi}|\rangle \simeq 2$ at $m/g = 4$



$$\langle\overline{\phi}\rangle \simeq -2: \quad Z_{00} \simeq Z_{10} \simeq Z_{01} \simeq Z_{11} \quad \Rightarrow \quad Z_{pp} \simeq -Z_{pa}$$
$$\langle\overline{\phi}\rangle \simeq +2: \quad Z_{00} \simeq 1, Z_{10} \simeq Z_{01} \simeq Z_{11} \simeq 0 \quad \Rightarrow \quad Z_{pp} \simeq Z_{pa}$$

$\mathbb{Z}(2)$ broken phase

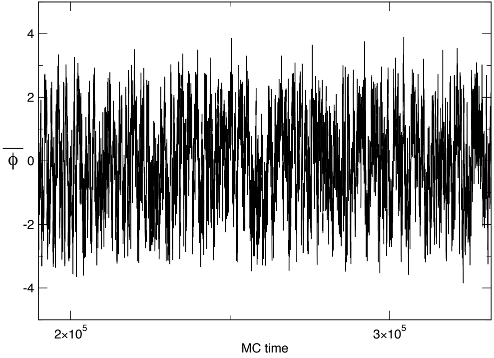
$\mathbb{Z}(2)$ broken phase with $\langle|\overline{\phi}|\rangle \simeq 2$ at $m/g = 4$



supersymmetry restored (at least in the limit $L \to \infty$)

$\mathbb{Z}(2)$ symmetric phase

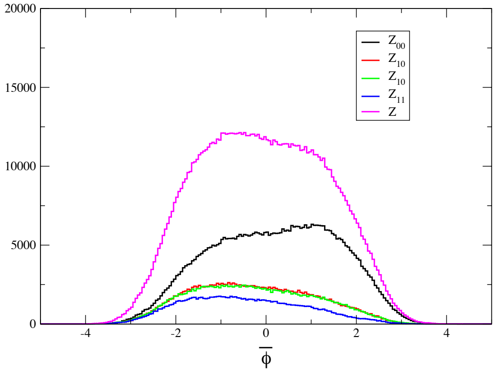
$\mathbb{Z}(2)$ symmetric phase with $\langle\overline{\phi}\rangle \simeq 0$ at $m/g = 0.16$



