Phase transitions in strong QED$_3$

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SFB 634

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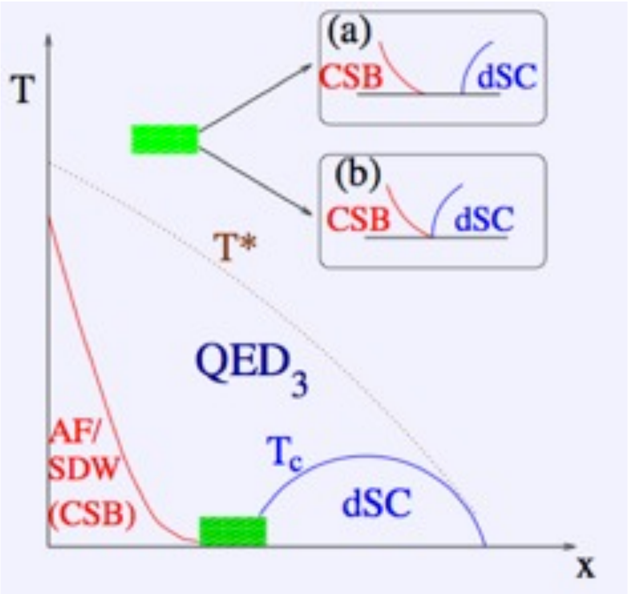
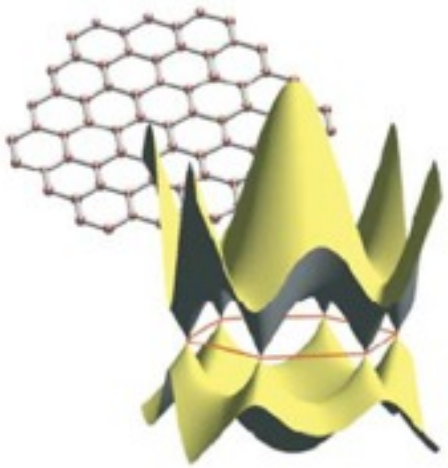
HELMHOLTZ
GEMEINSCHAFT

Overview

1. Introduction to QED_3

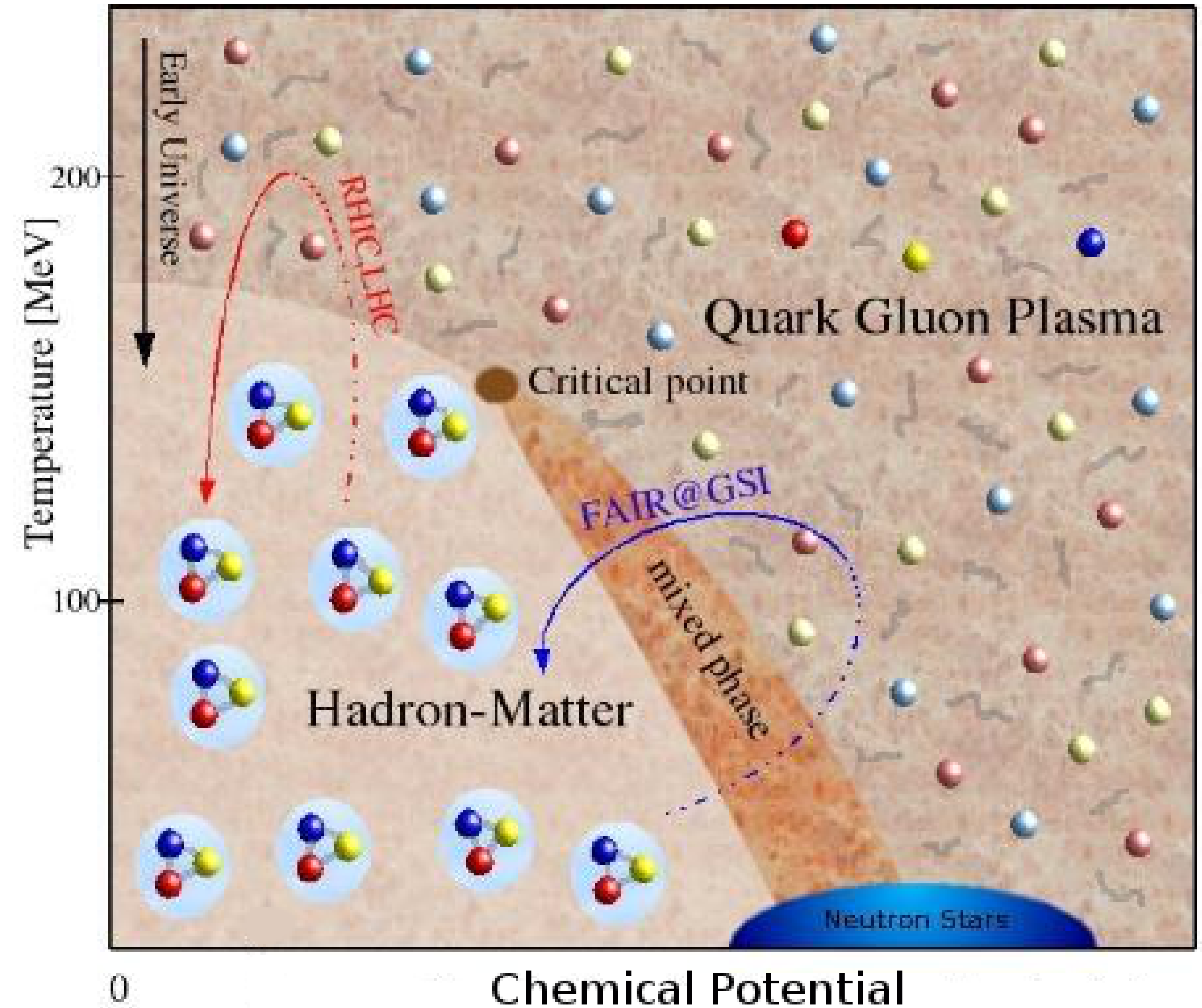


2. QED_3 and high T_c superconductors



3. QED_3 and graphene

QCD phase diagram



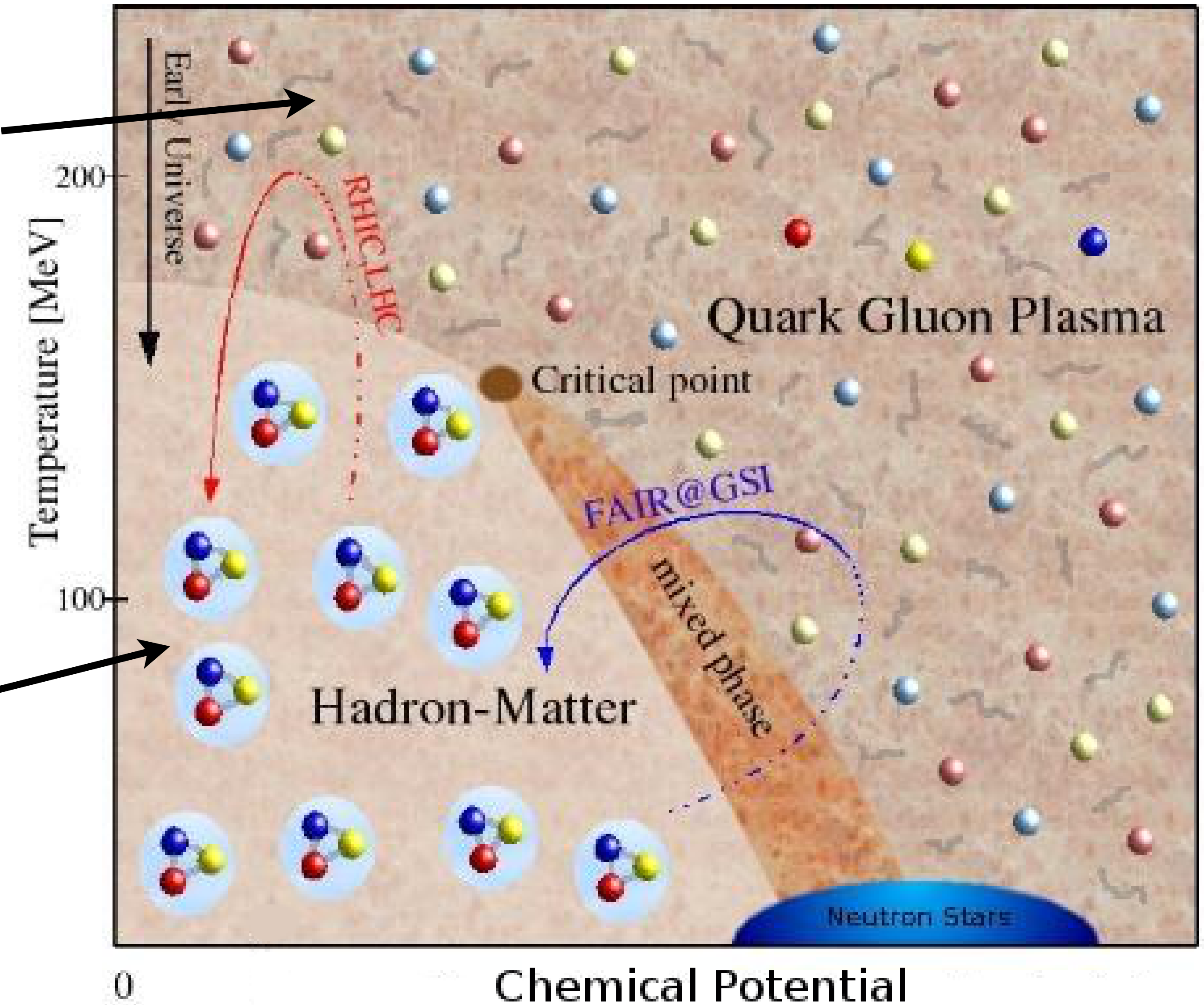
Interesting open questions:

- Details of phase transitions
- Existence and location of critical point
- Properties of quarks and gluons in different phases
- Consequences for astrophysics

QCD phase diagram

Quarks de-confined and (almost) massless

Quarks confined and massive



Interesting open questions:

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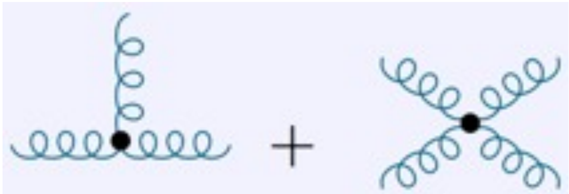
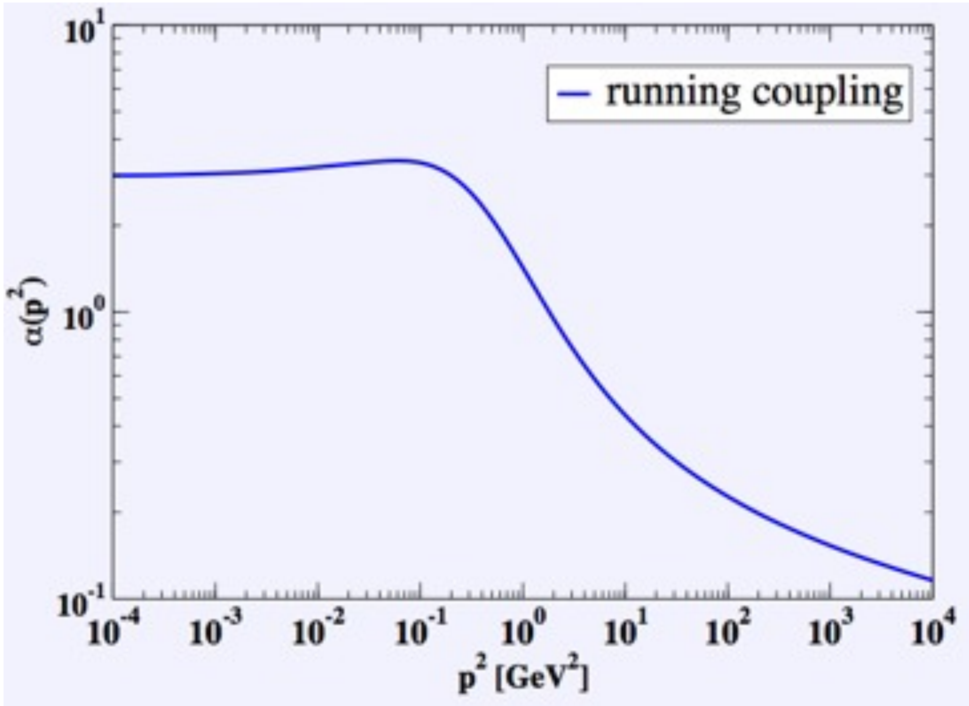
Strong QFTs: QCD vs QED3

