

# Aspects of confinement and the center symmetry phase transition from QCD correlation functions

On the status of studies of Landau gauge QCD Green functions  
within functional continuum methods

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Jena, Dec. 1, 2012



# Outline

- 1 Fundamental Concepts
  - BRST quartets & Kugo–Ojima confinement criterion
  - Gribov horizon & Zwanziger condition
- 2 Infrared Structure of Landau gauge Yang-Mills theory
  - Infrared Behavior of Gluons and Ghosts
  - Two-loop terms in the gluon propagator DSE
- 3 Coupling matter to gluons
  - Quark propagator and quark-gluon vertex
  - The effect of an IR divergent quark-antiquark interaction kernel
- 4 Center symmetry phase transition
- 5 Conclusions and Outlook

## CONFINEMENT

implies

- a non-perturbative RG invariant confinement scale

$$\Lambda = \mu \exp \left( - \int^g \frac{dg'}{\beta(g')} \right) \xrightarrow{g \rightarrow 0} \mu \exp \left( - \frac{1}{2\beta_0 g^2} \right)$$

- infrared singularities  $\iff$  continuum approach



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# Covariant Gauge Theory and BRST Quartets

Gauge theory in covariant gauges: **Unphysical degrees of freedom!**

**QED:** Physical states obey Lorentz condition.

$$\partial_\mu A^\mu |\Psi\rangle = 0 \quad (\text{Gupta} - \text{Bleuler}).$$

⇒ Two physical massless photons.

Time-like photon (i.e. negative norm state!) cancels longitudinal photon in physical states!

# Covariant Gauge Theory and BRST Quartets

## Faddeev-Popov Quantization & BRST in QCD:



Selfinteraction of gluons  $\Rightarrow$   
cancelation between four fields:  
forward & backward pol. gluons,  
ghost & antighosts,  
the elementary BRST quartet!

Global ghost field as ‘gauge parameter’:

**BRST symmetry of the gauge-fixed action!**

$$\begin{aligned}\delta_B A_\mu^a &= D_\mu^{ab} c^b \lambda, & \delta_B q &= -igt^a c^a q \lambda, \\ \delta_B c^a &= -\frac{g}{2} f^{abc} c^b c^c \lambda, & \delta_B \bar{c}^a &= \frac{1}{\xi} \partial_\mu A_\mu^a \lambda,\end{aligned}$$

Becchi–Rouet–Stora & Tyutin (BRST), 1975



# Covariant Gauge Theory and BRST Quartets

BRST symmetry of the gauge-fixed generating functional:

- Via Noether theorem: BRST charge operator  $Q_B$
- generates ghost # graded algebra  $\delta_B \Phi = \{iQ_B, \Phi\}$
- $\mathcal{L}_{GF} = \delta_B (\bar{c} (\partial_\mu A^\mu + \frac{\alpha}{2} B))$  is BRST exact.

BRST algebra:  $Q_B^2 = 0, [iQ_c, Q_B] = Q_B,$

BRST cohomology:

Positive definite subspace  $\mathcal{V}_{\text{pos}} = \text{Ker}(Q_B)$  contains  $\text{Im}Q_B$ .

Hilbert space: cohomology  $\mathcal{H} = \frac{\text{Ker}Q_B}{\text{Im}Q_B} \simeq \mathcal{V}_s$  space of BRST singlets

forw. & backw. gluons, ghosts & antighosts : elementary BRST quartet  
(c.f. Gupta–Bleuler mechanism in QED)

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# Kugo–Ojima confinement criterion

⇒ Hypothesis: Physical states are BRST singlets!

(BRST cohomology: Hilbert space  $\mathcal{H} = \frac{\text{Ker } Q_{BRST}}{\text{Im } Q_{BRST}}$ .)

Time-like and longitudinal gluons (in elementary BRST quartet)  
removed from asymptotic states as in QED, but:

Transverse gluons and quarks also members of BRST quartets,  
*i.e.* kinematically confined,  
if **ghost propagator is highly infrared singular!**  
(⇒ Kugo–Ojima confinement criterion)



# Kugo–Ojima confinement criterion

Realization of KO scenario depends on **global gauge structure**:

Globally conserved current ( $\partial^\mu J_\mu^a = 0$ )

$$J_\mu^a = \partial^\nu F_{\mu\nu}^a + \{Q_B, D_\mu^{ab} \bar{c}^b\}$$

with charge

$$Q^a = G^a + N^a.$$

---

**QED:** MASSLESS PHOTON states in both terms.

Two different combinations yield:

unbroken global charge  $\tilde{Q}^a = G^a + \xi N^a$ .

spont. broken displacements (photons as Goldstone bosons).

---

**No massless** gauge bosons in  $\partial^\nu F_{\mu\nu}^a$ :  $G^a \equiv 0$ .

(QCD, e.w. Higgs phase, ...)



# Kugo–Ojima confinement criterion

**QCD:** Well-defined (in  $\mathcal{V}$ ) unbroken global charge

$$Q^a = N^a = \{ Q_B, \int d^3x D_0^{ab} \bar{c}^b \}$$

With  $D_\mu^{ab} \bar{c}^b(x) \xrightarrow{x^0 \rightarrow \pm\infty} (\delta^{ab} + u^{ab}) \partial_\mu \bar{\gamma}^b + \dots$

⇒ Kugo-Ojima Confinement Criterion:  $u^{ab}(0) = -\delta^{ab}$

where

$$\int dx e^{ip(x-y)} \langle 0 | T D_\mu c^a(x) g(A_\nu \times \bar{c})^b(y) | 0 \rangle =: (g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) u^{ab}(p^2),$$

Sufficient condition in Landau gauge:

