



Andreas Schmitt

Mathematical Sciences and STAG Research Centre  
University of Southampton  
Southampton SO17 1BJ, United Kingdom



## Dense nuclear and quark matter from holography

- motivation: theoretical challenges in dense QCD and relevance for compact stars
- nuclear matter in the "decompactified" limit of the Sakai-Sugimoto model
- holographic quark-hadron continuity?

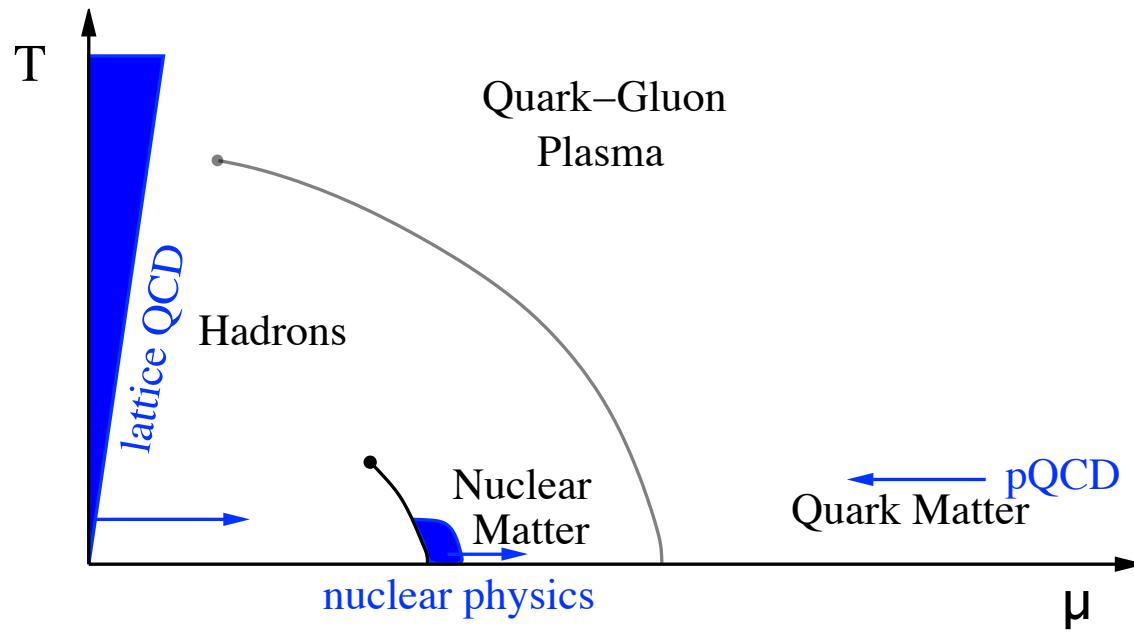
S.-w. Li, A. Schmitt, Q. Wang, PRD 92, 026006 (2015)

F. Preis, A. Schmitt, JHEP 1607, 001 (2016); EPJ Web Conf. 137, 09009 (2017)

K. Bitaghsir Fadafan, F. Kazemian, A. Schmitt, JHEP 1903, 183 (2019)

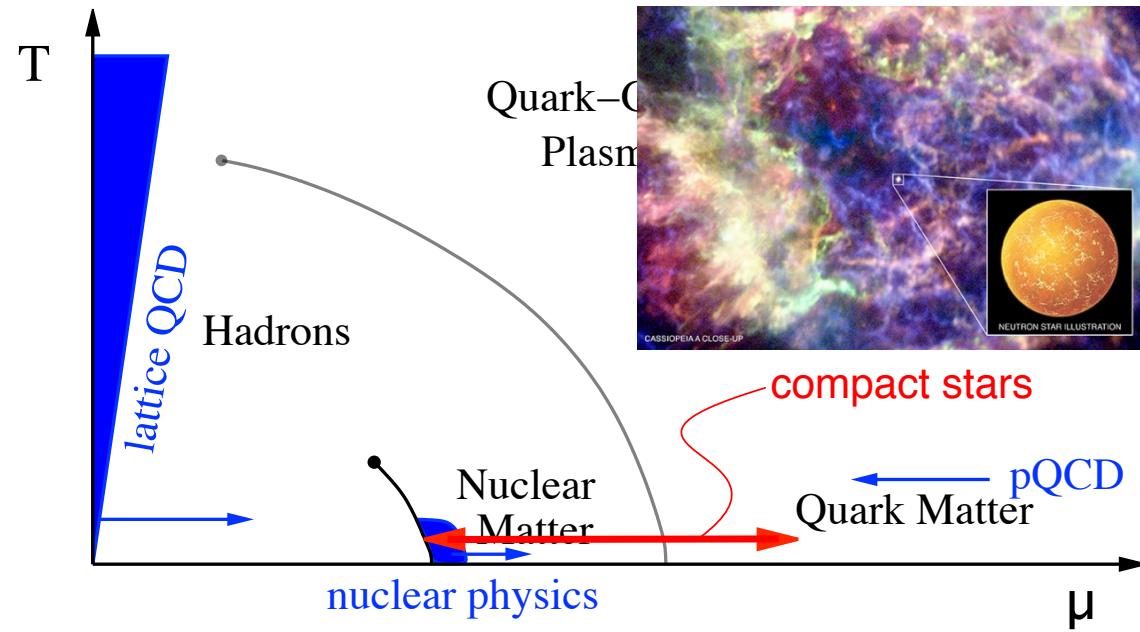
N. Kovensky, A. Schmitt, in preparation

## Dense QCD: theoretical approaches (page 1/2)



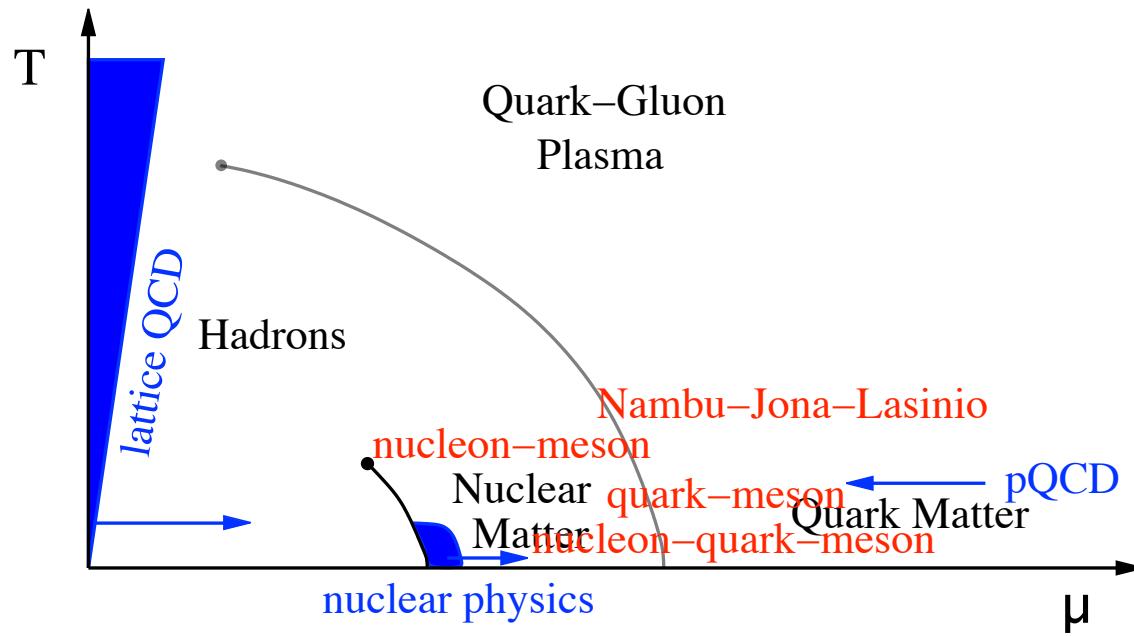
- lattice QCD: sign problem at nonzero  $\mu$  (some recent progress)
- perturbative QCD: restricted to ultra-high densities
- “standard” nuclear physics:  
restricted to densities  $\lesssim$  nuclear saturation density  $n_0$

## Dense QCD: theoretical approaches (page 1/2)



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## Dense QCD: theoretical approaches (page 2/2)

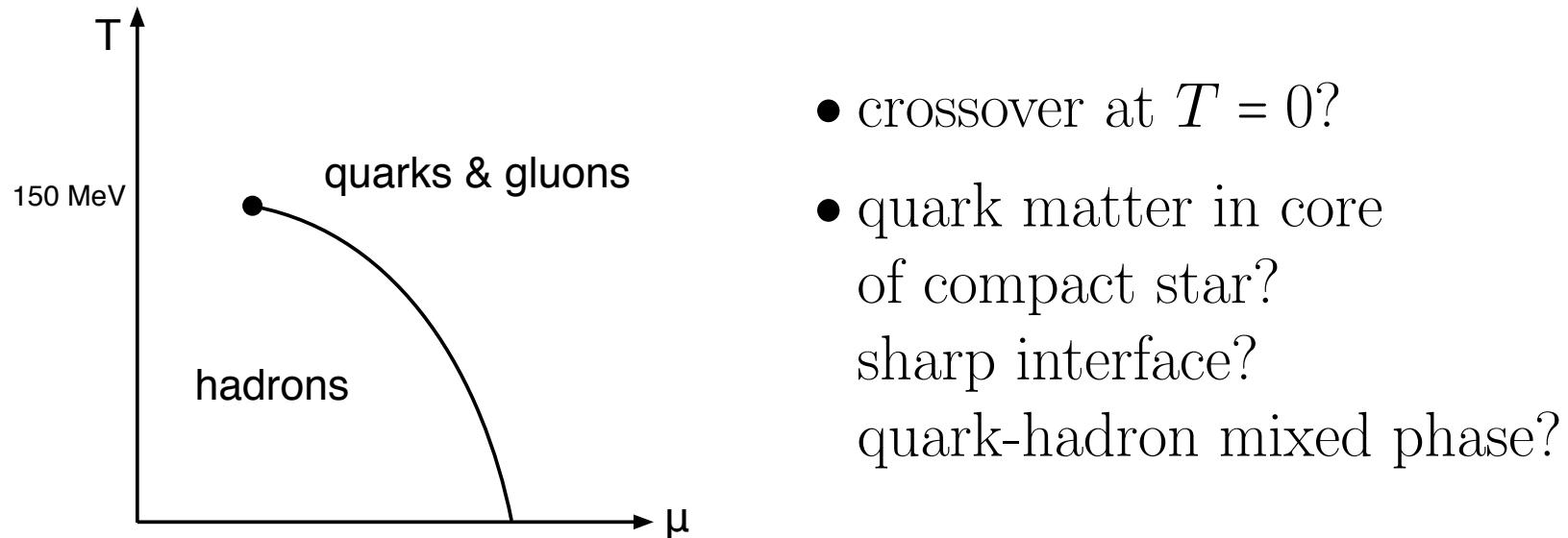


- Nambu–Jona-Lasinio (usually no nuclear matter)
  - quark-meson (no nucleons), nucleon-meson (no quarks)
  - nucleon-quark-meson (patched together, many parameters)
- even without rigor: models for compact stars hard to construct!

