

Crystalline chiral condensates in dense quark matter

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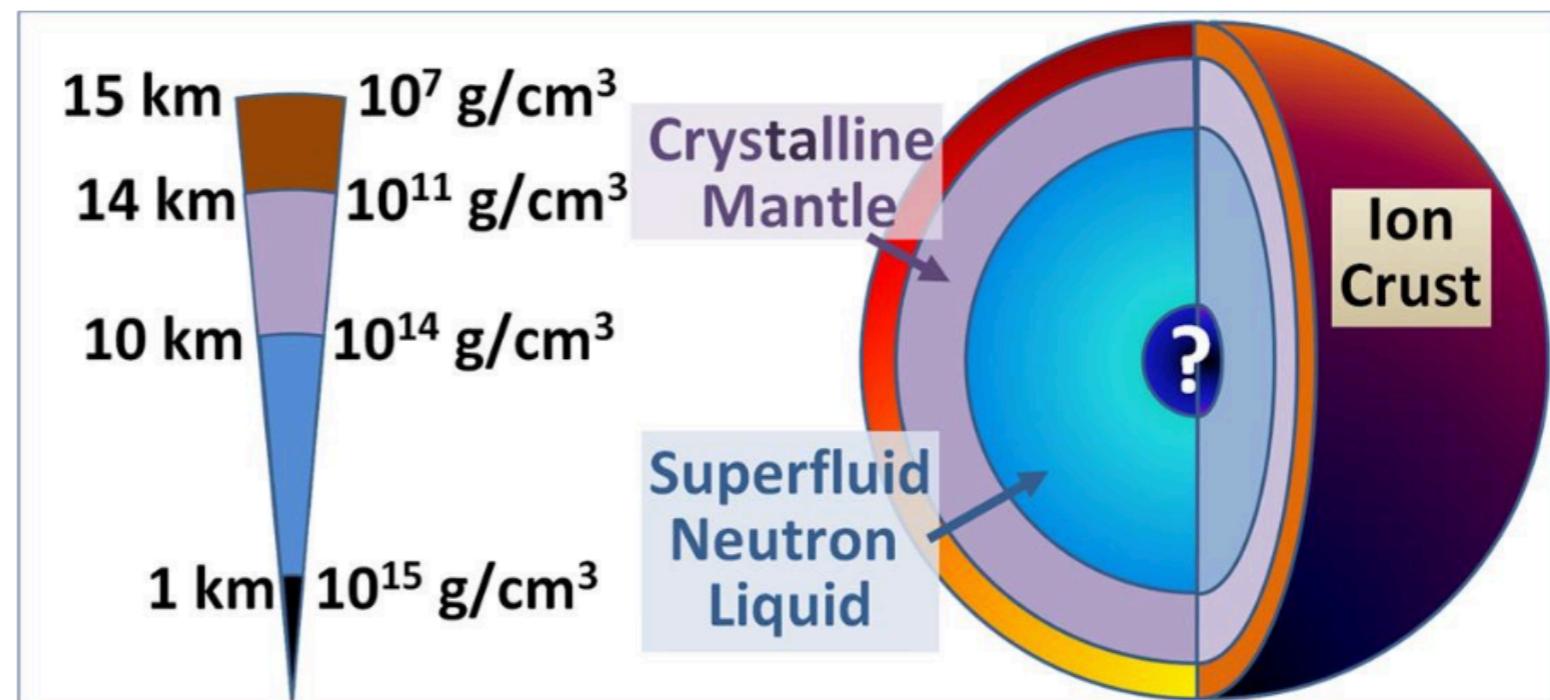


Today's menu

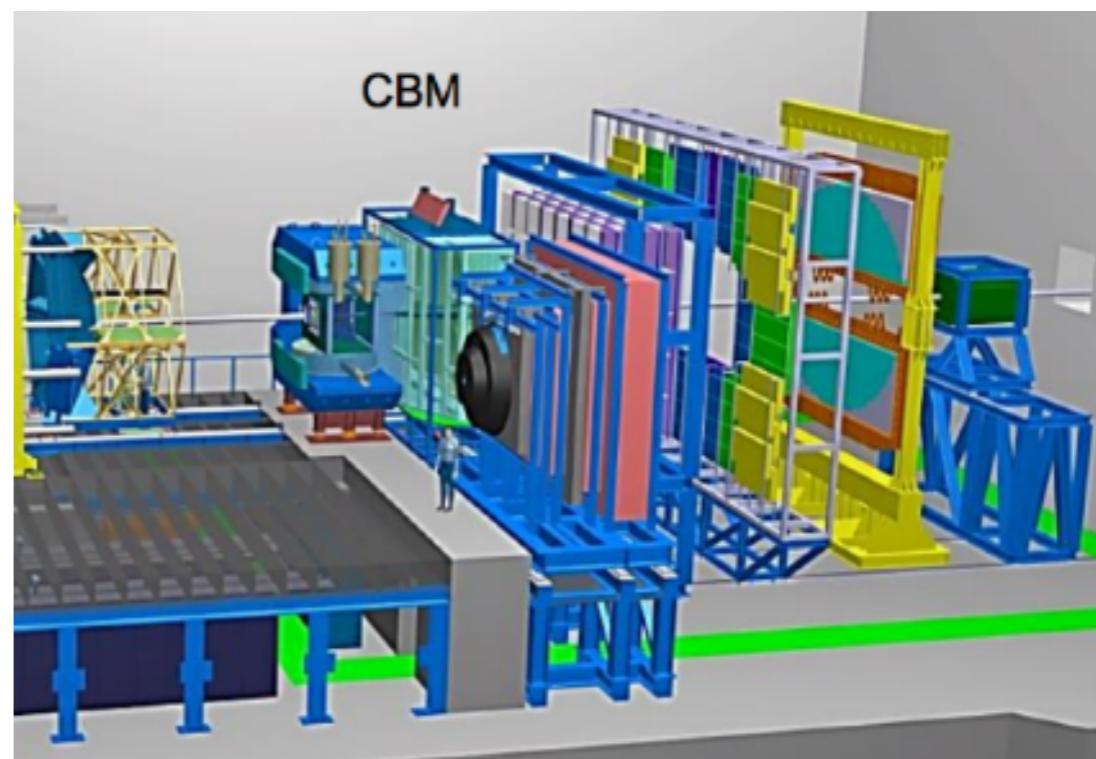
- Why crystalline chiral condensates?
- How to study them: model approaches
- Some results

Motivation: dense QCD

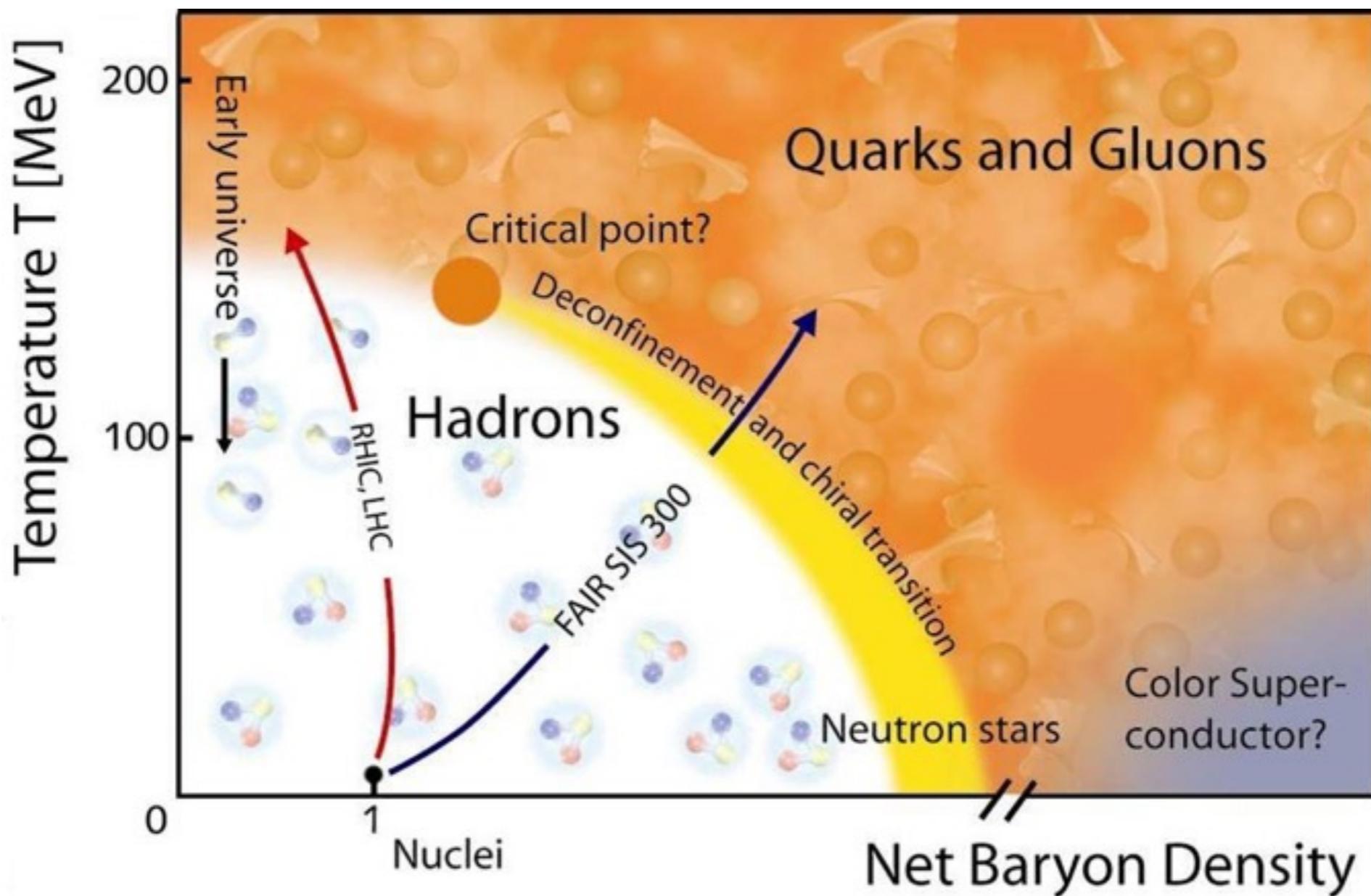
- Compact stars



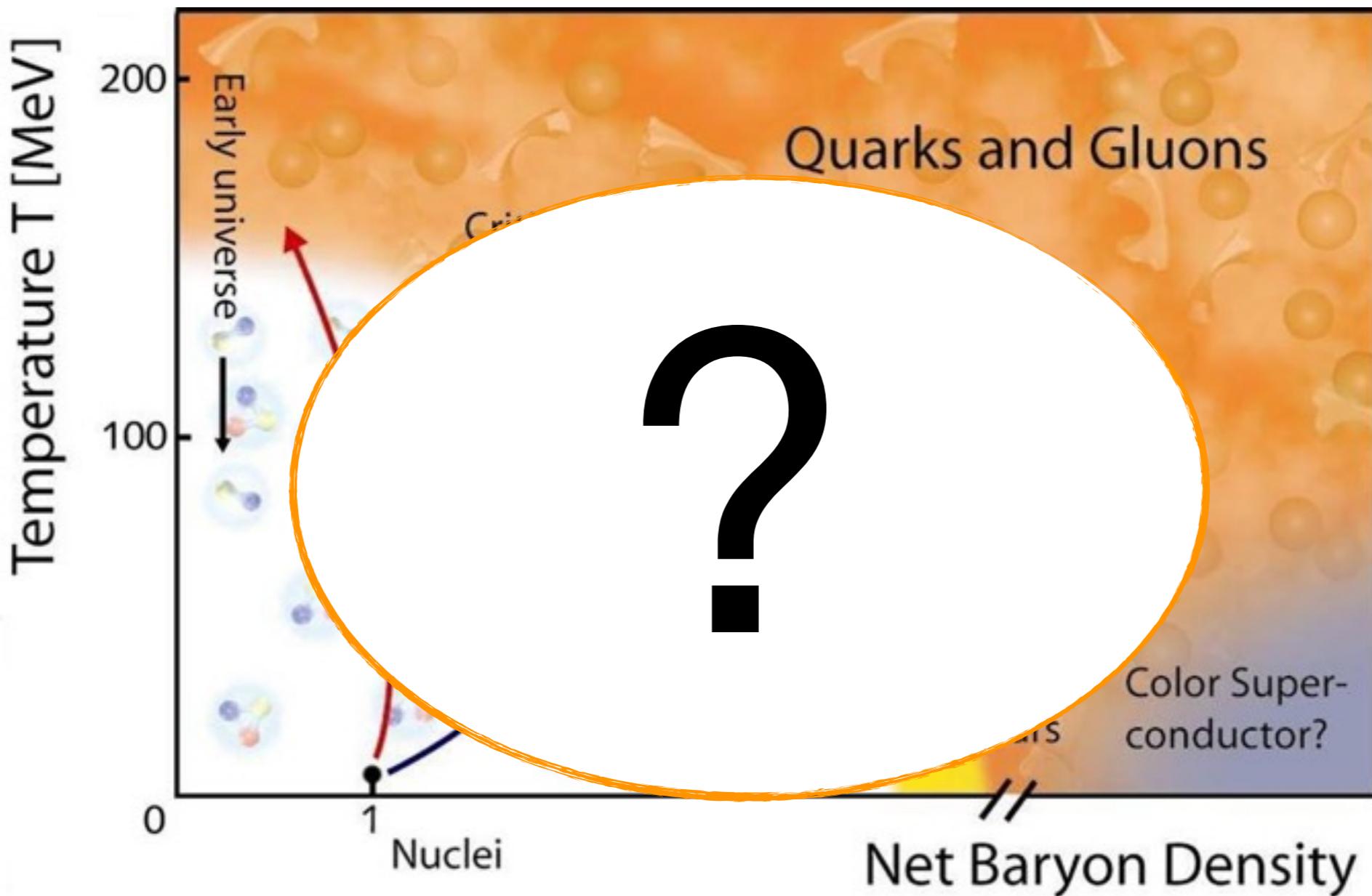
- Heavy ion collisions
(RHIC BES, FAIR, NICA..)
and the search for a Critical Point



The QCD phase diagram people have in mind

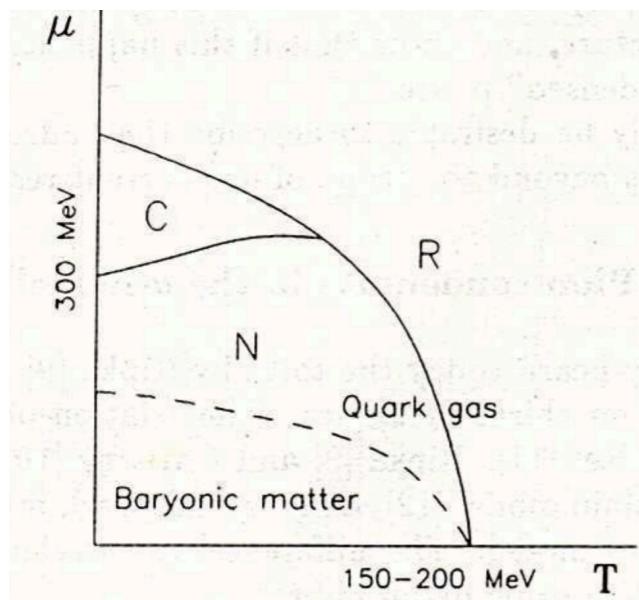


But if you actually think about it,
it's more like

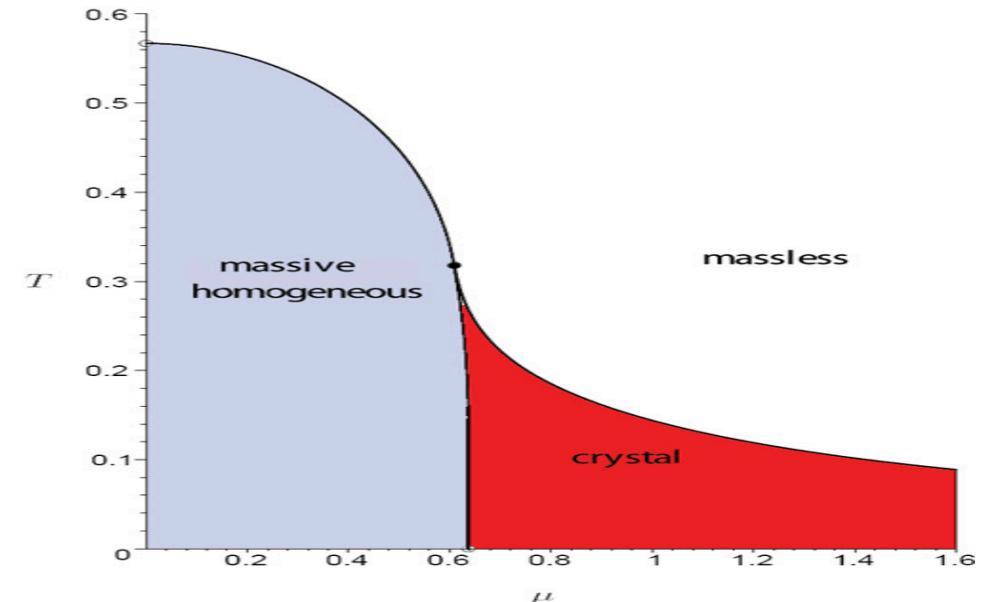


(some) inhomogeneous phases in strong interaction matter

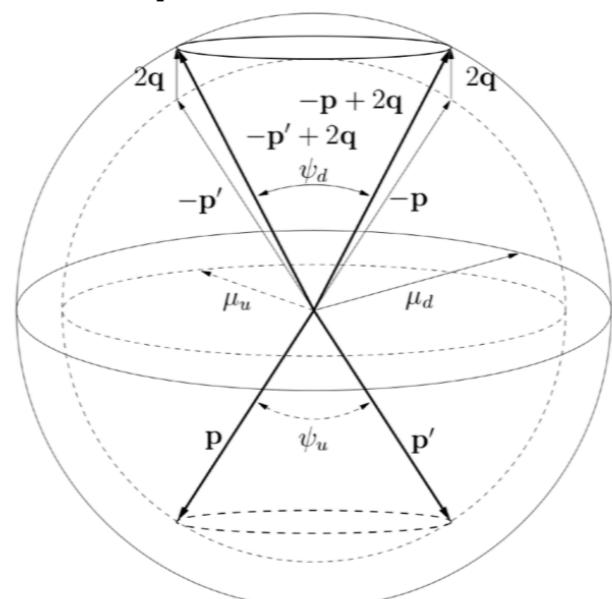
Pion condensation



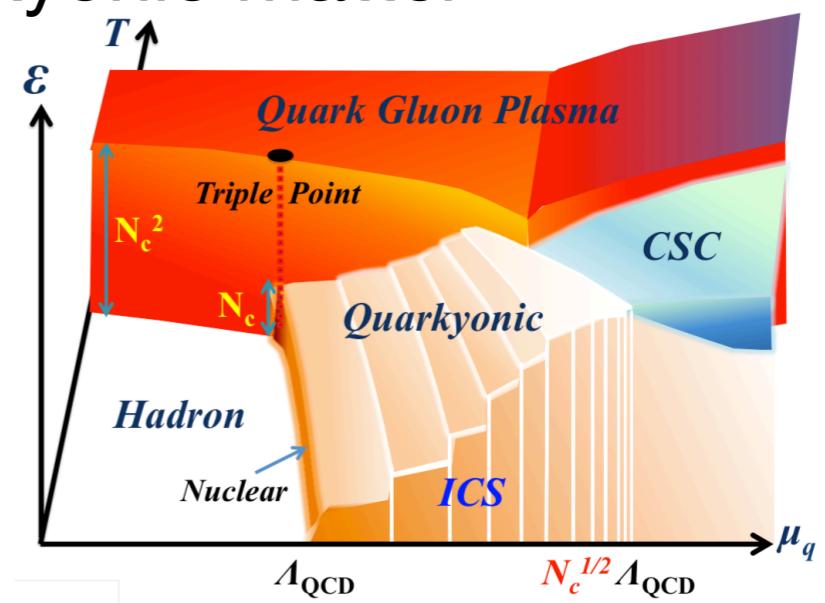
Toy models (Gross-Neveu)...



