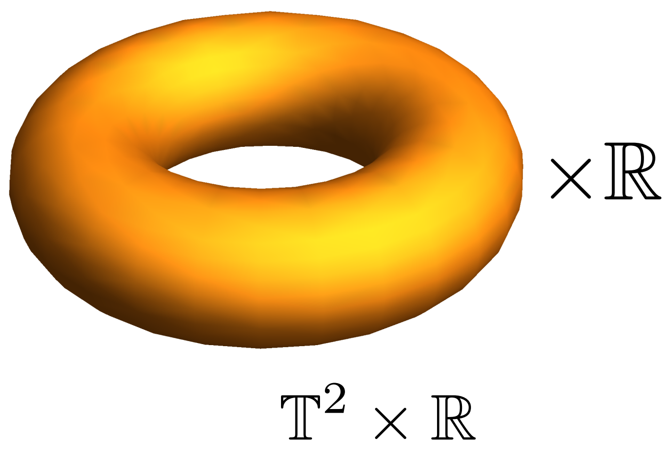

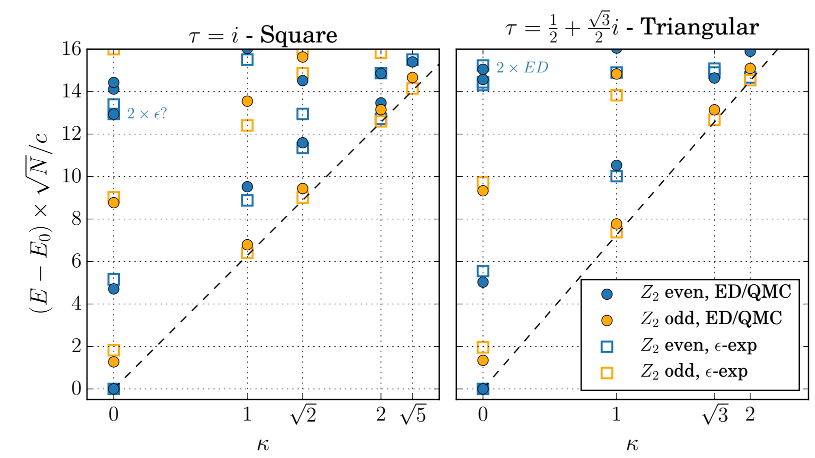

Energy Spectroscopy of Quantum Critical Systems

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universität innsbruck
Theoretische Physik

Support: FWF, SFB FoQuS Foundations and Applications of Quantum Science, FOR 1807

Strongly Interacting Field Theories (SIFT) 2019, Jena, 11/7/2019

Outline of this talk

- Introduction: Quantum Many Body Systems / Spectroscopy
- Torus Energy Spectra and Quantum Critical Points ?
- Spectrum of the standard 2+1D Ising transition
- Topological Phase Transitions
- Gross-Neveu-Yukawa fixed points
- Outlook

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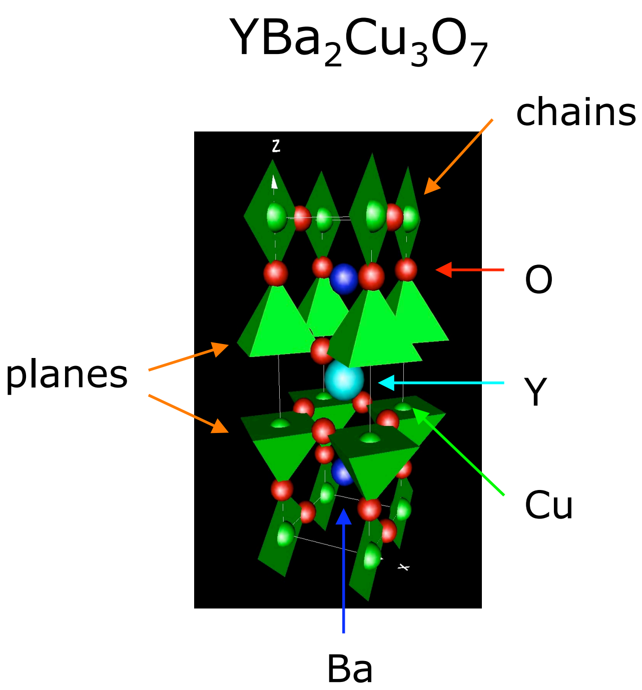
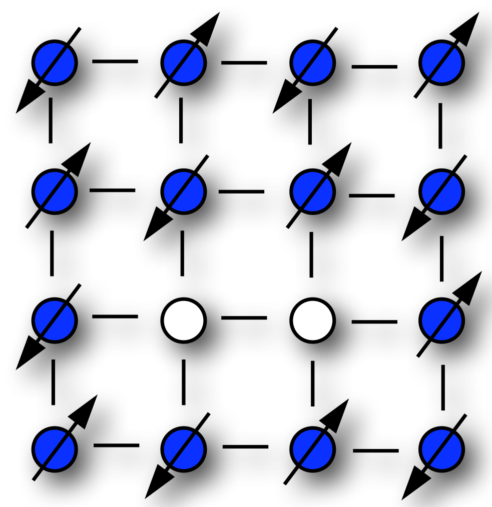
Quantum Matter:
Strongly correlated electrons in solids

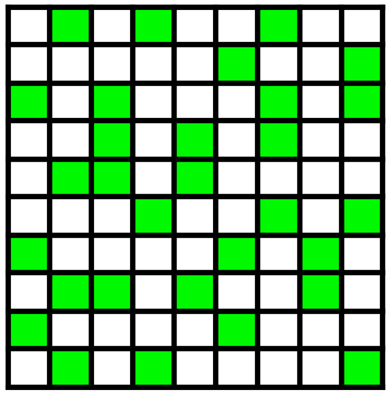
- High Tc superconductors & Quantum Magnets

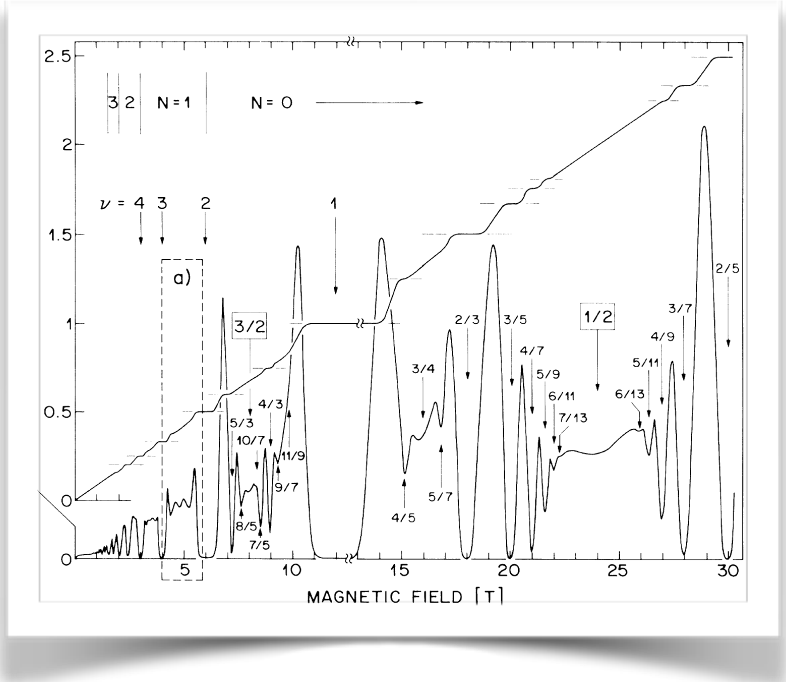




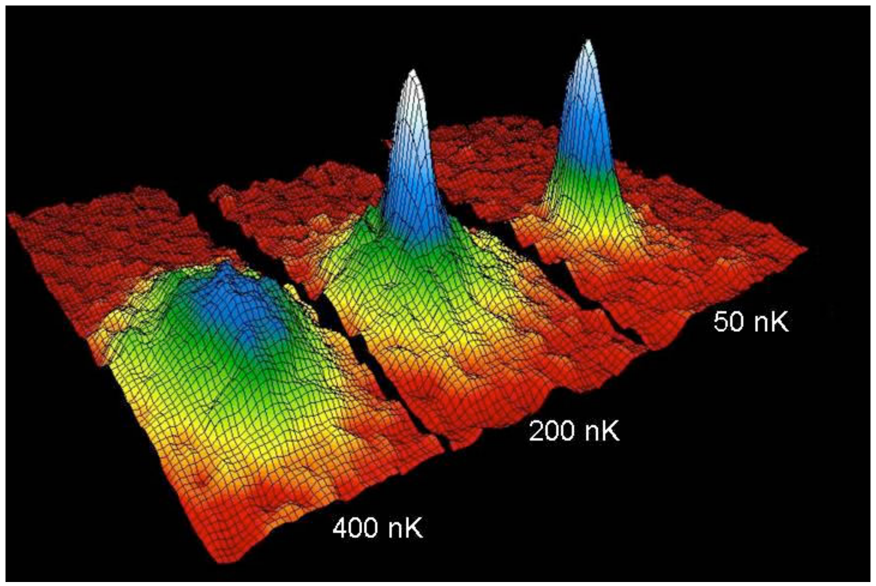
- Fractional Quantum Hall Effect

$$\nu = 1/3$$



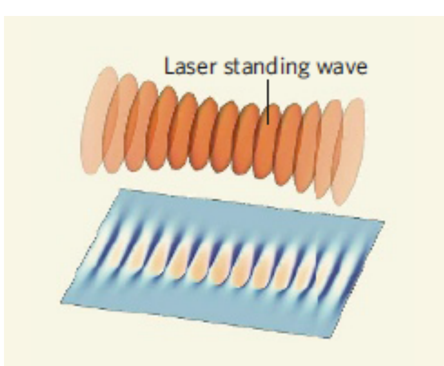


Quantum Matter:
Ultracold atomic gases



From weakly interacting Bose gases

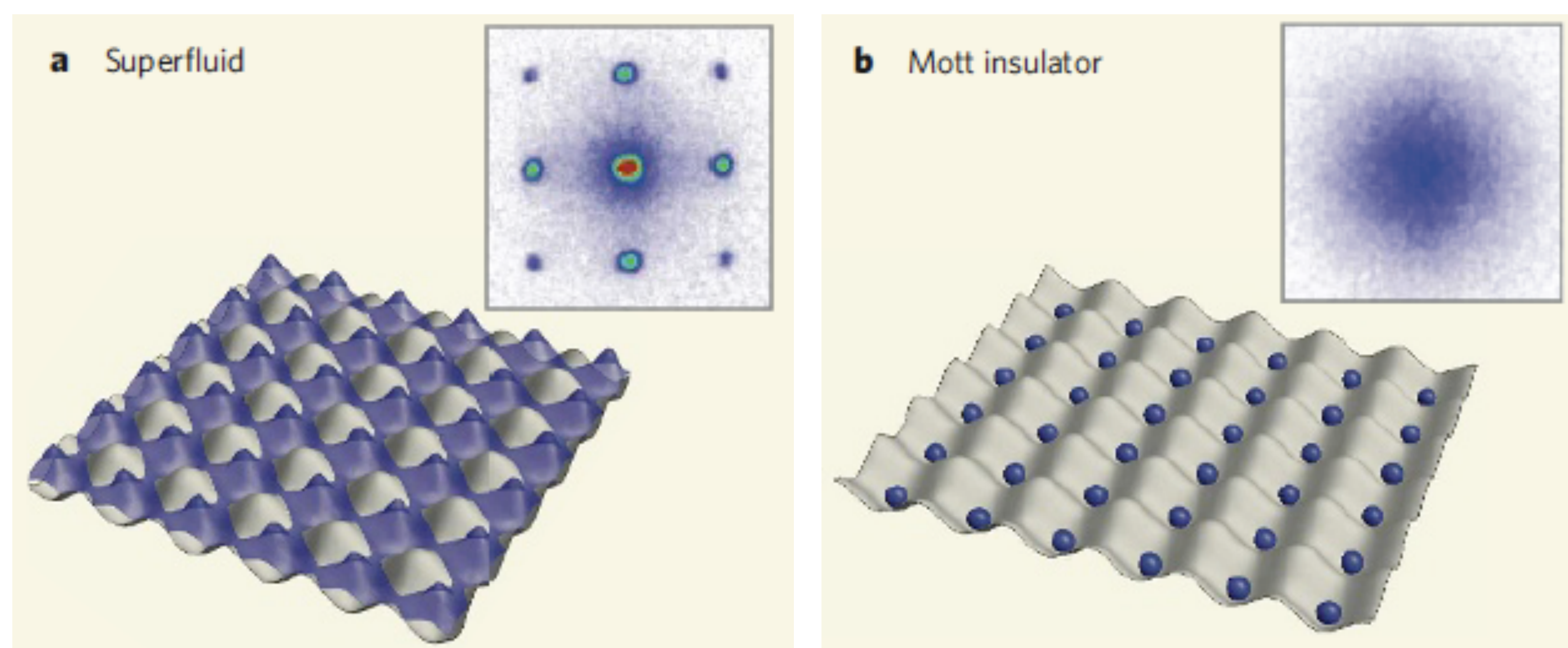
to strongly interacting gases in optical lattices



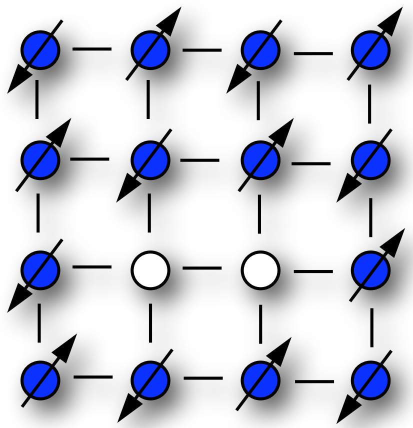
$$H = -J \sum_{\langle i,j \rangle} (b_i^\dagger b_j + h.c.) + \frac{U}{2} \sum_i n_i (n_i - 1)$$

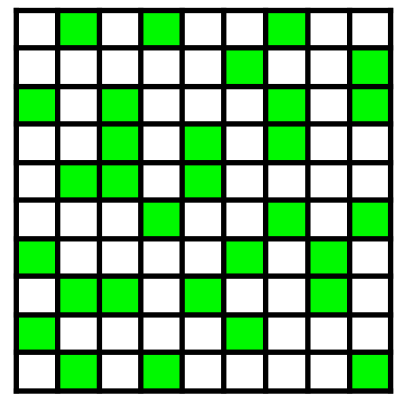
D. Jaksch et al., PRL (1998)

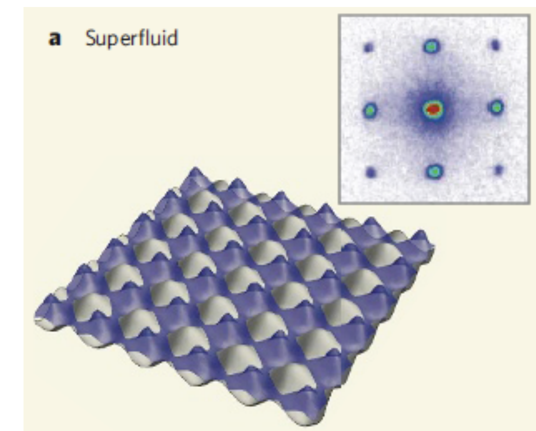
M. Greiner et al., Nature (2002)



Quantum Matter







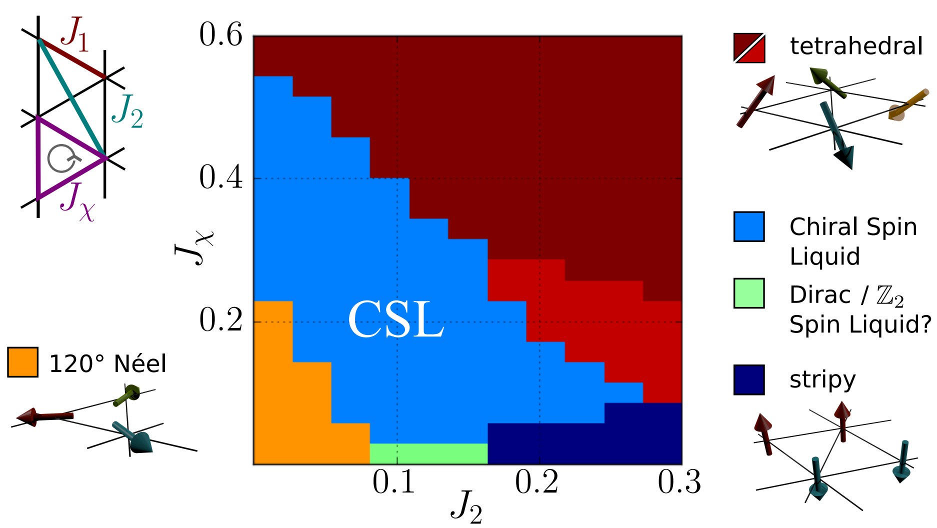
- We would like to understand phase diagrams of complex systems, but whose Hamiltonians are often reasonably well known.

- Quantum phase transitions occur. What is their universality class & field theoretical description ?