# Quark-hadron continuity at high density?

order parameter	Polyakov loop (confinement)	chiral condensate
spontaneously breaks	$\mathbb{Z}_{N_c}$	$SU(N_f) \times SU(N_f)$
symmetry exact for	pure Yang-Mills ( $m_q = \infty$ )	chiral limit ( $m_q = 0$ )

→ no qualitative difference between hadronic and quark matter

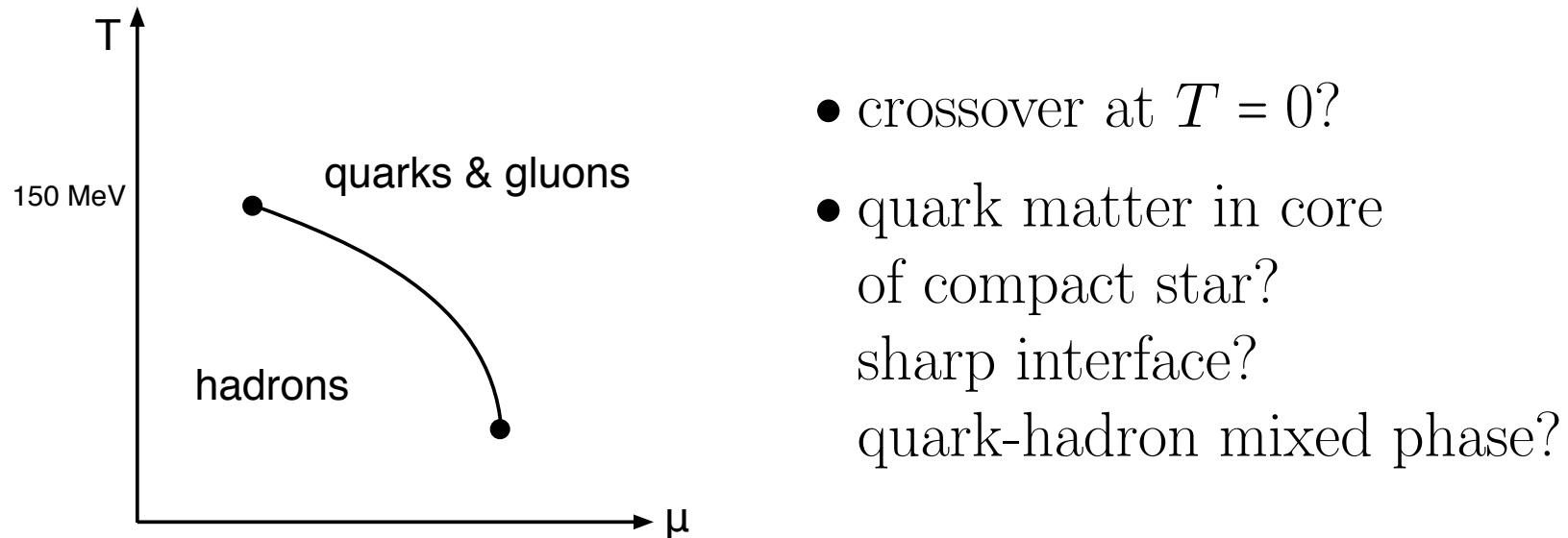


(for simplicity ignore Cooper pairing)

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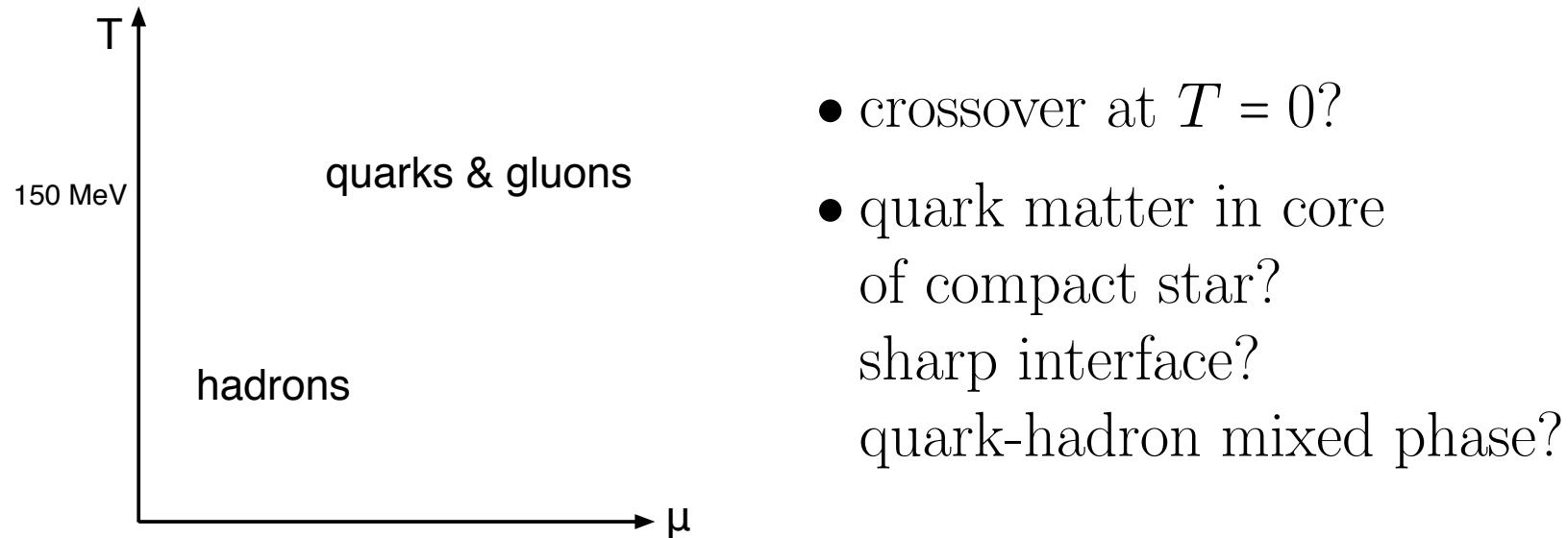


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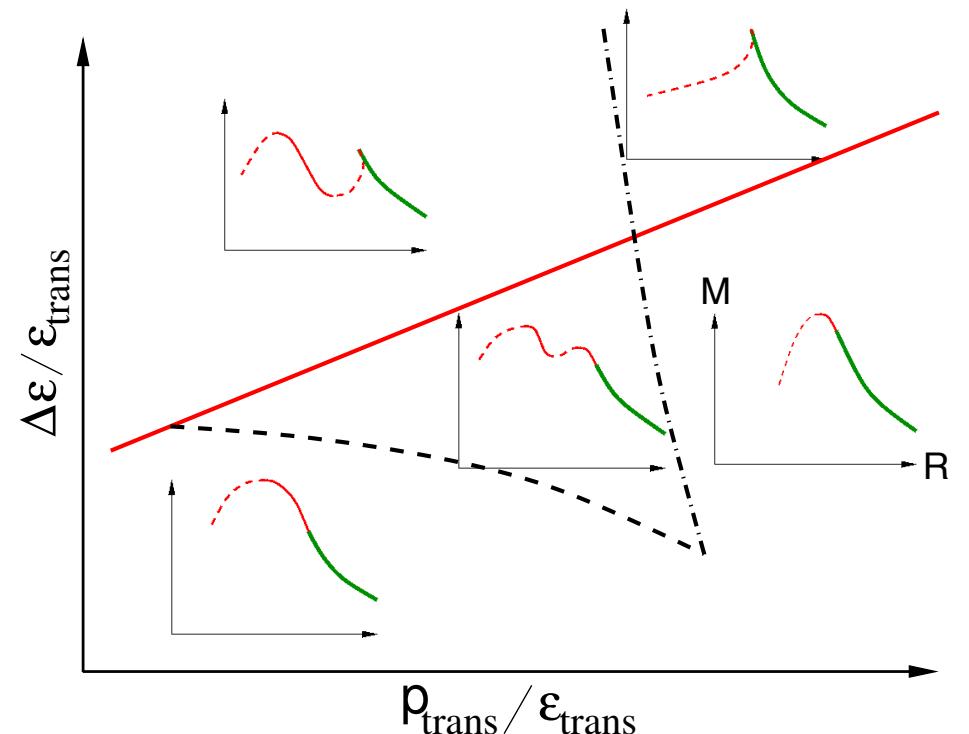
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(for simplicity ignore Cooper pairing)

# Observable consequences of first-order transition?

- qualitative difference in mass/radius curve  
M. G. Alford, S. Han and M. Prakash,  
PRD 88, 083013 (2013)
- sequential 1st-order transitions?  
M. G. Alford and A. Sedrakian, PRL  
119, 161104 (2017)

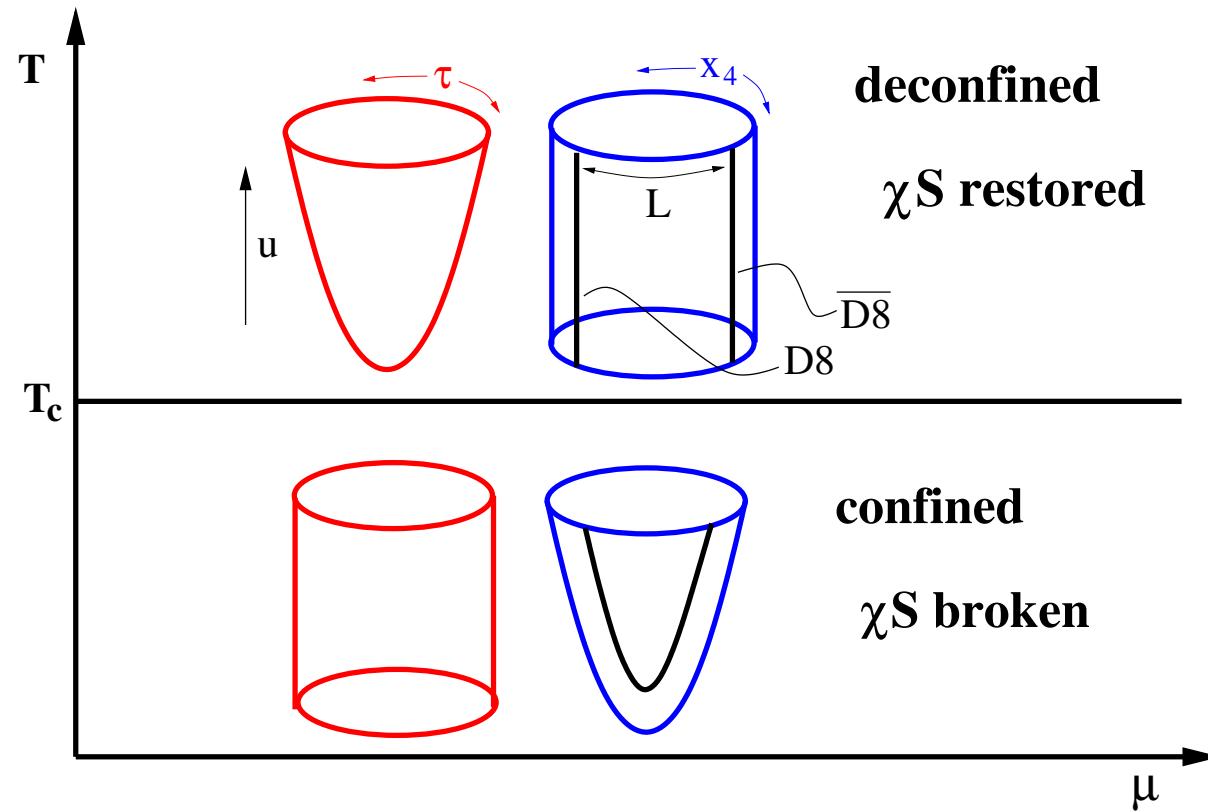


- different gravitational wave signal in neutron star mergers?  
E. R. Most *et al.*, PRL 122, 061101 (2019)
- gravitational wave from bubble nucleation during supernova?  
G. Cao and S. Lin, arXiv:1810.00528 [nucl-th]