Color-superconductivity

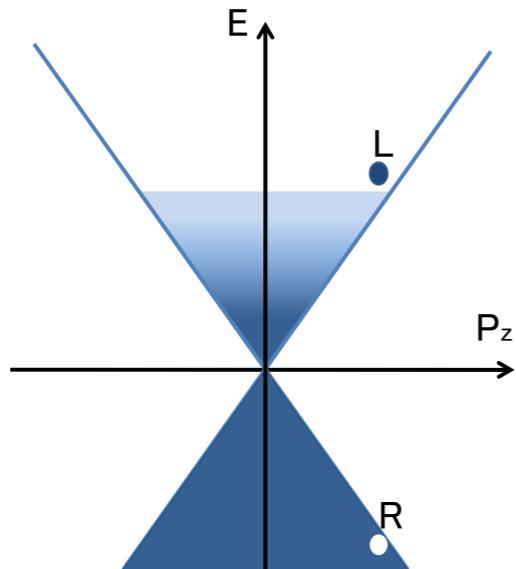


Quarkyonic matter



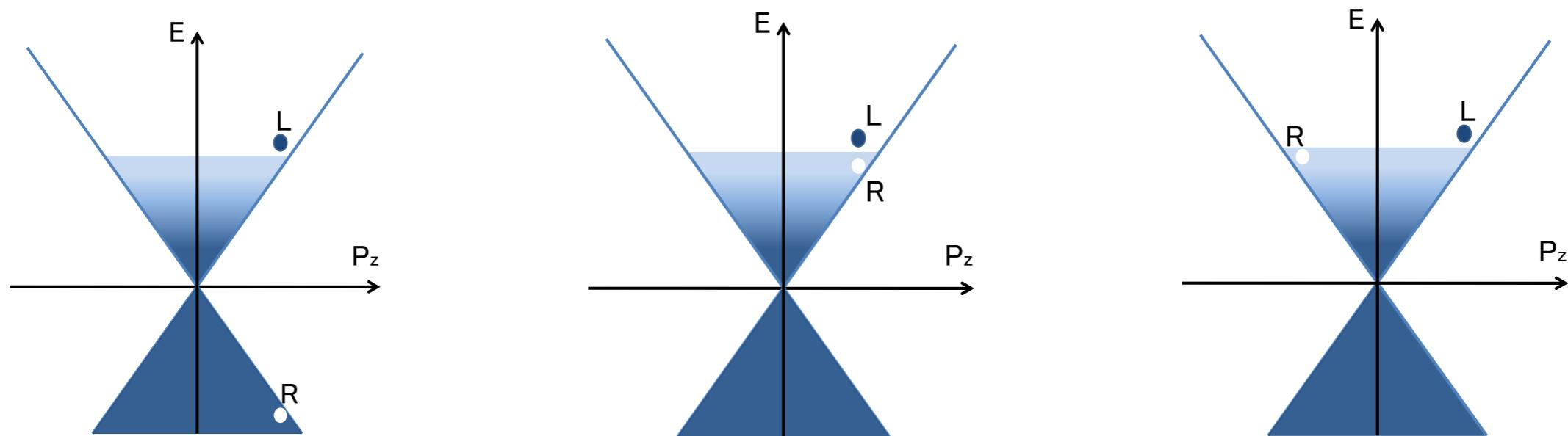
Inhomogeneous chiral condensates

Instead of the standard particle-antiparticle condensate...



Inhomogeneous chiral condensates

...particle-hole pairing at the Fermi surface



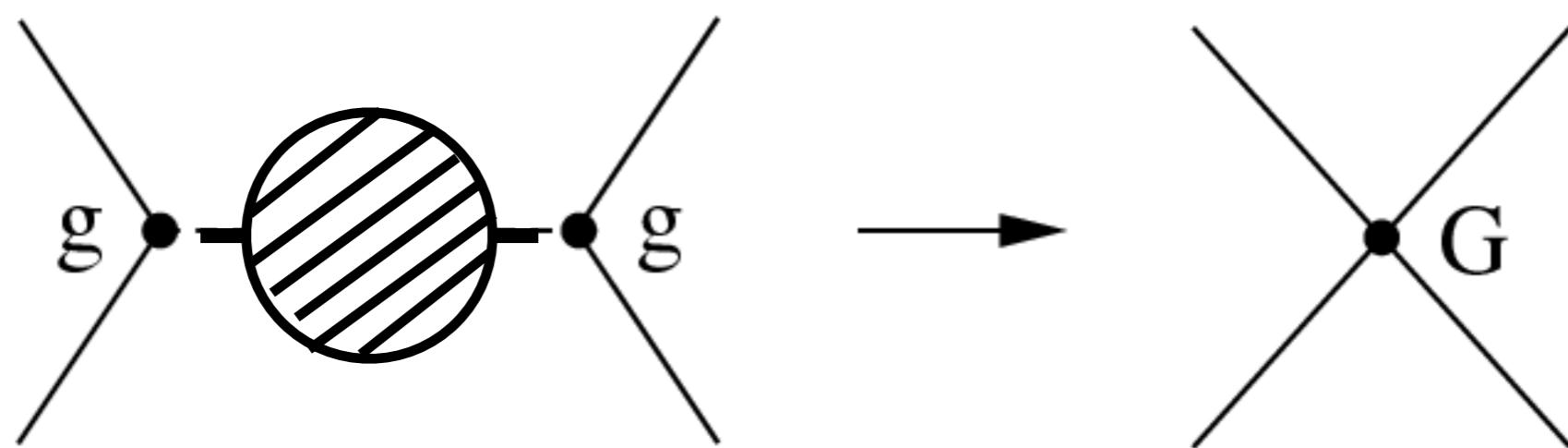
- Can occur at finite density: could be relevant at intermediate densities, close to the chiral phase transition

A concrete setup: NJL model

- Simple model with the relevant features for a (more or less realistic) description of dense QCD (symmetries, dynamical mass generation/XSB...)
- Non-renormalizable, simplifying assumptions to make things tractable (MFA..)
- A good starting point to investigate qualitative features and as input for more refined calculations

Nambu—Jona-Lasinio (NJL) model

Complicated quark-gluon interaction replaced by effective four-fermion vertex with fixed coupling constant G



Simplest version: 2 flavor, scalar-pseudoscalar interaction

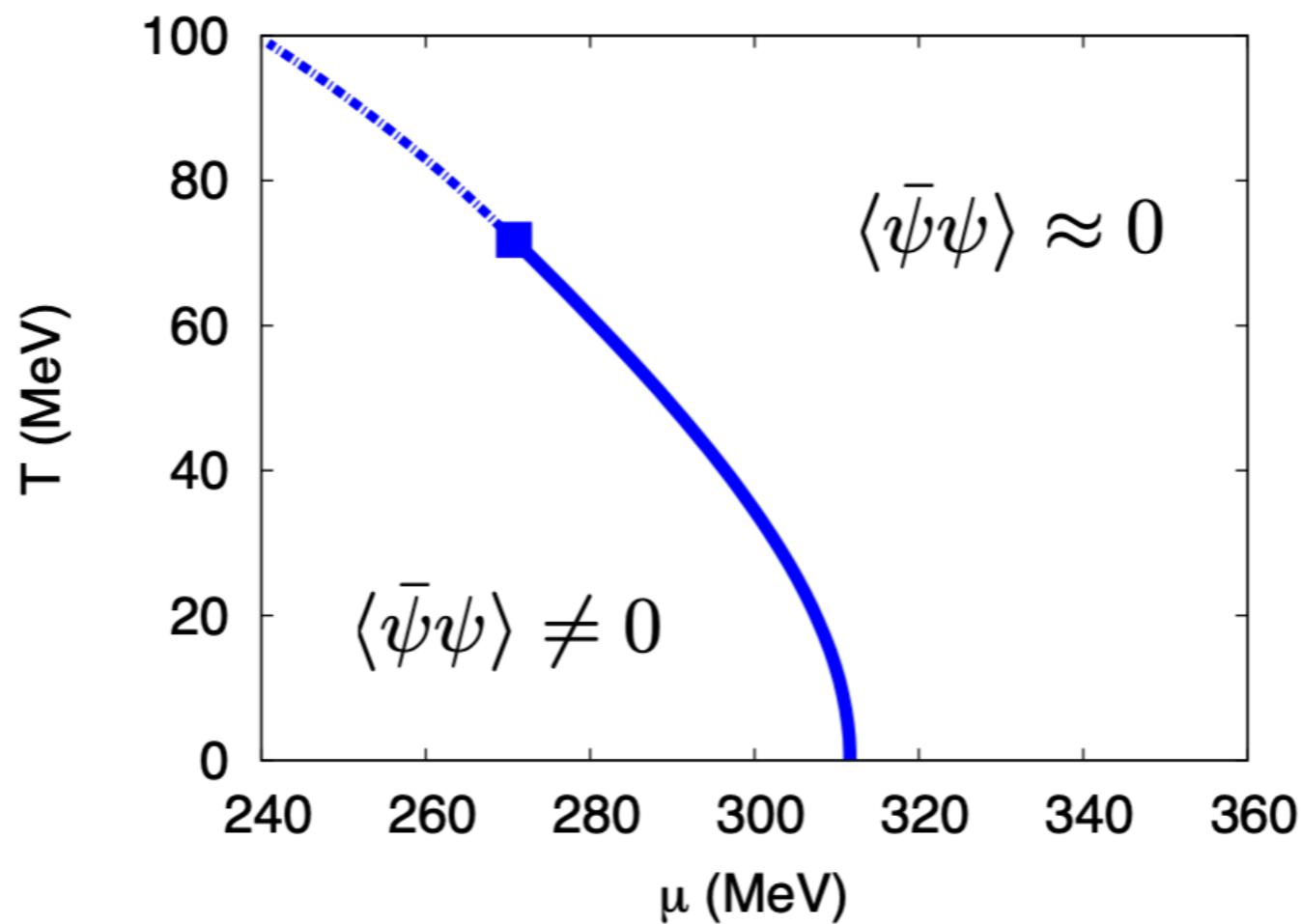
$$\mathcal{L}_{NJL} = \bar{\psi} i\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5 \tau^a \psi)^2]$$

Mean-field approximation

- Typical assumption: mean-field approximation $(\bar{\psi}\psi) \approx \langle \bar{\psi}\psi \rangle$
- A constant mean-field chiral condensate acts as constituent quark mass:
$$M_q = m - 2G\langle \bar{\psi}\psi \rangle$$
- Neglecting fluctuations, it is possible to obtain the free energy of the system as a trace over the inverse quark propagator:

$$\Omega \sim \frac{T}{V} \text{Tr} \log \left(\frac{S^{-1}(M_q)}{T} \right)$$

NJL phase diagram*



*Assuming spatially homogeneous condensates!!!

Inhomogeneous chiral condensates in NJL

- Allow for a spatially modulated chiral condensate

$$\langle \bar{\psi} \psi \rangle = S(\mathbf{x}) \quad \langle \bar{\psi} i\gamma^5 \tau_a \psi \rangle = P_a(\mathbf{x})$$

(we can also build $M(\mathbf{x}) = -2G(S(\mathbf{x}) + iP_3(\mathbf{x}))$) (Chiral limit)

- Diagonalize the mean-field quark Hamiltonian in momentum space

$$\mathcal{H}_{\vec{p}_m, \vec{p}_n} = \begin{pmatrix} -\vec{\sigma} \cdot \vec{p}_m \delta_{\vec{p}_m, \vec{p}_n} & \sum_{\vec{q}_k} M_{\vec{q}_k} \delta_{\vec{p}_m, \vec{p}_n + \vec{q}_k} \\ \sum_{\vec{q}_k} M_{\vec{q}_k}^* \delta_{\vec{p}_m, \vec{p}_n - \vec{q}_k} & \vec{\sigma} \cdot \vec{p}_m \delta_{\vec{p}_m, \vec{p}_n} \end{pmatrix}$$

Inhomogeneous chiral condensates in NJL

- Then, minimize the thermodynamic potential

$$\begin{aligned}\Omega(T, \mu; M(\vec{x})) &= -\frac{T}{V} \text{Log} \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp \left(\int_{x \in [0, \frac{1}{T}] \times V} (\mathcal{L}_{MF} + \mu \bar{\psi} \gamma^0 \psi) \right) \\ &= -\frac{TN_c}{V} \sum_n \text{Tr}_{D,f,V} \text{Log} \left(\frac{1}{T} (i\omega_n + \mathcal{H}_{MF} - \mu) \right) + \frac{1}{V} \int_V \frac{|M(\vec{x}) - m|^2}{4G_s}\end{aligned}$$

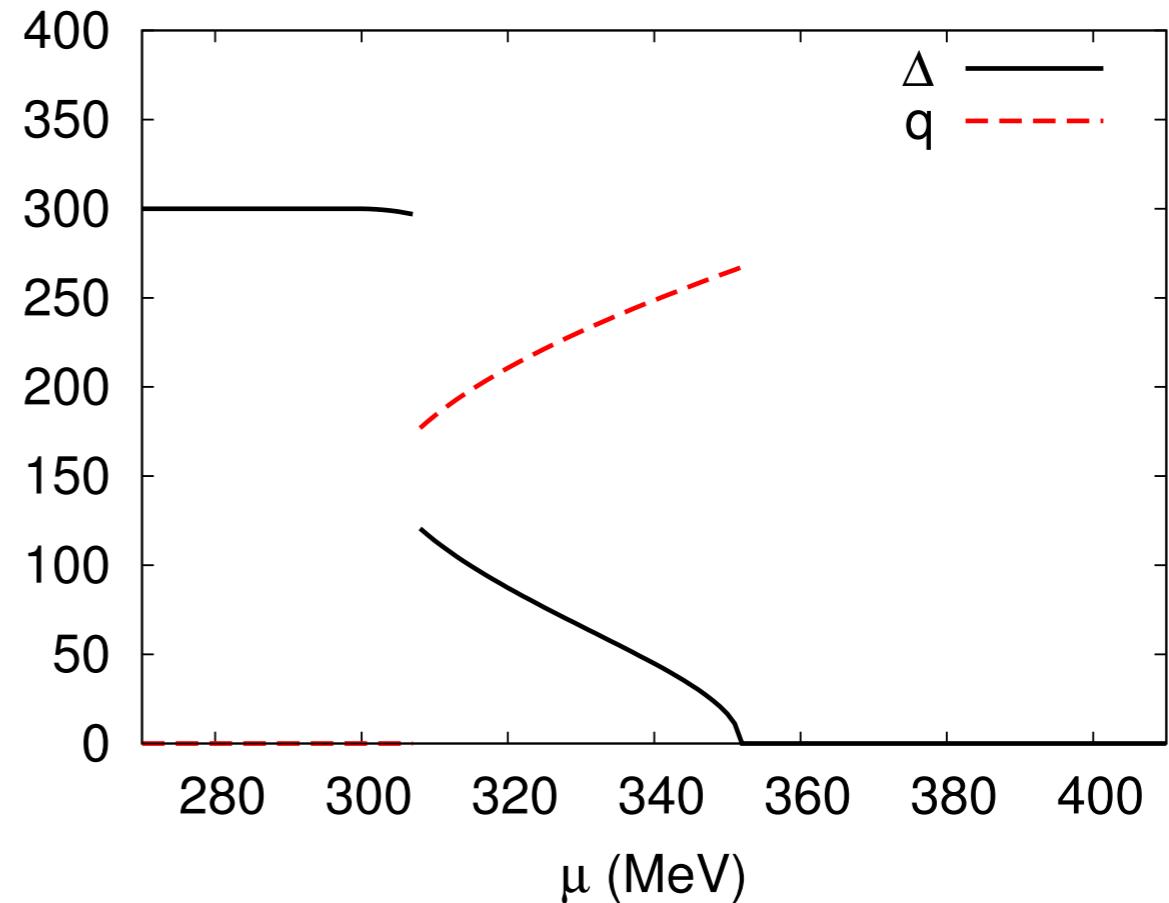
with respect to the mass function $M(x)$

- Not so easy for an arbitrary $M(x)$!
- To make the problem tractable, assume specific ansatz for the functional form of M

Chiral density wave

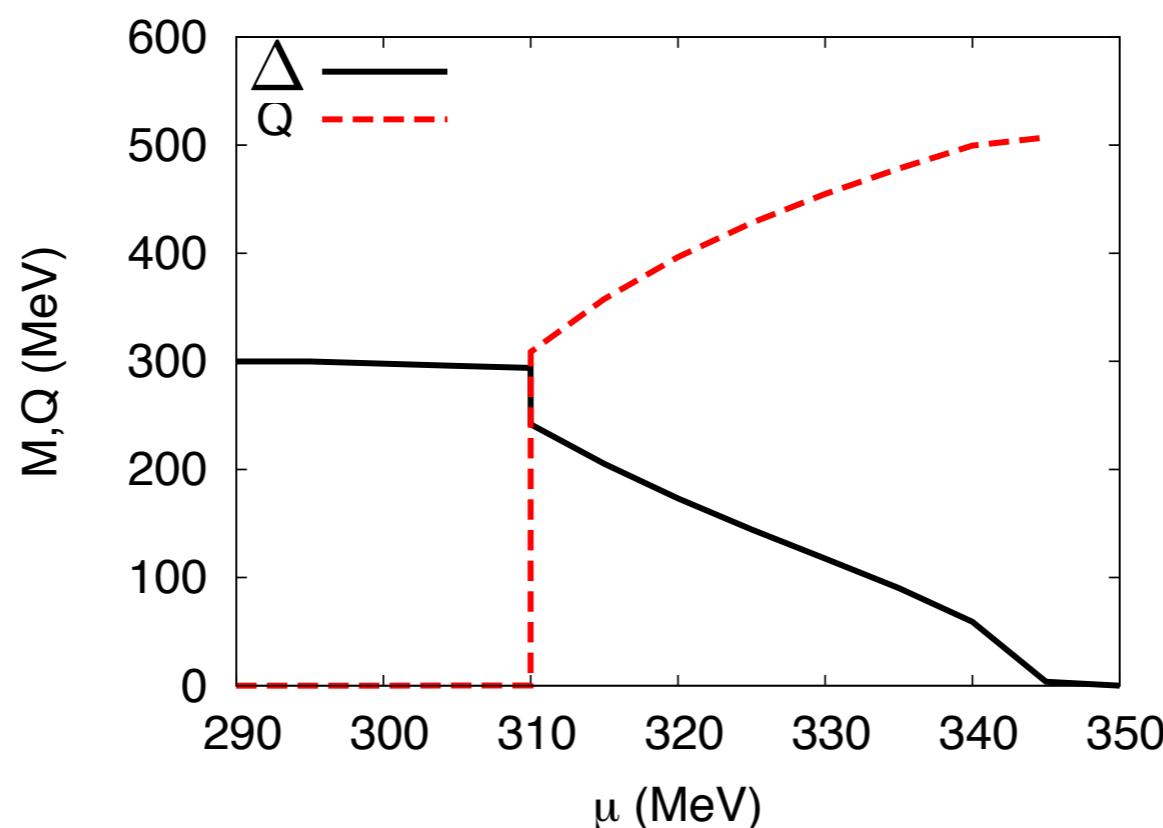
- Simplest ansatz: 1D plane wave (FF-type): $M(x) = \Delta e^{iqz}$
- Analytical expression known for the eigenvalue spectrum
- Order parameters: amplitude Δ , wave number q
- Minimizing free energy
varying chemical potential
($T=0$)

Special feature:
constant density!