$\Rightarrow \mathbb{Z}(2)$ exact only in the limit $a \to 0$

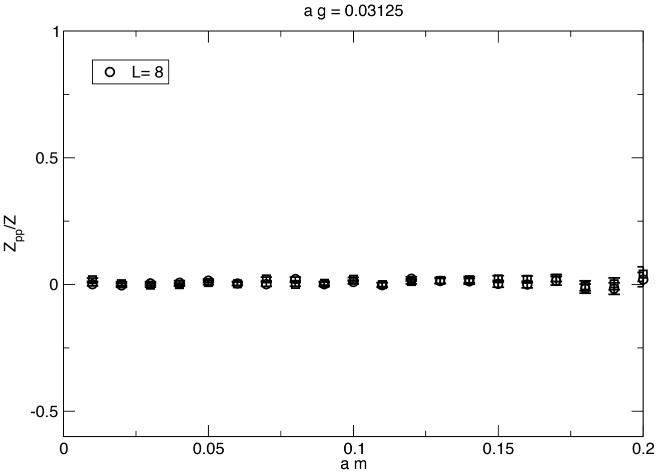
$\mathbb{Z}(2)$ symmetric phase

$\mathbb{Z}(2)$ symı

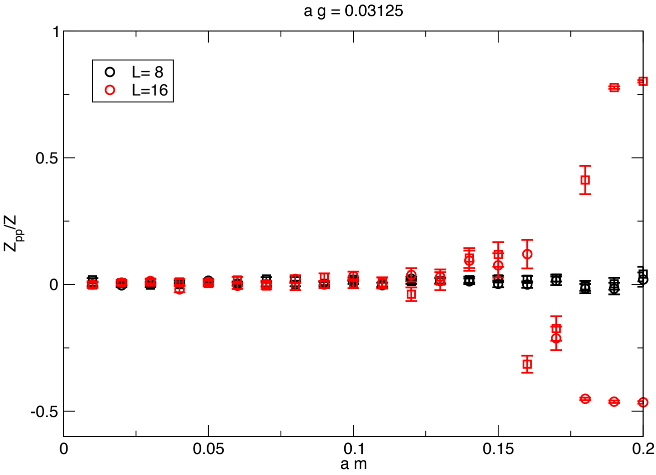


- $\langle\bar{\phi}\rangle \simeq 0 \quad \Rightarrow \quad Z_{00} \simeq Z_{10} + Z_{01} + Z_{11} \quad \Rightarrow \quad Z_{pp} \simeq 0$
- supersymmetry broken

