$\mathcal{N}$=1 SUPERSYMMETRIC SU(3) GAUGE THEORY WITH A TWIST

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MOTIVATION

- The **standard model** of particle physics is only an effective theory with unsolved problems
  $\Rightarrow$ Search for a more fundamental theory:
  One possibility is the **extension with supersymmetry**
- Here we focus on the strong interaction sector and study an important building block of supersymmetric quantum chromodynamics (Super-QCD): **4-dimensional $\mathcal{N}$=1 Super-Yang-Mills theory (SYM) with gauge group SU(3)**

THE THEORY

- The $\mathcal{N}=1$ SYM theory contains two **fields**:
  - Gauge potential $A_\mu(x)$
  - Majorana field $\lambda(x)$

  both transforming in the **adjoint representation**
- They describe two members of the same super-multiplet:
  - A gauge boson, called **gluon**
  - Its fermionic superpartner, named **gluino**
- **Supersymmetry** transforms those fields into each other:

$$\delta_\epsilon A_\mu = \mathrm{i}\bar{\epsilon}\gamma_\mu\lambda, \quad \delta_\epsilon\lambda = \mathrm{i}\Sigma_{\mu\nu}F^{\mu\nu}\epsilon$$

- Continuum on-shell **Lagrange density**:

$$\mathcal{L}_{\mathsf{SYM}} = \mathrm{tr}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mathrm{i}}{2}\bar{\lambda}\not{D}\lambda - \frac{m_{\mathsf{g}}}{2}\bar{\lambda}\lambda\right)$$

- **Gluino mass term** breaks supersymmetry softly

CHIRAL SYMMETRY

In comparison to QCD, the SU(N_c) SYM theory possesses a different **breaking pattern of chiral symmetry** for $m_{\mathsf{g}}=0$:

- **Global chiral U(1)$_A$ symmetry**: $\lambda \mapsto \mathrm{e}^{\mathrm{i}\alpha\gamma_5}\lambda$
- Due to the **anomaly**, only a $\mathbb{Z}_{2N_c}$ remnant symmetry $\lambda \mapsto \mathrm{e}^{2\pi\mathrm{i}n\gamma_5/2N_c}\lambda$ with $n \in \{1,\ldots,2N_c\}$ survives
- Gluino condensate $\langle\bar{\lambda}\lambda\rangle \neq 0$ leads to a **spontaneous breaking** of the remnant symmetry to a $\mathbb{Z}_2$ symmetry
  $\Rightarrow$ The theory contains N_c physically equivalent vacua

LATTICE FORMULATION

- Non-perturbative methods like lattice simulations are required to investigate the mass spectrum and confinement
- **Method of Curci & Veneziano** with **Wilson fermions**:
  - Problem: **supersymmetry** is **broken explicitly** at finite lattice spacing $a \Rightarrow$ lattice artifact $\sim m_{\mathsf{g}}$
  - Solution: add explicit **gluino mass counter-term** and **fine-tune** its bare mass m such that the gluino becomes massless in the continuum limit
  - Problem: Due to confinement, the gluino can't be measured directly to find the correct mass parameter
  - Solution: **adjoint pion** mass squared, $m^2_{\mathsf{a}\text{-}\pi} \propto m_{\mathsf{g}}$
    - ⋆ Not a physical particle
    - ⋆ Defined in a partially quenched approximation
  - Fine-tuning to critical gluino mass $m_{\mathsf{crit}} \Rightarrow$ **in continuum limit: restoration** of supersymmetry & chiral symmetry

TWISTED DIRAC OPERATOR

- Wilson-Dirac operator with deformation resembling a **twisted** mass:

$$D_{\mathsf{W}}^{\mathsf{mtw}}(x,y) = (4+m+\mathrm{i}\mu\gamma_5)\delta_{x,y} - \frac{1}{2}\sum_{\mu=\pm1}^{\pm4}(\mathbb{1}-\gamma_\mu)\mathcal{V}_\mu(x)\,\delta_{x+\hat{\mu},y}$$
$$= (4+M\,\mathrm{e}^{\mathrm{i}\alpha\gamma_5})\delta_{x,y} - \frac{1}{2}\sum_{\mu=\pm1}^{\pm4}(\mathbb{1}-\gamma_\mu)\mathcal{V}_\mu(x)\,\delta_{x+\hat{\mu},y}$$