LOFF-type modulations

- Second-simplest one: 1D cosine (“LOFF”)
$$M(\mathbf{x}) = \Delta \cos(qz)$$
- Numerical diagonalization in momentum space required:
Computationally intensive, but still doable on my laptop
- Qualitatively similar behavior to the CDW for the order parameters

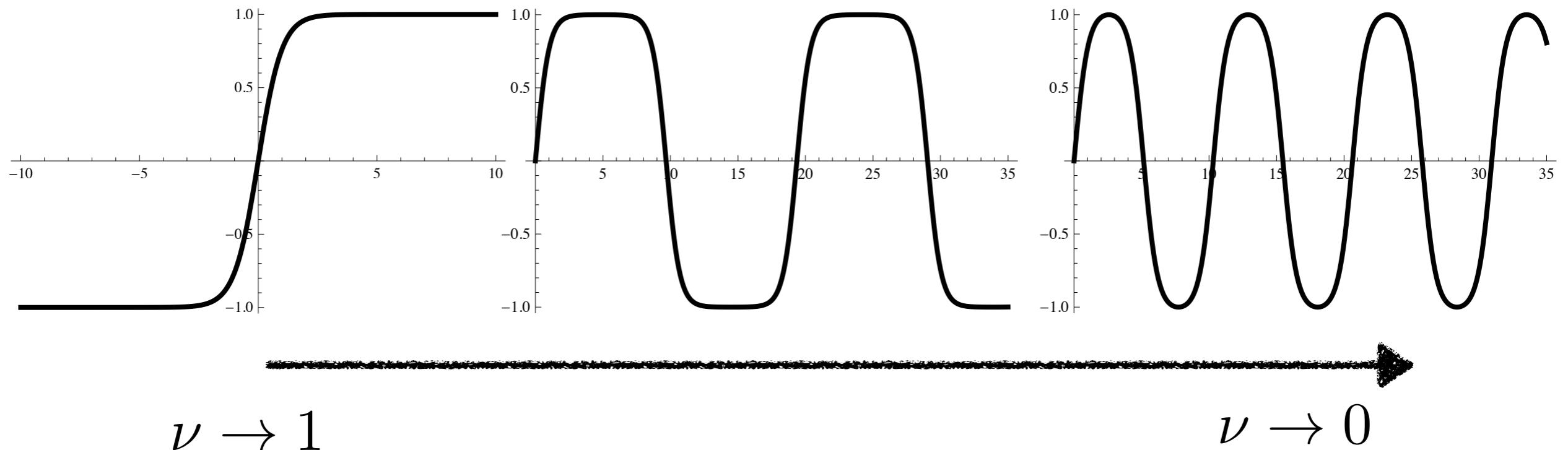


Real-kink crystal

- A more generic one-dimensional structure expressed in terms of Jacobi elliptic functions:

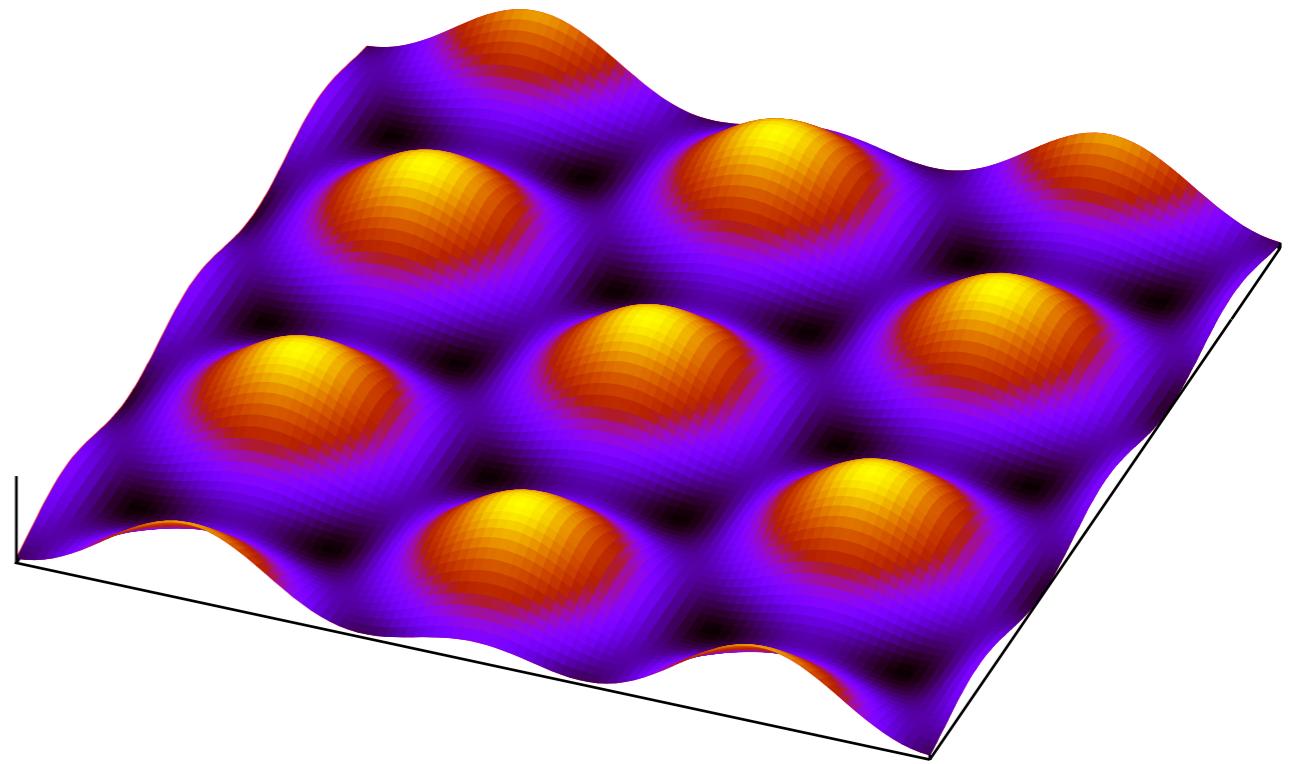
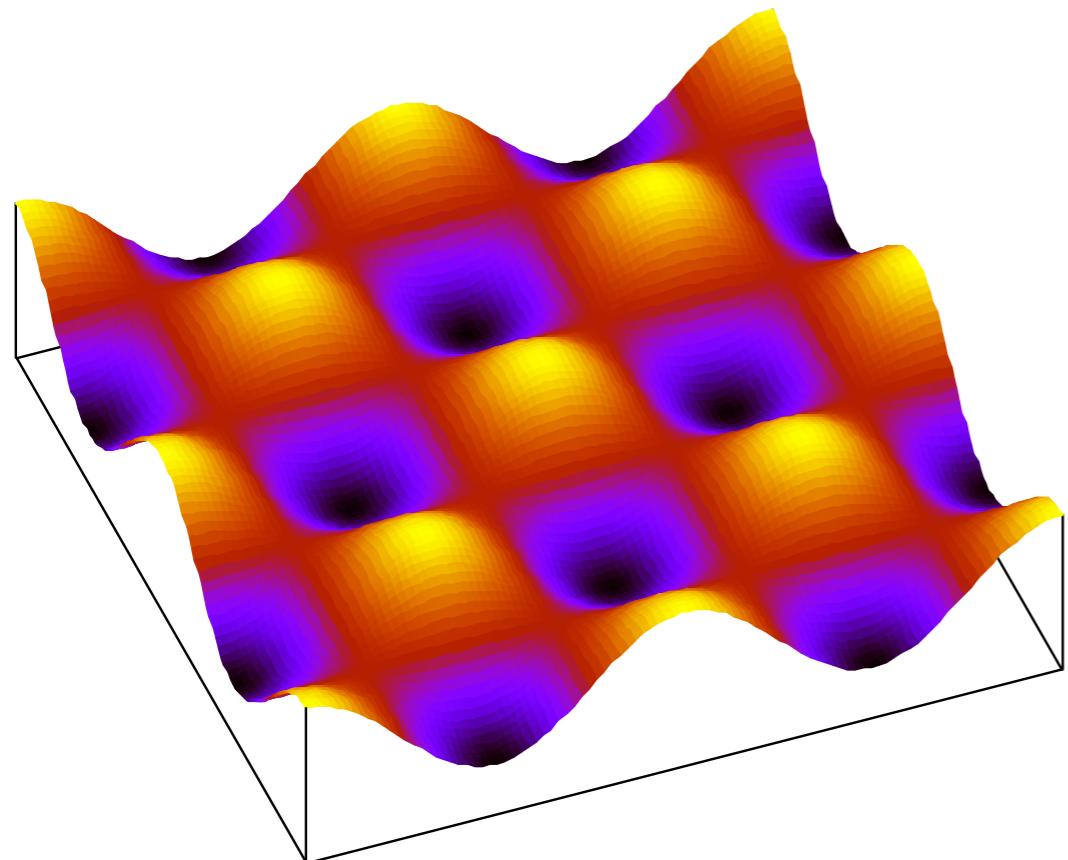
$$M(z) = \Delta \sqrt{\nu}(\Delta z, \nu)$$

- Parameters: Δ, ν



Two-dimensional modulations

- Different lattice structures



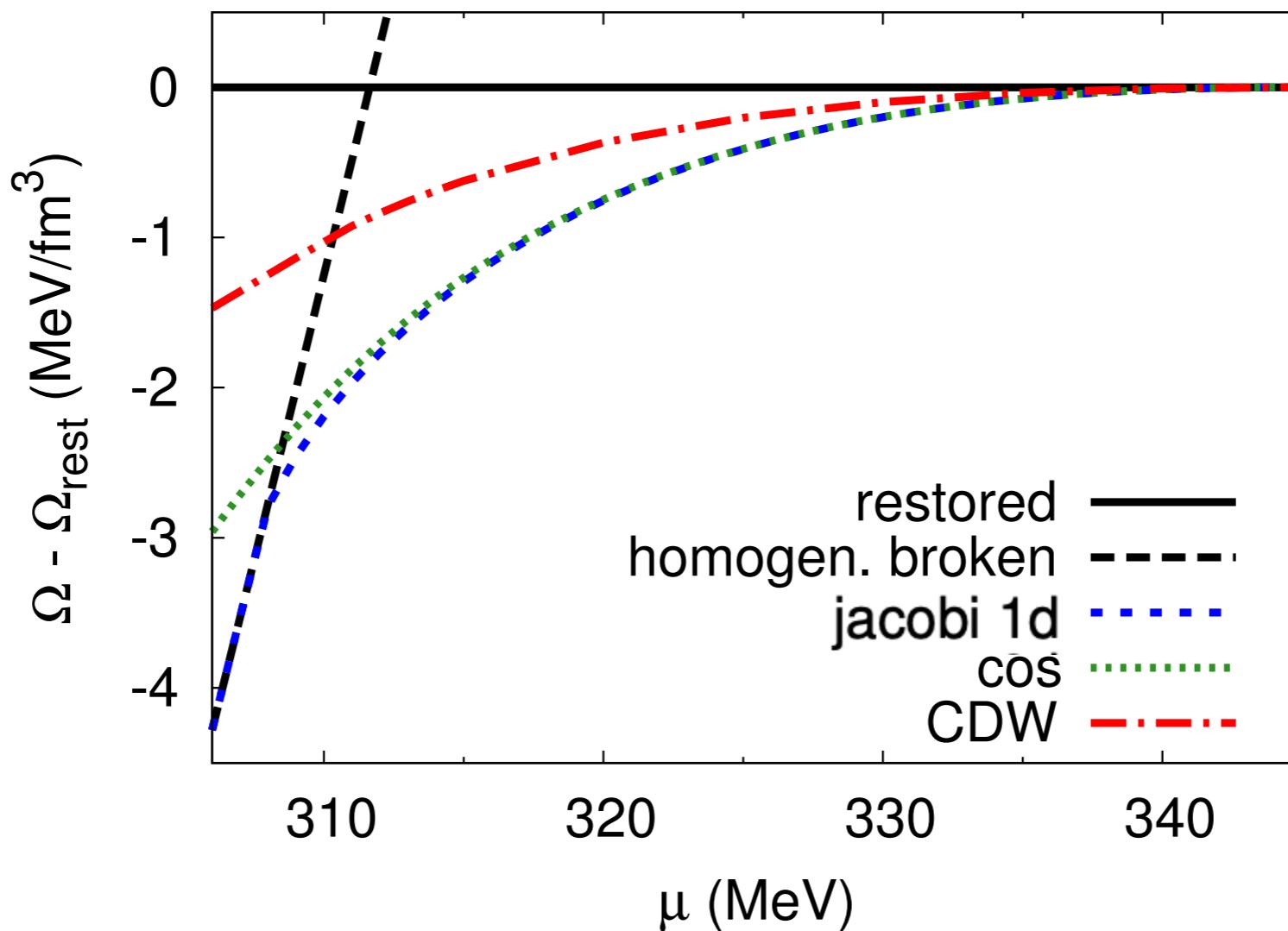
- Still numerically doable (on a cluster)
- Qualitatively similar results to 1D mods for order parameters

Free energy comparison

- What is the favored phase in the inhomogeneous window?
compare free energies for different modulations at T=0

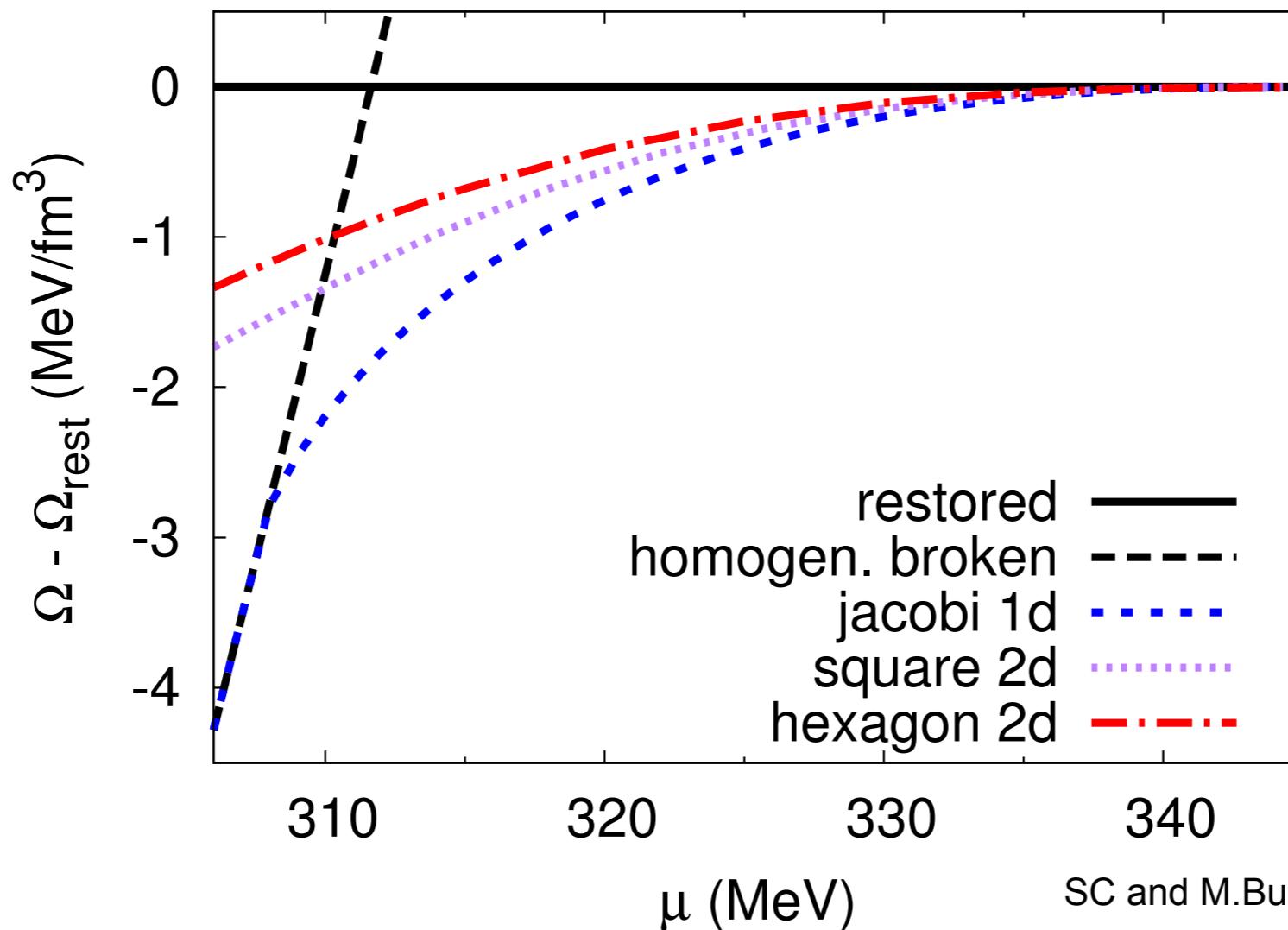
Free energy comparison

- What is the favored phase in the inhomogeneous window?
compare free energies for different modulations at T=0.
For 1D modulations...



Free energy comparison

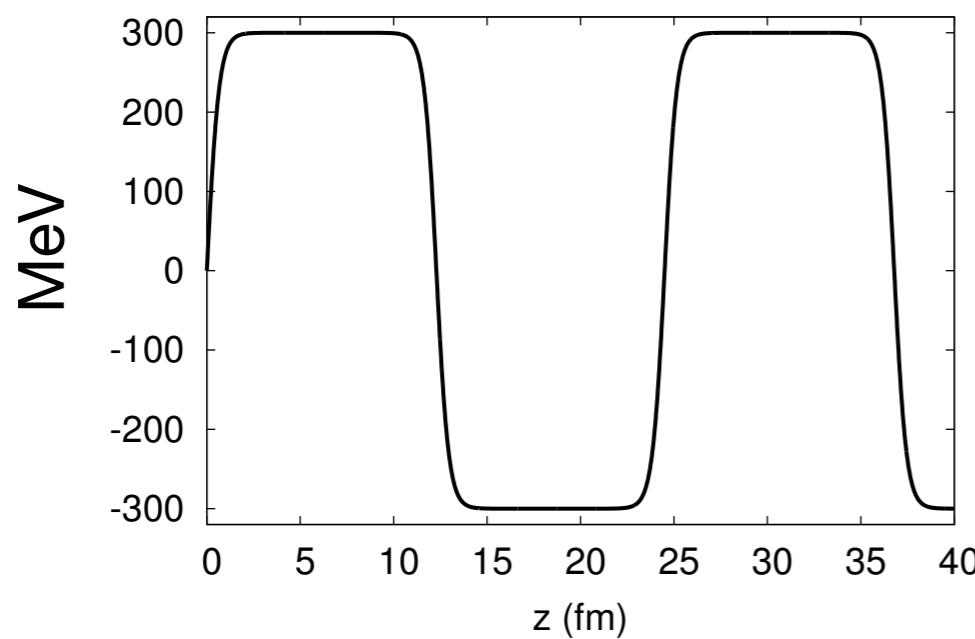
- What is the favored phase in the inhomogeneous window? compare free energies for different modulations at T=0. Including also 2D modulations..



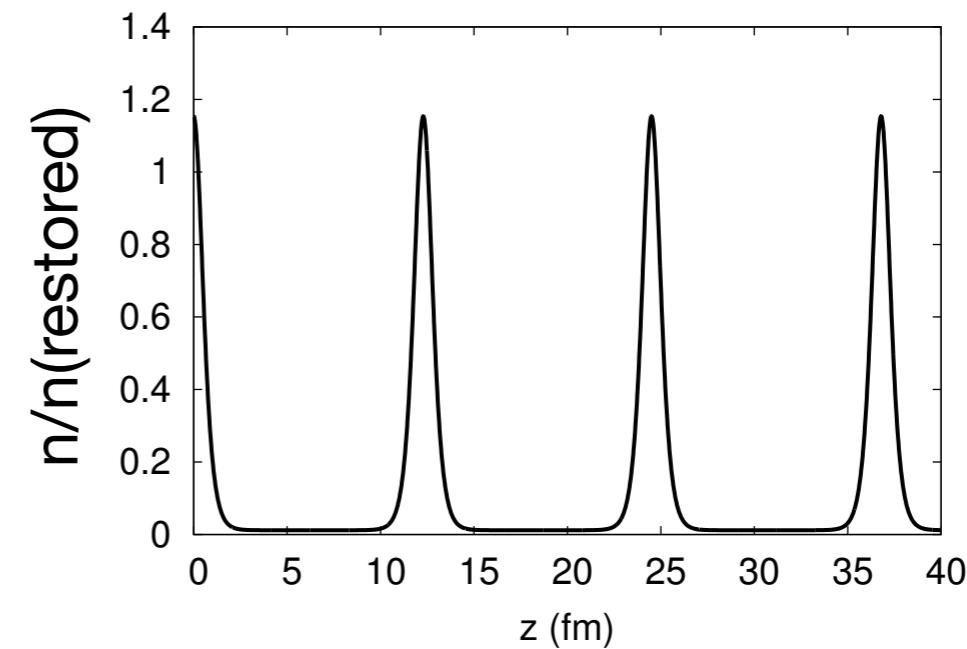
Condensate and density

- If the chiral condensate is spatially modulated, the density of the system becomes inhomogeneous as well
- For the real kink crystal