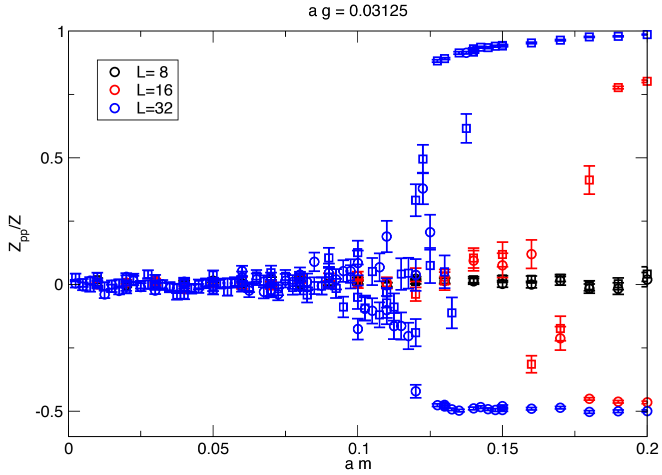
Scanning for the transition



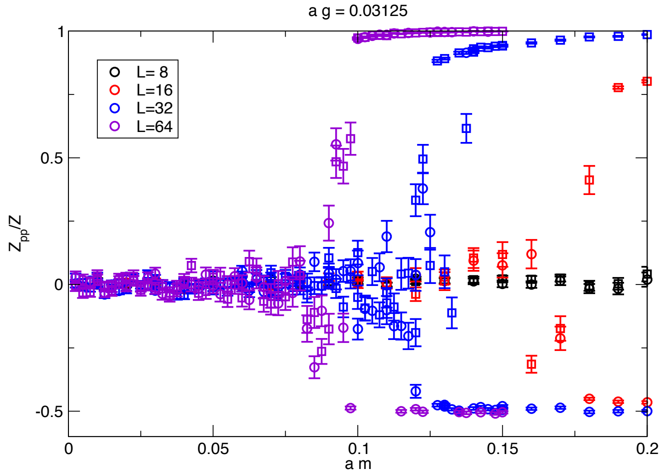
Scanning for the transition



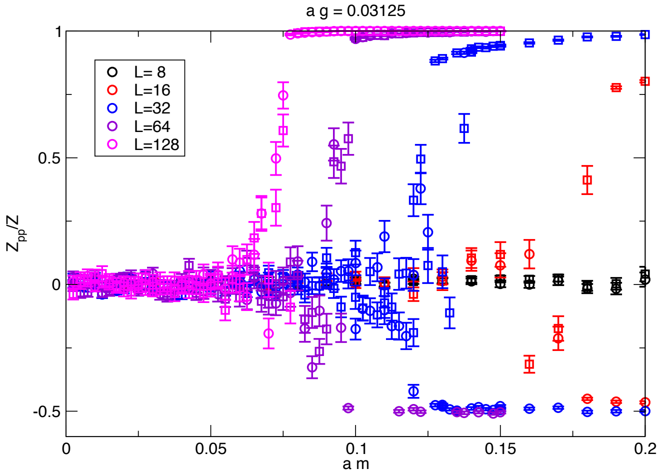
Scanning for the transition



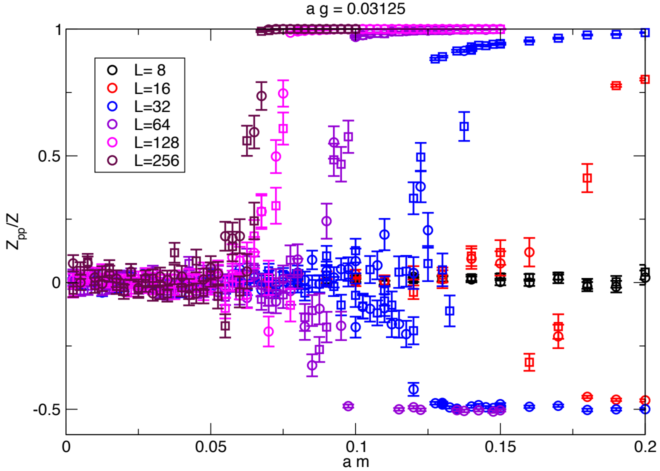
Scanning for the transition



Scanning for the transition

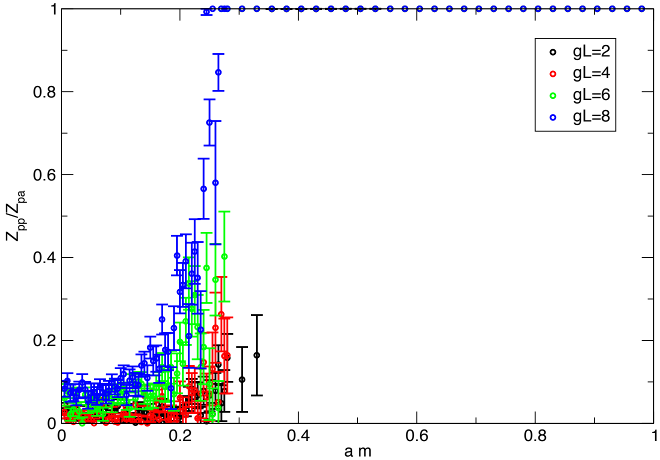


Supersymmetry breaking phase transition



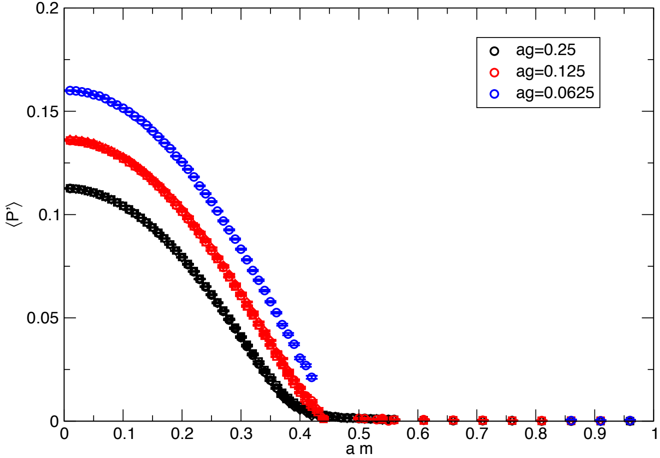
- bosonic vacuum preferred ($\mathbb{Z}(2)_\chi$ broken at $a \neq 0$)
- $am_c \simeq 0.05$ for $L \to \infty$

Supersymmetry breaking phase transition



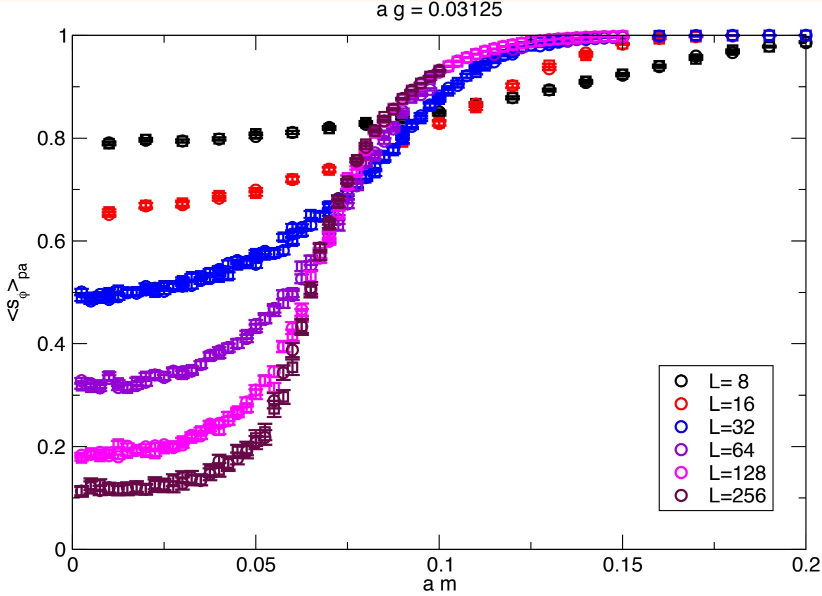
- Witten index $W \propto Z_{pp}/Z_{pa}$ as an order parameter