QCD

- non-Abelian



- dynamical generation of scale
- asymptotically free



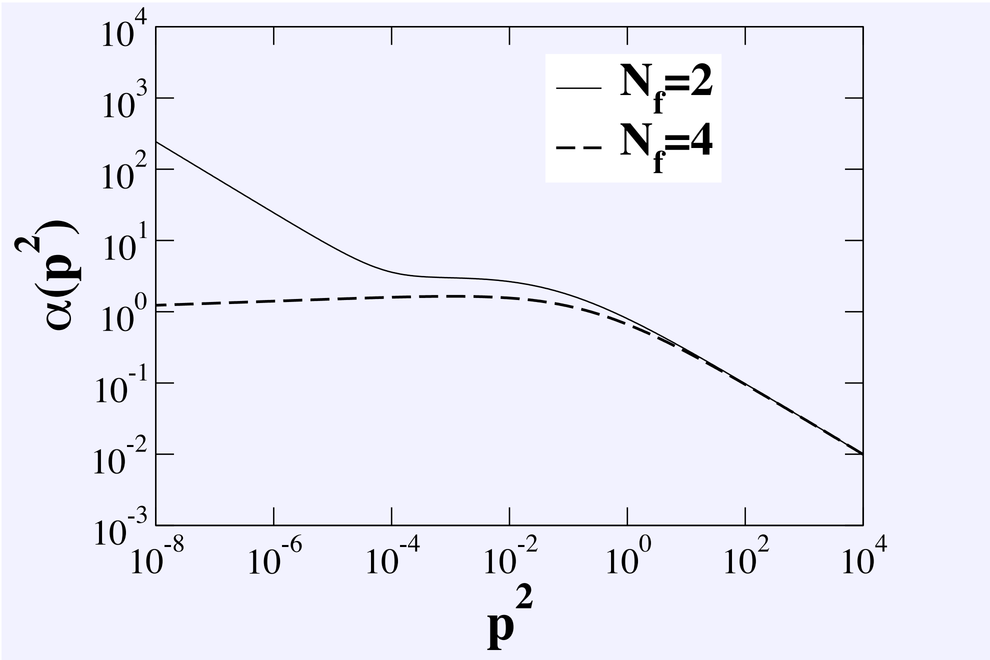
- Confinement and DχSB

QED3

- Abelian
- scale set by coupling

$$\alpha = N_f\, e^2/8$$

- asymptotically free



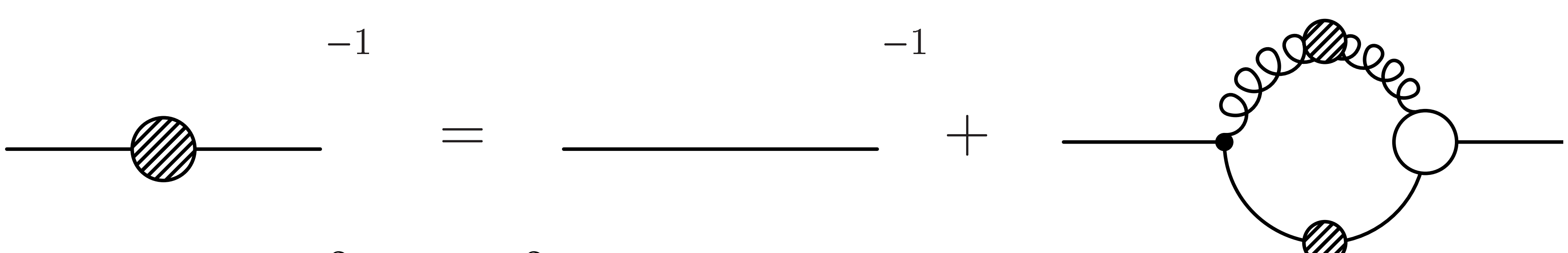
- Confinement and DχSB

Properties of QCD: Dynamical mass generation

Dynamical quark masses
via weak and strong force

Yoichiro Nambu,
Nobel prize 2008

	u	d	s	c	b	t
M_{weak} $[MeV/c^2]$	3	5	80	1200	4500	176000
M_{strong} $[MeV/c^2]$	350	350	350	350	350	350
M_{total} $[MeV/c^2]$	350	350	450	1500	4800	176000



$$S^{-1}(p) = [i\not{p} + M(p^2)]/Z_f(p^2)$$

Properties of QCD: Dynamical mass generation

Dynamical quark masses
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Input parameters in N_f=2+1 QCD

	u	d	s	c	b	t
M_{weak} $[MeV/c^2]$	3	5	80	1200	4500	176000
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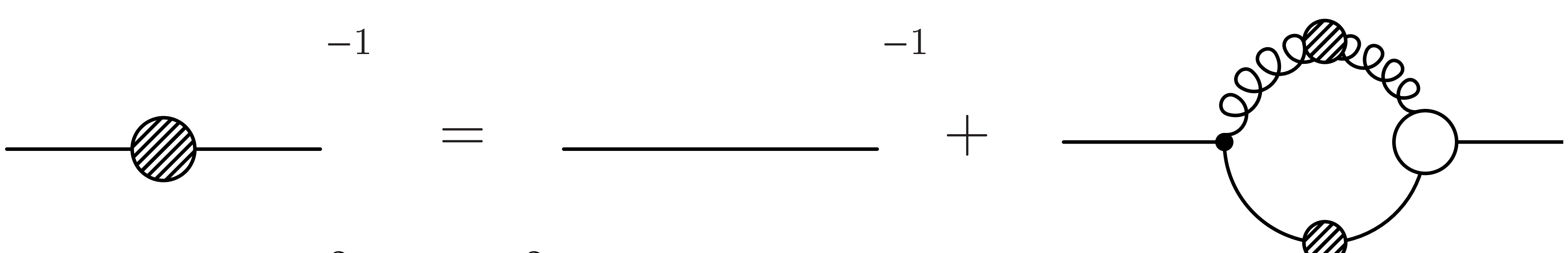


$$S^{-1}(p) = [i\not{p} + M(p^2)]/Z_f(p^2)$$

Properties of QED3: Chiral Symmetry

$$S = \int d^3x \left(\sum_{N_f} \bar{\Psi} i \not{D} \Psi - \frac{1}{4} F_{\mu\nu} F_{\mu\nu} \right)$$

- Four component spinors
- Clifford algebra

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}, \qquad \mu, \nu = 0..2$$

- Generators for chiral symmetry:

$$\gamma_3, \gamma_5, [\gamma_3, \gamma_5] \to U(2N_f)$$

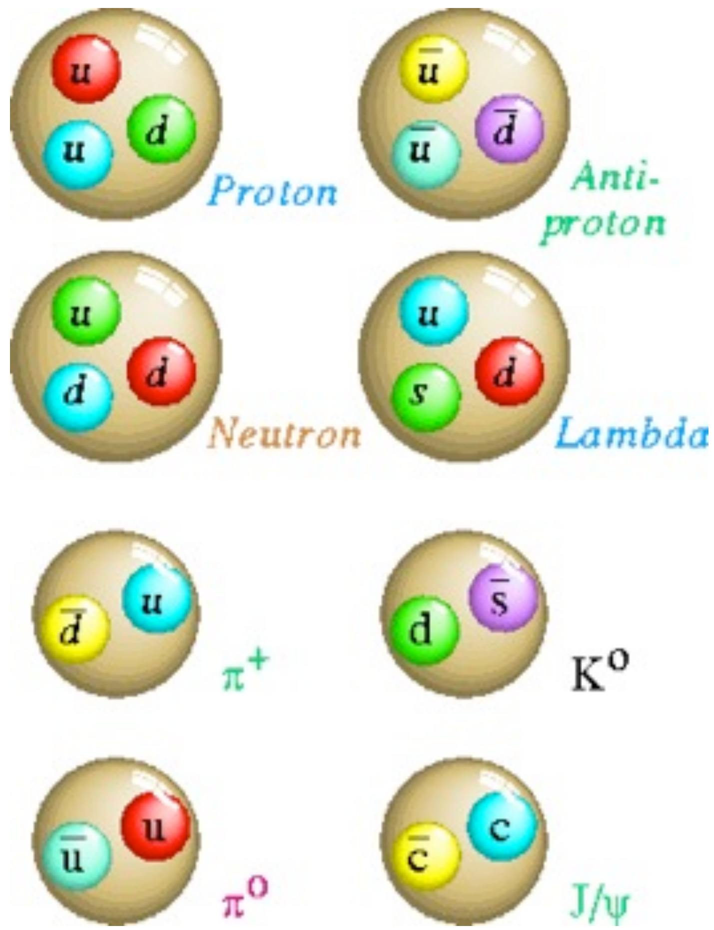
- Chiral symmetry breaking

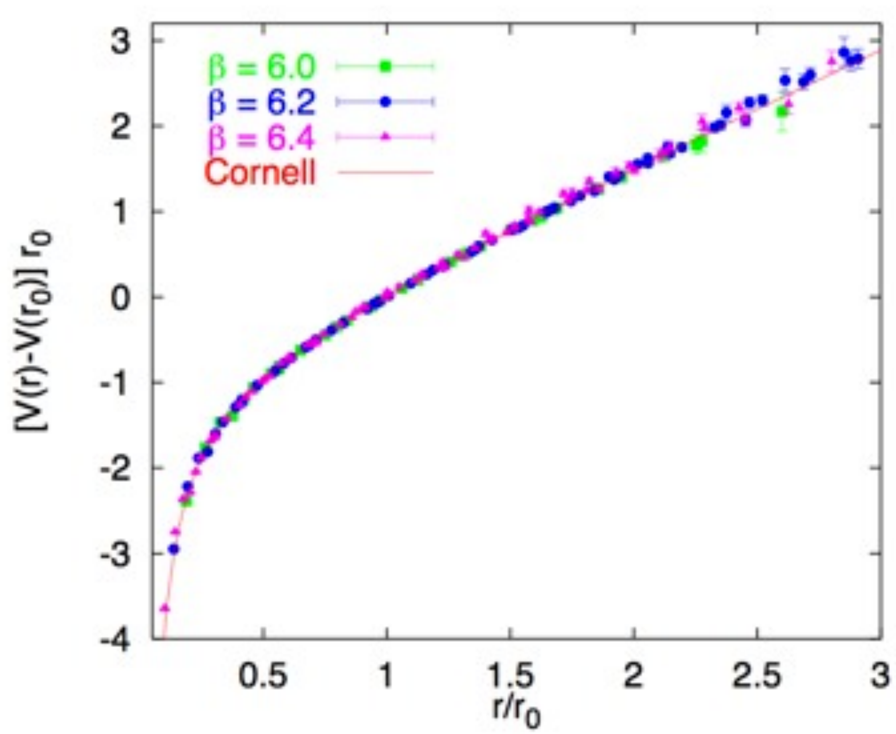
$$U(2N_f) \to U(1) \times U(1) \times SU(N_f) \times SU(N_f)$$

Properties of QCD: Confinement

Quark confinement

Millenium-Prize (1 Mio Dollar)
Clay Mathematics Institute





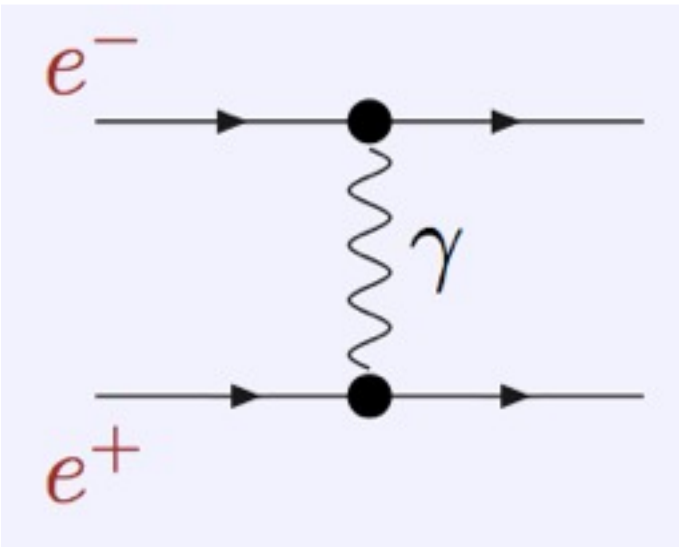
Bali, Phys. Rept 343 (2001)

- baryons, mesons (and glueballs ?)
- linear rising potential
- related to center symmetry

Jeff Greensite, Lecture Notes in Physics 821 (2011) 1.

Properties of QED3: ‘Confinement’

Logarithmically rising potential:



$$V(r) \sim \int d^2k \, \frac{1}{k^2} \, e^{ikr} \sim ln(r)$$

- massless one boson exchange
- ‘geometrical confinement’
- dressed Polyakov loop does NOT show confinement

  (similar to NJL model...)

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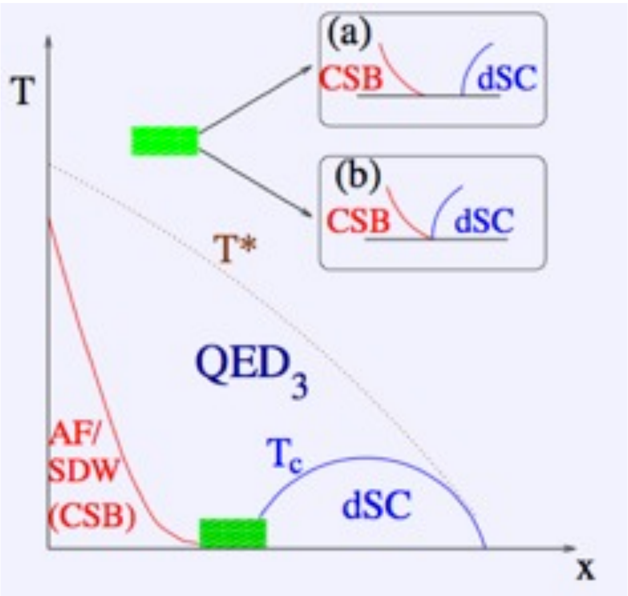

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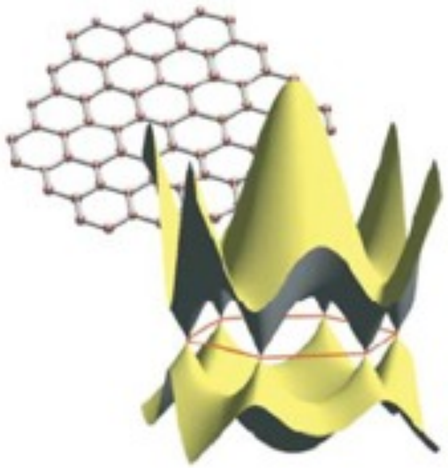
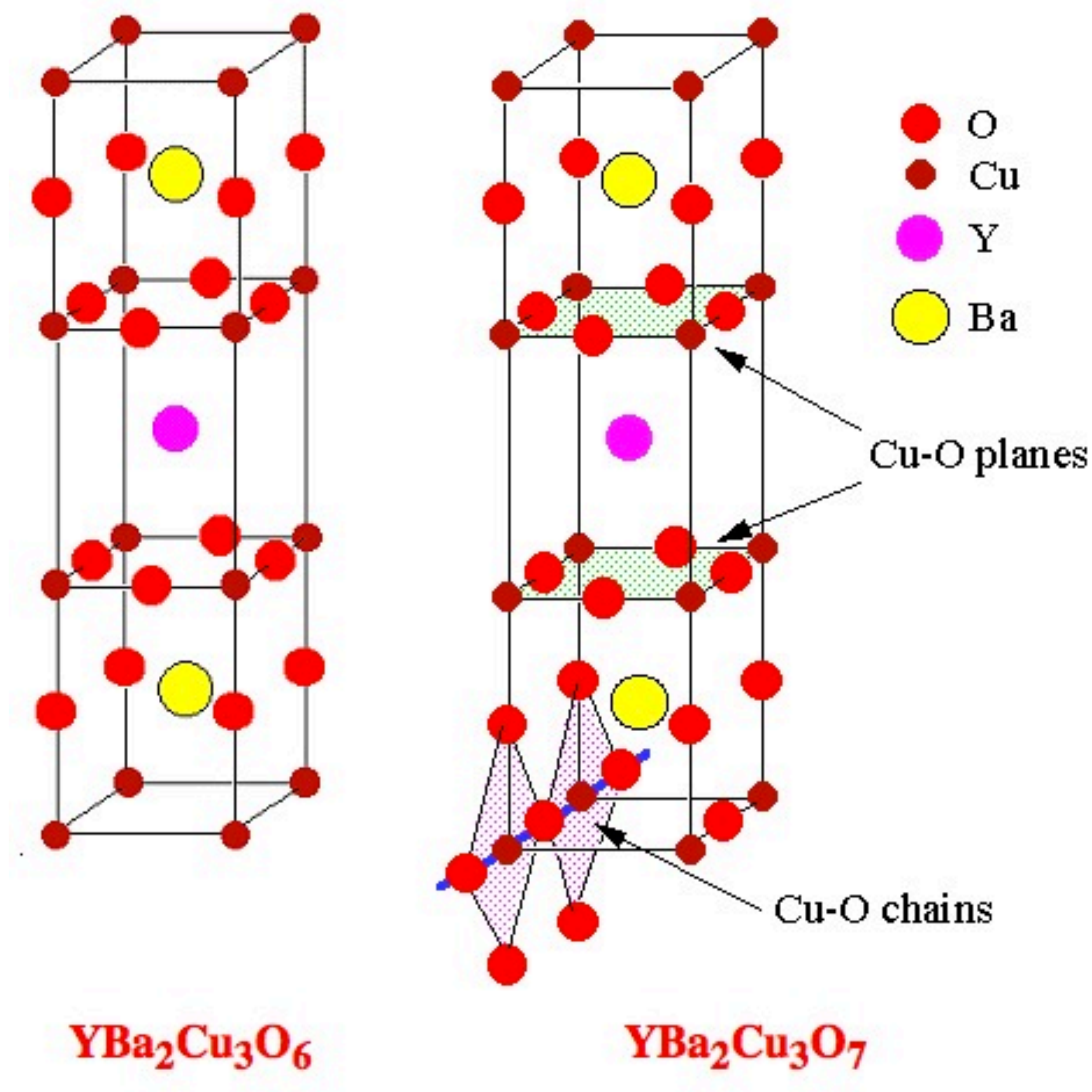


