QUANTUM CRITICALITY OF 2D FERMI SYSTEMS WITH $\mathbb{Z}_3$-SYMMETRIC QUADRATIC BAND TOUCHING

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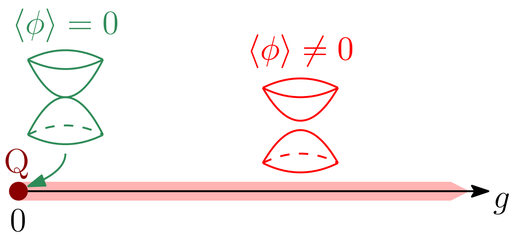
Introduction

Field-theoretic predictions (1-loop RG) for semimetals in $(2+1)$D with local 4-Fermi interaction g:

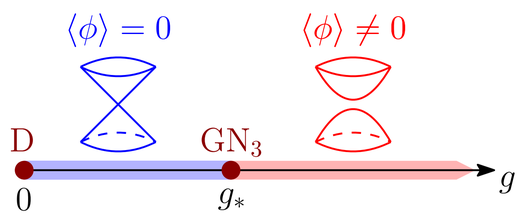
Quadratic band touching (QBT)



[Sun *et al.*, Phys. Rev. Lett. **103**, 046811 (2009)]

▶ unstable for all $g > 0$

Dirac semimetal (DSM)

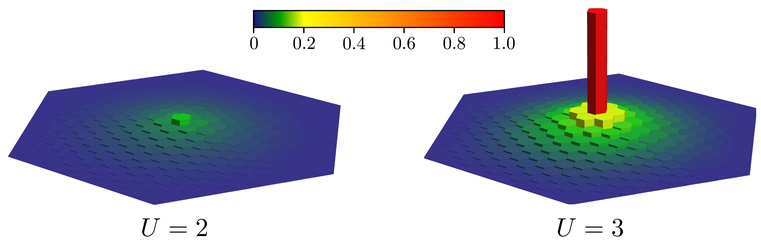


[Herbut, Phys. Rev. Lett. **97**, 146401 (2006)]

▶ stable for small $g < g_*$

Quantum Monte Carlo (QMC) for Hubbard model of spin-$\frac{1}{2}$ fermions on honeycomb bilayer

…nearest-neighbour tight-binding spectrum has QBTs at K-points of 1st Brillouin zone
…Order parameter: Antiferromagnetic structure factor