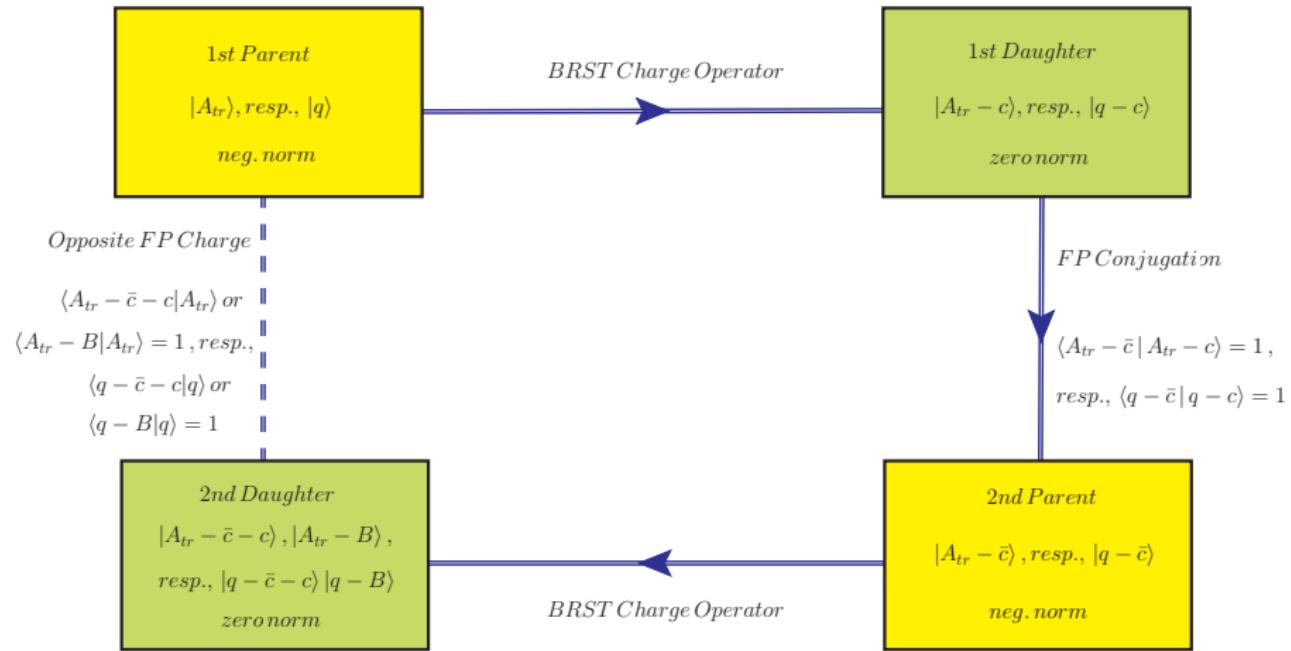
Ghost propagator more sing. than simple pole!

If fulfilled: Physical States  $\equiv$  BRST singlets  $\equiv$  color singlets!



# Kugo–Ojima confinement criterion

Non-perturbative BRST quartets of transverse gluons, resp., quarks:

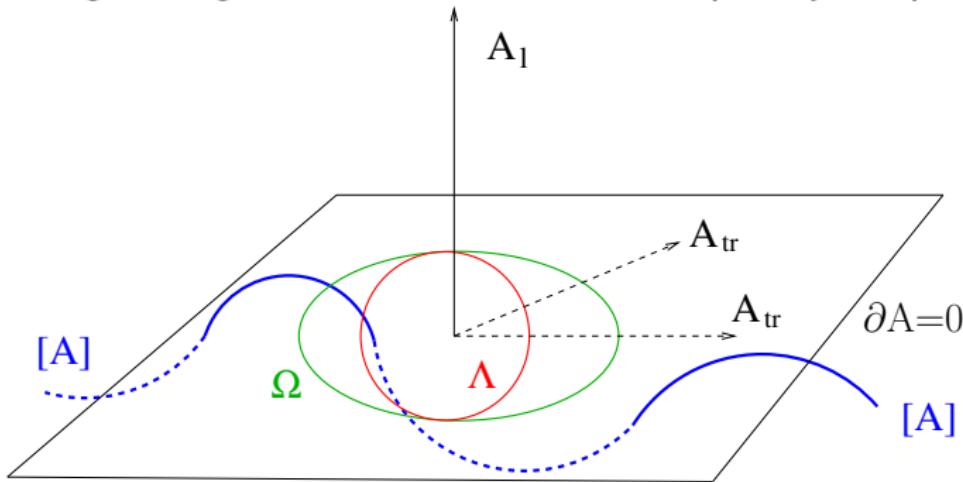


N. Alkofer and R.A., Phys. Lett. B 702 (2011) 158 [arXiv:1102.2753 [hep-th]];  
PoS QCD-TNT-II (2011) 002 [arXiv:1112.4483 [hep-th]].



# Gribov horizon & Zwanziger condition

Gauge fixing in YM theories never completely unique:



$\Lambda$  topologically non-trivial as complete config. space also is!

Landau gauge:

$$\Gamma = \{A : \partial \cdot A = 0\}$$

Minimal Landau gauge:  $\Omega = \{A : \|A\|^2 \text{ minimal}\}$

First Gribov region:  $\Omega = \{A : \partial \cdot A = 0, \partial \cdot D(A) \geq 0\}$

Fundam. Modular Region:  $\Lambda = \{A : \text{global extrema}\}$

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NO GRIBOV COPIES

Relevant configuration space:  $\Lambda / SU(N_c)$

Gribov: Cut off integral at boundary  $\partial\Omega$

Zwanziger: Ambiguities resolved due to additional  
IR boundary condition on ghost prop.

$$\lim_{k^2 \rightarrow 0} (k^2 D_{\text{Ghost}}(k^2))^{-1} = 0.$$

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# Kugo-Ojima vs. Gribov-Zwanziger

- In Landau gauge: Kugo-Ojima and Gribov-Zwanziger lead to practically same infrared constraints.
- Results in positivity violation for transverse gluons: non-pert. realization of Oehme–Zimmermann superconvergence relation (antiscreening contradicts positivity of gluon spectral density).

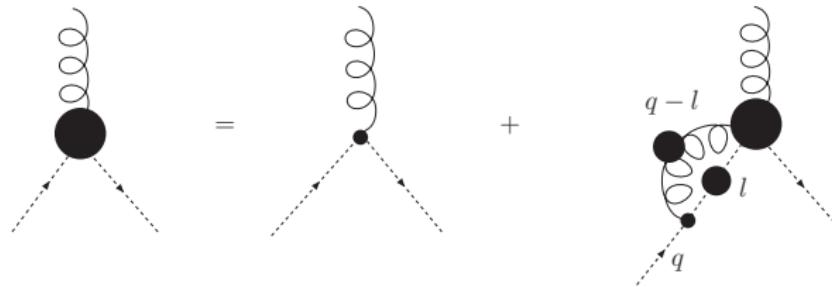
R. Oehme and W. Zimmermann, Phys. Rev. **D21** (1980) 471.

- Kugo-Ojima requires Lorentz-covariant gauge, but fails e.g. also in the Lorentz-cov. Maximally Abelian gauge.
- Gribov-Zwanziger applies to Landau and Coulomb gauge (where  $\Lambda$  is compact and convex), but e.g. not to Maximally Abelian gauge (where Gribov region is unbounded).
- Recently: generalized (“refined”) Gribov-Zwanziger scheme which results in infrared finite Landau gauge Green functions, D. Dudal, S. Sorella, N. Vandersickel, *et al.*



# Infrared Structure of Landau gauge Yang-Mills theory

- Starting point in gauges with transverse gluon propagator:  
Ghost-Gluon-Vertex fulfills Dyson-Schwinger equation



- Transversality of gluon  $I_\mu D_{\mu\nu}(I - q) = q_\mu D_{\mu\nu}(I - q) \Rightarrow$   
**Bare Vertex** for  $q_\mu \rightarrow 0$
- No anomalous dimensions in the IR

J.C. Taylor, Nucl. Phys. B 33 (1971) 436;

C. Lerche, L. v. Smekal, PRD 65 (2002) 125006.

Recently:

Solution of DSEs for YM propagators *and* ghost-gluon vertex!

