# Generalized ‘t Hooft anomalies



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SIFT 2019

BASED ON ARXIV: 1906.10315 AND 1805.12290, 1909.09027 WITH ERICH POPPITZ

# Outline



- Introduction.
- 1-form ‘t Hooft anomalies: construction and examples.
- BCF ‘t Hooft anomalies: construction and consequences.

# Introduction

# Why a ‘t Hooft anomaly?



- ‘t Hooft anomalies: study nonperturbative phenomena in strongly coupled QFT.
- They put sever constraints on the IR spectrum of asymptotically free theories.
- ‘t Hooft anomalies of $0-$form symmetries were known since the 80s.
- New generalized ‘t Hooft anomalies $\Rightarrow$

  new constraints on the spectrum of a theory.

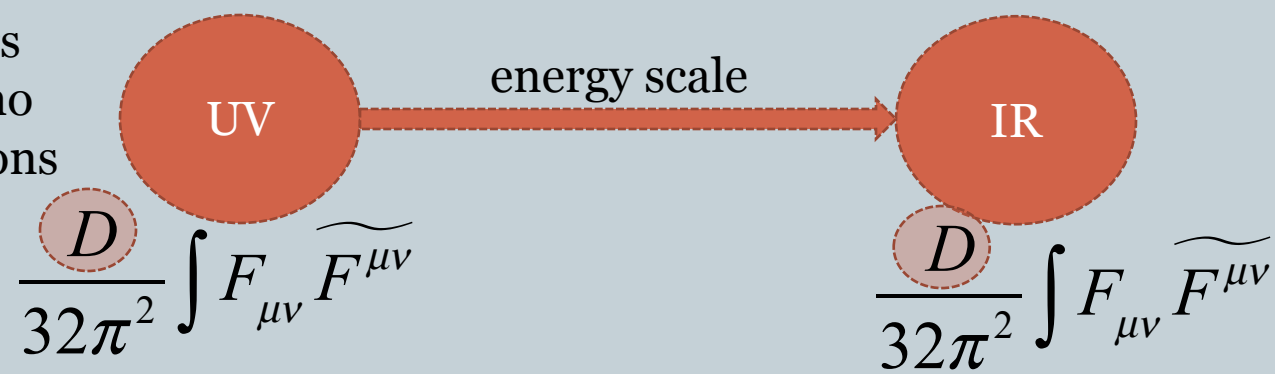
Gaiotto, Kapustin, Seiberg, Willett, 2014
Gaiotto, Kapustin, Komargodski, Seiberg, 2017

# What is a ‘t Hooft anomaly?



- Given a global symmetry $G$ of a QFT, we may try to gauge $G$. (turn on a **background gauge** of $G$ )
- **If obstructed** (anomalous), the theory has an **‘t Hooft anomaly**. ‘t Hooft, 1980 Frishman, Schwimmer, Banks, Yankielowicz 1981
- ‘t Hooft anomaly is RG invariant, useful for asymptotically free theories.

using spectators
or Stora-Zumino
descent equations



# What is a ‘t Hooft anomaly?



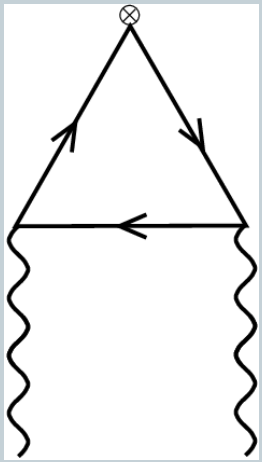
- **The anomaly is unremovable phase** in the partition function upon the transformation:

$$\chi \to U\chi, \quad U = e^{i\varepsilon^a T^a}$$

$$[D\chi][D\bar{\chi}] \to e^{-2i\varepsilon DQ^T}[D\chi][D\bar{\chi}] \qquad Q^T = \frac{1}{32\pi^2}\int F_{\mu\nu}\widetilde{F_{\mu\nu}}$$

G is a 0-form symmetry

fields of G



- In the t**raditional** $(0-\text{form})$ anomalies we take

$$Q^T \in Z$$

# What is a ‘t Hooft anomaly?



- The UV/IR matching of the anomaly: (1) CFT, (2) composite massless fermions, (3) Goldstone bosons. **Trivial gapped vacuum is excluded**.