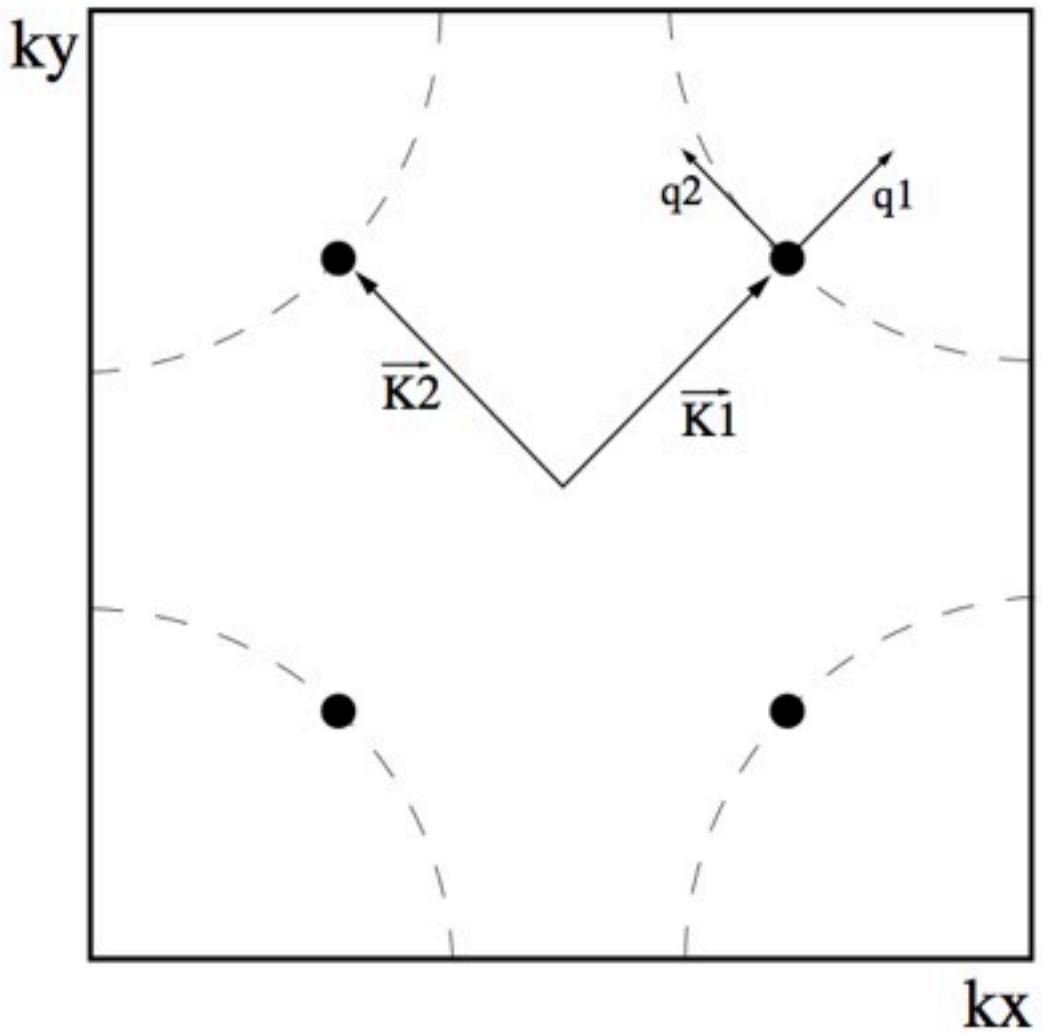
High-temperature superconductors

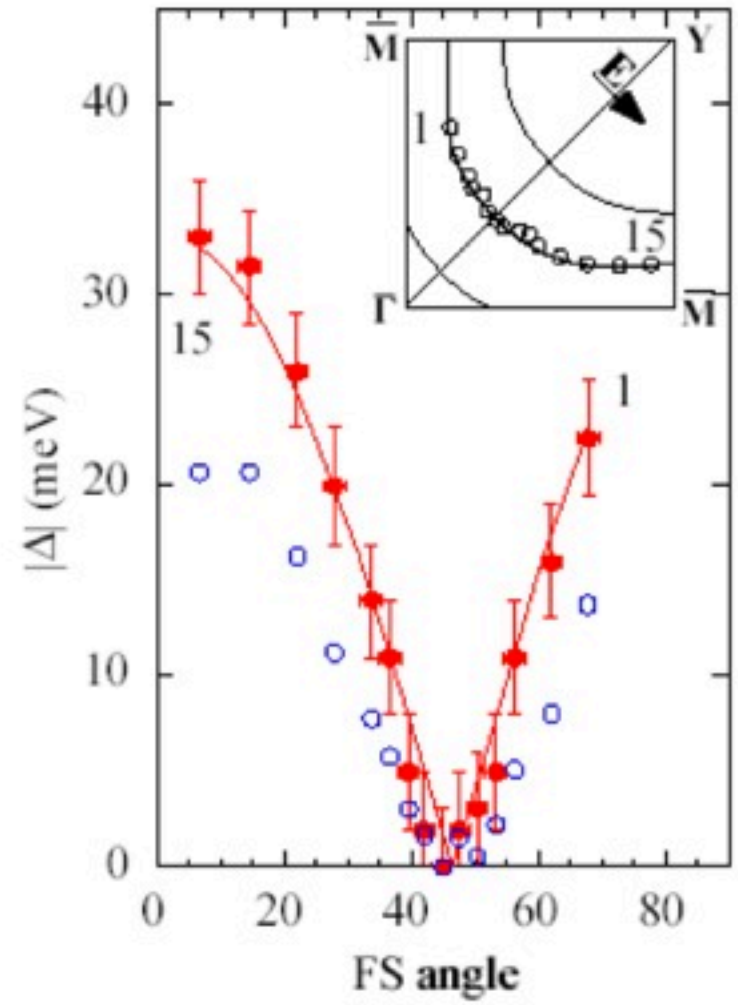
- superconducting CuO_2-planes
- doping is important
- normal state is insulator
- order parameter has d-wave symmetry



Fermi surface

Schematic Fermi surface:





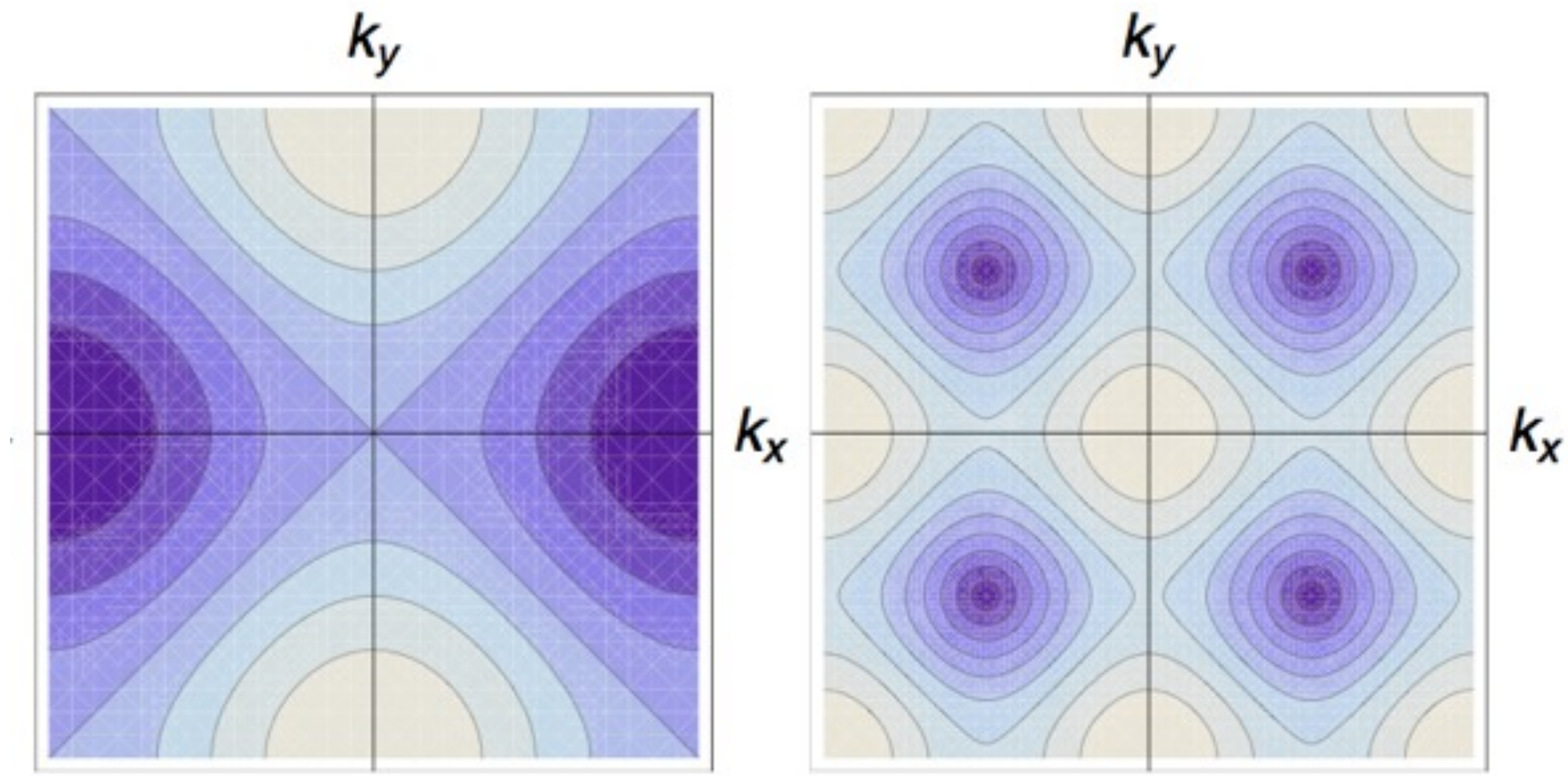
Ding et al. PRB 54 R9678 (1996)

- Gap has nodes on Fermi surface: d-wave symmetry
- Measured via ARPES experiments

Damascelli,Hussain, Shen, Rev. Mod. Phys. 75 (2003)

Disperson relation

Effective BCS-like Hamiltonian: 2d-Quasiparticles



$\Delta_{\vec{k}}$ $\qquad\qquad$ $\sqrt{\epsilon_{\vec{k}}^2 + \Delta_{\vec{k}}^2}$

- Linear expansion around nodes:

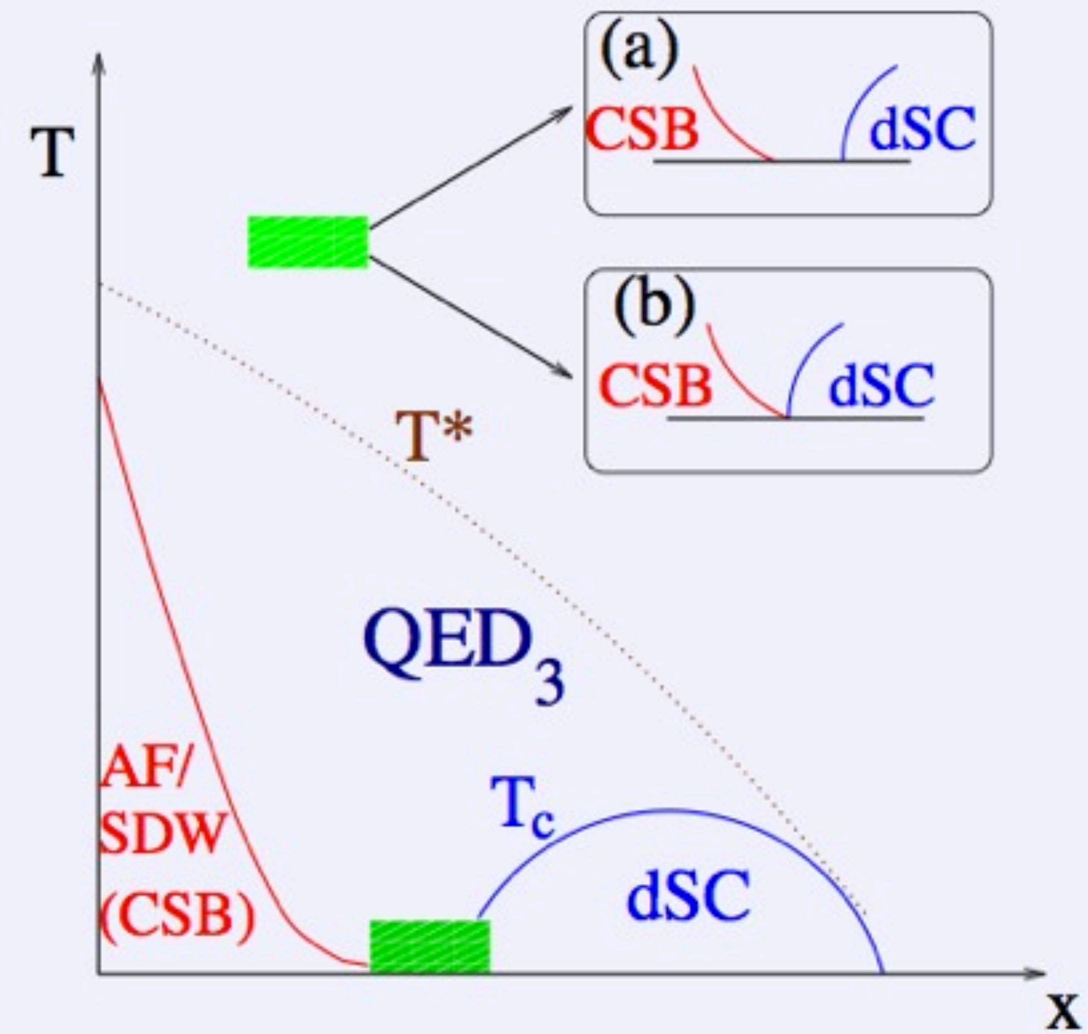
$$\epsilon_{\vec{k}} = v_f q_1 + \ldots$$
$$\Delta_{\vec{k}} = v_\Delta q_2 + \ldots$$

Anisotropy!