$$M(z) \sim \langle \bar{\psi} \psi \rangle$$



$$n(z) \sim \langle \psi^\dagger \psi \rangle$$

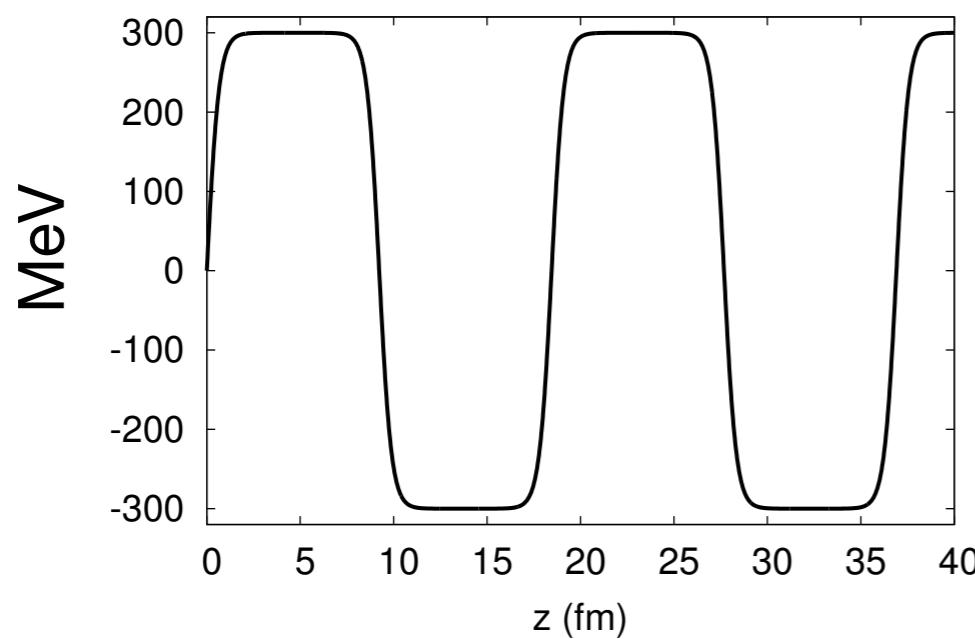


$$\mu \sim 308 \text{ MeV}$$

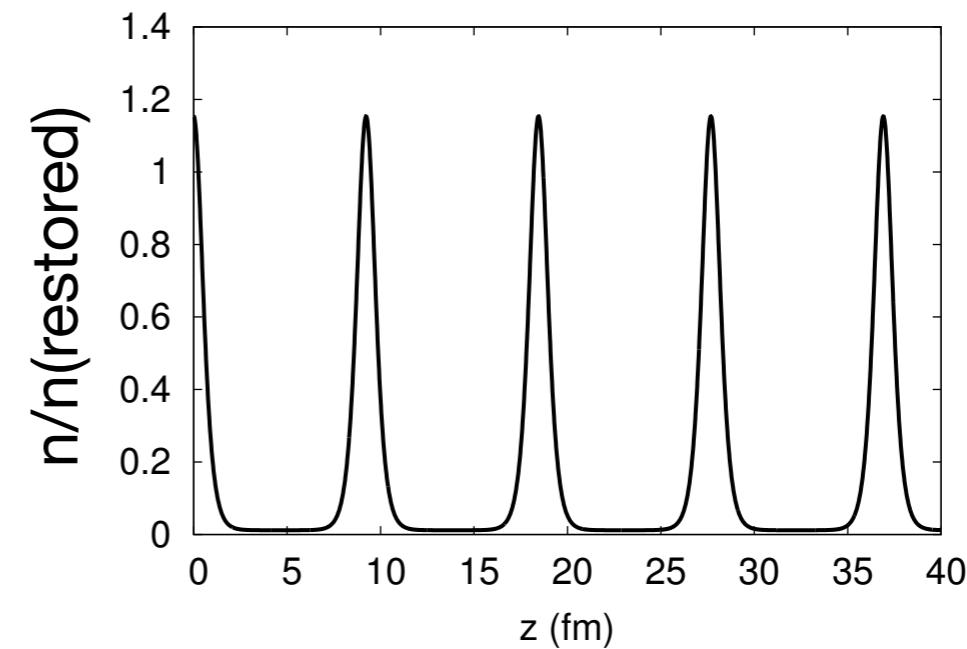
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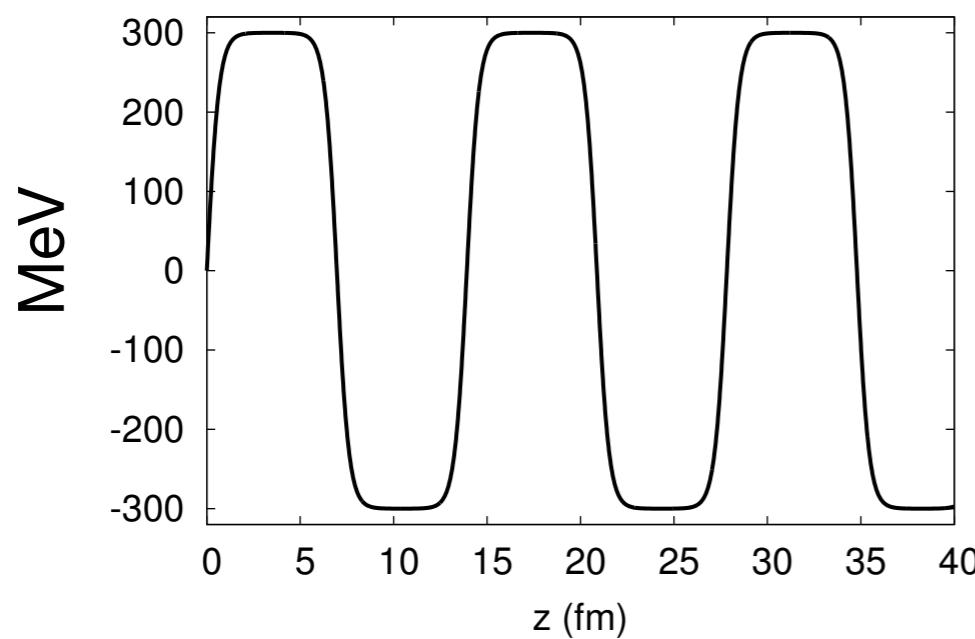
$$n(z) \sim \langle \psi^\dagger \psi \rangle$$



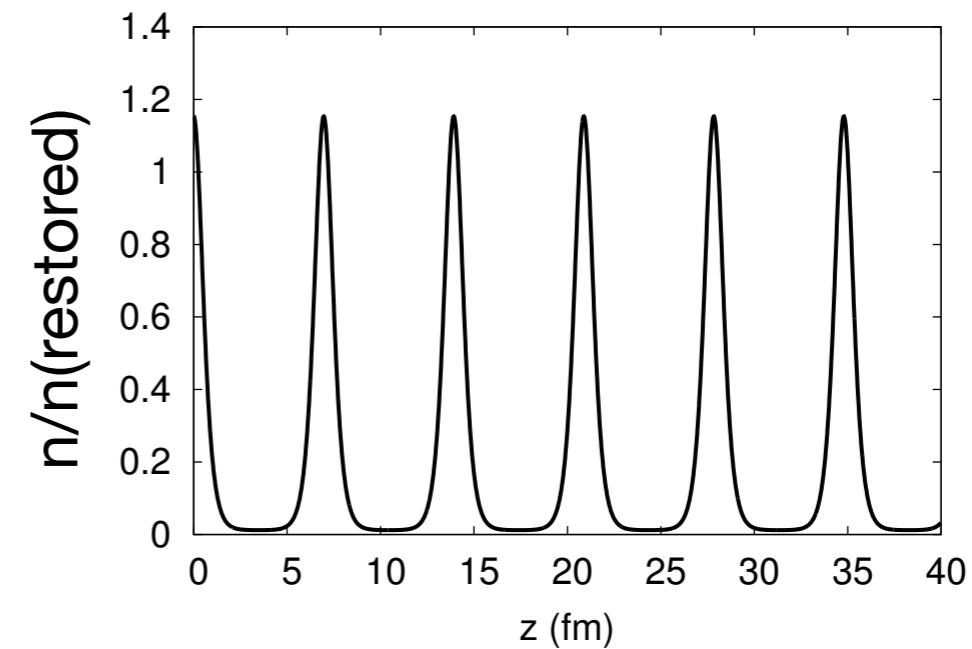
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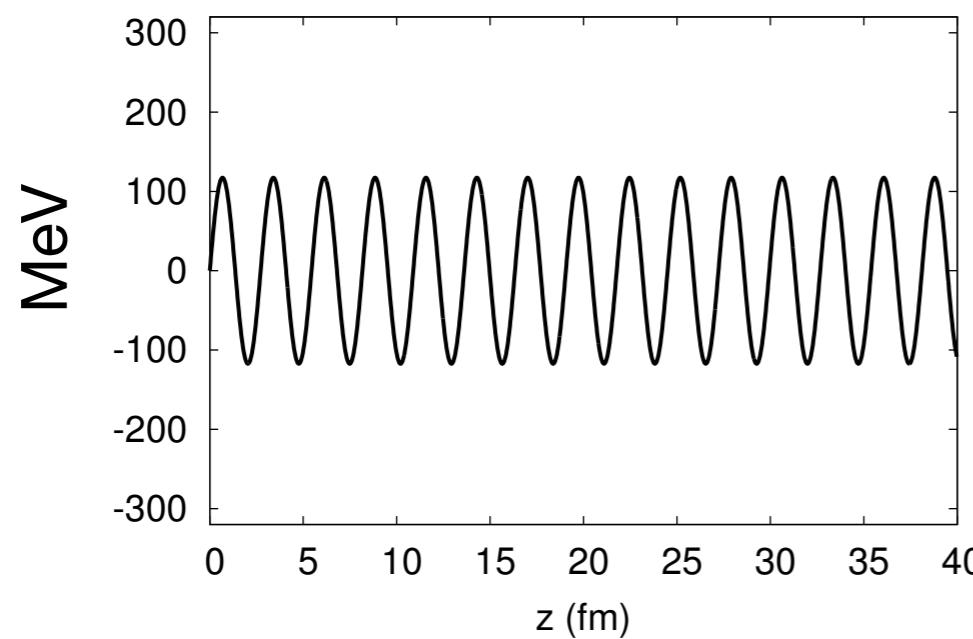
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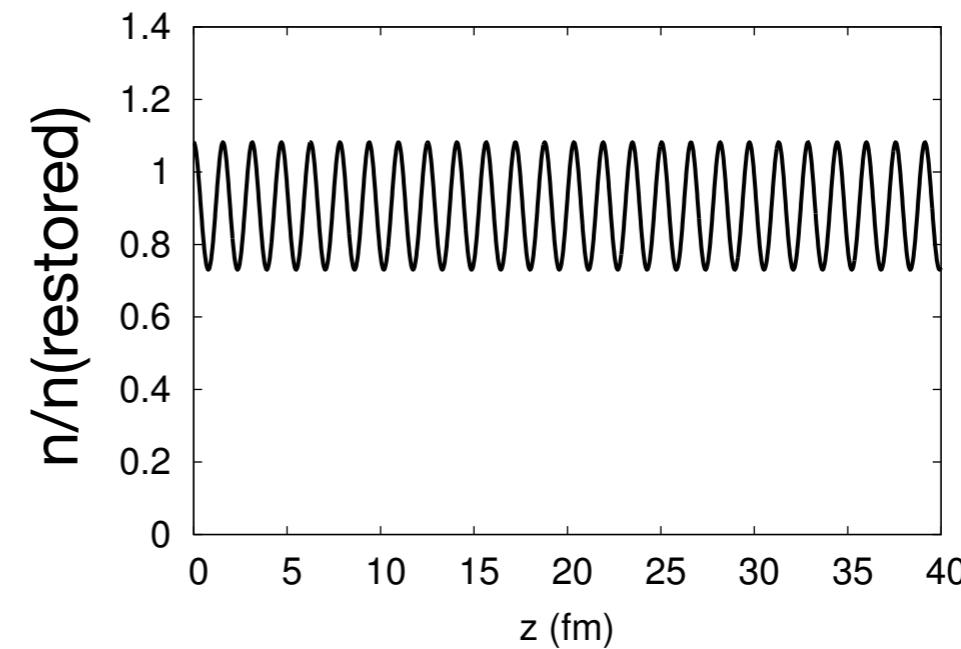
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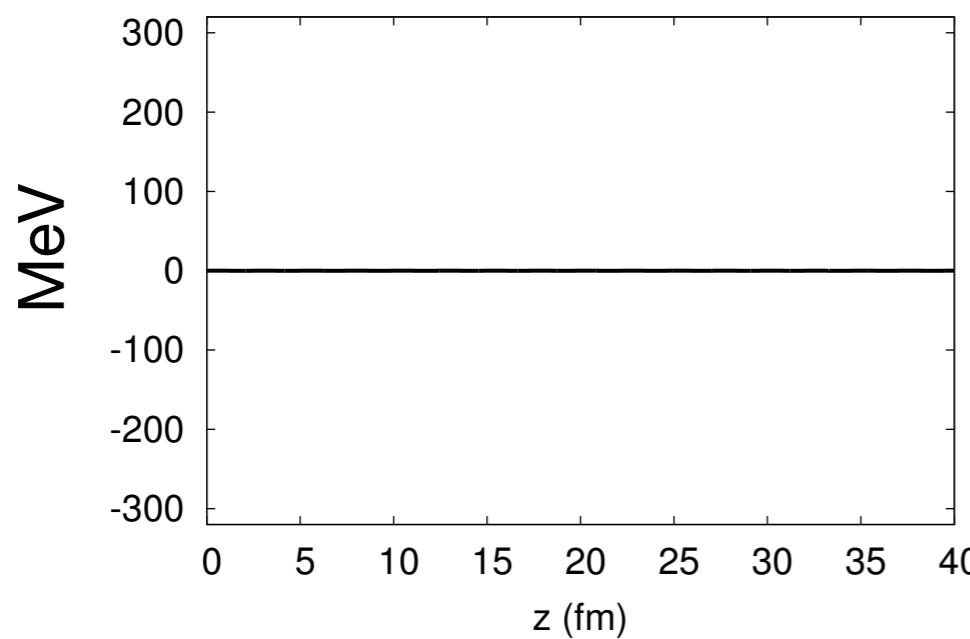


$$\mu \sim 320 \text{ MeV}$$

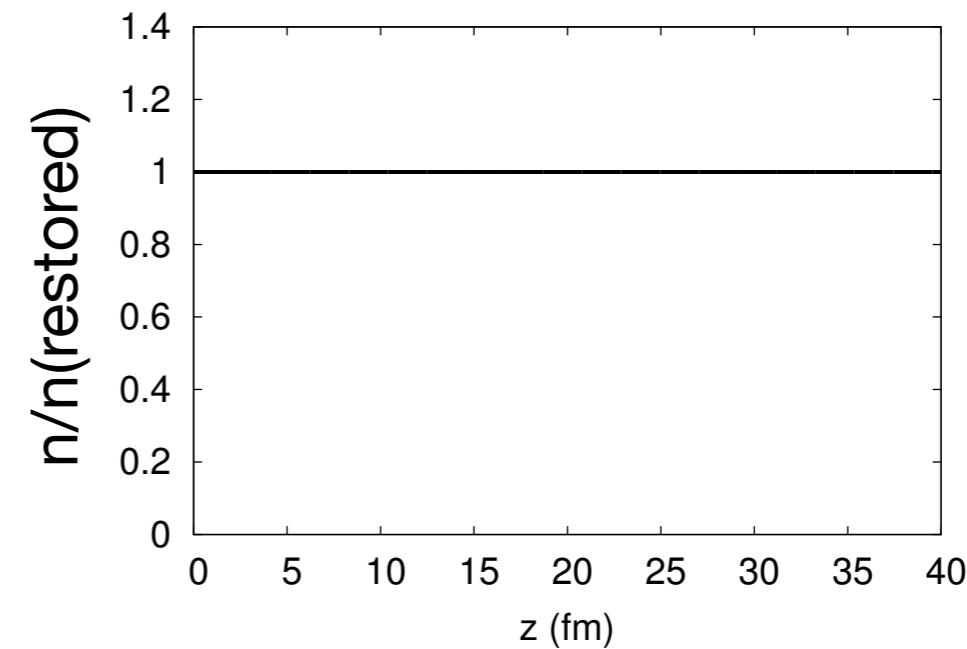
Condensate and density

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- For the real kink crystal

$$M(z) \sim \langle \bar{\psi} \psi \rangle$$



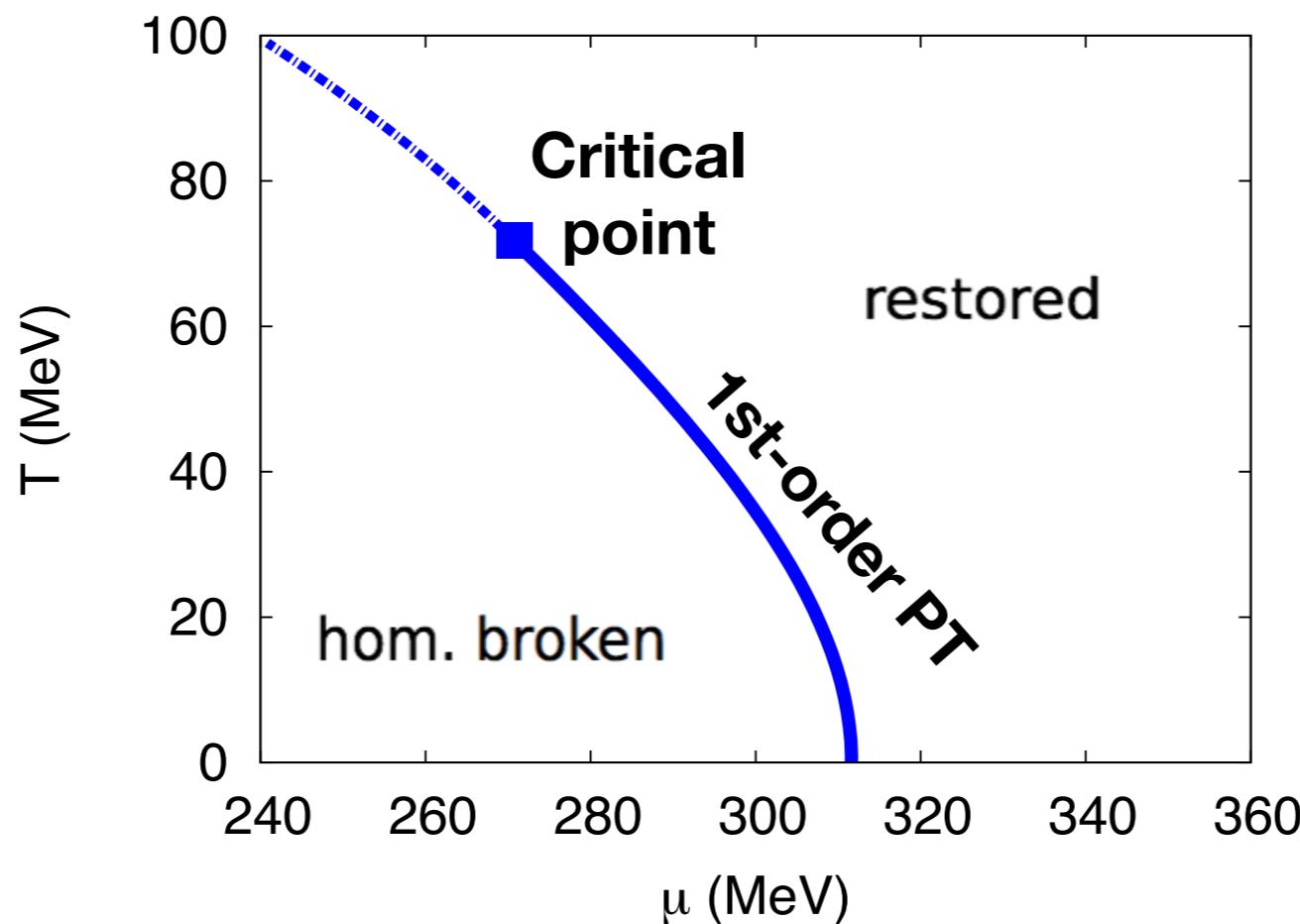
$$n(z) \sim \langle \psi^\dagger \psi \rangle$$



$$\mu \sim 350 \text{ MeV}$$

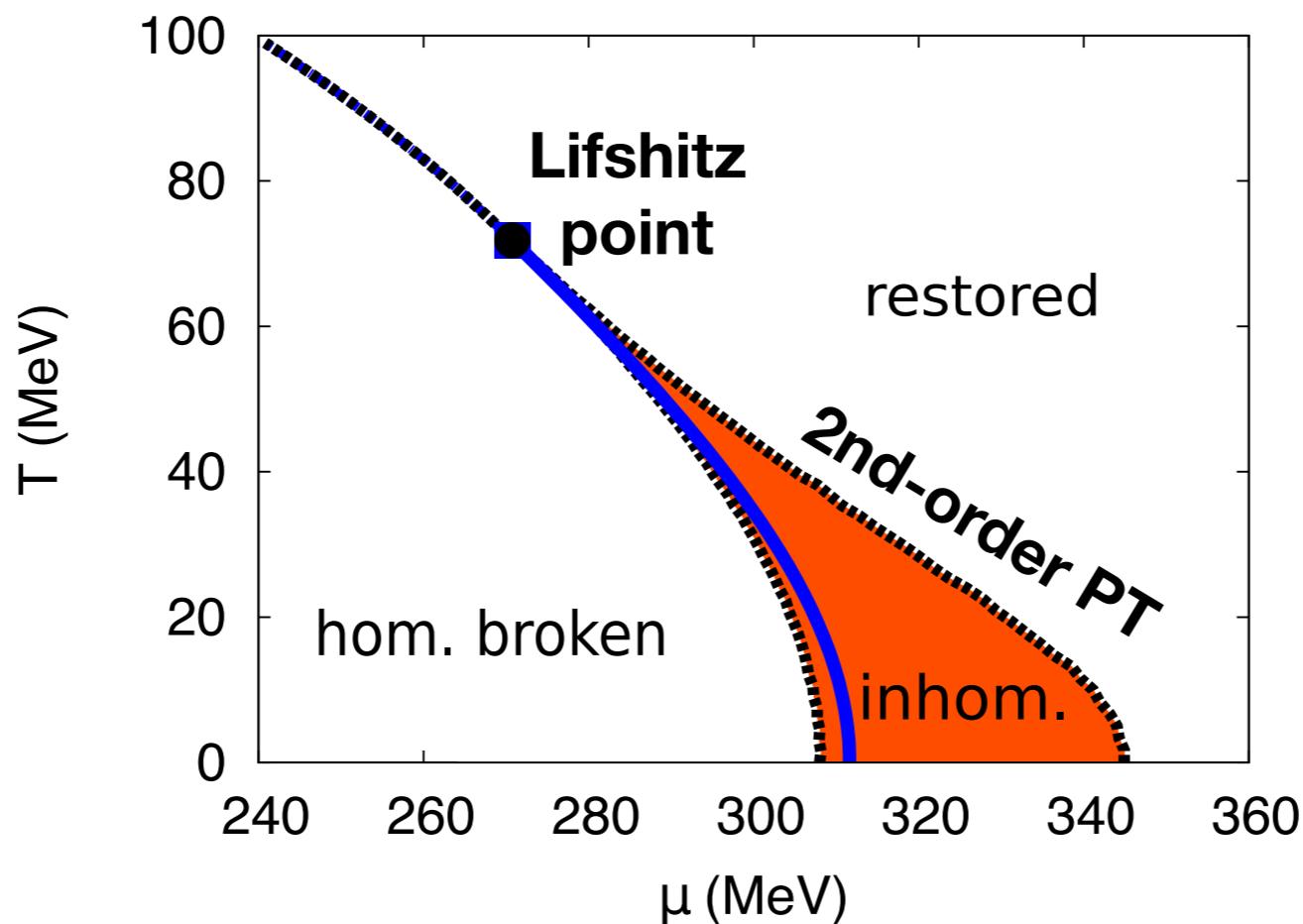
NJL phase diagram

- Allowing for inhomogeneous phases, we go from this...



