

Dynamics on the edge: charge fractionalization and anyonic exclusion

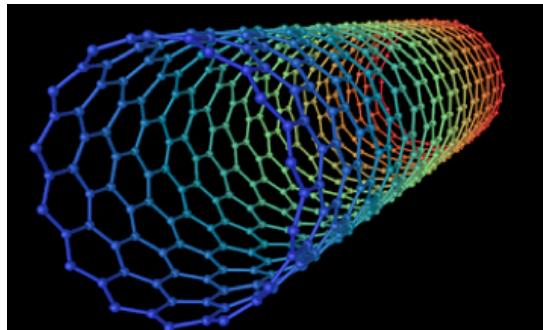
SIFT 2019 Strongly Interacting Field Theories,
November 7th 2019

Bernd Rosenow

Universität Leipzig

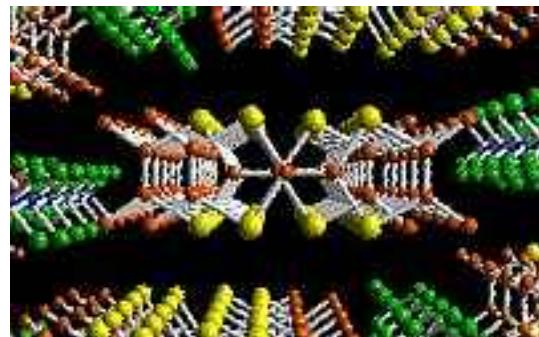
Work done in collaboration with B.I. Halperin, I. Levkivskyi, M. Milletari, and A. Schneider

Where do we encounter 1d physics?



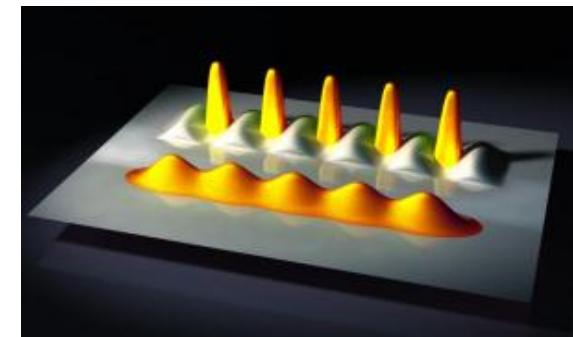
www.wikipedia.org

carbon nanotube



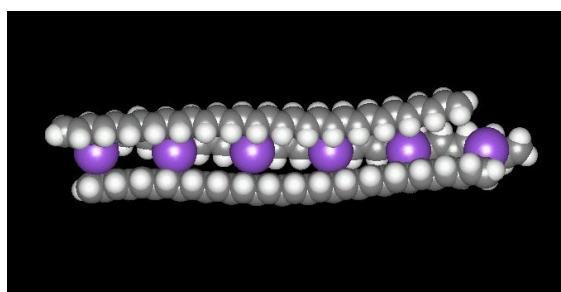
www.superconductivity.org

organic Bechgaard salts



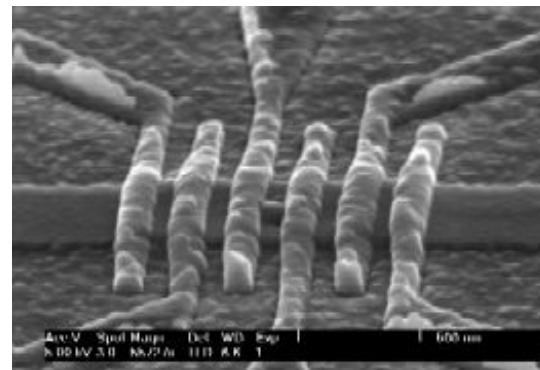
Credit: University of Innsbruck

cold atoms



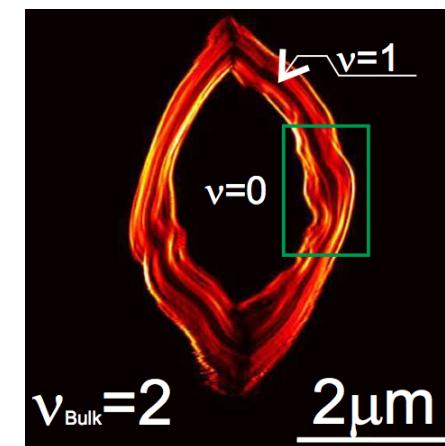
ncnr.nist.gov

Polyacetelene



www.ptb.de

quantum wires

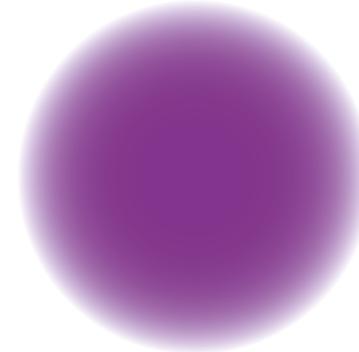


Pascher et al., PRX 4, 011014 (2014)

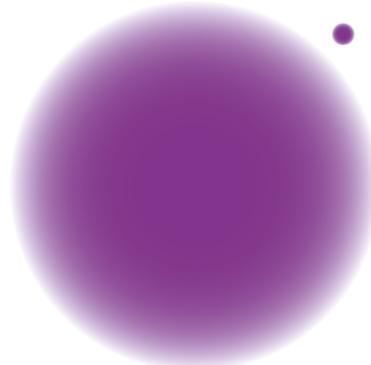
quantum Hall edges

Free electrons in 3d

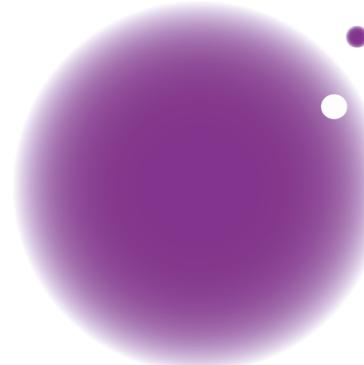
ground state: Fermi sphere



$N+1$ particle excited state

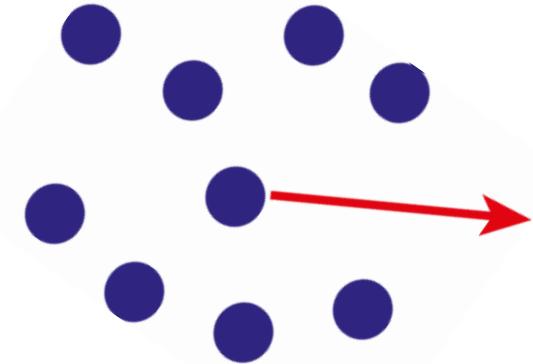


N particle excited state



Peculiarity of one dimension

3d: nearly free quasi-particles possible
(Fermi liquid)



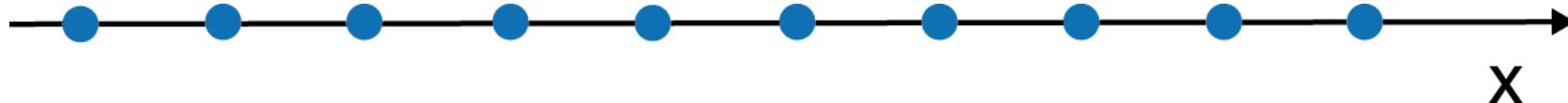
1d: individual electron cannot move without pushing all electrons



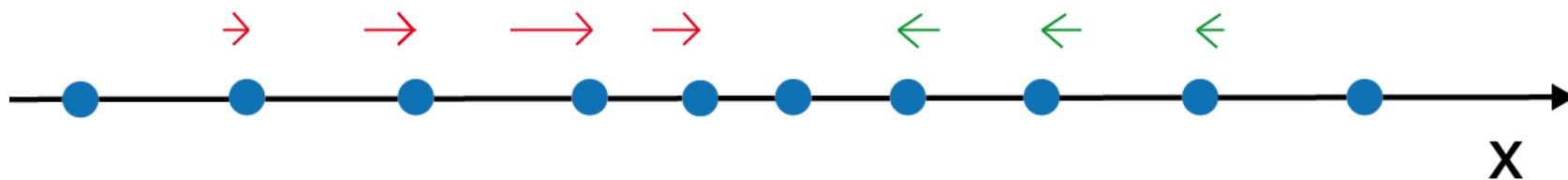
⇒ only collective excitations in 1d

Electrons in one dimension - Luttinger liquid (LL)

classical ground state: electron crystal (spinless electrons only in this talk)



excitations: displacement of electrons by $\phi(x)$ from equilibrium position

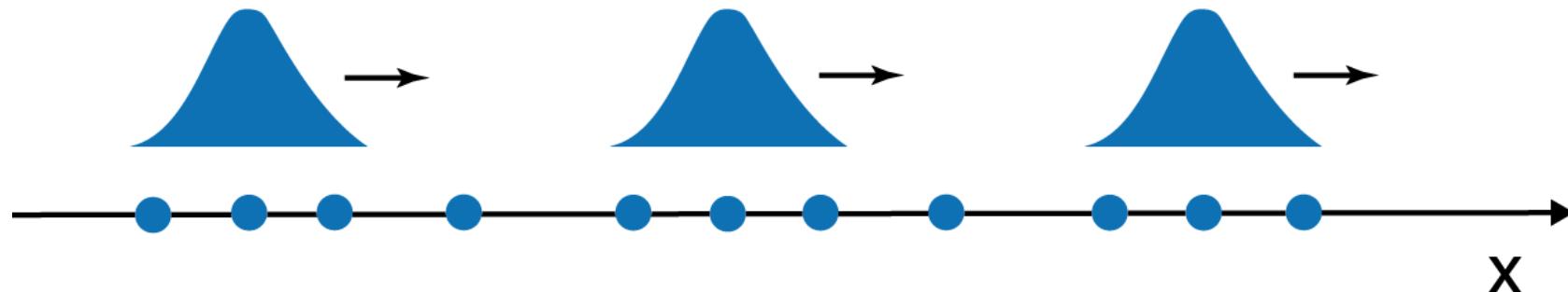


charge density

$$\rho(x, t) = \rho_0 - \frac{1}{2\pi} \partial_x \phi(x, t)$$

Luttinger liquid - phonon excitations

excitations: lattice vibrations (phonons) of electron crystal



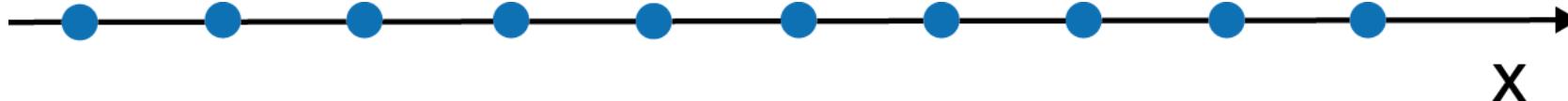
charge density $\rho(x,t)$ has left and right moving contributions

$$\rho = \rho_L + \rho_R \quad , \quad (\partial_x \pm \partial_t) \rho_{L/R} = 0 \quad , \quad H = \frac{u\pi}{K} \int dx (\rho_L^2 + \rho_R^2)$$

