Hydrodynamics, Spontaneously Broken Symmetries, and Holography

Based on: [Ammon, Baggioli, SG, Grieninger, JHEP ’19], [Alberte, Ammon, Jiménez-Alba, Baggioli, Pujolas, PRL ’17]

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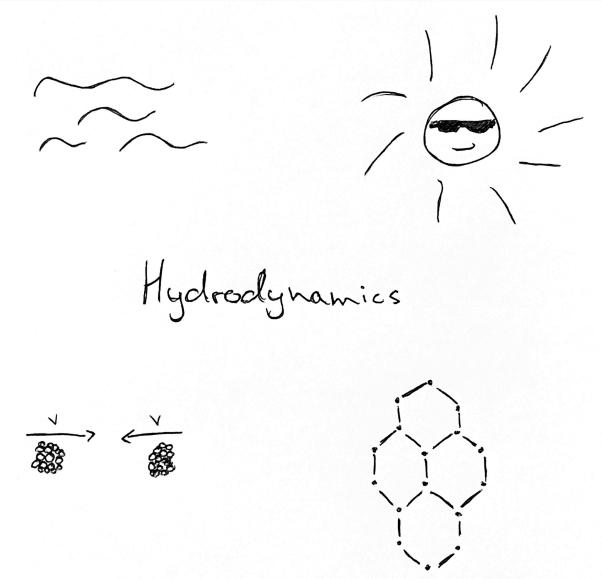
Hydrodynamics

Hydrodynamics gives insight into long-wavelength and long-time behaviour of a system, e.g. sound modes.

The hydrodynamic equations are conservation equations.

The World of Hydrodynamics



Hydrodynamics in QFT

Relativistic hydrodynamics effectively describes small momentum and small frequency fluctuations in QFT with finite temperature.

Conservation equations

$$\partial_\mu \langle T^{\mu\nu} \rangle = 0, \qquad \partial_\mu \langle J^{\mu} \rangle = 0.$$

One point functions of symmetry currents are expressed as derivative expansions, e.g.[1]

$$\langle T^{\mu\nu} \rangle = \varepsilon u^\mu u^\nu + p\Delta^{\mu\nu} - \eta \Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{d}\eta_{\alpha\beta}\partial_\rho u^\rho\right) + \mathcal{O}(\partial^2).$$

[1] Here we have assumed conformal invariance.

Derivative Expansion

Ideal part:

$$\langle T^{\mu\nu}\rangle = \varepsilon u^\mu u^\nu + p\Delta^{\mu\nu} - \eta\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{d}\eta_{\alpha\beta}\partial_\rho u^\rho\right) + \mathcal{O}(\partial^2),$$

where

- $u^\mu(t,\vec{x})$: velocity of the fluid
- $\varepsilon(T,\mu)$: energy density
- $p(T,\mu)$: pressure
- $\eta(T,\mu)$: shear viscocity

and $\Delta^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$ is a projector.

Thermodynamics enters through the equation of state for $p(T,\mu)$.

Derivative Expansion

Viscous part:

$$\langle T^{\mu\nu} \rangle = \varepsilon u^\mu u^\nu + p\Delta^{\mu\nu} - \eta \Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{d}\eta_{\alpha\beta}\partial_\rho u^\rho\right) + \mathcal{O}(\partial^2),$$

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and $\Delta^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$ is a projector.

Determines fluid response to perturbation.

Derivative Expansion

Truncation:

$$\langle T^{\mu\nu}\rangle = \varepsilon u^\mu u^\nu + p\Delta^{\mu\nu} - \eta\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{d}\eta_{\alpha\beta}\partial_\rho u^\rho\right) + \mathcal{O}(\partial^2),$$

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Terms beyond one derivative not well understood

Constraints on Derivative Expansion

Form of derivative expansion is constrained by

- Equations of motion
- Frame choice
- Onsager relations
- Positivity of local entropy production

Finding consistent derivative expansions is very challenging – terms can be, and have been, overlooked.

Holographic techniques discovered new transport phenomena, i.e. the chiral vortical effect. [Erdmenger, Haack, Kaminski, Yarom, '08], [Benerjee et. al, '08] [Son, Surowka, '09], [Landsteiner, Megias, Melgar, Pena-Benitez, '11], [Gooth et. al, '17]

Symmetries

Symmetries are often idealisations – the real world breaks them!

Explicit symmetry breaking:

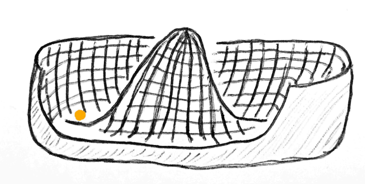
- Global symmetry of QFT broken by external effects,
  for example by external magnetic field
- Corresponding symmetry current no longer conserved
- Beyond the regime of hydrodynamics

Spontaneous symmetry breaking (SSB):
Ground state of QFT no longer invariant under global symmetry.