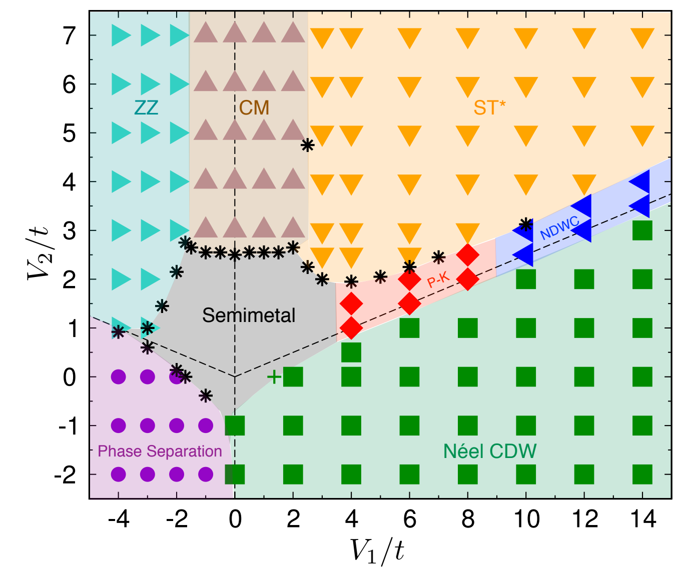
- New tools welcome to diagnose/characterize QFTs at phase transitions

Example of Microscopic Condensed Matter Models

- From microscopic models:



Phys. Rev. B. **95**, 035141 (2017)



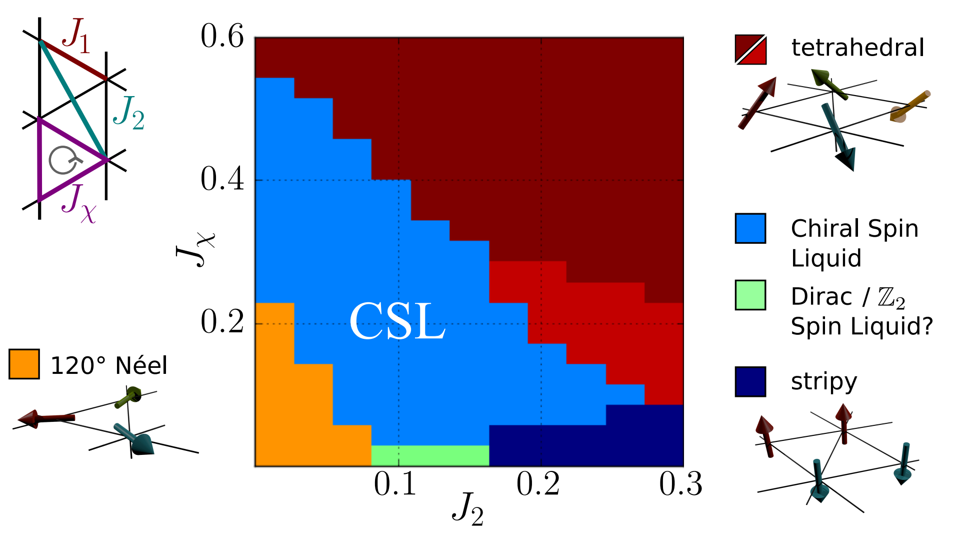
Phys. Rev. B **92**, 085146 (2015)

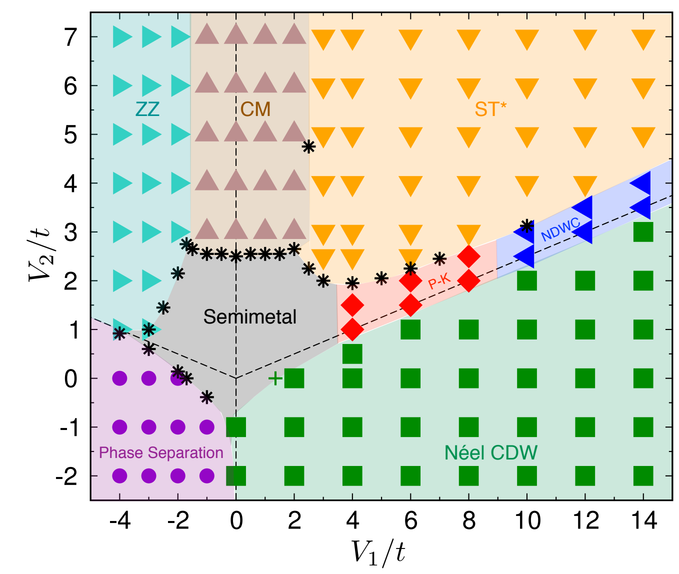
$$\mathcal{H} = J_1 \sum_{\langle i,j\rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle i,j\rangle\rangle} \vec{S}_i \cdot \vec{S}_j + J_\chi \sum_{i,j,k\in\triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

$$\mathcal{H} = -t\sum_{\langle ij\rangle}(c_i^\dagger c_j + h.c.) + V_1 \sum_{\langle ij\rangle}(n_i - 1/2)(n_j - 1/2) + V_2 \sum_{\langle\langle ij\rangle\rangle}(n_i - 1/2)(n_j - 1/2)$$

- To quantum phase transitions: Wilson Fisher CFTs, QED$_3$, Gross Neveu, …

Quantum Matter



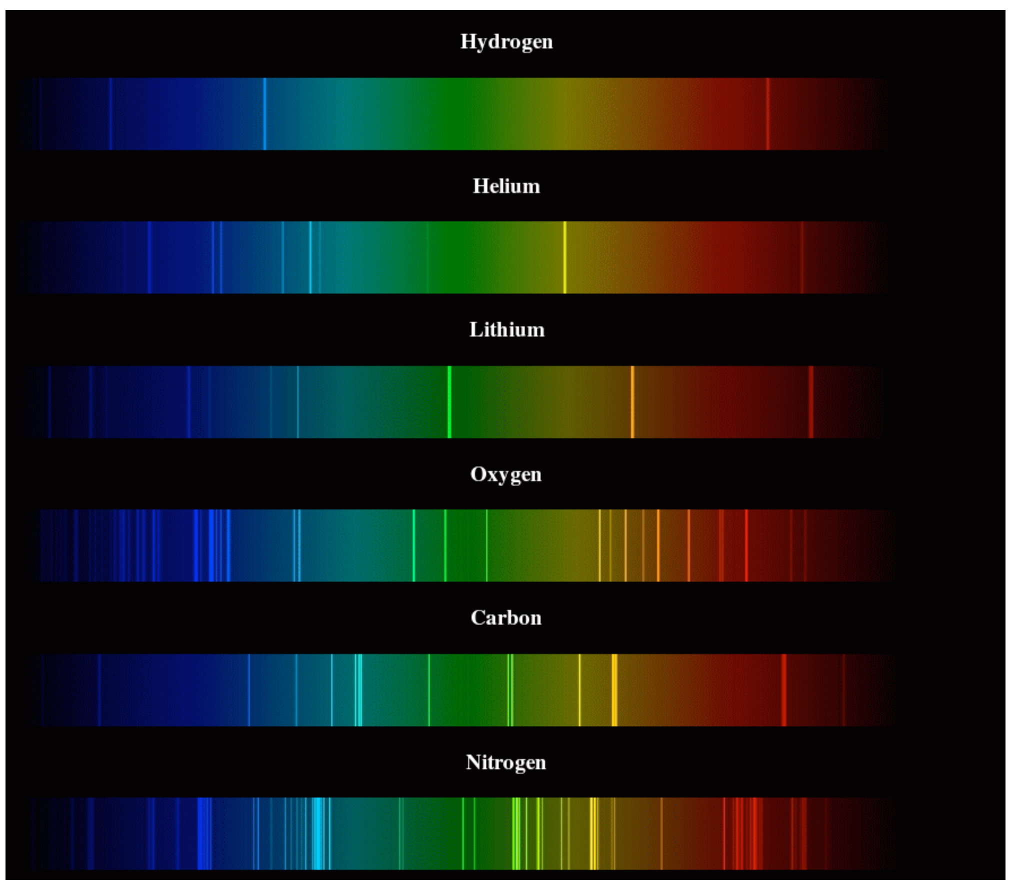


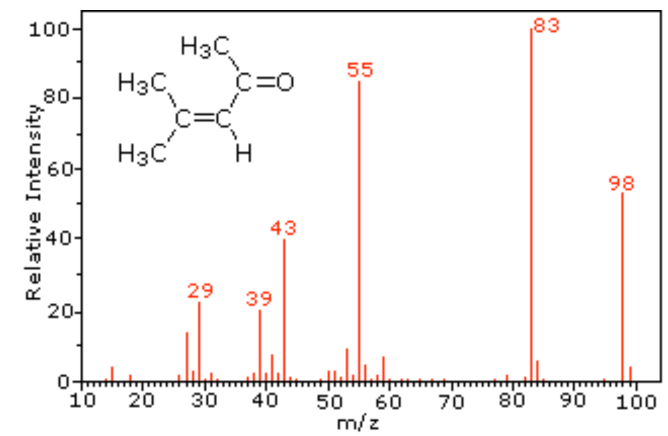
- Standard Approach: Simulate system on a computer, calculate correlation functions, order parameter, and determine critical exponents. Can work very well, but does not have to…

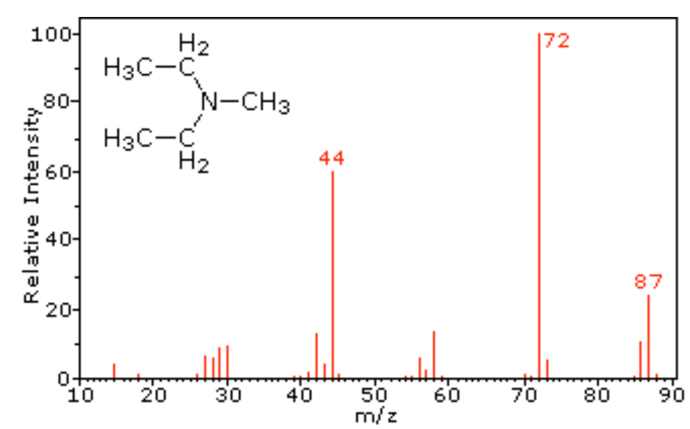
- Here want to investigate whether the Energy Spectrum of a quantum many body system at criticality reveals its universality class (Spectroscopy) ?

Spectroscopy in other areas:

- For example in optics and and mass spectroscopy one measures spectra, and then compares with a catalogue of known spectra to infer the nature of an "unknown" substance.







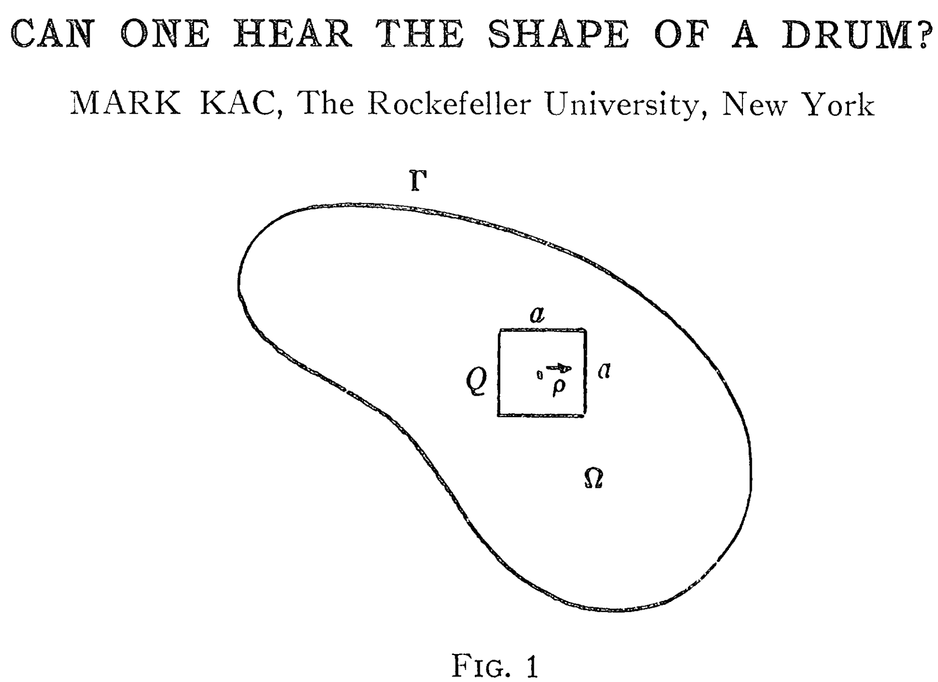
http://www.astro.rug.nl

https://www2.chemistry.msu.edu

- Can we do the same with Quantum Field Theories at Quantum Critical Points ?

“Can one hear the shape of a drum” ?

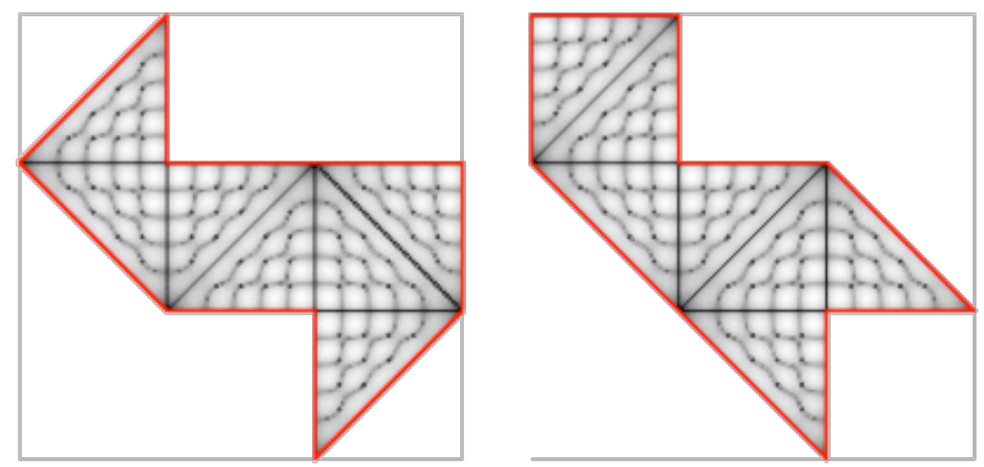
- Can one infer the shape of a domain from the spectrum of the Laplacian ?
  (not unambiguously, there are non-congruent shapes with the same spectrum)


CAN ONE HEAR THE SHAPE OF A DRUM?

MARK KAC, The Rockefeller University, New York

FIG. 1

Amer. Math. Monthly 73, 1-23, 1966.



http://mathworld.wolfram.com/IsospectralManifolds.html

- We would ask a related, but somewhat different question:
  Given a shape, can we “hear” the nature of the (massless) field theory confined to this shape ?

Exact Diagonalization: Main Idea

- Solve the Schrödinger equation of a quantum many body system numerically

$$\mathcal{H}|\psi\rangle = E|\psi\rangle$$

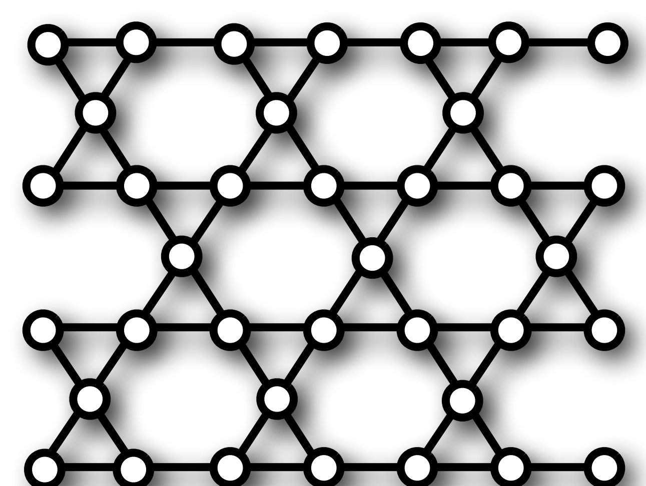
- Sparse matrix, but for quantum many body systems the vector space dimension grows exponentially!

- But the amount of information one is able to extract is worth the effort:

Powerful Quantum Mechanics Toolbox

Hilbert space sizes

- The Hilbert space of a quantum many body system grows exponentially in general
- For N spin 1/2 particles, the complete Hilbert space has dim=2^N states
- 10 spins dim=1‘024
- 20 spins dim=1‘048‘576
- 30 spins dim=1‘073‘741‘824
- 40 spins dim=1’099’511’627’776
- 50 spins dim=1‘125‘899‘906‘842’624 ...
- The quantum mechanical wave function is a vector in this Hilbert (vector) space and we would like to know the ground state and a few other low lying eigenstates



$|\uparrow\rangle$ or $|\downarrow\rangle$

- Current limit for S=1/2 spin models on regular lattice, 50 spins ! (MPI + Supercomputer) actual dimension ~ 3 x 10^{11}.