- Two neutral spin 1/2 quasiparticles combined in four-spinors interacting with topological excitations of the gap function

Three phases:

- DχSB $\leftrightarrow$ Anti-Ferromagn.
- Symmetric $\leftrightarrow$ pseudogap
- 'Higgs'-phase $\leftrightarrow$ superconducting

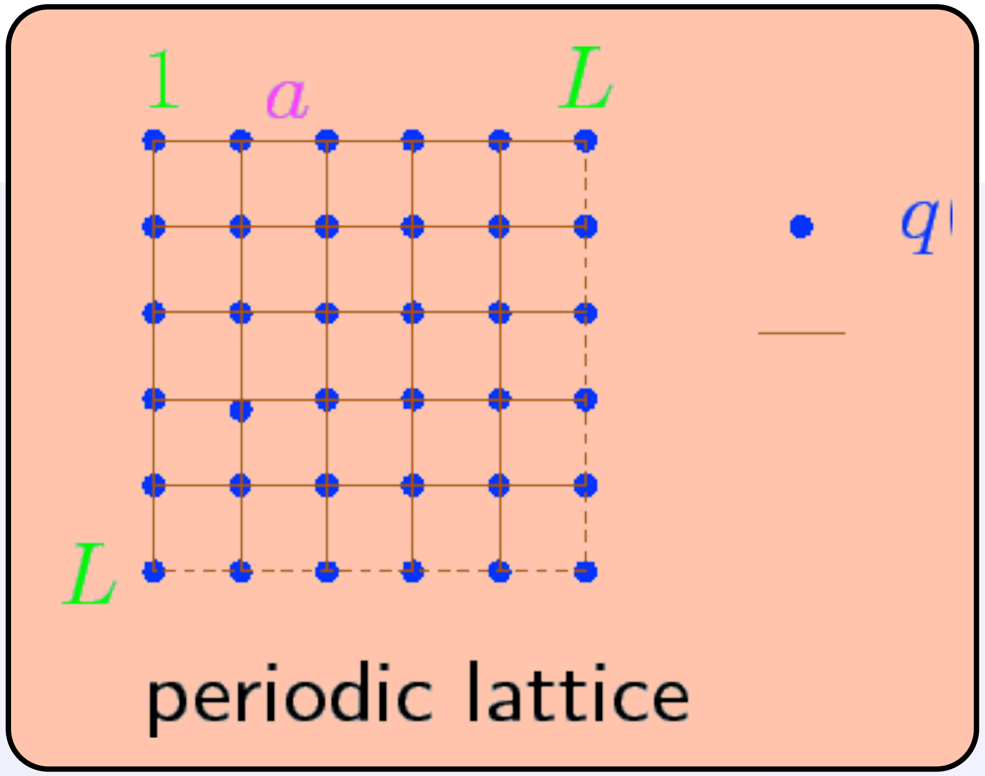


Open questions:

- critical number of fermion flavours $N_f^c \gtrless 2$?
- effects of anisotropies ?

Franz, Tesanovic and Vafek, PRB **66** (2002) 054535, PRB **65** (2002) 180511;
Franz and Tesanovic, PRL **87**, (2001) 257003.
Herbut, PRB **66**, 094504 (2002), PRL **88** (2002) 047006

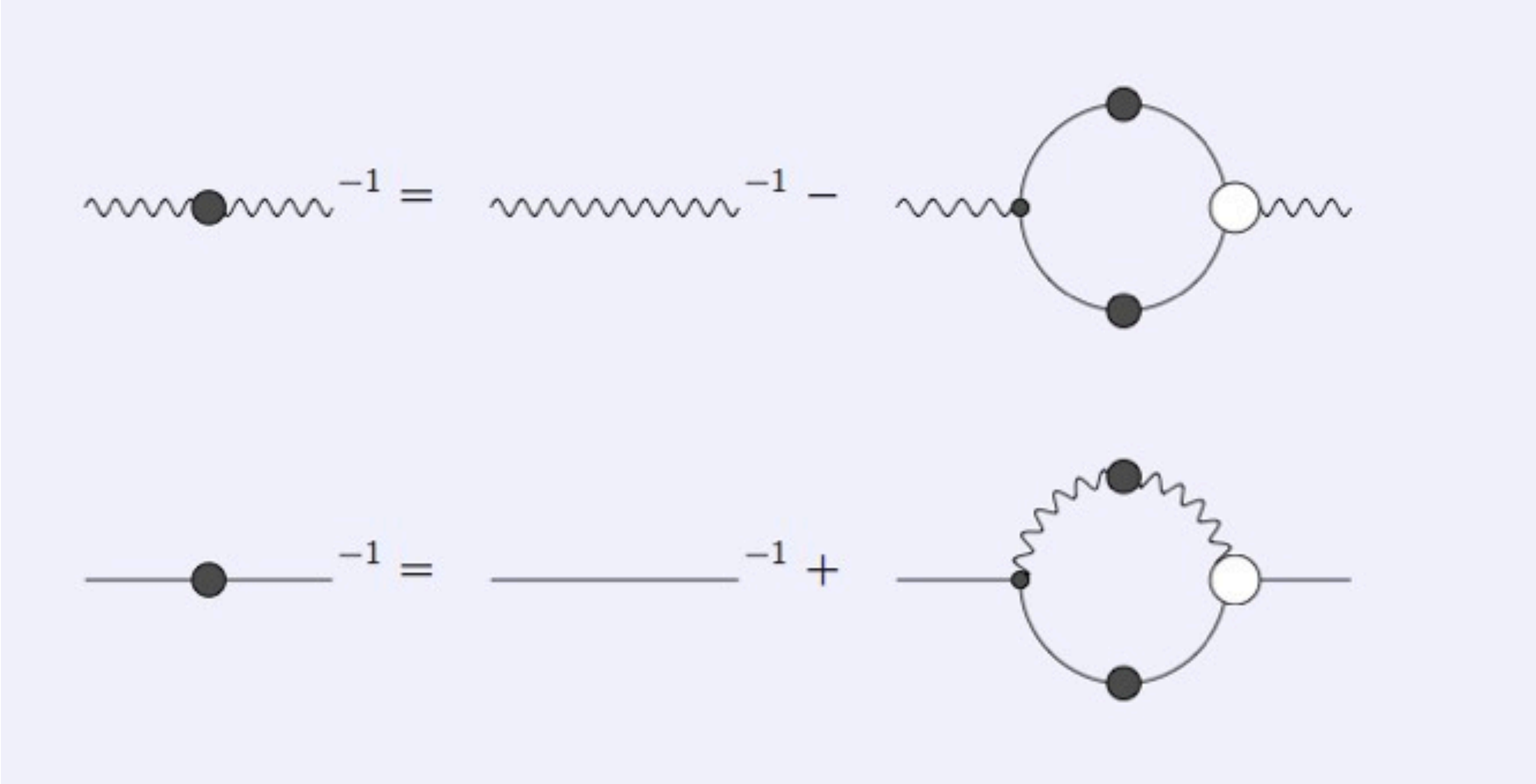
Lattice QCD vs. DSE/FRG: Complementary!



- Lattice simulations
  - Ab initio
  - Gauge invariant
- Functional approaches:
  Dyson-Schwinger equations (DSE)
  Functional renormalisation group (FRG)
  - Analytic solutions at small momenta
    CF, J. Pawlowski, PRD 80 (2009) 025023
  - Space-Time-Continuum
  - Chiral symmetry: light quarks and mesons
  - Multi-scale problems feasible: e.g. $(g\text{-}2)_\mu$
    T. Goecke, C.F., R. Williams, PLB 704 (2011); PRD 83 (2011)
  - Chemical potential: no sign problem

relevant for QED_3

DSEs of QED3 in Landau gauge



- **Transverse photon** $$D_{\mu\nu}(p^2) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{1}{Z_3 + \Pi(p^2)}$$
- **Quark propagator** $$S(p) = \frac{i\not{p}\, A(p^2) + B(p^2)}{p^2 A^2 + B^2}$$
- **Quark-photon vertex:** $$k_\mu \Gamma_\mu(p,q) = S^{-1}(p) - S^{-1}(q)$$

Ball, Chiu, PRD 22 (1980) 2542
Curtis, Pennington, PRD 42 (1990) 4165-4169

Analytic solutions: PT vs deep infrared

large momenta (PT): $\Pi(p^2) = \frac{\alpha}{p}$ $A(p^2) \to 1$ $B = 0$

small momenta:

Chirally broken phase	Chirally symmetric phase
$A(p^2) \sim const$	$A(p^2) \sim (p^2)^{\kappa}$
$\Pi(p^2) \sim const$	$\Pi(p^2) \sim (p^2)^{-1/2-\kappa}$
$B(p^2) \sim const$	$B(p^2) \sim 0$

C. F., Alkofer, Dahm and Maris PRD **70**, 073007 (2004)

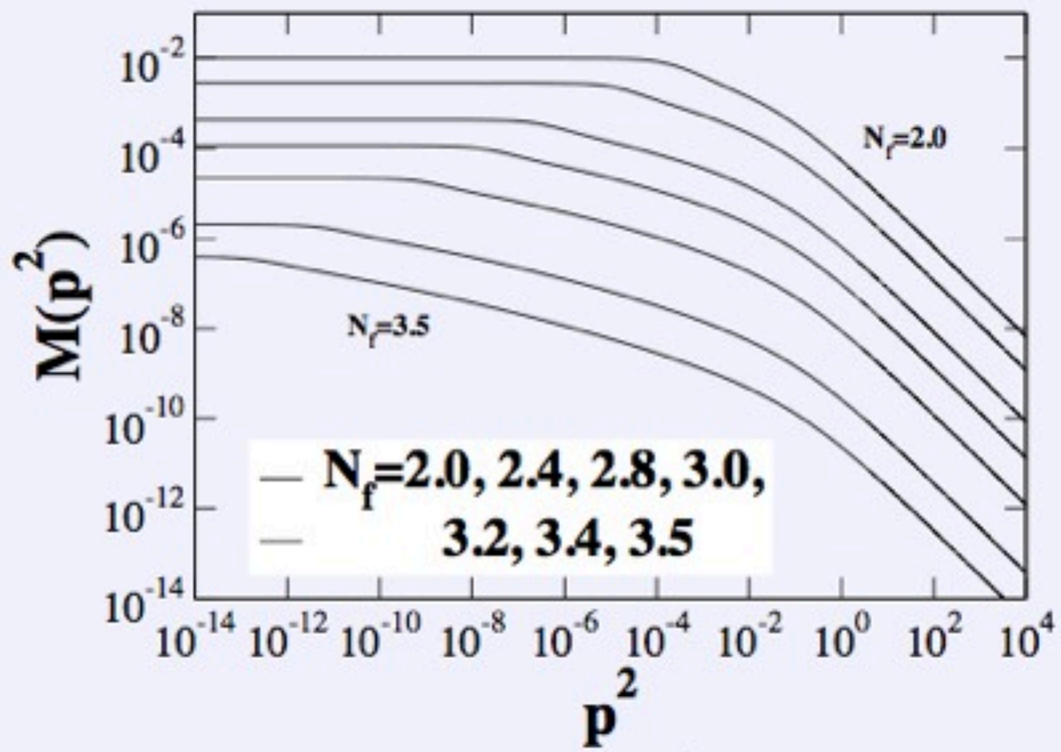
Deconfined

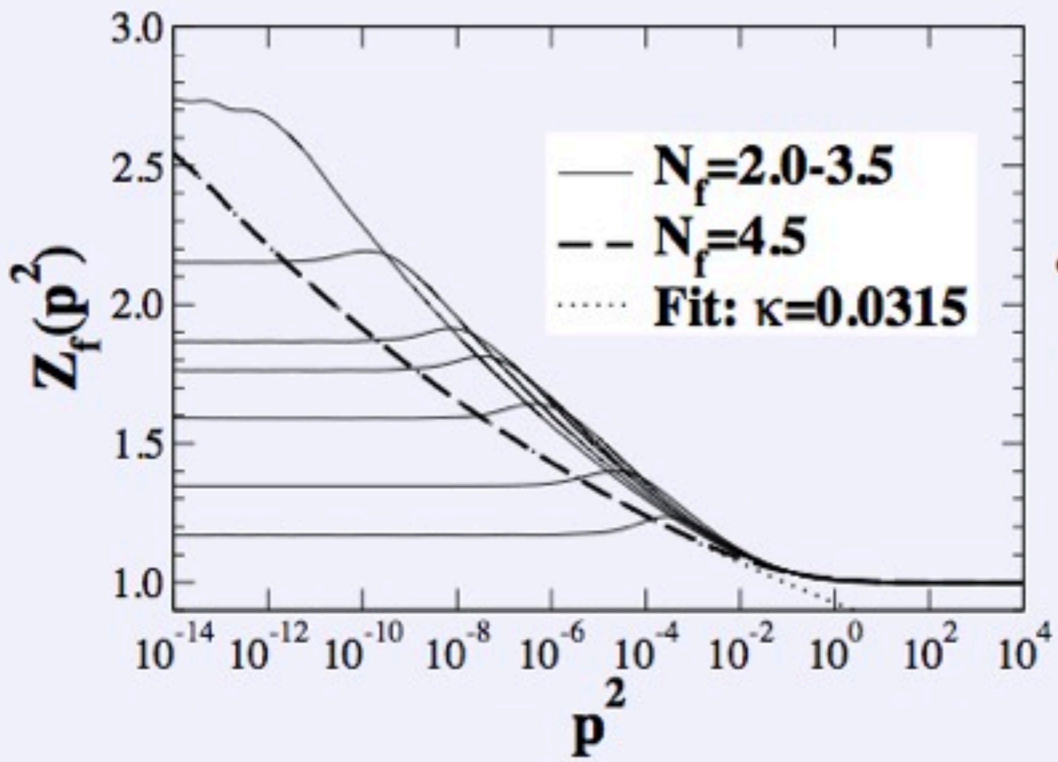
- Symmetric phase: 'almost-conformal' infrared behaviour with running coupling:

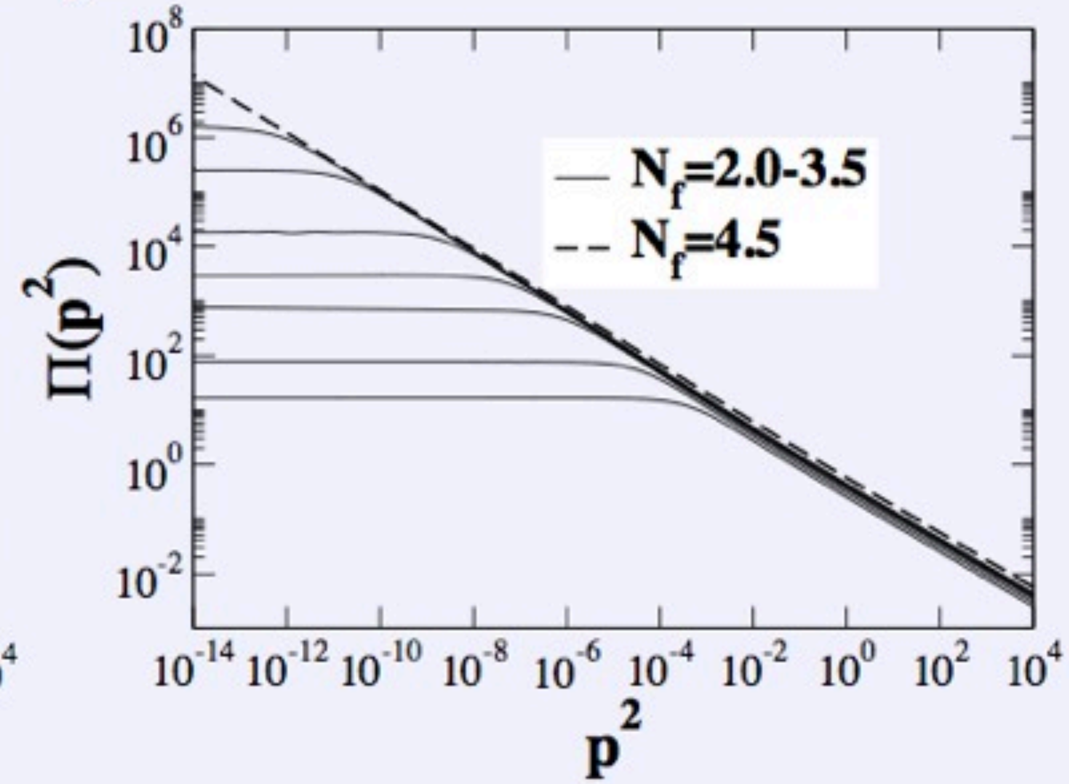
$$\alpha(p^2) = \alpha/(p(1+\Pi(p)) \sim (p^2)^{\kappa}$$

$$\kappa = \frac{0.115}{N_f} + \frac{0.044}{N_f^2} + O(1/N_f^3)$$

Numerical solutions

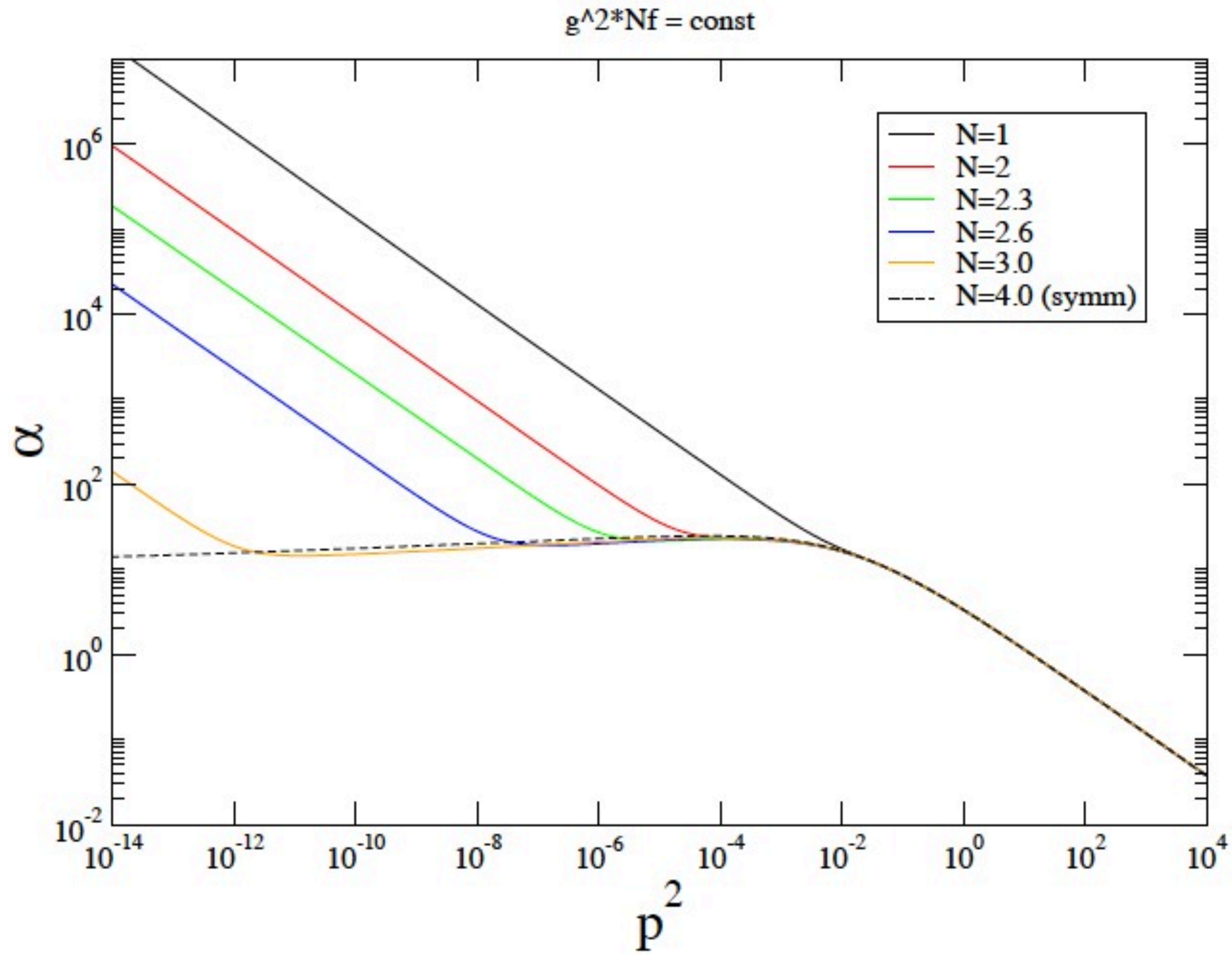






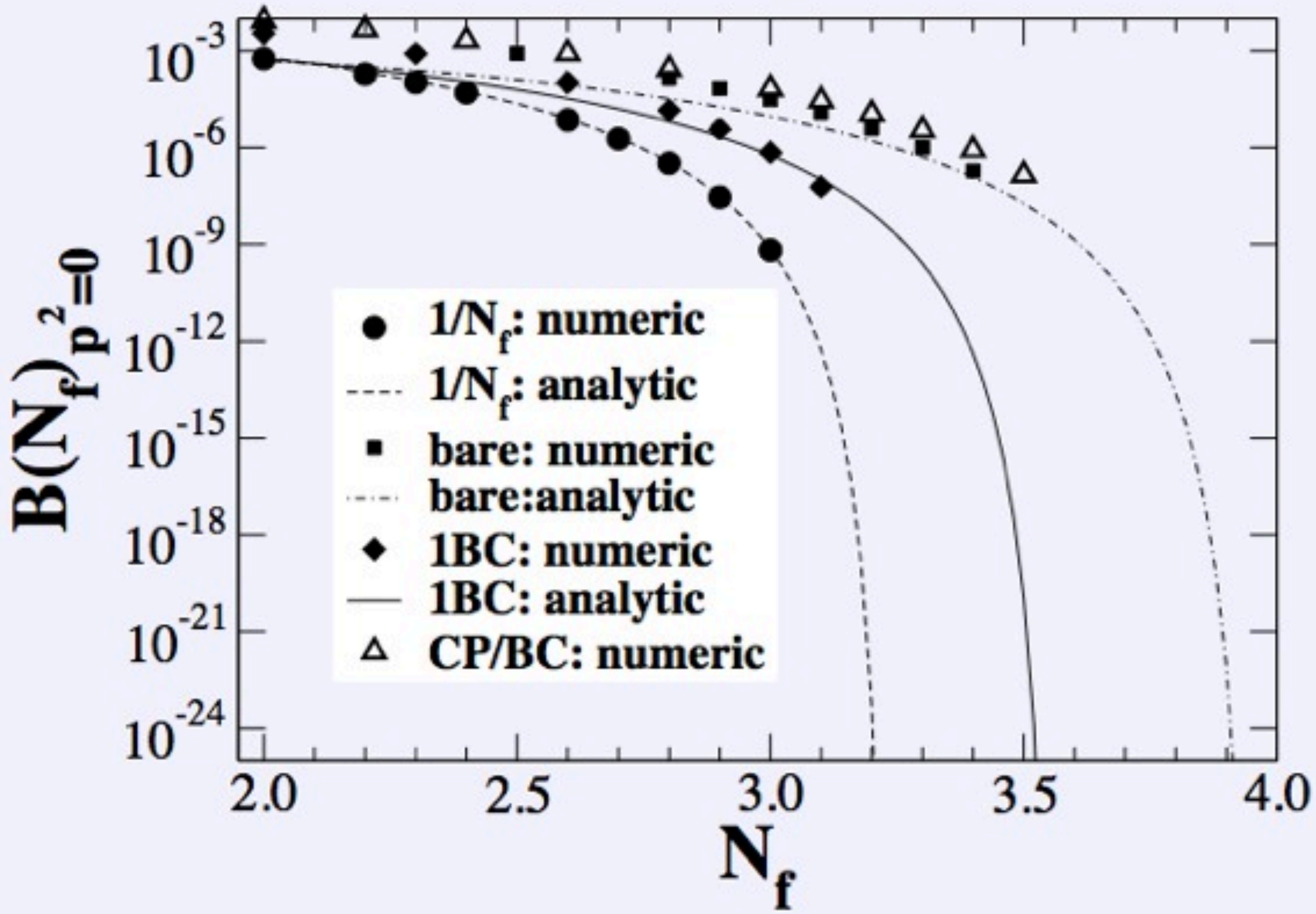
C. F., Alkofer, Dahm and Maris PRD **70**, 073007 (2004)

Running coupling



- broken phase: IR scale set by generated fermion mass

Phase transition: Miransky scaling



$$B(N_f)_{p^2=0} \sim \exp\left(\frac{-2\pi N_f}{\sqrt{N_f(N_f - N_f^c)}}\right); \qquad N_f^c > 2$$

C. F., Alkofer, Dahm and Maris PRD **70**, 073007 (2004)

Friday, November 30, 2012

Finite volume: DSEs on a torus

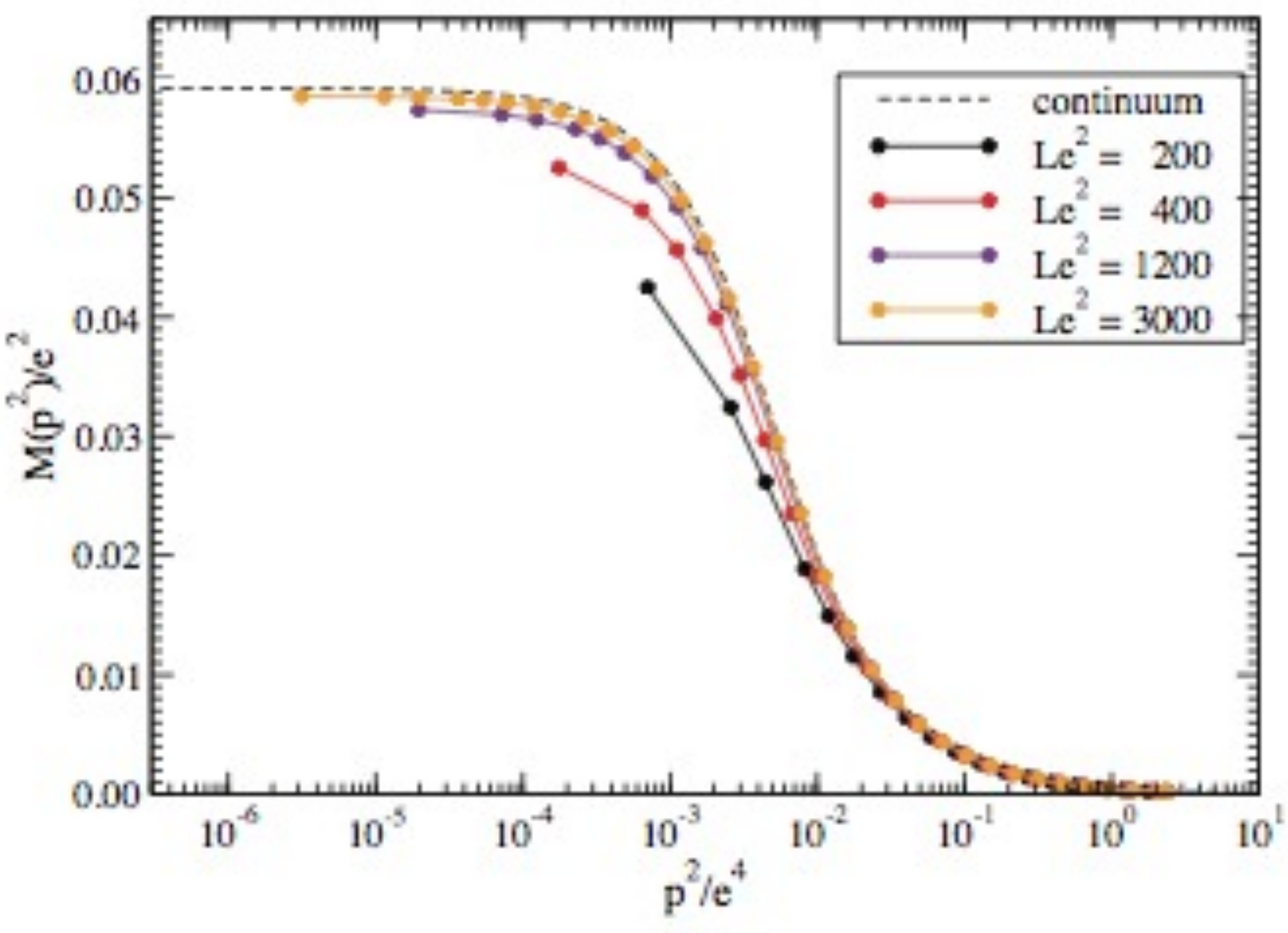
- critical N_f much smaller on lattice - why ?
- separation of scales: dynamical mass << α - volume effects ?