Spontaneous Symmetry Breaking: a Simple Example

SSB of continuous global symmetry:

- Vacuum state lives in ‘sombrero’ potential $V((\phi^*\phi)^2)$



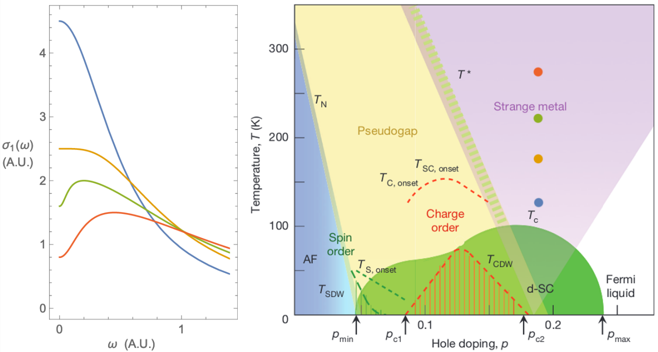
- New massless degrees of freedom: Goldstone bosons
- Symmetry currents still conserved; hence suitable for hydrodynamics.

SSB of Translational Invariance

Consider spontaneous breaking of spatial translational invariance.

Why?

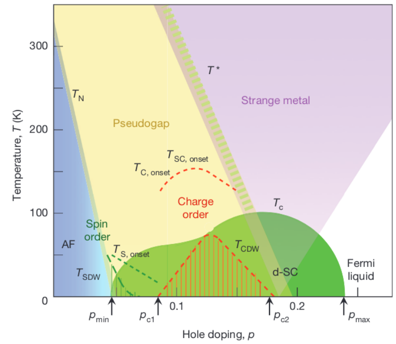
SSB of Translational Invariance: Motivation



[Delacrétaz, Goutéraux, Hartnoll, Karlsson, '16], [Keimer, Kivelson, Norman, Uchida, Zaanen, '15]

Shift and broadening of peaks in optical conductivity of 'strange metals' potentially explained using hydrodynamics with pseudo-SSB of translations.

SSB of Translational Invariance: Motivation



[Keimer, Kivelson, Norman, Uchida, Zaanen, '15]

Strange metals believed to arise from quantum critical point (QCP).

Pseudo-SSB may be imprint of symmetry breaking of QCP, which could also affect other phases.

SSB of Translational Invariance: Motivation

Can holography shed light on these questions?

A first step is to study pure spontaneous symmetry breaking.

SSB of Translational Invariance: Goldstone Mode

Consider spontaneous breaking of spatial translational invariance in a $(2+1)$-dimensional QFT.

Goldstone bosons associated with this spontaneous breaking are the phonons

$$\Phi = \begin{pmatrix} \Phi_x \\ \Phi_y \end{pmatrix}.$$

Since the phonons are massless fields, they will contribute to the hydrodynamics.

Phonon Contribution: Derivative Expansion

Derivative expansion of spatial components of conserved current becomes[2]

$$\langle T_{ij}\rangle = \delta_{ij}\left[p-(\kappa+G)\partial\cdot\langle\Phi\rangle\right] - 2G\left[\partial_{(i}\Phi_{j)} - \delta_{ij}\partial\cdot\langle\Phi\rangle\right] - \sigma_{ij} + \mathcal{O}(\partial^2),$$

where $\partial\cdot\langle\Phi\rangle$ is the divergence of the expectation value of Φ.

[2] We have chosen a frame where $u^\mu = (1,0,0)$.

Phonon Contribution: Derivative Expansion

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where $\partial \cdot \langle \Phi \rangle$ is the divergence of the expectation value of Φ.

Terms which appeared without spontaneous breaking; $\sigma_{ij}(t, \vec{x})$ is the viscous contribution.

[2] We have chosen a frame where $u^\mu = (1, 0, 0)$.

Phonon Contribution: Derivative Expansion

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Additional contributions due to presence of Goldstone bosons Φ_i.

New coefficients:

- $G(T,\mu)$: shear elastic modulus
- $\kappa(T,\mu)$: bulk elastic modulus

[2]We have chosen a frame where $u^\mu = (1,0,0)$.

Phonon Contribution: Dynamics I

Additional dynamic equations for phonons: 'Josephson relation',

$$\partial_t \langle \Phi_i \rangle = u_i.$$

Derivation:

- Hamiltonian density at finite velocity: $\mathcal{H} = \mathcal{H}_0 + u^i T^0{}_i$
- From Goldstone theorem: $[\Phi_i(x), T^0{}_j(y)] = i\delta_{ij}\delta(x-y) + \ldots$
- Hence $\partial_t \langle \Phi_i \rangle = i \langle [\Phi_i, \hat{H}] \rangle = \ldots = u_i$.