A. Wietek & AML, PRE 2018

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Operator spectrum in conformal field theories

- A local operator has a scaling dimension:

$$\mathcal{O}_i \to \Delta_i = \text{scaling dimension}$$

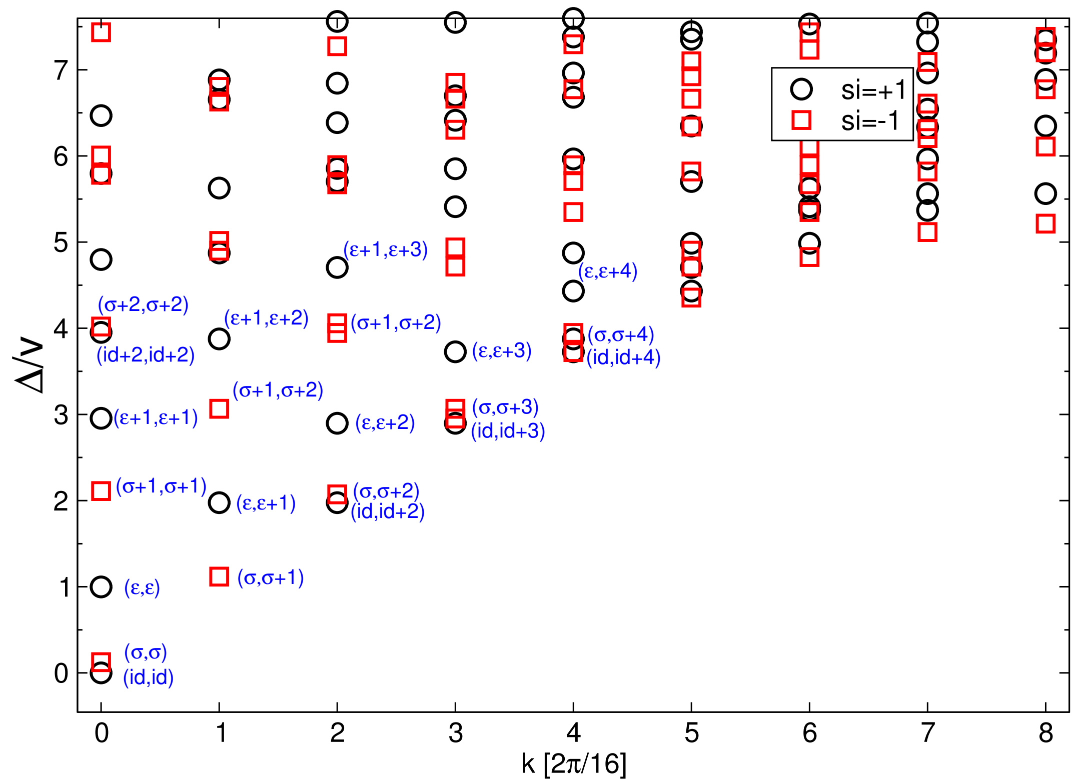
- The scaling dimension determines the decay of the 2-point correlation function:

$$\langle \mathcal{O}_i(x)\mathcal{O}_i(0)\rangle = \frac{c}{|x|^{2\Delta_i}}$$

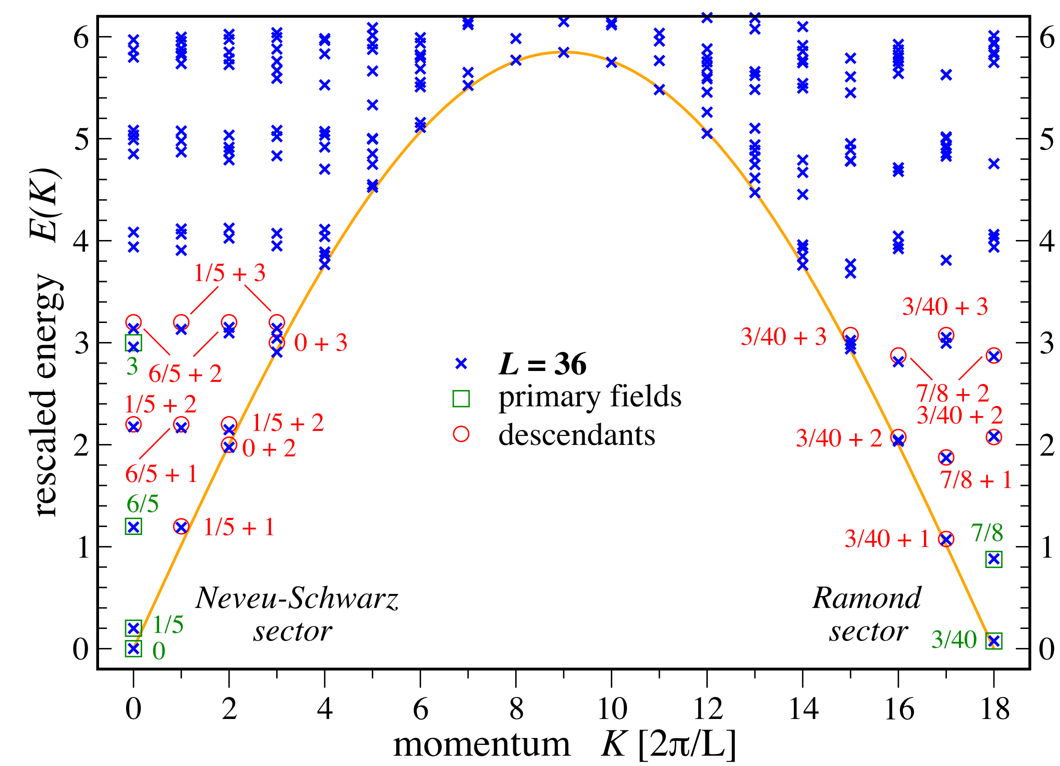
- It seems interesting and important to know the various fields with their corresponding scaling dimensions.
- Where can we find those in numerics ?

1D Torus (Circle) Energy Spectra

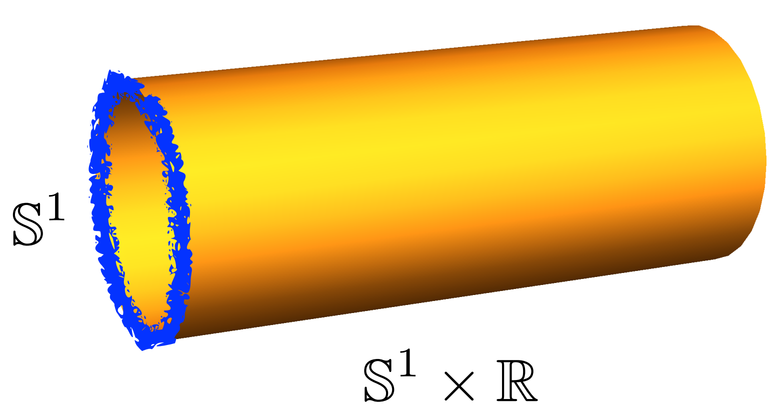
- For CFTs energy spectra of finite size (1+1D) systems arrange into conformal towers !



TFI chain L=16
2D Ising CFT Spectrum



A. Feiguin et al. PRL 2007
tricritical Ising CFT Spectrum in anyon chains



$$\mathbb{R}^2 \leftrightarrow \mathbb{S}^1 \times \mathbb{R}$$

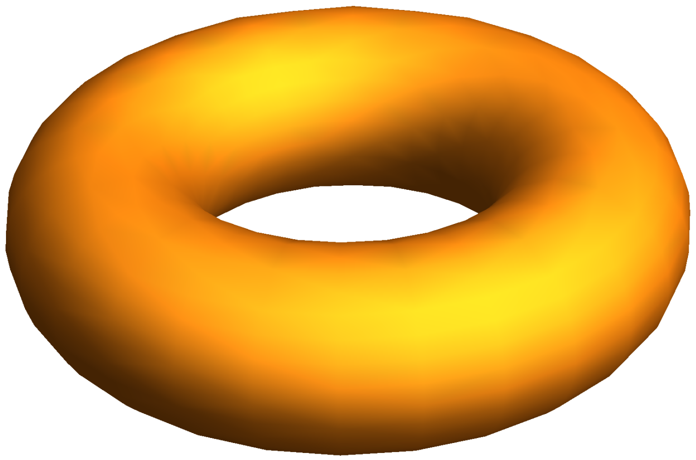
- Spectrum of scaling dimensions of CFT maps to Hamiltonian spectrum on a circle.

Energy spectra and CFTs in more than 1+1D ?

- In more than 1+1D, this relation does not hold for tori anymore, only for the sphere !

$$\mathbb{R}^d \leftrightarrow S^{d-1} \times \mathbb{R} \quad (\neq \mathbb{T}^{d-1} \times \mathbb{R},\ d > 2)$$

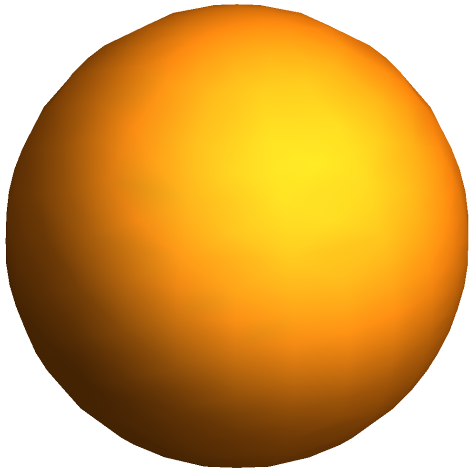
- First mapping: radial quantisation, can reveal scaling dimensions in higher d, but not easily accessible to numerics (although several efforts over the decades).



$\times\mathbb{R}$

$\mathbb{T}^2 \times \mathbb{R}$

space x time



$\times\mathbb{R}$

$\mathbb{S}^2 \times \mathbb{R}$

space x time

Energy spectra and CFT in more than 1+1D ?

- In more than 1+1D, this is not expected to hold anymore for tori !

$$\mathbb{R}^d \leftrightarrow S^{d-1} \times \mathbb{R} \quad (\neq \mathbb{T}^{d-1} \times \mathbb{R},\ d > 2)$$

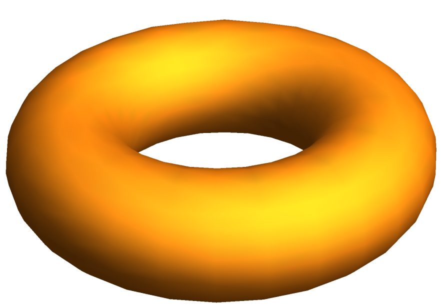
- First mapping: radial quantization, can reveal scaling dimension in higher d, but not easily accessible to numerics (although several efforts over the decades).

- What about energy spectra on tori, which are numerically accessible?

  - Is there a universal low-energy spectrum (and is it accessible numerically) ?

  - How does it look like ?

  - Any analogy to the spectrum of scaling dimensions ?



$\mathbb{T}^2 \times \mathbb{R}$

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- Torus Energy Spectra and Quantum Critical Points ?
- Spectrum of the standard 2+1D Ising transition

M. Schuler, S. Whittsitt, L.-P. Henry, S. Sachdev & AML
Phys. Rev. Lett. 2016

S. Whittsitt, M. Schuler,, L.-P. Henry, AML & S. Sachdev
Phys. Rev. B. 2017

- Topological Phase Transitions
- Gross-Neveu-Yukawa fixed points
- Outlook

2+1D “standard” Ising CFT

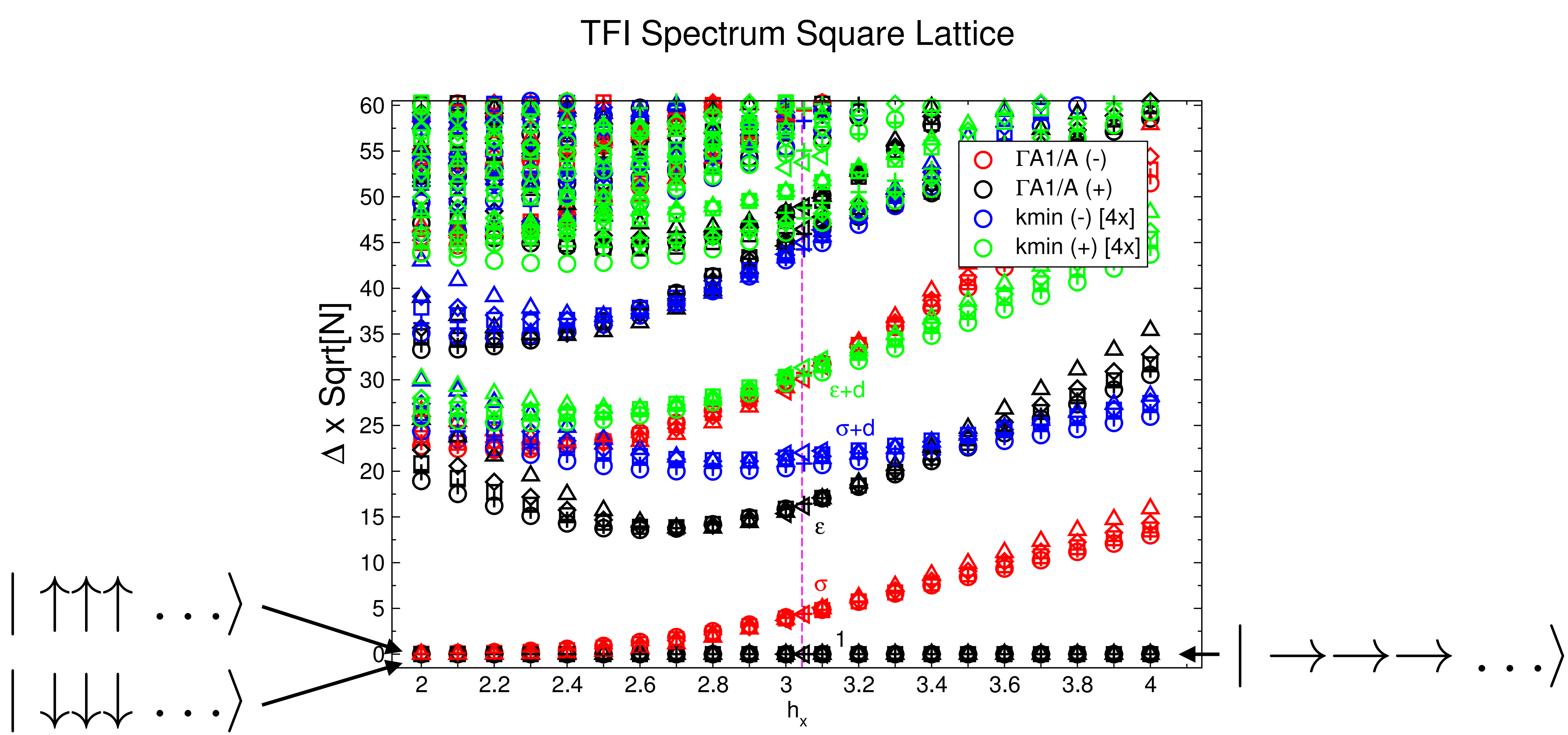
- We want to investigate the torus energy spectrum at a quantum critical point.
- While we do not expect to find the exact spectrum of scaling dimensions, the spectrum is still expected to be universal, i.e. UV cutoff independent.
- The spectrum could however depend on the IR-cutoff (shape of torus) (c.f. “hearing the shape of the drum”)
- We start with a Z_2 symmetry breaking transition, and consider the transverse field Ising (TFI) model as a particular microscopic realization

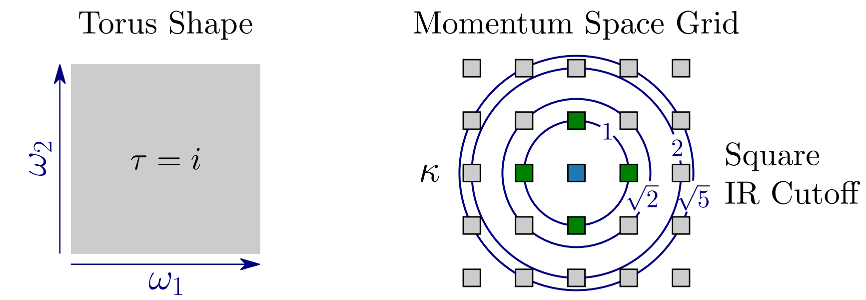
$$H_{\mathrm{TFI}} = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$

“Raw” energy spectrum across the transition

- small field: approx. 2-fold degeneracy due to Z_2-symmetry breaking.

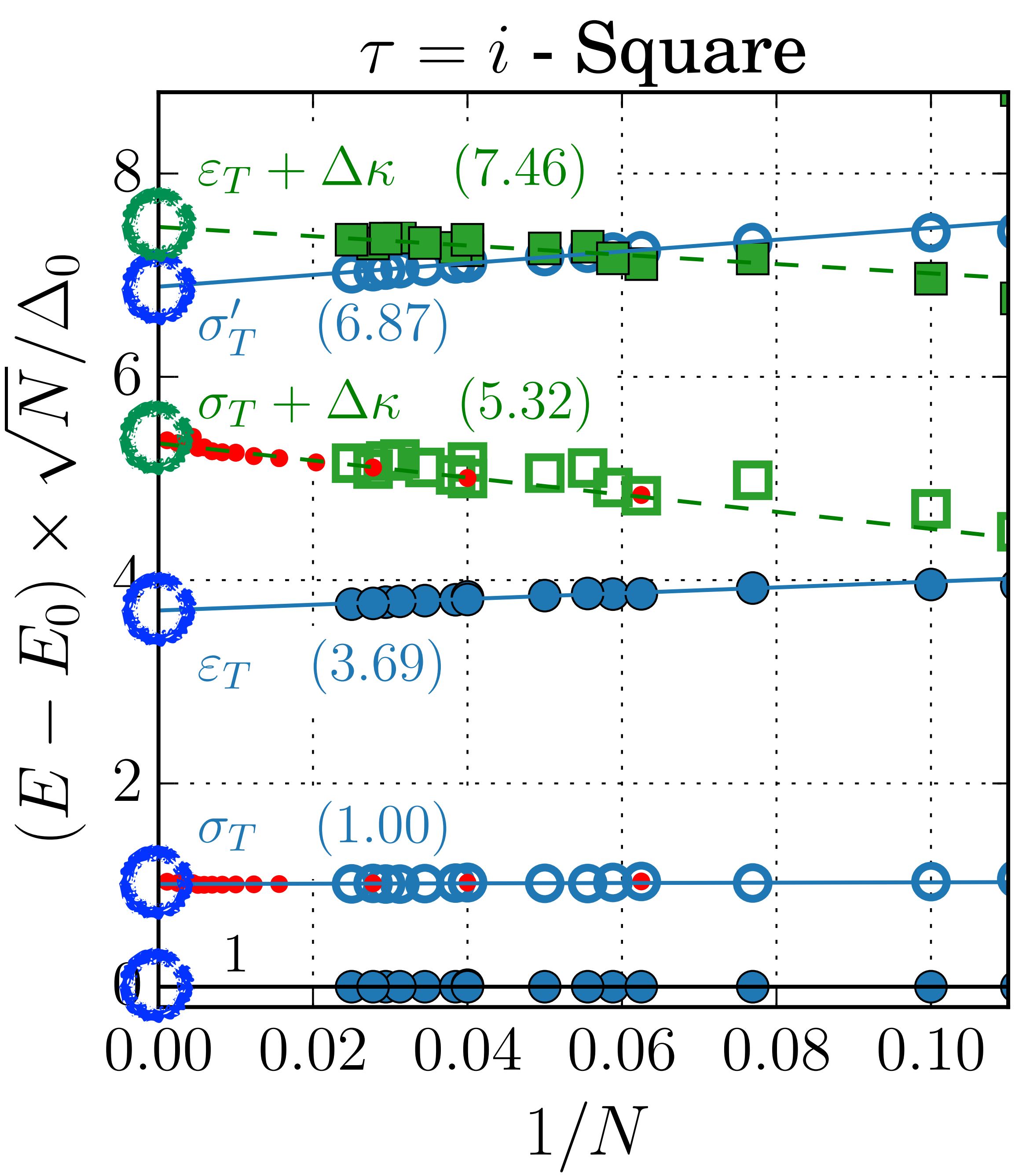
- large field: unique ground state in paramagnetic phase.

