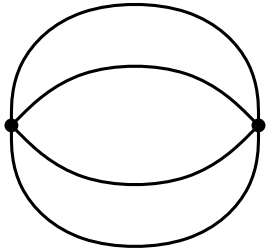
PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS

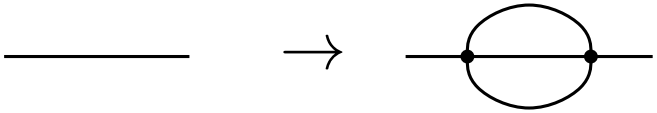
The melonic large-N limit: from SYK to tensor field theory

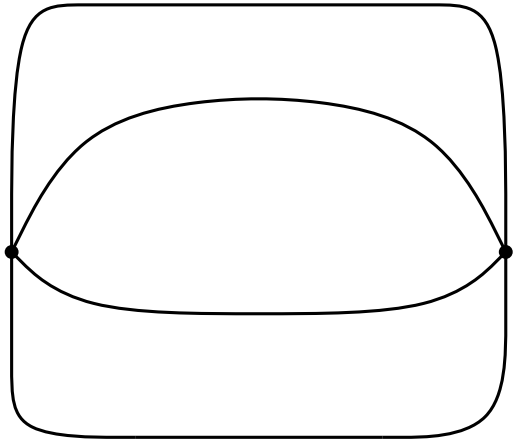
Sylvain Carrozza

Strongly-Interacting Field Theories
Jena – November 7-9 2019

Melon diagrams







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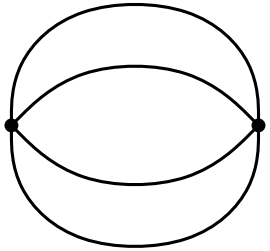

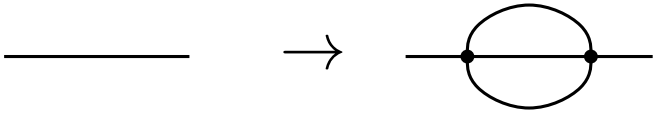

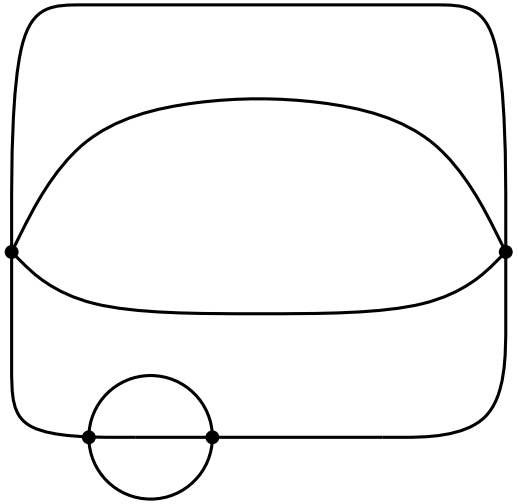

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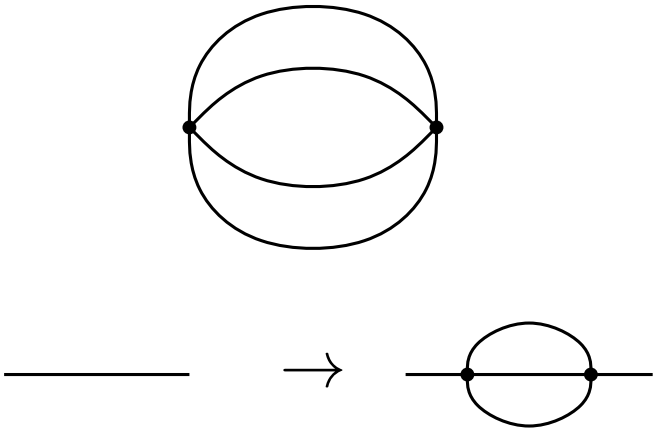

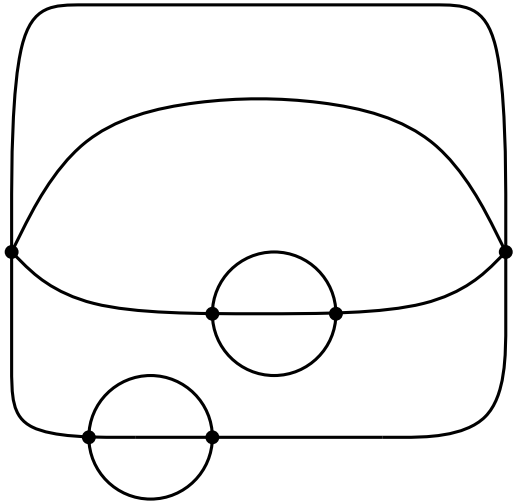

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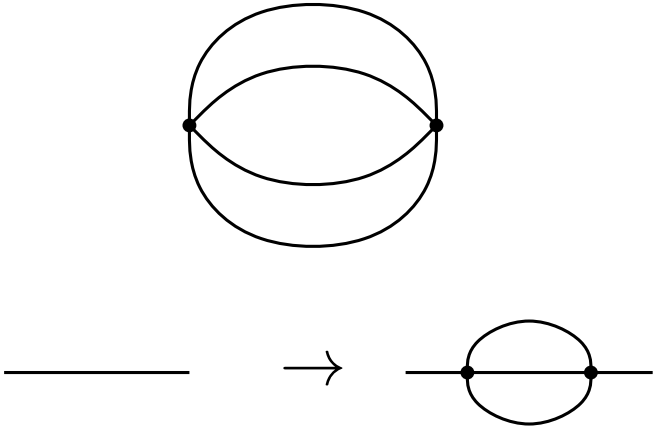

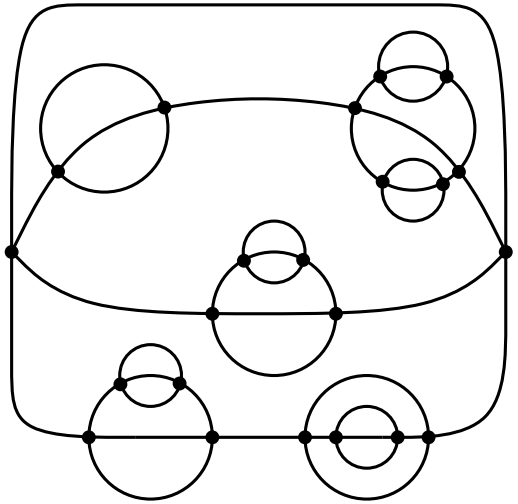

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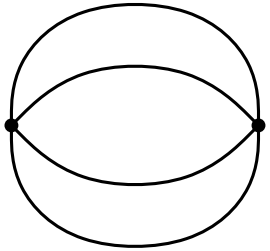