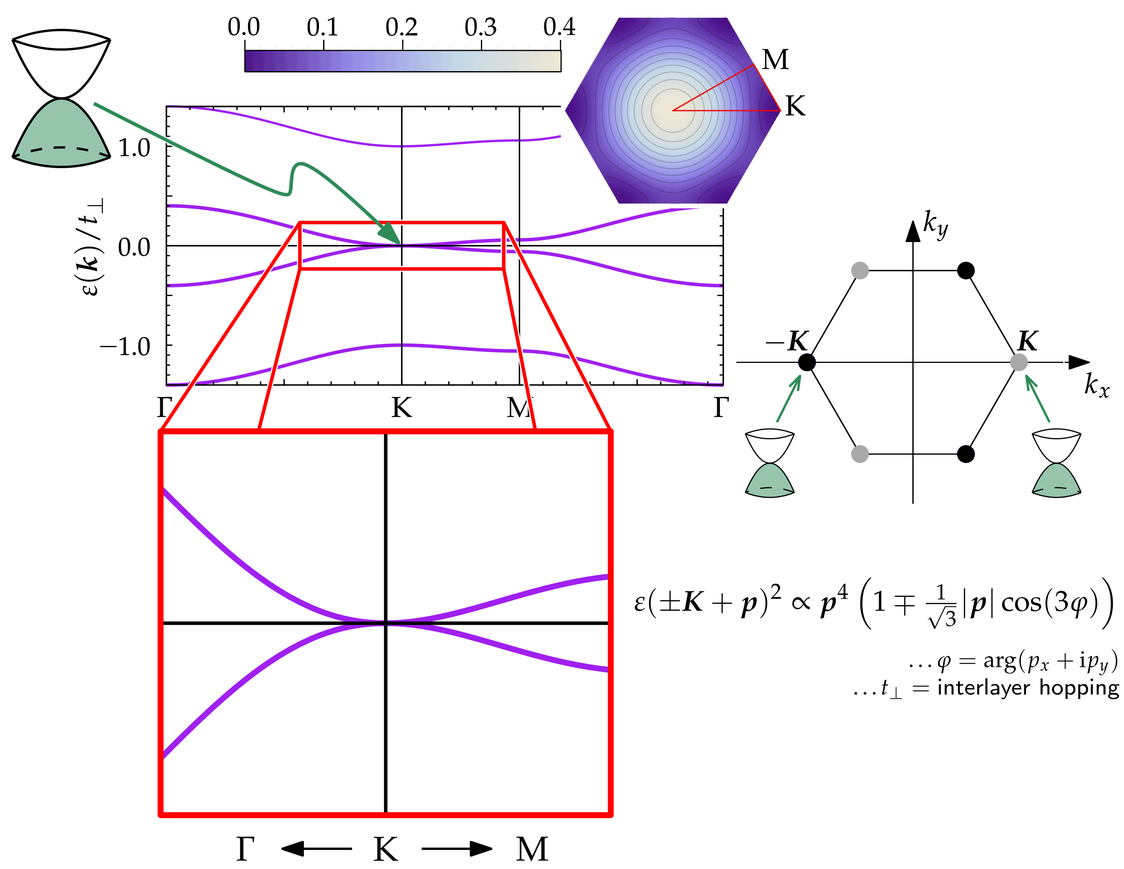
[Pujari *et al.*, Phys. Rev. Lett. **117**, 086404 (2016)]

▶ semimetal survives (sufficiently) weak interactions(!)

AIM: Improved RG equations

Effective Field Theory

▶ Rotational symmetry broken explicitly: $O(2) \to \mathbb{Z}_3$ …realizable only by irrelevant (higher-order) term!

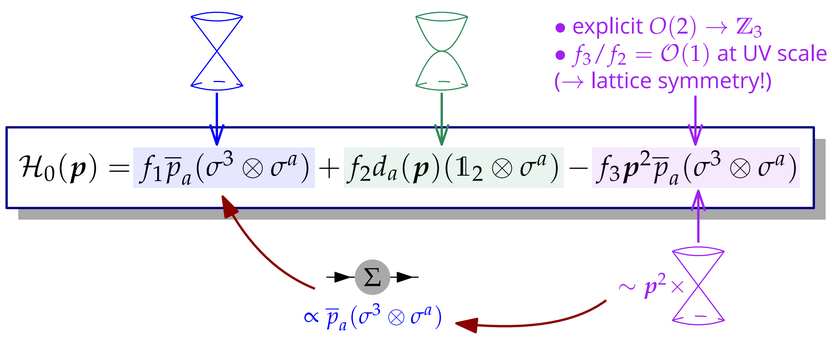


$$\varepsilon(\pm\boldsymbol{K}+\boldsymbol{p})^2 \propto \boldsymbol{p}^4\left(1 \mp \tfrac{1}{\sqrt{3}}|\boldsymbol{p}|\cos(3\varphi)\right)$$

…$\varphi = \arg(p_x + \mathrm{i}p_y)$
…$t_\perp$ = interlayer hopping

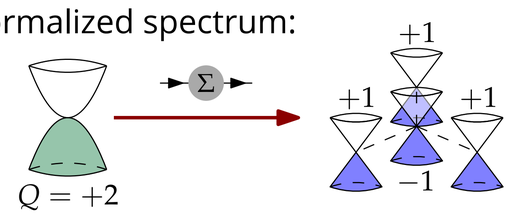
▶ Minimal Lagrangian (free part): $\mathcal{L}_0 = \psi^\dagger\left[\partial_\tau + \mathcal{H}_0(-\mathrm{i}\nabla)\right]\psi$

$(\tau, \boldsymbol{x})$ …$(2+1)$D Euclidean spacetime coordinates
ψ …4-spinors – 2 K-points ($\mathcal{T}$-symmetry partners!) × 2 touching bands