- E.g. QCD with 3 fundamentals: $G = U(1)_B \times SU(3)_L \times SU(3)_R$

- Anomalies: $\left[SU(3)_L\right]^3 = N_c \quad , \quad U(1)_B\left[SU(3)_L\right]^2 = N_c$ .

- The matching is via Goldstone bosons:

$$SU(3)_L \times SU(3)_R \to SU(3)_V$$ Witten, 1983

# What is a ‘t Hooft anomaly?



- The option $Q^T \in Z$ is **not** the most general one.
- In 2014 it was realized that $Q^T \notin Z$ leads to new generalized anomalies.
- Generalized ‘t Hooft anomalies can play an important role. Gaiotto, Kapustin, Seiberg, Willett, 2014

# Part I: 1-form ‘t Hooft anomaly

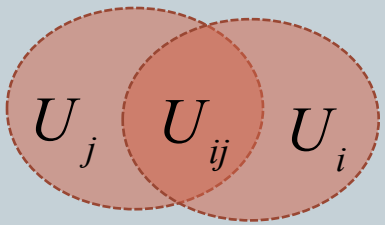
# 1-form ‘t Hooft anomaly



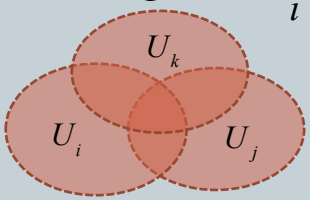
- We take a manifold $\mathtt{M}$ with fermion $\chi$ in Rep. $R$ of a group $G$ defined on a collection of covers $\{U_i\}$ of $\mathtt{M}$.

- We also take transition functions $g_{ij}^{R}$ of $G$ on the overlap $U_{ij} = U_i \cap U_j$ :

$$\chi_i = \left(g_{ij}^{R}\right)^{-1} \chi_j$$




- The cocycle condition $g_{ij}^{R} g_{jk}^{R} g_{ki}^{R} = 1$ on $U_i \cap U_j \cap U_k$ .

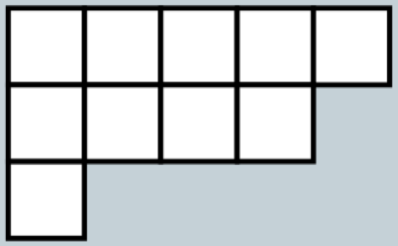


# 1-form ‘t Hooft anomaly



- If we take $G = SU(N)$ and $g_{ij}$ in the defining Rep.:

$$g_{ij}^{R} \sim \underbrace{g_{ij} g_{ij} \cdots g_{ij}}_{\text{number of boxes}=n}$$



number of boxes=$n$

- Only the Nality=$n$ mod $N$ matters.

- **Side note**: a similar construction on the lattice

$$\chi_x^* \left( U_{x,\mu} \right)^n \chi_{x+\mu}$$

# 1-form ‘t Hooft anomaly



- We consider the center elements:

$$Z_N : g_{ij} \sim e^{\frac{i2\pi k}{N}} I, \quad k = 0,1,..,N-1$$

- In Rep. $R$: $\quad g^R_{ij} \sim e^{\frac{i2\pi n}{N}k} I$.