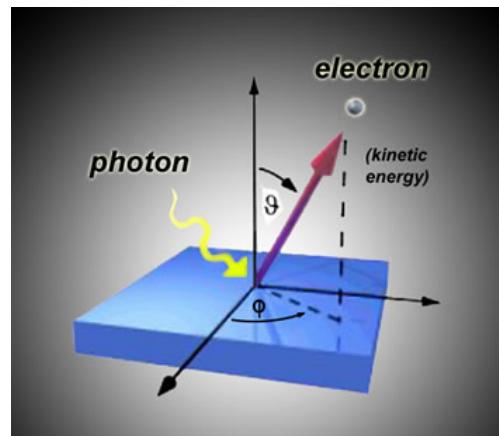
$$\phi = \phi_L + \phi_R \quad , \quad [\phi_R(x), \phi_R(x')] = -[\phi_L(x), \phi_L(x')] = i\pi K \text{sgn}(x - x')$$

K=1 noninteracting electrons, K < 1 for repulsive, K > 1 for attractive interactions

Electron density of states in a LL



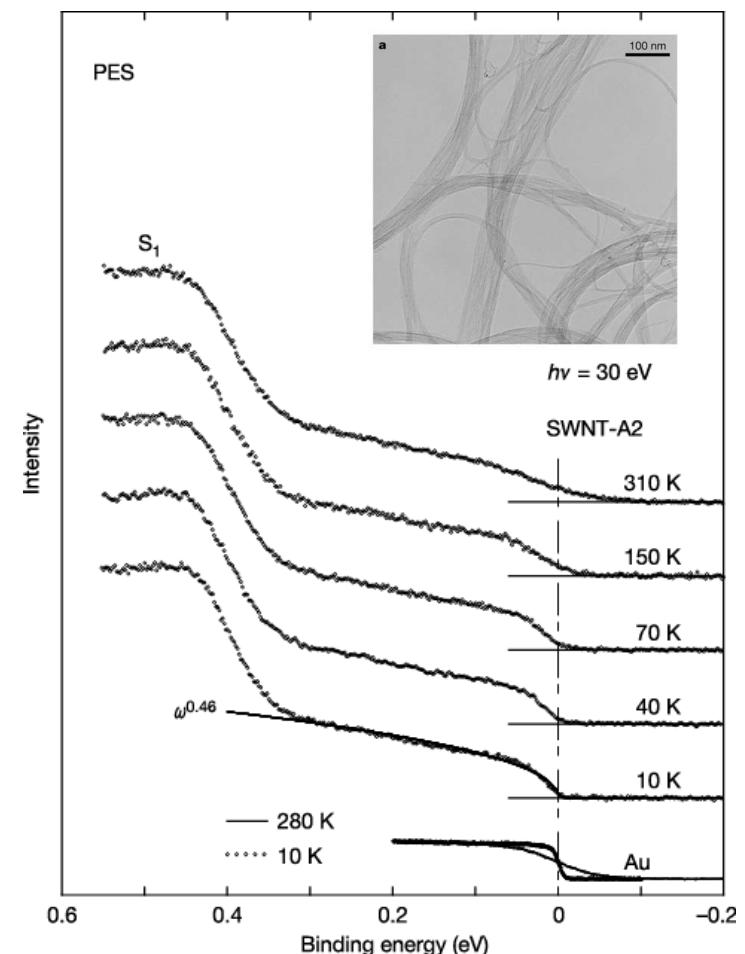
- strong correlations between electron positions
 - adding extra electrons difficult at low energy
- ⇒ reduced density of states $\rho(\epsilon)$ at low energy



$$\rho(E) \propto E^{(1/K+K-2)/4}$$

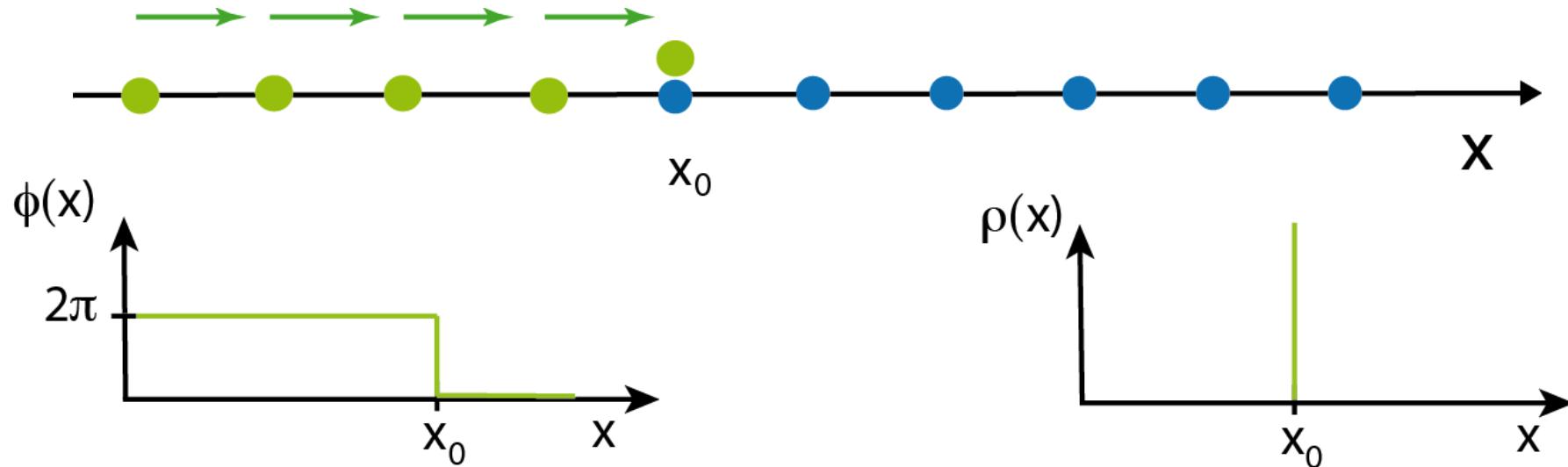
single wall carbon nanotubes
Hiroyoshi Ishii et al., Nature
426, 540 (2003)

<http://www.lbl.gov/Science-Articles/Archive/sabl/2006/Jul/04.html>



Luttinger liquid - bosonization

creation of electron at x_0 : move electrons at $x < x_0$ one position to the right



remember:

$$e^{i\hat{p}a/\hbar} |x\rangle = |x+a\rangle$$

$$\phi = \phi_L + \phi_R , \quad [\phi_R(x), \phi_R(x')] = -[\phi_L(x), \phi_L(x')] = i\pi K \text{sgn}(x - x')$$

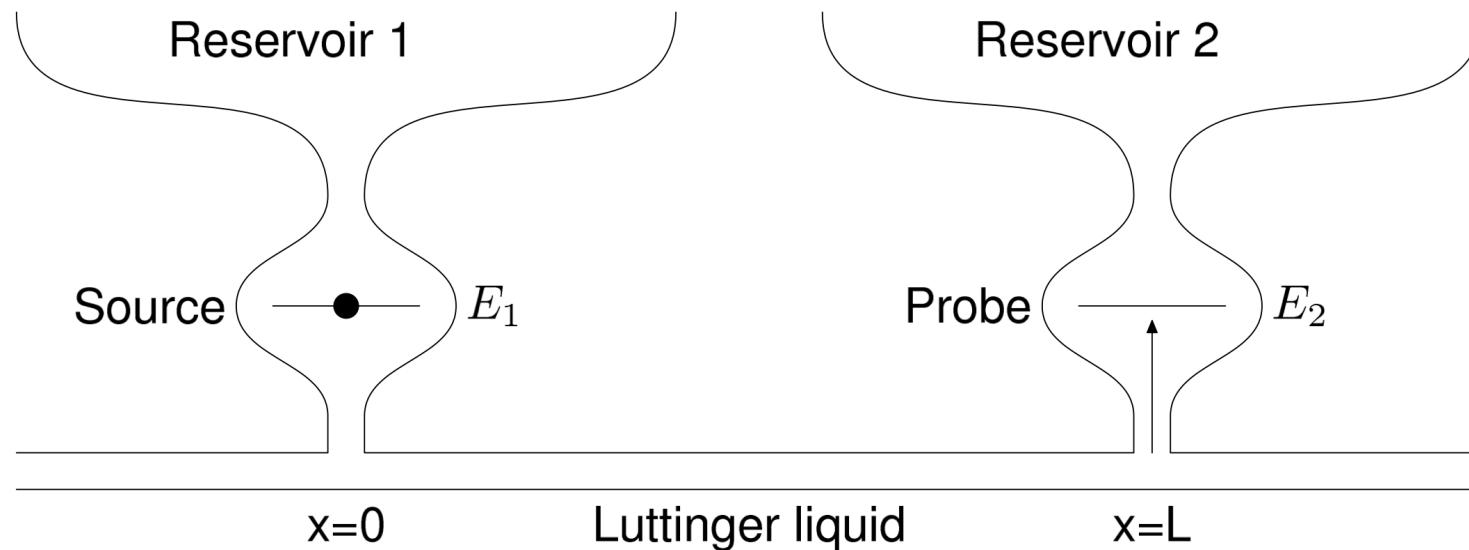
$$\Rightarrow e^{i2\pi \int_{-\infty}^x d\tilde{x} \Pi_{L/R}(\tilde{x})} = e^{\pm \frac{i}{K} \phi_{L/R}} \quad \text{is translation operator for } \phi_{L/R}$$

electron operator

$$\psi_{R,L}(x) \propto \exp[i(K_{\pm}\phi_R(x) + K_{\mp}\phi_L(x))]$$

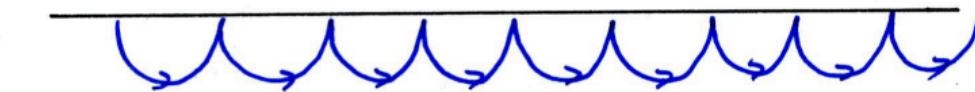
$$K_{\pm} = (K^{-1} \pm 1)/2$$

Injection of high-energy electrons - what to expect



- regular (chaotic) systems: equilibration towards a thermal state
 - quadratic Hamiltonian of an integrable system: nothing?
- however: initial state is not an eigenstate \Rightarrow expect some time evolution to a non-equilibrium (steady?) state