  - Adjoint representation $[\mathcal{V}_\mu(x)]_{ab} = 2\,\mathrm{tr}\left[\mathcal{U}_\mu^\dagger(x)T_a\mathcal{U}_\mu(x)T_b\right]$
- **Remnant $\mathbb{Z}_{2N_c}$ symmetry** from the anomalous chiral U$(1)_A$ symmetry
  ↳ Particular directions favored by gluino condensate
- Add **parity-breaking mass** μ resembling a twisted mass
  ↳ Possibility to get closer to chiral symmetry and supersymmetry at finite lattice spacing
- Mass term of Wilson-Dirac operator breaks chiral symmetry explicitly and generates a condensate $\langle\bar{\lambda}\lambda\rangle$
- μ-mass leads to a condensate $\langle\bar{\lambda}\gamma_5\lambda\rangle$
- A breaking in this direction can be achieved by using a maximal twist, i. e. vary μ at fixed m_{crit}
- Interesting observables are the (unphysical) a-π and a-a mesons

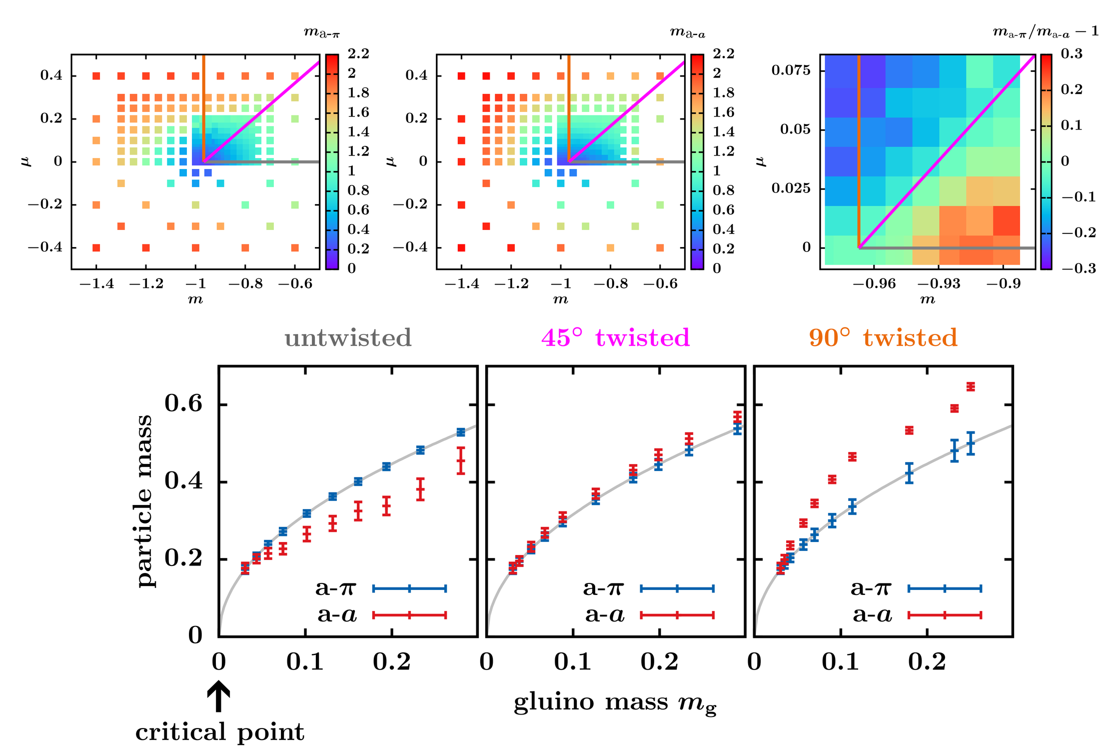

Fig. 1: Parameter scan on a $8^3\times16$ lattice with $\beta=5.4$.
Top row: Mass of a-π (left), a-a (middle) and their subtracted quotient $m_{\mathsf{a}\text{-}\pi}/m_{\mathsf{a}\text{-}a}-1$ zoomed to the region near the chiral point $(m_{\mathsf{crit}},\mu_{\mathsf{crit}})=(-0.967,0)$ (right). The colored lines mark the three directions summarized in the table below.
Bottom row: Mass of a-π and a-a for $\alpha=0°$ (left), $\alpha=45°$ (middle) and $\alpha=90°$ (right) with extrapolations in the gluino mass m_{g} to the chiral point.