Poster of M. Q. Huber; M. Q. Huber, L. v. Smekal, arXiv:1211.6092.

# Infrared Structure of Landau gauge Yang-Mills theory

DSEs for YM propagators:

$$\text{---} \bullet \text{---}^{-1} = \text{---} \bullet \text{---}^{-1} - \frac{1}{2} \text{---} \bullet \text{---} - \frac{1}{2} \text{---} \bullet \text{---}$$
$$- \frac{1}{6} \text{---} \bullet \text{---} - \frac{1}{2} \text{---} \bullet \text{---}$$
$$+ \text{---} \bullet \text{---} + N_F \text{---} \bullet \text{---}$$
$$\text{---} \bullet \text{---}^{-1} = \text{---} \bullet \text{---}^{-1} - \text{---} \bullet \text{---}$$

# Infrared Structure of Landau gauge Yang-Mills theory

Gluon propagator DSE:  
(without matter)

$$\text{Diagram with a loop} = \text{Diagram without loop} + \text{Diagram (a)} - \text{Diagram (b)} + \text{Diagram (c)} + \text{Diagram (d)}$$

The equation shows the Dyson-Schwinger equation for the gluon propagator. On the left is a diagram consisting of a wavy line with a black dot and a loop attached. This is followed by an equals sign. To the right of the equals sign is another wavy line with a black dot, followed by a plus sign. Then comes a diagram labeled (a) which shows a wavy line with a black dot and a loop attached, with a minus sign in front of it. Next is a plus sign, followed by diagram (b), which is similar to (a) but with two loops attached to the wavy line. Another plus sign follows, then diagram (c), which has three loops attached. Finally, there is a plus sign and diagram (d), which has four loops attached.

IR leading: ghost loop (a)

UV leading: ghost loop (a) + gluon loop (b)

⇒ 2-loop terms, sunset (c) and squint (d), qualitatively unimportant

Quantitatively?



# Infrared Structure of Landau gauge Yang-Mills theory

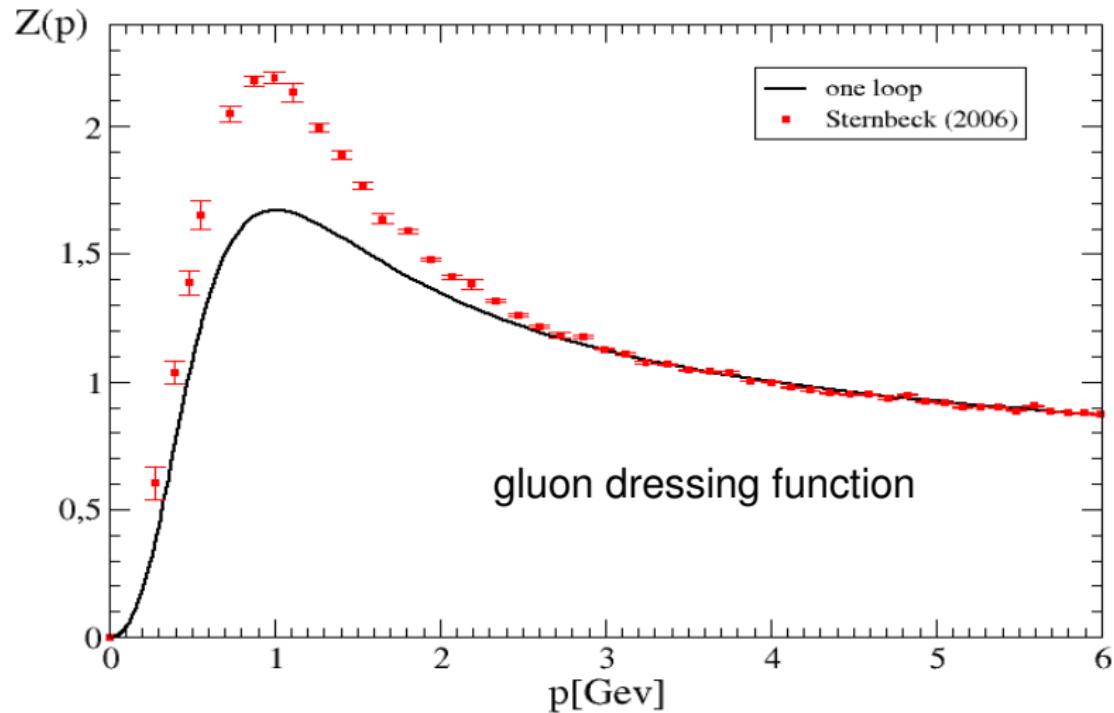
## Sunset diagram:

- Overlapping divergence!
- non-perturbative renormalization?

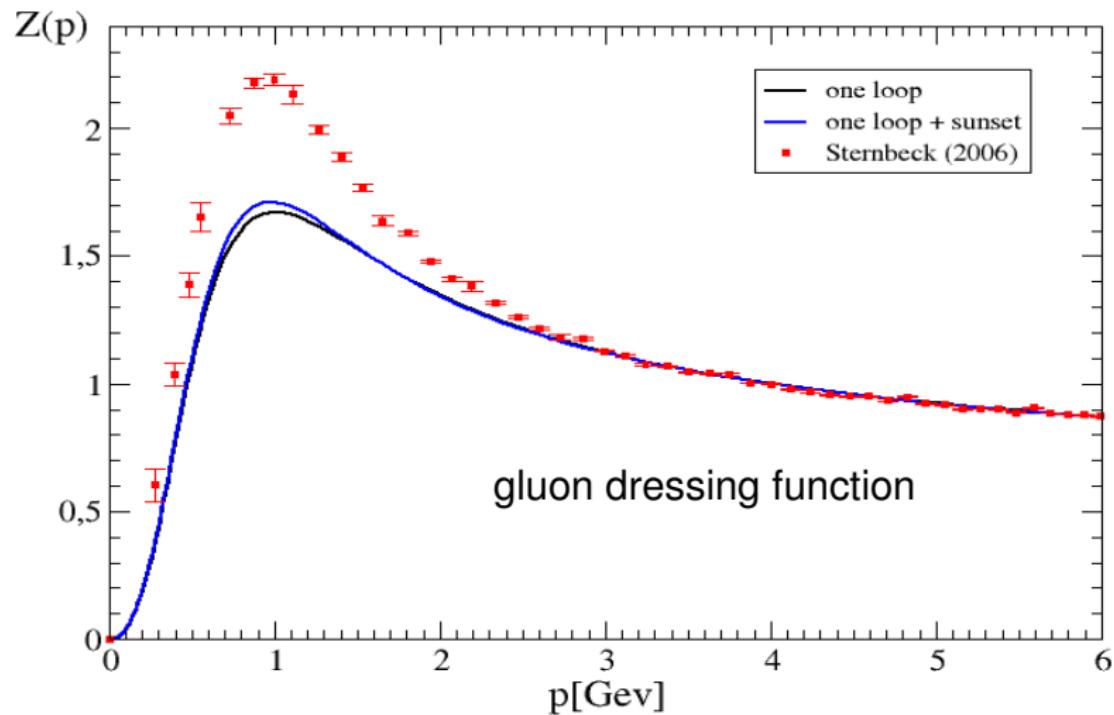
RA, M.Q.Huber, V.Mader, A.Windisch, PoS **QCD-TNT-II** (2011) 003 [1112.6173];  
V. Mader, A. Windisch and RA, in preparation.