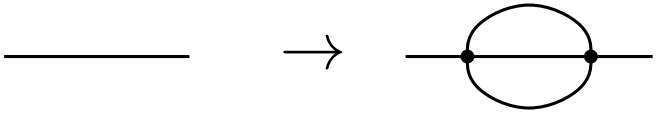

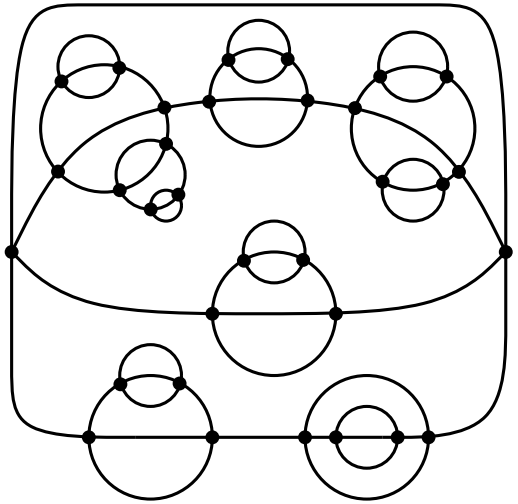

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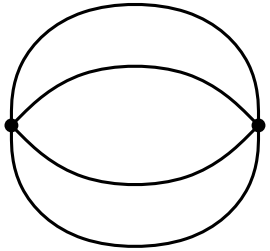

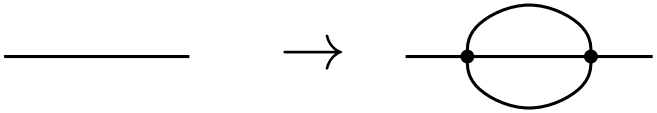

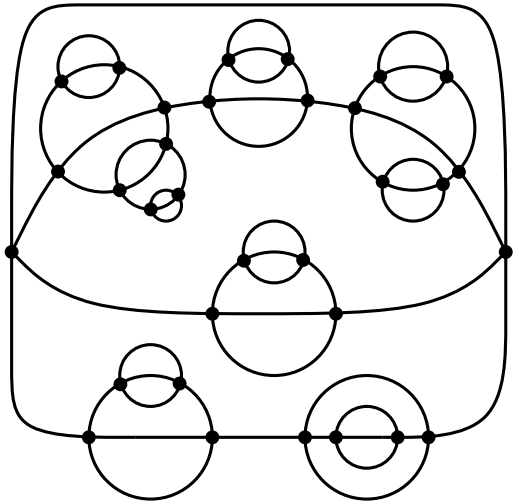

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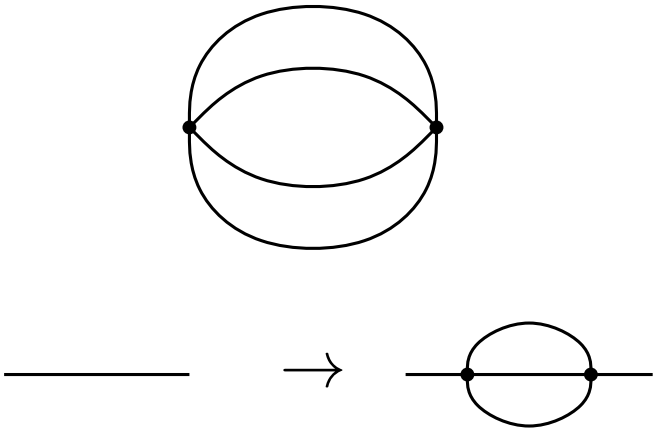
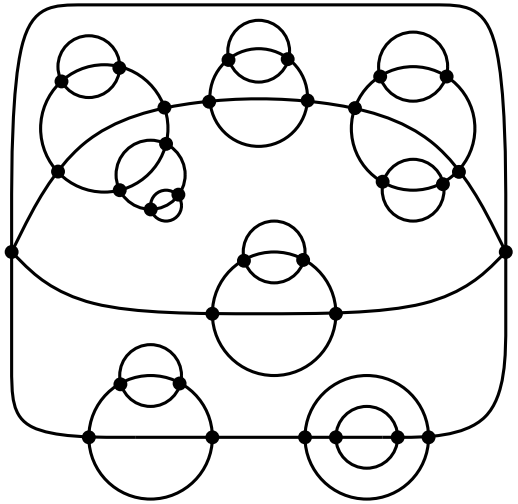




Leading-order Feynman diagrams in a variety of large-N theories:

Tensor models

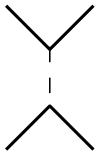
Sachdev-Ye-Kitaev model

Structural glass / machine learning

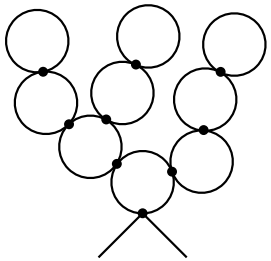
Three generic families of large N theories

Vector ϕ_a

$\frac{\lambda}{N}(\phi_a\phi_a)^2$



Bubble diagrams

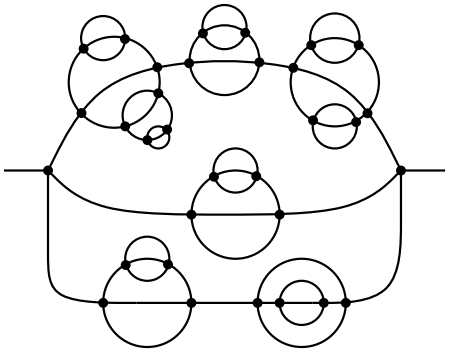


Easy

Tensor T_{abc}

$\frac{\lambda}{N^{3/2}} T_{aeb} T_{bfc} T_{ced} T_{dfa}$

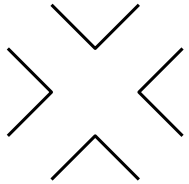
Melon diagrams



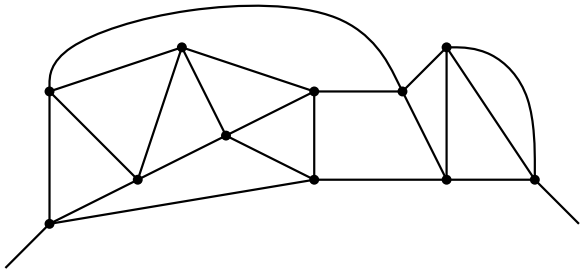
Tractable

Matrix M_{ab}

$\frac{\lambda}{N} M_{ab} M_{bc} M_{cd} M_{da}$



Planar diagrams



Hard

Melonic regime $\Rightarrow$ closed and often solvable systems of Schwinger-Dyson equations.