## Can holography help?

- dual of QCD: probably exists, but currently out of reach
- reliable strong-coupling calculation (usually infinite coupling)
- successful (qualitative) predictions for heavy-ion collisions  
(supersymmetric YM plasma instead of quark-gluon plasma)
- Sakai-Sugimoto model
  - E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998)
  - T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)
    - top-down approach with only 3 parameters
    - dual to large- $N_c$  QCD, however in inaccessible limit
    - successfully applied to meson, baryon, glueball spectra
- holographic approach to neutron stars
  - D3/D7: C. Hoyos *et al.*, PRL 117, 032501 (2016); E. Annala *et al.*, JHEP 1812, 078 (2018)
  - V-QCD: N. Jokela *et al.*, JHEP 1903, 041 (2019); C. Ecker *et al.*, 1908.03213 [astro-ph.HE]
  - Sakai-Sugimoto with pointlike baryons: T. Hirayama *et al.*, 1902.08477 [hep-ph]

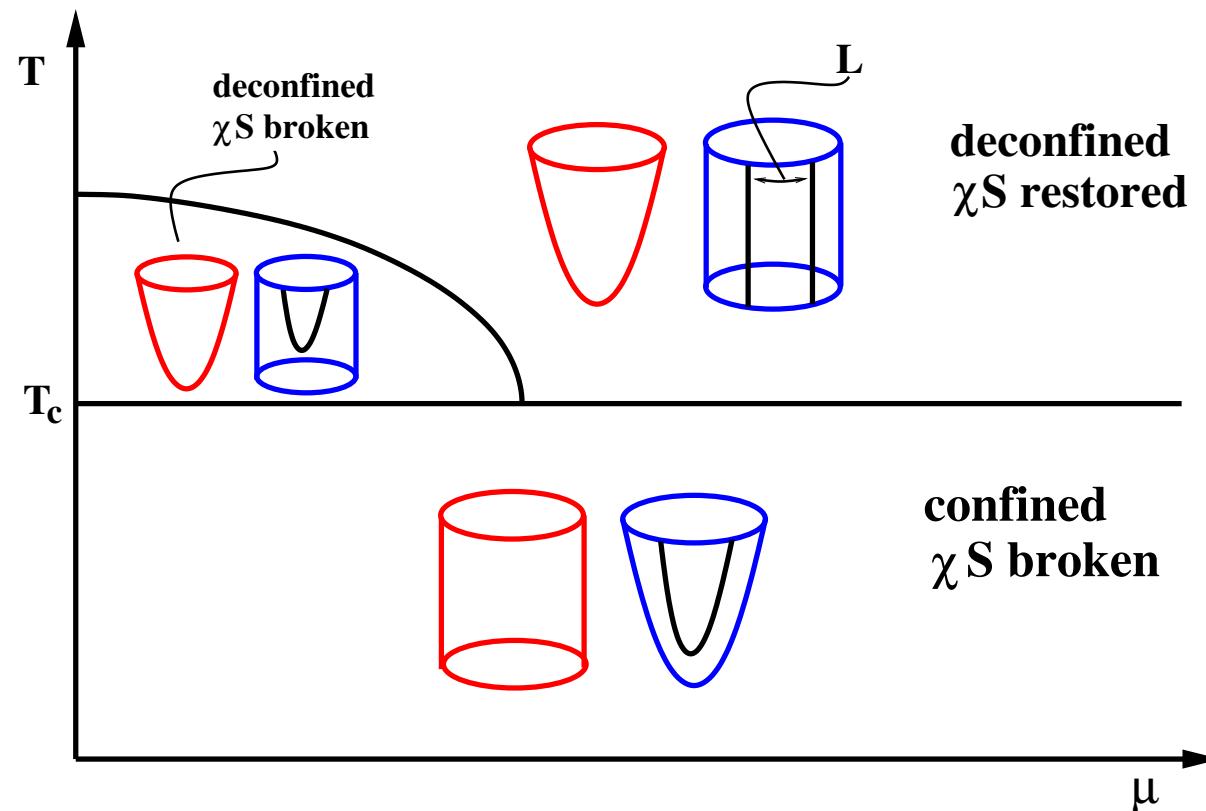
## Phases in the Sakai-Sugimoto model (page 1/3)



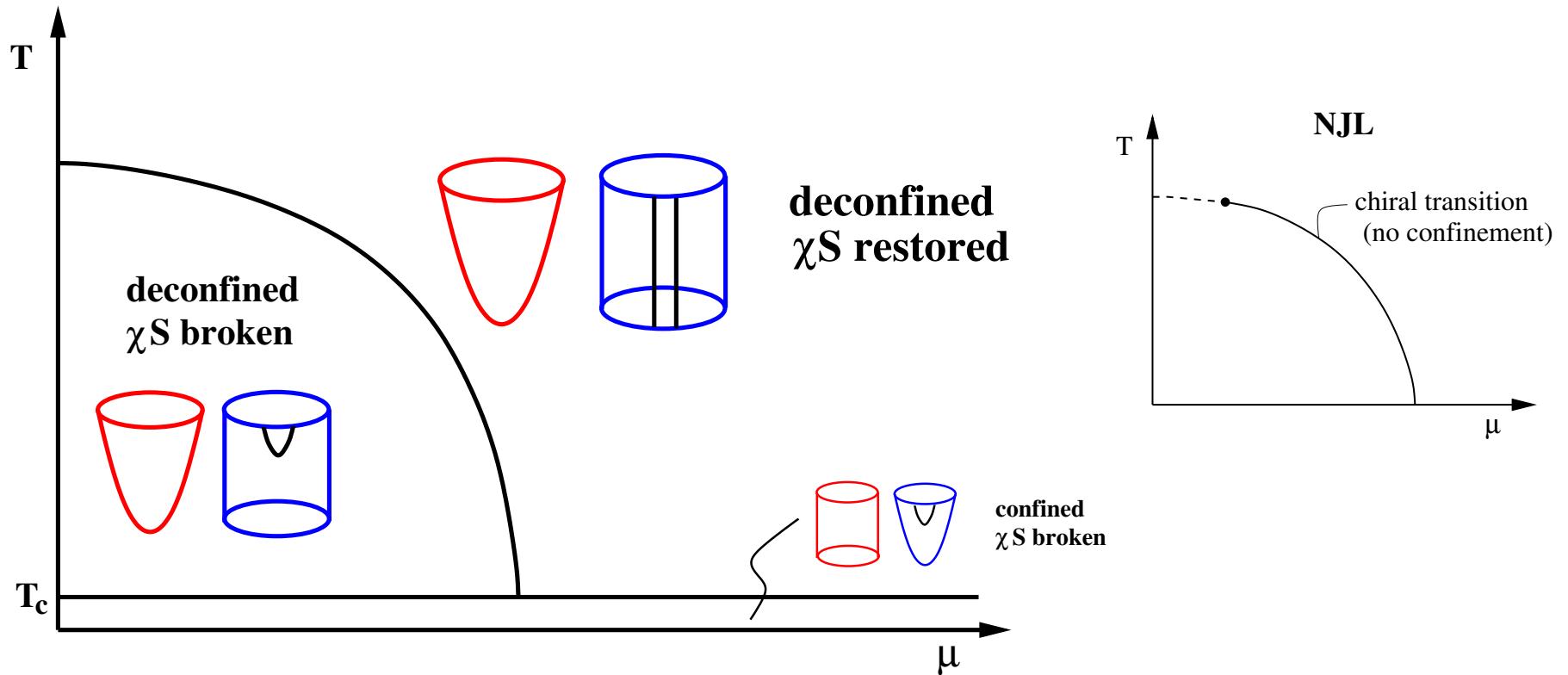
- in probe brane ("quenched") approximation: phase transition unaffected by quantities on flavor branes ( $\mu, B, \dots$ )
- not unlike expectation from large- $N_c$  QCD

## Phases in the Sakai-Sugimoto model (page 2/3)

- less “rigid” behavior for smaller  $L$
- deconfined, chirally broken phase for  $L < 0.3\pi/M_{KK}$   
 O. Aharony, J. Sonnenschein, S. Yankielowicz, Annals Phys. 322, 1420 (2007)  
 N. Horigome, Y. Tanii, JHEP 0701, 072 (2007)



## Phases in the Sakai-Sugimoto model (page 3/3)



- “decompactified” limit  $\rightarrow$  gluon dynamics decouple
- “NJL-like” dual field theory

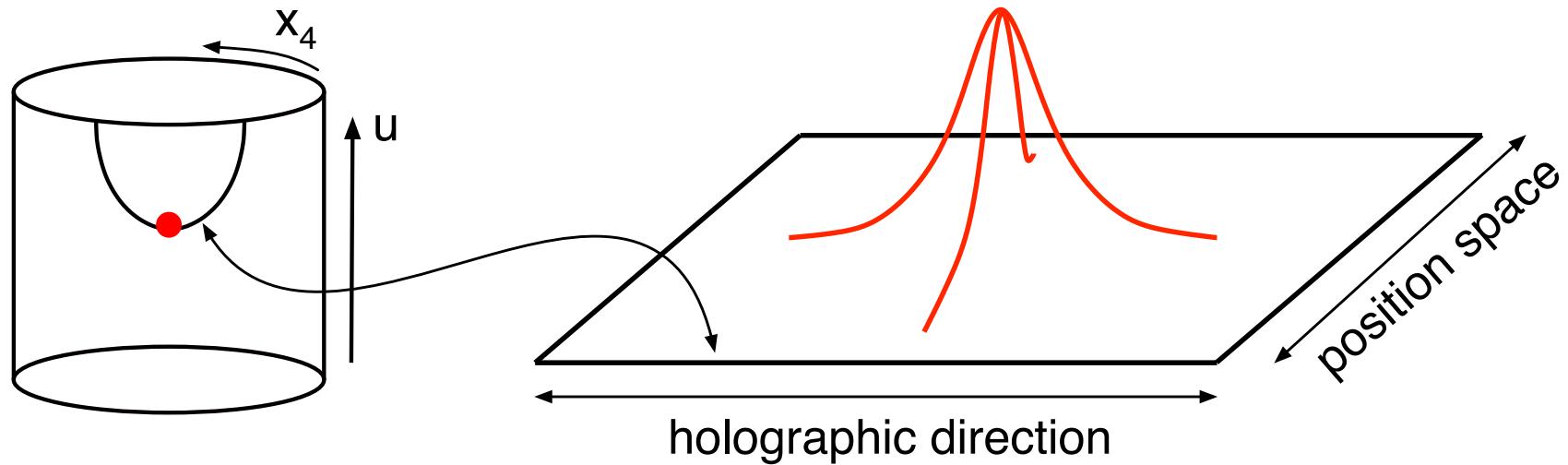
E. Antonyan, J. A. Harvey, S. Jensen, D. Kutasov, hep-th/0604017