- $\widetilde{\text{MOM}}$  (verified by BPHZ)

# Infrared Structure of Landau gauge Yang-Mills theory



# Infrared Structure of Landau gauge Yang-Mills theory



# Infrared Structure of Landau gauge Yang-Mills theory

- Include squint diagram (in progress):
  - also quantitatively unimportant?
  - qualitatively: cancelation of spurious divergencies!?!?
- Mismatch to lattice data at intermediate  $p^2$ 
  - resolved in ERG  
C.S. Fischer, A. Maas, J.M. Pawłowski, Ann. Phys. **324** (2009) 2408.
  - in DSEs:  
zero crossing of 3-gluon vertex function  
Poster of M. Q. Huber; M. Q. Huber, L. v. Smekal, arXiv:1211.6092.

# Coupling matter to gluons

DSEs for Landau gauge QCD propagators:

$$\text{---}^{-1} = \text{---}^{-1} - \frac{1}{2} \text{---} \text{---} - \frac{1}{2} \text{---} \text{---}$$

$$- \frac{1}{6} \text{---} \text{---} + \frac{1}{2} \text{---} \text{---}$$

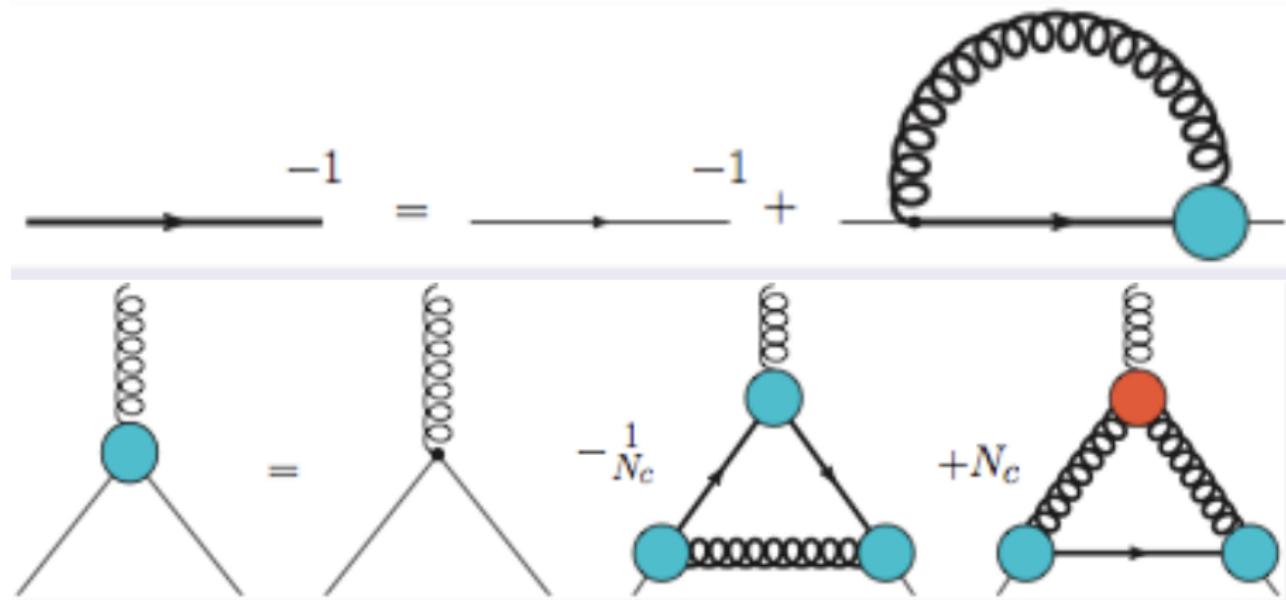
$$+ \text{---} + \mathbf{N}_f \text{---}$$

$$\text{---}^{-1} = \text{---}^{-1} - \text{---} \text{---}$$

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# Coupling matter to gluons

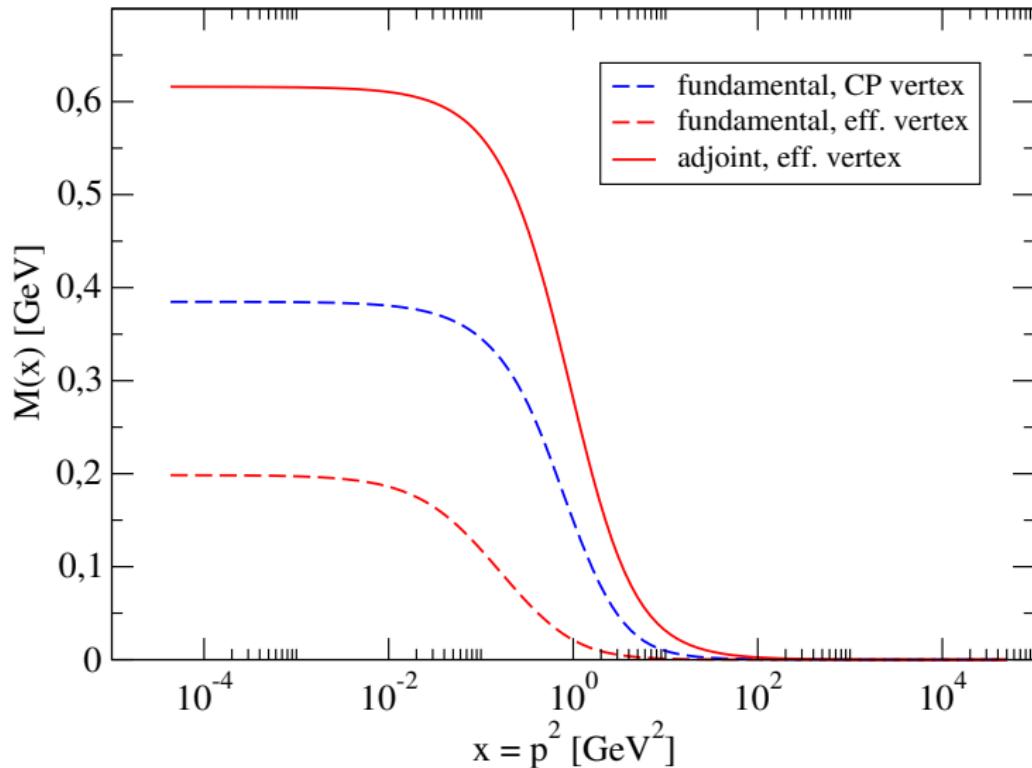
DSEs for quark propagator and quark-gluon vertex:



cf. poster of M. Hopfer

# Coupling matter to gluons

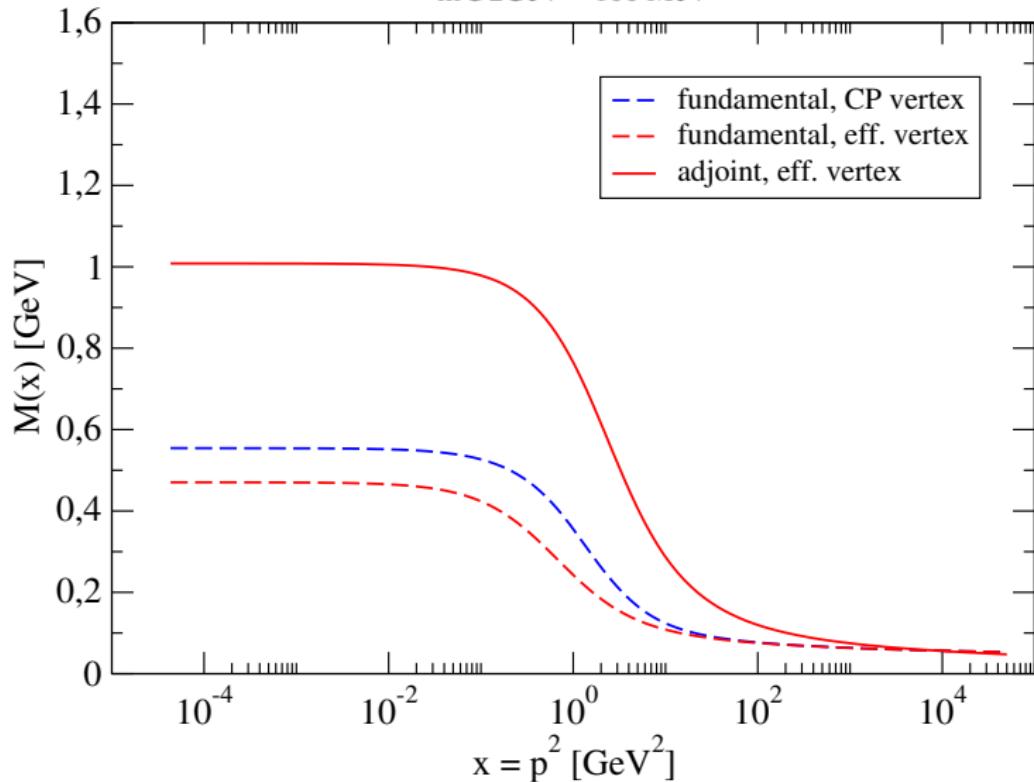
Quark mass function with models for QGV:  
Chiral Limit



# Coupling matter to gluons

Quark mass function with models for QGV:

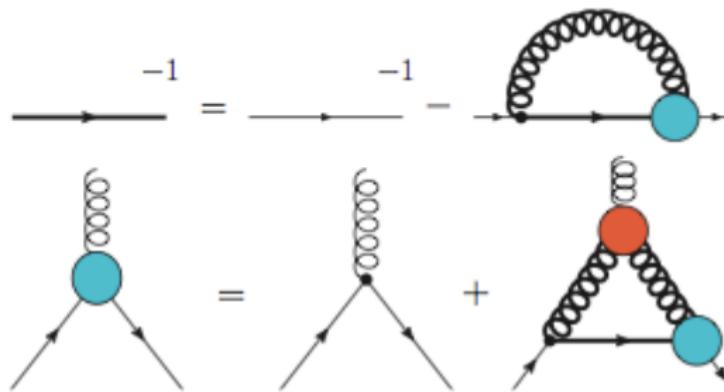
$m@2\text{GeV} = 100 \text{ MeV}$



# Coupling matter to gluons

Solving for the quark-gluon vertex:  
Preliminary results for a simplified system

- self-consistent solution of the quark-gluon vertex DSE  
in a truncation including all 12 tensor structures

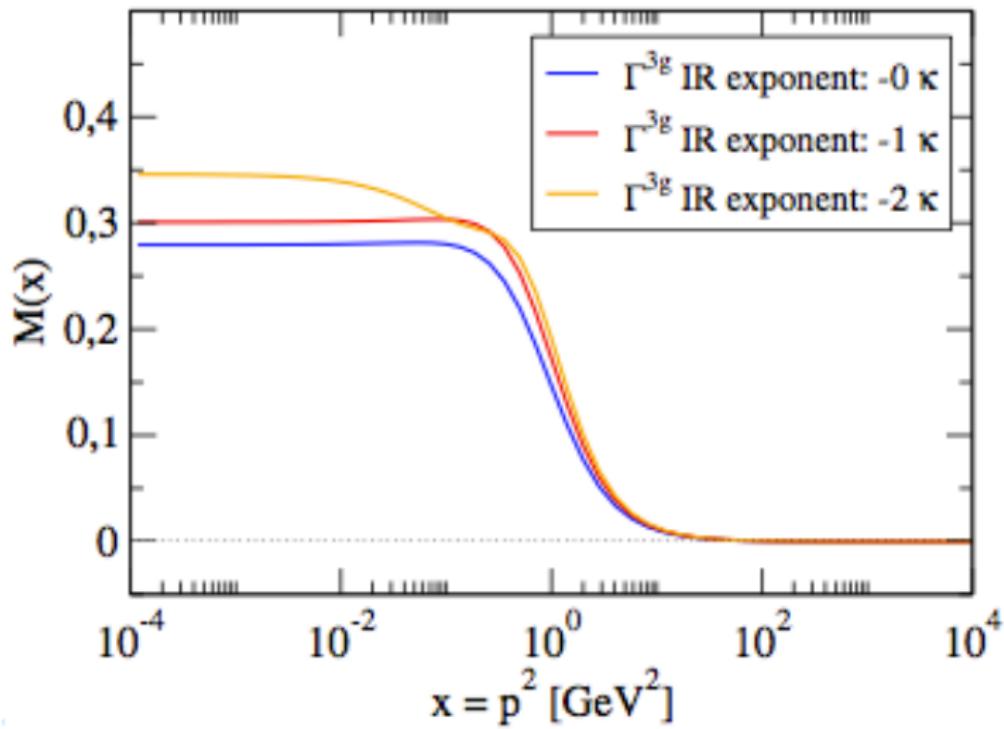


cf. poster of M. Hopfer

- scaling-type gluon propagator
- model for three-gluon vertex (cf. MQ Huber)