- If $\gcd(N,n) = p > 1$, then the faithful Rep. is $SU(N)/Z_p$ and the cocycle condition:

$$g^R_{ij} g^R_{jk} g^R_{ki} = 1 \quad \Longrightarrow \quad g_{ij} g_{jk} g_{ki} = e^{i\frac{2\pi}{p} n_{ijk}}$$

# 1-form 't Hooft anomaly

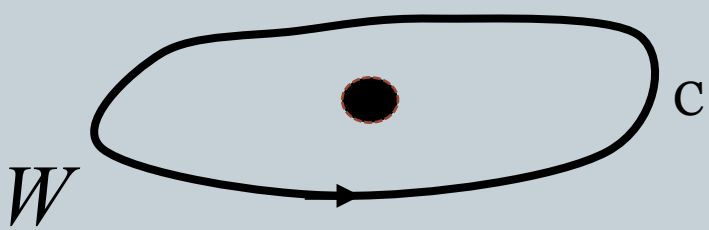


- E.g. faithful Rep. of $SU(N)$ with adjoint matter is $SU(N)/Z_N$.

- E.g. faithful Rep. of $SU(6)$ with matter in the 3-index antisymmetric is $SU(6)/Z_3$.

- $Z_p$ acts on non-contractible Wilson's loops (**1-form center symmetry**).

Gaiotto, Kapustin, Seiberg, Willett, 2014




$$W \xrightarrow{Z_p} e^{\frac{i2\pi k}{p}} W,\ k = 0,1,..,p-1$$

# 1-form ‘t Hooft anomaly



- Now, we turn on a background gauge field of $G$:

- Since the fermions are uncharged under $Z_p$, we can turn on fractional fluxes (‘t Hooft twists) that respect

$$g_{ij} g_{jk} g_{ki} = e^{i\frac{2\pi}{p} n_{ijk}}$$

‘t Hooft, 1981

- Take $\mathbb{M}$ a 4-torus with gauge fields in the 1-2 plane (similar expression in the 3-4 plane)

$$A_1 = \frac{2\pi m_{12}}{L^2} H.\nu x^2, \qquad A_2 = 0$$

# 1-form ‘t Hooft anomaly



- These gauge fields are non periodic. They are related via gauge transformations:

$$A_j = \left(g_{ij}\right)^{-1} A_i \left(g_{ij}\right) + \left(g_{ij}\right)^{-1} d\left(g_{ij}\right) \qquad g(x^1) = e^{\frac{i2\pi m_{12}}{L} H.\nu x^1}$$

- We have $H.\nu \in Z_N$, then we must choose $m_{12}$ so the fermions are uncharged under $Z_p$ and $g(0) = g(L)$.

- The minimum value of $m_{12}$ gives $Q^T{}_{\text{minimum}}$.

# 1-form ‘t Hooft anomaly



- If the theory fails to be invariant under a $0-$form global symmetry in the background of a $Z_p$ flux
  $\Rightarrow$ we have a $0-$form/1-form mixed anomaly.

- The 1-form anomaly can also be revealed by coupling the theory to TQFT and introducing 2-form field valued in $Z_N$ .

Seiberg, Kapustin 2014

# 1-form ‘t Hooft anomaly



- E.g. take $SU(N)$ QCD(adj) with $N_f$ Weyl fermions: global $SU(N_f)\times Z_{2NN_f}$ and 1-form $Z_N^{(1)}$ .
-

$$\chi \to U\chi,\ U = e^{i\frac{2\pi}{2NN_f}}$$

$$[D\chi][D\bar{\chi}] \to e^{-i2\pi Q_T}[D\chi][D\bar{\chi}] \qquad Q^T{}_{\text{minimum}} = \frac{1}{N}$$

- For $N_f = 1$ (SYM) we saturate the anomaly via $Z_{2N} \to Z_2$ (gaugino condensate).

Bergner, Lopez, Piemonte, 2018

Gaiotto, Kapustin, Seiberg, Willett, 2014
Gaiotto, Kapustin, Komargodski, Seiberg, 2017

# 1-form 't Hooft anomaly



- For $SU(N_c = 2)$ and $N_f = 2$ there are 3 scenarios:

M.A., Poppitz, 2018
Cordova, Dumetriscu, 2018
Bi, Senthil, 2018
Poppitz, Ryttov, 2019

(1) $SU(2) \times Z_8 \to SO(2) \times Z_2$

(2) $SU(2) \times Z_8 \to SU(2) \times Z_4 \otimes \mathrm{TQFT}$

(3) CFT

Currently under investigation by the lattice groups at Jena and MIT.