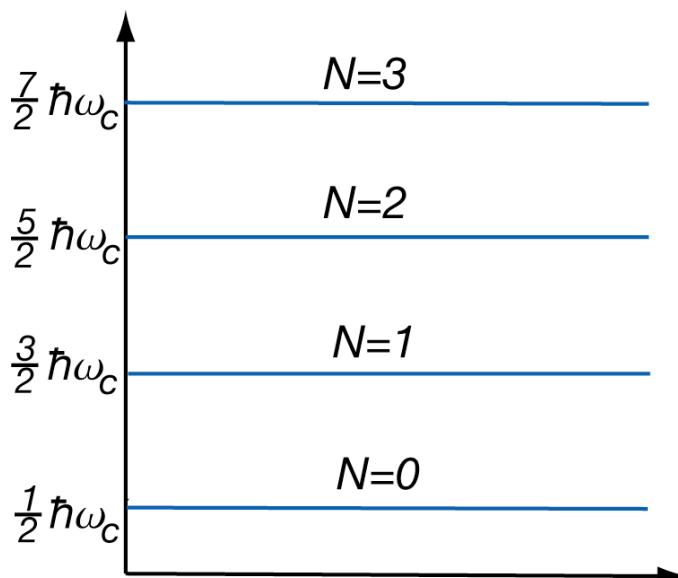
Edge states - chiral Luttinger liquid



cyclotron frequency $\omega_c = eB/m$
skipping orbit along edge



closed orbits with quantized energy



$$E_N = (N + 1/2) \hbar \omega_c \text{ in the bulk}$$

degeneracy: one electron per flux quantum

filling factor ν : number of electrons per flux quantum

$$H = \frac{u\pi}{\nu} \int dx \rho_R^2$$

$$[\phi_R(x), \phi_R(x')] = i\pi\nu \text{sgn}(x - x')$$

integer $\nu \Rightarrow$ integer quantum Hall effect K. Von Klitzing, G.Dorda, and M.Pepper, PRL 1980

with integer edge states

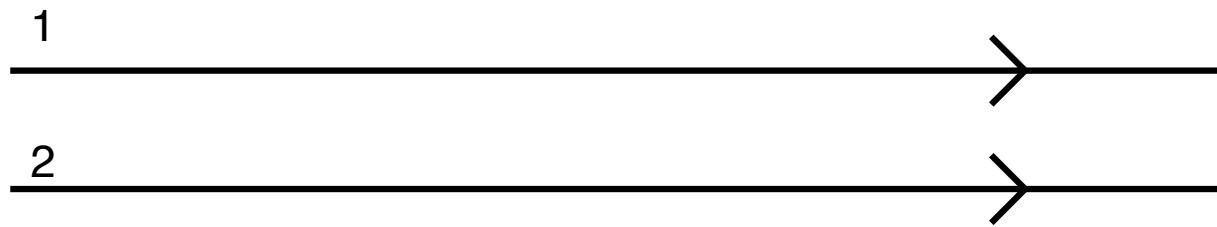
B.I. Halperin, PRB 1982

Outline

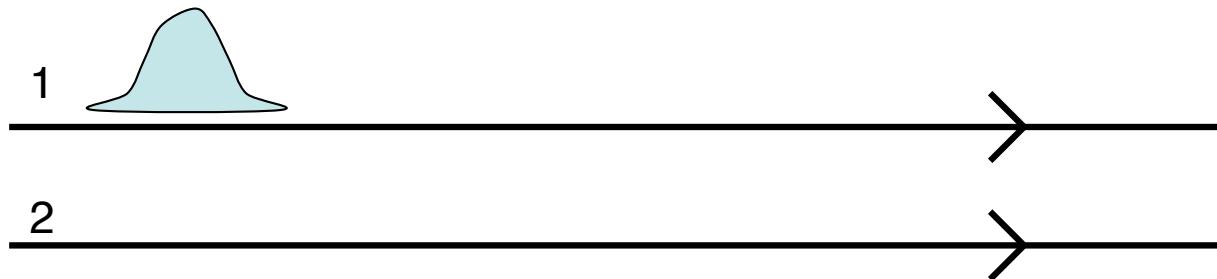
- Charge fractionalization
- Non-equilibrium bosonization, equilibration, and shot noise
- Anyonic exclusion

The $\nu = 2$ quantum Hall edge

At bulk filling $\nu = 2$, there are two co-propagating edge states



$$H = \pi \int dx (v_1 \rho_1^2 + v_2 \rho_2^2 + v_{12} \rho_1 \rho_2) \quad [\phi_{1/2}(x), \phi_{1/2}(x')] = i\pi \text{sgn}(x - x')$$



Due to coupling v_{12} , electrons injected into edge 1 are not eigenstates

Charge fractionalization

E. Berg, Y. Oreg, E.-A. Kim, and F. von Oppen, Phys. Rev. Lett. 102, 236402 (2009).

I. Neder,¹ Phys. Rev. Lett. 108, 186404 (2012).

I.P. Levkivskyi and E.V. Sukhorukov, Phys. Rev. B 85, 075309 (2012).

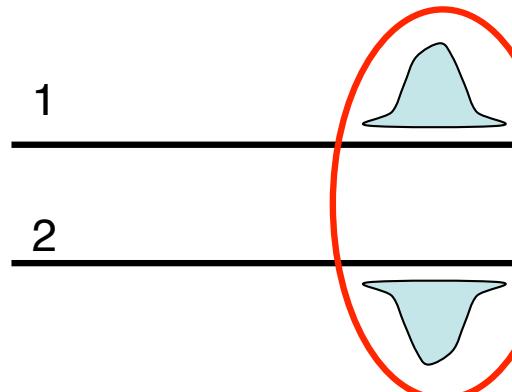
I.P.² Levkivskyi and E.V. Sukhorukov, Phys. Rev. Lett. 109, 246806 (2012). inject electron into edge 1

For charge fractionalization in momentum resolved tunneling, see
Diagonalize H by Bogoliubov transformation

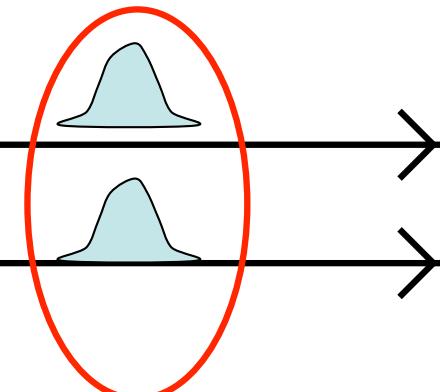
H. Steinberg, G. Barak, A. Yacoby, L.N. Pfeiffer, K.W. West, B.I. Halperin, K. Le Hur, Nature Physics 4, 116–119 (2008). $\tan(2\theta) = v_{12}/(v_1 - v_2)$

$$\begin{pmatrix} \tilde{\rho}_1 \\ \tilde{\rho}_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}$$

neutral mode $\tilde{\rho}_2$



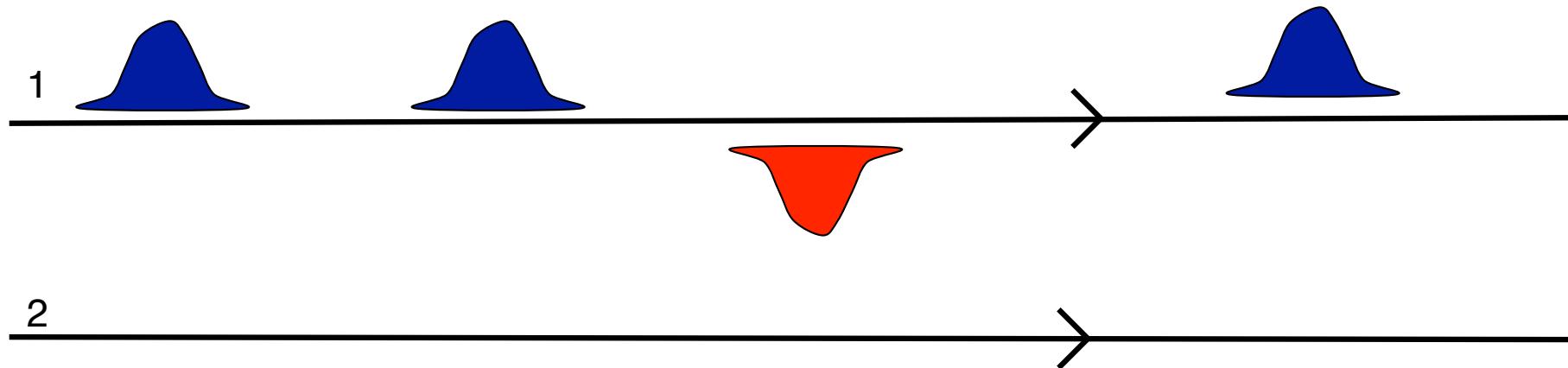
charge mode $\tilde{\rho}_1$



fractional charges $\pm e^* = \pm (e/2) \sin(2\theta)$

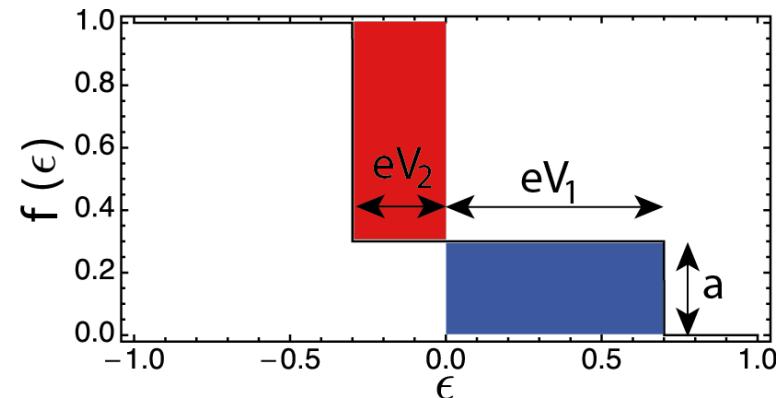
physics behind non-equilibrium
steady state?

Quantum Quench Protocol



prepare edge 1 with a spatially random sequence of charge pulses, described by a double step distribution function

for times $0 < t < t_Q$, switch on interaction between edge 1 and 2



$$H_{\text{int}} = \theta(t)[1 - \theta(t_Q)]2\pi \int dx v_{12}\rho_1\rho_2$$

Outline

- Charge fractionalization
- Non-equilibrium bosonization, equilibration, and shot noise
- Anyonic exclusion

Model

$$H = 2\pi \int dx (v_1 \rho_1^2 + v_2 \rho_2^2 + v_{12} \rho_1 \rho_2) \quad \rho_i = \frac{1}{2\pi} \partial_x \phi_i$$

$$\tan(2\theta) = v_{12}/(v_1 - v_2)$$

$$[\phi_m(x), \phi_n(x')] = i\pi \delta_{mn} \text{sgn}(x - x')$$

operator relation $\psi_j = \frac{1}{\sqrt{2\pi a}} e^{i\phi_j}$ valid in non-equilibrium

but relation $\langle e^{i\varphi_1} e^{-i\varphi_2} \rangle = e^{[\varphi_1, \varphi_2]} e^{-\frac{1}{2} \langle (\varphi_1 - \varphi_2)^2 \rangle}$ breaks down

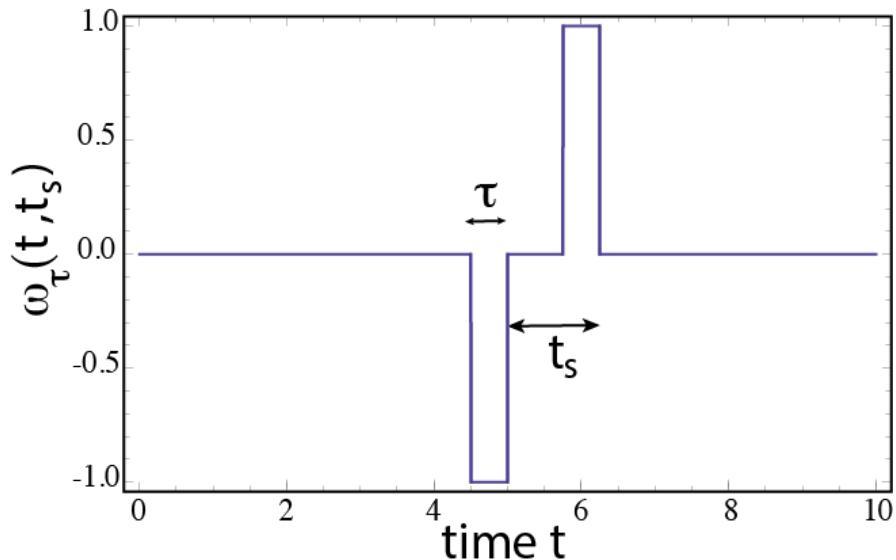
Non-equilibrium bosonization

interaction effect on fermions described by scattering phase

$$\delta_\tau = 2\pi \frac{e^*}{e} \omega_\tau(t, t_s)$$

$$t_s = \frac{v_{12}}{v_2 \sin 2\theta} t_Q$$

window function $\omega_\tau(t, t_s)$ describes propagation of $\pm e^*$ pulses on edge 2