As a consequence $\partial_\mu \langle \Phi_i \rangle$ is zeroth order in derivatives.

Phonon Contribution: Dynamics II

Derivative expansion of Josephson relation

$$\partial_t \langle \Phi_i \rangle = u_i + \dots,$$

where ellipsis denotes higher orders in derivatives.

Taking additional derivatives (curl and divergence) gives separate equation for each $\langle \Phi_i \rangle$:[3]

$$\begin{aligned}
\partial_t(\partial_x \langle \Phi_y \rangle) &= \partial_x u_y + G\xi_y \partial_x^2(\partial_x \langle \Phi_y \rangle) + \dots \\
\partial_t(\partial_x \langle \Phi_x \rangle) &= \partial_x u_x + (\kappa + G)\xi_x \partial_x^2(\partial_x \langle \Phi_x \rangle) + \dots
\end{aligned}$$

Note: ξ_i are new coefficients which relate to Goldstone diffusion.

[3] We assumed momentum in x-direction, with fluctuations in both spatial directions.

Hydrodynamic Modes

The hydrodynamic equations decompose into two sectors, transverse and parallel to momentum.

Solving set of equations in frequency space leads to hydrodynamic modes.

Modes of Transverse Sector

Pair of sound modes with dispersion relation

$$\omega = \pm c_T k - i D_T k^2,$$

and coefficients

- Speed of transverse sound: $c_T^2 = \frac{G}{\chi_{\pi\pi}}$
- Diffusion constant: $D_T = \frac{1}{2}\left(G\xi_y + \frac{\eta}{\chi_{\pi\pi}}\right)$

where $\chi_{\pi\pi}$ is the momentum susceptibility, which relates velocity to momentum via $T^0{}_i = \chi_{\pi\pi} u_i$.

Modes of Longitudinal Sector I

1. Pair of sound modes with dispersion relation

$$\omega = \pm c_L k - i D_p k^2,$$

and coefficients

- Speed of longitudinal sound: $c_L^2 = \frac{\partial p}{\partial \varepsilon} + \frac{\kappa + G}{\chi_{\pi\pi}}$
- Diffusion constant: $D_p = \frac{1}{2}\frac{\eta}{\chi_{\pi\pi}} + \frac{1}{2}\,\frac{(\kappa+G)^2\,\xi_x}{\kappa+G+(\partial p/\partial\varepsilon)\,\chi_{\pi\pi}}$

  [Ammon, Baggioli, SG, Grieninger, '19]

Modes of Longitudinal Sector II

2. Diffusive mode with dispersion relation

$$\omega = -iD_\Phi k^2,$$

and diffusion constant

$$D_\Phi = (\kappa + G)\,\frac{(\partial p/\partial \varepsilon)\,(\xi_x\,\chi_{\pi\pi})}{(\kappa + G + (\partial p/\partial \varepsilon)\,\chi_{\pi\pi})}.$$

[Ammon, Baggioli, SG, Grieninger, '19]

Holographic Duality

Holographic Duality

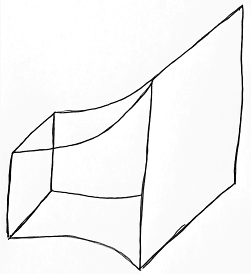
Conjectured duality between $(d+1)$-dimensional gravitational theories in asymptotically Anti-de Sitter (AdS) spacetimes and d-dimensional QFTs living on flat conformal boundary.

When QFT is strongly coupled, dual gravity description is weakly coupled; and vice-versa.

Holography provides tool-set to study strongly coupled quantum phenomena.

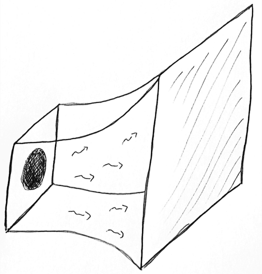
Simplest case:

Pure AdS $\Longleftrightarrow$ Vacuum state of QFT in flat-space.



Finite temperature:

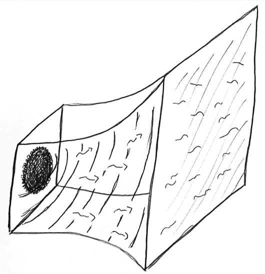
Black hole in asymptotic AdS $\Longleftrightarrow$ QFT at finite temperature.



Levels of Holography

Dynamics at finite temperature:

Pertubations of black hole (QNMs) $\Longleftrightarrow$ Poles of Green's functions.



Poles of retarded Green's functions are frequency modes of QFT.
Modes with zero frequency in limit of zero momentum correspond to hydrodynamic modes!