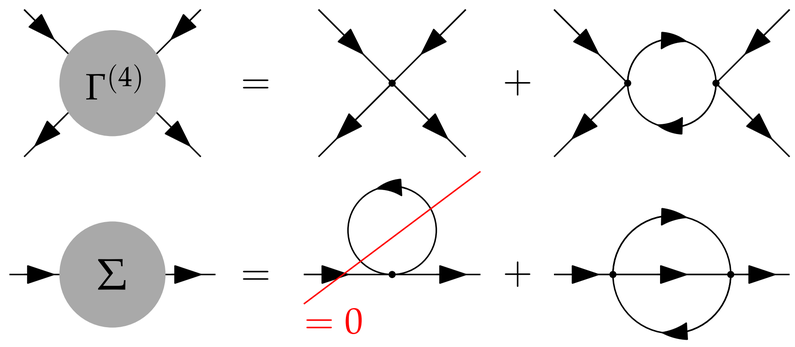
…$d_a(\boldsymbol{p}) = \left(p_x^2 - p_y^2, 2p_xp_y\right)$
…$\overline{p}_a = (p_x, -p_y)$

Renormalized spectrum:



[Pujari *et al.* (2016)]

Computational Details

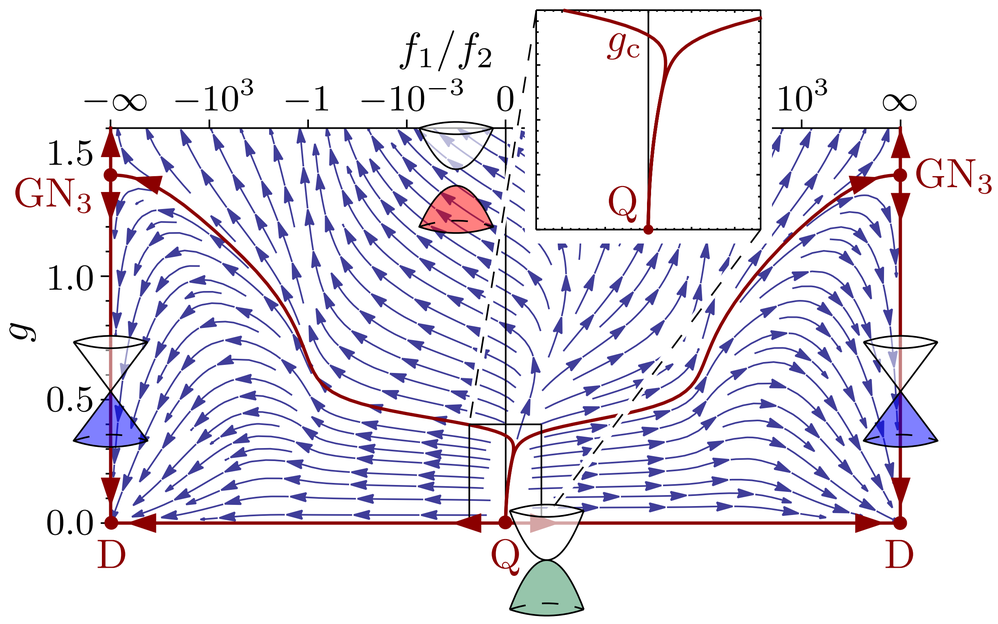


Challenges: 2-loop, but low symmetry – $O(3) \supset O(2) \supset \mathbb{Z}_3$
Trick: Evaluate in real space (exploit diagramme topology!)