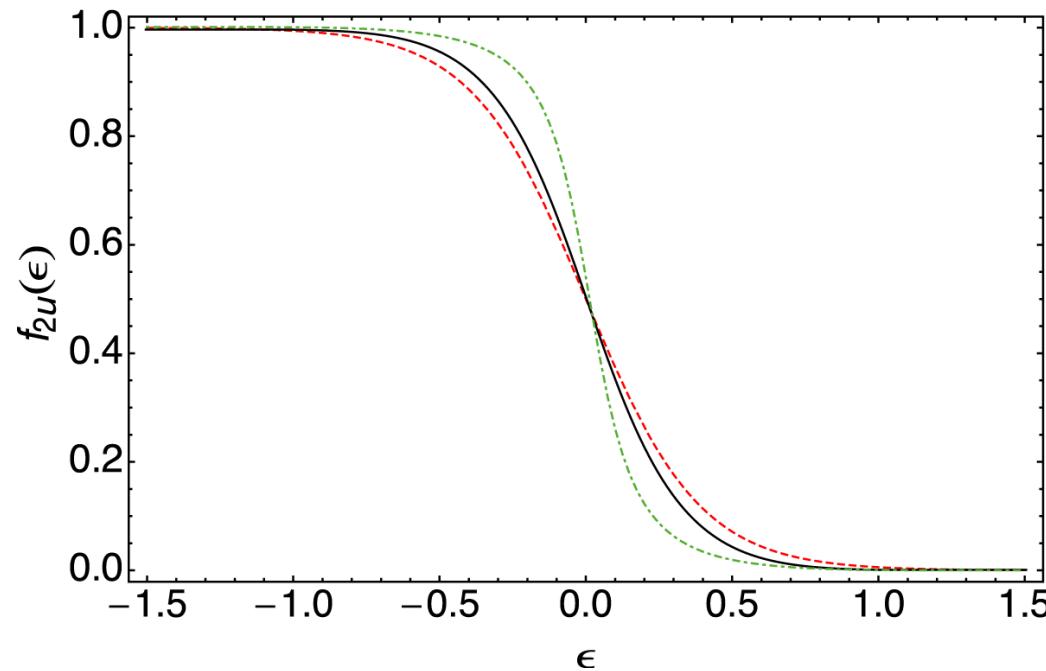
Green function $G_2^<(\tau) = G_0^<(\tau) \bar{\Delta}_\tau(\delta)$

determinant of single fermion operator

$$\bar{\Delta}_\tau(\delta) = \frac{\det [1 + (e^{-i\delta_\tau} - 1)f(\epsilon)]}{\det [1 + (e^{-i\delta_\tau} - 1)\theta(-\epsilon)]}$$

Gutman, Gefen & Mirlin, PRB (2010); Levkivskyi & Sukhorukov, PRL (2009); Neder & Ginossar, PRL (2008), PRB (2912); Klich, in *Quantum Noise in Mesoscopic Systems*, ed. Nazarov (2003).

steady state distribution function



steady state distribution of mode 2 for $a=1/2$ and $\theta=0.47$

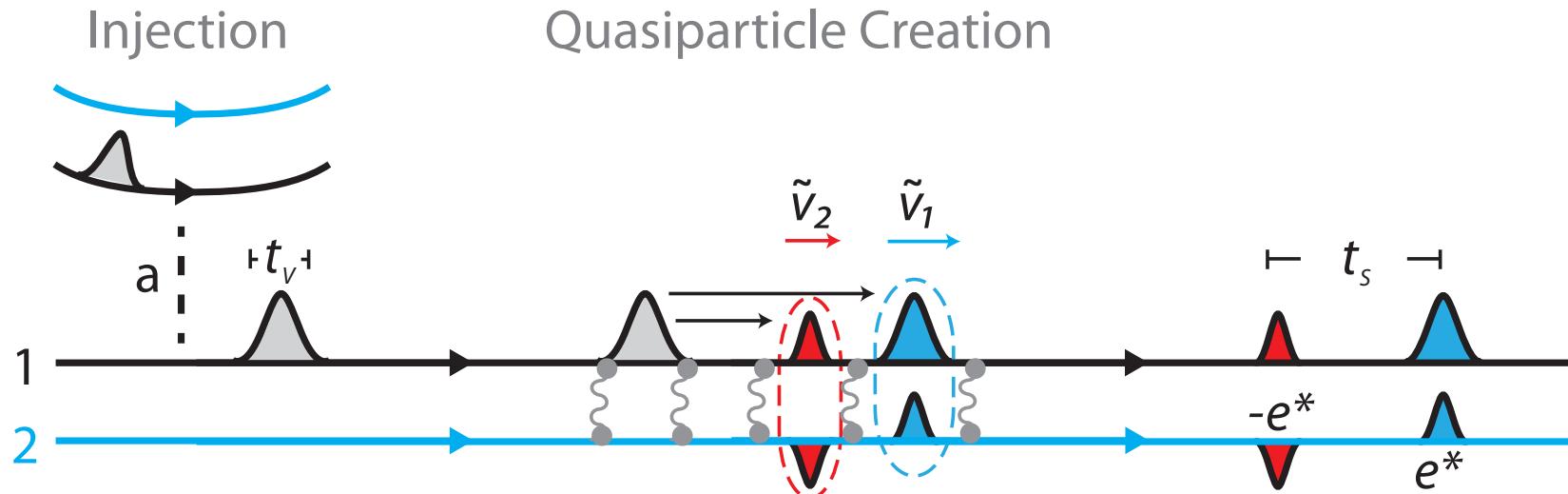
black line exact result

green line Gaussian approximation

red line fully equilibrated distribution at $T^* = eV\sqrt{(3/2)a(1-a)}/\pi$

Milletari & Rosenow, PRL 111, 136807 (2013)