# Coupling matter to gluons

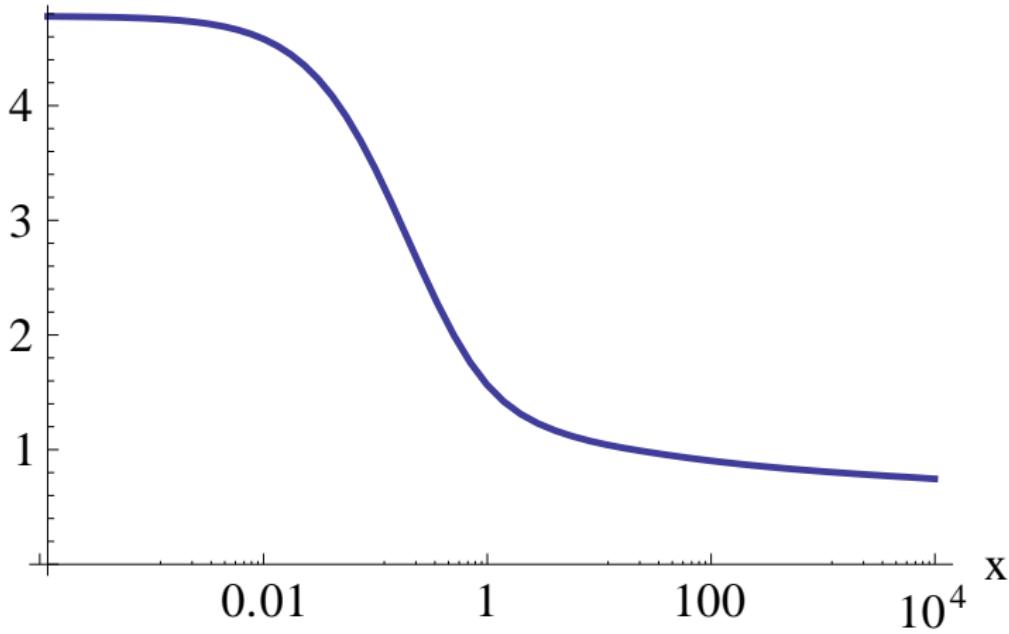
Quark mass function with calculated QGV:



# Coupling matter to gluons

Leading tensor structure, calculated QGV,  
symm. momenta  $x = p_1^2 = p_2^2 = p_3^2$ : Significant IR enhancement!

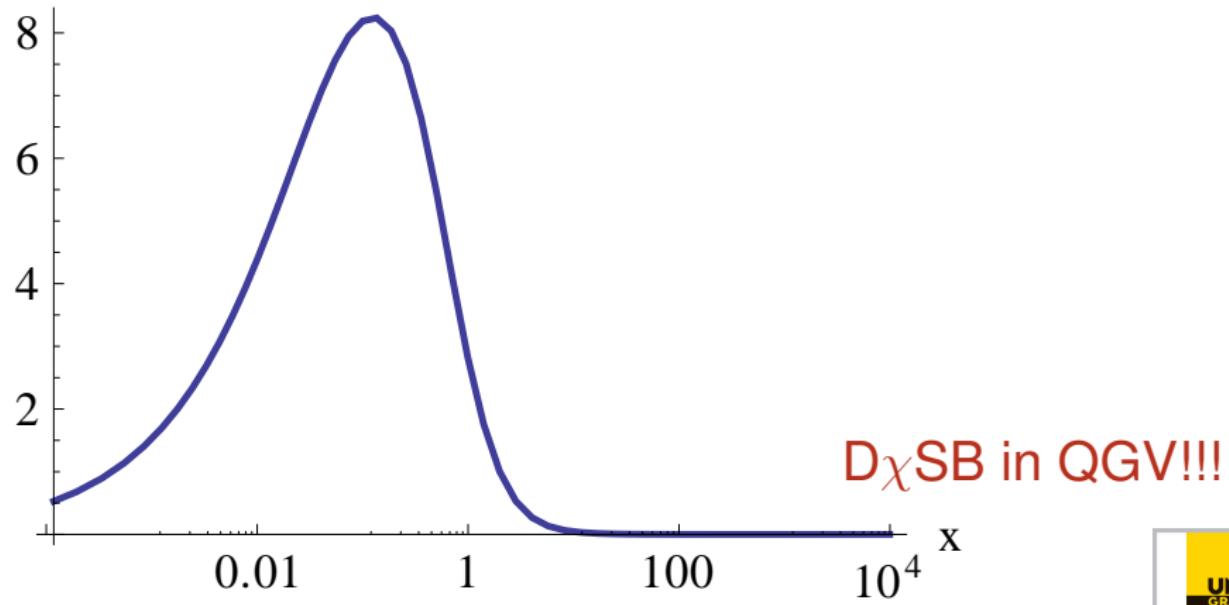
$$\lambda_1(x)$$



# Coupling matter to gluons

Subleading  $D\chi_{\text{SB}}$  tensor structure, calculated QGV:

$$x^{1/2} \lambda_3(x)$$

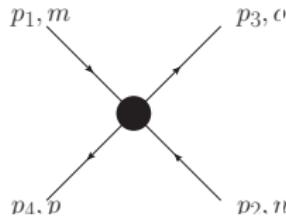


# Assuming an IR divergent 4-point function

M. Mitter, RA, in preparation

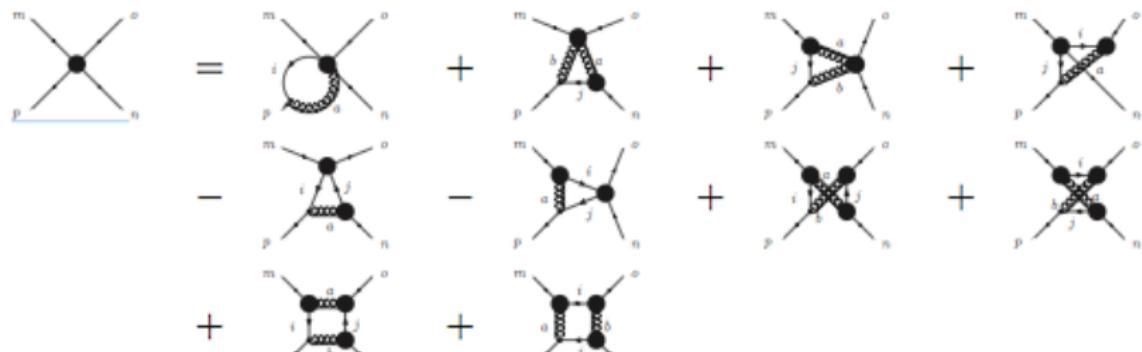
How is Confinement described by Green's functions?