Outline

The SYK model

SYK model

[Sachdev, Ye, Georges, Parcollet '90s...; Kitaev '15, Maldacena, Stanford, Polchinski, Rosenhaus...]

Sachdev-Ye-Kitaev model

Disordered system of N Majorana fermions ψ_a in $d = 0 + 1$

$$H \sim J_{abcd}\, \psi_a\, \psi_b\, \psi_c\, \psi_d\,, \qquad \langle J_{abcd} \rangle = 0\,, \quad \langle J^2_{abcd} \rangle \sim \frac{\lambda^2}{N^3}$$

Interesting properties, for both condensed matter and high-energy physicists:

- solvable at large N
- emergent conformal symmetry at strong coupling
- same symmetry breaking as in Jackiw-Teitelboim quantum gravity $\rightarrow$ toy-models of quantum black holes
- maximal quantum chaos

[Maldacena, Shenker, Stanford '15]

Bi-local formalism

[Jevicki, Suzuki, Yoon '16...]

- Annealed partition function:

$$\langle \mathcal{Z} \rangle_J = \langle \int [\mathcal{D}\psi_i]\, \mathrm{e}^{-S_{\mathrm{SYK}}[\psi]} \rangle_J \,, \qquad S_{\mathrm{SYK}} = \int \mathrm{d}t \left(\frac{1}{2}\psi_a \partial_t \psi_a - \frac{1}{4!} J_{abcd} \psi_a \psi_b \psi_c \psi_d \right)$$

- Collective bi-local field:

$$\tilde{G} := \frac{1}{N} \sum_i \psi_i(t_1)\psi_i(t_2) \,, \qquad \langle \mathcal{Z} \rangle_J = \int \mathcal{D}\tilde{G}\, \mathcal{D}\tilde{\Sigma}\, \mathrm{e}^{-N S_{\mathrm{eff}}[\tilde{G},\tilde{\Sigma}]} \,,$$

$$S_{\mathrm{eff}}[\tilde{G},\tilde{\Sigma}] = \ln \mathrm{Pf} \left(\partial_t - \tilde{\Sigma} \right) - \frac{1}{2} \int \mathrm{d}t_1 \mathrm{d}t_2 \left(\tilde{\Sigma}(t_1,t_2)\tilde{G}(t_1,t_2) - \frac{\lambda^2}{4} \tilde{G}(t_1,t_2)^4 \right)$$

- Large-N physics from saddle-point $\Rightarrow$ 2-point function:

$$G(t_1,t_2) = \frac{1}{N} \sum_i \langle \psi_i(t_1)\psi_i(t_2) \rangle$$

Saddle-point equation

$$G(t_1, t_2) = G_{\text{free}}(t_1, t_2) + \lambda^2 \int \mathrm{d}t\mathrm{d}t' \, G_{\text{free}}(t_1, t) \, [G(t, t')]^3 \, G(t', t_2)$$

- Infrared / strong-coupling limit ($\lambda|\omega| \gg 1$, or $\lambda\beta \gg 1$ if β finite):

$$\lambda^2 \int \mathrm{d}t \, G(t_1, t) \, [G(t, t_2)]^3 = -\delta(t_1 - t_2)$$

- Emergent conformal invariance: reparametrization $t \mapsto f(t)$

$$G(t_1, t_2) \mapsto |f'(t_1) f'(t_2)|^{1/4} G(f(t_1), f(t_2))$$

- Conformal solution:

$$G(t_1, t_2) = -\left(\frac{1}{4\pi\lambda^2}\right)^{1/4} \frac{\mathrm{sgn}(t_1 - t_2)}{|t_1 - t_2|^{2\Delta}}, \qquad \Delta = \frac{1}{4}$$

Symmetry breaking

$$G(t_1, t_2) = G_{\text{free}}(t_1, t_2) + \lambda^2 \int \mathrm{d}t\mathrm{d}t' \, G_{\text{free}}(t_1, t) \, [G(t, t')]^3 \, G(t', t_2)$$

- Spontaneous breaking to $\mathrm{SL}(2, \mathbb{R}) \Rightarrow$ Goldstone modes $f \in \mathrm{Diff}(\mathbb{R})/\mathrm{SL}(2, \mathbb{R})$?
- G_{free} term $\Rightarrow$ explicit breaking $\Rightarrow$ non-zero Schwarzian effective action

$$S_{\text{eff}}[f] \sim \frac{1}{\lambda} \int \mathrm{d}t \, \{f, t\}$$

[Maldacena, Stanford '16]

($\{f, t\} = \frac{f'''}{f'} - \frac{3}{2} \left(\frac{f''}{f'}\right)^2$ is the *Schwarzian derivative*)

- The Schwarzian action governs very interesting physics in
  - SYK $\rightarrow$ maximal quantum chaos;
  - 2d quantum gravity $\rightarrow$ non-trivial dynamics of asymptotic fields.

$\Rightarrow$ near AdS_2 / near CFT_1 correspondence.