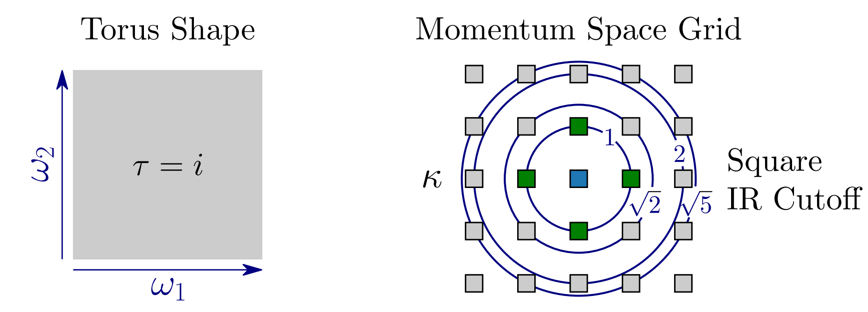




Detailed finite size scaling

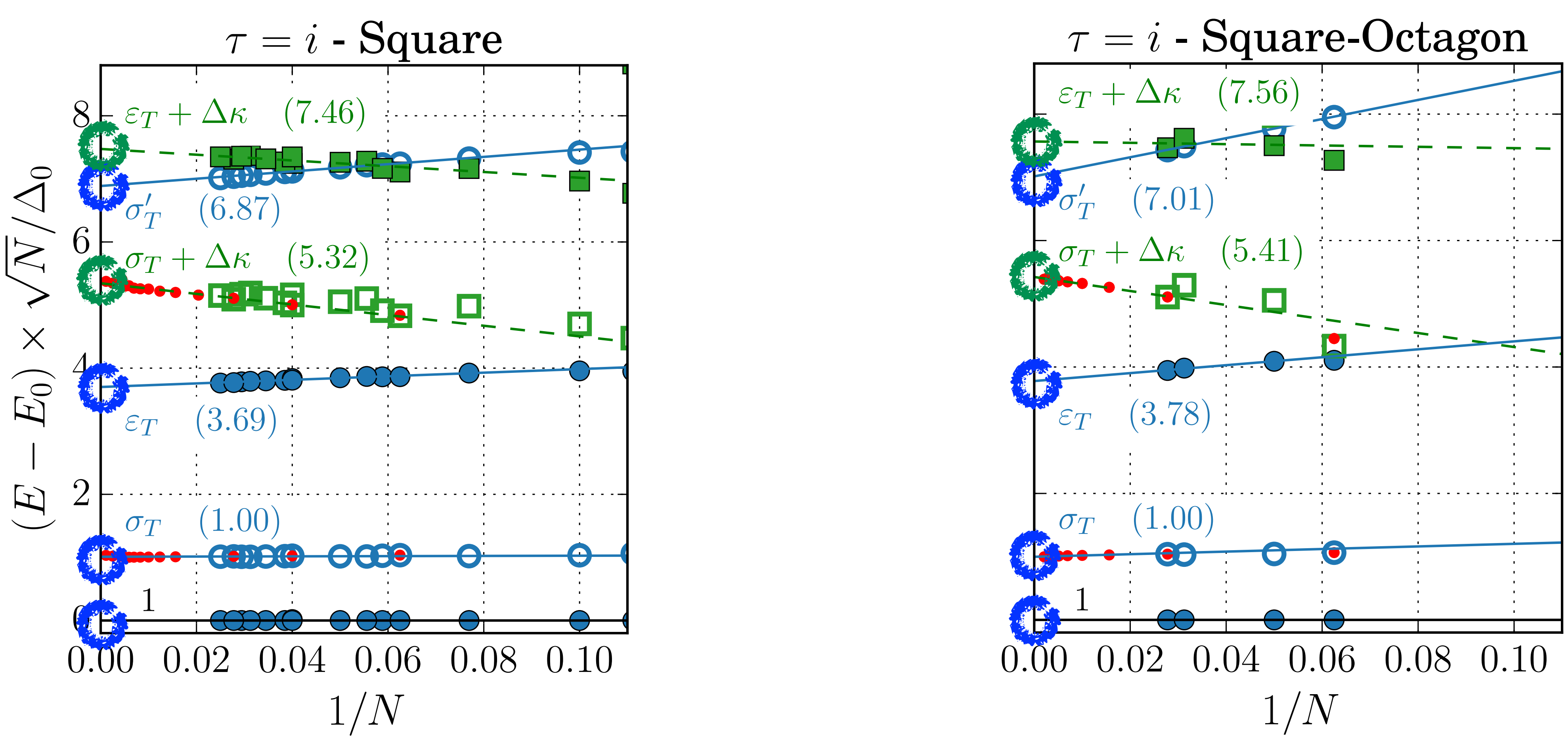
- Square lattice at critical transverse field h_c:



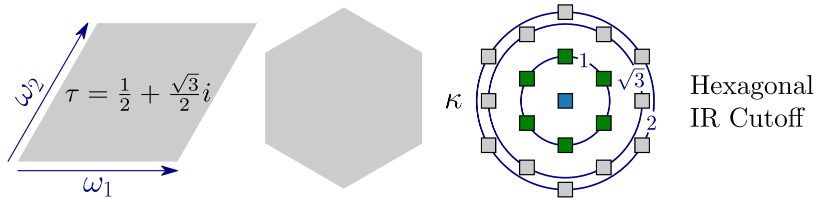


Comparison with a different lattice

- Square lattice and Square-Octagon lattice at their critical point:

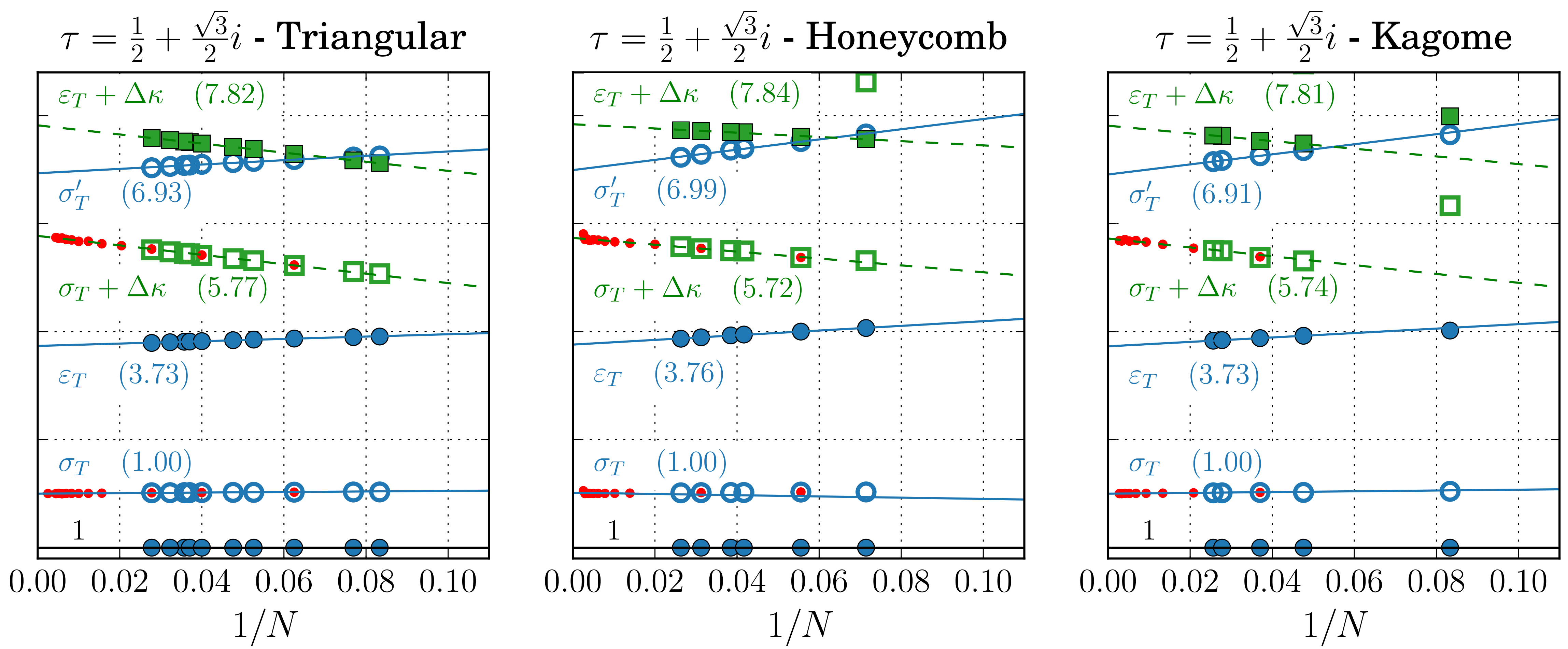


- The spectra are identical after finite-size extrapolation!
  This is thus the genuine 3D Ising CFT spectrum on a square torus !



Comparison with different modular parameter

- Triangular, honeycomb and kagome lattice at their critical point:



- The spectra are identical after finite-size extrapolation!
  This is thus the genuine Ising CFT spectrum on a hexagonal torus !

S. Whittsitt, M. Schuler,, L.-P. Henry, AML & S. Sachdev
Phys. Rev. B. 2017

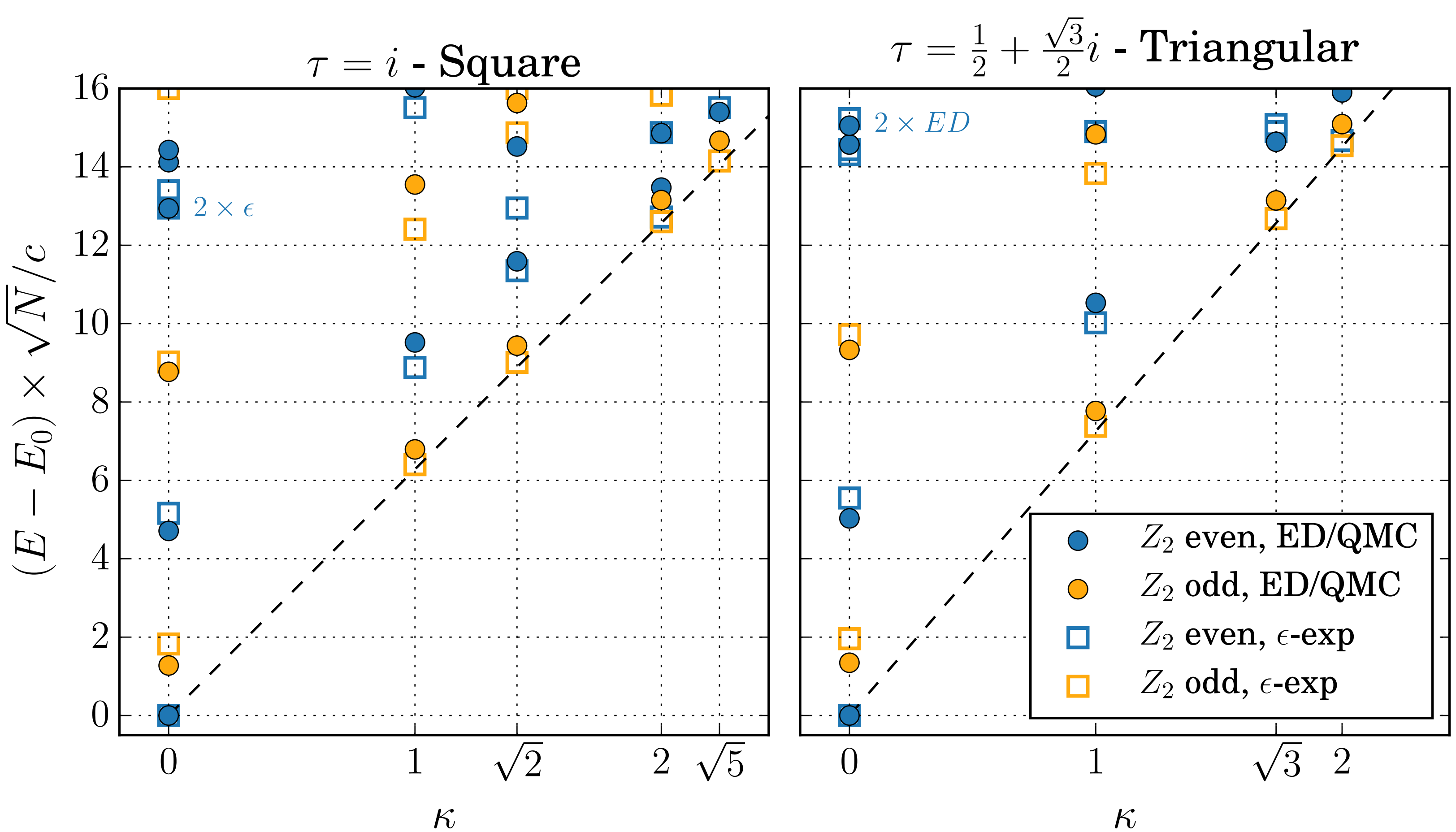
Analytical approach: (4-epsilon)-expansion

- Work done by S. Withsett and S. Sachdev. Lowest non-trivial order in epsilon.

$$\mathcal{H} = \int d^d x \left[\frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{s}{2}\phi^2 + \frac{u}{4!}\phi^4\right]$$

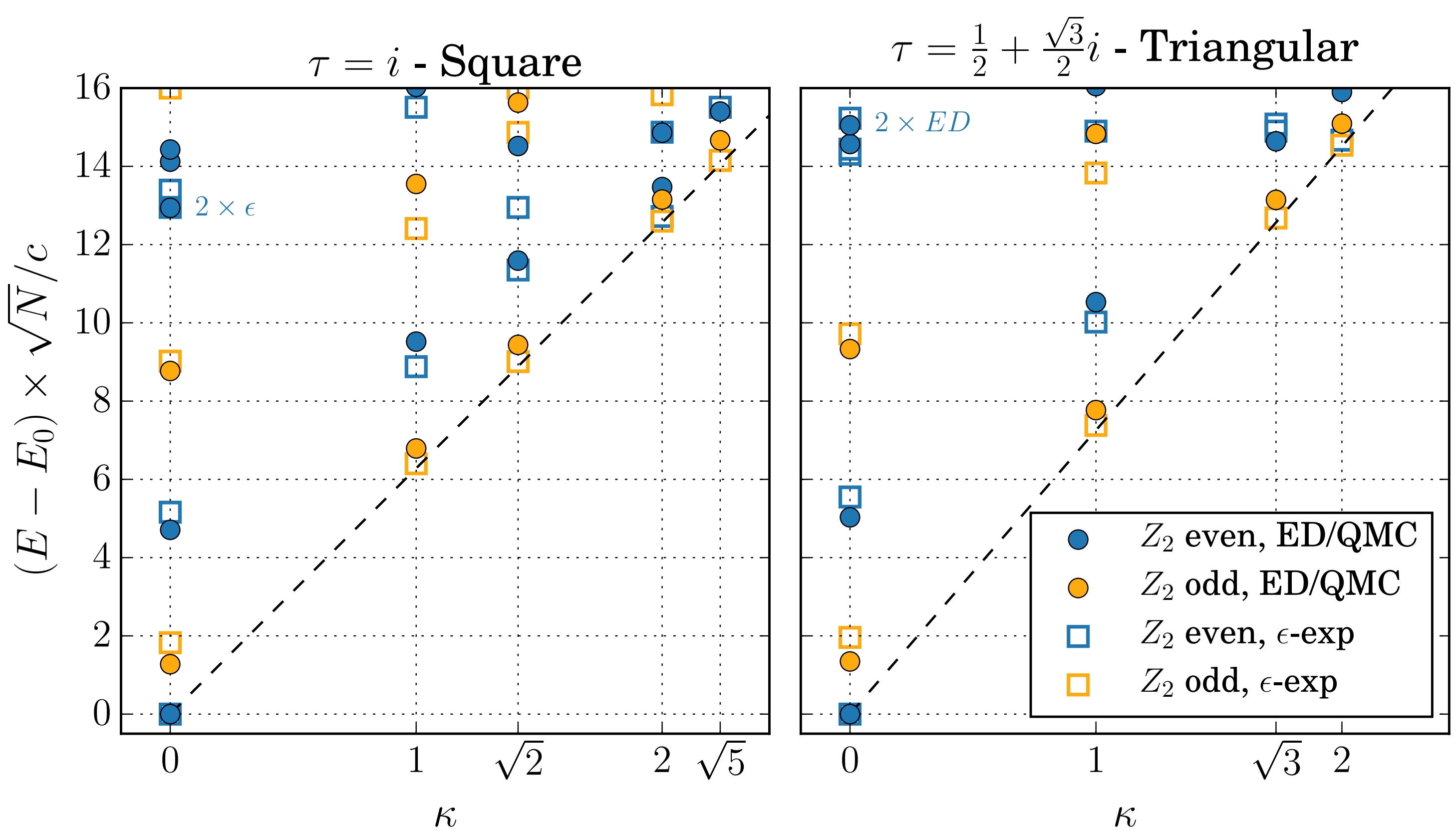
- Rather good agreement between analytics and numerics.