J. L. Davis, M. Gutperle, P. Kraus, I. Sachs, JHEP 0710, 049 (2007)

F. Preis, A. Rebhan and A. Schmitt, Lect. Notes Phys. 871, 51 (2013)

## Adding baryons

- baryons in AdS/CFT: wrapped D-branes with  $N_c$  string endpoints  
E. Witten, JHEP 9807, 006 (1998); D. J. Gross, H. Ooguri, PRD 58, 106002 (1998)
- baryons in Sakai-Sugimoto:
  - D4-branes wrapped on  $S^4$
  - equivalently: instantons on D8-branes ( $\rightarrow$  skyrmions)  
T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843-882 (2005)  
H. Hata, T. Sakai, S. Sugimoto, S. Yamato, Prog. Theor. Phys. 117, 1157 (2007)



# Approximations for holographic nuclear matter

- Pointlike approximation  
O. Bergman, G. Lifschytz, M. Lippert, JHEP 0711, 056 (2007)
- Finite-width, but non-interacting, instantons  
K. Ghoroku, K. Kubo, M. Tachibana, T. Taminato and F. Toyoda, PRD 87, 066006 (2013)  
S.-w. Li, A. Schmitt, Q. Wang, PRD 92, 026006 (2015)  
F. Preis, A. Schmitt, JHEP 1607, 001 (2016); EPJ Web Conf. 137, 09009 (2017)
- Interacting instantons from exact two-instanton solution  
(this talk) K. Bitaghsir Fadafan, F. Kazemian, A. Schmitt, JHEP 1903, 183 (2019)  
beyond chiral limit: N. Kovensky, A. Schmitt (in preparation)
- see also: “Homogeneous ansatz”  
M. Rozali, H. H. Shieh, M. Van Raamsdonk and J. Wu, JHEP 0801, 053 (2008)  
S.-w. Li, A. Schmitt, Q. Wang, PRD 92, 026006 (2015)  
M. Elliot-Ripley, P. Sutcliffe and M. Zamaklar, JHEP 1610, 088 (2016)

## Setup

- D8-brane action

$$S = \underbrace{T_8 V_4 \int_{x^\mu} \int_z e^{-\Phi} \sqrt{\det(g + 2\pi\alpha' F)}}_{\text{Dirac-Born-Infeld (DBI)}} + \underbrace{\frac{N_c}{8\pi^2} \int_{x^\mu} \int_z \hat{A}_0 \text{Tr}[F_{ij} F_{kz}] \epsilon_{ijk}}_{\text{Chern-Simons (CS)}}$$

- gauge fields in the bulk ( $\rightarrow$  global symmetry at the boundary)

$$N_f = 2 : \quad F_{\mu\nu} = \hat{F}_{\mu\nu} + F_{\mu\nu}^a \sigma_a$$

- abelian part  $U(1)$ : chemical potential  $\mu = \hat{A}_0(z = \pm\infty)$

- non-abelian part  $SU(2)$ : baryon number (instantons)

$$N_B = -\frac{1}{8\pi^2} \int_{\vec{x}} \int_z \text{Tr}[F_{ij} F_{kz}] \epsilon_{ijk}$$

## Many-instanton approximation (page 1/2)

- ansatz for interacting many-instanton system from flat-space instantons

$$F^2 \simeq \sum_{n=1}^{N_I} F_0^2(n) + \frac{1}{2} \sum_{n=1}^{N_I} \sum_{n \neq m}^{N_I} \mathcal{I}(m, n)$$

- single instanton with width  $\rho = \rho_0 u_c^{3/4}$  and deformation  $\gamma = \gamma_0 u_c^{3/2}$

$$F_0^2(n) \sim \frac{\rho^4}{[(\vec{x} - \vec{x}_n)^2 + \frac{z^2}{\gamma^2} + \frac{\rho^2}{\gamma^2}]^4}$$

- 2-body interaction from exact 2-instanton solution  $F(m, n)$

ADHM: M. F. Atiyah, N. J. Hitchin, V. G. Drinfeld and Y. I. Manin, PLA 65, 185 (1978)

nucleon interaction in Sakai-Sugimoto: K. Y. Kim and I. Zahed, JHEP 0903, 131 (2009)

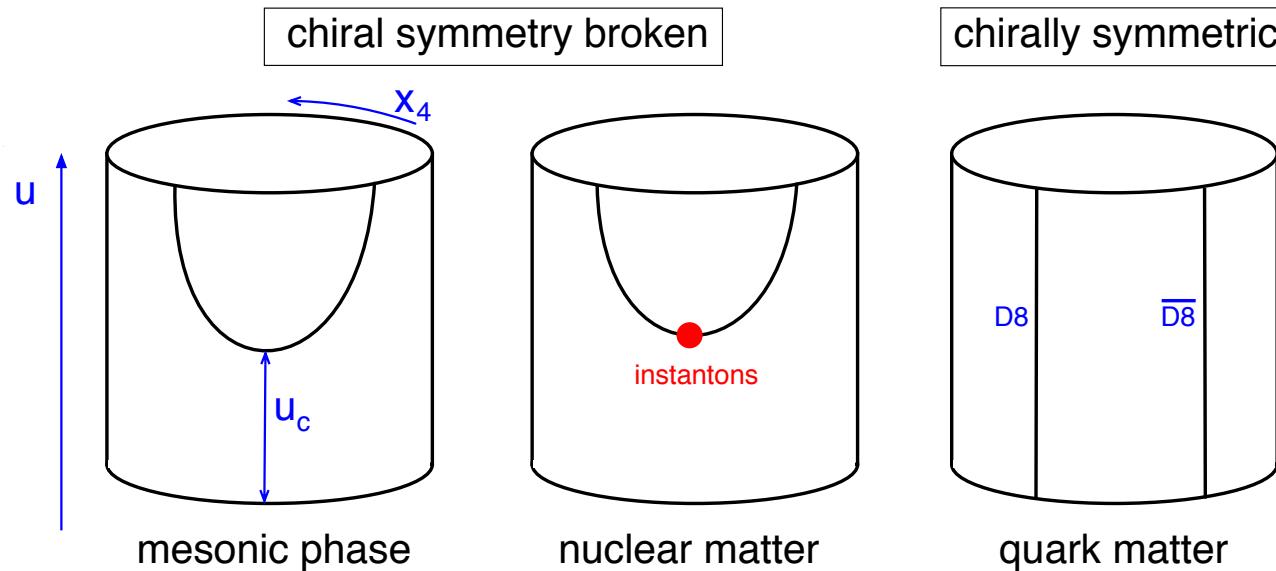
K. Hashimoto, T. Sakai and S. Sugimoto, Prog. Theor. Phys. 122, 427 (2009)

$$\mathcal{I}(m, n) = F^2(m, n) - F_0^2(m) - F_0^2(n)$$

- choose lattice structure in  $\mathbb{R}^3$ , apply nearest-neighbor approximation, neglect layers in holographic direction
- assume  $SU(2)$  orientation of instantons to be the same
- take spatial average  $F^2 \rightarrow \langle F^2 \rangle$

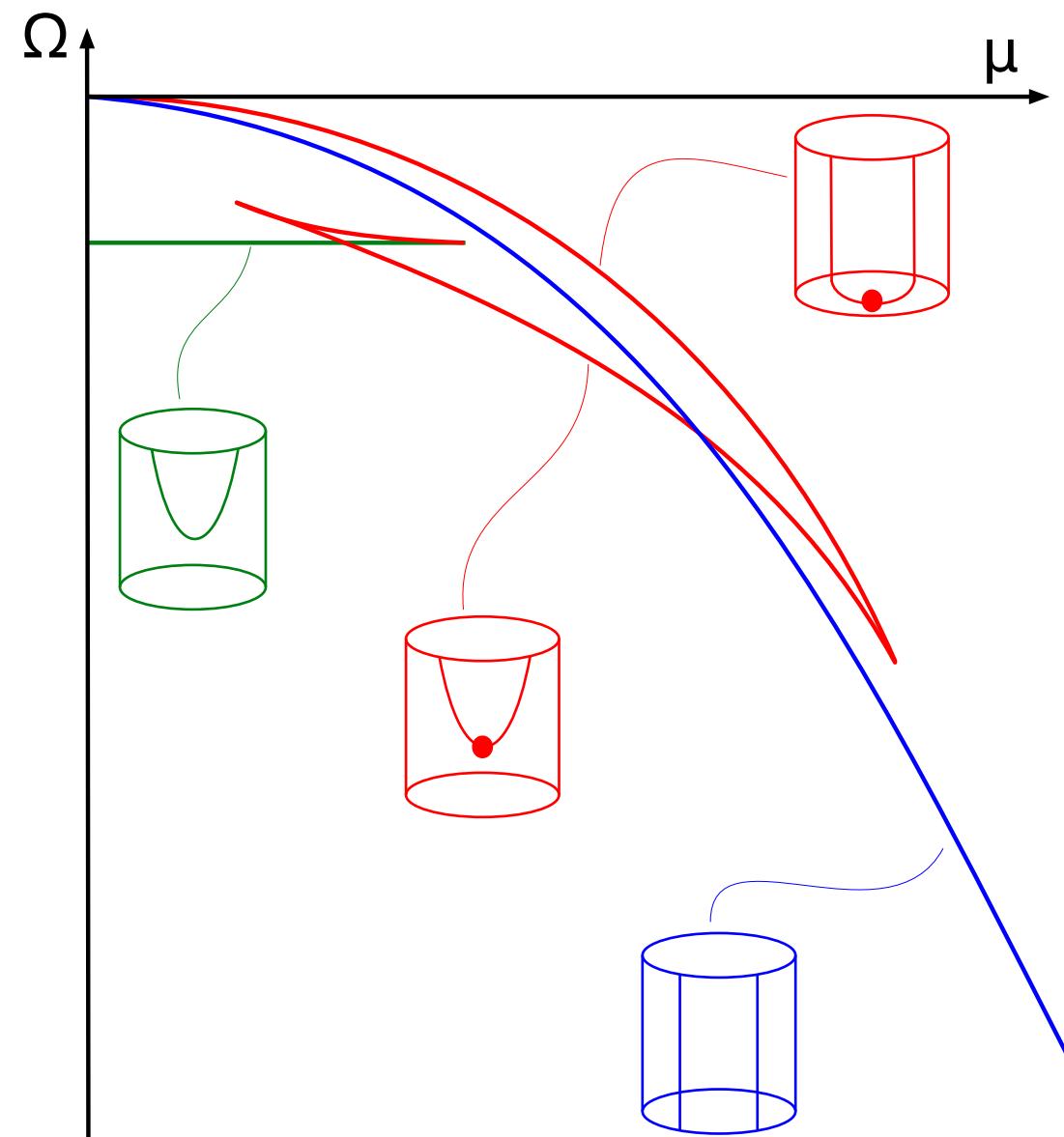
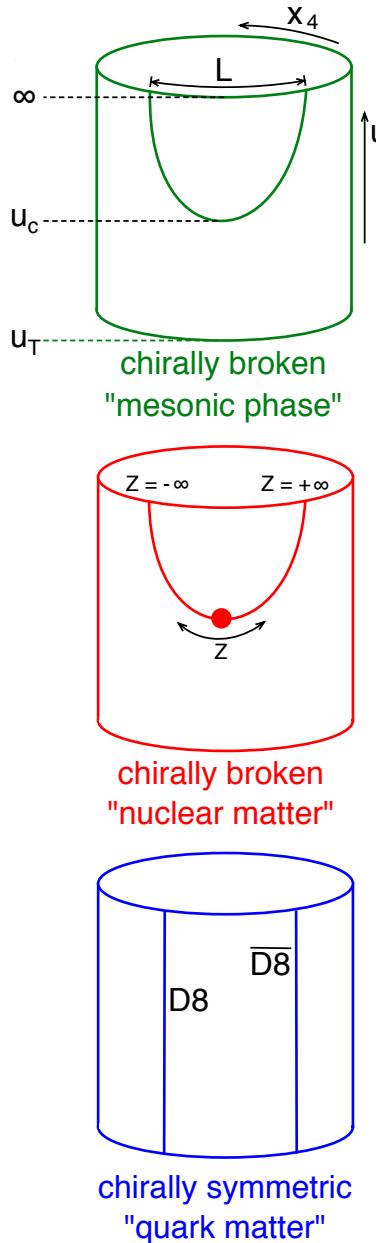
## Many-instanton approximation (page 2/2)