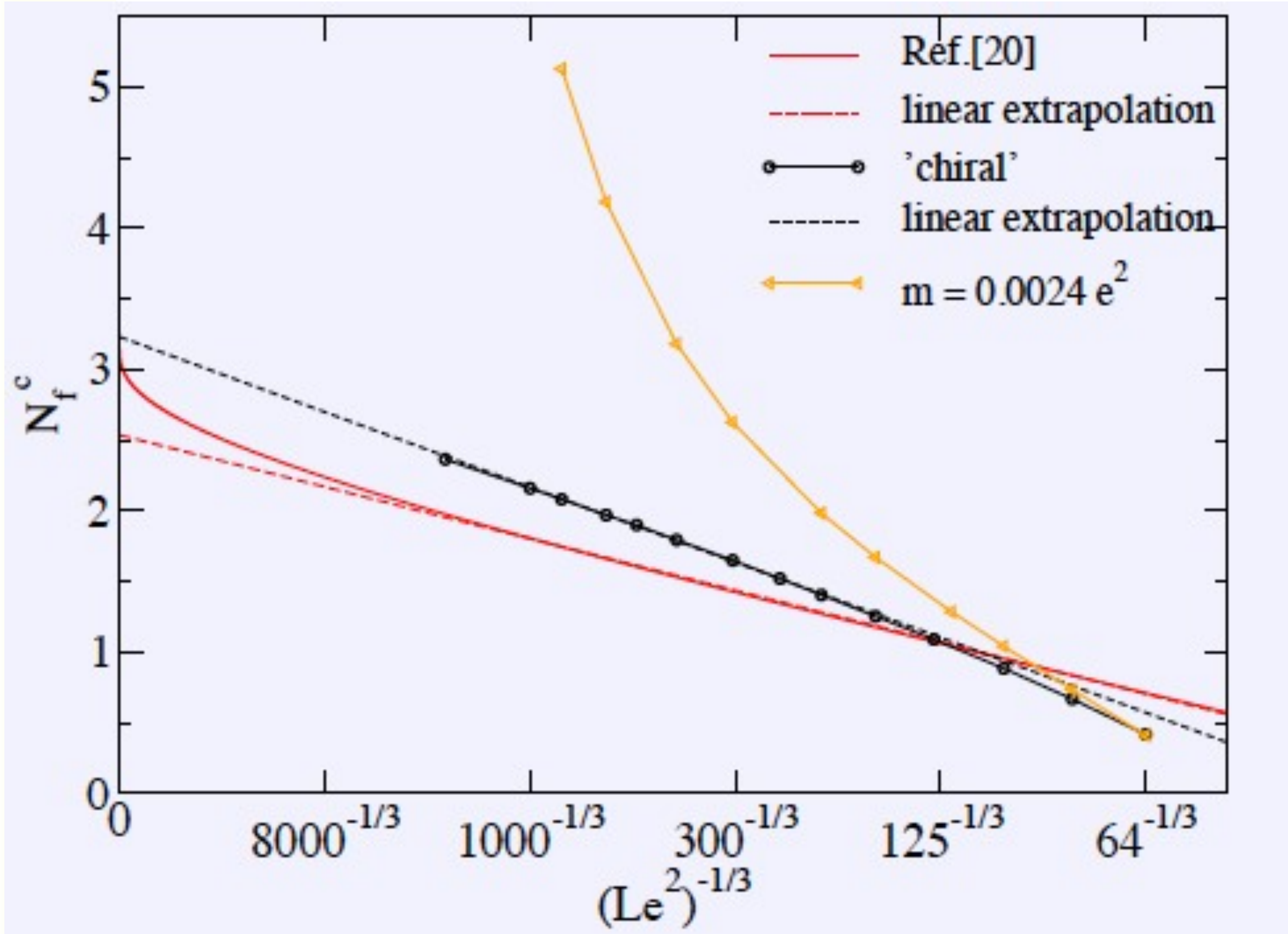
- Put DSEs on torus with (anti-)periodic boundary conditions
- Integrals become Matsubara sums

$$\int d^4p \to \left(\frac{2\pi}{L}\right)^4 \sum_{j_1, j_2, j_3, j_4} = \left(\frac{2\pi}{L}\right)^4 \sum_{j,l}$$

Formalism well known from QCD...

C.F., Alkofer and Reinhardt, PRD 65 (2002) 094008
C.F., Gruter and Alkofer, Annals Phys.321 (2006) 1918
C.F. and Pennington, PRD 73 (2006) 034029
C.F., Maas, Pawlowski and von Smekal, Annals Phys. 322 (2007) 2916
Luecker, C.F. and Williams, PRD 81 (2010) 094005





Continuum: $N_f^c = 3.5 - 4.0$

Lattice: $N_f^c \approx 1.5$

} Volume effect!

Goecke, C.F. and Williams, PRB **79**, 064513 (2009)
'Ref[20]': Gusynin and Reenders, PRD **68** (2003) 025017
Hands, Kogut, Scorzato and Strouthos, PRB **70** (2004) 104501
Strouthos and Kogut, arXiv:0808.2714 [cond-mat.supr-con]

Anisotropy

- Recall: high temperature superconductors governed by (large) anisotropy

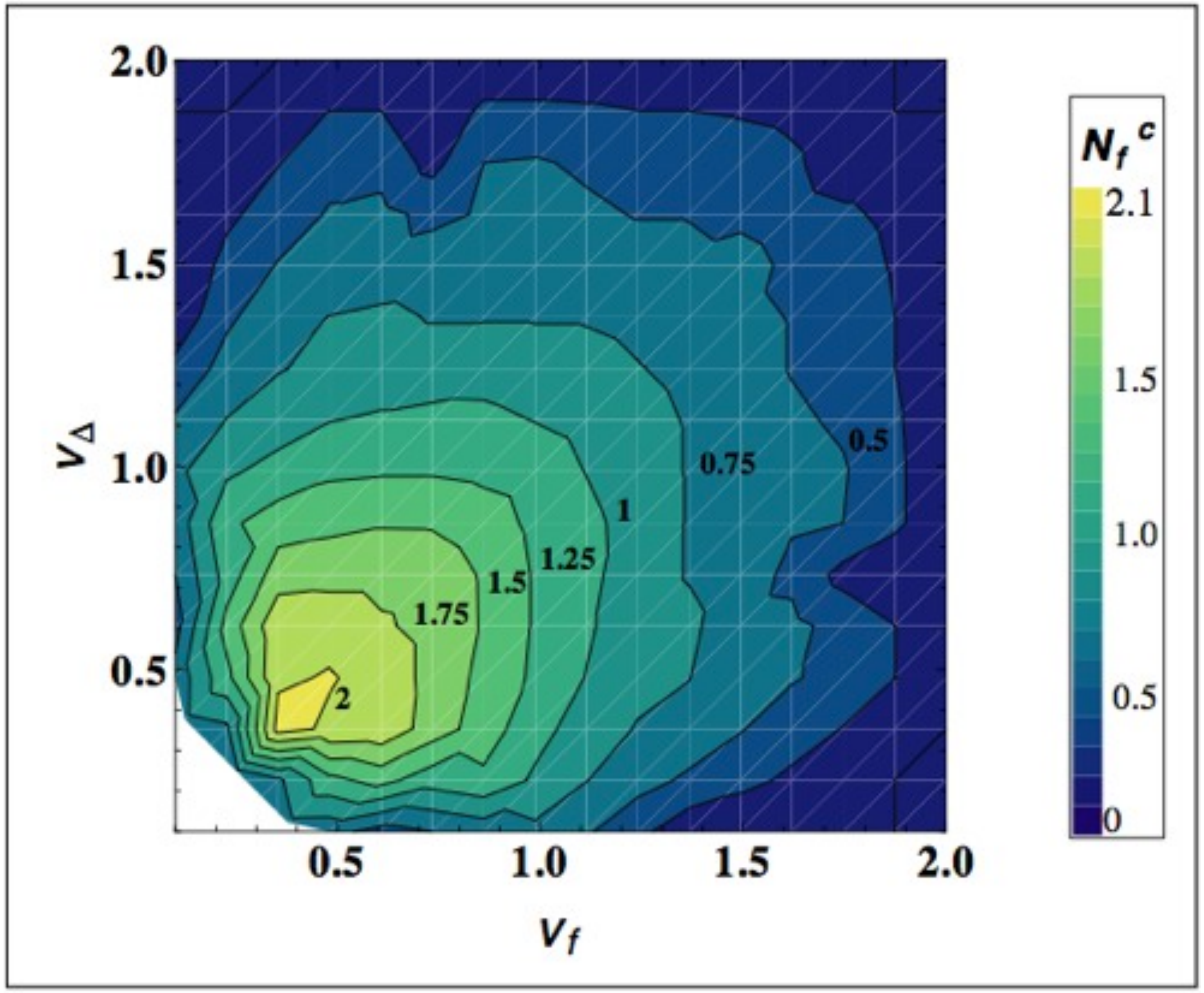
$$\epsilon_{\vec{k}} = v_f q_1 + \ldots$$
$$\Delta_{\vec{k}} = v_\Delta q_2 + \ldots$$

- Define metric-like quantity...

$$(g^{\mu\nu}) = \begin{pmatrix} 1 & & \\ & (v_F)^2 & \\ & & (v_\Delta)^2 \end{pmatrix}$$

- ... and modify Lagrangian accordingly

$$S = \int d^3x \left(\sum_{N_f} \bar{\Psi} i \gamma_\mu \sqrt{g_{\mu\nu}} (\partial_\nu + i A_\nu) \Psi - \frac{1}{4} F_{\mu\nu} F_{\mu\nu} \right)$$



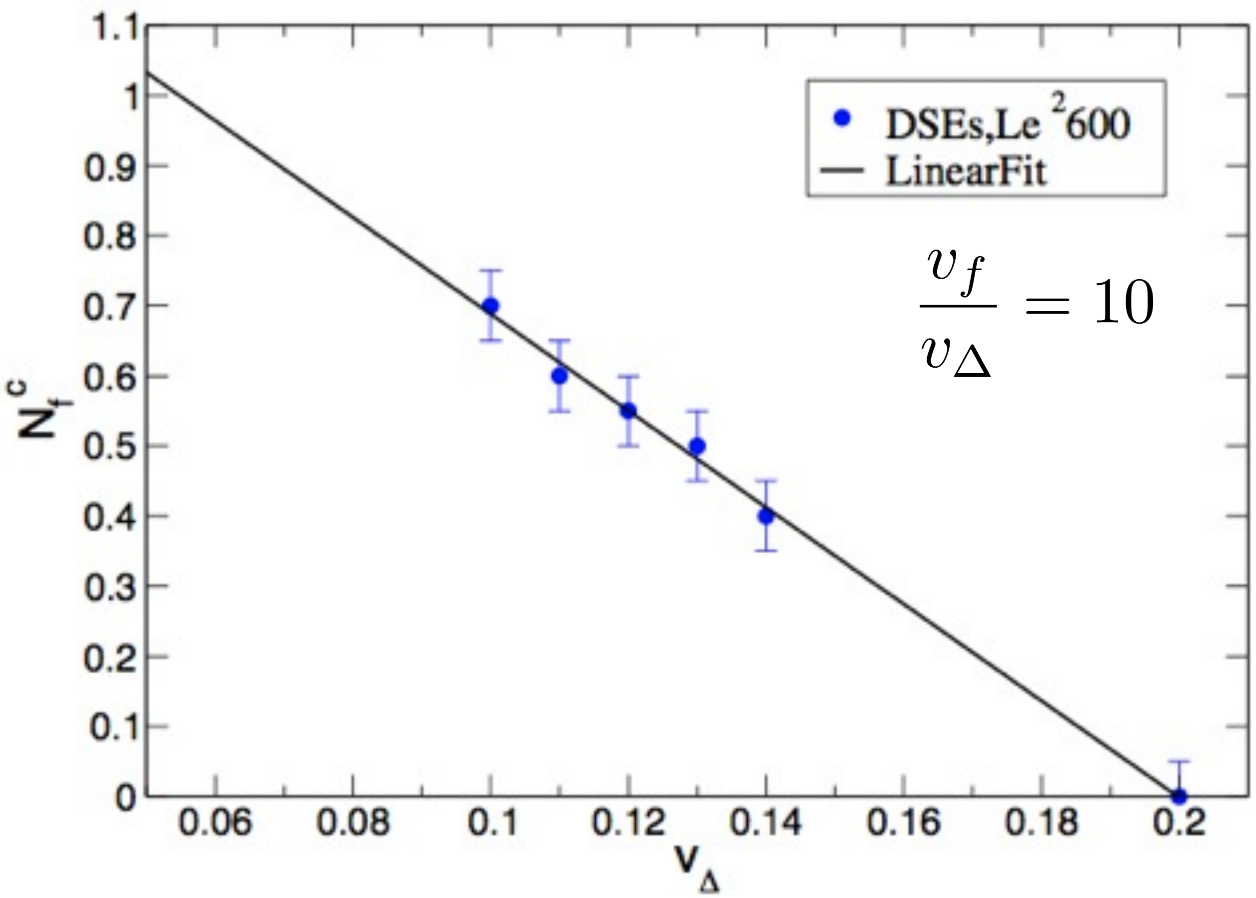
Bonnet, C.F. and Williams, PRB 84 (2011) 024520