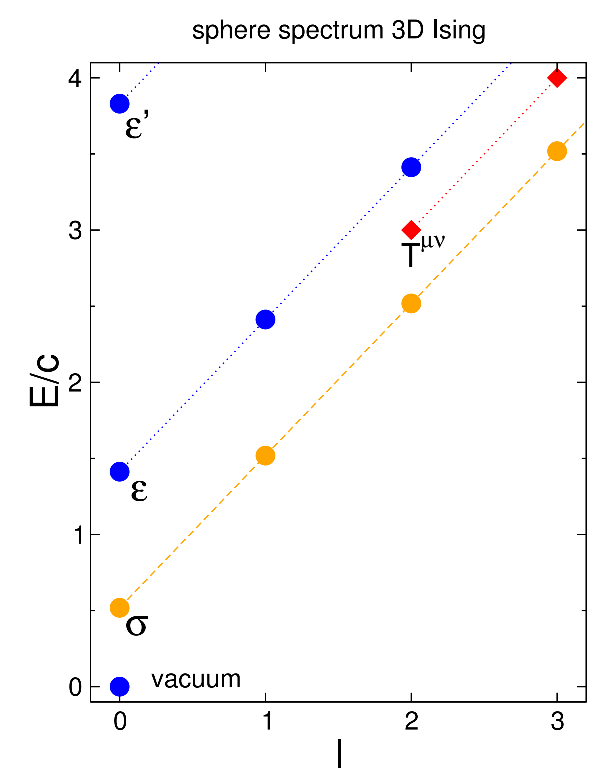
- Zero-mode is most important in (4-epsilon)-expansion, anharmonic oscillator.



Comparison between torus and sphere spectra

- Torus spectra at low energy per sector resemble the spectrum on the sphere:



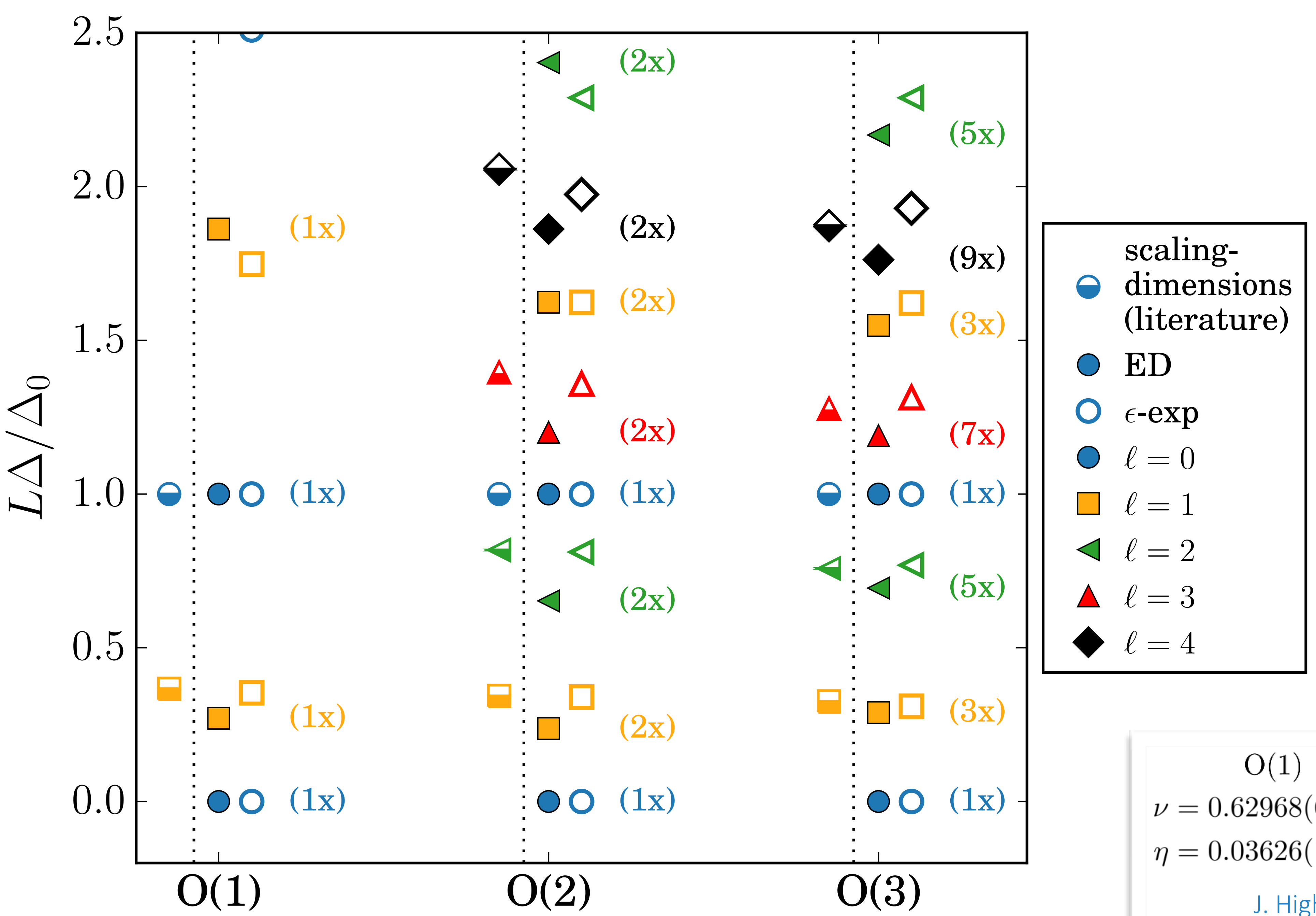


- We believe this handwaving resemblance might be more generally the case: "light states on the sphere have a light analogon on the torus"

- But likely *no* state operator correspondence on the torus.

Wilson-Fisher Z_2 / O(2) / O(3) Results

- Universality classes differ by amount of multiplets and their size!



	O(1)	O(2)	O(3)
$\nu =$	0.62968(66)	0.67195(97)	0.7113(21)
$\eta =$	0.03626(10)	0.03810(20)	0.03750(50)

J. High Energ. Phys. (2014) 2014: 91

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  M. Schuler, S. Whittsitt, L.-P. Henry, S. Sachdev & AML
  Phys. Rev. Lett. 2016
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Confinement transition

- Z_2 spin liquids are among the simplest topological phases.

- The are phases with a four-fold ground state degeneracy on a torus, but the degeneracy is topological, and not related to symmetry breaking.

- One of the simplest incarnations of this phase appears in the Toric Code model by Kitaev.

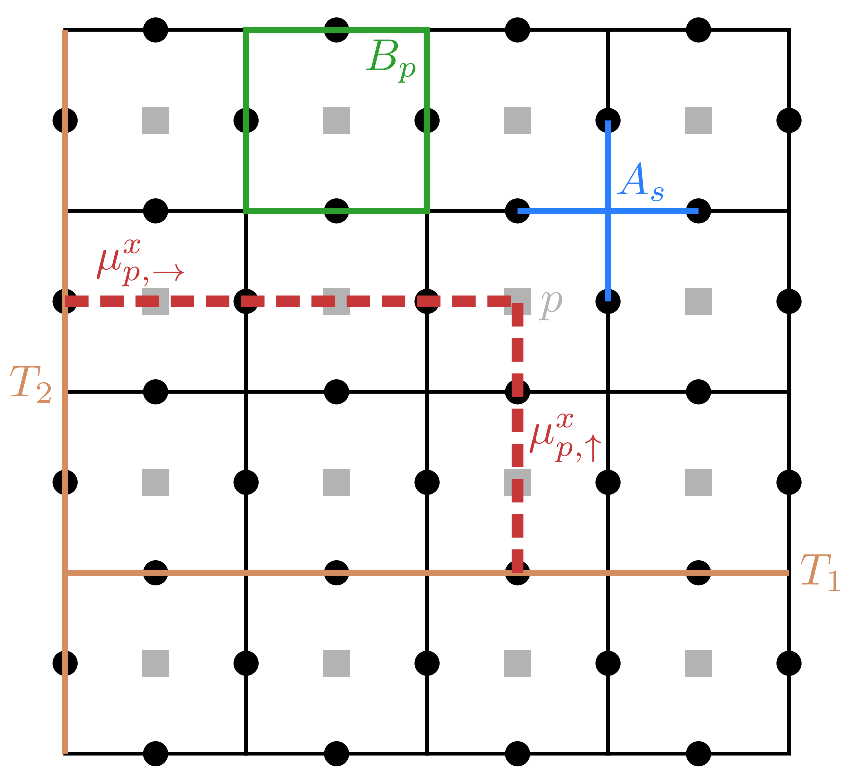
- By an appropriate perturbation the topological phase (“deconfined”) gives way to a simple paramagnetic phase (“confined”). The transition is a confinement transition and is expected to be in the 2+1D = 3D Ising universality class.

- Q: Is the torus spectrum at criticality identical to the symmetry breaking case ?

Toric code in a magnetic field

- We study the following microscopic model
  (but results will be independent of specific model):

- Toric code with a longitudinal magnetic field (S. Trebst et al., J. Vidal et al, …):



$$H = -J\sum_s A_s - J\sum_p B_p - h\sum_i \sigma_i^x$$

$$A_s = \prod_{i\in s}\sigma_i^x,\ B_p = \prod_{i\in p}\sigma_i^z$$

$$\mu_p^z = B_p$$

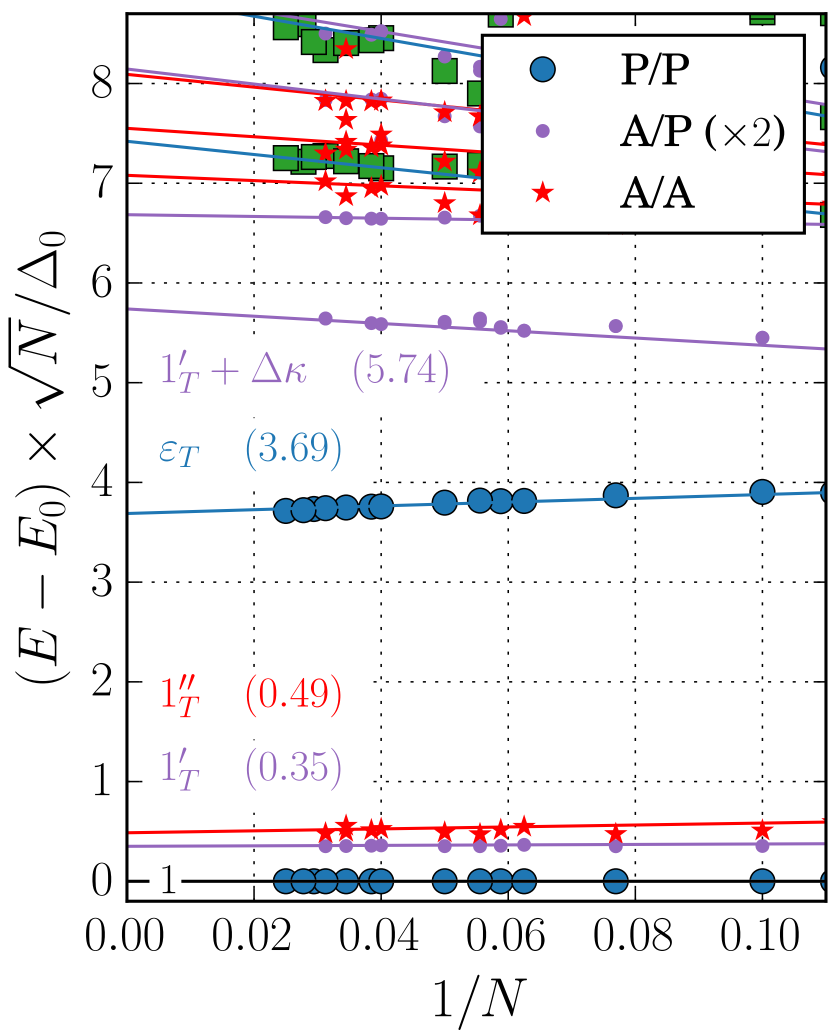
$$\mu^x_{p,\rightarrow(\uparrow)} = \prod_{i\in c_{p\rightarrow(\uparrow)}} \sigma_i^x$$

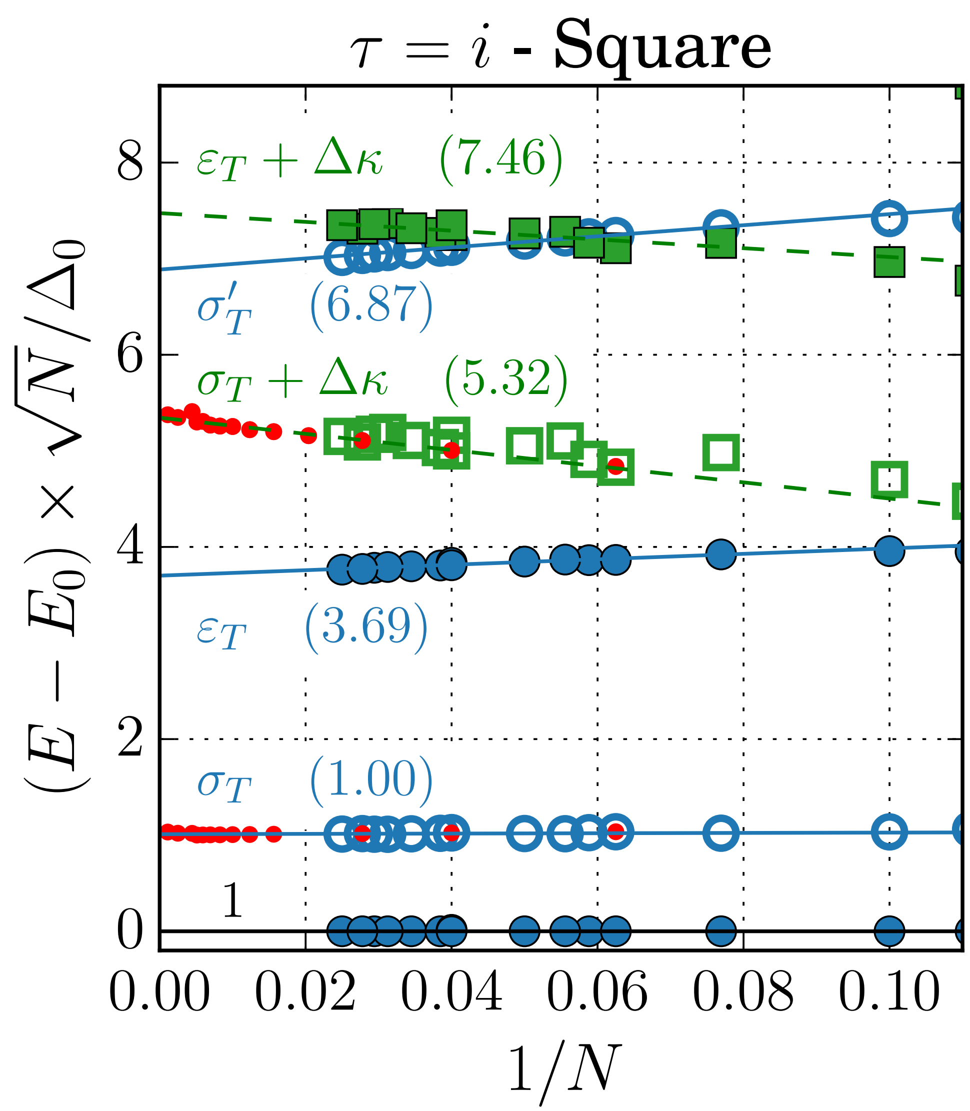
TFI boundary conditions imposed by T_1,T_2 loops !

$$H_{TFI} = -h\sum_{\langle p,q\rangle}\mu_p^x\mu_q^x - J_p\sum_p \mu_q^z + const.$$

Numerics at criticality

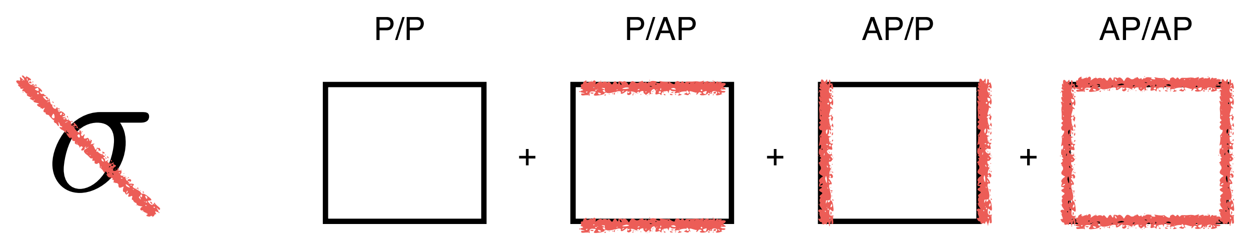
- Left: data for the TC at criticality, Right: Symmetry breaking





- The spectra at criticality do not agree ! What is going on ?

The Ising* transition



- The explanation is that the operator content of the two transitions are different:

- In the Z_2 symmetry breaking case we have Z_2 even and odd levels and only one set of boundary conditions (fixed by the lattice model).