NJL phase diagram

- Allowing for inhomogeneous phases, we go
...to this



- So far: results tied to specific Ansätze for $M(x)$
- Many of them requiring brute-force numerical diagonalizations in momentum space
- Is this the only way?

Ginzburg-Landau analysis

- Systematic expansion of the free energy in terms of the order parameter and its gradients
- Reliable if amplitudes and gradients are small
 - > close to the Critical/Lifshitz point

$$\Omega_{\text{GL}} = \Omega[0] + \frac{1}{V} \int d\mathbf{x} \left[\alpha_2 M^2 + \alpha_4 (M^4 + (\nabla M)^2) + \alpha_6 \left(M^6 + 3(\nabla M)^2 M^2 + \frac{1}{2}(\nabla M^2)^2 + \frac{1}{2}(\nabla^2 M)^2 \right) + \alpha_8 \left(M^8 + 14M^4(\nabla M)^2 - \frac{1}{5}(\nabla M)^4 + \frac{18}{5}M(\nabla^2 M)(\nabla M)^2 + \frac{14}{5}M^2(\nabla^2 M)^2 + \frac{1}{5}(\nabla^3 M)^2 \right) + \dots \right]$$

D.Nickel, Phys.Rev.Lett.103:072301,2009

H.Abuki, D.Ishibashi, K.Suzuki, Phys.Rev.D85:074002,2012

Ginzburg-Landau analysis

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Restored +

Ginzburg-Landau analysis

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Restored + “homogeneous” +

Ginzburg-Landau analysis

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Restored + “homogeneous” + gradient terms

- In principle straightforward: for each order add all possible independent terms (considering gradients are of the same order as M)

Ginzburg-Landau analysis

$$\Omega_{\text{GL}} = \Omega[0] + \frac{1}{V} \int d\mathbf{x} \left[\alpha_2 M^2 + \alpha_4 (M^4 + (\nabla M)^2) + \alpha_6 \left(M^6 + 3(\nabla M)^2 M^2 + \frac{1}{2}(\nabla M^2)^2 + \frac{1}{2}(\nabla^2 M)^2 \right) + \alpha_8 \left(M^8 + 14M^4(\nabla M)^2 - \frac{1}{5}(\nabla M)^4 + \frac{18}{5}M(\nabla^2 M)(\nabla M)^2 + \frac{14}{5}M^2(\nabla^2 M)^2 + \frac{1}{5}(\nabla^3 M)^2 \right) + \dots \right]$$

- GL coefficients $\alpha_n(T, \mu)$ are independent from the shape of the modulation
-> can be computed relatively easily in a chirally restored background!

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- GL coefficients $\alpha_n(T, \mu)$ are independent from the shape of the modulation
-> can be computed relatively easily in a chirally restored background!
- But: calculating the relative prefactors between terms of the same order is an extremely tedious task..

Ginzburg-Landau analysis

Already non-trivial result at lowest order:

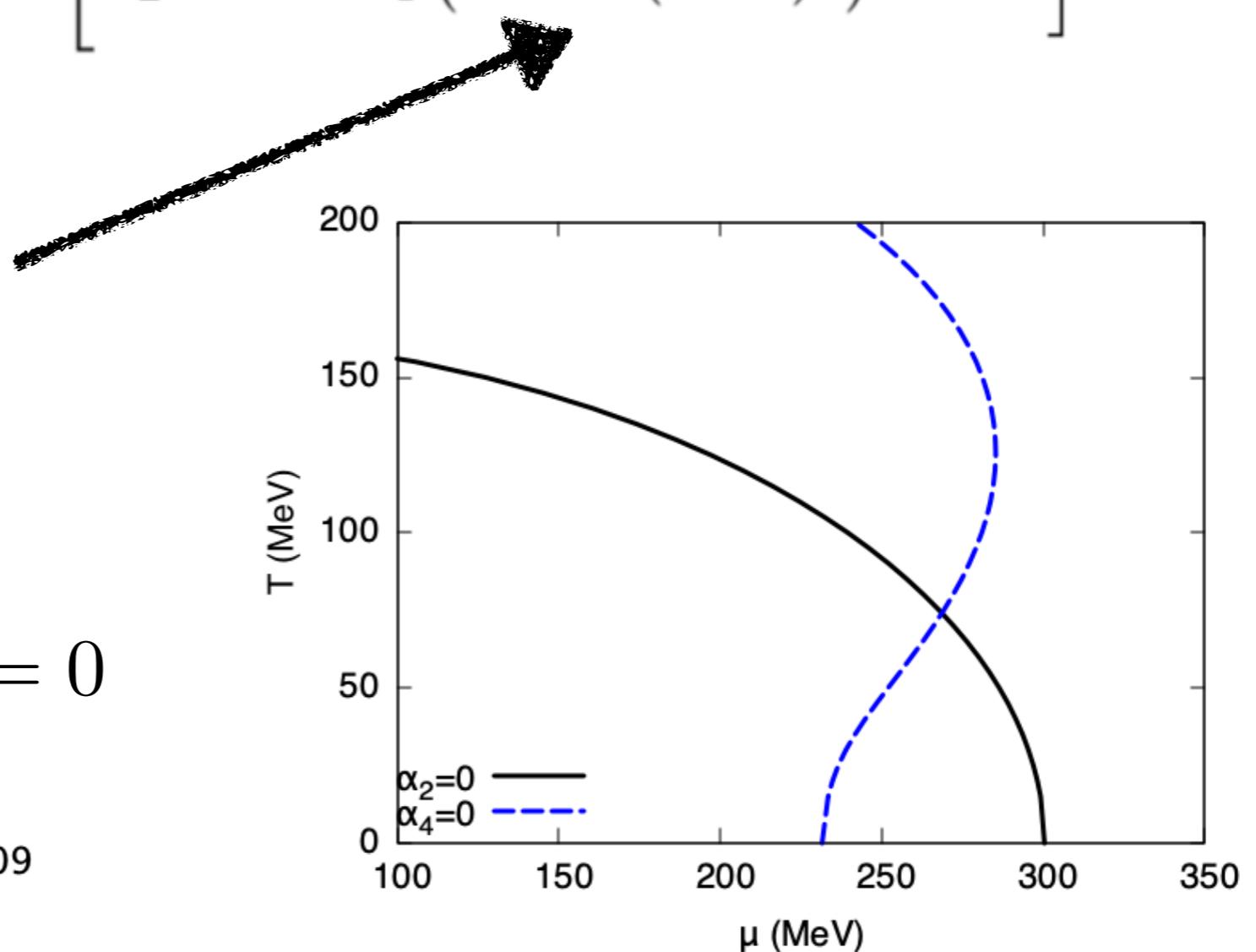
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Critical/Lifshitz point
coincide

and are located at
the point where

$$\alpha_2(T, \mu) = \alpha_4(T, \mu) = 0$$

D.Nickel, Phys.Rev.Lett.103:072301,2009

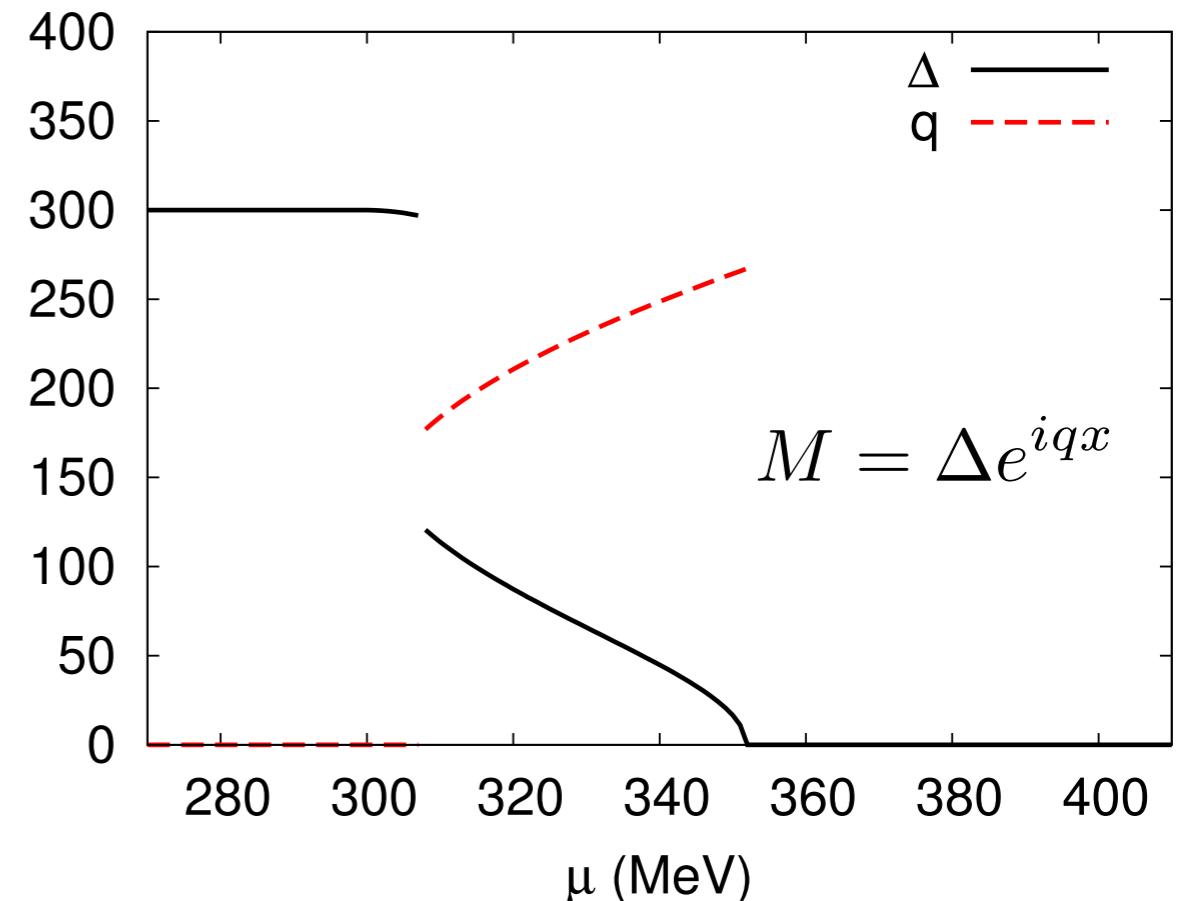


Improved Ginzburg-Landau

- Can we do better ?
Recall the typical behavior
of the order parameters
(eg. CDW)

$$M \sim \Delta$$

$$\nabla M \sim q\Delta$$



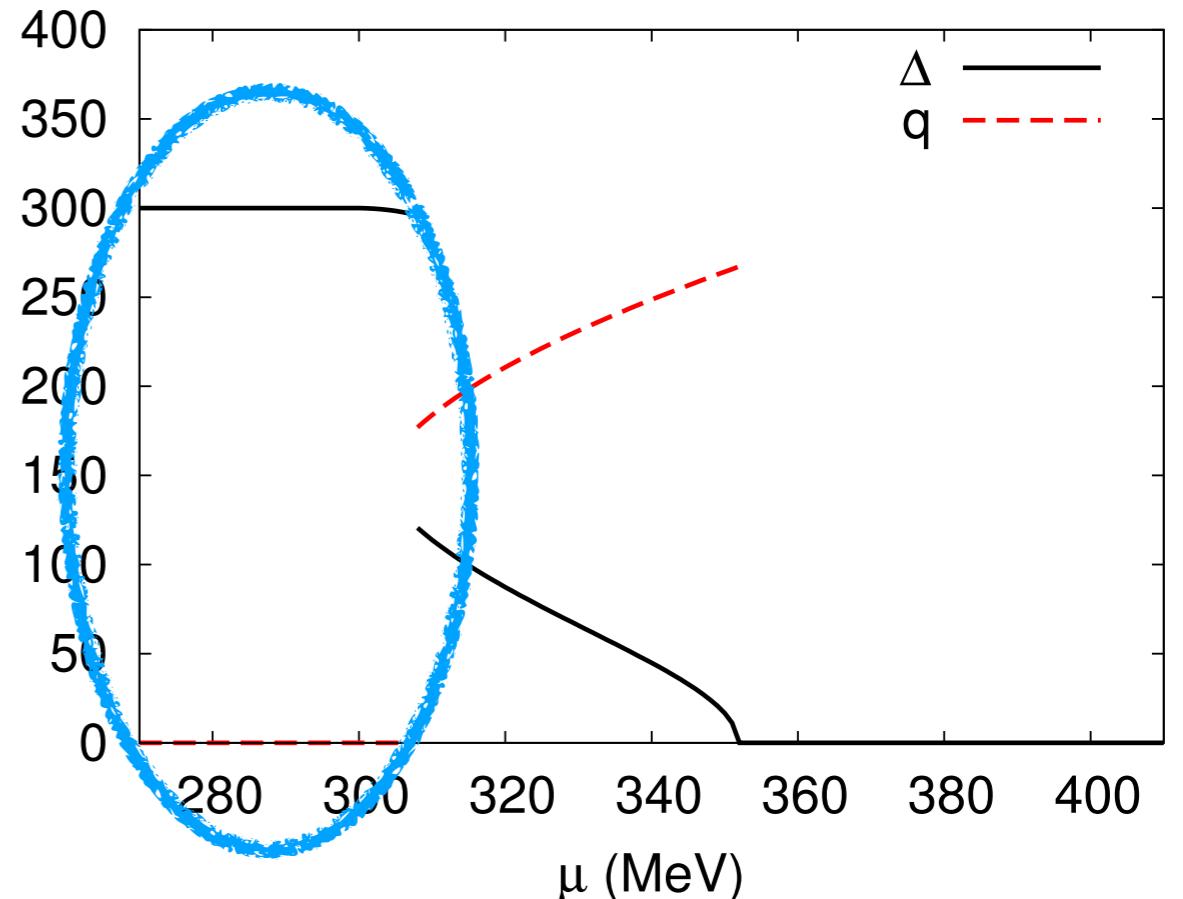
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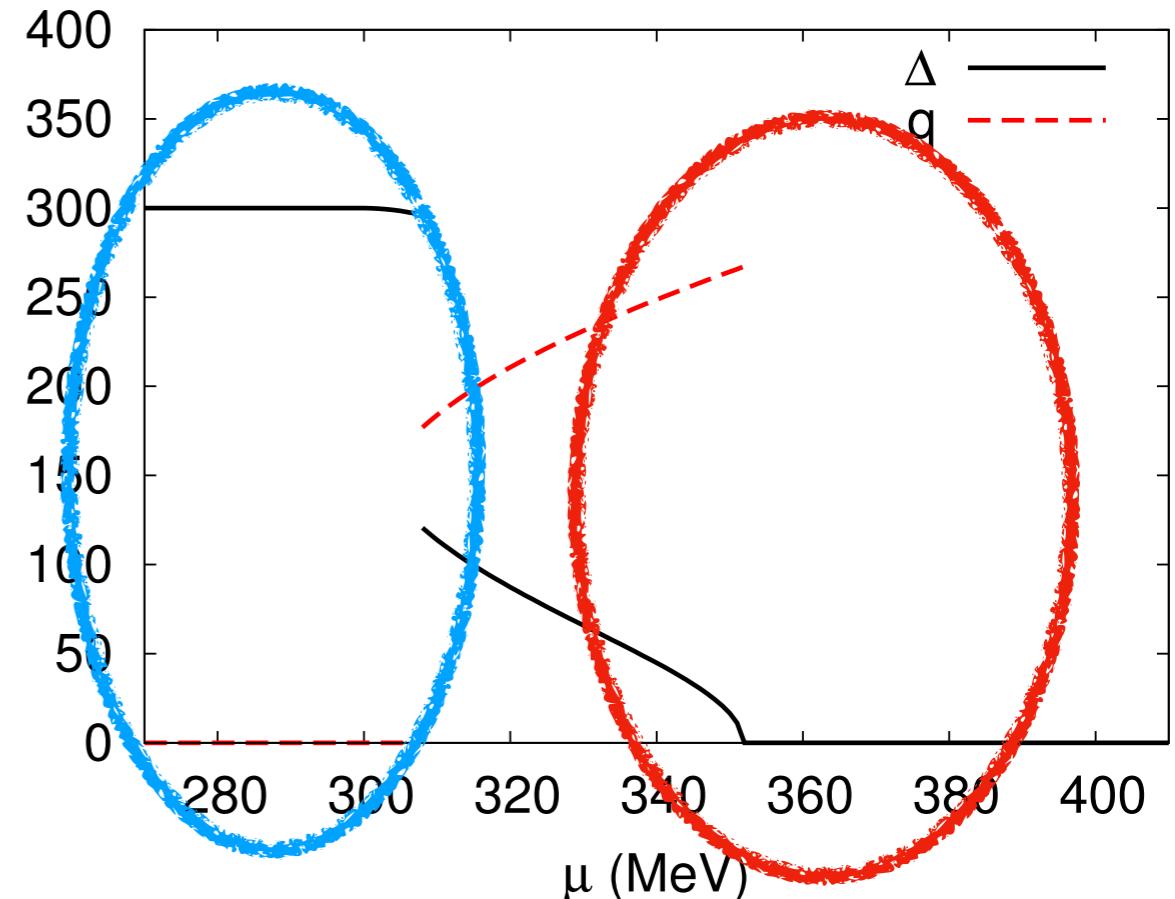
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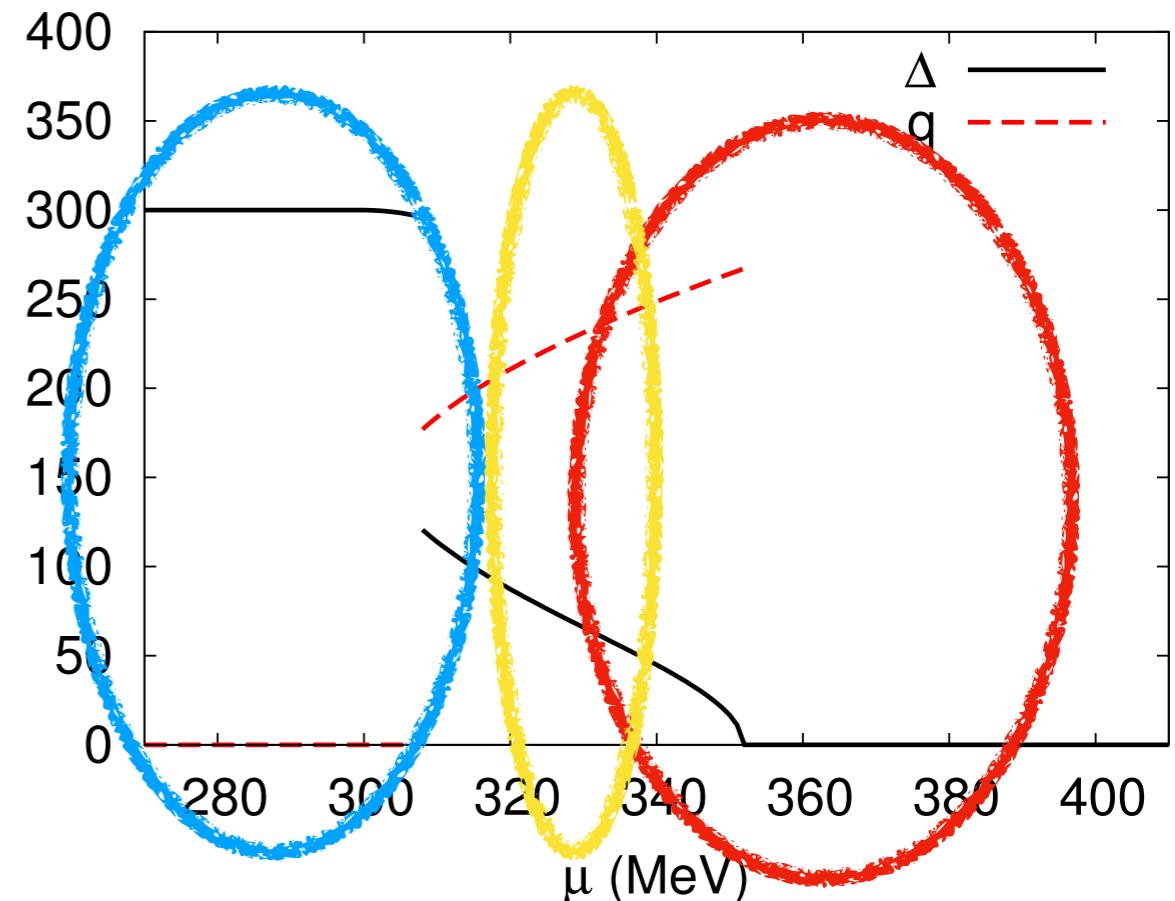
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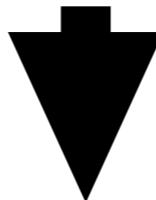
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Improved Ginzburg-Landau

$$\Omega_{\text{GL}} = \Omega[0] + \frac{1}{V} \int d\mathbf{x} \left[\alpha_2 M^2 + \alpha_4 (M^4 + (\nabla M)^2) + \alpha_6 \left(M^6 - 3(\nabla M)^2 M^2 + \frac{1}{2}(\nabla M^2)^2 + \frac{1}{2}(\nabla^2 M)^2 \right) \right. \\ \left. + \alpha_8 \left(M^8 - 14M^4(\nabla M)^2 - \frac{1}{5}(\nabla M)^4 + \frac{18}{5}M(\nabla^2 M)(\nabla M)^2 + \frac{14}{5}M^2(\nabla^2 M)^2 + \frac{1}{5}(\nabla^3 M)^2 \right) + \dots \right]$$