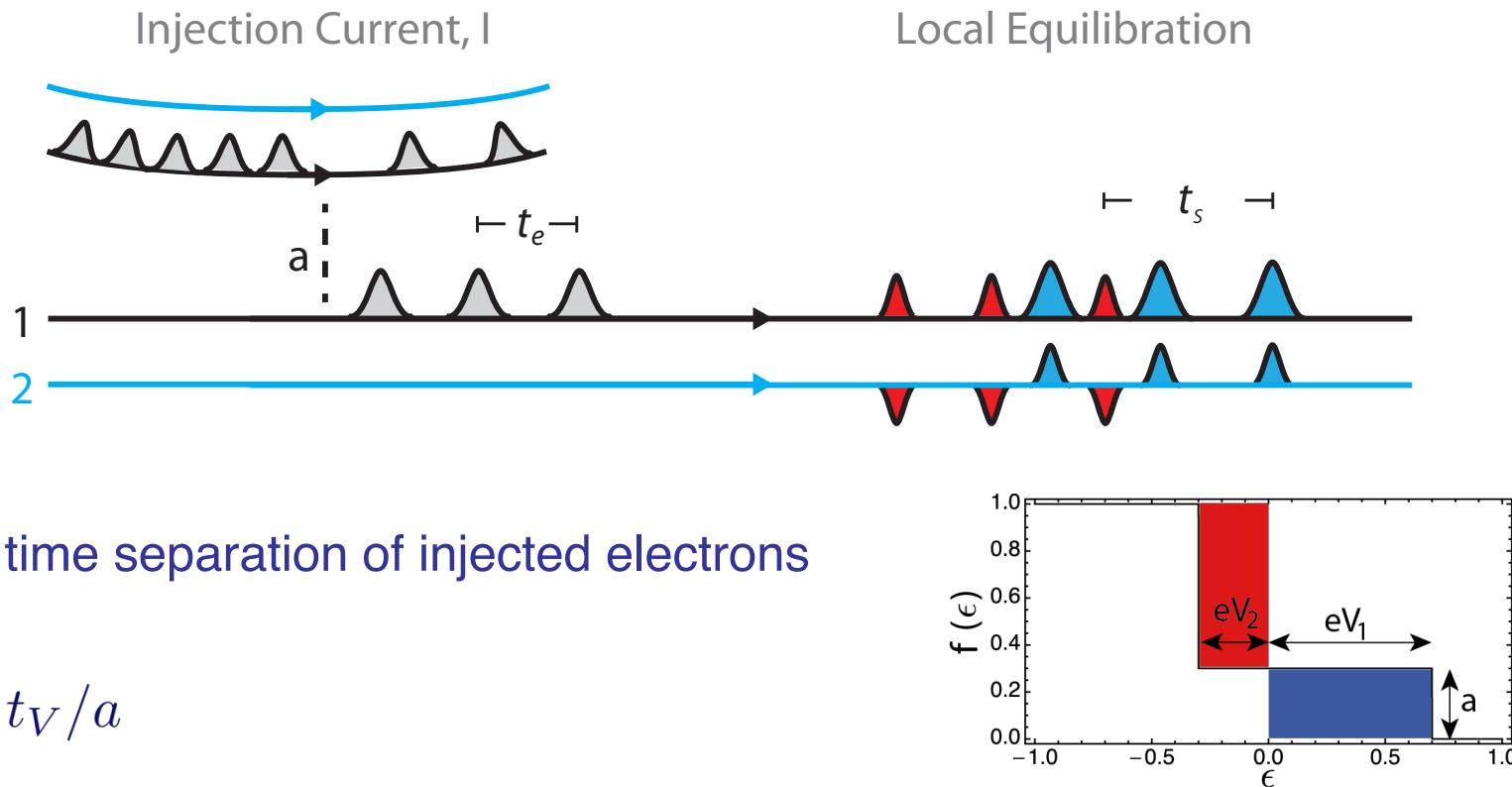
Quasi-particle creation – regime A



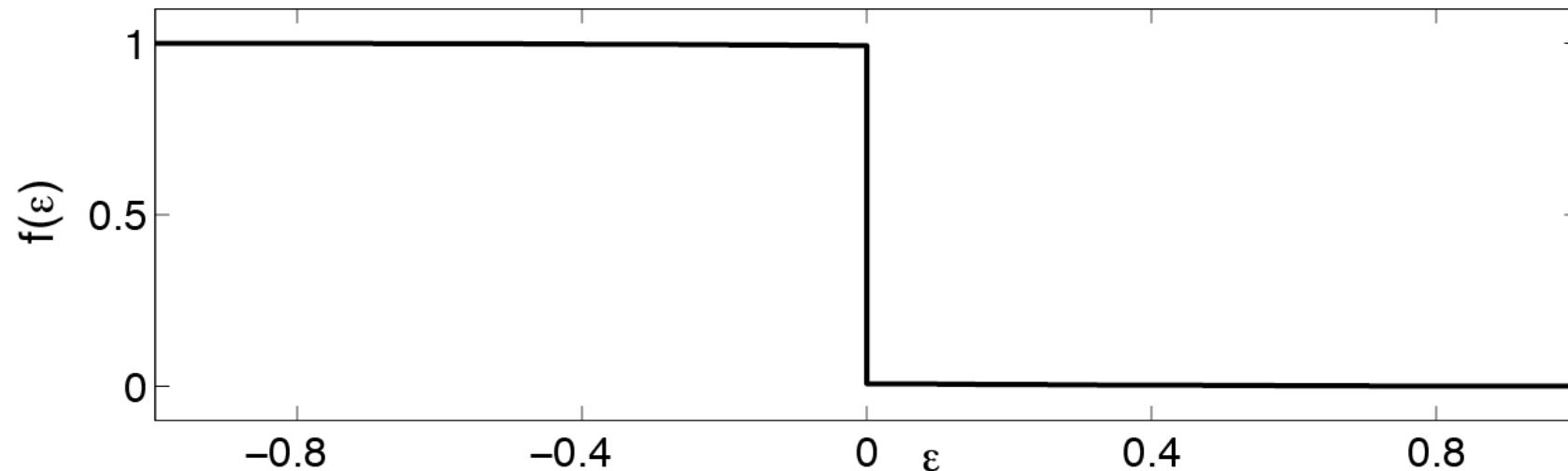
$$t_V = \frac{\hbar}{eV}$$

$$t_s = \frac{v_{12}}{v_2 \sin 2\theta} t_Q$$

Local Equilibration – regime B



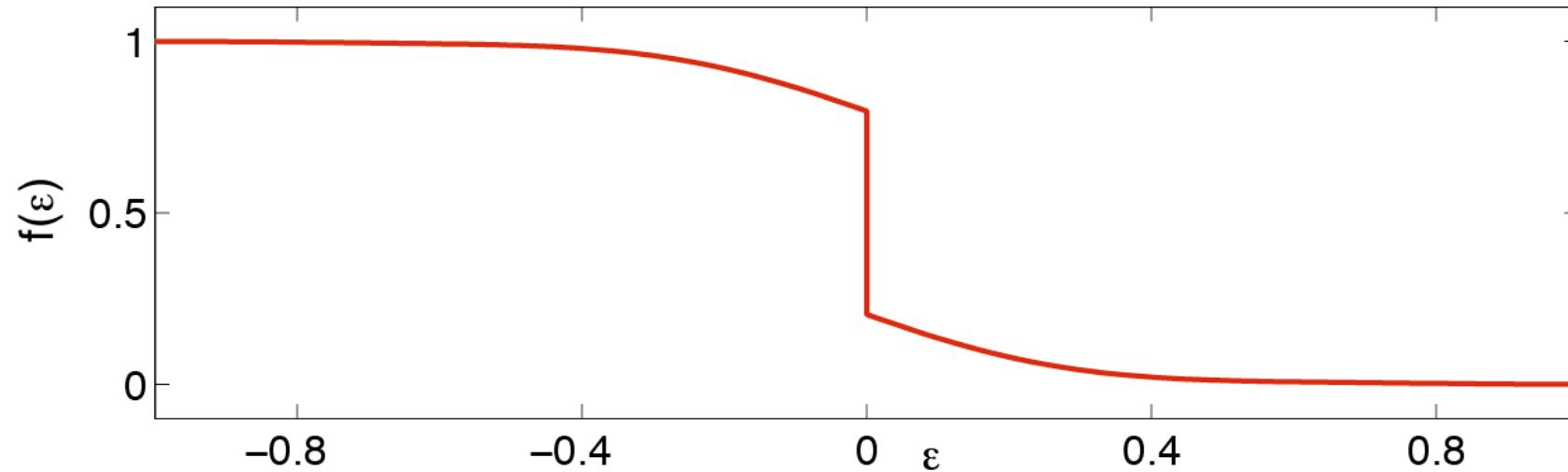
Regime A: $t_s \leq t_V$



$$f_{2u}(\epsilon) = \bar{\Delta}_{t_s}^{\text{sep}}(\delta) \theta(-\epsilon) + \frac{1 - \bar{\Delta}_{t_s}^{\text{sep}}(\delta)}{2} \begin{cases} 2 - (\epsilon/eV + 1)^2 & -eV < \epsilon < 0 \\ (\epsilon/eV - 1)^2 & 0 \leq \epsilon < eV \\ 2 \theta(-\epsilon) & \text{else} \end{cases}$$

Schneider, Milletari & Rosenow, SciPost Phys. 2, 007 (2017)

Regime B: $t_v \leq t_s \leq t_e$



$$\begin{aligned}f_2(\epsilon) = & e^{-\frac{t_s}{t_\phi}} \left(\theta(-\epsilon) + \frac{1}{\pi} \text{Si}(t_s \epsilon) - \frac{1}{2} \right) \\& + \frac{1}{2} - \frac{1}{\pi} \text{Im} [\text{Ci}(t_s \epsilon + it_s/t_\phi)] - \frac{1}{\pi} \text{Re} [\text{Si}(t_s \epsilon + it_s/t_\phi)] \\& + \frac{1}{2} - \frac{1}{\pi} \tan^{-1}(t_\phi \epsilon)\end{aligned}$$

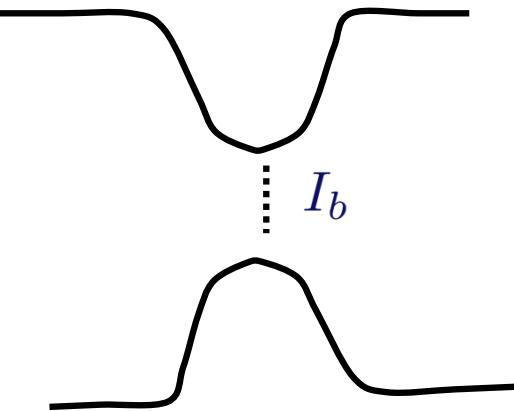
Schneider, Milletari & Rosenow, SciPost Phys. 2, 007 (2017)

Shot noise and Fano factor

weak scattering is Poisson process

with $\langle (\delta N_{\Delta t})^2 \rangle = \langle N_{\Delta t} \rangle$

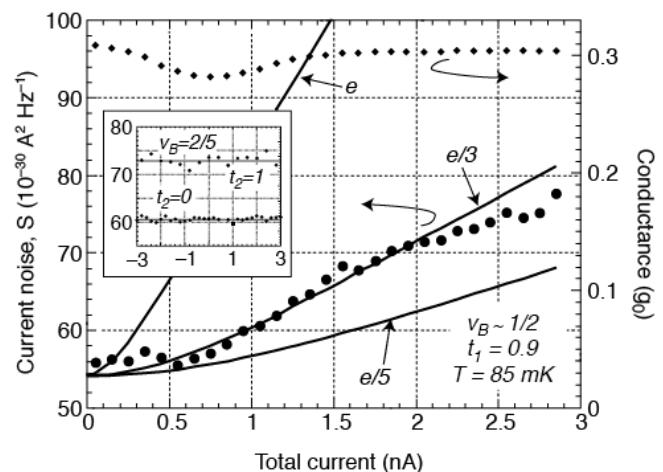
$$S \equiv \langle \delta I^2 \rangle \Delta t = q^2 \left\langle \left(\frac{\delta N_{\Delta t}}{\Delta t} \right)^2 \right\rangle \Delta t = q I_b$$



Fano factor

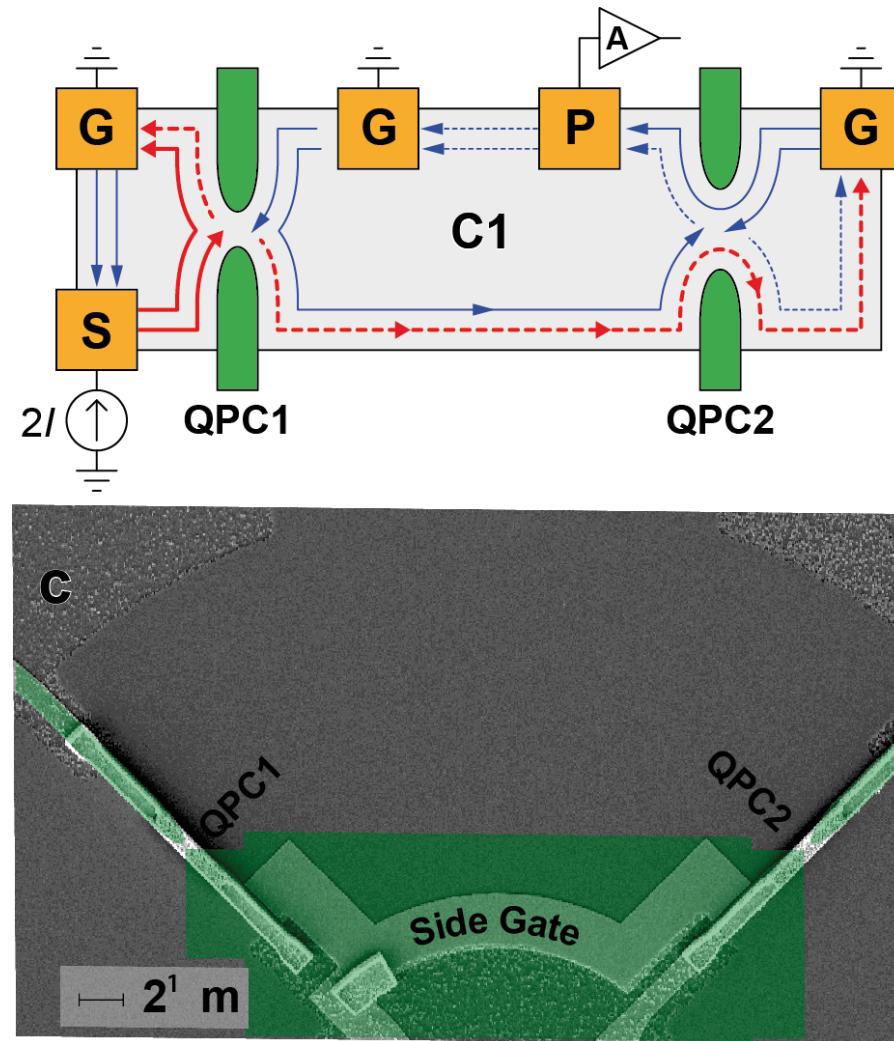
$$F \equiv \frac{S}{e I_b} = \frac{q}{e}$$

measures quasi-particle charge



M. Reznikov et al., Nature 399, 238 (1999)
 $e/3$ Laughlin charges

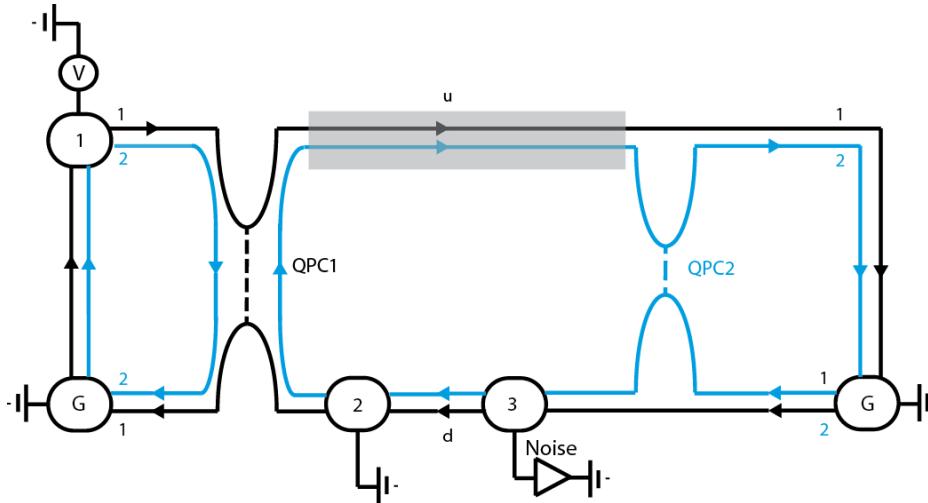
Experiment: Neutral Edge Modes in Integer Quantum Hall regime