- ▶ Assume quark 4-point function to be maximally IR singular,  
i.e.,  $\propto 1/k^4$ :



$$p_1 \rightarrow p_3 \propto \frac{1}{(p_1 - p_3)^4} \Big|_{\text{reg.}}$$

- ▶ Put e.g. DSE for 4-quark function:



# Consequences an IR divergent 4-point function

- ▶ For simplicity: Analysis first for fundamentally charged scalar!
- ▶ Consistency requirements:

☺ Boundedness of higher  $n$ -point functions to  $1/k^4 \Rightarrow$  matter-gluon vertex less singular  $\Rightarrow$  **colour structure**

$$p_1 \rightarrow p_3 \propto \frac{\delta_{mo}\delta_{np}}{(p_1 - p_3)^4} \Big|_{\text{reg.}}$$

☺ One-gluon exchange fails to reproduce this colour structure!  
☺ **All 4-point functions** (4-gluon, ghost-gluon, matter-gluon, matter-ghost) inherit the  $1/k^4$  singularity in specific colour channels.  
☺ **Higher  $n$ -point functions** contain contributions  $\propto 1/k^4$  with  $k$  being the momentum transfer between two coloured clusters.  
☺ Propagators and 3-point functions protected by cancellations.

- ▶ Decoupling theorem circumvented by IR singularities:  
**One heavy fundamental charge induce changes in the IR behaviour of YM Green's functions!?**



# Consequences an IR divergent 4-point function

- ▶ Assumption of confining IR singularity in matter-matter scattering kernel leads to several wanted features.
- ▶ Especially Casimir scaling!
- ▶ No decoupling of infinitely heavy charges?
- ▶ Further to be clarified:
  - Absence of van-der-Waals forces?
  - $N$ -ality?
  - Relation to dynamical chiral symmetry breaking / restoration?
  - ...

# Center symmetry phase transition

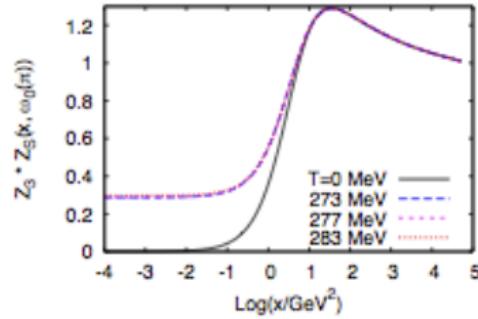
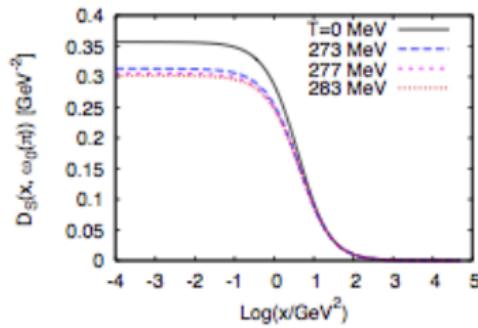
M. Mitter, M. Hopfer, BJ Schaefer, RA, in preparation

Investigate QCD correlation functions at  $T \neq 0$ :  
Exploit dependence on boundary conditions!

Propagator of fund. scalar:

- gluon propagator from lattice data  
C.S. Fischer, A. Maas, J.A. Mueller, Eur. Phys. J. **C 68** (2010) 165 [1003.1960].
- vacuum vertex model

Anti-periodic boundary conditions:



# Center symmetry phase transition

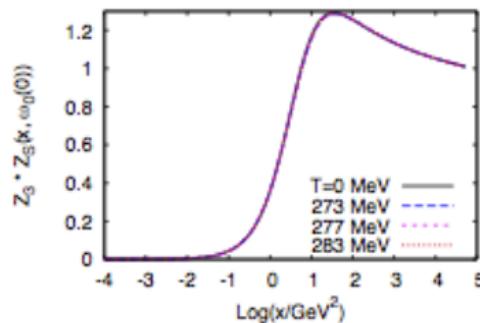
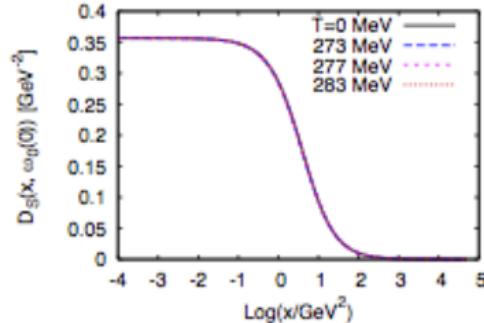
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Periodic boundary conditions:



# Center symmetry phase transition

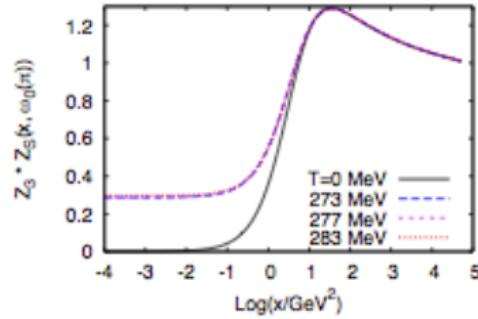
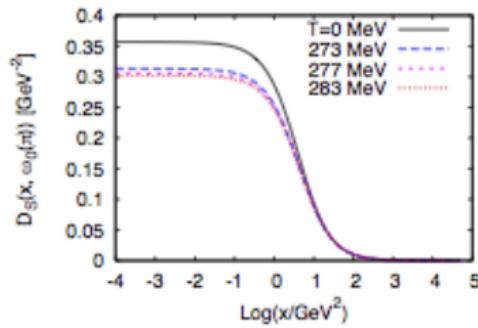
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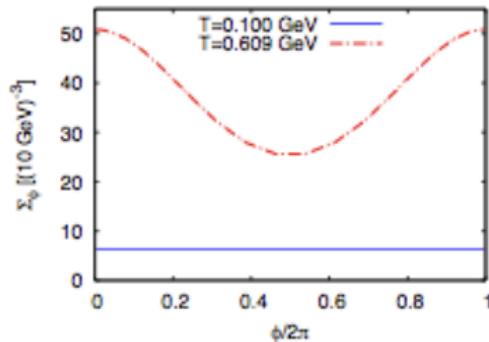
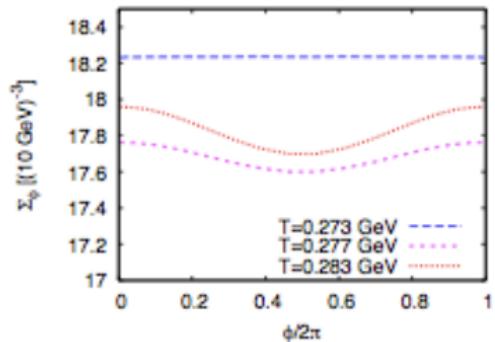
Anti-periodic boundary conditions:



# Center symmetry phase transition

Dual order parameter for scalar QCD:

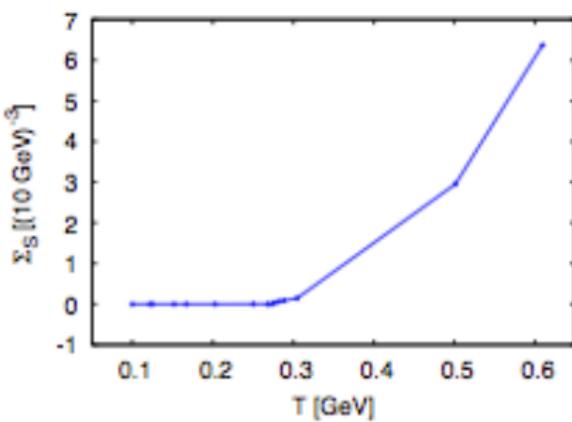
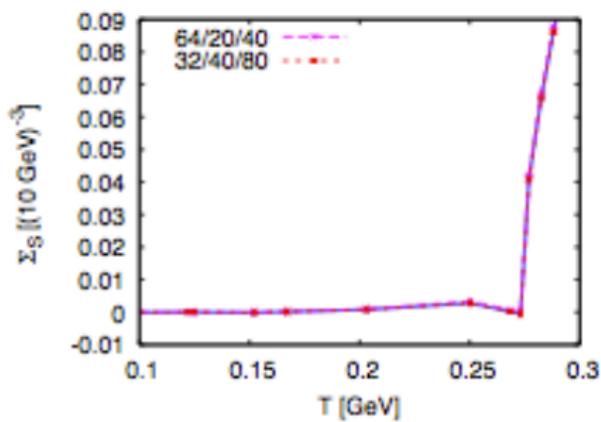
$$\begin{aligned}\Sigma_\phi &= T \sum_{\omega_n(\phi)} D_{S,\phi}^2(\vec{0}, \omega_n(\phi)) \xrightarrow{z} T \sum_{\omega_n(\phi)} D_{S,\phi+\arg(z)}^2(\vec{0}, \omega_n(\phi)) \\ &= T \sum_{\omega_n(\phi+\arg(z))} D_{S,\phi+\arg(z)}^2(\vec{0}, \omega_n(\phi + \arg(z))) = \Sigma_{\phi+\arg(z)} \\ \Sigma_{\phi+2\pi} &= \Sigma_\phi\end{aligned}$$



# Center symmetry phase transition

Center transition in scalar QCD:

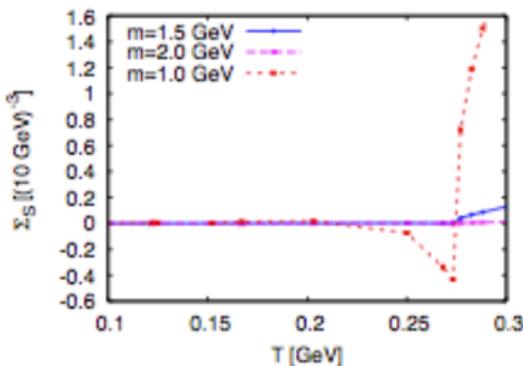
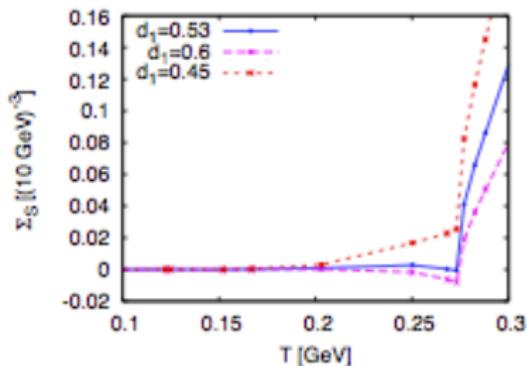
$$\Sigma_S = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi} \Sigma_\phi , \quad \Sigma_\phi = T \sum_{\omega_n(\phi)} D_{S,\phi}^2(\vec{0}, \omega_n(\phi))$$



# Center symmetry phase transition

Vertex model / mass dep. of center transition in scalar QCD:

$$A(x, y, z) = \tilde{Z}_3 \frac{D_S^{-1}(x) - D_S^{-1}(y)}{x - y} d_1 \left\{ \left( \frac{\Lambda^2}{\Lambda^2 + k^2} \right) + \frac{k^2}{\Lambda^2 + k^2} \left( \frac{\beta_0 \alpha(\mu) \ln [k^2/\Lambda^2 + 1]}{4\pi} \right)^{2\delta} \right\}$$

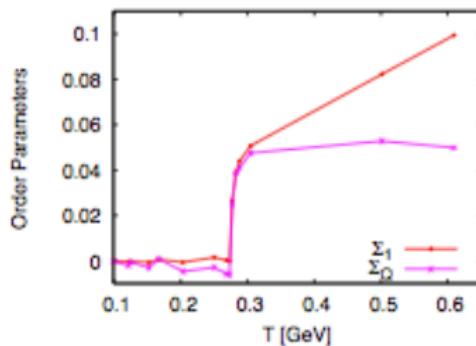
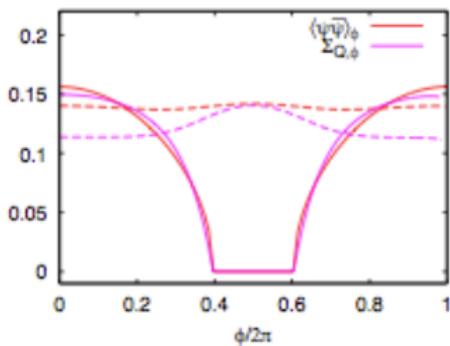


# Center symmetry phase transition

Alternative order parameter in QCD (with quarks):

$$\Sigma_Q = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi} \Sigma_{Q,\phi}, \quad \Sigma_{Q,\phi} = T \sum_{\omega_n(\phi)} \frac{1}{4i} \text{tr} \left[ S_{Q,\phi}(\vec{0}, \omega_n(\phi)) \right]^2$$

- no regularization necessary for bare quark mass  $m_0 \neq 0$
- different definitions of crossover in unquenched QCD possible



# Conclusions and Outlook

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- ☺ Chiral symmetry dynamically broken! In 2- and **3**-point function!
- ☺ Quark/matter confinement: Analysis of IR divergencies!  
Color structure, Casimir scaling, no decoupling of heavy d.o.f., ...
- ☺ Center symmetry phase transition:  
sensitivity to quark-gluon vertex, dual order parameter, ...
- ☺ Quark/matter-gluon vertex ( $T = 0$ ):
  - quark/matter confinement,  $D\chi_{\text{SB}}$ ,  $U_A(1)$
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