- solve EOMs for  $U(1)$  gauge field  $\hat{A}_0(u)$  and embedding  $x_4(u)$
- minimize free energy wrt  $u_c$ ,  $N_I$  for given  $\mu$  (and  $T$ )

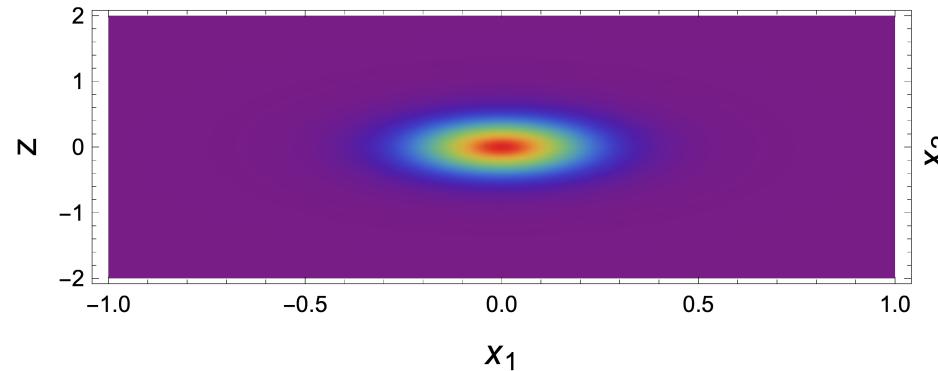
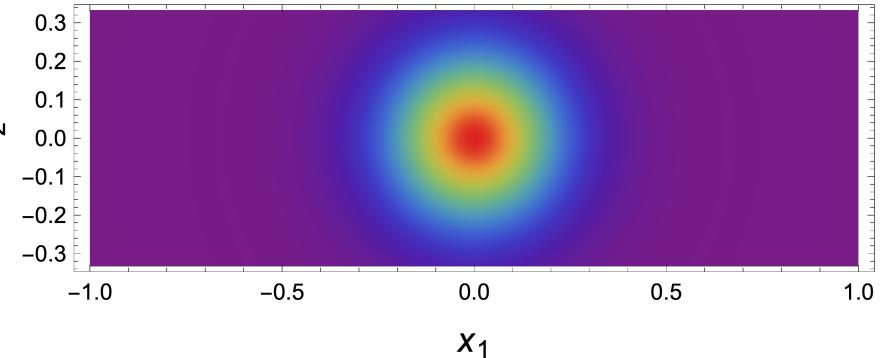


- compare free energy of all three phases
- 5 parameters ( $M_{KK}, L, \lambda, \rho_0, \gamma_0$ ):  
scan parameter space F. Preis, A. Schmitt, JHEP 1607, 001 (2016)  
or fit parameters to low-density nuclear matter properties  
K. Bitaghsir Fadafan, F. Kazemian, A. Schmitt, JHEP 1903, 183 (2019)

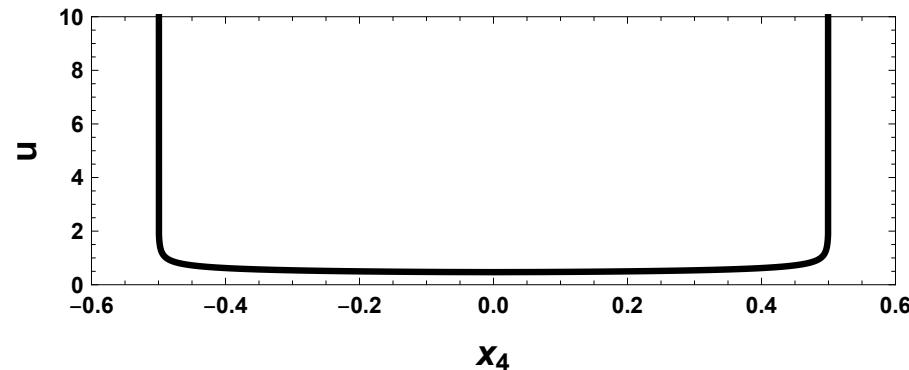
## Main result (schematically)



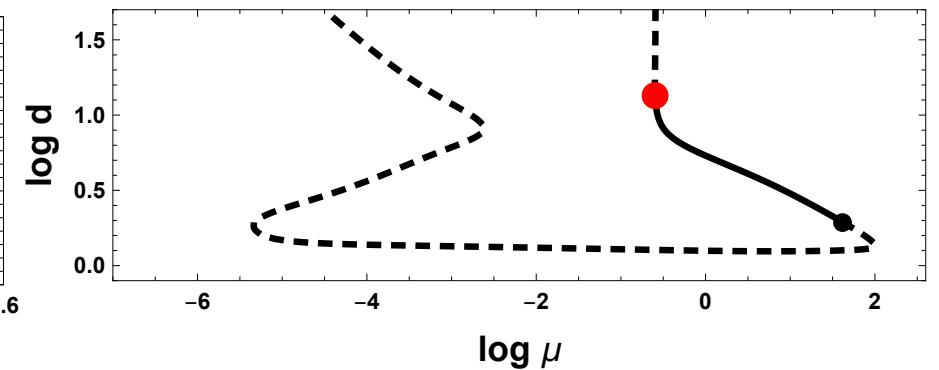
## Instanton profiles

deformation from  $\text{SO}(4)$ 

instanton lattice

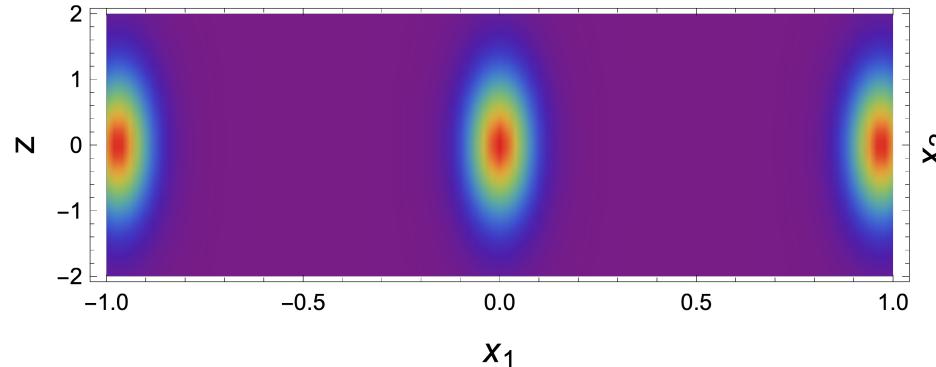
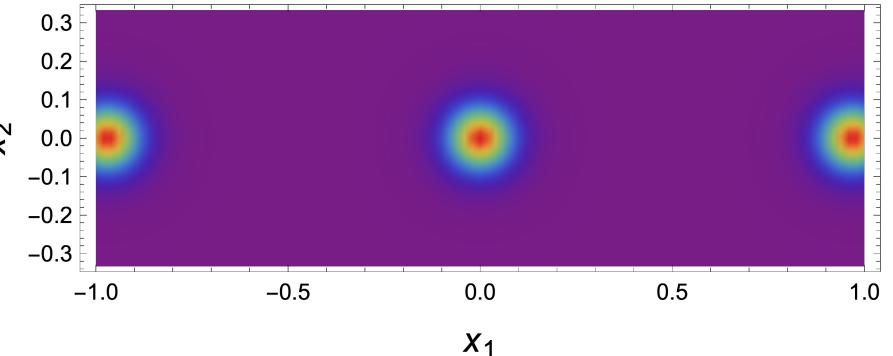