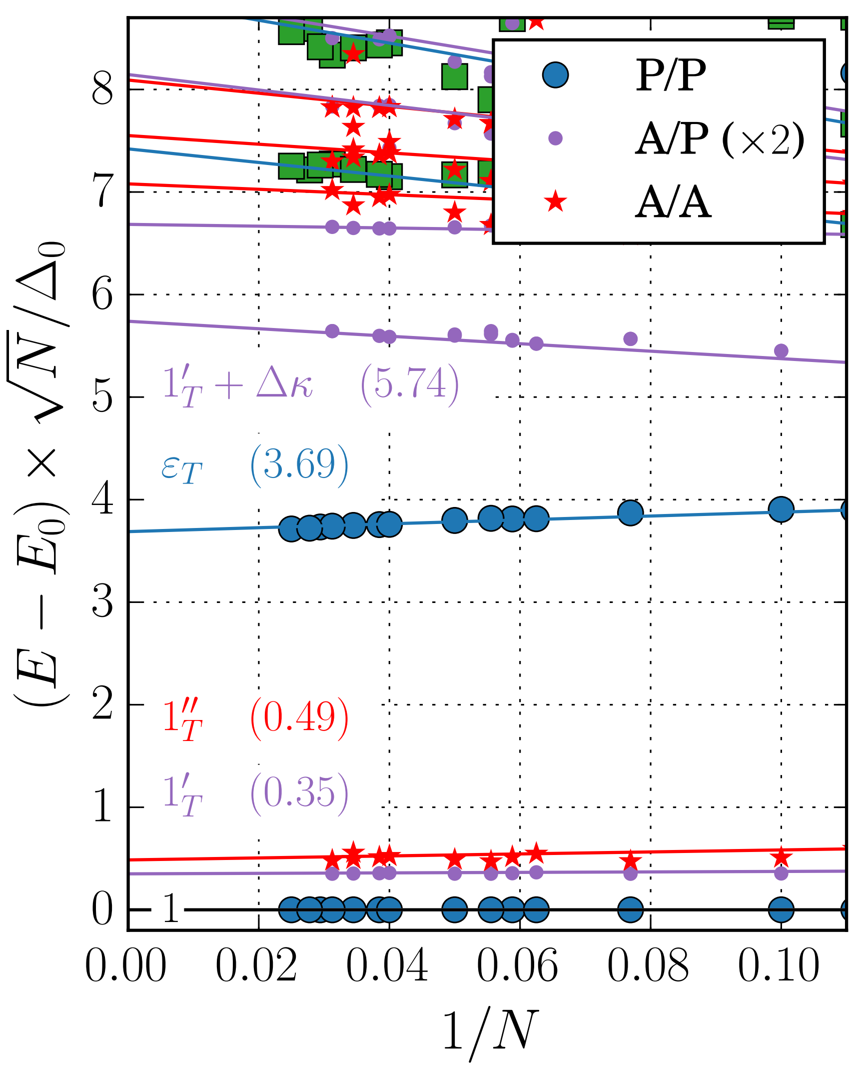
- In the confinement transition (Ising*), only Z_2 even levels are allowed, and for periodic boundary conditions in the Toric Code, four different boundary conditions of the CFT become simultaneously apparent.

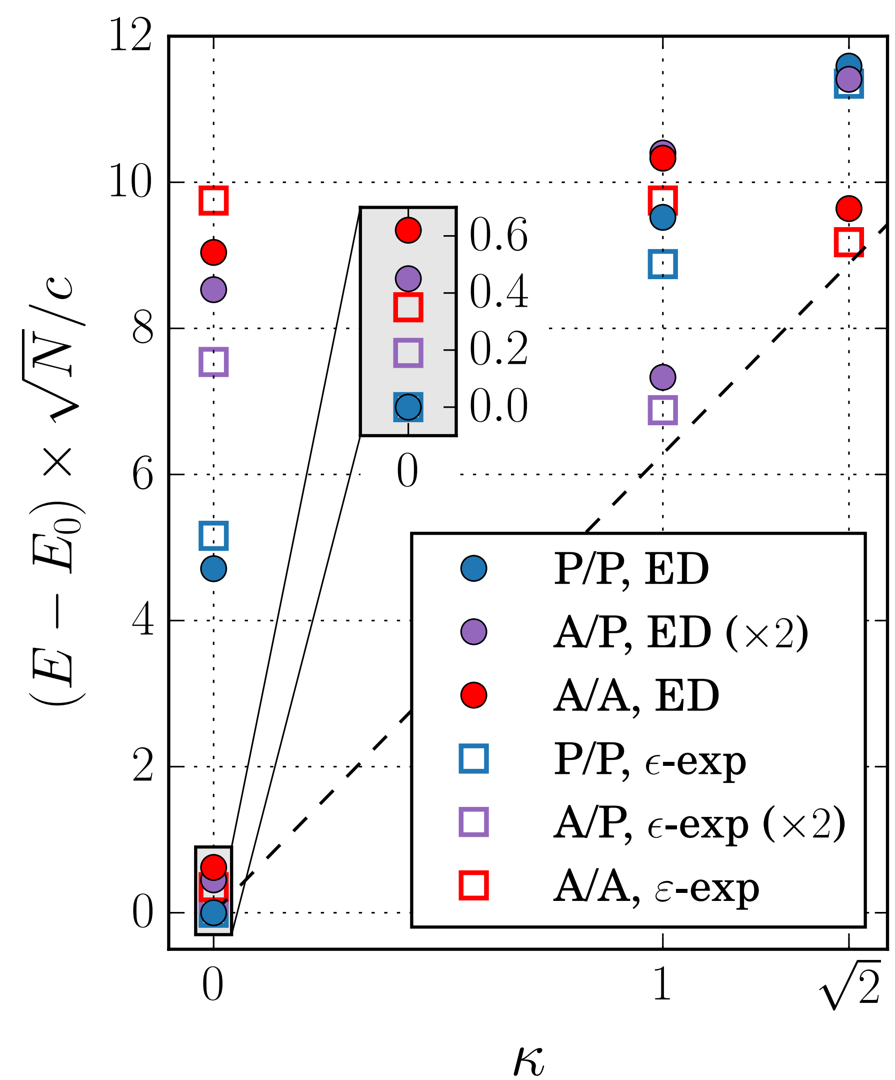
- This can be understood at the microscopic level in the Toric Code Hamiltonian and is supported by general field theoretical considerations.

- In the Ising* case the magnetic sector is completely absent, and the torus energy spectrum reflects this fact.

The Ising* transition

- comparison between numerics and epsilon-expansion:
- At criticality the 4 “topological sectors” scale also as 1/L , but are much closer together than the next level above them.





Toric code with Ising interactions

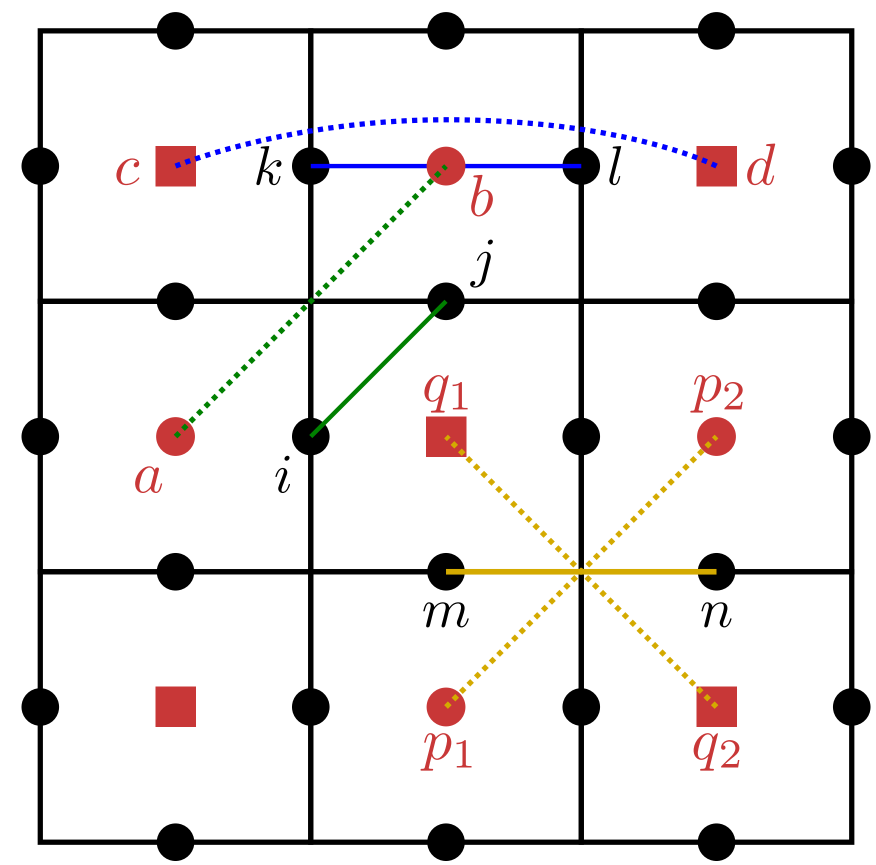
- Want to study a possible quantum phase transition between Z_2 topological order and spontaneous global Z_2 symmetry breaking.

- Toric code plus additional Ising interactions:

$$H = -J\sum_s A_s - J\sum_p B_p$$

$$\boxed{-J_I \sum_{\langle i,j\rangle} \sigma_i^x \sigma_j^x - J_{I_2} \sum_{\langle\langle i,j\rangle\rangle} \sigma_i^x \sigma_j^x}$$

$$A_s = \prod_{i\in s} \sigma_i^x \qquad B_p = \prod_{i\in p} \sigma_i^z$$



$$\sigma_i^x \sigma_j^x \longrightarrow 2\mu_a^x \mu_b^x$$

$$\sigma_k^x \sigma_l^x \longrightarrow \mu_c^x \mu_d^x$$

$$\sigma_m^x \sigma_n^x \longrightarrow 2\mu_{p_1}^x \mu_{p_2}^x \mu_{q_1}^x \mu_{q_2}^x$$

Toric code with Ising interactions

- Toric code plus additional Ising interactions:

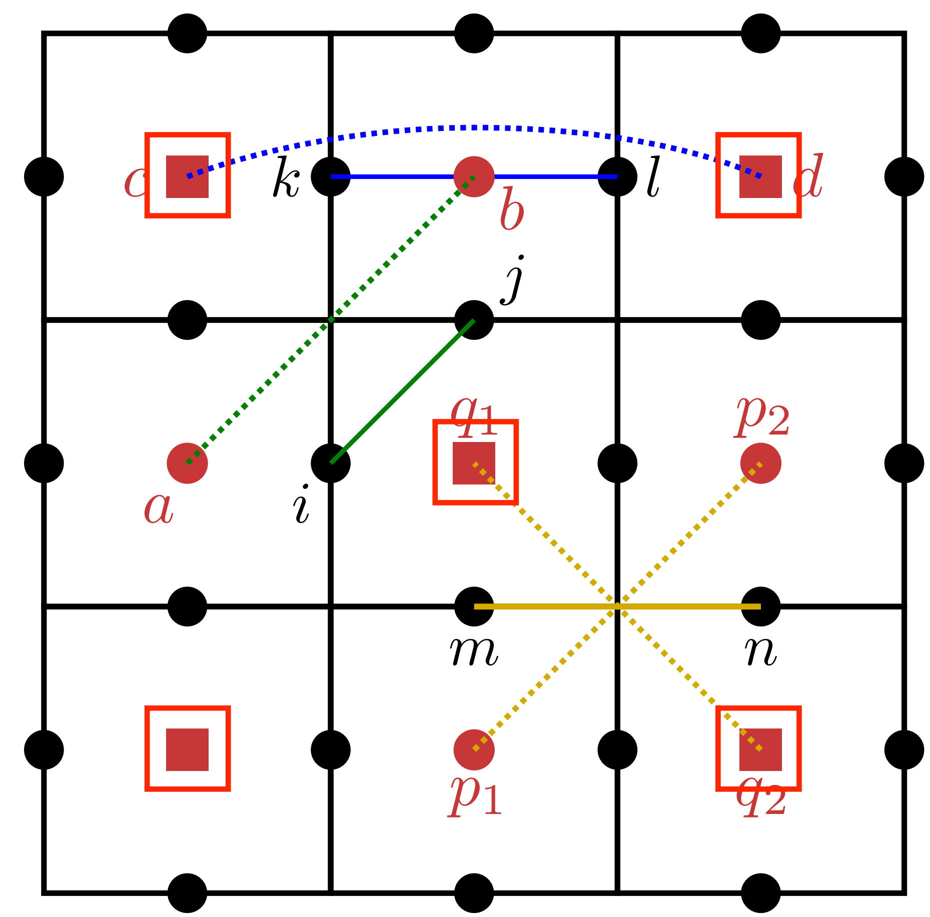
$$H = -J\sum_s A_s - J\sum_p B_p - J_I \sum_{\langle i,j\rangle} \sigma_i^x\sigma_j^x - J_{I_2}\sum_{\langle\langle i,j\rangle\rangle}\sigma_i^x\sigma_j^x$$

$$A_s = \prod_{i\in s}\sigma_i^x \qquad B_p = \prod_{i\in p}\sigma_i^z$$

- Maps onto a particular 2+1D quantum Ashkin-Teller (AT) model:

$$H_{AT} = -J\sum_i \mu_i^z - 2J_I \sum_{\langle\langle i,j\rangle\rangle}\mu_i^x\mu_j^x - J_{I_2}\sum_{\langle\langle\langle i,j\rangle\rangle\rangle}\mu_i^x\mu_j^x - 2J_{I_2}\sum_i \mu_i^x\mu_{i+\hat{\mathbf{x}}}^x\mu_{i+\hat{\mathbf{y}}}^x\mu_{i+\hat{\mathbf{x}}+\hat{\mathbf{y}}}^x \tag{A6}$$

- This model has a two checkerboard lattice spatial structure, yielding the two AT-sublattices



$$\sigma_i^x\sigma_j^x \longrightarrow 2\mu_a^x\mu_b^x$$

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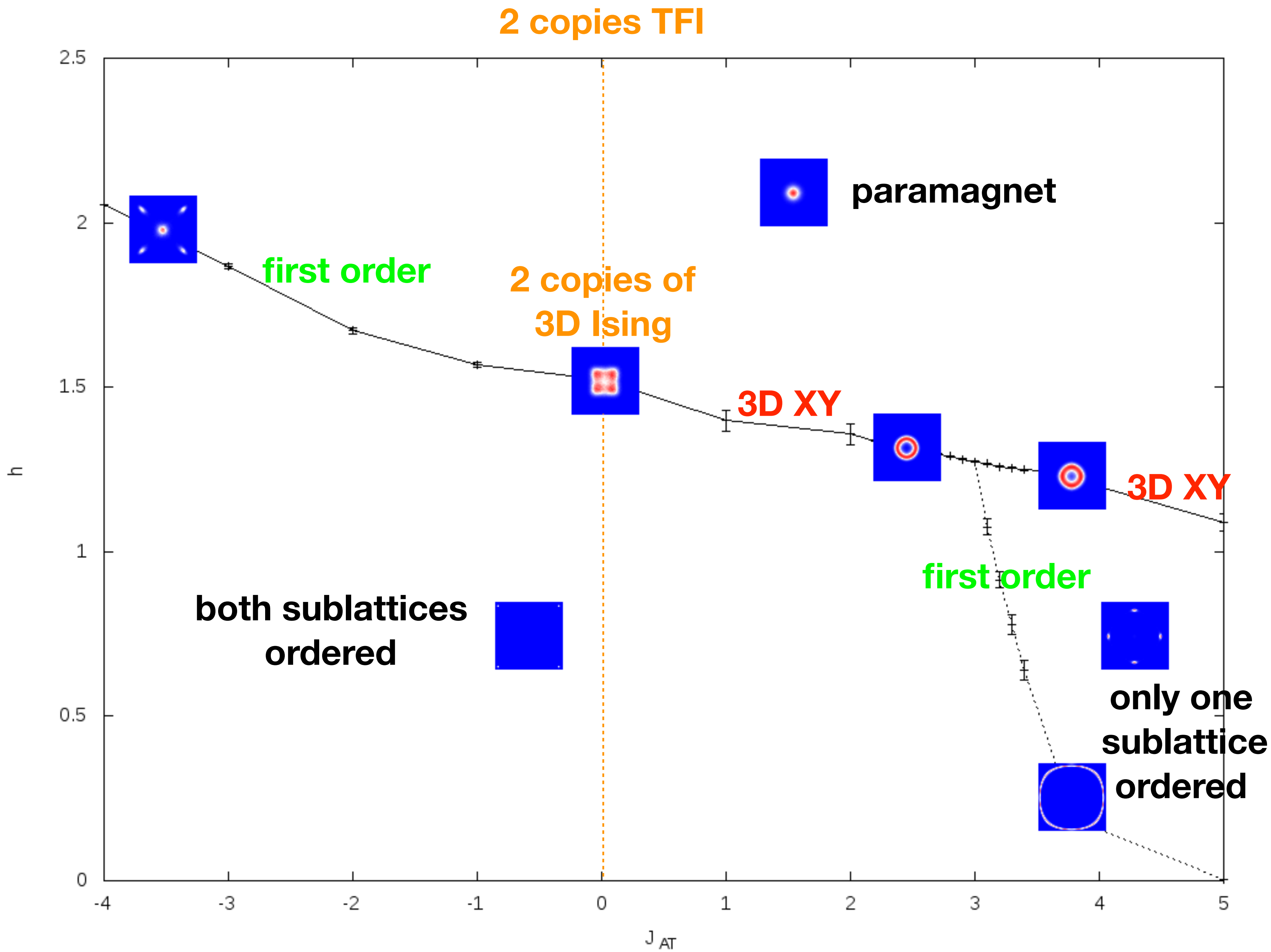
$$\sigma_m^x\sigma_n^x \longrightarrow 2\mu_{p_1}^x\mu_{p_2}^x\mu_{q_1}^x\mu_{q_2}^x$$

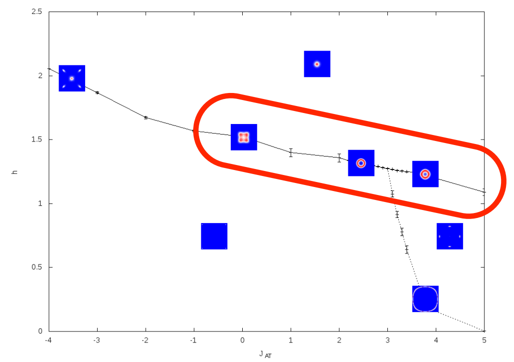
Phase diagram of the Quantum Ashkin-Teller model

- Rather poorly studied in the past, so here we perform a new QMC study:

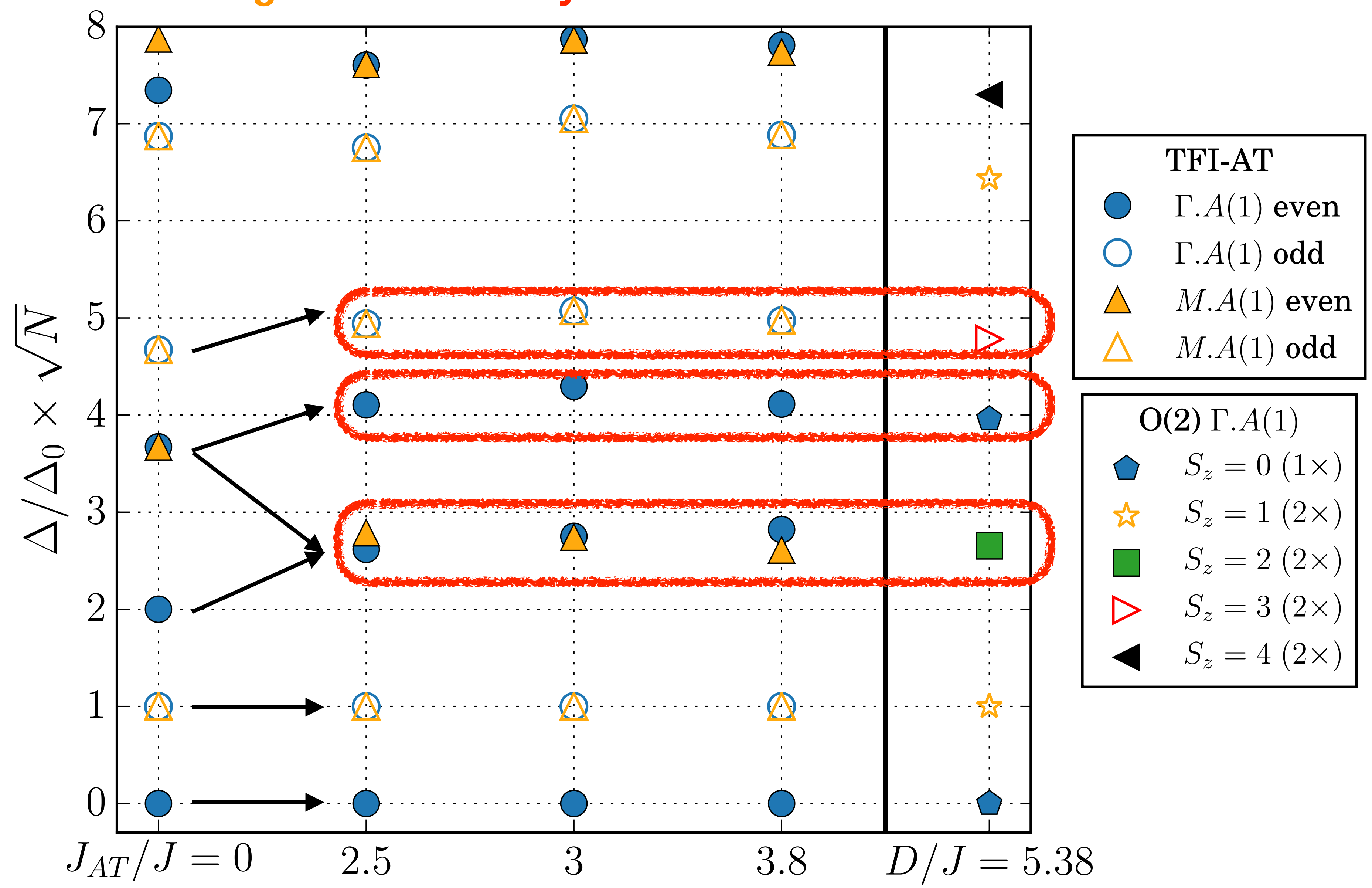


- Phase structure in agreement with QFT results of N_c=2 ϕ^4 theory with cubic anisotropy.

Spectroscopy of QCP

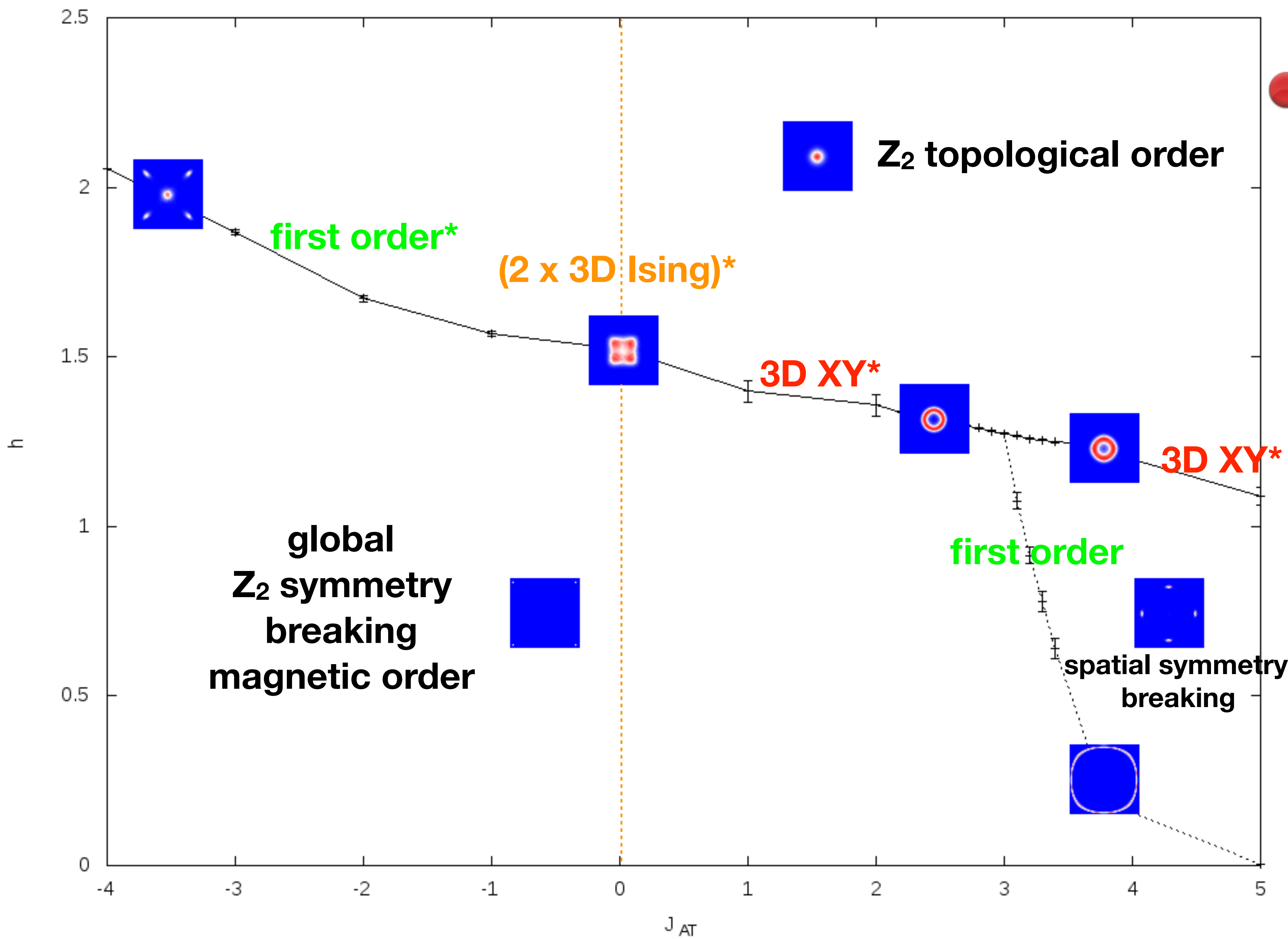


- ED Torus Spectra in the Quantum Ashkin-Teller model at criticality:



Phase diagram of the Toric Code + Ising interactions

- Translate the Ashkin-Teller results back to the Toric code + Ising:



- The direct transition between Z_2 topological order and Z_2 symmetry breaking can be:
  - first order
  - (2 x 3D Ising)* unstable fixed point
  - 3D XY* !

Outline of this talk

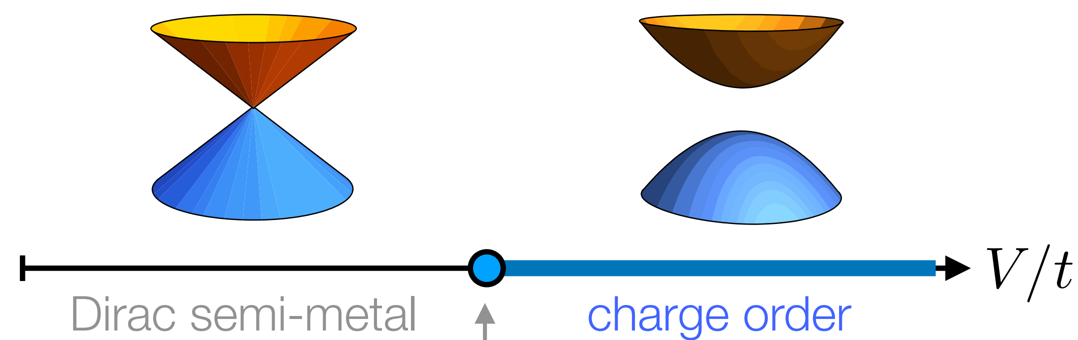
- Introduction: Quantum Many Body Systems / Spectroscopy
- Torus Energy Spectra and Quantum Critical Points ?
- Spectrum of the standard 2+1D Ising transition
- Topological Phase Transitions
- Gross-Neveu-Yukawa fixed points

M. Schuler, S. Hesselmann, T.C. Lang, S. Wessel & AML
arXiv:1907.05373

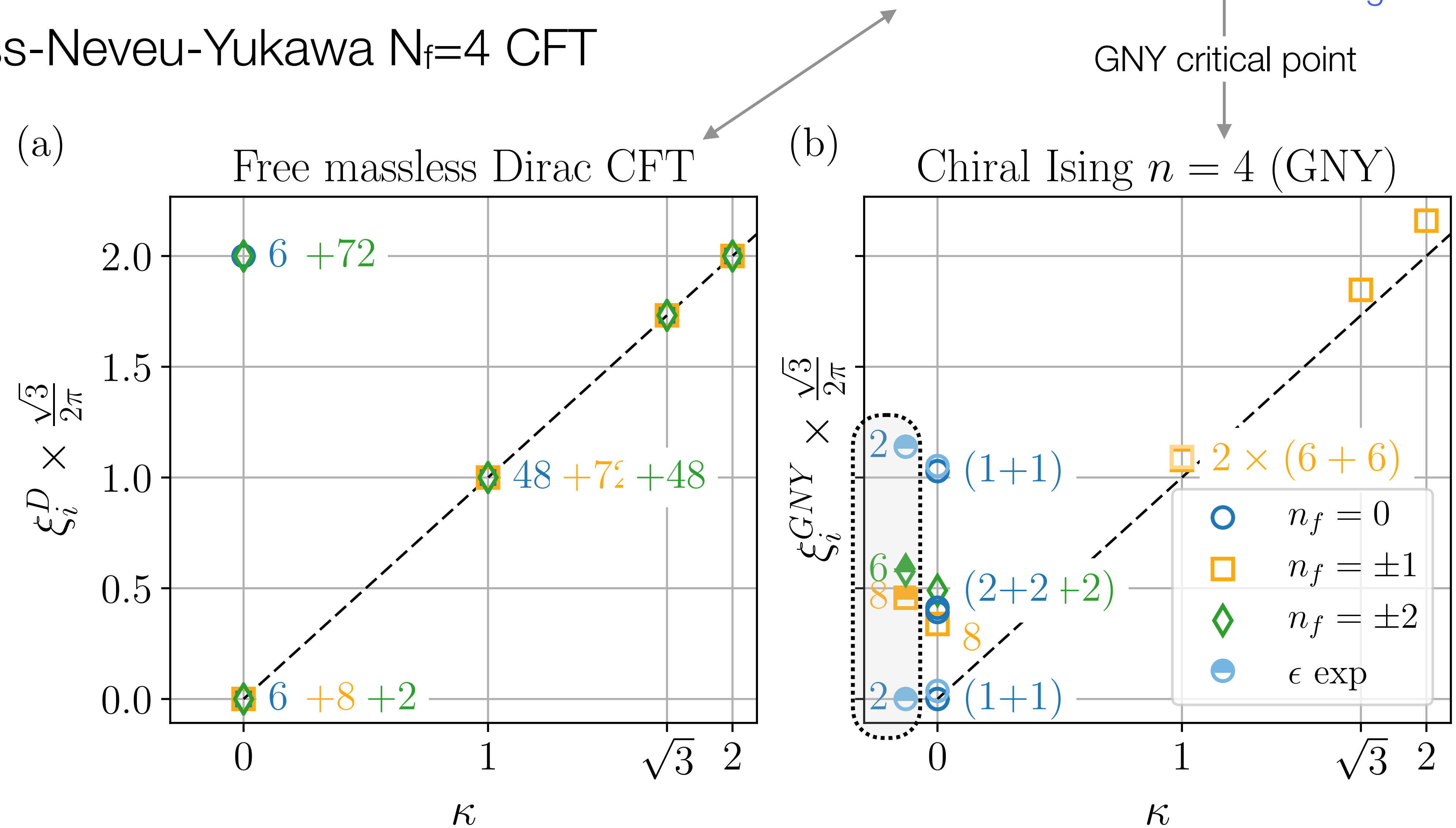
- Outlook

Gross-Neveu-Yukawa: Chiral Ising

- Spinless fermions on a honeycomb lattice:



- Gross-Neveu-Yukawa N_f=4 CFT



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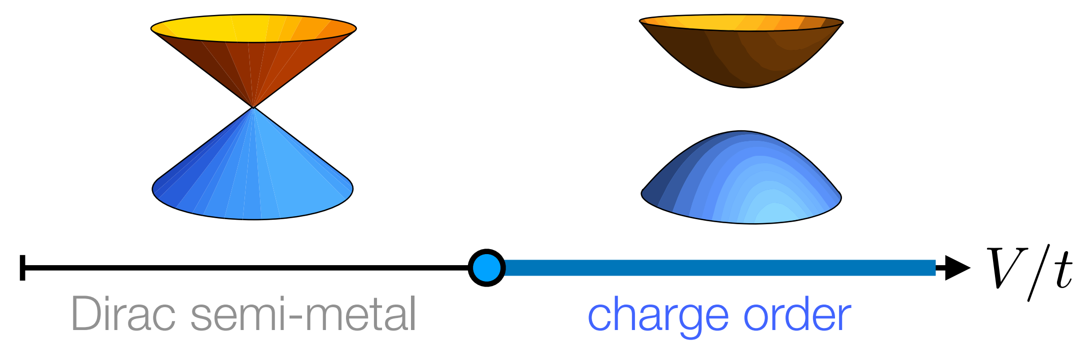
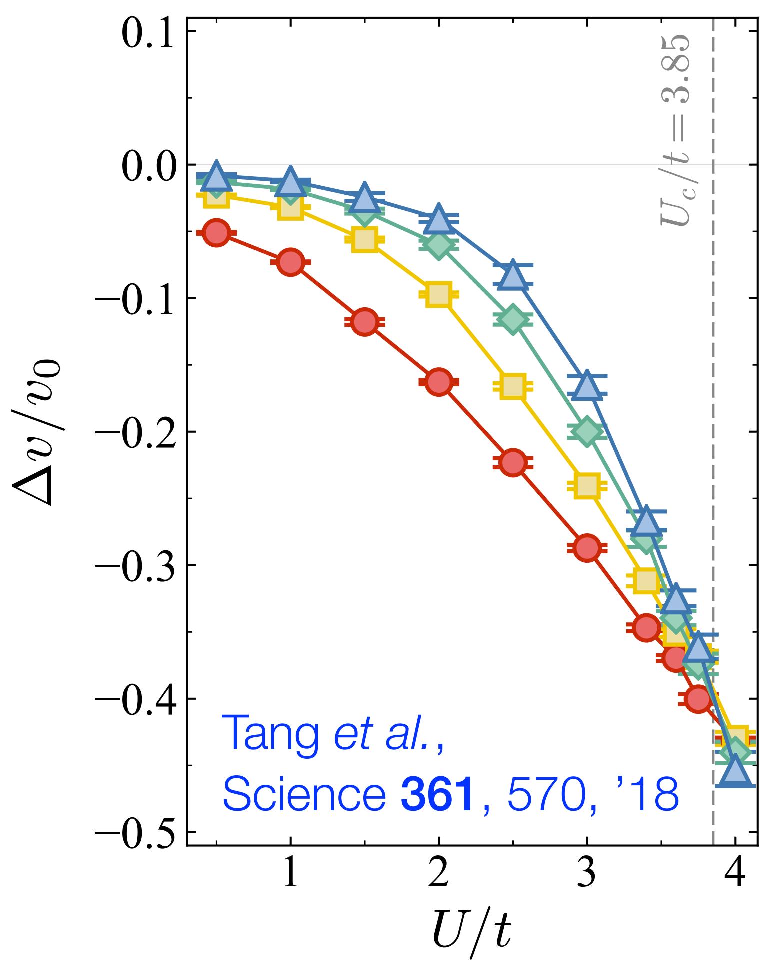
Gross-Neveu-Yukawa: Chiral Ising

- Can one infer the "speed of light" from energy spectrum ?