Supersymmetry breaking phase transition

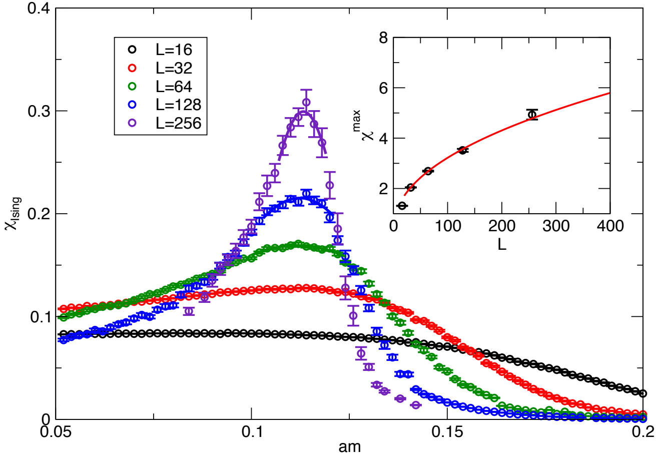


- Ward identity $\langle P' \rangle$ as an order parameter, volumes $gL = 2, 4, 6, 8$

$\mathbb{Z}(2)$ breaking phase transition

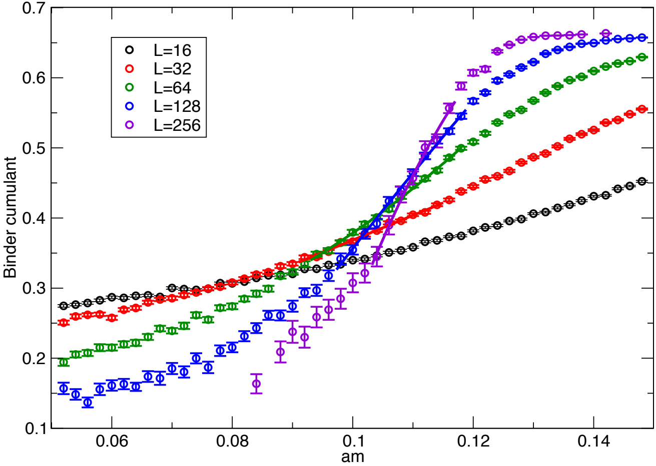


$\mathbb{Z}(2)$ breaking phase transition



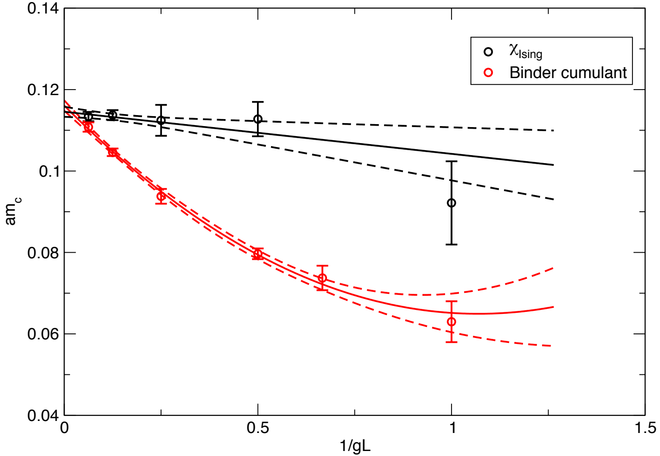
- 'Ising susceptibility' χ_m with $m = 1/V \sum_x \text{sign}[\phi_x]$