flavor brane embedding

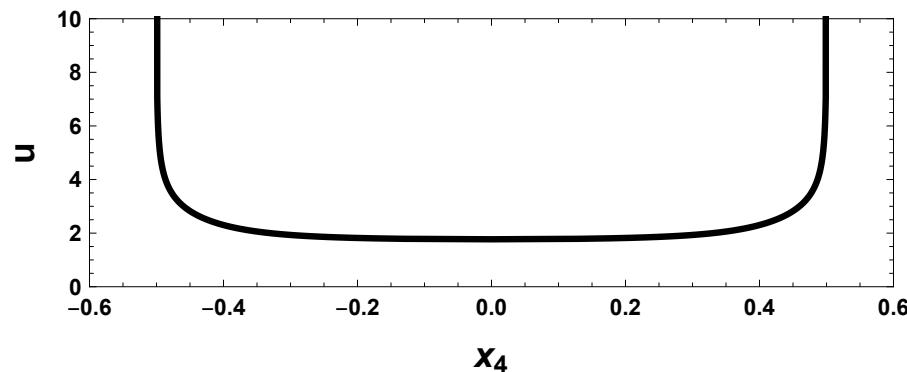


instanton overlap

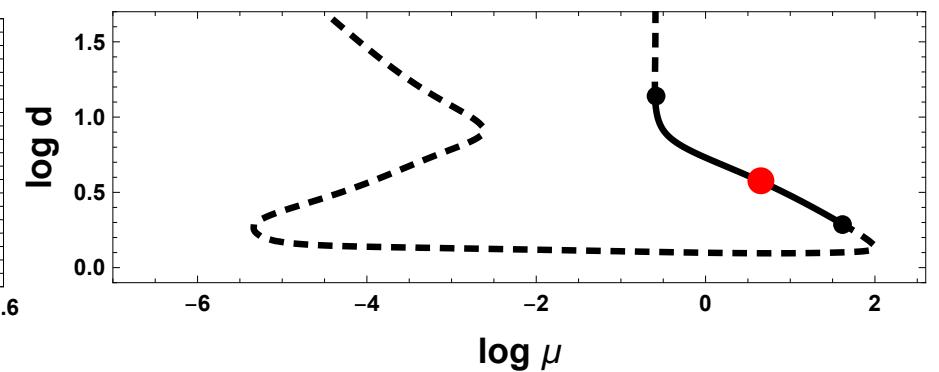
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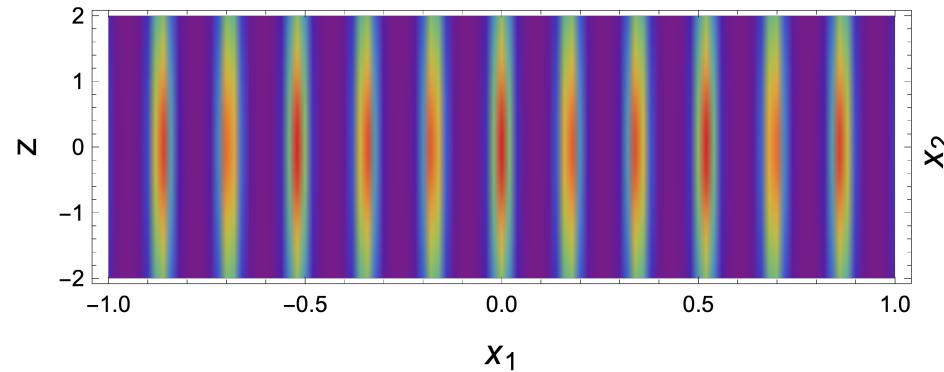
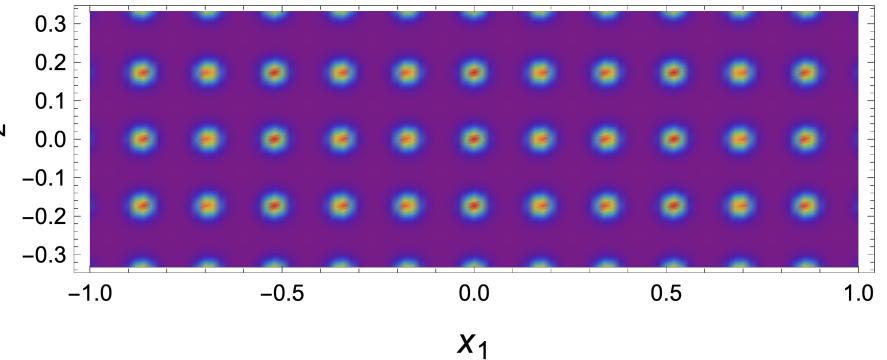


flavor brane embedding

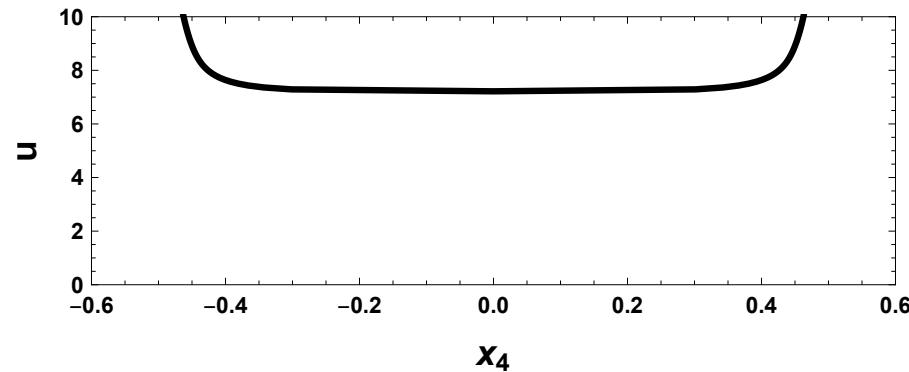


instanton overlap

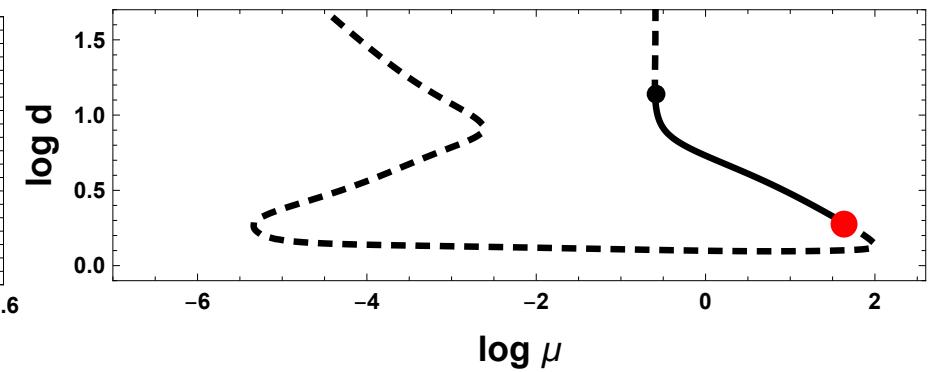
## Instanton profiles

deformation from  $SO(4)$ 

instanton lattice

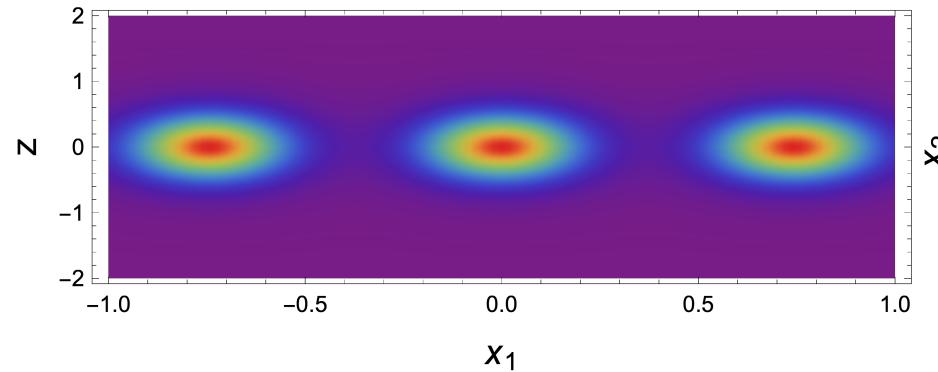
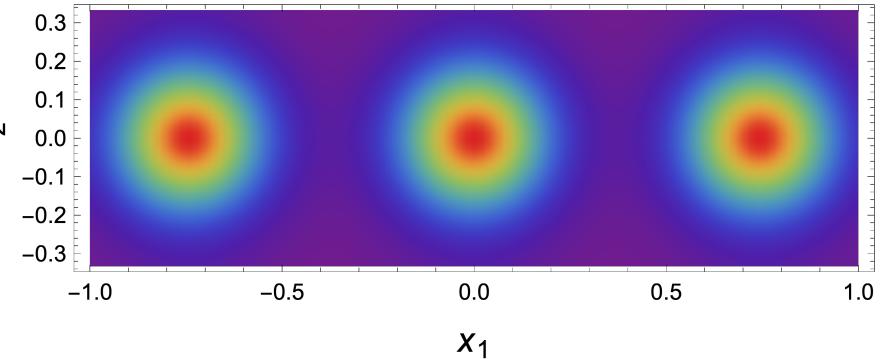


flavor brane embedding

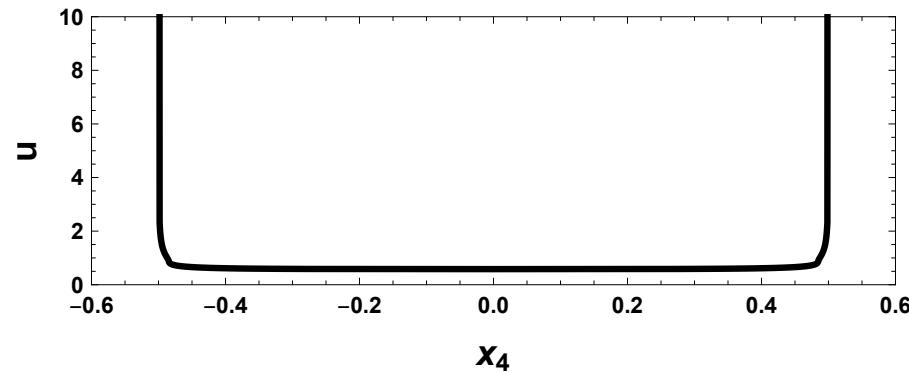


instanton overlap

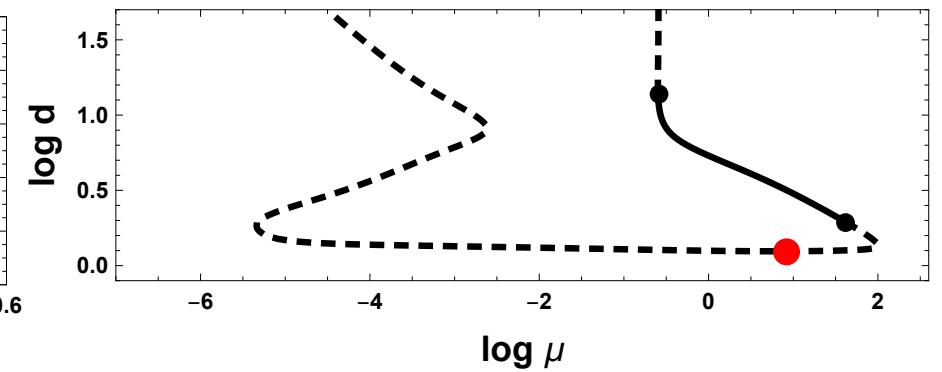
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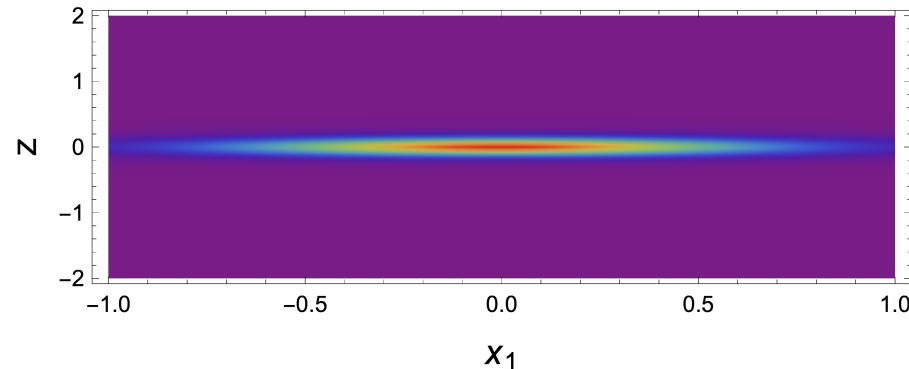
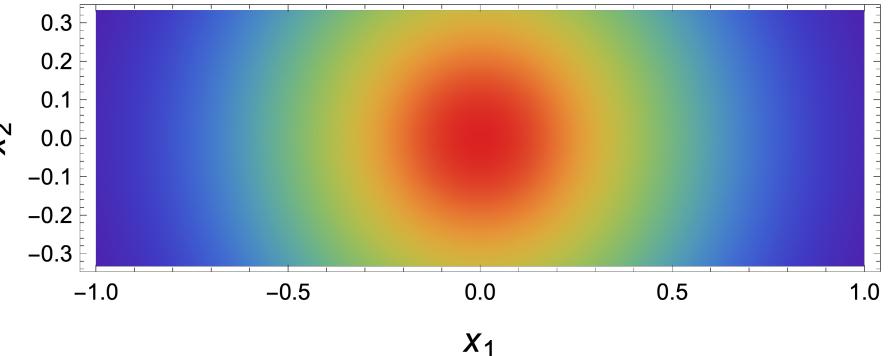


