# Exploring the phases of Yang-Mills theory with adjoint matter through the gradient flow

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2. The gradient flow and renormalisation
3. Thermal super Yang-Mills
4. (Near) conformal field theories
5. Gradient flow and RG flow
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# The Yang-Mills gradient flow

- A gradient flow (curve of steepest descent) in a linear space $M$ is a curve $\gamma : \mathbb{R} \to M$, such that for a functional $S : M \to \mathbb{R}$

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$$\partial_t B_\mu = D_\nu G_{\nu\mu}, \quad B_\mu|_{t=0} = A_\mu: \ \texttt{flow of gauge fields}$$
$$\partial_t \chi = D_\mu D^\mu \chi, \quad \chi|_{t=0} = \psi: \ \texttt{flow of fermion fields}$$

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- These evolve the fields to local minima of the action

What does the flow imply for the quantum theory?

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# The gradient flow in QFT

- The flow has a smoothening effect on the fields, which are Gaussian-like smeared over an effective radius $r_t = \sqrt{8t}$

$$B_\mu(t,x) = \int d^D y \; K_t(x-y) A_\mu(y) + \text{non linear terms},$$

$$K_t(z) = \int \frac{d^D p}{(2\pi)^D} \mathrm{e}^{ipz} \mathrm{e}^{-tp^2} \quad \text{(at leading order)}$$

- $\mathrm{e}^{-tp^2}$ is UV cut-off for $t > 0$. It remains at all orders in perturbation theory [Lüscher and Weisz,arXiv:1405.3180]

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- D+1 dimensional QFT with flow time as spurious dimension
- t-propagator is the heat kernel $K$.
- BRS-Ward identities $\rightarrow$ no counter-terms for the gauge fieds
- Fermions get an extra multiplicative renormalisation

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We used this to investigate the phase structure of SU(2) and SU(3) SYM

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# Brief review of minimal SYM

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- Expected to have mass gap, confinement and spontaneous breaking of chiral symmetry
- Low energy degrees of freedom: glueballs, meson-like states, baryon-like (see Sajid Ali's poster)

# At zero temperature

$\mathrm{Z}_{N_c}$ Centre symmetry

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## Chiral symmetry

- Anomaly free $Z_{2N_c}$ symmetry
- Condensate $< \bar{\lambda}\lambda > \neq 0 \Rightarrow Z_{2N_c} \rightarrow Z_2$
- A domain wall interpolates $N_c$ degenerated vacua
- Chern-Simons theory on domain wall with deconfined quarks

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We computed the PL and the flowed condensate at different $T$

$\rightarrow$ bound is saturated for SU(2). Evidence for SU(3)

# Computing the condensate

✗ Additive renormalisation constant needed because of the Wilsonian fermion discretisation

$$\langle\lambda\lambda\rangle_{\mathrm{R}} = Z_{\lambda\lambda}(\beta)(\langle\lambda\lambda\rangle_{\mathrm{B}} - \mathbf{b_0}).$$

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✓ Another way out is the gradient flow

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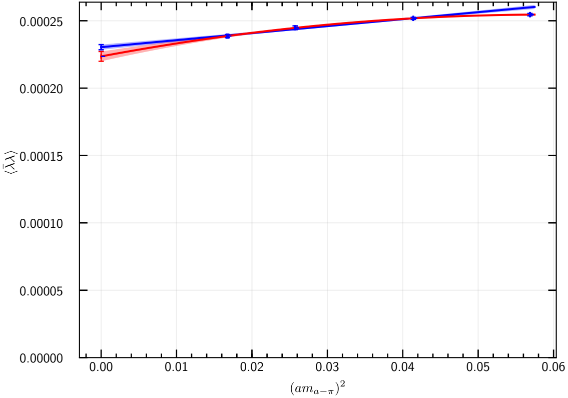
…The inversion and the fermion *adjoint* flow are the most expensive part of the numerics

# Results: SU(2)




Non-vanishing condensate at zero temperature in the chiral / supersymmetric limit