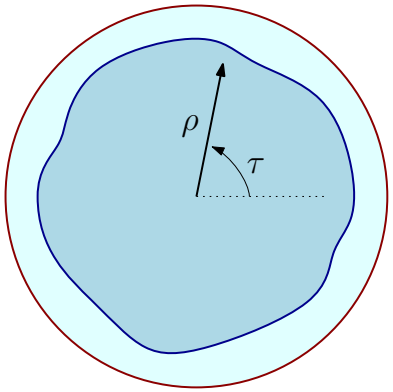
Jackiw-Teitelboim gravity

- Near-horizon geometry of near-extremal Reissner–Nordström BHs $\sim AdS_2 \times S_2$

- Near-horizon dynamics captured by **2d dilaton Jackiw-Teitelboim gravity**

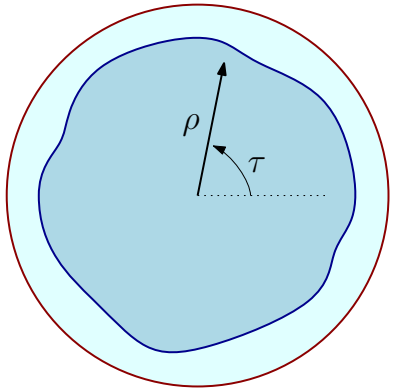
$$
\begin{aligned}
S[g,\phi] \sim & -\frac{S_0}{2\pi}\left[\frac{1}{2}\int_{\mathcal{M}}\sqrt{g}R + \int_{\partial\mathcal{M}}\sqrt{h}K\right] && \textit{topological} \\
& -\frac{1}{2}\int_{\mathcal{M}}\sqrt{g}\phi\,(R+2) && \textit{bulk} = \textit{hyperbolic} \\
& -\int_{\partial\mathcal{M}}\sqrt{h}\phi_b\,(K-1) && \textit{"boundary graviton"}
\end{aligned}
$$

Euclidean solutions = cut-outs from the hyperbolic plane



$$ds^2 = d\rho^2 + \sinh^2\rho\, d\tau^2$$

Jackiw-Teitelboim gravity



$$ds^2 = d\rho^2 + \sinh^2\rho \, d\tau^2$$

- Non-trivial zero modes of bulk piece $\Rightarrow$ $\mathrm{Diff}(\mathbb{S}^1)/\mathrm{SL}(2,\mathbb{R})$ symmetry.
- Explicit breaking by boundary $\Rightarrow$ non-trivial boundary dynamics.
- Effective Schwarzian action:

$$S_{eff}[\tau] = -\gamma \int_0^\beta \mathrm{d}u \{\tan\frac{\tau(u)}{2}, u\}$$

[Maldacena, Stanford, Yang '16...]

($\{f,t\} = \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$ is the *Schwarzian derivative*)

Near AdS_2 / near CFT_1 duality

Boundary		Bulk
Strongly-coupled SYK-like QM	$\longleftrightarrow$	AdS_2 JT gravity
Conformal breaking	$\longleftrightarrow$	Boundary graviton
Conformal spectrum	$\longleftrightarrow$	Matter

Physical relevance: Near-horizon dynamics of near-extremal charged black holes.

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Physical relevance: Near-horizon dynamics of near-extremal charged black holes.

Sum over topologies with arbitrary number of boundaries can be performed explicitly:

$$\boxed{\text{JT quantum gravity} \Leftrightarrow \text{random matrix model}}$$

[Saad, Shenker, Stanford '19; Stanford, Witten '19]

Tensor models

Pre-history

Statistical approach to quantum gravity

1) Feynman expansion $\leftrightarrow$ Weighted sum of discrete geometries

2) criticality $\rightarrow$ continuum limit $\rightarrow$ random geometry

Successful template. random matrices quantum gravity in $d = 2$.

[Ambjørn, Brézin, David, Durhuus, Fröhlich, Itzykson, Jónsson, Kazakov, Parisi, Zuber... '80s]

Higher dimensions. Motivated the theory of random tensors.

[Ambjørn, Durhuss, Jónsson '91; Boulatov '92; Ooguri '92; Freidel, Oriti, Gurau, Bonzom, Rivasseau... '10s]

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Challenge. Continuum limit of melons $\rightarrow$ "branched polymers" (random continuous tree)

$d_{\mathrm{H}} = 2$ and $d_{\mathrm{S}} = 4/3$ [Ambjørn, Durhuus, Jonsson '90; Bialas, Burda '96; Gurau, Ryan '13]

How can we escape melonic universality in this set-up?

FRG approach: [Eichhorn, Koslowski, Lumma, Peireira...]

Zoo of melonic limits

$a, b, c, \ldots = 1, \ldots, N$

- *colored* tensor models: 4 tensor fields, $O(N)^6$ symmetry

$$T^{(0)}_{abc}\, T^{(1)}_{dfa}\, T^{(2)}_{ebd}\, T^{(3)}_{cfe}$$

- *uncolored* tensor models: 1 tensor field, $O(N)^3$ symmetry

$$T_{aeb}\, T_{cfb}\, T_{ced}\, T_{afd}$$

- *irreducible* tensor models: 1 tensor field, $O(N)$ symmetry

$$T_{aeb}\, T_{bfc}\, T_{ced}\, T_{dfa}$$

- multi-matrix models, with large number of matrices

[Ferrari, Schaposnik Massolo, Valette, Rivasseau...]

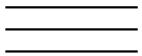
$$\mathrm{Tr}(M^{(i)} M^{(j)} M^{(i)} M^{(j)})$$

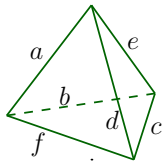
- disordered systems: SYK, p-spin models...

$$J_{abcd}\, \psi_a \psi_b \psi_c \psi_d$$

Brief history

$$\mathcal{F}(\lambda) = \ln \int \mathrm{d}T \, \exp\left(-T_{abc}\, T_{abc} + \frac{\lambda}{N^{\alpha}}\, T_{aeb}\, T_{bfc}\, T_{ced}\, T_{dfa} \right)$$





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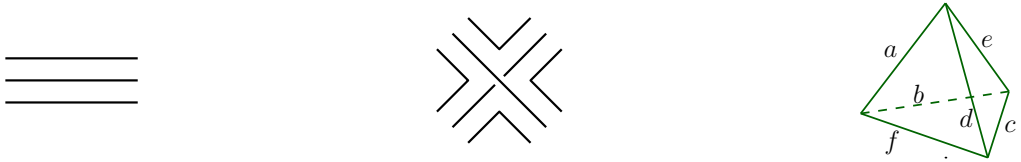

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$$= \sum_{\omega \in \mathbb{N}/2} N^{3-\omega} \, \mathcal{F}_\omega(\lambda) = N^3 \left(\mathcal{F}_0(\lambda) + \frac{1}{\sqrt{N}} \mathcal{F}_{1/2}(\lambda) + \frac{1}{N} \mathcal{F}_1(\lambda) + \cdots \right)$$

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- 2010 – 2015: solution assuming **no** permutation symmetry on the indices

  $\rightarrow \mathrm{O}(N)^3$ symmetry of the action

  [Gurau '10; Bonzom, Rivasseau, Riello, ... SC, Tanasa '15]

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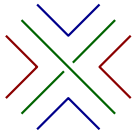
- 2017 – 2019: tensors with (anti)-symmetrization of the indices finally understood!