$\mathbb{Z}(2)$ breaking phase transition



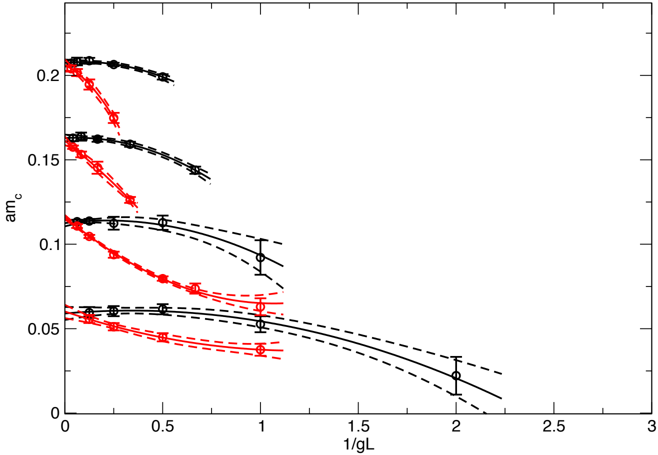
- Binder cumulant

$\mathbb{Z}(2)$ breaking phase transition



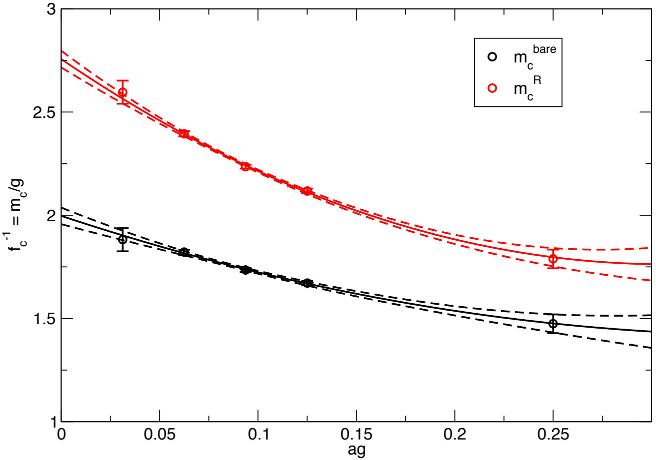
- Thermodynamic limit

$\mathbb{Z}(2)$/SUSY breaking phase transition



- Thermodynamic limit

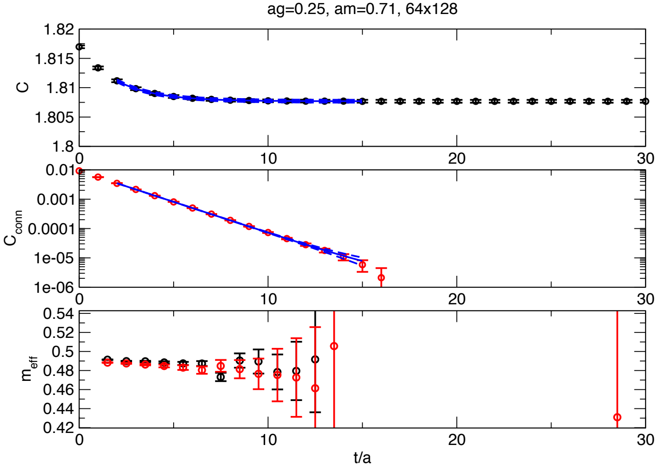
$\mathbb{Z}(2)$/SUSY breaking phase transition