- Assume isotropic volume effects:

$$N_f^c(V=\infty) \approx 3N_f^c(V)$$

- v_f > c anticipated

$$\rightarrow N_f^c(V=\infty) > 2$$



Experiment:

$$\mathrm{YBa_2Cu_3O_7} : \frac{v_f}{v_\Delta} = 14$$

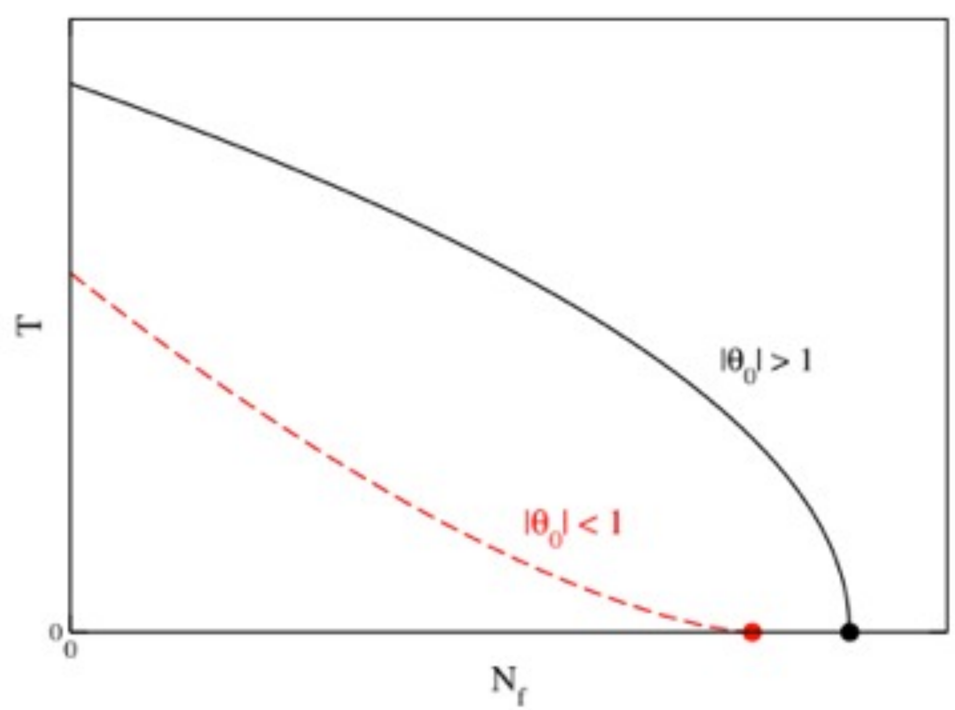
$$\mathrm{Bi_2Sr_2CaCu_2O_8} : \frac{v_f}{v_\Delta} = 19$$

Chiao et al., PRB 62 (2000) 3554

Universal power law corrections to Miransky scaling:

$$T_{cr} \sim k_0 |N^c_{f,0} - N_f|^{-\frac{1}{\Theta_0}} \exp\left(- \frac{a}{\sqrt{|N^c_{f,0} - N_f|}} \right)$$

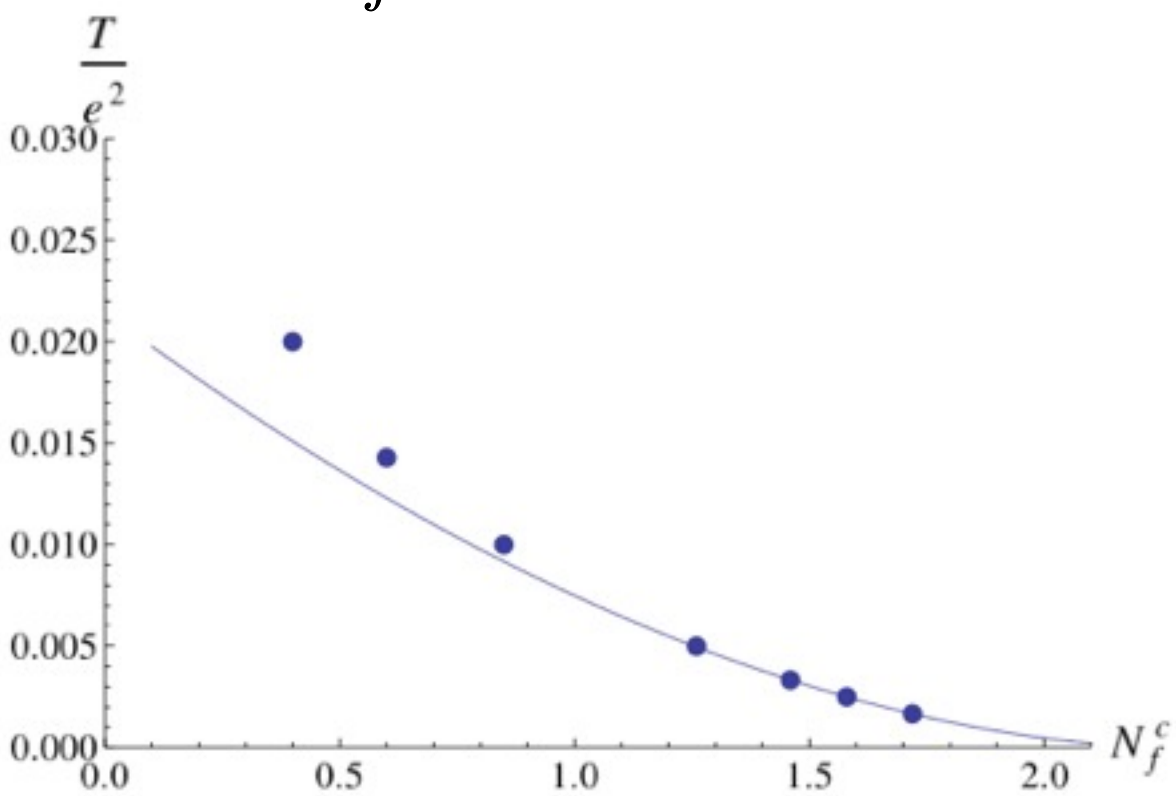
Two generic cases for critical exponent:



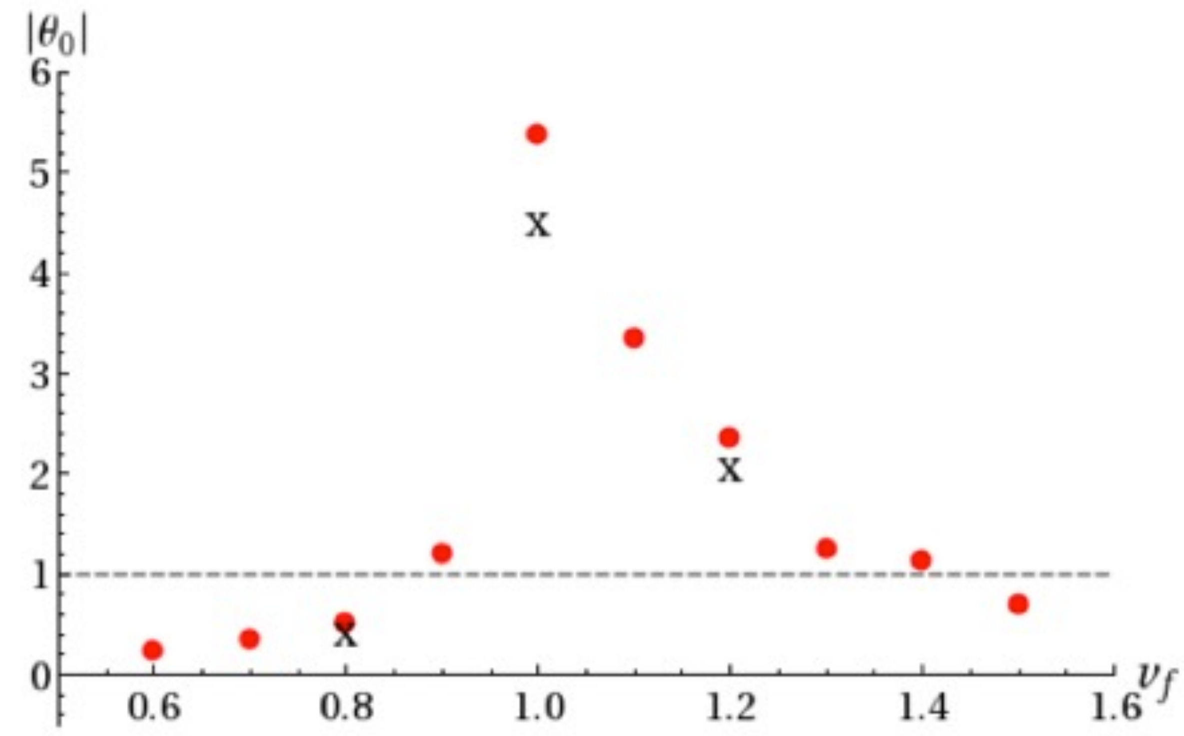
Braun, CF, Gies, PRD 84, 034045 (2011)
Braun, Gies, JHEP 1005 (2010) 060

Finite T and scaling in anisotropic QED3

$$v_f = v_\Delta = 0.8$$



$$v_f = v_\Delta$$



- Scaling observed
- Critical exponent is controlled by anisotropy
- Physical case numerically not yet accessible

Bonnet and CF, PLB 718, (2012) 532

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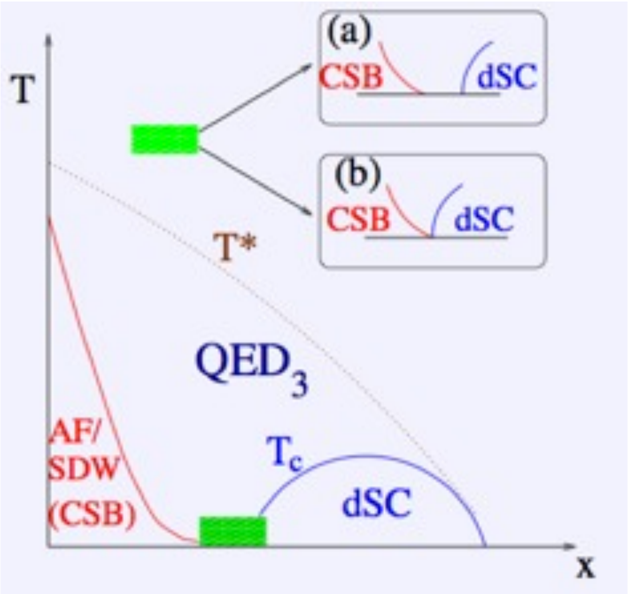

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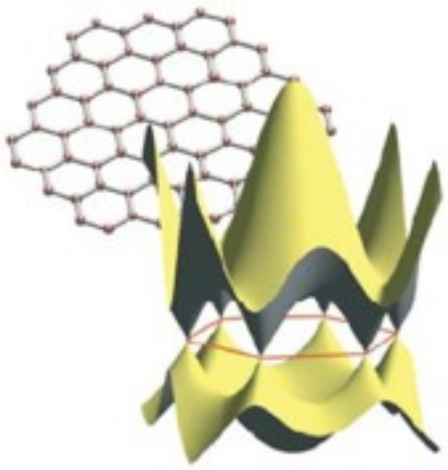
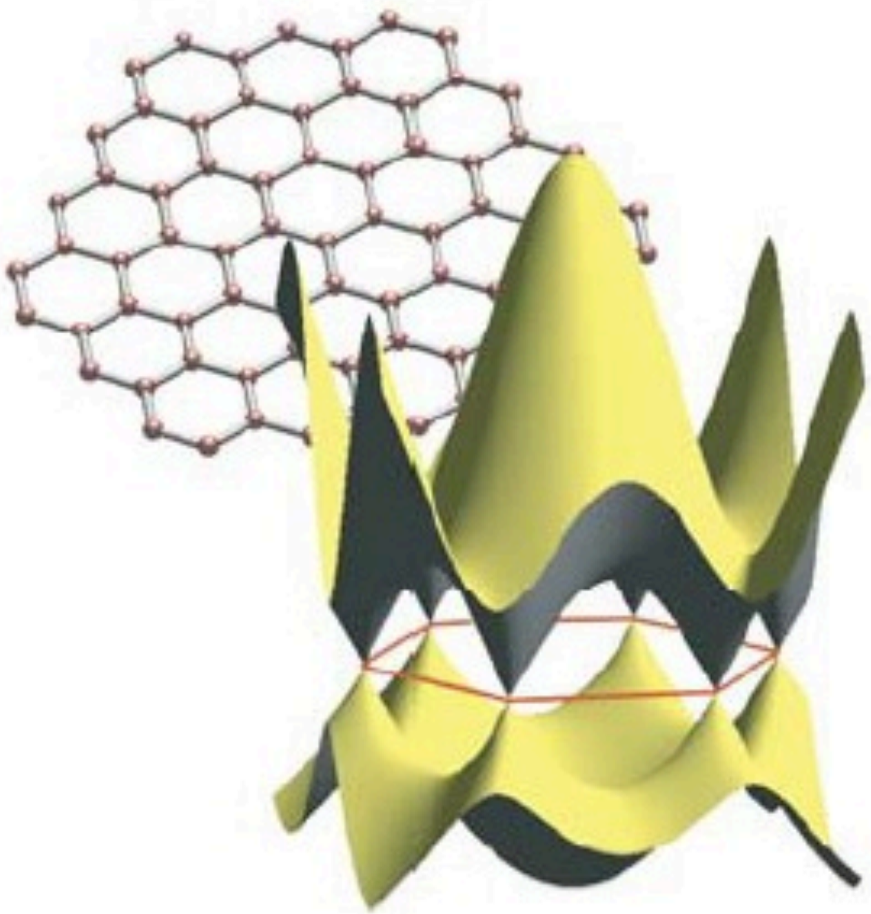


Graphene

- a single layer of carbon atoms



→ honeycomb lattice of carbon atoms in a two-dimensional plane

→ electronic band structure with two bands (yellow) that intersect at corners of a hexagonal Brillouin zone (red)

www.als.lbl.gov/als/science/sci_archive/154graphene.html

- unusual nature of low-energy excitations

→ ***massless fermions with linear dispersion***:

$$\varepsilon \sim v\, p \quad , \quad v \sim c/300$$

[Wallace, 1947; Semenoff, 1984]

Quantum critical point

- graphene analogue of fine structure constant

$$\alpha_g \sim e^2 / (4\pi\varepsilon_0\hbar v)$$

→ coupling reduced on a substrate with large dielectric constant: weakly-coupled semimetallic phase

→ freely suspended in vacuum or air:

$$\alpha_g \gtrsim 1$$ ***strong coupling!***

- if substrate is removed:

→ dynamical ***semimetal-insulator transition***?

→ phase transition involving ***chiral symmetry breaking***?

→ quantum critical point showing properties of a nearly ***perfect fluid***?