  $\rightarrow \mathrm{O}(N)$ symmetry of the action

  [Benedetti, SC, Gurau, Kolanowski, Pozsgay...]

Melonic dominance in $O(N)^3$ models

$$\frac{\lambda}{N^{3/2}} T_{aeb} T_{cfb} T_{ced} T_{afd}$$



$$A(G) \sim N^{-\omega} \text{ with } \omega = 3 + \tfrac{3}{2} V - F$$

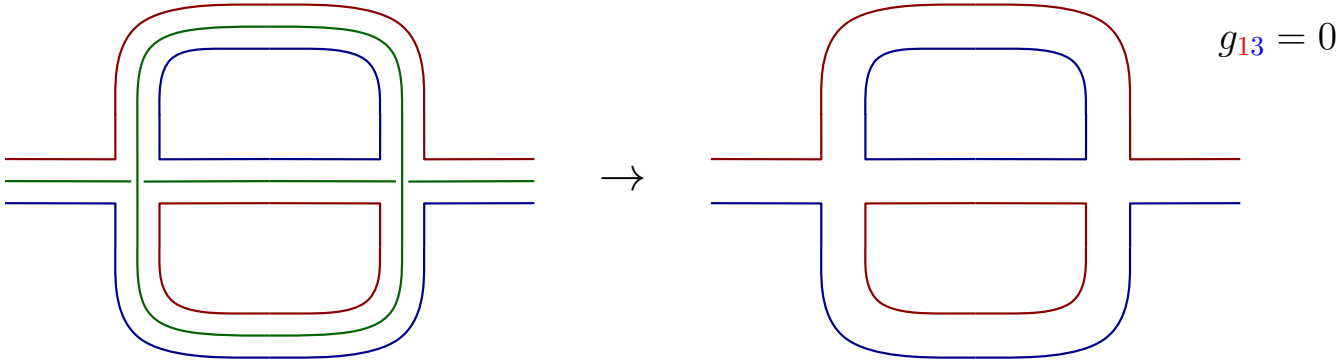
Existence of the large N melonic limit

$$\omega \geq 0$$

$$G \text{ leading order } \Leftrightarrow \omega = 0 \Leftrightarrow G \text{ is a melon diagram}$$

Idea of proof: melons are "super-planar" i.e. they have planar jackets

$$\omega := g_{13} + g_{12} + g_{23} \in \frac{\mathbb{N}}{2}$$



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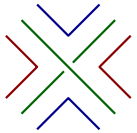

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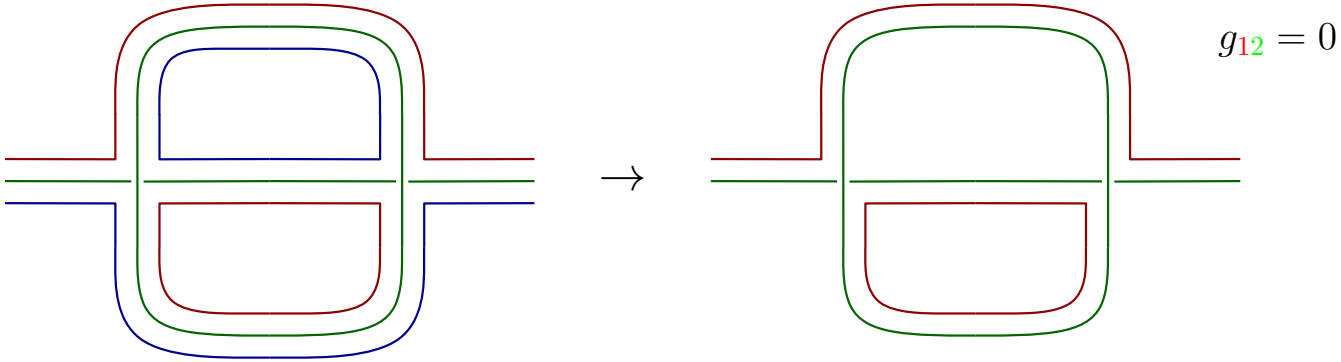
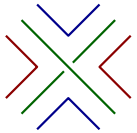


Melonic dominance in $O(N)^3$ models

$$\frac{\lambda}{N^{3/2}} T_{aeb} T_{cfb} T_{ced} T_{afd}$$



$$A(G) \sim N^{-\omega} \text{ with } \omega = 3 + \tfrac{3}{2} V - F$$

Existence of the large N melonic limit

$$\omega \geq 0$$

$$G \text{ leading order } \Leftrightarrow \omega = 0 \Leftrightarrow G \text{ is a melon diagram}$$

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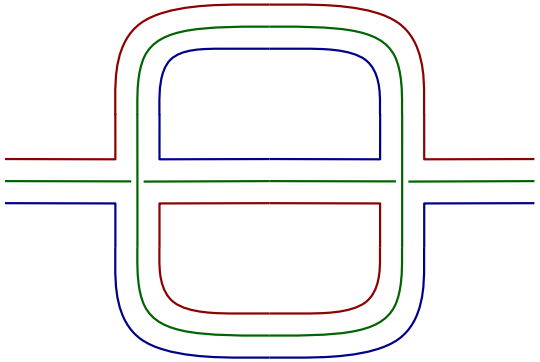
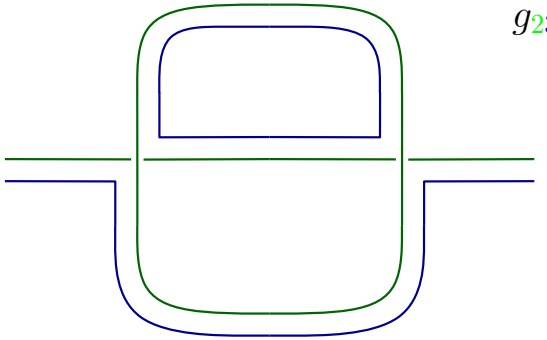