Improved Ginzburg-Landau

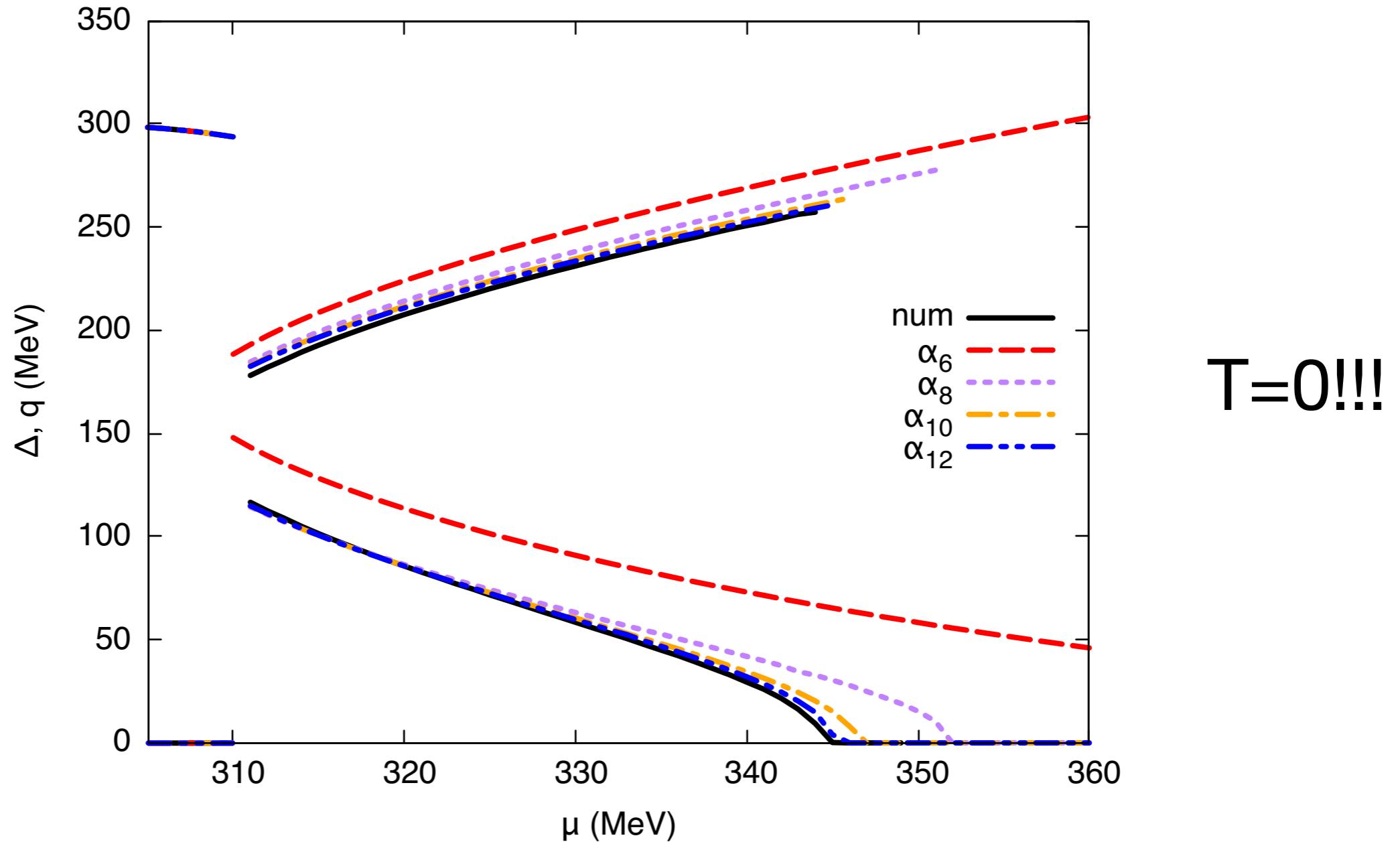
$$\Omega_{\text{GL}} = \Omega[0] + \frac{1}{V} \int d\mathbf{x} \left[\alpha_2 M^2 + \alpha_4 (M^4 + (\nabla M)^2) + \alpha_6 \left(M^6 - 3(\nabla M)^2 M^2 + \frac{1}{2}(\nabla M^2)^2 + \frac{1}{2}(\nabla^2 M)^2 \right) \right. \\ \left. + \alpha_8 \left(M^8 - 14M^4(\nabla M)^2 - \frac{1}{5}(\nabla M)^4 + \frac{18}{5}M(\nabla^2 M)(\nabla M)^2 + \frac{14}{5}M^2(\nabla^2 M)^2 + \frac{1}{5}(\nabla^3 M)^2 \right) + \dots \right]$$



straightforward to compute

$$\Omega_{\text{IGL}} = \frac{1}{V} \int d\mathbf{x} \left[\Omega_{\text{hom}}(\overline{M^2}) + \alpha_6 \left(3(\nabla M)^2 M^2 + \frac{1}{2}(\nabla M^2)^2 \right) \right. \\ \left. + \alpha_8 \left(14M^4(\nabla M)^2 - \frac{1}{5}(\nabla M)^4 + \frac{18}{5}M(\nabla^2 M)(\nabla M)^2 + \frac{14}{5}M^2(\nabla^2 M)^2 \right) + \sum_{n \geq 1} \tilde{\alpha}_{2n+2} (\nabla^n M)^2 \right]$$

Does it work ?



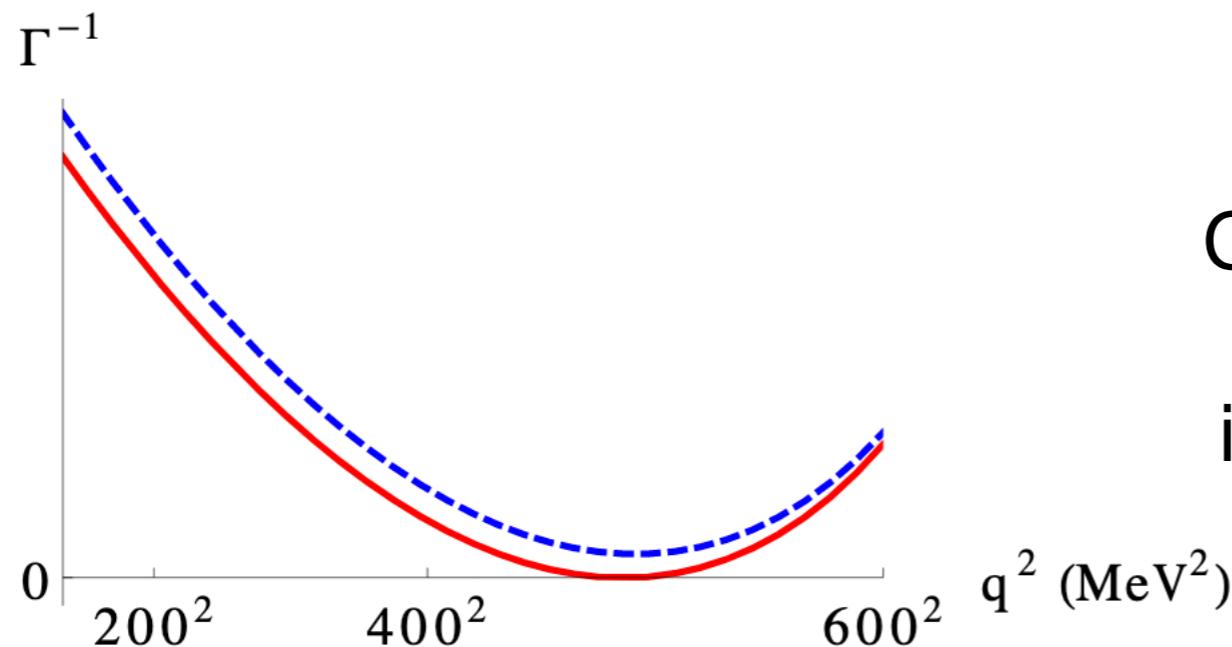
Stability analysis

Similar spirit to the (I)GL analysis: expand the free energy and look at the second-order piece

$$\Omega^{(2)} = 2G^2 \sum_{\mathbf{q}_k} \left\{ |\delta\phi_{S,\mathbf{q}_k}|^2 \Gamma_S^{-1}(\mathbf{q}_k^2) + |\delta\phi_{P,\mathbf{q}_k}|^2 \Gamma_P^{-1}(\mathbf{q}_k^2) \right\}$$



Look for where the correlation functions in either condensation channel changes sign



Can be used to determine
the phase boundary
inhomogeneous-restored

Model extensions and inhomogeneous phases

So far: simplest NJL model

- Two flavor quark matter
- Scalar-pseudoscalar interaction channel only
- Chiral limit

Can we do better?

Model extensions and inhomogeneous phases

Some extensions I won't discuss much:

- Coupling with Polyakov loop (PNJL)
- Magnetic fields
- Interplay with color-superconductivity
- Isospin-asymmetric matter

Vector interactions

Repulsive vector interaction channel:

$$\mathcal{L} = \mathcal{L}_{NJL} - G_V (\bar{\psi} \gamma^\mu \psi)^2$$

Vector interactions

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Mean-field: density-dependent shift of the chemical potential

Vector interactions

Repulsive vector interaction channel:

$$\mathcal{L} = \mathcal{L}_{NJL} - G_V (\bar{\psi} \gamma^\mu \psi)^2$$

Mean-field: density-dependent shift of the chemical potential
For inhomogeneous phases: spatially dependent!

$$\tilde{\mu}(\mathbf{x}) = \mu - 2G_V n(\mathbf{x})$$

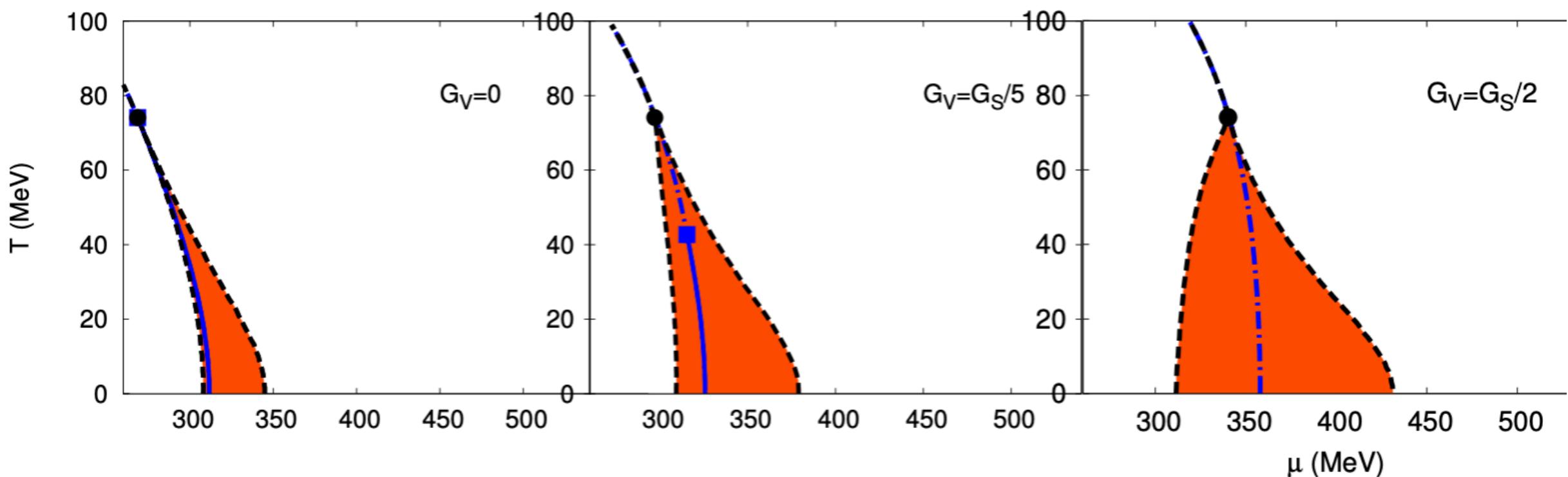
Technically challenging!
As first approximation assume

$$n(\mathbf{x}) \rightarrow \bar{n} = \langle n(\mathbf{x}) \rangle_{\mathbf{x}}$$

Vector interactions

Constant density approximation:
the inhomogeneous phase enlarges dramatically!

CP falls below the LP and **disappears**
inside the inhomogeneous phase



Vector interactions

Going beyond constant density approximation:
vector interactions could alter hierarchy of favored spatial modulations
according to their density profile

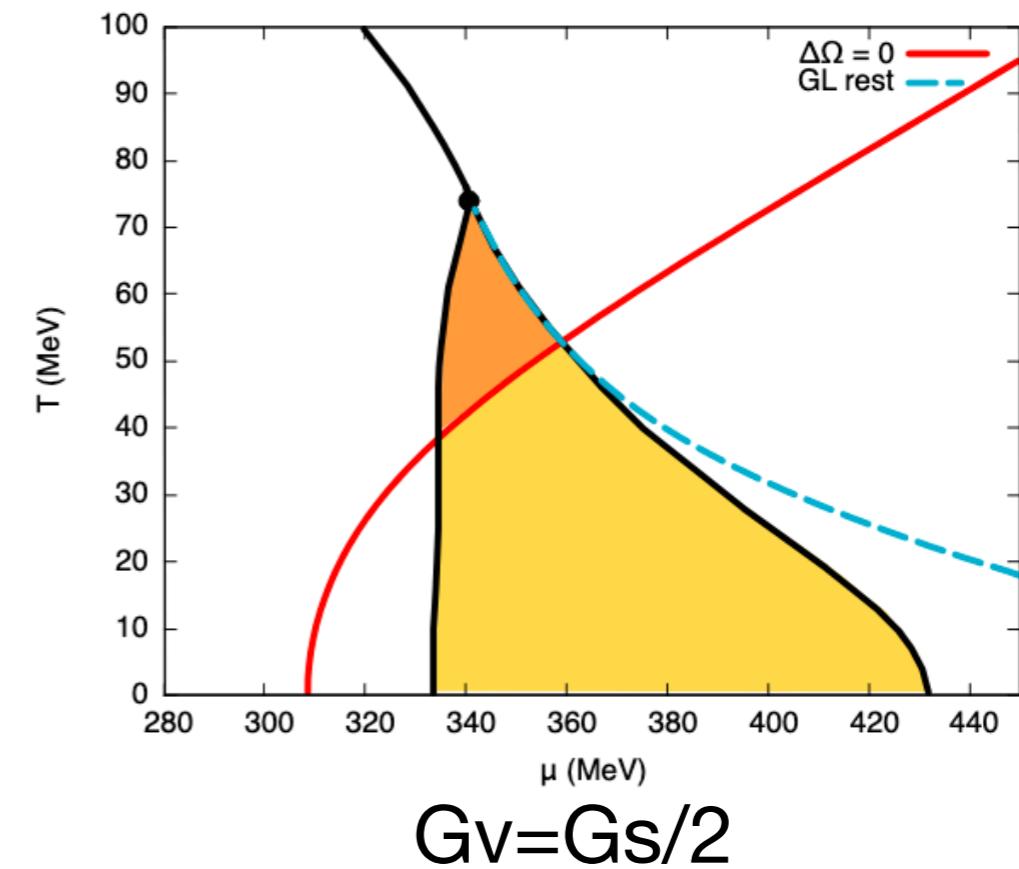
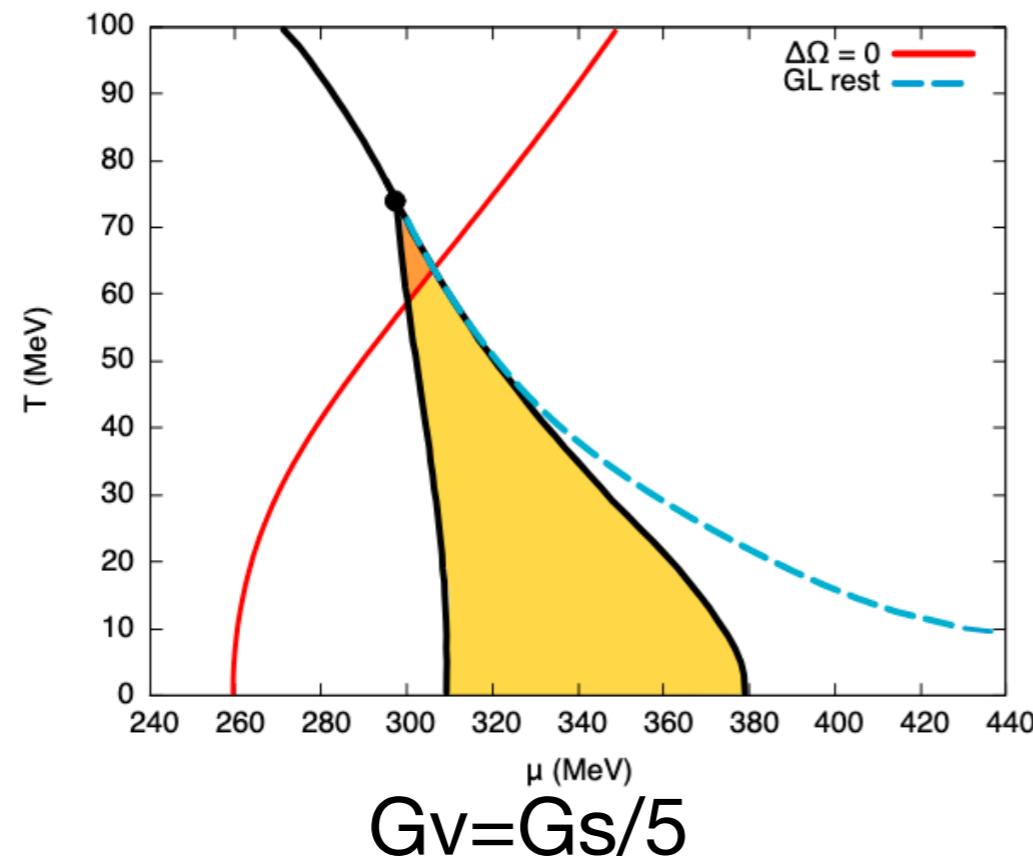
-> is the RKC still favored over a CDW?