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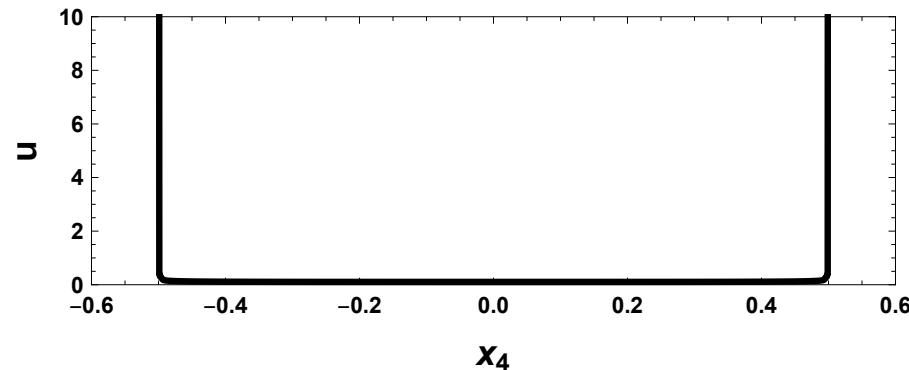


instanton overlap

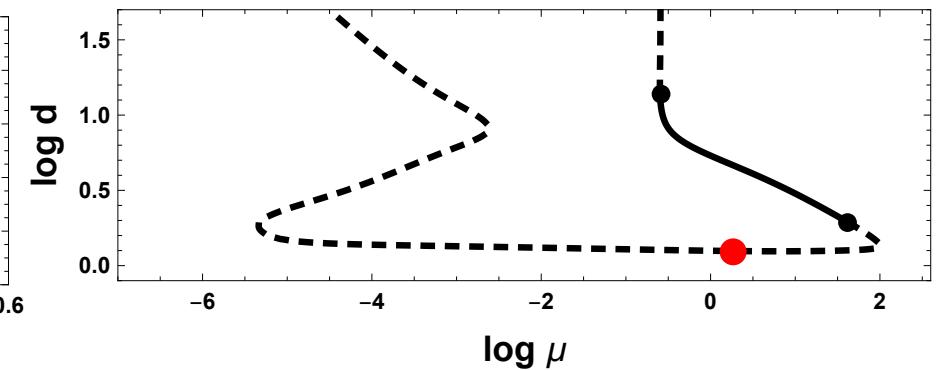
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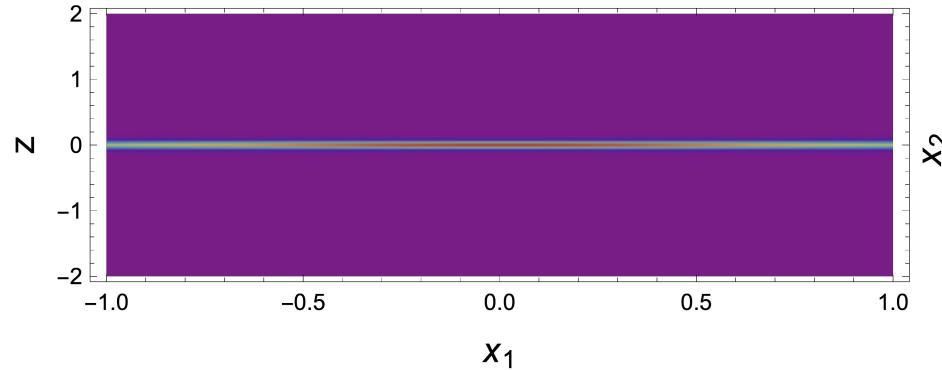
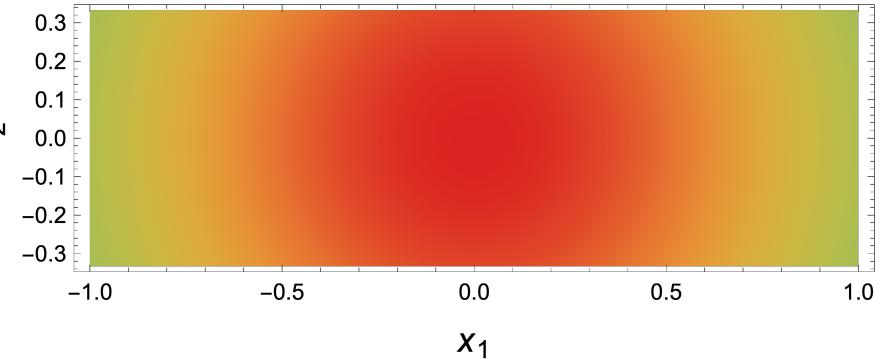


flavor brane embedding

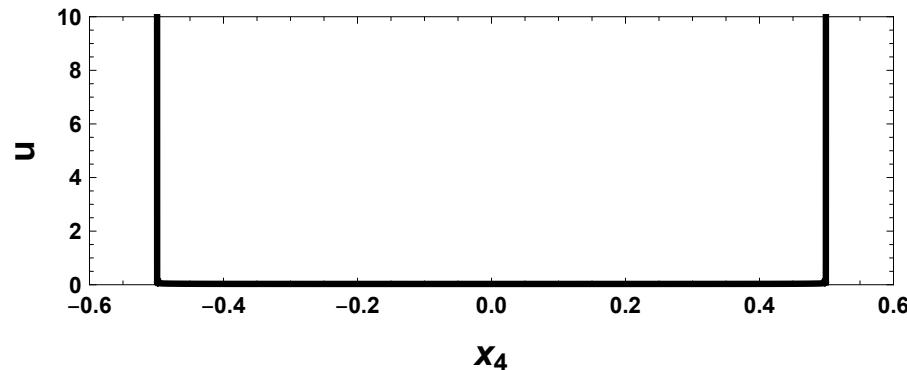


instanton overlap

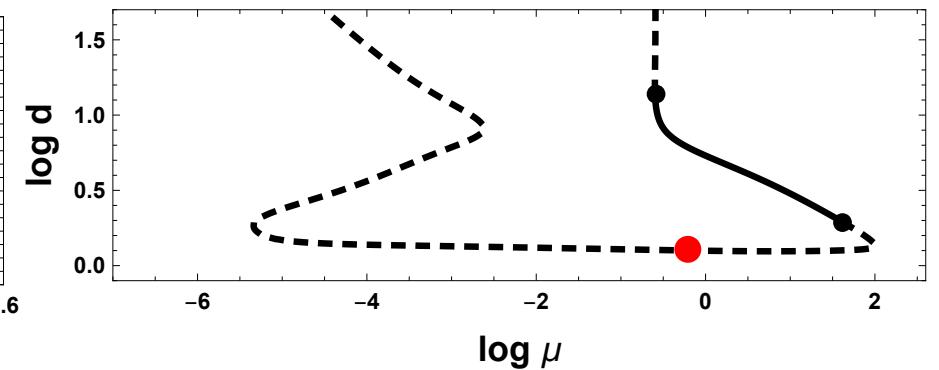
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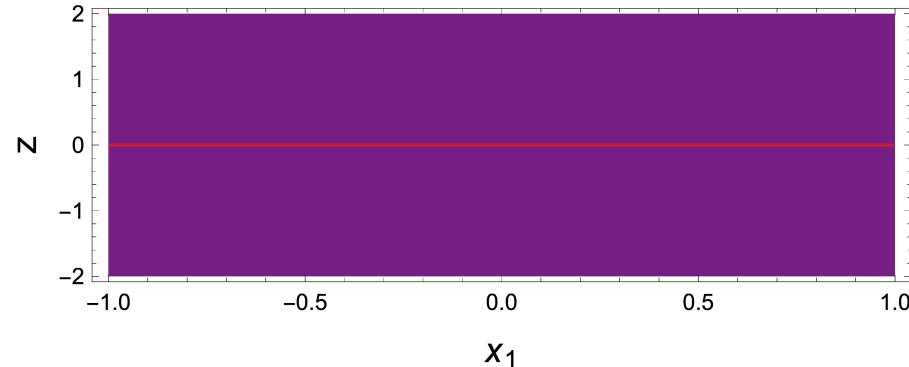
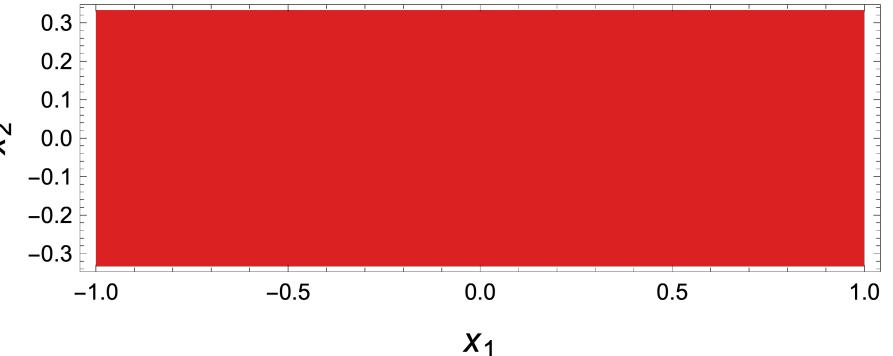


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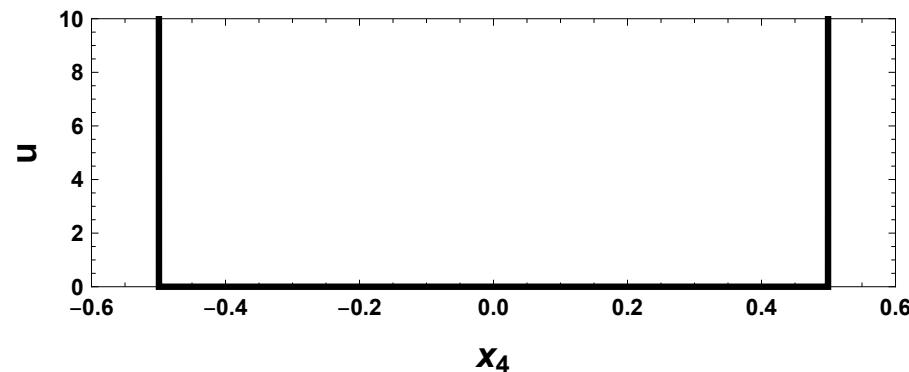