Translational Breaking in Holography: Our Model

Construct gravity model such that dual QFT has desired properties.

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + \frac{3}{\ell^2} - m^2\,V(X)\right],$$

where $g_{\mu\nu}$ is a black hole metric; R is the Ricci scalar; ℓ is the radius of curvature of AdS; and

$$V(X) = X^N, \quad X = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^I,$$

with scalar field $\phi^I = x^I$. [Alberte, Ammon, Jiménez-Alba, Baggioli, Pujolas, JHEP '17]

Gives spacetime of Schwarzschild black hole in asymptotic AdS, with massive graviton.

If $N < 5/2$: Translational symmetry of dual QFT is explicitly broken.
If $N > 5/2$: Dual QFT exhibits spontaneously broken translational invariance. [Alberte, Ammon, Jiménez-Alba, Baggioli, Pujolas, PRL '17]

Method

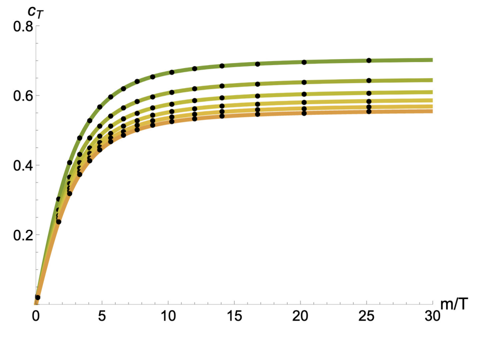
1. Compute (numerically) QNMs of gravity theory
2. Identify hydrodynamic modes
3. Extract coefficients
4. Compare QNM results to hydrodynamic formulas

Results: Transverse Sector

[Alberte, Ammon, Jiménez-Alba, Baggioli, Pujolas, PRL '17]

Propagating mode of transverse sector:

$$\omega = \pm c_T k - i D_T k^2$$



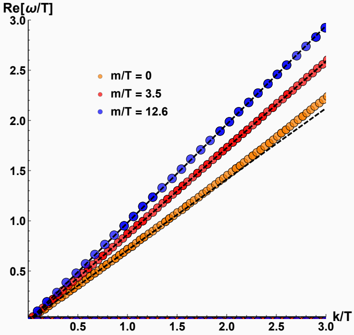
Function of m/T, which is the dimensionless SSB scale.

Results: Longitudinal Sector I

[Ammon, Baggioli, SG, Grieninger, JHEP '19]

Propagating mode mode of longitudinal sector:

$$\omega = \pm c_L k - i D_p k^2$$



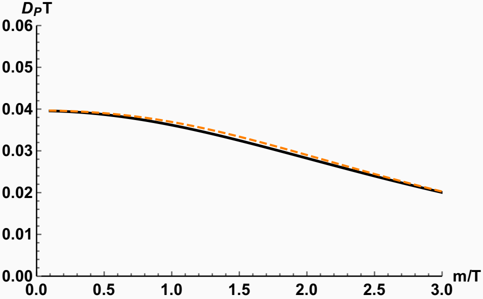
Speed of sound given by the slope; exponent $N = 3$.

Results: Longitudinal Sector II (Sound Diffusion)

[Ammon, Baggioli, SG, Grieninger, JHEP '19]

Propagating mode of longitudinal sector:

$$\omega = \pm c_L k - i D_p k^2$$



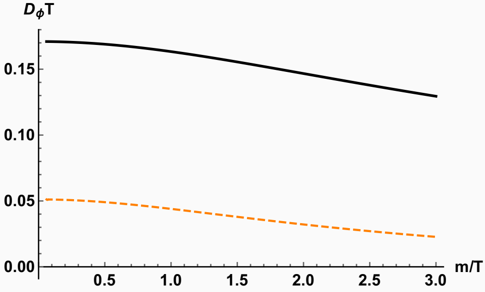
Made dimensionless by multiplication with temperature T; $N = 5$.

Results: Longitudinal Sector III (Diffusive Mode)

[Ammon, Baggioli, SG, Grieninger, JHEP '19]

Diffusive mode of longitudinal sector

$$\omega = -iD_\Phi k^2$$



Discrepancy between QNMs and hydrodynamic relation; $N = 5$.

Potential Reasons for Discrepancy

What could be going on:

- Mistake?
- Missing thermodynamic relations?
- New hydrodynamic effect?

Conclusion and Outlook

Hydrodynamics is interesting!
Symmetry breaking is hard!
Holography provides novel approach!

Future directions:

- Find reason behind discrepancy
- Investigate pseudo-spontaneous symmetry breaking from first-principles
- Fluid/Gravity approach to derivative expansion

Thank you for your attention!