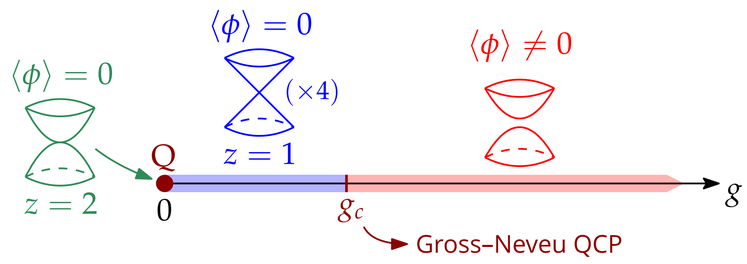
[Groote, Körner, and Pivovarov, Nucl. Phys. B. **542**, 515 (1999)]

Results

▶ Loop-corrected RG phase portrait

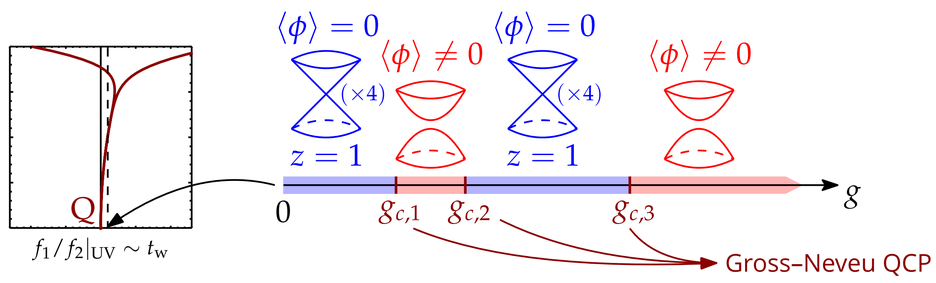


▶ Fate of $\mathbb{Z}_3$-invariant QBTs in deep IR ($T = 0$)



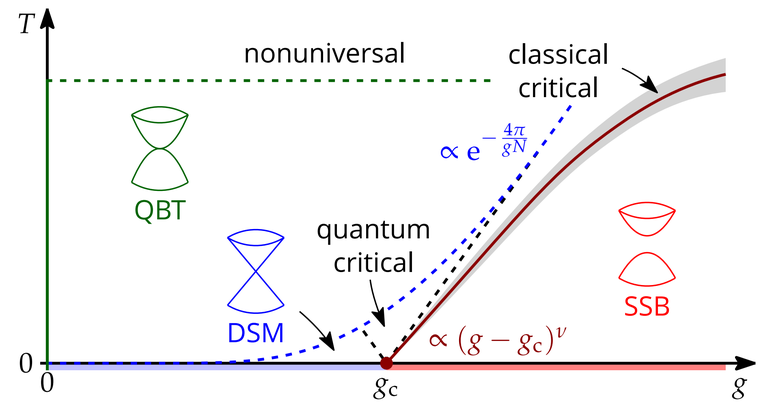
…QMC observations reproduced within explicit RG calculation ✓

▶ Finite (small) trigonal warping and 'reentrance



…Prediction: rich sequence of quantum phase transitions for "generic" trajectory through theory space

▶ Finite temperature $T > 0$ …T imposes RG scale at which flow is truncated



bilayer graphene: $g \sim 2\pi/\ln(T_*/T_c) \sim 0.6 \gtrsim g_c \sim 0.4$

$\sim t^2/t_\perp \sim 20\,\mathrm{eV}$ $\sim 5\,\mathrm{K}$

…Predictions: T-dependent crossover between $z = 1$ and $z = 2$ (observable in transport quantities – e.g., Hall coefficient)
…**bilayer graphene nearly critical – $g \sim g_c$?**

Remark (single-channel approximation). 4-Fermi part of Lagrangian restricted to $\mathcal{L}_{\mathrm{int}} = -\frac{1}{2}g[\psi^\dagger(\sigma^z\otimes\sigma^z)\psi]^2$ = leading instability (RG @ 1-loop) in t–V model for AB-stacked honeycomb bilayer [Vafek, Phys. Rev. B **82**, 205106 (2010)]
Remark (running of f_3/f_2). Irrelevant coupling $f_3/f_2 = -(2\sqrt{3})^{-1}$ (corresponds to tight-binding on AB-stacked honeycomb bilayer) held fixed; running of f_3/f_2 has negligible effect on qualitative features of phase diagrammes.

Further information: S. R., M. Vojta, and L. Janssen, Phys. Rev. B **98**, 245128 (2018) (*Editor's Suggestion*)

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