(4) (excluded) $SU(2) \times Z_8 \to SU(2) \times Z_8 \otimes \mathrm{TQFT}$

Proposed by: Bi, Senthil, 2018

Excluded by: Wan, Wang, 2018
Cordova, Ohmori, 2019

# 1-form ‘t Hooft anomaly



- In scenario (2) the $0-$form anomalies are matched via composite massless fermions~ $\chi\chi\chi$ transforming in fund. of $SU(2)$. M.A., Poppitz, 2018

- The breaking $Z_8 \to Z_4$ is probed via $\chi\chi\chi\chi$, while $\langle \chi\chi \rangle = 0$

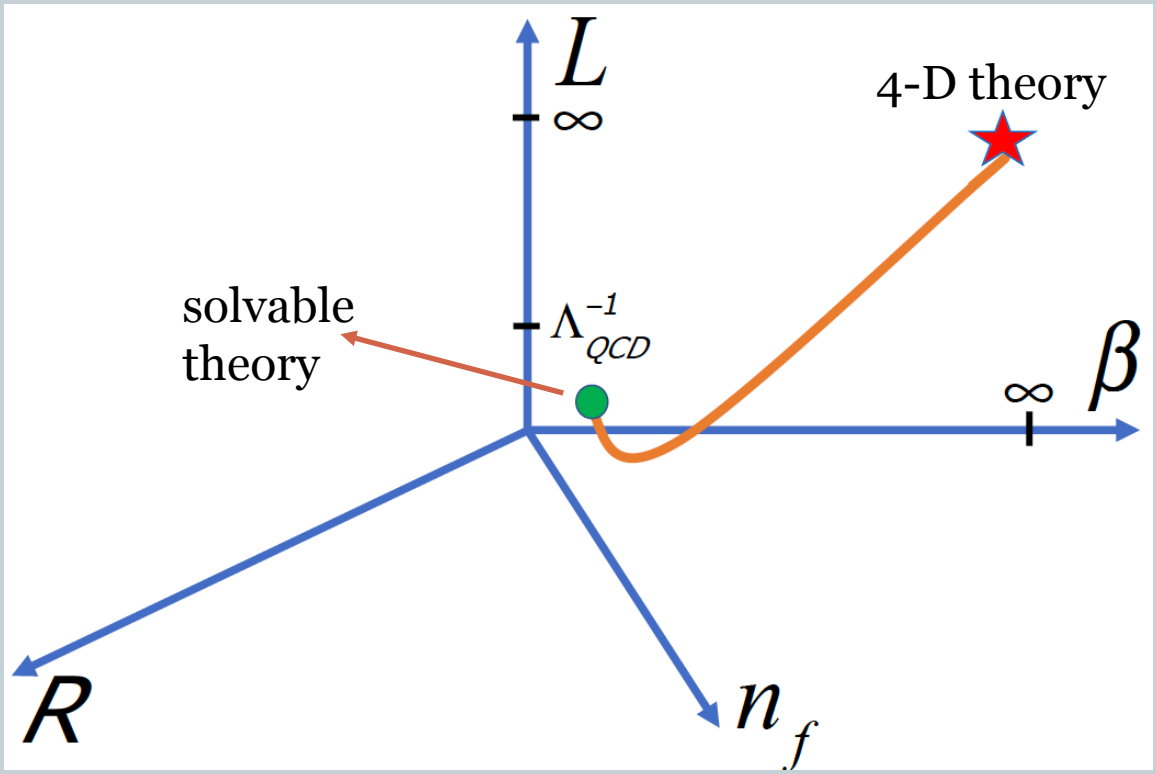
- This is adiabatically connected to the semiclassical theory on $R^3 \times S^1_L$ (massless fundamental fermion).