- Continuum limit: 1-loop renormalised critical coupling

Boson mass spectrum

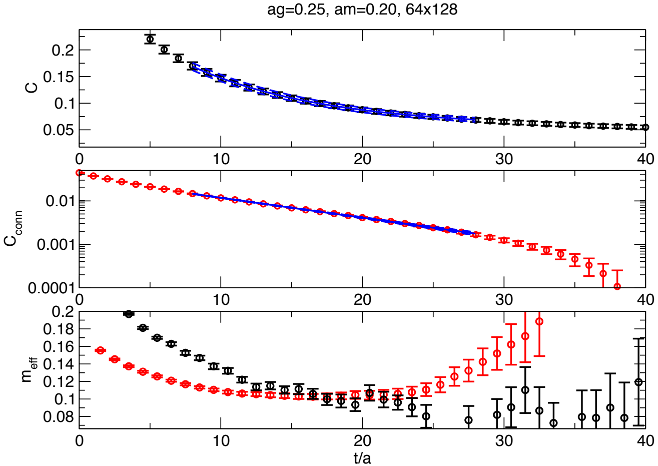
Boson field correlator in the SUSY symmetric/$\mathbb{Z}(2)$ broken phase:



perfect fit with $e^{-M_B t}$ plus constant shift

Boson mass spectrum

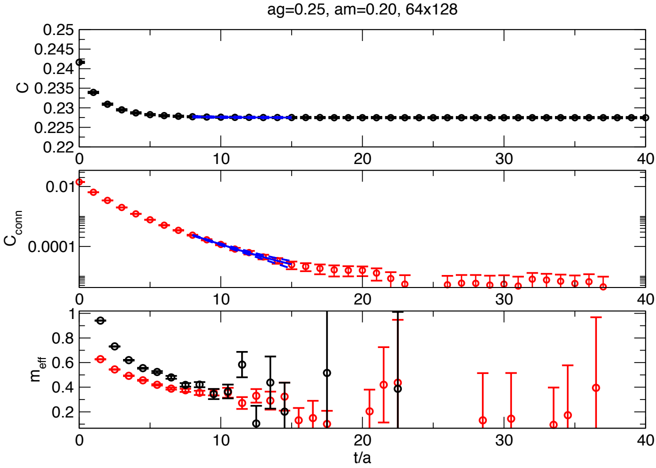
Boson field correlator in the SUSY broken/$\mathbb{Z}(2)$ symmetric phase, $\mathbb{Z}(2)$ odd state ϕ:



perfect fit with $A_0 e^{-M_B^{(0)} t} + A_1 e^{-M_B^{(1)} t}$

Boson mass spectrum

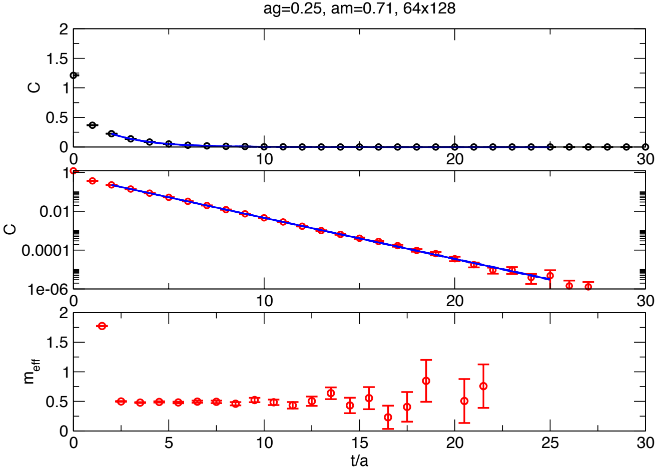
Boson field correlator in the SUSY broken/$\mathbb{Z}(2)$ symmetric phase, $\mathbb{Z}(2)$ even state ϕ:



perfect fit with $e^{-M_B^{(2)}t}$ plus constant shift

Fermion mass spectrum

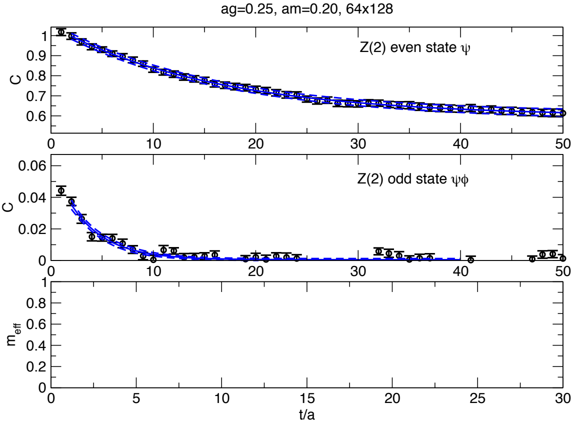
Fermion field correlator in the SUSY symmetric/$\mathbb{Z}(2)$ broken phase (spans $\mathcal{O}(10^7)$ due to fermion loop algorithm):