$\alpha=0°$	$\mu=0$	$m_{\mathsf{a}\text{-}\pi} > m_{\mathsf{a}\text{-}a}$	gray line
$\alpha=45°$	$\mu=m-m_{\mathsf{crit}}$	$m_{\mathsf{a}\text{-}\pi} \approx m_{\mathsf{a}\text{-}a}$	magenta line
$\alpha=90°$	$m=m_{\mathsf{crit}}$	$m_{\mathsf{a}\text{-}\pi} < m_{\mathsf{a}\text{-}a}$	orange line

LOW-ENERGY SPECTRUM

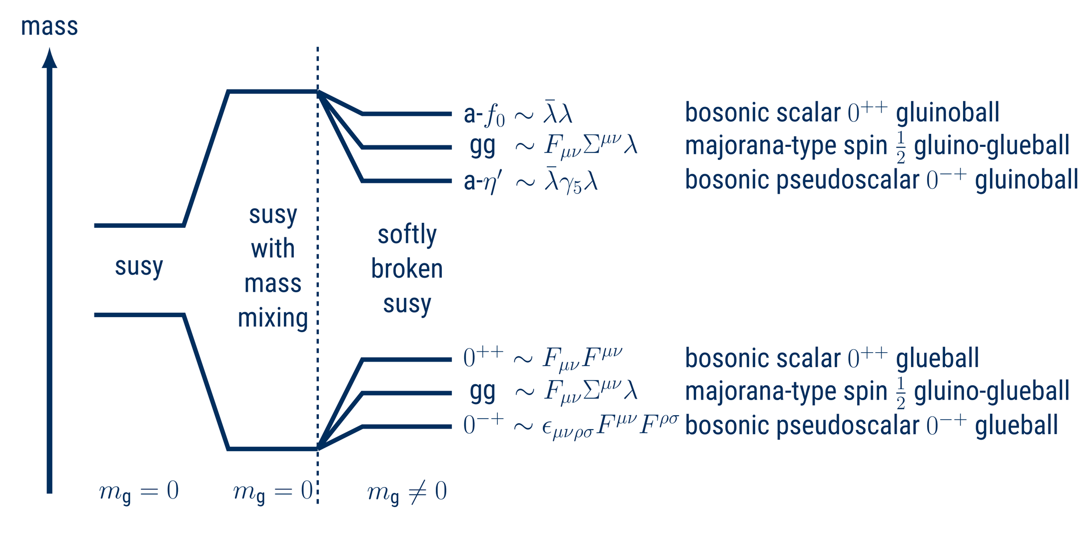

Fig. 2: The two lowest supermultiplets predicted by effective field theory.

MESONIC CORRELATORS

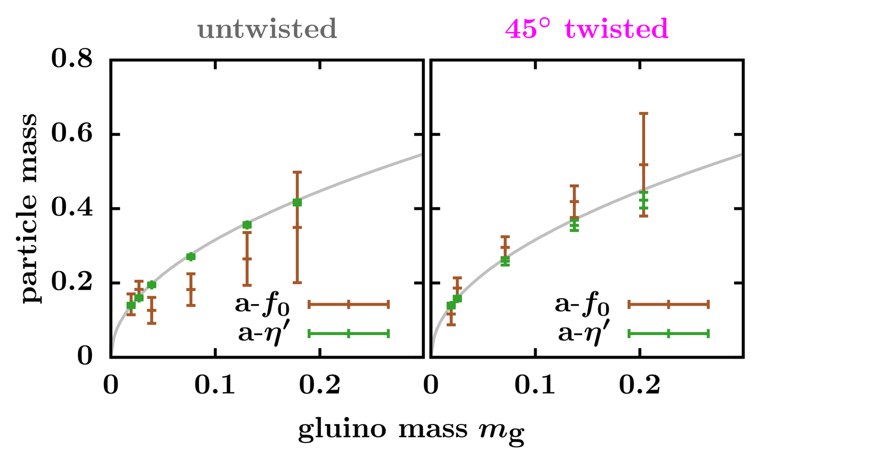

Fig. 3: Measurement of the physical mesonic states a-η' and a-f_0 on a $8^3\times16$ lattice at $\beta=5.0$ for twist angles $\alpha=0°$ (left) and $\alpha=45°$ (right). In comparison to Fig. 1, these observables contains disconnected contributions.