H. Inoue, A. Grivnin, N. Ofek, I. Neder, M. Heiblum, V. Umansky, and D. Mahalu, PRL 112, 166801 (2014)

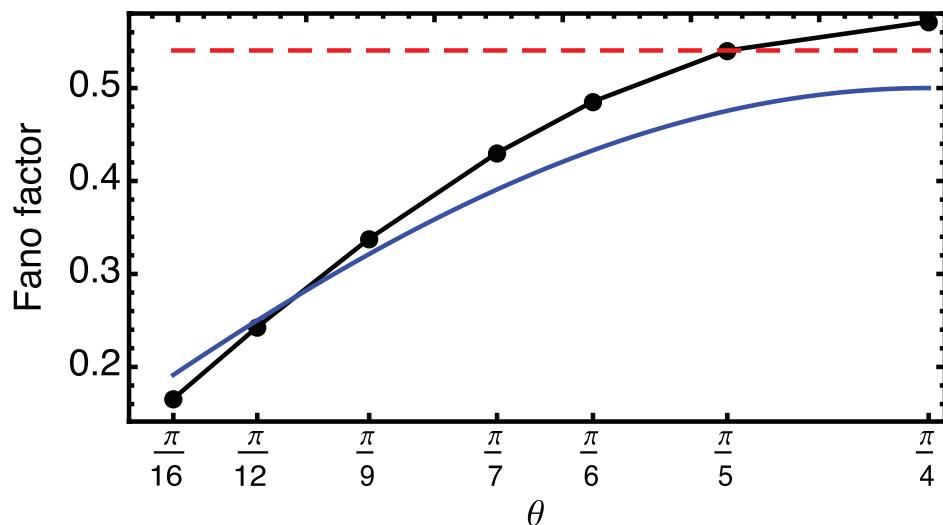
see also: H. Kamata, N. Kumada, M. Hashisaka, K. Muraki, and T. Fujisawa, Nat. Nanotechnol. 9, 177 (2014)

Theoretical results: shot noise



$$\mathcal{H}_{\text{QPC}2} = t_2 \psi_{2u}^\dagger(x) \psi_{2d}(x) + h.c.$$

$$t_s = x_0(\tilde{v}_2^{-1} - \tilde{v}_1^{-1})$$

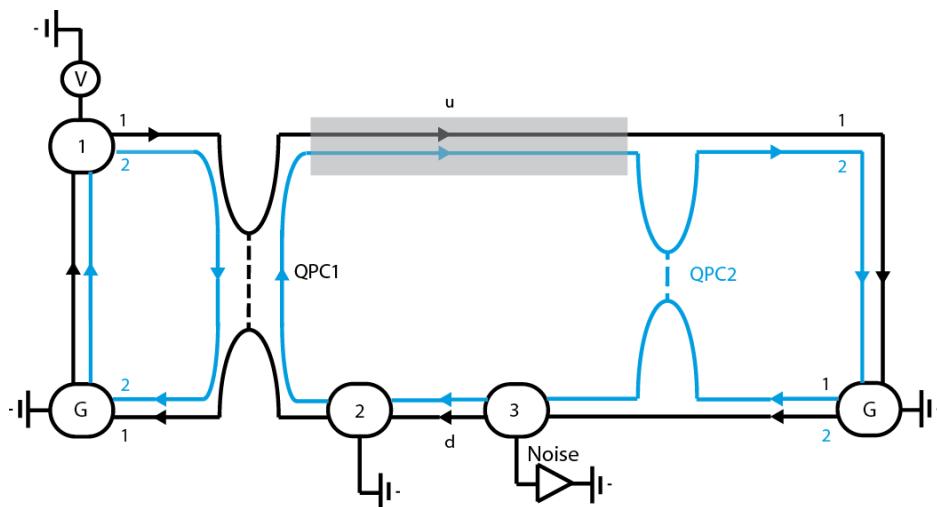


blue line: fractionalization model

$$F = \frac{e^*}{e} \equiv \frac{1}{2} \sin 2\theta$$

black line: non-equilibrium bosonization
red line: thermal state

Theoretical results: shot noise



$$\mathcal{H}_{\text{QPC}2} = t_2 \psi_{2u}^\dagger(x) \psi_{2d}(x) + h.c.$$

$$t_s = x_0(\tilde{v}_2^{-1} - \tilde{v}_1^{-1})$$

$$S_{\omega \rightarrow 0} = \frac{2e^2}{h} \frac{|t_2|^2}{2\pi} \int_{\epsilon} G_{2u}^<(\epsilon) G_{2d}^>(\epsilon) + G_{2d}^<(\epsilon) G_{2u}^>(\epsilon)$$

regime A

$$S_{\omega \rightarrow 0} \propto a \sin(2\theta)^2 t_s^2 (eV)^3$$

regime B

$$S_{\omega \rightarrow 0} \propto a eV \log(t_s eV) \quad \text{limit of small } a$$

regime C

$$S_{\omega \rightarrow 0} \propto a eV \log(1/a)$$

Schneider, Milletari & Rosenow, SciPost Phys. 2, 007 (2017)

See also I. P. Levkivskyi, E. V. Sukhorukov, Phys. Rev. Lett. 109, 246806 (2012)

Comparison to experiment

- Parameters

$$x_0 = 8 \mu\text{m}$$

$$v_{12} = 4.6 \times 10^4 \frac{\text{m}}{\text{s}}$$

- Maximum likelihood fit

$$\theta = 0.37 \pm 0.03$$

$$\tan(2\theta) = \frac{v_{12}}{v_1 - v_2}$$

$$g = 0.101 \pm 0.012$$

$$g = \frac{v_{12}^2}{4 v_1 v_2}$$

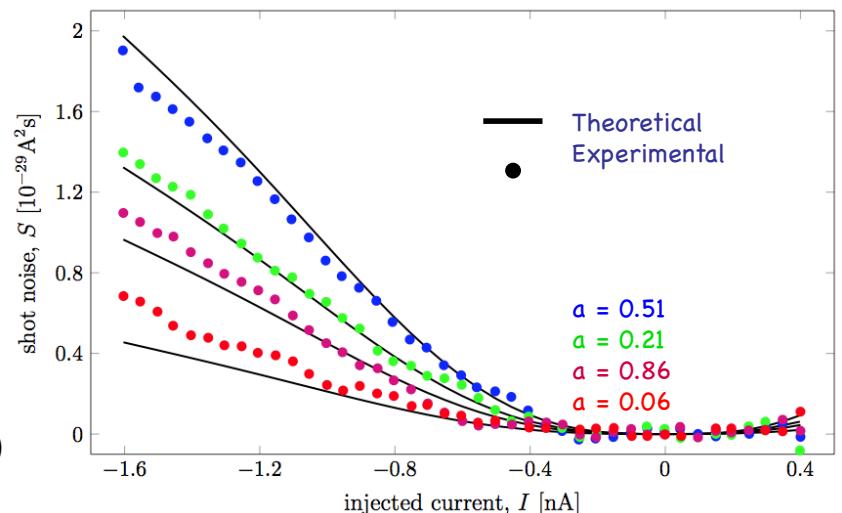
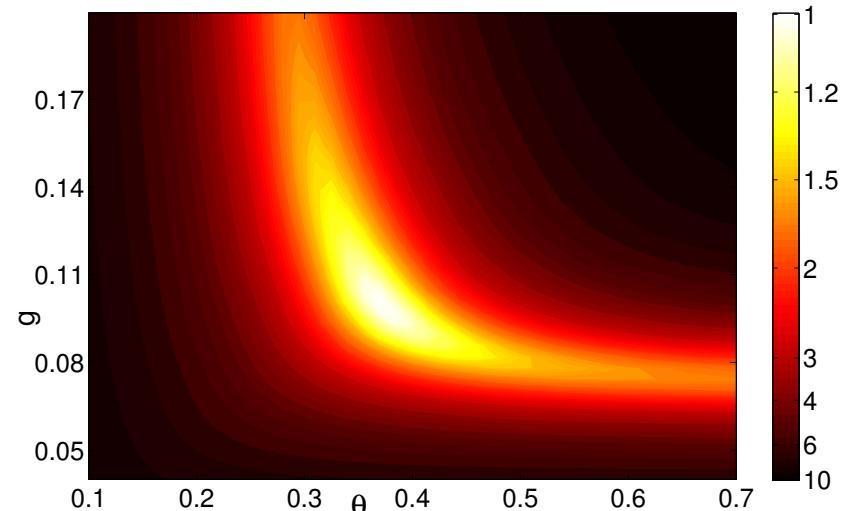
- Fermi Velocities

$$v_1 = (3.16 \pm 0.22) \times 10^5 \text{ m/s}$$

$$v_2 = (2.65 \pm 0.16) \times 10^5 \text{ m/s}$$

Schneider, Milletari & Rosenow, SciPost Phys. 2, 007 (2017)

H. Inoue, A. Grivnin, I. Neder, M. Heiblum, V. Umansky, D. Mahalu, PRL 112, 166801 (2014)



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Extending the notion of quantum statistics - anyons