Vector interactions

Going beyond constant density approximation:
vector interactions could alter hierarchy of favored spatial modulations
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-> is the RKC still favored over a CDW?

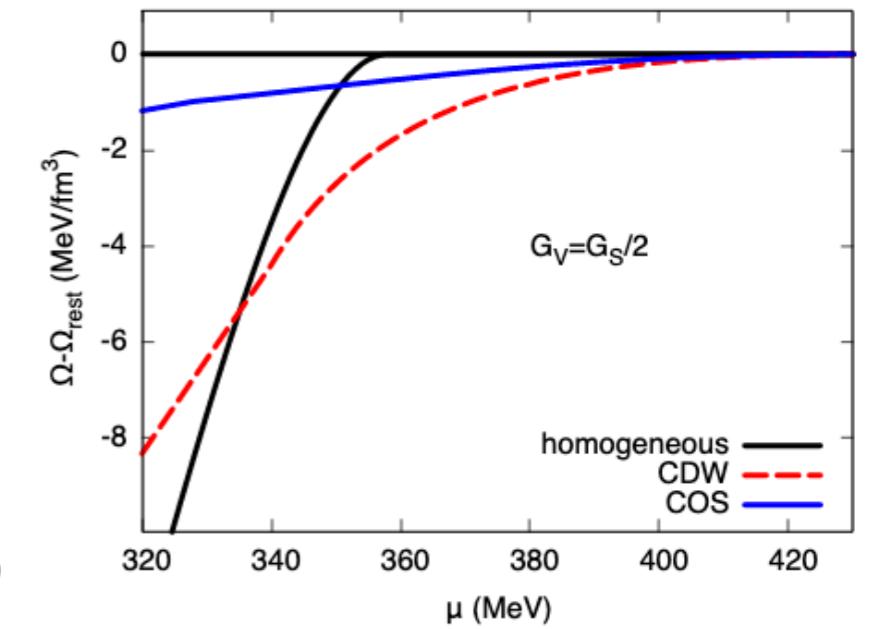
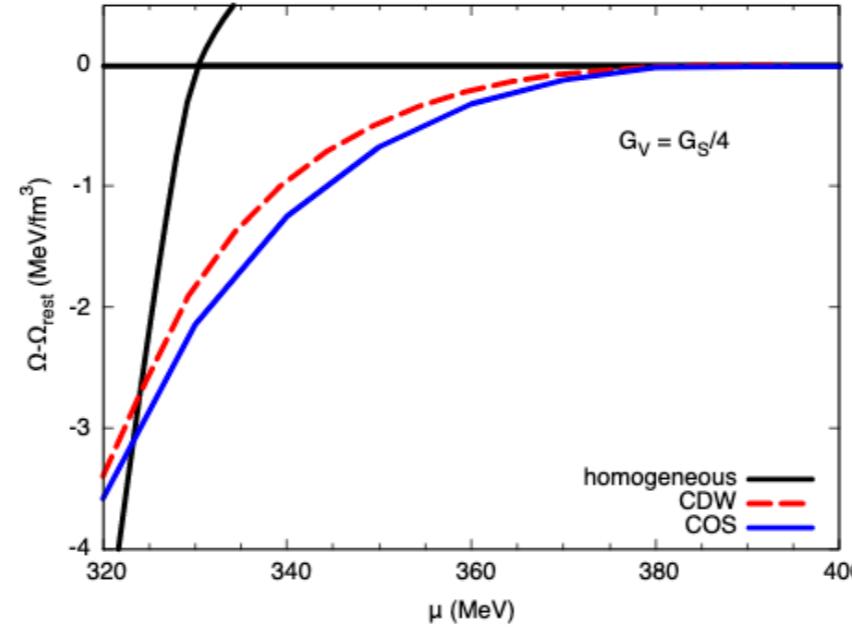
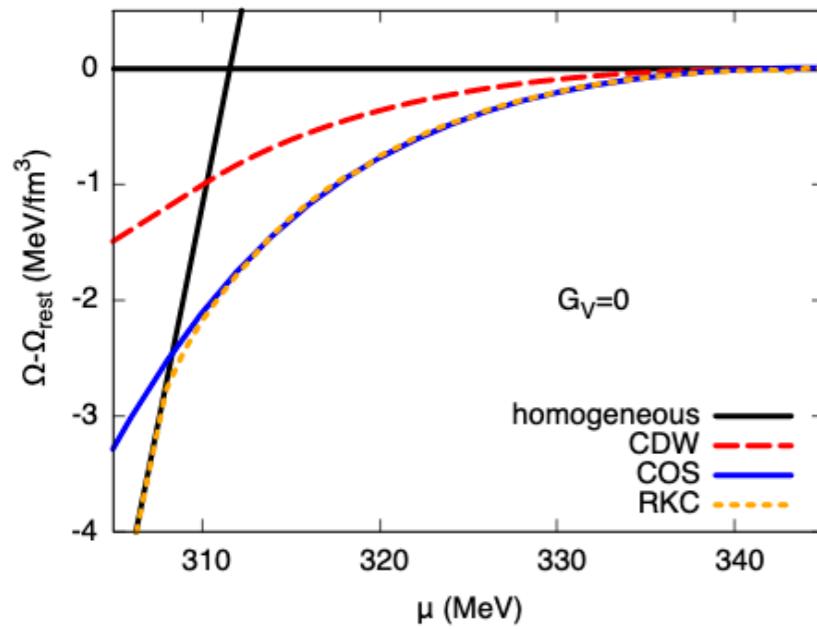
Close to LP: GL analysis...



Vector interactions

Going beyond constant density approximation:
vector interactions could alter hierarchy of favored spatial modulations
according to their density profile

-> is the RKC still favored over a CDW?
...or numerically at T=0



Going away from the chiral limit

Less straightforward: in the restored phase $M = M_0 \neq 0$

Issues of self-consistency with some solutions (eg. CDW)

-> Work again within a modulation-agnostic GL approach:
expand around $M(\mathbf{x}) = M_0 + \delta M(\mathbf{x})$

$$\Omega[M] = \Omega[M_0] + \frac{1}{V} \int d^3x (\alpha_1 \delta M(\mathbf{x}) + \alpha_2 \delta M^2(\mathbf{x}) + \alpha_3 \delta M^3(\mathbf{x}) + \alpha_{4,a} \delta M^4(\mathbf{x}) + \alpha_{4,b} (\nabla \delta M(\mathbf{x}))^2 + \dots)$$

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CEP : $\alpha_1 = \alpha_2 = \alpha_3 = 0$ → CP and LP split?
PLP : $\alpha_1 = \alpha_2 = \alpha_{4,b} = 0$

Going away from the chiral limit

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CEP : $\alpha_1 = \alpha_2 = \alpha_3 = 0$

PLP : $\alpha_1 = \alpha_2 = \alpha_{4,b} = 0$

CP and LP split?

No!

$$\alpha_3 = 4M_0 \alpha_{4,b}$$

Three-flavor quark matter

Add strange quarks with KMT interaction

$$\mathcal{L} = \bar{\psi} (i\gamma^\mu \partial_\mu - \hat{m}) \psi + \mathcal{L}_4 + \mathcal{L}_6$$

$$\mathcal{L}_4 = G \sum_{a=0}^8 [(\bar{\psi} \tau_a \psi)^2 + (\bar{\psi} i\gamma_5 \tau_a \psi)^2]$$

$$\mathcal{L}_6 = -K [\det_f \bar{\psi} (1 + \gamma_5) \psi + \det_f \bar{\psi} (1 - \gamma_5) \psi]$$

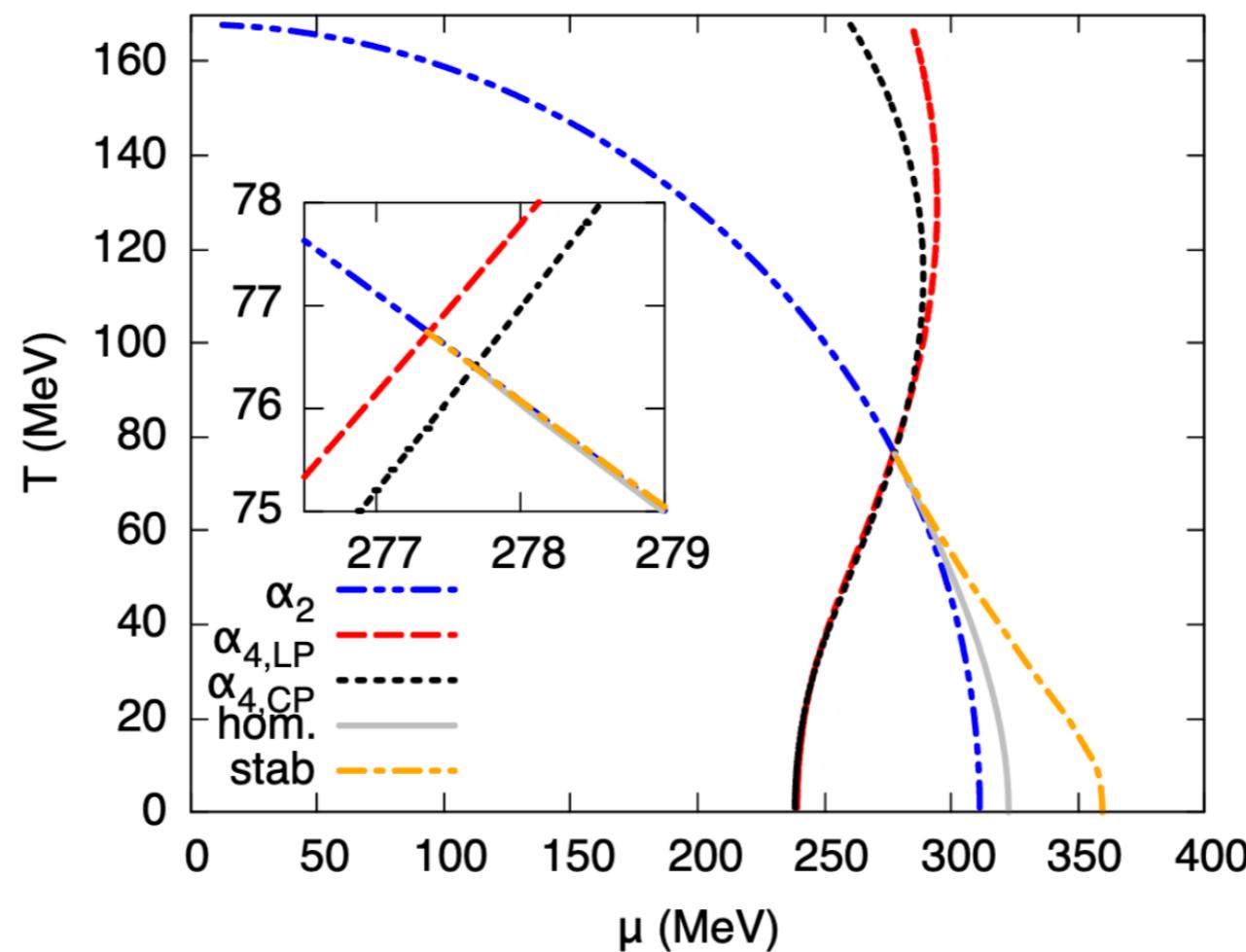
Again modulation-agnostic GL expansion:

$$\begin{aligned} \omega_{GL}(\Delta_\ell, \Delta_s) = & \alpha_2 |\Delta_\ell|^2 + \alpha_{4,a} |\Delta_\ell|^4 + \alpha_{4,b} |\nabla \Delta_\ell|^2 + \dots \\ & + \beta_1 \Delta_s + \beta_2 \Delta_s^2 + \beta_3 \Delta_s^3 + \beta_{4,a} \Delta_s^4 + \beta_{4,b} (\nabla \Delta_s)^2 + \dots \\ & + \gamma_3 |\Delta_\ell|^2 \Delta_s + \gamma_4 |\Delta_\ell|^2 \Delta_s^2 + \dots , \end{aligned}$$

Three-flavor quark matter

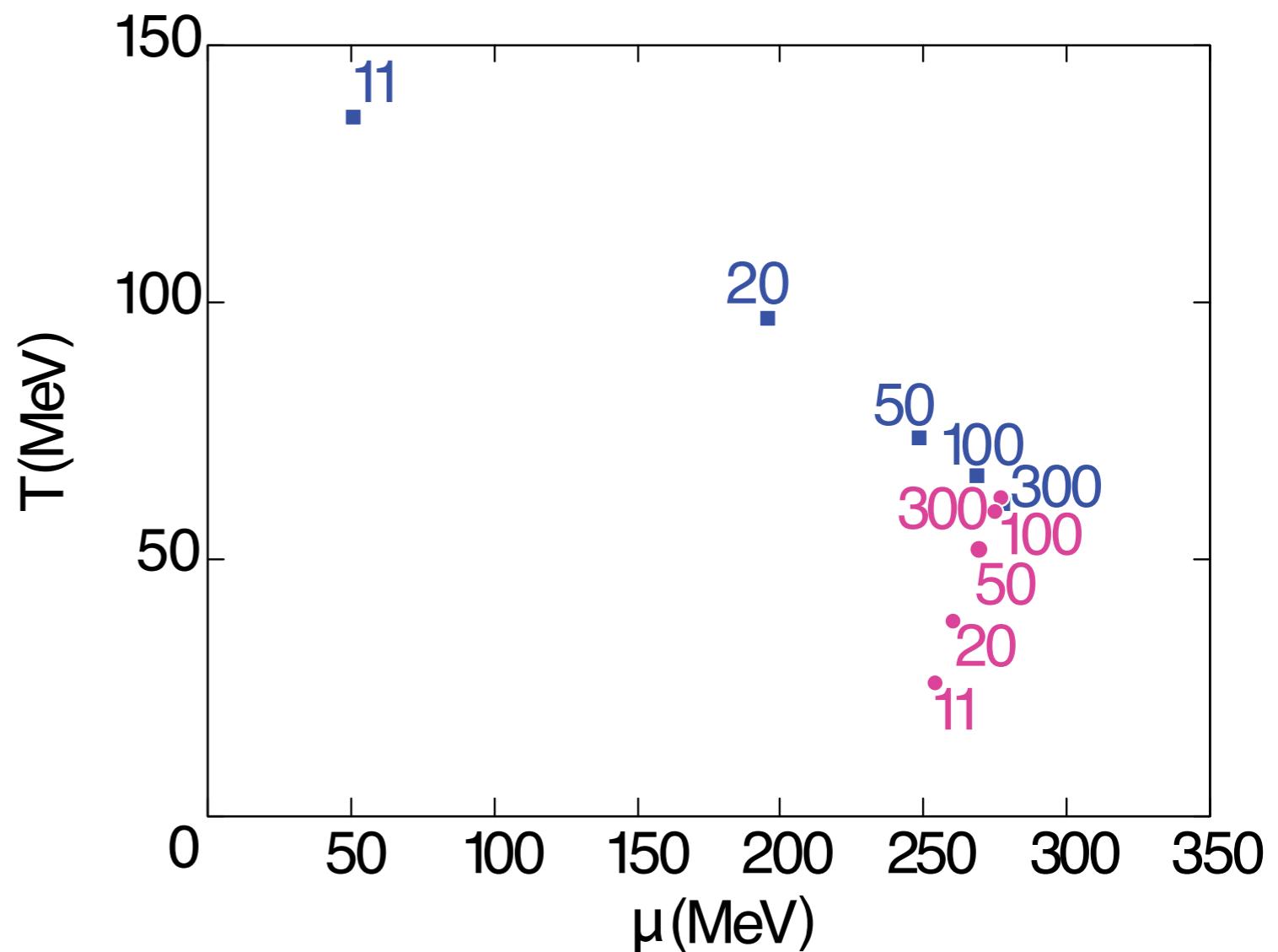
CP and LP split!

For a reasonable parameter set, LP **above** CP



Varying the current strange quark mass

Lowering m_s :
CP moves to the T axis
LP doesn't follow!



Some other things I didn't talk about

- Details on the model regularization
- Inhomogeneous “continents”
- Consequences for compact stars phenomenology
- Fluctuations

Inhomogeneous phases in the Quark-meson model

$$\mathcal{L}_{\text{QM}} = \bar{\psi} (i\gamma^\mu \partial_\mu - g(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi})) \psi + \mathcal{L}_{\text{M}}^{\text{kin}} - U(\sigma, \vec{\pi})$$

$$\mathcal{L}_{\text{M}}^{\text{kin}} = \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi})$$

$$U(\sigma, \vec{\pi}) = \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2 - v^2)^2$$

Inhomogeneous phases in the Quark-meson model

$$\mathcal{L}_{QM} = \bar{\psi} (i\gamma^\mu \partial_\mu - g(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi})) \psi + \mathcal{L}_M^{\text{kin}} - U(\sigma, \vec{\pi})$$

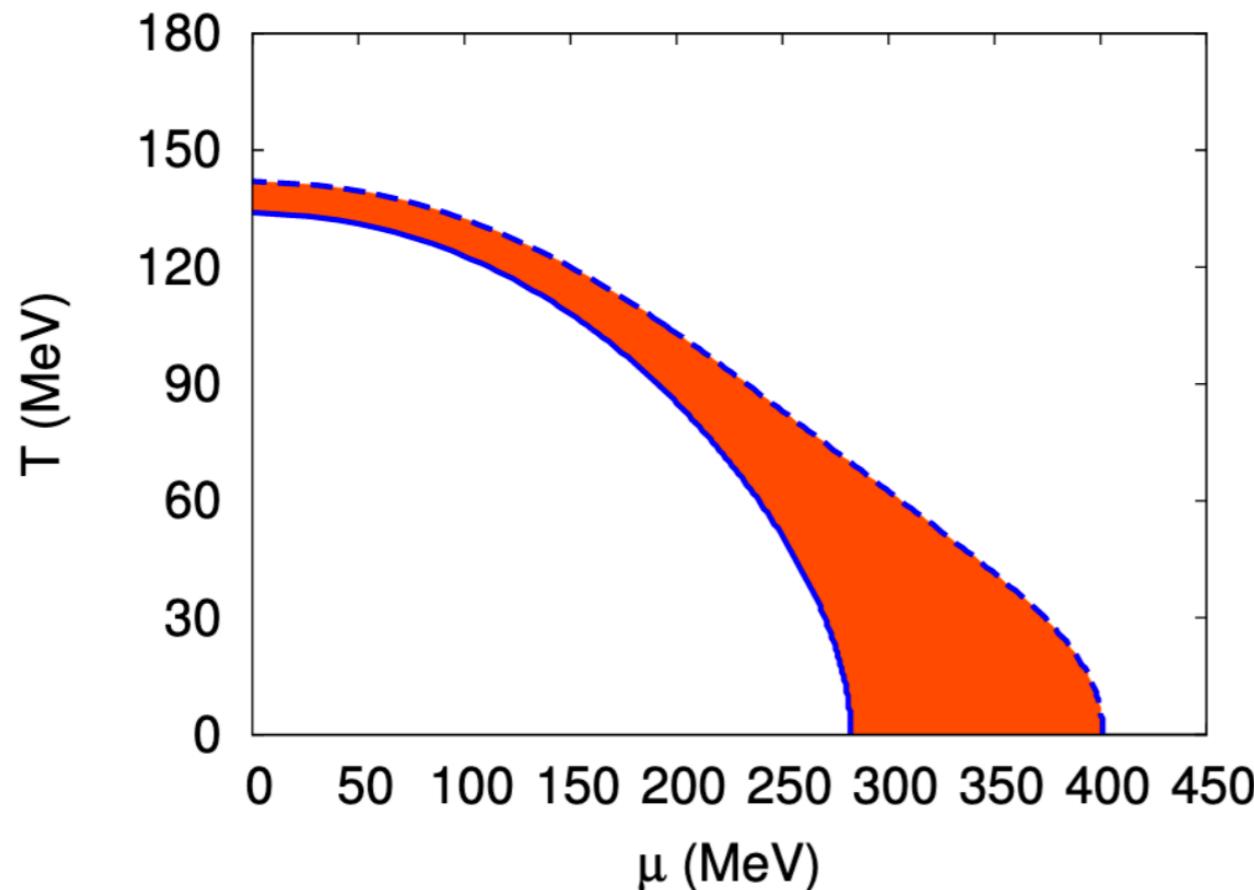
Formally quite similar to the NJL model in MFA

However, renormalizable!
A better starting point to study the role of fluctuations

The role of quark vacuum fluctuations

- Until few years ago typically discarded
- As a consequence, unrealistic phase structure already for homogeneous phases (no CP)

For inhomogeneous phases:



Inhomogeneous at $\mu = 0$?!
Not very likely...