instanton overlap

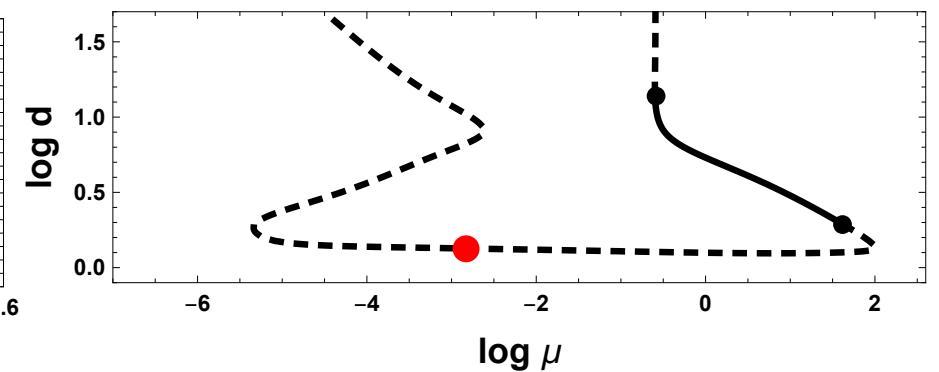
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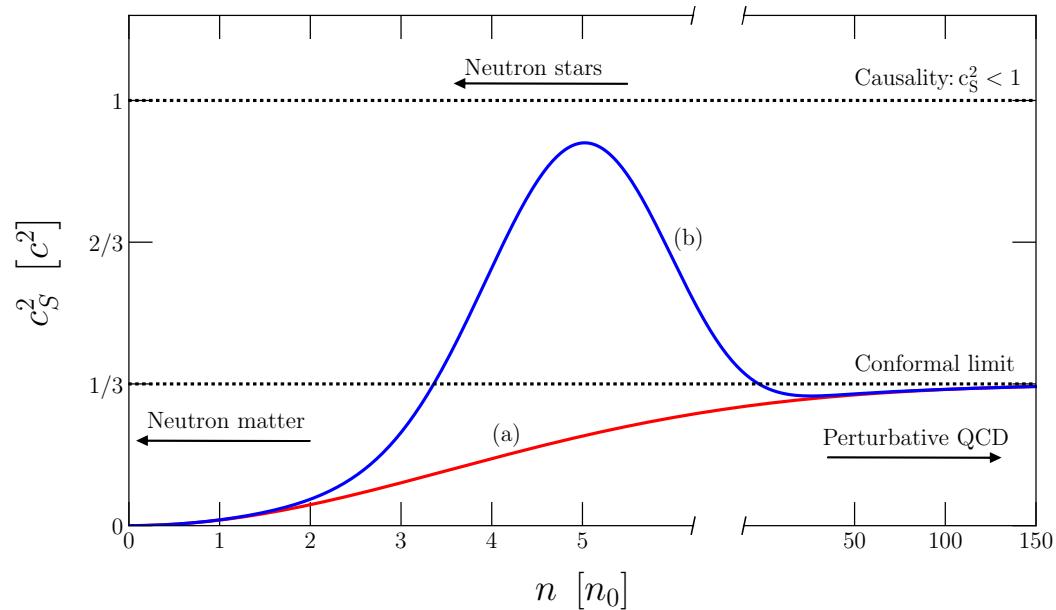


instanton overlap

## Observations

- hadron and quark phases connect continuously  
(instanton interactions crucial!)
  - geometrically: continuous transformation between connected and disconnected flavor branes
  - instantons smear out in spatial direction and become infinitesimally thin in holographic direction
  - continuity at zero chemical potential due to masslessness of quarks  
(like a second-order phase transition)
- instantons avoid overlapping by becoming smaller at high density
- actual quark-hadron transition is of first order and at extremely large  $\mu \simeq 30 \text{ GeV}$  (compact stars:  $\mu \lesssim 0.5 \text{ GeV}$ )

# Speed of sound

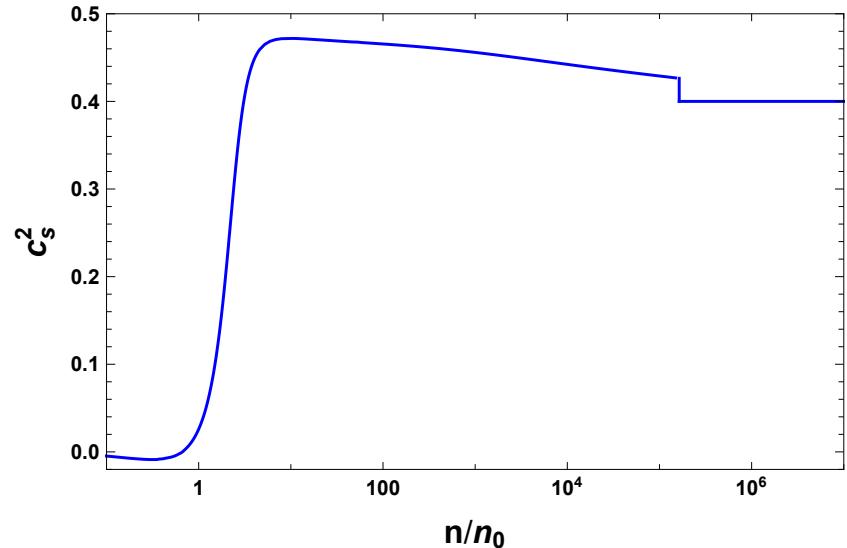


sound speed  
 $\leftrightarrow$  stiffness of matter  
 $\leftrightarrow$  neutron star masses  
 schematic plot from I. Tews *et al.*,  
*Astrophys. J.* 860, 149 (2018)

- Sakai-Sugimoto: non-monotonic speed of sound

K. Bitaghsir Fadafan, F. Kazemian,  
 A. Schmitt, *JHEP* 1903, 183 (2019)

- see also: quarkyonic speed of sound  
 L. McLerran, S. Reddy,  
*PRL* 122, 122701 (2019)



## Beyond chiral limit (page 1/2)

N. Kovensky, A. Schmitt, in preparation

- quarks as strings stretching from color to flavor branes  
 → separation of branes gives current quark mass (e.g., D3-D7)  
 N. Evans, A. Gebauer, K. Y. Kim and M. Magou, JHEP 1003, 132 (2010)
- Sakai-Sugimoto: D4-D8, no dimension left to separate branes  
 → less straightforward to introduce quark mass
- add mass correction to action, as in chiral perturbation theory

$$S_m = -m_q \int d^4x \text{Tr } \mathcal{O}$$

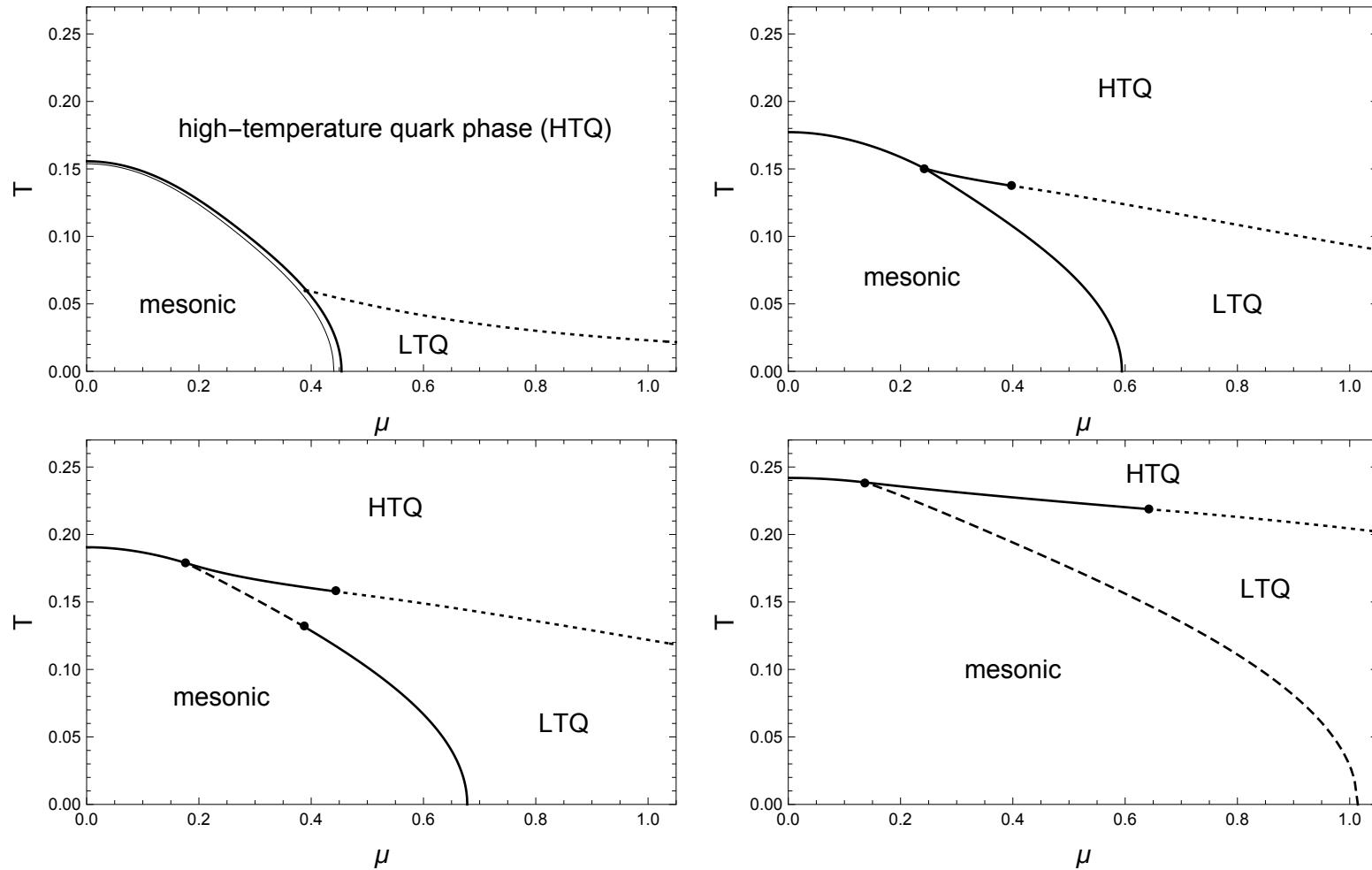
- chiral condensate from open Wilson line operator  
 O. Aharony and D. Kutasov, PRD 78, 026005 (2008)

$$\mathcal{O} = \bar{\psi}(x_4 = -L/2) \mathcal{P} \exp \left( i \int_{L/2}^{L/2} dx_4 a_4 \right) \psi(x_4 = L/2)$$

- $\langle \mathcal{O} \rangle$  computed from string worldsheet action  
 R. McNees, R. C. Myers and A. Sinha, JHEP 0811, 056 (2008)

## Beyond chiral limit (page 2/2)

Phase diagrams for increasing quark mass (without baryons):



- Very similar to D3-D7 (unlike chiral limit) and to "heavy QCD"  
 N. Evans, A. Gebauer, K. Y. Kim and M. Magou, JHEP 1003, 132 (2010)  
 M. Fromm, J. Langelage, S. Lottini and O. Philipsen, JHEP 1201, 042 (2012)

## Summary

- location and nature of the quark-hadron transition at large baryon densities is unknown (sign problem)
- a potential first-order quark-hadron transition has observable consequences for neutron star physics
- Sakai-Sugimoto model allows for consistent treatment of nuclear and quark matter
- if instanton interactions are included, nuclear and quark matter phases are continuously connected

## Outlook

- include nonzero quark masses  
N. Kovensky, A. Schmitt, in preparation
- isospin asymmetry → from symmetric nuclear matter to neutron star matter
- nonzero temperature and/or magnetic field → phase diagrams
- equation of state → neutron star mass/radius, deformability