Quantum critical point

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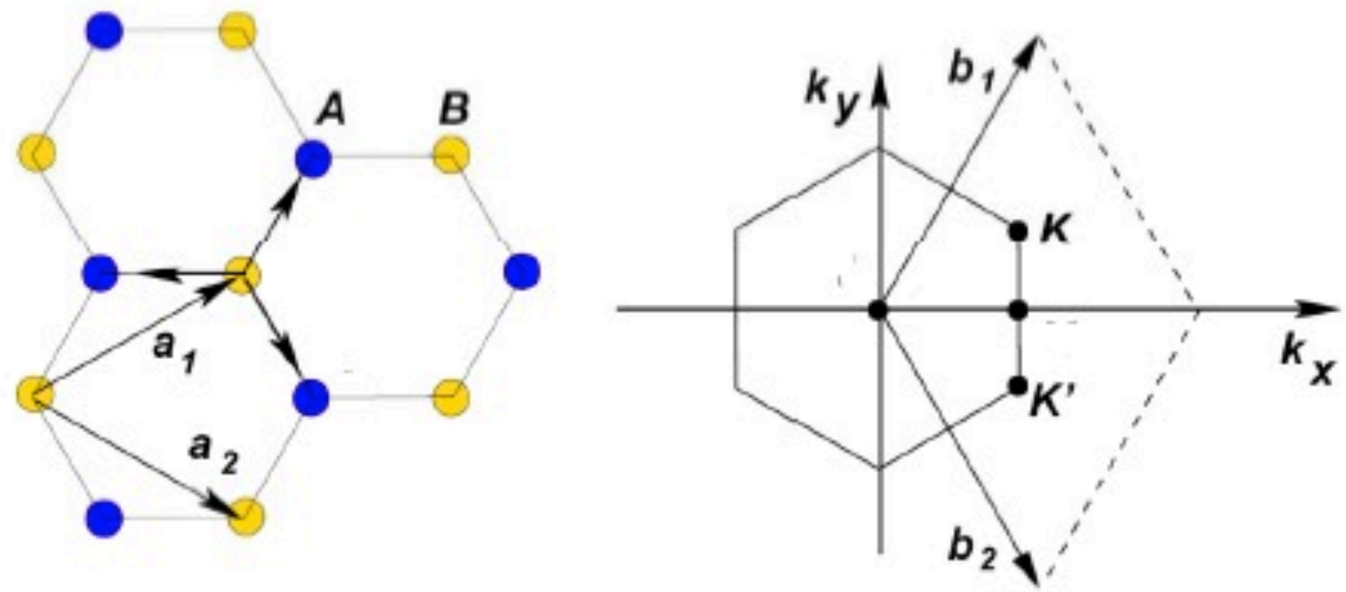
Gamayun et al, 2007, 2010; Drut, Lähde, 2009; Son,2007

QED$_3$ as an effective theory for graphene

- low-energy degrees of freedom

→ 2 atoms per unit cell × 2 zeros per zone × 2 spin components per electron



→ ***N* = 2 *four-component massless Dirac fermions*** with action (ħ=1)

$$S = -\sum_{a=1}^{N}\int dt\, d^2x\,(\bar{\psi}_a\gamma^0\partial_0\psi_a + v\bar{\psi}_a\gamma^i\partial_i\psi_a + iA_0\bar{\psi}_a\gamma^0\psi_a) + \frac{1}{2g^2}\int dt\, d^3x\,(\partial_i A_0)^2$$

- velocity of photon practically infinite compared to fermion velocity (c/300)

→ ***instantaneous Coulomb interaction***

Herbut, PRL 97 (2006) 146401

DSE for Fermions

$$S^{-1}(p_0,\vec{p}) = p_0\gamma^0 - \vec{\not{p}}\,A(p_0,\vec{p}) - B(p_0,\vec{p})$$

- Vector dressing function A renormalizes fermi velocity

$$v_f(p) = v_f A(p)$$

- Solve DSEs with bare vertex and one-loop photon

$$D(q_0,\vec{q}) = \frac{2\pi}{|\vec{q}| + \Pi(q_0,\vec{q})} \qquad \Pi(q_0,\vec{q}) = \frac{\pi e^2 N_f}{4\varepsilon}\frac{\vec{q}^2}{\sqrt{\hbar^2 v_F^2 \vec{q}^2 - q_0^2}}$$

DSE for Fermions

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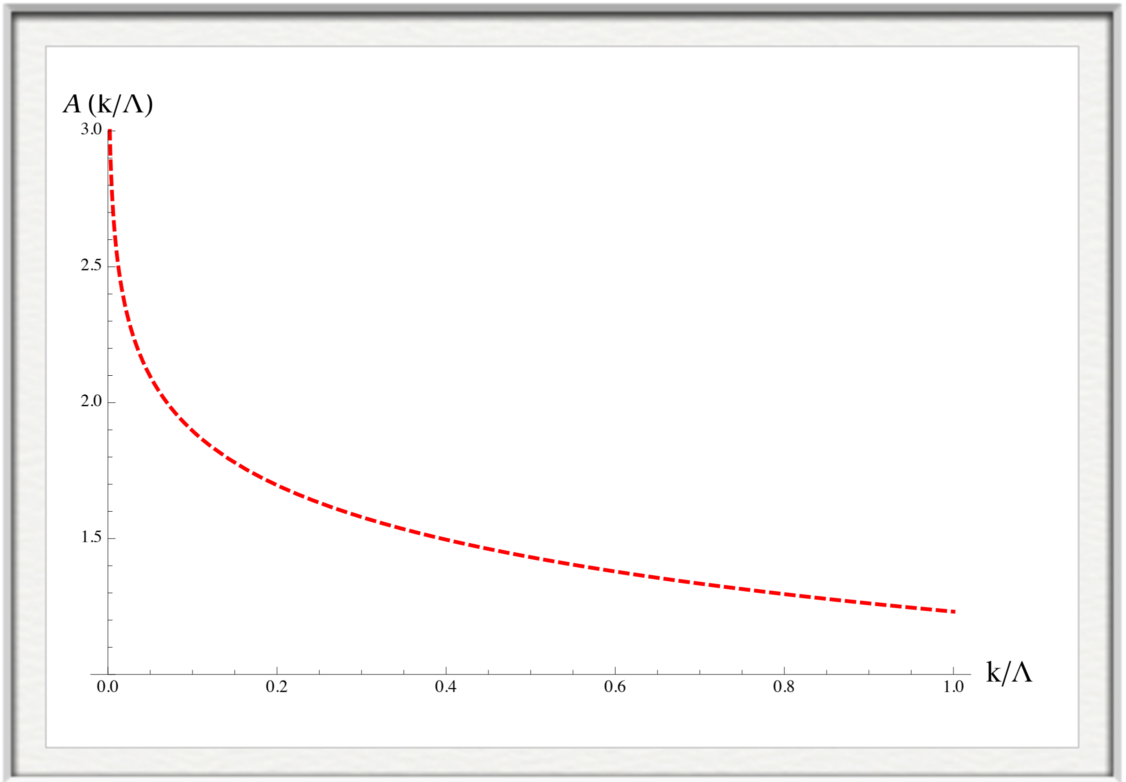
Gonzalez, Vozmediano et al, 1994

- Solve DSEs with bare vertex and one-loop photon

$$D(q_0,\vec{q}) = \frac{2\pi}{|\vec{q}| + \Pi(q_0,\vec{q})} \qquad \Pi(q_0,\vec{q}) = \frac{\pi e^2 N_f}{4\varepsilon}\frac{\vec{q}^{\,2}}{\sqrt{\hbar^2 v_F^2 \vec{q}^{\,2} - q_0^2}}$$

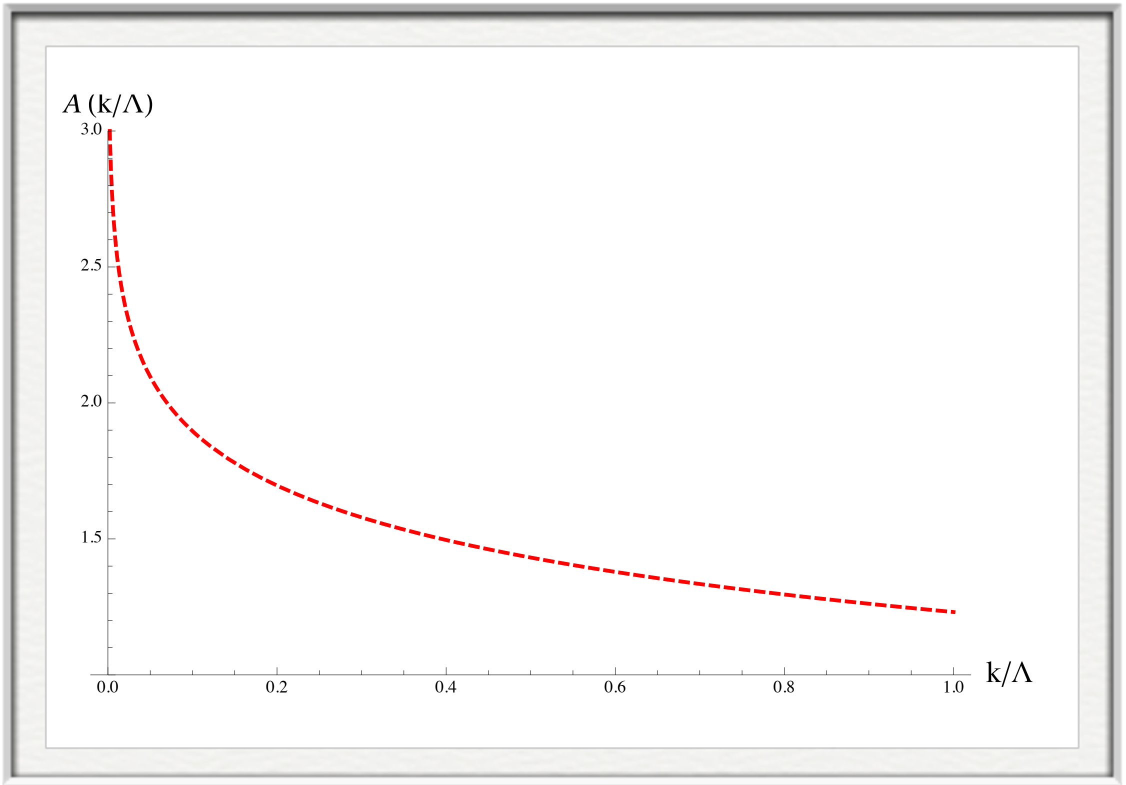
Results



Analytic result in symmeric phase:

$$v_F(p) = v_F\left[1 + f_1(\alpha)\ln\frac{\Lambda}{p} + f_2(\alpha)\right]$$

- Symmetric phase: infrared divergence → experiment ?!
- Critical coupling: $\alpha_c \sim 2.7$ → larger than value 2.19 for suspended graphene !



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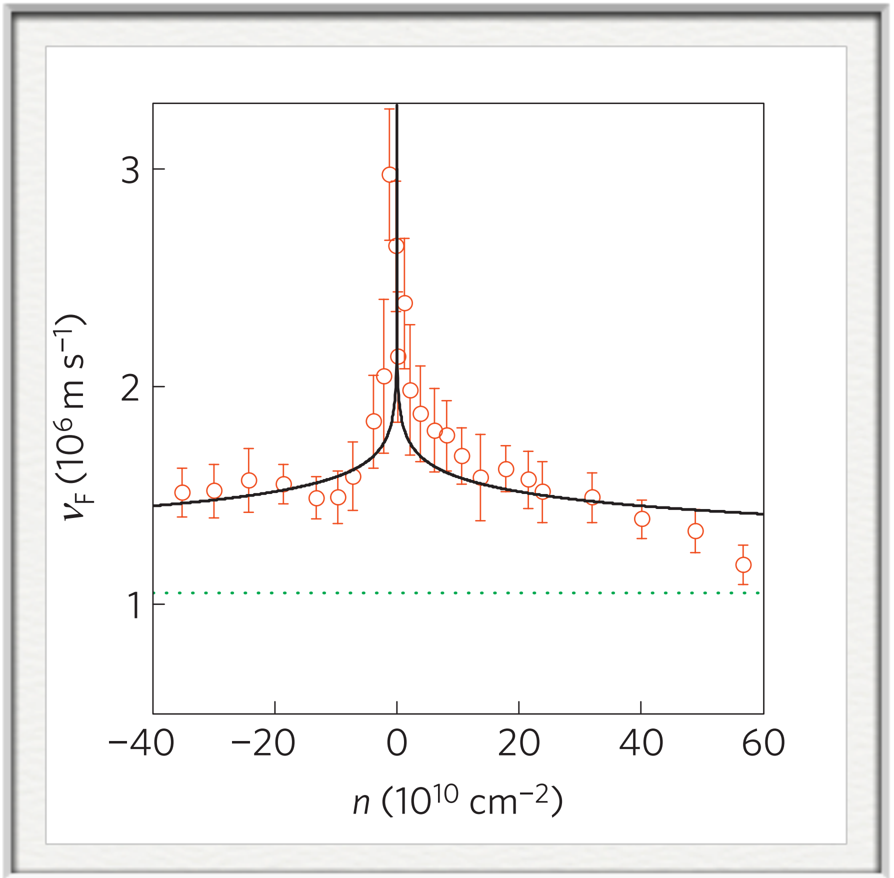


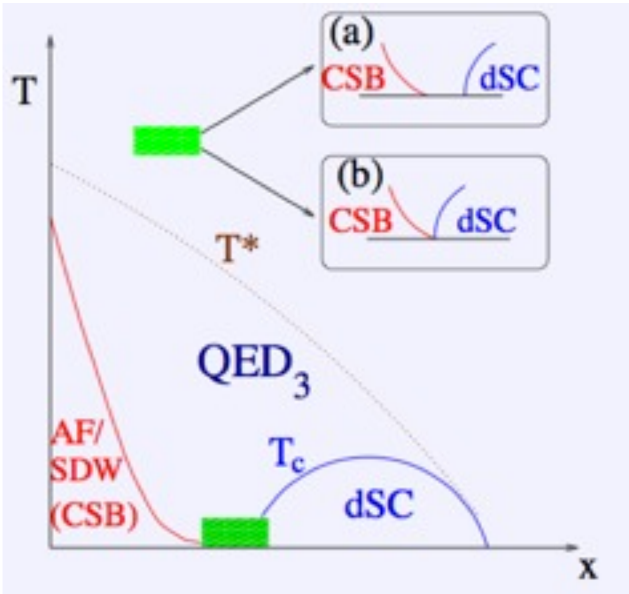
Popovici, CF, von Smekal, in preparation

Elias, Gorbachev, Mayorov et al, Nature Physics , 2011

Summary

QED$_3$

- Analytic and numerical solutions from DSEs
- Transition at N_{fc} with Miransky scaling
- Large volume effects: extremely difficult for lattice
- Anisotropies taken into account
- $N_{fc} > 2$: at zero temperature direct transition from dSC to AF



instantaneous QED$_3$

- large effects due to running fermion velocity
- critical coupling too large: suspended graphene remains semi-metalic