$\rightarrow$



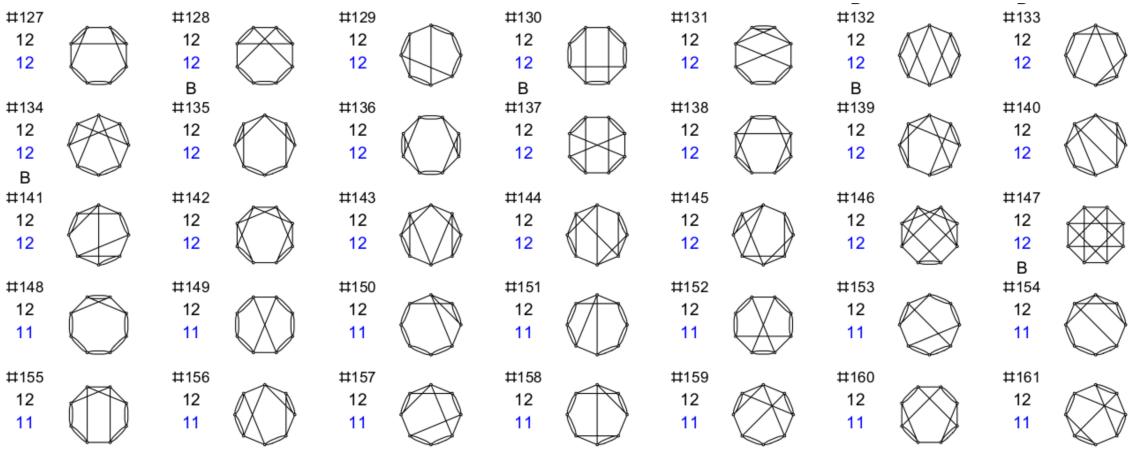
$g_{23} = 0$

Conjecture (Klebanov–Tarnopolsky)

The large N expansion exists for $\mathrm{O}(N)$ symmetric **traceless** tensors.

Evidence. Explicit numerical check of all diagrams up to order λ^8.

[Klebanov, Tarnopolsky, JHEP '17]



Proof and further generalizations.

1. $\mathrm{O}(N)$ symmetric traceless or antisymmetric [Benedetti, SC, Gurau, Kolanowski, Commun. Math. Phys. '19]
2. $\mathrm{O}(N)$ mixed symmetric traceless [SC, JHEP '18]
3. $\mathrm{Sp}(N)$ irreducible [SC, Pozsgay, Nucl. Phys. B '19]

Theorem

The large N expansion exists for arbitrary **irreducible tensor representations**.

Melonic dominance ($d = 0$)

The full connected 2-point function verifies:

$$\langle T_{a_1 a_2 a_3} T_{b_1 b_2 b_3} \rangle_{\mathrm{c}} = \left(\underbrace{K_0(\lambda)}_{\text{melons}} + \mathrm{O}(1/\sqrt{N}) \right) \mathbf{P}_{a_1 a_2 a_3, b_1 b_2 b_3}$$

- Holds whenever $T_{a_1 a_2 a_3}$ is **irreducible**.
- $K_0(\lambda)$ non-perturbative in λ:

$$K_0(\lambda) = 1 + \mathrm{cte}\, \lambda^2\, K_0(\lambda)^4$$

SYK-like tensor models

- **Sachdev-Ye-Kitaev** model = disordered system of N Majorana fermions

  [Sachdev, Ye, Georges, Parcollet '90s...; Kitaev '15, Maldacena, Stanford, Polchinski, Rosenhaus...]

$$H_{\rm int} \sim J_{i_1 i_2 i_3 i_4}\, \psi_{i_1}\, \psi_{i_2}\, \psi_{i_3}\, \psi_{i_4}\,, \qquad \left\langle J_{i_1 i_2 i_3 i_4} \right\rangle \sim 0\,, \quad \left\langle J^2_{i_1 i_2 i_3 i_4} \right\rangle \sim \frac{J^2}{N^3}$$

- Many interesting properties:
  - solvable at large N
  - emergent conformal symmetry at strong coupling
  - same symmetry breaking as in Jackiw-Teitelboim quantum gravity $\rightarrow$ toy-models of quantum black holes
  - maximal quantum chaos

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- Same melonic large N limit as tensor models [Witten '16]

$\rightarrow$ **SYK-like quantum-mechanical models**:

- same qualitative properties at large N and strong coupling;
- **no disorder**.

$\rightarrow$ New class of **QFTs** with solvable large N limits.

Klebanov–Tarnopolsky model

Tensor quantum mechanics of N^3 Majorana fermions: [Klebanov, Tarnopolsky '16]

$$S = \int dt \left(\frac{\mathrm{i}}{2} \psi_{i_1 i_2 i_3} \partial_t \psi_{i_1 i_2 i_3} + \frac{\lambda}{4 N^{3/2}} \psi_{i_1 i_2 i_3} \psi_{i_4 i_5 i_3} \psi_{i_4 i_2 i_6} \psi_{i_1 i_5 i_6} \right)$$

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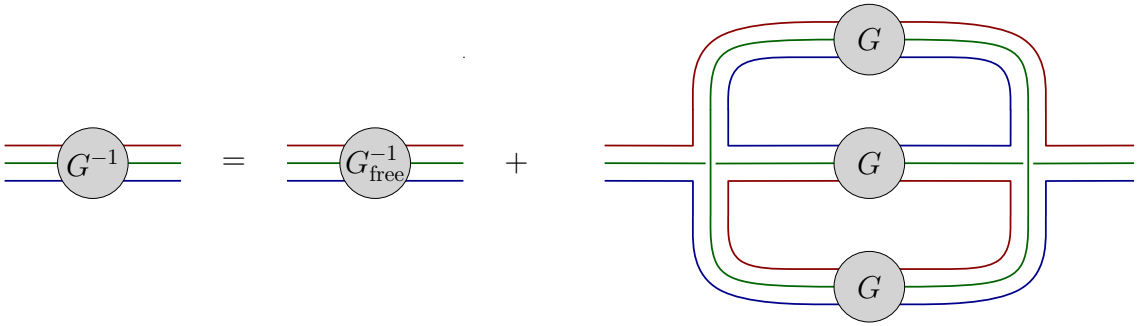
- Melonic dominance at large $N \Rightarrow$ closed Schwinger-Dyson equation: [SC, Tanasa '15]



