Competition of inhomogeneous chiral phases with homogeneous 2SC phases in low-energy models of QCD

Phillip Lakaschus

Institute for Theoretical Physics, Goethe University Frankfurt am Main

CRC-TR 211
Strong-interaction matter under extreme conditions

Introduction

- We present preliminary results on the competition between inhomogeneous chiral phases with homogeneous 2SC phases in the framework of a NJL model using the mean-field approximation
- Two different inhomogeneous Anstze are employed: The chiral-density wave (CDW) and a solitonic Ansatz using Jacobi-elliptic functions
- Two outlooks are provided on how to continue research in this direction

The NJL Model

We consider the NJL Model including a scalar-isosinglet and pseudoscalar-isotriplet $\bar{q}q$ channel, as well as a scalar qq channel:

$$\mathcal{L} = \bar{\psi}(i\not{\partial} + \mu\gamma_0)\psi + g\left[(\bar{\psi}\psi)^2 + \bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right] + g_\Delta\,(\bar{\psi}_c i\gamma^5\tau^2\lambda^a\psi)(\bar{\psi}i\gamma^5\tau^2\lambda^a\psi_c)$$

A Hubbard-Stratonovich transformation is performed which yields the bosonized action:

$$S = \int_x \Big\{\bar{\psi}(i\gamma^\mu\partial_\mu + \mu\gamma_0 + \sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau})\psi + \frac{\Delta_a}{2}(\bar{\psi}_c i\gamma_5\tau^2\lambda^a\psi) + \frac{\Delta_a^*}{2}(\bar{\psi}i\gamma_5\tau^2\lambda^a\psi_c) - \frac{\sigma^2+\vec{\pi}^2}{4g} - \frac{|\Delta_A|^2}{4g_\Delta}\Big\}$$

The CDW Ansatz is given as $\sigma(\vec{x}) = M/g\cos\vec{q}\cdot\vec{x}$, $\pi_a(\vec{x}) = \delta_{a3}M/g\sin\vec{q}\cdot\vec{x}$ and the solitonic Ansatz as $-2g(\sigma(x) + i\vec{\pi}(x)) = M\nu\dfrac{\mathrm{sn}(Mx|\nu)\mathrm{cn}(Mx|\nu)}{\mathrm{dn}(Mx|\nu)}$.