Example: quantum Hall effect

Leinaas & Myrheim
Halperin
Arovas, Schrieffer & Wilczek

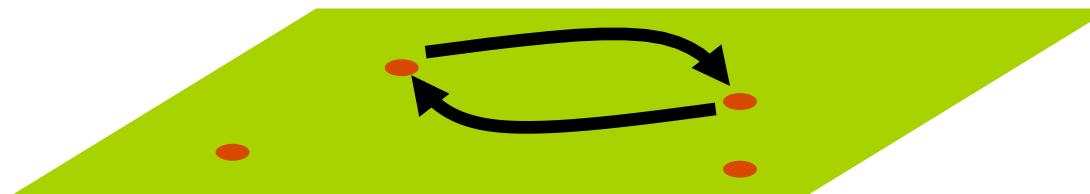
A ground state:

$$\Psi(r_1, \dots, r_{N_e}; R_1, \dots, R_4)$$

Electrons

Laughlin quasi-particles

Energy gap



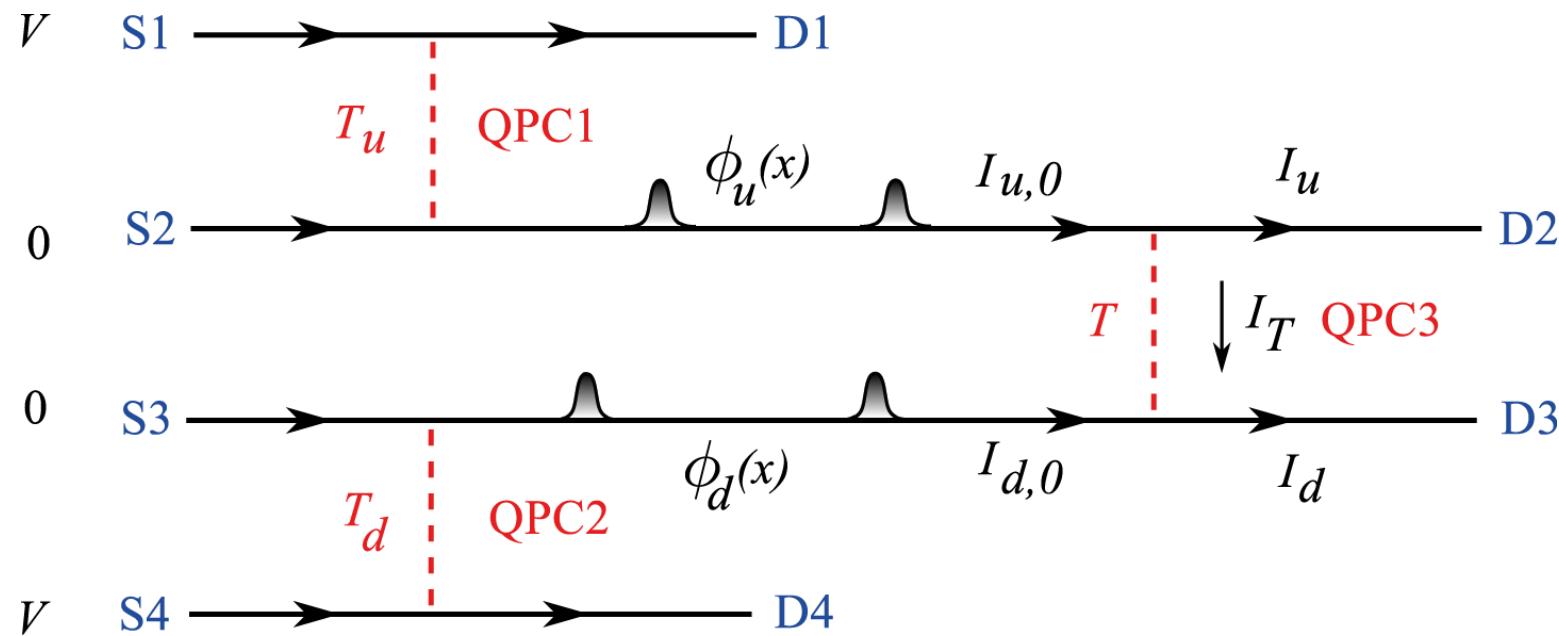
For Abelian states:

Adiabatically interchange the position of two excitations

$$\Psi \rightarrow e^{i\theta} \Psi$$

$$\theta = \pi/3 \text{ for } \nu = 1/3 \text{ Laughlin state}$$

Anyon Collider



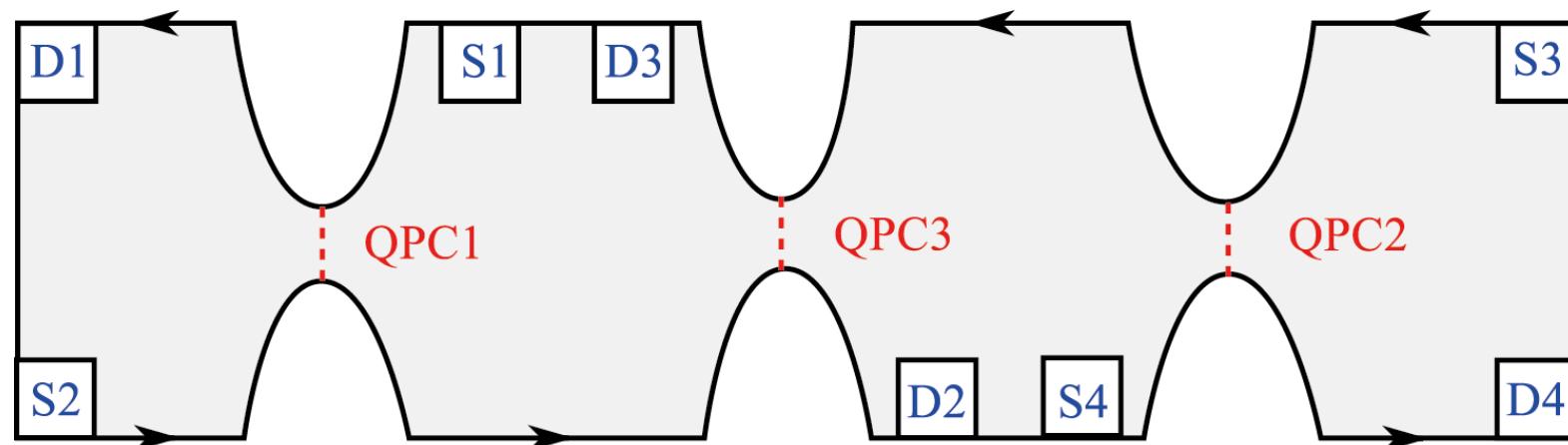
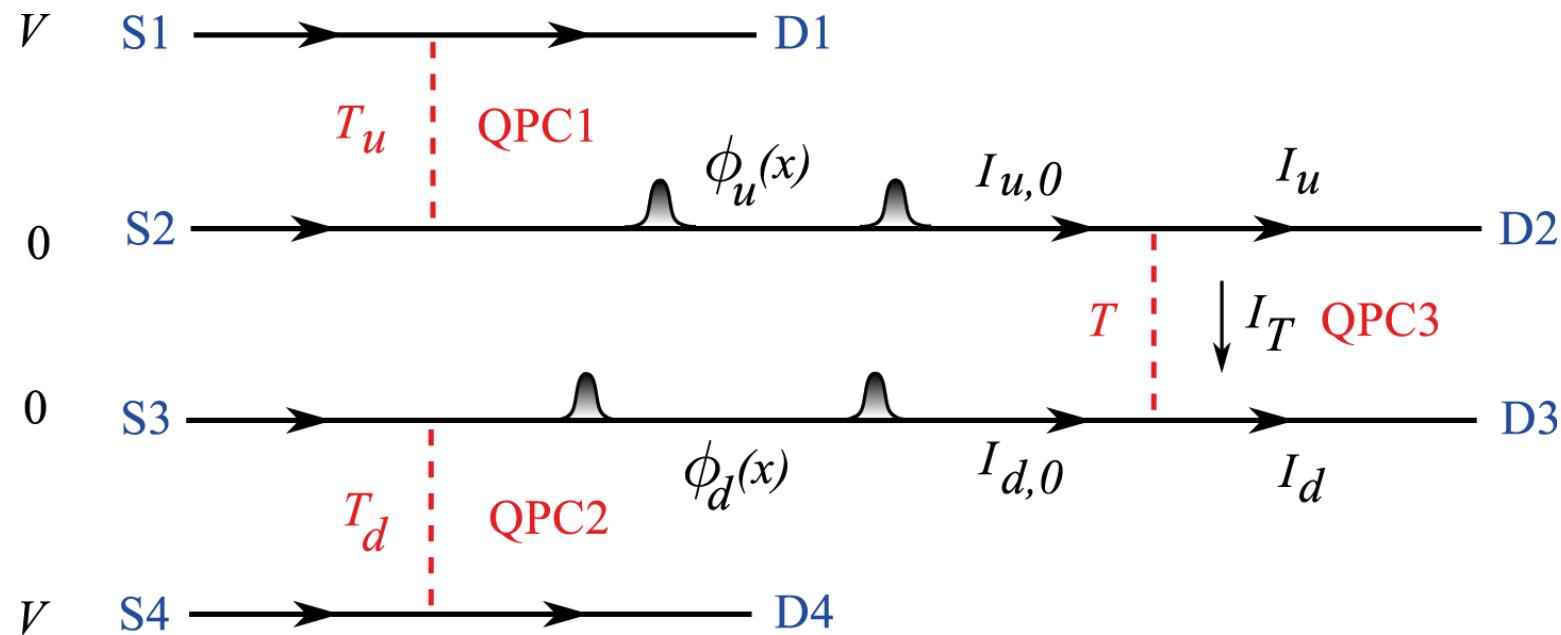
QPC1, QPC2: creation of diluted beams of anyons for $T_u, T_d \ll 1$

QPC3: collision of diluted anyon beams

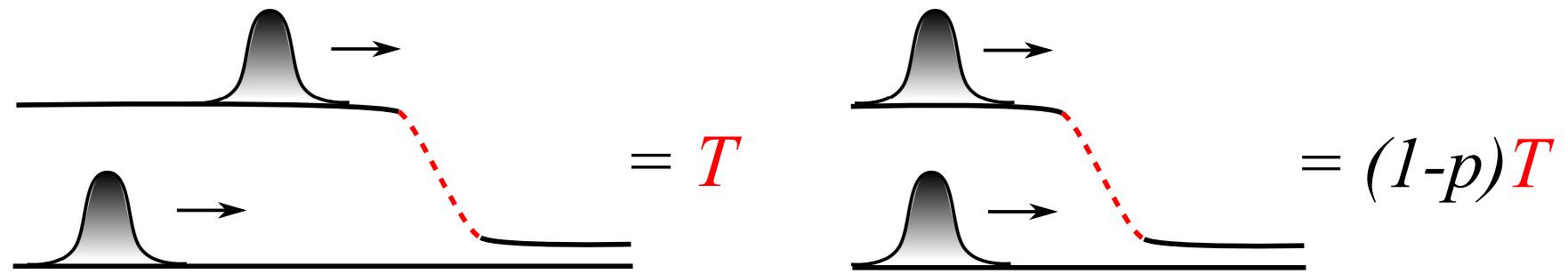
current fluctuations in D2, D3 depend on spatial exclusion of anyons

Rosenow, Levkivskyi & Halperin, PRL 116, 156802 (2016)

Anyon collider



Spatial exclusion



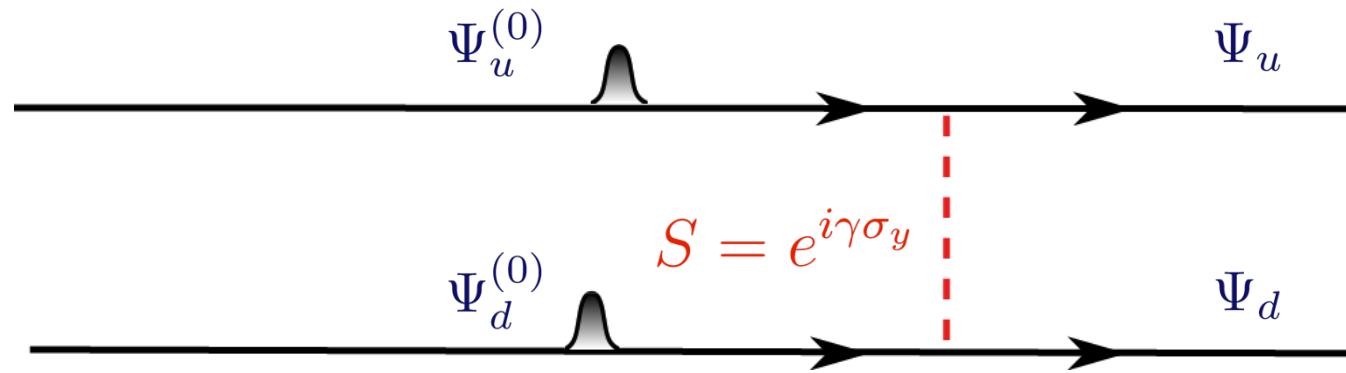
single particle tunneling probability T

reduced tunneling probability $(1-p)T$ when two particles continue together

How is anyonic exchange statistics related to spatial exclusion?