perfect fit with $e^{-M_F t}$

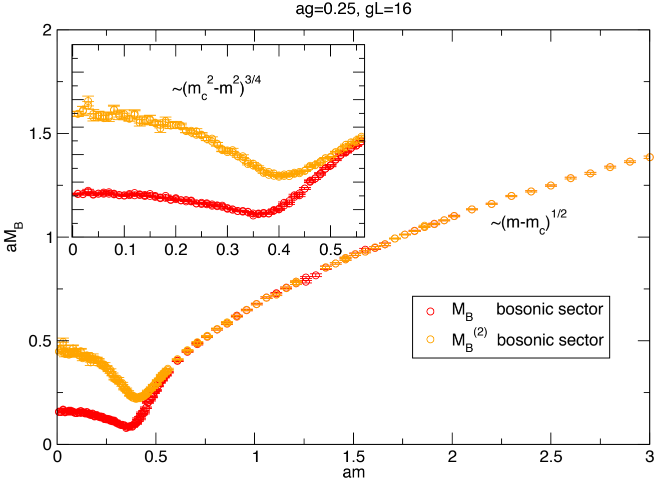
Fermion mass spectrum

Fermion field correlator in the SUSY broken/$\mathbb{Z}(2)$ symmetric phase, $\mathbb{Z}(2)$ even/odd state ψ and $\psi\phi$:

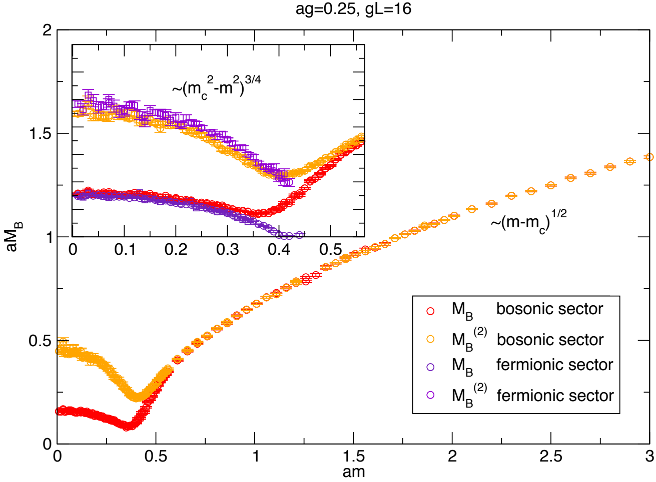


good fits with $A_0 e^{-M_F^{(0)} t} + A_1 e^{-M_F^{(1)} t}$ and $e^{-M_F^{(2)} t}$