# 1-form 't Hooft anomaly



- Scenario (2) would imply an adiabatic continuity:

$$SU(2)\times Z_8 \to SU(2)\times Z_4 \otimes \text{TQFT}$$




Active line of research over a decade. M. Ünsal, 2009

(See M. Ünsal's talk on quantum distillation)

# 1-form ‘t Hooft anomaly



- E.g. SU(6) with matter in the 3-index antisymmetric Rep. with global $G = SU(n_f) \times Z_{6n_f}$ and 1-form $Z_3^{(1)}$

Yamaguchi, 2018
M.A., 2019

- Vanishing fermion bilinear: $\chi\chi = 0$

Bolognesi, Konishi, Luzio, 2019

$$\chi \to U_{Z_{6n_f}} \chi, \; U_{Z_{6n_f}} = e^{i\frac{2\pi}{6n_f}}$$

$$[D\chi][D\overline{\chi}] \to e^{-2\pi i Q^T} [D\chi][D\overline{\chi}] \qquad Q^T = \frac{1}{3}$$

# 1-form ‘t Hooft anomaly



- No composite fermions are allowed.
- Take $n_f = 1$, the 1-form anomaly is matched via the breaking $Z_6 \to Z_2$ (3-degenerate ground states).
- A CFT is unlikely since $\dim(R) = 20 < \dim(adj) = 35$ .
- This theory is connected to the semiclassical theory on $R^3 \times S^1_L$ (also admits 3 ground states). M.A., 2019

# Part II: BCF ‘t Hooft anomaly

# BCF 't Hooft anomaly



- The theory might not have a center symmetry. Yet, there can be non-trivial new anomalies.

  M.A., Poppitz, 2019. See also: Shimizu and K. Yonekura, 2017; Tanizaki, 2018.

- Consider a $SU(N_c)$ vector-like theory with $N_F$ fermions in Rep. of Nality $n^c$.

- The good global symmetry $G = \dfrac{SU(N_F)_L \times SU(N_F)_R \times U(1)_B \times Z_{2\dim(R^f)T_R}}{Z_{N_f} \times Z_2}$

| | $SU(N_c)$ | $SU_L(N_f)$ | $SU_R(N_f)$ | $U_B(1)$ | $\mathbb{Z}_{2\dim(\mathcal{R}^f)T_{\mathcal{R}^c}}$ |
|---|---|---|---|---|---|
| $\chi$ | $\mathcal{R}^c$ | $\mathcal{R}^f$ | 1 | 1 | 1 |
| $\tilde{\chi}$ | $\overline{\mathcal{R}^c}$ | 1 | $\overline{\mathcal{R}^f}$ | $-1$ | 1 |

# BCF ‘t Hooft anomaly



- Thinking about transition functions:

$$\chi_i = \left(g_{ij}{}^{R^c}\right)^{-1}\left(g_{ij}{}^{R^f}\right)^{-1}\left(g_{ij}{}^{U(1)_B}\right)^{-1}\chi_j$$

- Cocycle conditions:

$$g^c_{ij}g^c_{jk}g^c_{ki} = e^{i\frac{2\pi}{N_c}n^{(c)}_{ijk}}, g^f_{ij}g^f_{jk}g^f_{ki} = e^{i\frac{2\pi}{N_f}n^{(f)}_{ijk}}, g_{ij}^{U(1)_B}g_{jk}^{U(1)_B}g_{ki}^{U(1)_B} = e^{-in_c\frac{2\pi}{N_c}n^{(c)}_{ijk}-in_f\frac{2\pi}{N_f}n^{(f)}_{ijk}}$$

# BCF ‘t Hooft anomaly



- We turn on ‘t Hooft fluxes compatible with the transition functions:

$$A_1^c = \frac{2\pi m_{12}^c}{L^2} H^c . v^c x^2 , A_2^c = 0$$

$$A_1^f = \frac{2\pi m_{12}^f}{L^2} H^f . v^f x^2 , A_2^f = 0$$

$$A_1^B = \left( n_c \frac{2\pi m_{12}^c}{L^2 N_c} + n_f \frac{2\pi m_{12}^f}{L^2 N_f} \right) x^2 , A_2^B = 0$$