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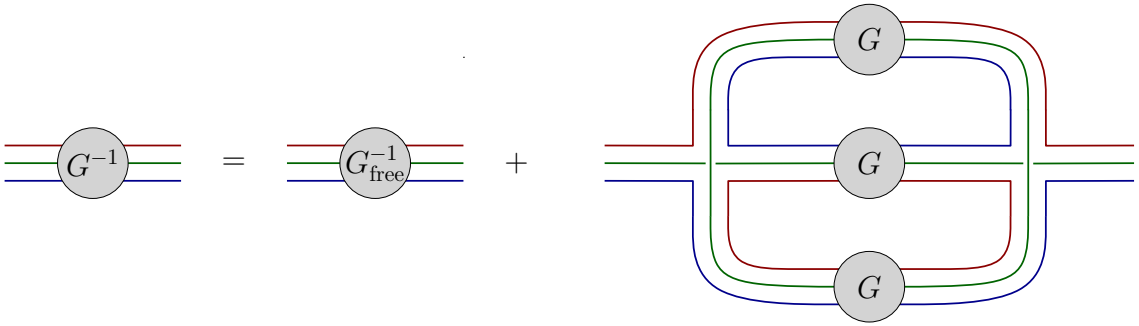

- Same melonic equation as in SYK:

$$\langle T(\psi_{a_1 a_2 a_3}(t_1) \psi_{b_1 b_2 b_3}(t_1)) \rangle \equiv G(t_1, t_2) \prod_{i=1}^{3} \delta_{a_i, b_i}$$

$$G(t_1, t_2) = G_{\text{free}}(t_1, t_2) + \lambda^2 \int \mathrm{d}t \mathrm{d}t' \, G_{\text{free}}(t_1, t) \, [G(t, t')]^3 \, G(t', t_2)$$

SYK Vs SYK-like tensor models

- SYK and SYK-like tensor models lead to similar IR physics:
  - near-conformality
  - chaos
  - conformal spectra from ladder diagrams

- Specificities of SYK-like tensor models:
  - no disorder $\Rightarrow$ fully quantum
  - standard large N theory $\Rightarrow$ gauging + natural generalizations to higher dimension
  - no simple collective path-integral formalism
    (but see 2PI effective action: [Benedetti, Gurau '18])
  - $\#\{\text{states}\} \sim \exp(\text{cte} \times N^3) \Rightarrow$ open numerical challenges

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N	Number of singlets
1	2
2	36
3	595 354 780

Real $\mathrm{O}(2\mathrm{N})^3$

(Klebanov-Tarnopolsky)

N	Number of singlets
1	3
2	39
3	170 640

Complex $\mathrm{USp}(2\mathrm{N})$ symmetric

(SC-Pozsgay)

Large-N tensor field theory

Unlike SYK, tensor models naturally fit in the framework of **local quantum field theory**.

Natural research programme:

Investigate the properties of melonic large N QFTs in $d \geq 2$.

Bosonic: [Giombi, Klebanov, Tarnopolsky '17 ; Giombi, Klebanov, Popov, Prakash, Tarnopolsky '18; Benedetti, Deleporte '18 ; Benedetti, Gurau, Harribey '19...]

Fermionic: [Prakash, Sinah '17 ; Benedetti, SC, Gurau, Sfondrini '17...]

Why it is interesting:

- only diagrams that proliferate are melons $\Rightarrow$ non-perturbative analytic control (summable)
- melons are bi-local $\Rightarrow$ anomalous dimensions $\Rightarrow$ non-trivial CFTs and RG flows
- 4-point functions = sums of ladder diagrams $\Rightarrow$ non-perturbative access to the spectrum

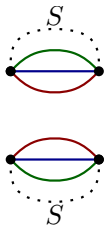
1st example: four-fermion theory, $d = 2$

Action

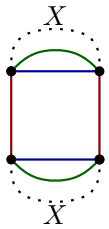
Tensorial Gross-Neveu model, with e.g. $O(N)$ Majorana fermions.

$$S_N = \frac{1}{2}\int d^2x\, \psi_{i_1 i_2 i_3} \partial\!\!\!/ \psi_{i_1 i_2 i_3}$$

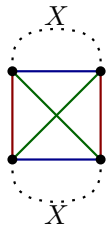
$$- \frac{\lambda_0}{4N^3}\int d^2x$$



$$- \sum_{X=S,V,P} \frac{\lambda_1^X}{4N^2}\int d^2x$$



$$- \sum_{X=S,V,P} \frac{\lambda_2^X}{4N^{3/2}}\int d^2x$$



Spinorial contractions with: $\mathbf{1}$ (S), γ_μ (V) or γ_5 (P) insertions.

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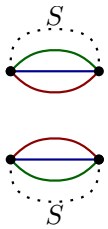
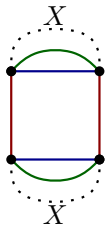
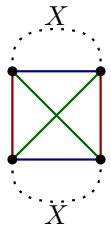
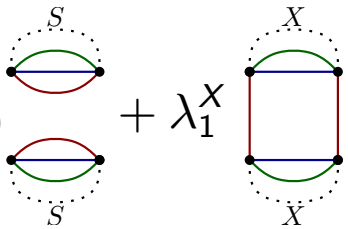
$$-\frac{\lambda_0}{4N^3}\int d^2x$$ 

$$-\sum_{X=S,V,P}\frac{\lambda_1^X}{4N^2}\int d^2x$$ 

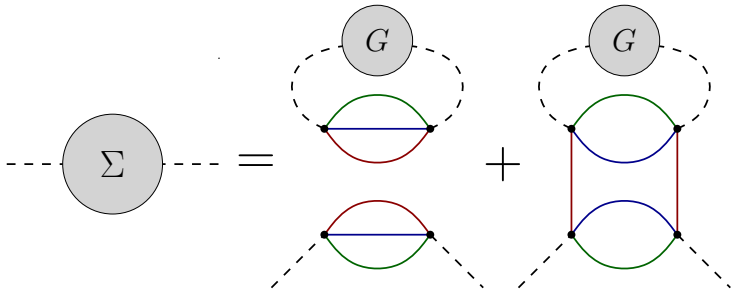
$$-\sum_{X=S,V,P}\frac{\lambda_2^X}{4N^{3/2}}\int d^2x$$ 

Spinorial contractions with: $\mathbf{1}$ (S), γ_μ (V) or γ_5 (P) insertions.

