WIs and Baryonic States in $\mathcal{N}$=1 SUSY Yang-Mills theory on the lattice

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The Model

$\mathcal{N}=1$ SUSY Yang-Mills Theory

The action:

$$S_{\rm SYM}=\mathrm{Re}\int d^4x\, d^2\theta \mathrm{Tr}[W^\alpha W_\alpha]=\int d^4x\left\{\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}+\frac{1}{2}\bar\lambda^a\gamma_\mu\mathcal{D}_\mu\lambda^a\right\}$$

- Field strength tensor:
  $F_{\mu\nu}=-\mathrm{i}gF^a_{\mu\nu}T^a=\partial_\mu A_\nu-\partial_\nu A_\mu+[A_\mu,A_\nu]$
- Covariant derivative in adjoint representation:
  $\mathcal{D}_\mu\lambda^a=\partial_\mu\lambda^a+g\,f_{abc}A^b_\mu\lambda^c,\qquad a=1,\dots,N_c^2-1\,,$
- Vector supermultiplet:
  1) Gauge field $A^a_\mu(x)$, "Gluon"
  2) Majorana-spinor field $\lambda^a(x)$, $\bar\lambda=\lambda^T C$, "Gluino"
- SUSY transformations (on-shell):

$$\delta A^a_\mu=-2g\bar\lambda^a\gamma_\mu\varepsilon\,,$$
$$\delta\lambda^a=-\frac{\mathrm{i}}{g}\sigma_{\mu\nu}F^a_{\mu\nu}\varepsilon$$

- In contrast to QCD:
  1) λ is Majorana spinor field, "$N_f=\frac{1}{2}$"
  2) adjoint representation of SU(N_c)
- Gluino mass term $m_{\tilde g}\,\bar\lambda^a\lambda^a$ breaks SUSY softly

Motivation

- SYM: simplest model with SUSY and local gauge invariance
- Part of the supersymmetrically extended Standard Model
- Possible connection to ordinary QCD
- Similar to QCD:
  1) Asymptotic freedom
  2) Confinement
  3) Numerical lattice simulation of bound states

Solution of non-perturbative Problems:

- Spontaneous breaking of chiral symmetry $Z_{2N_c}\to Z_2$
  $\longleftrightarrow$ Gluino condensate $<\lambda\lambda>\neq 0$
- Spectrum of bound states $\to$ Supermultiplets
- Confinement of static quarks
- Spontaneous breaking of SUSY?
- SUSY restoration on the lattice
- Check predictions from effective Lagrangeans (Veneziano, Yankielowicz . . .)

Spontaneous breaking of chiral symmetry

U(1)$_\lambda$: $\lambda'=\mathrm{e}^{-\mathrm{i}\varphi\gamma_5}\lambda\,,\ \bar\lambda'=\bar\lambda\,\mathrm{e}^{-\mathrm{i}\varphi\gamma_5}\leftrightarrow$ R-symmetry, $J_\mu=\bar\lambda\gamma_\mu\gamma_5\lambda$

Anomaly: $\partial^\mu J_\mu=\frac{N_c g^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$ breaks U(1)$_\lambda\to Z_{2Nc}$

Spontaneous breaking $Z_{2N_c}\to Z_2$
by Gluino condensate $<\lambda\lambda>\neq 0$
$\leftrightarrow$ first order phase transition at $m_{\tilde g}=0$

$N_c=2:\quad <\lambda\lambda>=\pm C\Lambda^3$

Spectrum of bound states

Expect colour neutral bound states of gluons and gluinos
Predictions from effective Lagrangeans:

Chiral supermultiplet (Veneziano, Yankielowicz)

- 0^- gluinoball a - η' $\sim$ $\bar\lambda\gamma_5\lambda$
- 0^+ gluinoball a - f_0 $\sim$ $\bar\lambda\lambda$
- spin $\frac{1}{2}$ gluino-glueball $\sim$ $\sigma_{\mu\nu}\,\mathrm{Tr}\,(F_{\mu\nu}\lambda)$

Generalization (Farrar, Gabadadze, Schwetz):

additional chiral supermultiplet

- 0^- glueball
- 0^+ glueball
- gluino-glueball

possible mixing and Baryonic states

Simulations

SUSY on the lattice

Lattice breaks SUSY. Restoration in the continuum limit?
Curci, Veneziano: use Wilson action, search for continuum limit with SUSY

$$S=-\frac{\beta}{N_c}\sum_p \mathrm{Re}\,\mathrm{Tr}\,U_p$$
$$+\frac{1}{2}\sum_x\left\{\bar\lambda^a_x\lambda^a_x-\kappa\sum_{\mu=1}^4\left[\bar\lambda^a_{x+\hat\mu}V_{ab,x\mu}(1+\gamma_\mu)\lambda^b_x+\bar\lambda^a_x V^t_{ab,x\mu}(1-\gamma_\mu)\lambda^b_{x+\hat\mu}\right]\right\}$$

$\beta=\frac{2N_c}{g^2},\quad \kappa=\frac{1}{2m_0+8}$ hopping parameter, m_0 : bare gluino mass

$$V_{ab,x\mu}=2\,\mathrm{Tr}\,(U^\dagger_{x\mu}T_aU_{x\mu}T_b),\quad \text{adjoint link variables}$$

We study gauge group SU(3).