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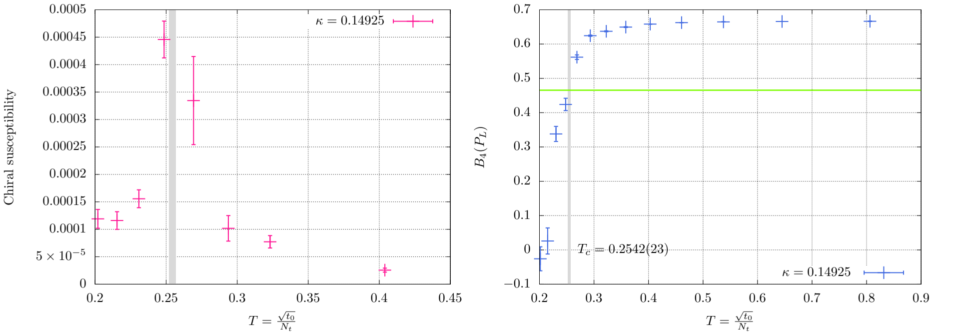


- Deconfinement critical temperature coincides with peak of chiral susceptibility

Deconfinement and chiral restoration phase transitions occur at the same critical temperature $T \sim 0.25$

# How can we understand/explain this observation?

Mathematically (Witten, 97):

* studied configuration of branes in M-theory, which is in universality class of $N = 1$ SYM

* showed that (QCD strings $\leftrightarrow$ fundamental strings) can end in (domain walls $\leftrightarrow$ D-branes)

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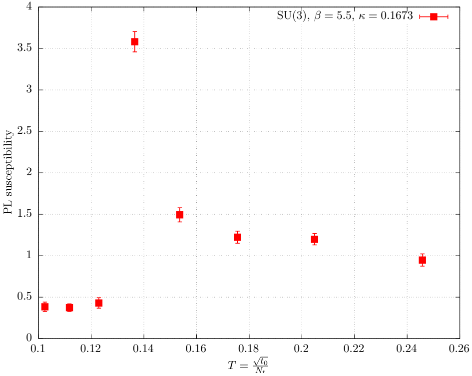
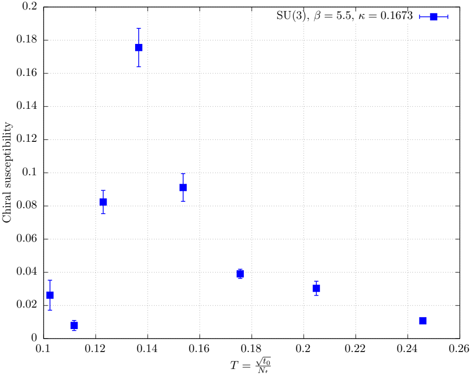
* Witten's observation holds only if chiral restoration and deconfinement occur simultaneously

# Preliminary results: SU(3)



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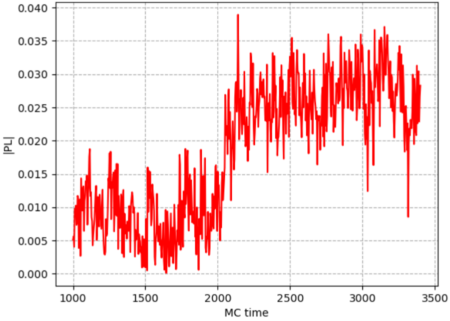


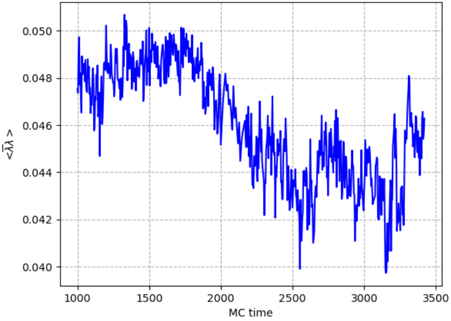

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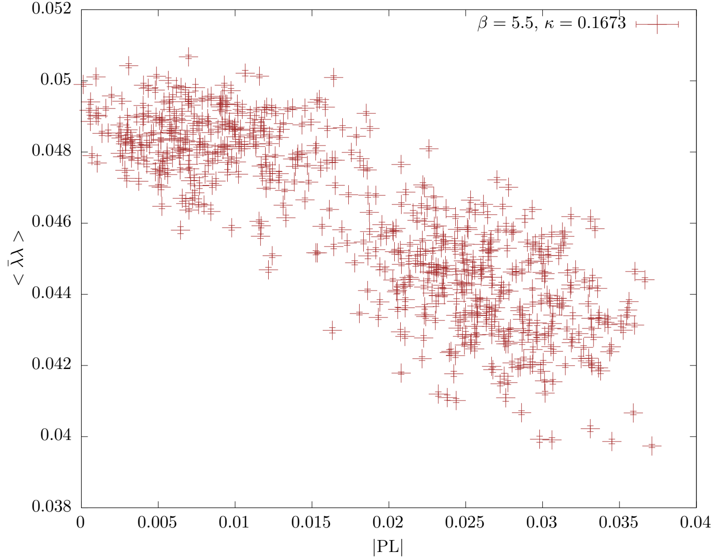

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  III. Conformal, e.g. theories with IR fixed point (FP)
  IV. Non-trivially gapped, i.e. topological QFT, BPS states...