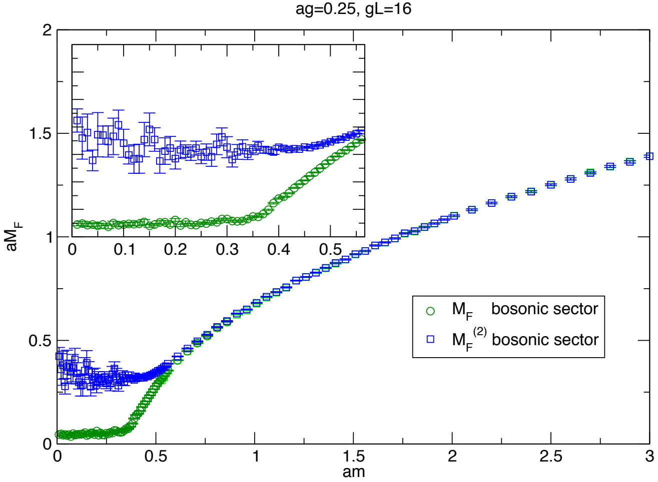
Boson mass spectrum



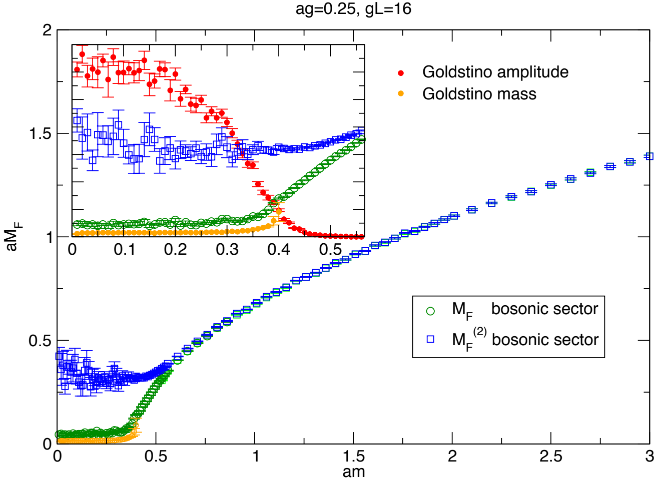
Boson mass spectrum



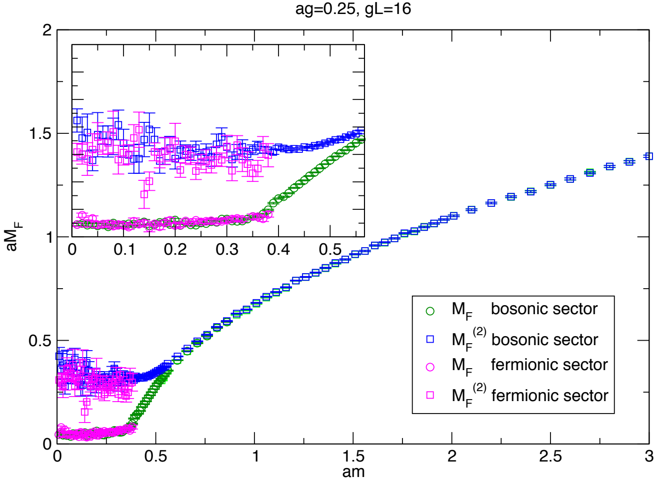
Fermion mass spectrum



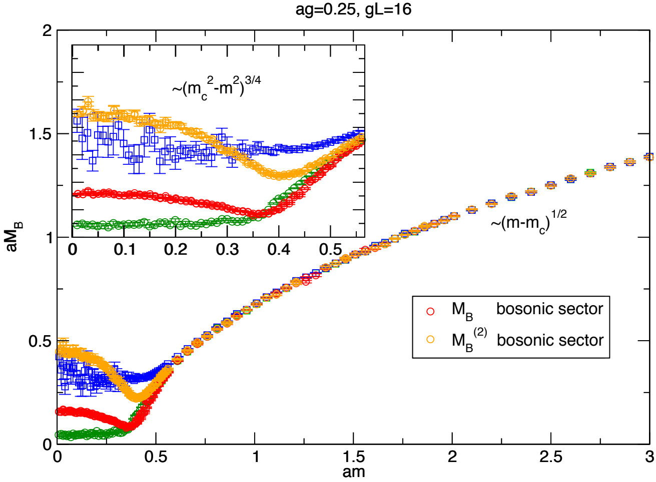
Fermion mass spectrum



Fermion mass spectrum



Combined mass spectrum



Conclusions

- Reformulation of Majorana fermions in terms of fermion loops:
  - separation into bosonic and fermionic sector,
  - avoids the fermion sign problem,
  - allows simulations without critical slowing down.
- Transfer matrix yields exact results for SUSY QM.
- Determination of the critical coupling for the $\mathbb{Z}(2)$/SUSY breaking phase transition for the $\mathcal{N} = 1$ WZ model.
- Determination of the particle mass spectrum above and below the transition.
- Complete non-perturbative description of a spontaneous supersymmetry breaking phase transition.