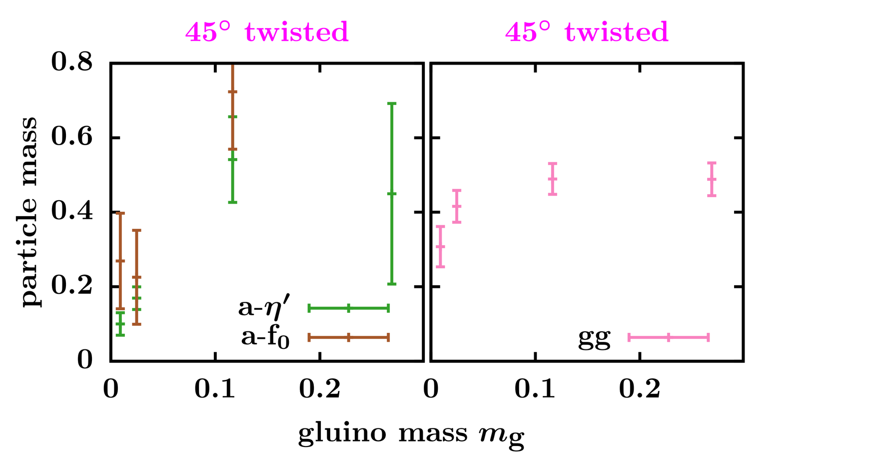

Fig. 4: Measurement of the physical mesonic states a-η' and a-f_0 (left) and the gluino-glue (right) on a $16^3\times32$ lattice at $\beta=5.0$ for twist angle $\alpha=45°$.

PFAFFIAN

- After integrating out the **Majorana fermions**:
  Pfaffian of the Dirac operator in the path integral
  $\Rightarrow$ rational hybrid Monte Carlo algorithm (**RHMC**)
- The Pfaffian is part of the Boltzmann weight in the functional integral and hence should be positive to avoid a **sign problem**
- Pf$(D_{\mathsf{W}}^{\mathsf{tw}}) \in \mathbb{C}$ for the twisted Wilson-Dirac operator
- In the **continuum theory** $m \to m_{\mathsf{crit}}$, $\mu \to 0$, $a \to 0$ the Pfaffian becomes real
- Numerical investigations reveal that **at finite lattice spacing** the phase of Pf$(D_{\mathsf{W}}^{\mathsf{tw}}) = |\mathrm{Pf}(D_{\mathsf{W}}^{\mathsf{tw}})|\cdot\mathrm{e}^{\mathrm{i}\alpha}$ is **negligible**

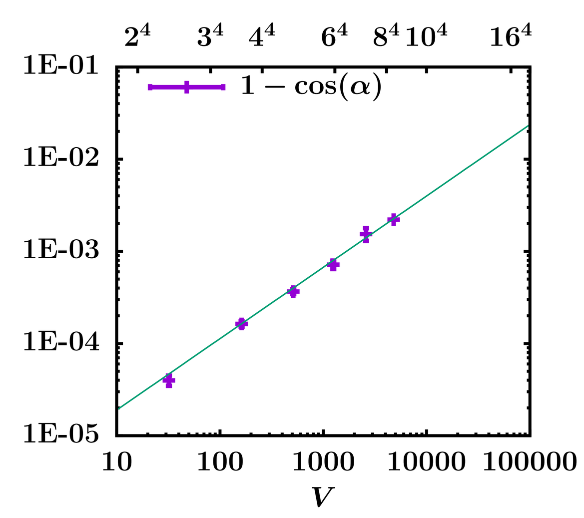

Fig. 5: Numerical values for the Pfaffian phase $\mathrm{e}^{\mathrm{i}\alpha}$ plotted as $1-\mathrm{Re}(\mathrm{e}^{\mathrm{i}\alpha})$ for the parameters $m=-0.85$, $\mu=0.10$, $m_{\mathsf{a}\text{-}\pi}\approx0.70$ and various lattice sizes V.

PUBLICATION

[1] M. Steinhauser, A. Sternbeck, B. Wellegehausen, and A. Wipf. "$\mathcal{N}=1$ Supersymmetric $SU(3)$ Gauge Theory – Pure Gauge sector with a twist". In: *PoS* LATTICE2018 (2018), p. 211. arXiv: 1811.01785 [hep-lat].

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