Fermion integration

Fermionic action

$$S_f=\frac{1}{2}\bar\lambda Q\lambda=\frac{1}{2}\lambda M\lambda\,,\qquad M\equiv CQ$$

Pfaffian

$$\int[d\lambda]\,\mathrm{e}^{-S_f}=\mathrm{Pf}(M)=\pm\sqrt{\det Q}$$

Effective gauge field action

$$S_{\rm eff}=-\frac{\beta}{N_c}\sum_p\mathrm{Re}\,\mathrm{Tr}\,U_p-\frac{1}{2}\log\det Q[U]$$

Include sign Pf(M) in the observables.

Monte Carlo algorithm

- Monte Carlo Simulation has been used to generate configurations.
- Each configuration contains numerical values of link variables (U).
- These configurations are used to compute correlation functions.

Sign Problem:
monitoring of sign Pf(M)

- through spectral flow
- by calculation of real negative eigenvalues of Q with Arnoldi

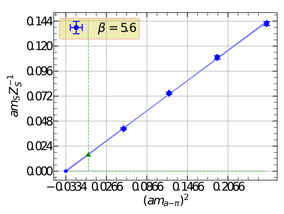
$\to$ Negative Pfaffians occur in our simulations near κ_c, but rarely.

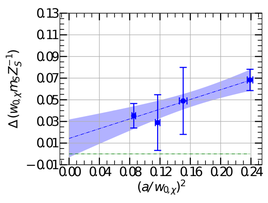
SUSY Ward identities on lattice

Ward identity is the quantum version of Noether Theorem
Renormalised SUSY WI on the lattice with non-zero gluino mass

$$\langle(\nabla_\mu S_\mu(x))Q(y)\rangle+Z_TZ_S^{-1}\langle(\nabla_\mu T_\mu(x))Q(y)\rangle$$
$$=m_SZ_S^{-1}\langle\chi(x)Q(y)\rangle+\langle X_S(x)Q(y)\rangle-\left\langle\frac{\delta Q}{\delta\bar\varepsilon(x)}\right\rangle.$$

- Z_S and Z_T are renormalization coefficients
- $S_\mu(x)$ and $T_\mu(x)$ are the Super and the Mixing currents.
- Q is an insertion operator.
- $\varepsilon(x)$ is the parameter of infinitesimal symmetry transformations.
- $\left\langle\frac{\delta Q}{\delta\bar\varepsilon(x)}\right\rangle$ is contact term, which is zero if Q is localised at space-time points different from x.
- $\langle X_S(x)Q(y)\rangle$ is introduced by the lattice regulator and vanishes in the continuum limit
- $m_SZ_S^{-1}\langle\chi(x)Q(y)\rangle$ is the mass term which break SUSY softly
- $am_SZ_S^{-1}$ is determine for each gauge ensemble
- Remnant gluino mass $\Delta(w_{0,\chi}m_SZ_S^{-1})$ in physical units is determined in the chiral limit
- The continuum extrapolation is performed, $\Delta(w_{0,\chi}m_SZ_S^{-1})$ is consistant with zero within error as expected from the theory

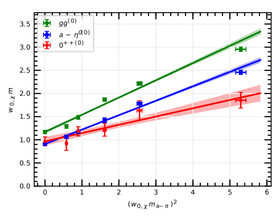


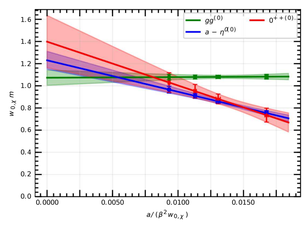


Results

Light particle spectrum

- Mass of light bound states is determined
- Variational analysis is used
- Mixing is considered
- Chiral extrapolations are performed
- Mass gap between different states due to soft breaking of SUSY is observed
- Mass degenerate chiral supermultiplet is formed as expected from predictions
- Gauge group SU(3) is considered



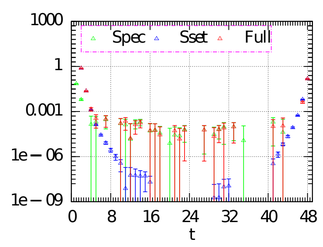


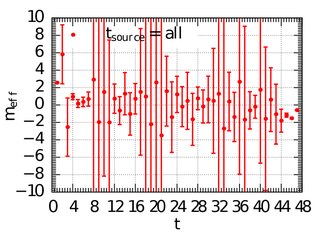
Baryonic States

Baryonic correlation functions can be constructed from Rarita Schwinger field

$$W_\mu(x)=t_{abc}\lambda_a(x)\left(\lambda^T_b(x)C\gamma_\mu\lambda_c(x)\right).$$

- Baryonic states are not predicted by the effective actions
- Constructed from three gluino fields in analogy with QCD
- Consists of two parts; the sunset piece and the spectacle piece
- Stochastic estimator technique is used for spectacle piece
- The lowest eigenvalues for the inverse of the Wilson-Dirac operator is used
- Computation of sunset piece is easy whereas the spectacle piece is rather challenging
- Preliminary results of correlation functions and the effective mass





Conclusion and Outlooks

- First order phase transition at $m_{\tilde g}=0$
- Determination of the mass spectrum of light bound states
- Chiral supermultiplet is formed
- Baryonic states are investigated
- SUSY WIs are being analysed
- SUSY is restored
- More states in SYM theory can be formulated

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