What happens if we include
vacuum quark fluctuations?

The role of quark vacuum fluctuations

Including quark fluctuations requires special care in defining vacuum parameters to fit to physical quantities

Inconsistent definitions lead to inconsistent phenomenology, especially (but not only) when dealing with inhomogeneous phases!

The role of quark vacuum fluctuations

Including quark fluctuations requires special care in defining vacuum parameters to fit to physical quantities

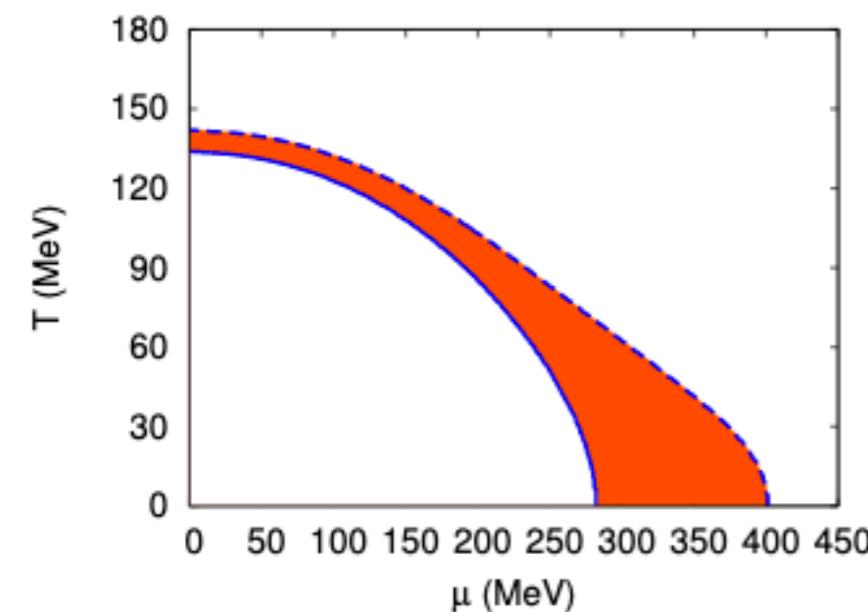
Inconsistent definitions lead to inconsistent phenomenology, especially (but not only) when dealing with inhomogeneous phases!

Fit to **pole** masses and **renormalized** decay constants

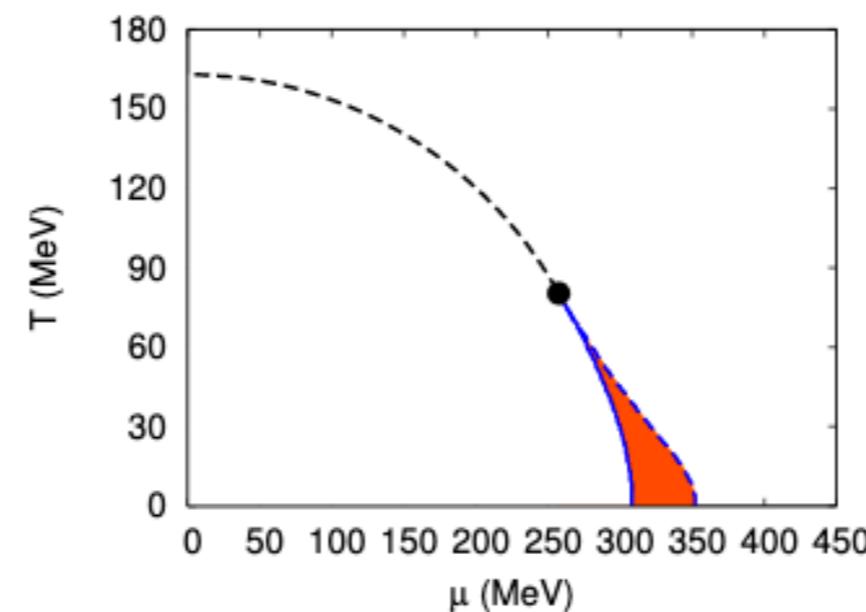
(instead of the commonly used
curvature masses - bare decay constant)

The role of quark vacuum fluctuations

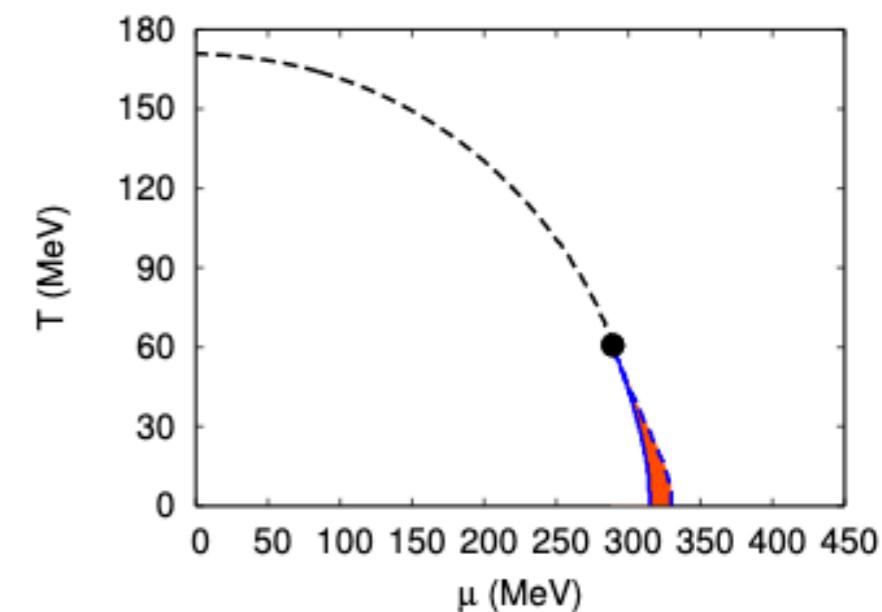
Including fluctuations....



$$\Lambda = 0$$

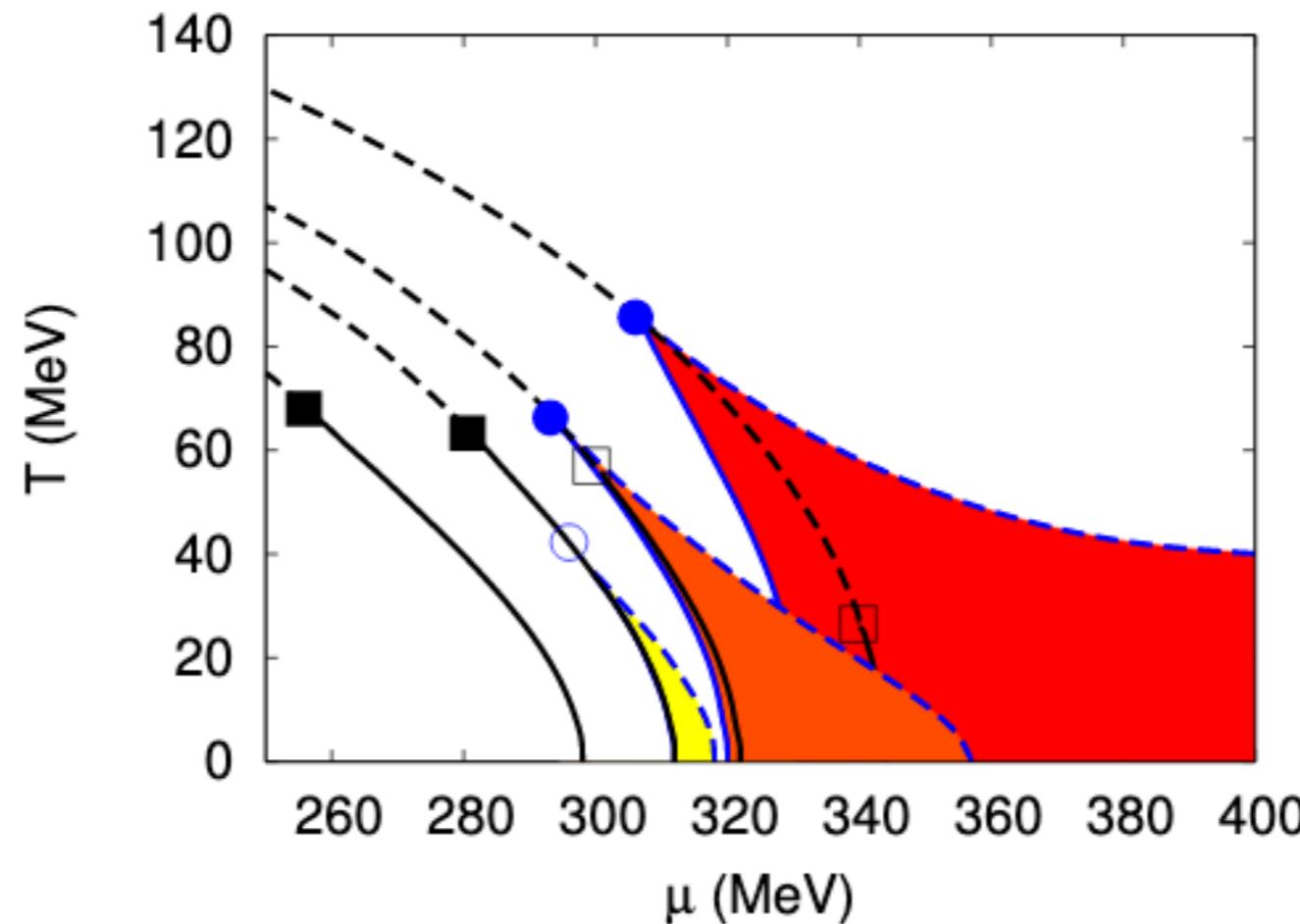


$$\Lambda = 600 \text{ MeV}$$



$$\Lambda \rightarrow \infty$$

Sigma mass influence

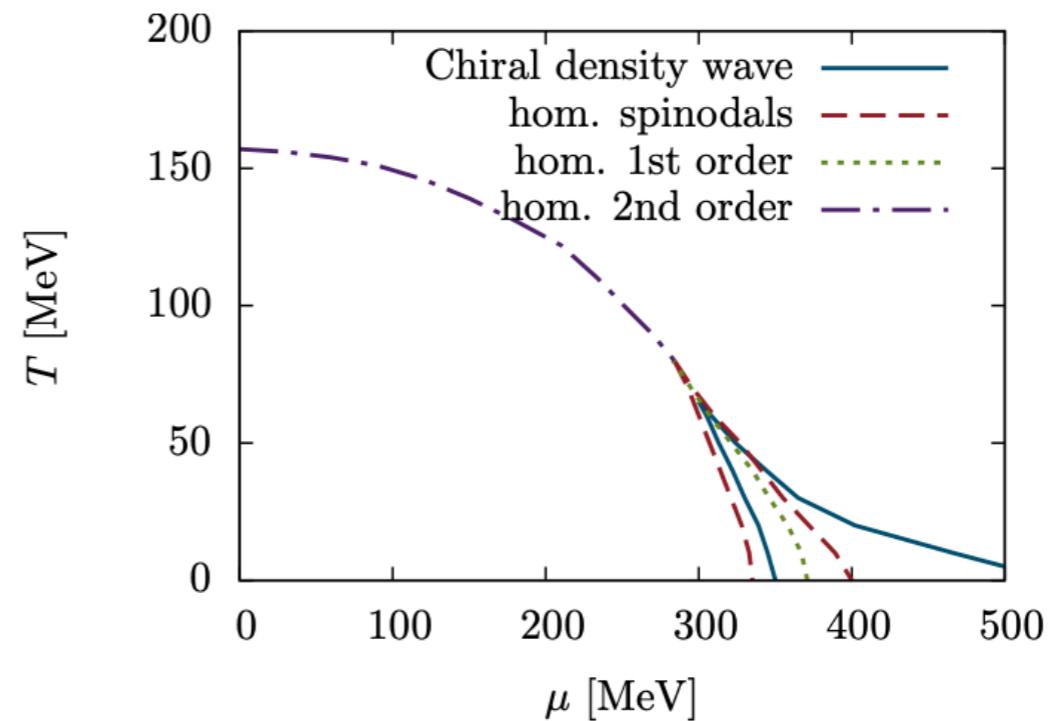


Renormalized limit, $m_\sigma = 550, 590, 610, 650$ MeV
($M_{vac} = 300$ MeV)

One might wonder...

Could inhomogeneous phases be a model “artifact” appearing in simplified quark models?

Unlikely, they appear also in Dyson-Schwinger studies

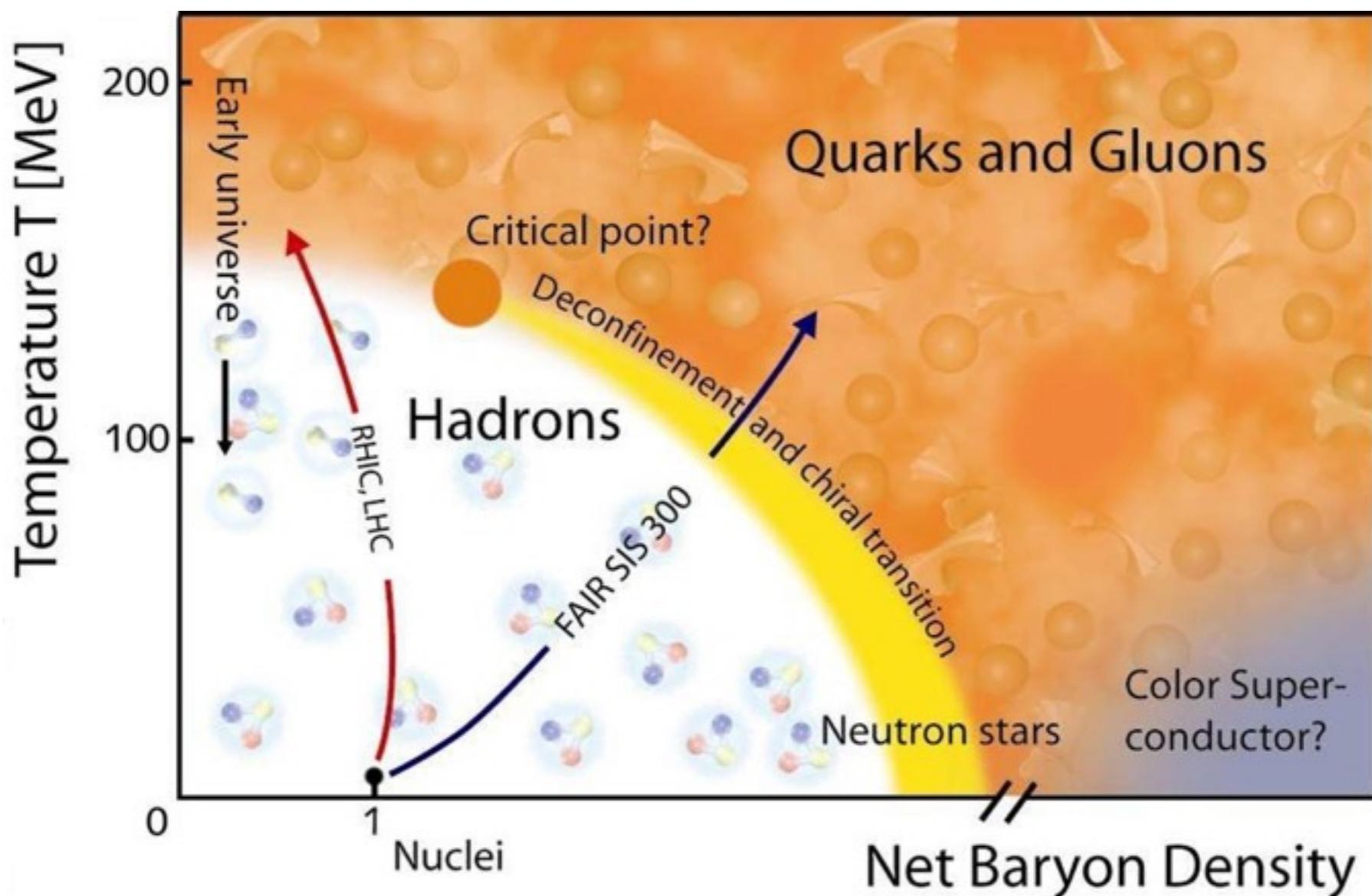


D.Müller, M.Buballa and J.Wambach,
Phys.Lett. B727 (2013) 240

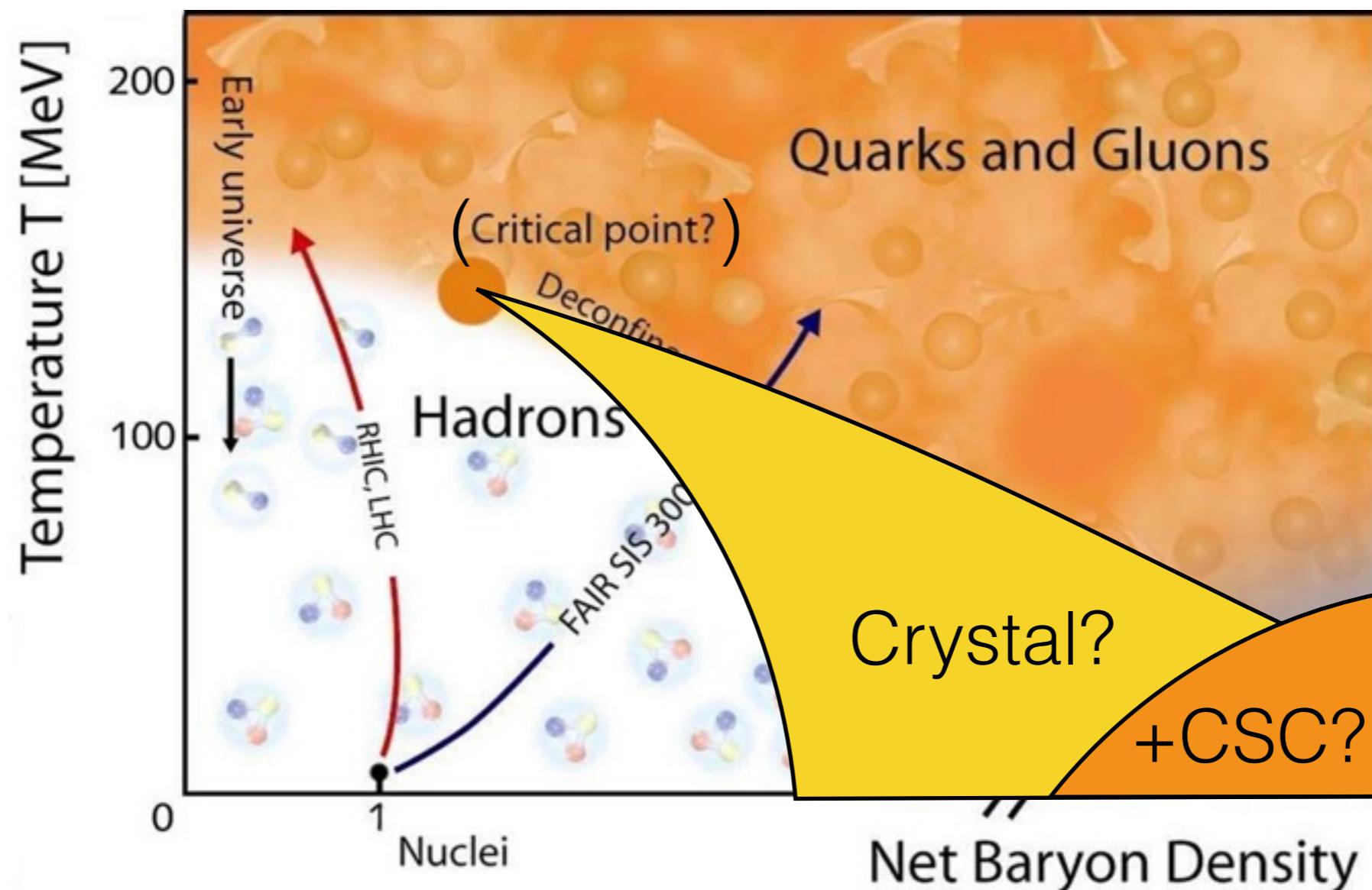
FRG approaches also seem to hint at their existence!

W.Fu, J.Pawlowski, F.Rennecke, arXiv:1909.02991

The QCD phase diagram people have in mind



The QCD phase diagram people *should* have in mind



Reviews: M.Buballa and SC,
Prog.Part.Nucl.Phys. 81 (2015) 39 - arXiv:1406.1367
Eur.Phys.J. A52 (2016) 57 - arXiv:1508.04361