Topological charges

$$Q^c = m_{12}^c m_{34}^c \left( 1 - \frac{1}{N_c} \right), Q^f = m_{12}^f m_{34}^f \left( 1 - \frac{1}{N_f} \right)$$

$$Q^B = \left( n_c \frac{m_{12}^c}{N_c} + n_f \frac{m_{12}^f}{N_f} \right) \left( n_c \frac{m_{34}^c}{N_c} + n_f \frac{m_{34}^f}{N_f} \right)$$

# BCF 't Hooft anomaly



- We examine the theory under a $Z_{q_g}$ global rotation in the background of 't Hooft twists.

- We assume that the traditional 't Hooft anomalies are matched by a set of massless composite fermions.

$$Z^{UV}\Big|_{Z_{q_g}} = e^{i\frac{2\pi}{q_g}\left(D^c_{UV}Q^c + D^f_{UV}Q^f + D^B_{UV}Q^B\right)}$$

$$Z^{IR}\Big|_{Z_{q_g}} = e^{i\frac{2\pi}{q_g}\left(D^c_{IR}Q^c + D^f_{IR}Q^f + D^B_{IR}Q^B\right)}$$

$$D^{c,f,B}_{UV} - D^{c,f,B}_{IR} = q_g \ell^{c,f,B}$$

Anomaly matching:

$$\frac{Z^{UV}\Big|_{Z_{q_g}}}{Z^{IR}\Big|_{Z_{q_g}}} = e^{i2\pi\left(\ell^c Q^c + \ell^f Q^f + \ell^B Q^B\right)} = 1$$

# BCF 't Hooft anomaly



- Anomaly matching condition:

$$\ell^c Q^c + \ell^f Q^f + \ell^B Q^B \in Z$$

- For 't Hooft twists this translates into:

$$N_c \ell^c - \ell^B n_c^2 \in N_c^2 Z,\ \ N_f \ell^f - \ell^B n_c^2 \in N_f^2 Z,\ \ \ell^B \in \Omega \frac{N_c N_f}{n_c n_f} Z$$

M.A., Poppitz, 2019

- **No** solution exists if (for $N_f \geq 2$):

$(i)\ \gcd(N_c, N_f) > 1$, or

$(ii)\ \gcd(N_c, n_c) > 1$

(1)composites don't form,
(2)composites are accompanied by partial breaking of $Z_{q_g}$ and/or TQFT

# BCF ‘t Hooft anomaly



- Give the fermions a small $SU(N_f)$ symmetric mass: $m < \Lambda$
- Introduce $\theta$ angle.
- In the background of ‘t Hooft twists:

$$S = \theta Q^c + \Theta_f Q^f + \Theta_B Q^B$$

- Under $\theta \to \theta + 2\pi r$ we require

$$\Delta S = 2\pi r Q^c + 2\pi s Q^f + 2\pi t Q^B \in 2\pi Z$$

- Which has a solution iff $N_c r - t n_c^2 \in N_c^2 Z,\ N_f s - t n_f^2 \in N_f^2 Z,\ t \in \Omega \frac{N_c N_f}{n_c n_f} Z$

# BCF ‘t Hooft anomaly



- This is the same condition found for the massless composites.
- Information about the fermions contents is encoded in the phase of the mass term.
- The anomaly implies the theory cannot be trivial as we vary $\theta$ : domain walls.
- We expect anomaly inflow on the DW (possible described by a TQFT).

# Future directions



- BCF anomalies in chiral gauge theories.
- The nature of the TQFT on the DW.
- Lattice simulations are very desirable to confirm some of these predictions.
- Using the full set of anomalies to explore the semi-classical theory on $R^3 \times S^1_L$.
- Are there more anomalies?