Using this action we derive the grand potential

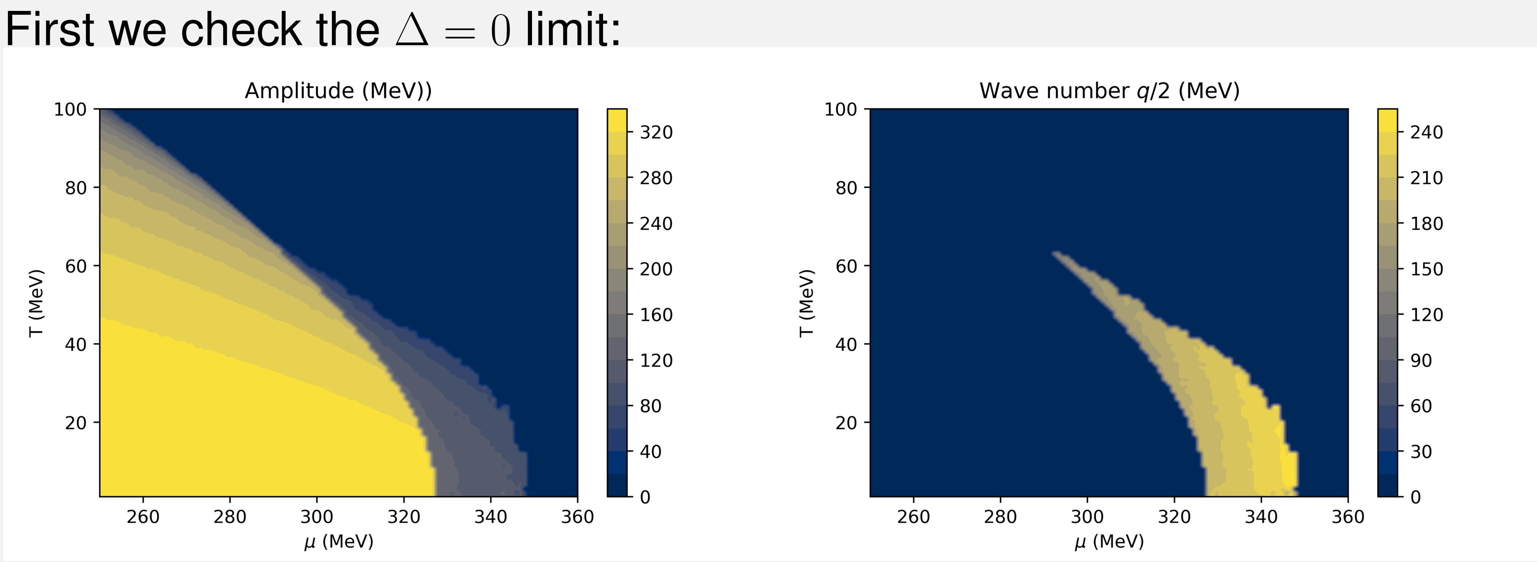
$$\Omega = \sum_{E_\lambda}\int\frac{d^3p}{(2\pi)^3}\left[E_\lambda + 2T\ln\left(1+\mathrm{e}^{-E_\lambda/T}\right)\right] + \frac{1}{4gL}\int_0^L dx(\sigma^2+\pi^2) + \frac{|\Delta|^2}{4g_\Delta}$$

Finally, we recast the momentum integral into an integral over the energies. This allows us to implement the inhomogeneity of the chiral condensate via the density of states. The density of states for the CDW and the solitonic Ansatz, ρ_{CDW} and ρ_{SN}, respectively, are taken from Ref. [1]

$$\Omega = -2\sum_\lambda\int_0^\infty dE\rho_{\mathrm{CDW/SN}}\left[E_\lambda + 2T\ln\left(1+\mathrm{e}^{-E_\lambda/T}\right)\right] + \frac{1}{4gL}\int_0^L dx(\sigma^2+\pi^2) + \frac{|\Delta|^2}{4g_\Delta}$$

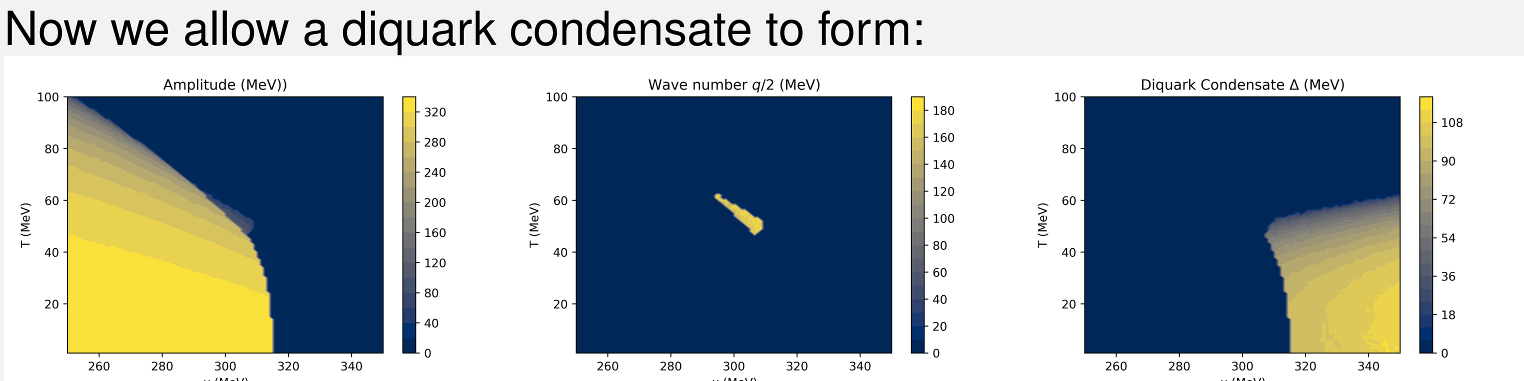
The phase diagram

First we check the $\Delta = 0$ limit:



This is in agreement with previous quark-meson model and NJL studies! See also Ref. [2].