GNY critical point

$$v = \frac{E(\Delta k(L), L) - E(0, L)}{\Delta k(L)}$$



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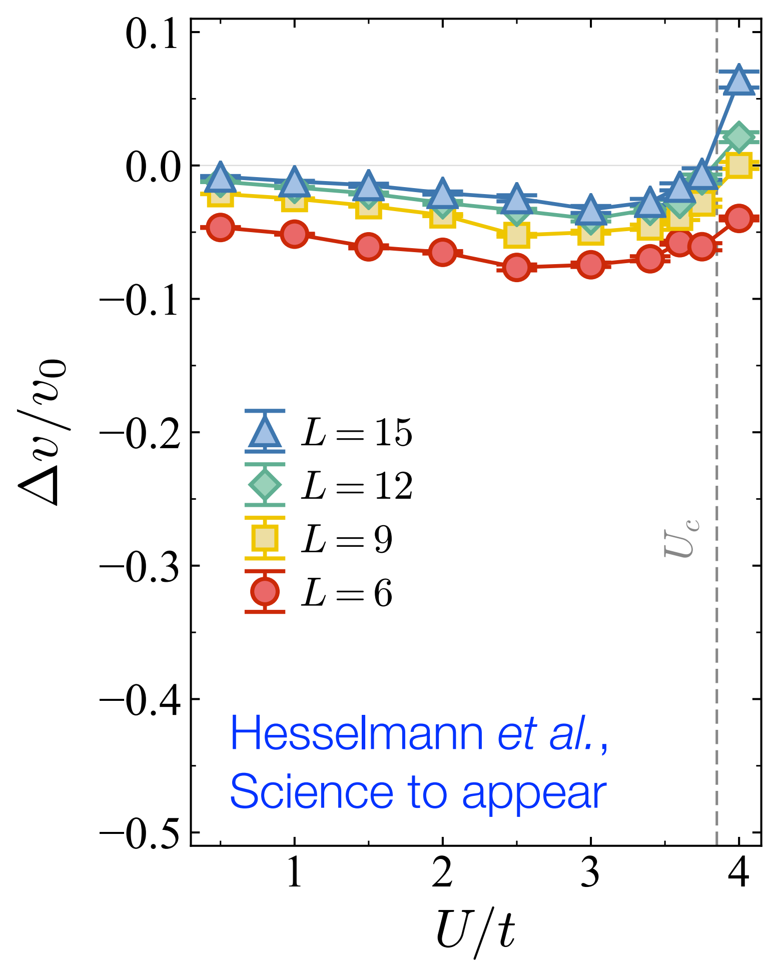

Gross-Neveu-Yukawa: Chiral Ising

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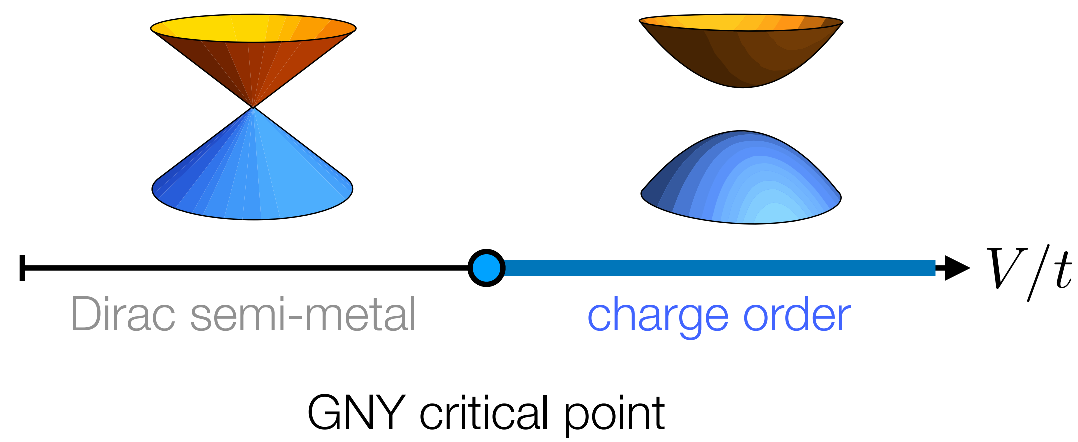

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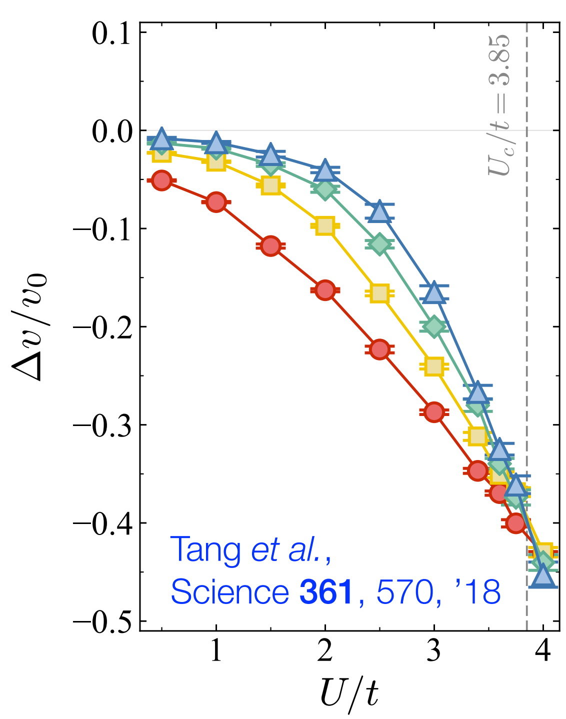
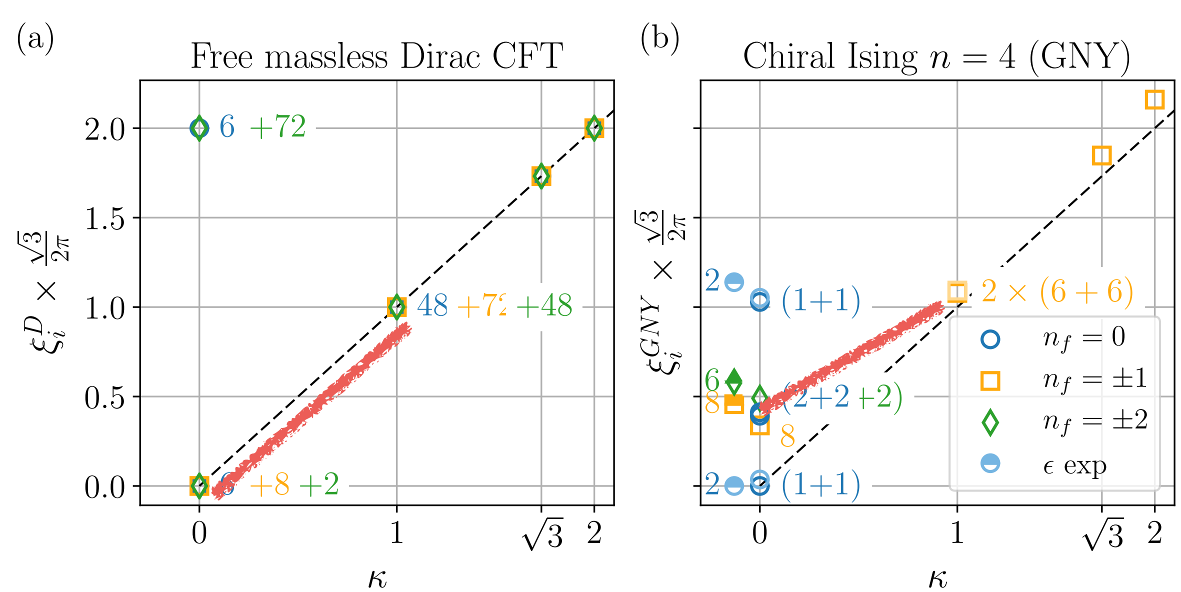
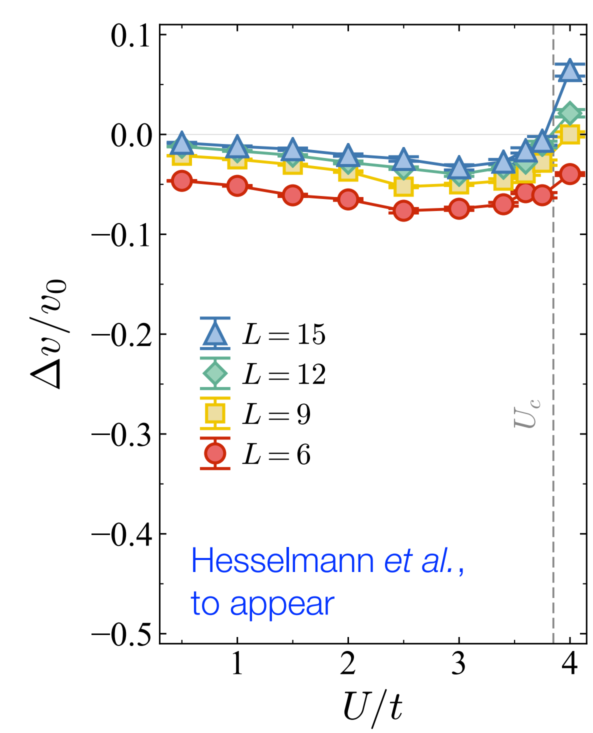




$$v = \frac{E(\Delta k(L), L)}{\Delta k(L)}$$



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Experimental Prospects ?

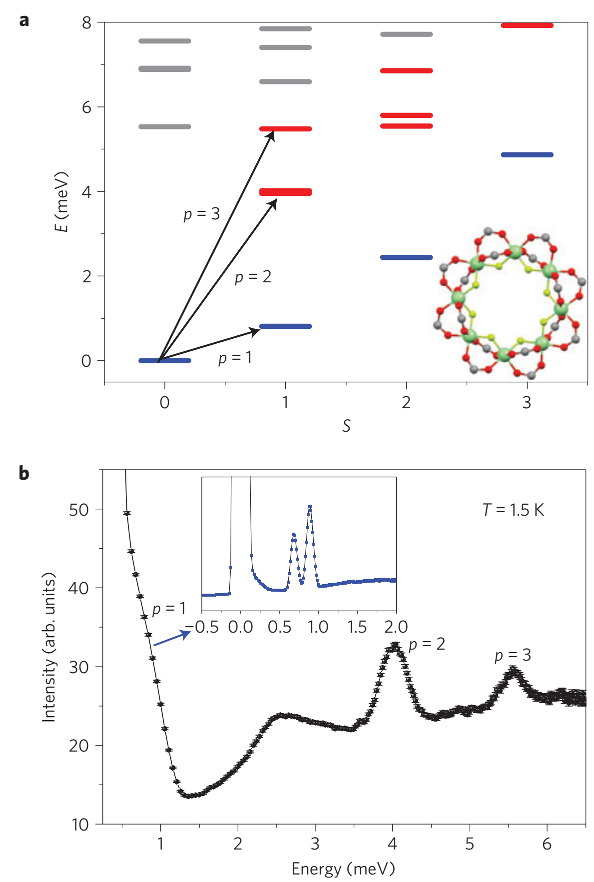
- In large, bulk materials, the many body energy spectrum is mostly extremely dense.
- In mesoscopic systems the finite energy spacing starts playing a role
- Our results show that the precise relative position of energy levels at the edge of the spectrum caries valuable information.
- Can one access some of this information using experimental probes ?

Experimental Prospects ?

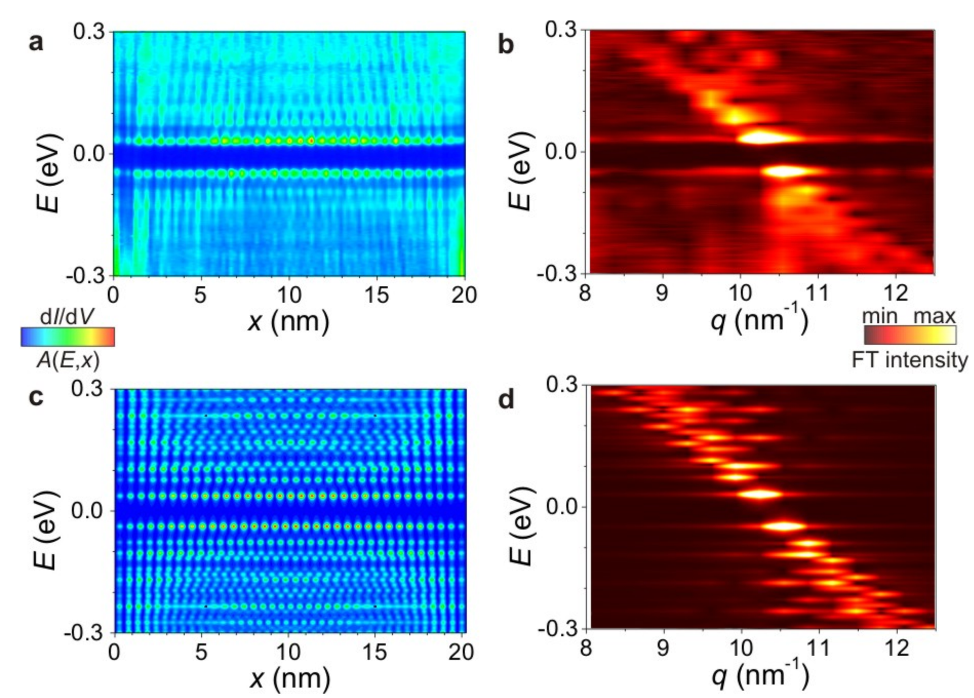
- Some of the energy levels can be seen in some inelastic scattering experiments on *mesoscopic* samples.

- in 1D, ring magnetic molecules provide a nice example of this approach.

- New STM experiments on interacting 1D metals reveal spin-charge separation based on real-space LDOS measurements.



M.L. Baker et al. Nat. Phys. 2012

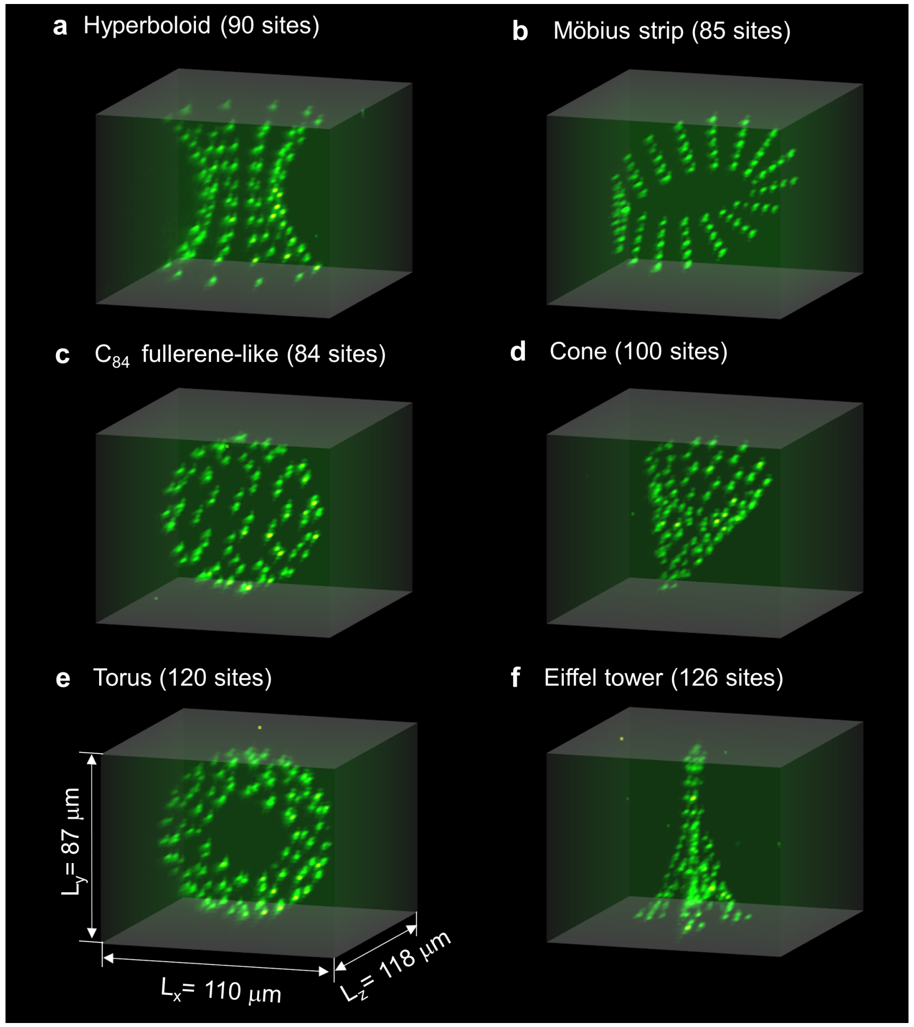


W. Jolie et al., Phys. Rev. X 2019

Experimental Prospects ?

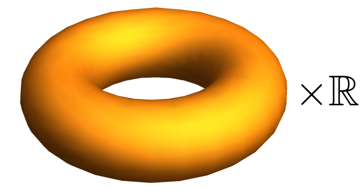
- In 2D tori are perhaps not readily available in condensed matter systems, but we plan to extend our analysis to systems with open boundaries, but the analysis might become involved.

- In synthetic (AMO) systems tori might become accessible for some Hamiltonians.

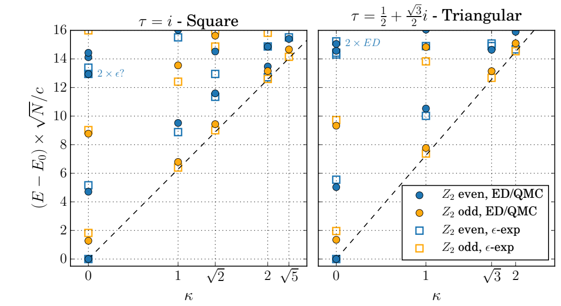


D. Barredo et al., Nature 2018

Conclusion / Outlook



$\mathbb{T}^2 \times \mathbb{R}$



- We have shown that the universal torus energy spectrum of the CFT describing quantum critical points is accessible numerically.

- The torus energy spectrum contains valuable information on the “operator content”. It is e.g. able to discriminate the Ising from the Ising* universality class, and 2 x Ising from 3D XY

- We have results for O(2)/O(3) Wilson-Fisher fixed points and for several Gross-Neveu-Yukawa critical points.

- Results from CFT side ?

- Spectra for QED_3, Fermi surface + U(1) gauge field ?

Collaborators

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Louis-Paul Henry
PostDoc ➤ Hamburg

- Harvard University

Seth Whitsitt
PhD Student
➤ JQI Maryland

Subir Sachdev

Thank you for your attention !