For two-anyon interference and exchange statistics, see e.g. Campagnano, Zilberberg, Gornyi, Feldman, Potter & Gefen, PRL(2012), PRB (2013).

Free fermions



$$T = \sin^2 \gamma$$

probability for tunneling across QPC

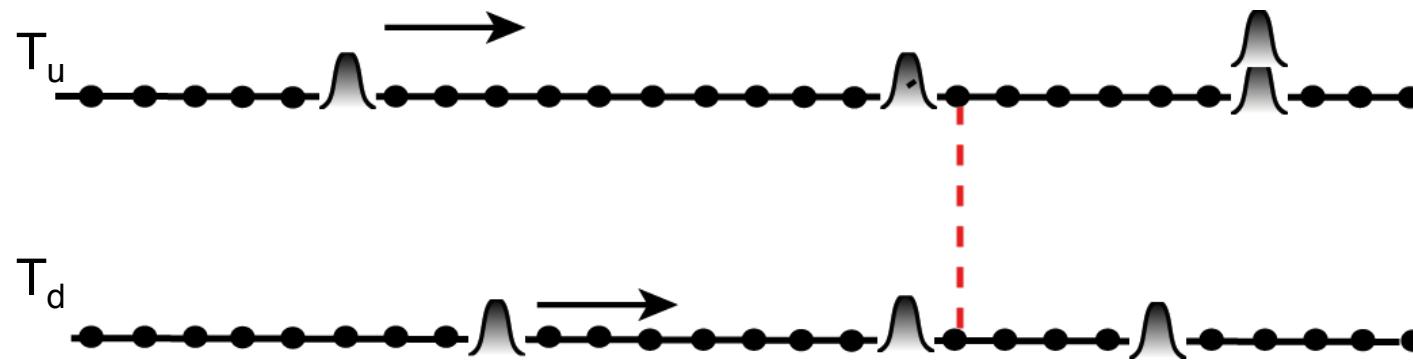
$$\langle \delta I_u \delta I_d \rangle_{\omega=0} = T(1-T) \frac{e^2}{h} \int d\epsilon \{ f_u(\epsilon)[1-f_u(\epsilon)] + f_d(\epsilon)[1-f_d(\epsilon)] \\ - f_u(\epsilon)[1-f_d(\epsilon)] - f_d(\epsilon)[1-f_u(\epsilon)] \}$$

incoming fermions with double step distributions $f_\alpha(\epsilon) = \theta(-\epsilon) + T_\alpha \theta(\epsilon) \theta(V - \epsilon)$

$$\langle \delta I_u \delta I_d \rangle_{\omega=0} = -T(1-T)V(e^2/h)(T_u - T_d)^2$$

vanishes for equal
bias currents $T_u = T_d$

Classical lattice model



Occupation probability T_u, T_d for incoming particles

single particle tunneling probability T

reduced tunneling probability $(1-p)T$ when two particles continue together

$$\langle \delta I_u \delta I_d \rangle_{\omega=0} = -T(1-p)TW \frac{e^3}{h^3} [(T_u T_d - T_d)^2 + T_u^2 (T_d - T_p) T_u T_d]$$

Current cross-correlations contain information about exclusion probability

Rosenow, Levkivskyi & Halperin, PRL 116, 156802 (2016)

Quantum mechanical anyons

$$A(t) = \zeta e^{i\phi_u(0,t)-i\phi_d(0,t)} , \quad I_T = ie^* (A^\dagger - A)$$

ζ tunneling amplitude, I_T tunneling current

equilibrium correlation function

$$\langle A(t)A^\dagger(0) \rangle_{\text{eq}} = |\zeta|^2 e^{i\pi\delta \text{sign}(t)} \frac{\tau_c^{2\delta}}{|t|^{2\delta}}$$

non-equilibrium contribution is generating function of a Poisson process

$$\langle A(t)A^\dagger(0) \rangle_0 = \langle A(t)A^\dagger(0) \rangle_{\text{eq}} \times \exp\left[-\frac{\langle I_{u,0} \rangle}{e^*} (1 - e^{-2\pi i \lambda}) t\right] \exp\left[-\frac{\langle I_{d,0} \rangle}{e^*} (1 - e^{2\pi i \lambda}) t\right]$$

I.P. Levkivskyi, Phys. Rev. B 93, 165427 (2016)

$\lambda=1/m$ for a Laughlin state, and $\lambda \neq 1/m$ in the presence of edge reconstruction

Quantum mechanical anyons

$$\langle I_T \rangle = e^* \int_{-\infty}^{\infty} dt \langle [A^\dagger(0), A(t)] \rangle_0$$

$$\langle (\delta I_T)^2 \rangle_{\omega=0} = (e^*)^2 \int_{-\infty}^{\infty} dt \langle \{A^\dagger(0), A(t)\} \rangle_0$$

$$\langle A(t)A^\dagger(0) \rangle_0 = \langle A(t)A^\dagger(0) \rangle_{\text{eq}} \times \exp \left[-\frac{\langle I_{u,0} \rangle}{e^*} (1 - e^{-2\pi i \lambda}) t \right] \exp \left[-\frac{\langle I_{d,0} \rangle}{e^*} (1 - e^{2\pi i \lambda}) t \right]$$

$\lambda = 1/m$ for a Laughlin state, and $\lambda \neq 1/m$ in the presence of edge reconstruction

Quantum mechanical anyons

$$\langle I_T \rangle = C \sin(\pi\delta) \text{Im} \left(I_+ + \frac{iI_-}{\tan \pi\lambda} \right)^{2\delta-1} [1 + O(\tau_c)]$$

$$\frac{\langle \delta I_T^2 \rangle_{\omega=0}}{(e^\star)^2} = \frac{C}{e^\star} \cos(\pi\delta) \text{Re} \left(I_+ + \frac{iI_-}{\tan \pi\lambda} \right)^{2\delta-1} [1 + O(\tau_c)]$$

with $C = e^\star 4|\zeta|^2 \tau_c^{2\delta} [\pi(1 - \cos 2\pi\lambda)/e^\star]^{-1+2\delta} \Gamma(1 - 2\delta)$

$$I_+ = |\langle I_{u,0} \rangle| + |\langle I_{d,0} \rangle| \quad I_- = \langle I_{u,0} \rangle - \langle I_{d,0} \rangle$$

$$\langle A(t)A^\dagger(0) \rangle_0 = \langle A(t)A^\dagger(0) \rangle_{\text{eq}} \times \exp \left[-\frac{\langle I_{u,0} \rangle}{e^\star} (1 - e^{-2\pi i \lambda}) t \right] \exp \left[-\frac{\langle I_{d,0} \rangle}{e^\star} (1 - e^{2\pi i \lambda}) t \right]$$

$\lambda = 1/m$ for a Laughlin state, and $\lambda \neq 1/m$ in the presence of edge reconstruction

Generalized fluctuation-dissipation relation

$$\langle \delta I_d \delta I_u \rangle_{\omega=0} = -\langle \delta I_T^2 \rangle_{\omega=0} + e^* \left(\underbrace{\langle I_{u,0} \rangle \frac{\partial}{\partial \langle I_{u,0} \rangle} - \langle I_{d,0} \rangle \frac{\partial}{\partial \langle I_{d,0} \rangle}}_{\text{Noise in incoming currents, transmitted through QPC3; Generalization of Johnson-Nyquist noise}} \right) \langle I_T \rangle$$

Current noise due to partitioning at QPC3

Noise in incoming currents, transmitted through QPC3;
Generalization of Johnson-Nyquist noise

Generalization of the fluctuation-dissipation theorem to a fully non-equilibrium situation in an interacting system

Rosenow, Levkivskyi & Halperin, PRL 116, 156802 (2016)

Generalized Fano factor

$$\langle \delta I_d \delta I_u \rangle_{\omega=0} = -\langle \delta I_T^2 \rangle_{\omega=0} + e^* \left(\langle I_{u,0} \rangle \frac{\partial}{\partial \langle I_{u,0} \rangle} - \langle I_{d,0} \rangle \frac{\partial}{\partial \langle I_{d,0} \rangle} \right) \langle I_T \rangle$$

To normalize cross-correlations, we divide by the second term which describes current cross-correlations due to a partial tunneling of fluctuations in incoming currents

$$P(I_-/I_+) = \frac{\langle \delta I_d \delta I_u \rangle_{\omega=0}}{e^* I_+ \frac{\partial}{\partial I_-} \langle I_T \rangle \Big|_{I_-=0}}$$

normalized noise power
for anyons ($T_u = T_d$)

$$P(0) = 1 - \frac{\tan \pi \lambda}{\tan \pi \delta} \frac{1}{1 - 2\delta} \xrightarrow{\lambda = \frac{1}{m}, \delta = \frac{1}{m}} \frac{-2}{m - 2}$$

Noise power in classical model

$$P_{\text{cl}}(T_u = T_d) = -(1 - p)T_u$$

Rosenow, Levkivskyi & Halperin, PRL 116, 156802 (2016)

Generalized Fano factor

Generally, a negative $P(0)$ is a robust signature of anyonic statistics

For $\nu=1/3$ Laughlin state, compare $P(0) = -2$ with $P_{\text{cl}} = -(1-p) T_u$

\Rightarrow suggests $p < 0$ or bunching for anyons

$$P(I_-/I_+) = \frac{\langle \delta I_d \delta I_u \rangle_{\omega=0}}{e^* I_+ \frac{\partial}{\partial I_-} \langle I_T \rangle \Big|_{I_-=0}}$$

normalized noise power
for anyons ($T_u = T_d$)

$$P(0) = 1 - \frac{\tan \pi \lambda}{\tan \pi \delta} \frac{1}{1 - 2\delta} \xrightarrow{\lambda=\frac{1}{m}, \delta=\frac{1}{m}} \frac{-2}{m-2}$$

Noise power in classical model

$$P_{\text{cl}}(T_u = T_d) = -(1-p)T_u$$

Rosenow, Levkivskyi & Halperin, PRL 116, 156802 (2016)

Conclusions

- non-equilibrium steady state for local injection of electrons into Luttinger liquid
- non-equilibrium bosonization allows exact calculation of noise in $\nu=2$ integer quantum Hall edge
- analytical description of thermalization after a quantum quench
- proposed anyon collider allows measurement of spatial exclusion, related to anyonic statistics
- negative cross-correlations for anyons are a clear signature of a reduced spatial exclusion as compared to fermions