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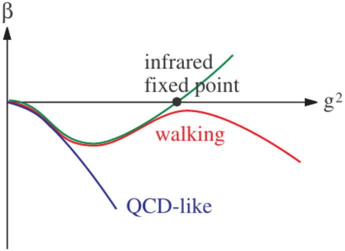
- For a YM theory with fermions, one has different scenarios depending on $\mathrm{N}_f$ and $\mathrm{N}_c$:

  1. Small $\mathrm{N}_f$: chiral symmetry breaking (IR massless)
  2. $\mathrm{N}_f^{\mathrm{l}} < N_f < N_f^{\mathrm{u}}$: Banks-Zaks (BZ) FP conformal window (IR conformal)
  3. $\mathrm{N}_f > N_f^{\mathrm{u}}$: not asymptotically free

# Conformal window




[Desy-Münster collaboration]

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- Methods to compute observables: Lattice Monte Carlo (LMC), conformal bootstrap, ...
- Within LMC: take mass-deformed theory, i.e. away from the FP and compute the anomalous dimensions from
  - ✓ Mass spectrum of the theory
  - ✓ Monte Carlo renormalisation group techniques
  - ✓ Spectral density of Dirac operator (mode number)
  - ✓ Recently: Gradient flow and RG flow [Carosso, Hasenfratz and Neil, PRL 121 no.20, 201601]

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- Near conformal QFTs are important for phenomenology, e.g. technicolor models
- Being able to study RG flow through the GF opens up the possibility to compute conformal data on the lattice

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- GF allows for blocked fields without having to know the blocked action

  [Carosso, Hasenfratz and Neil]

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- Relate blocked and flowed fields through $\phi_b(x_b) = b^{d_\phi + \eta/2}\phi_t(b x_b)$ and $\sqrt{t} \propto b$

$$\boxed{\frac{\langle \mathcal{O}_t(0)\mathcal{O}_t(x_0)\rangle}{\langle \mathcal{O}(0)\mathcal{O}(x_0)\rangle} = b^{2\Delta_{\mathcal{O}} - 2n_{\mathcal{O}}\Delta_\phi}, \quad \Delta_i = d_i + \gamma_i \text{ (canonical + anomalous dim)}}$$

# GF and RG flow II

- Get rid of $\Delta_\phi$ through conserved operator $\mathcal{V}$ $(\gamma_{\mathcal{V}} = 0)$

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- The mass anomalous dimension of the operator $\mathcal{O}$ can be then defined as

$$\boxed{\gamma_{\mathcal{O}}(\bar{t}) = \frac{\log(\mathcal{R}_{\mathcal{O}}(t_1)/\mathcal{R}_{\mathcal{O}}(t_2))}{\log\left(\sqrt{t_1}/\sqrt{t_2}\right)}}$$

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# Low energy adjoint QCD from GF

**Q1: For which number of flavours is SU(2) adjoint QCD (near-)conformal?**

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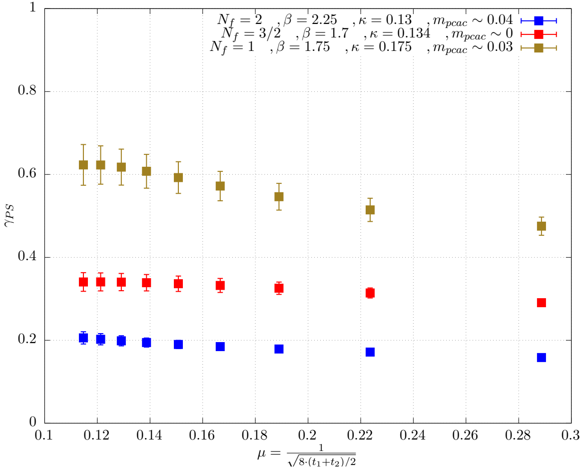
$\Rightarrow$ Look for freezing of $\gamma$ in the RG flow

$\Rightarrow$ Extrapolate $\gamma$ towards the deep IR

# Preliminary results



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- $N_f = 1$ has large anomalous dimension $\rightarrow$ may be relevant for BSM