Now we allow a diquark condensate to form:



- Both the the CDW Ansatz as well as the solitonic Ansatz yield very similar results
- Similar results using the CDW Ansatz have been obtained in Ref. [3]

Many questions remain:

- What is the most preferred inhomogeneous modulation?
- How do bosonic fluctuations affect inhomogeneous phases?

Work in progress!

Outlook 1: Finite-Mode Approach

- Spatial modulation of the chiral condensate could take an arbitrary shape → Use method that allows for arbitrary modulations
- One possibility is to apply lattice methods on low-energy effective models (see Marc Wagner's talk!)
- Another approach is the finite-mode approach [4]

Basic ideas of the finite-mode approach: expand fermionic fields as superpositions of plane waves: $\psi = \sum_{n_0,\vec{n}}\eta_{n_0,\vec{n}}\dfrac{\mathrm{e}^{-i(k_0x_0+\vec{k}\cdot\vec{x})}}{\sqrt{L_0V}}$

The path-integral measure then changes to

$$\int\mathcal{D}\psi\mathcal{D}\bar{\psi} \to \int\prod_{n_0,\vec{n}}d\eta_{n_0,\vec{n}}d\bar{\eta}_{n_0,\vec{n}}$$

The finite-mode approach can be applied for any theory with quadratic fermionic interactions:

$$S[\psi,\bar{\psi},\phi,\Delta] = \int d^Dx\,\left(\bar{\psi}Q(\phi,\Delta)\psi + \mathcal{V}\right)$$

In the mean-field approximation the effective action can then be written as:

$$S[\phi,\Delta] = \int d^4x\,\mathcal{V}(\phi,\Delta) - \frac{1}{2}\ln\det\left\langle k_0,\vec{k}\middle|\,Q^\dagger Q\,\middle|k_0',\vec{k}'\right\rangle$$

The last object is a finite-dimensional object in momentum space in the finite-mode approach, therefore it is possible to calculate the determinant numerically. NOTE: This works only if det Q is real and positive!

This approach is very useful for the investigation of inhomogeneous phases since it allows for arbitrary inhomogeneous modulations of the form

$$\sigma(\vec{x}) = \sum_j c_j\frac{\mathrm{e}^{-i\vec{p}\cdot\vec{x}}}{\sqrt{V}}$$

Preliminary idea!

Outlook 2: Functional Renormalization Group

- In the mean-field approximation fermionic fluctuations are integrated out while bosonic ones are not
- Use the FRG to include bosonic fluctuations and systematically study higher truncations
- A CDW Ansatz in the quark-meson model using the FRG is possible: See also the poster of Martin Steil
- Another approach is to look at instabilities which are signaled by zero crossings of the two-point function
- These instabilites do not give information about the specific spatial modulation but rather allows to identify the region in the phase diagram where such modulations are preferred
- In a previous quark-meson model study [5] such instabilities signaled by $\Gamma^{(2)}_{k,\pi}(\omega = 0, |\vec{p}_c|) = 0$ have been found in a region of the phase diagram where inhomogeneous phases are typically found in mean-field studies
- Our idea is to apply this analysis to the quark-meson-diquark model, which allows for the study of a phase diagram with two order parameters, the chiral condensate and the diquark gap

Suggestions and Feedback

I appreciate any feedback. Furthermore, I would love to hear any suggestions on how to continue this study!

Acknowledgements

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