1. $\lambda_0 > 0$, $\lambda_1^S > 0$, $\lambda_2 = 0$:
   - Tadpole diagrams $\Rightarrow$ non-perturbative generation of mass in the IR
   - Asymptotic freedom.
2. $\lambda_0 = \lambda_1^S = 0$ and $\lambda_2 \neq 0$:
   - Melonic Schwinger-Dyson equation
   - Free part still relevant at strong coupling $\Rightarrow$ no IR conformal regime

λ_0 

$+ \lambda_1^X$ $\Rightarrow$ Spontaneous generation of mass and **asymptotic freedom**



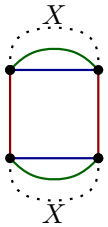
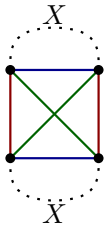
- Large-N Schwinger-Dyson equations:

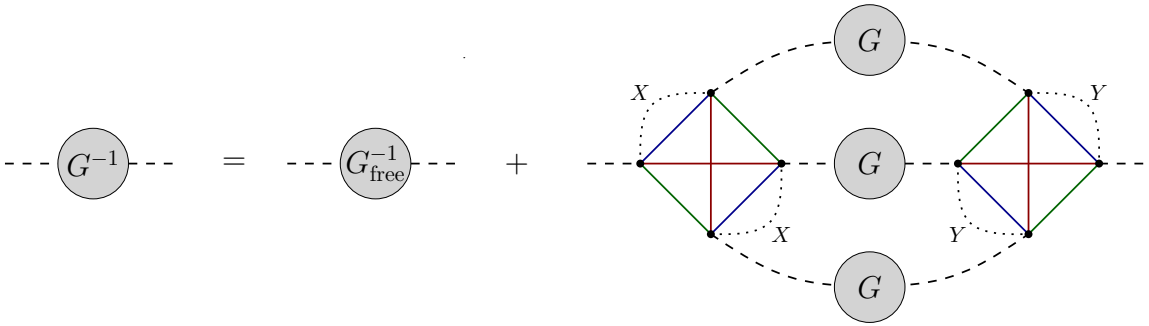
$$G^{-1}(x,x') = G_0^{-1}(x,x') - \Sigma(x,x')\,, \qquad \Sigma(x,x') = -(\lambda_0^S + \lambda_1^S)\mathrm{Tr}[G(x,x)]\delta(x,x')$$

- Gap equation $\rightarrow$ non-perturbative mass:

$$m = \Lambda \exp\left(-\frac{\pi}{\lambda_0^S + \lambda_1^S}\right)$$

- Callan-Symanzik $\Rightarrow$ asymptotic freedom: $\beta_1^S = -2(\lambda_1^S)^2/\pi < 0$

$$\sum_{X \neq S} \lambda_1^X$$  $$+ \lambda_2^X$$  $\Rightarrow$ **melonic Schwinger-Dyson equation**



- Same type of **bilocal** equation as in $d = 1$.
- Main difference: G_{free} **cannot be neglected** in the IR limit.
- Is there a non-trivial IR fixed point nonetheless?

  Best we could achieve analytically:

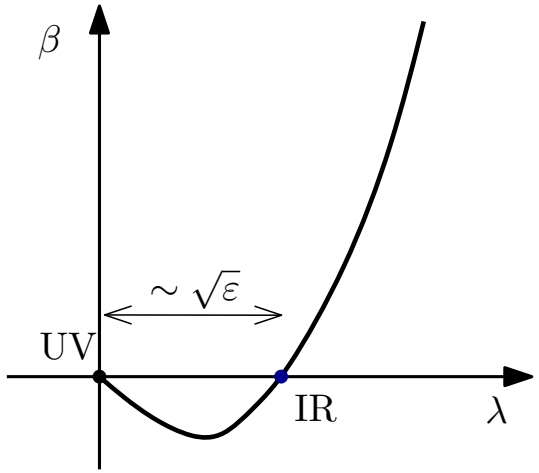
  perturbatively stable subsector with one effective coupling λ and

$$\beta_\lambda = \frac{3}{\pi^2}\lambda^3$$

Weakly interacting fixed point in $d = 2 - \varepsilon$

- Interesting feature in $d = 2 - \varepsilon$:

$$\beta_\lambda = \frac{3}{\pi^2}\lambda^3 \quad \rightarrow \quad \beta^{(\varepsilon)} = -\varepsilon\lambda + \frac{3}{\pi^2}\lambda^3$$



$\rightarrow$ weakly interacting IR fixed point $\lambda^* \sim \sqrt{\varepsilon}$.

$\rightarrow$ analogous to Wilson-Fisher in bosonic $\varphi^4_{4-\varepsilon}$.

Conjecture: governs the near-conformal regime of SYK in the limit $\varepsilon \rightarrow 1$.

2nd example: bosonic theory, $d = 3$

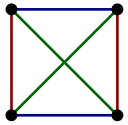
Bosonic theory

[Benedetti, Gurau, Harribey '19]

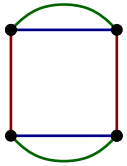
Euclidean bosonic tensor field theory in $d < 4$:

$$\mathcal{L} = \frac{1}{2}\varphi_{abc}(-\Delta)^{\zeta}\varphi_{abc} + \frac{m^{2\zeta}}{2}\varphi_{abc}\varphi_{abc}$$

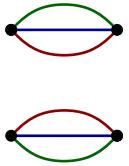
$$+ \frac{\lambda}{4N^{3/2}}$$



$$+ \frac{\lambda_P}{4N^2}$$



$$+ \frac{\lambda_D}{4N^3}$$



Infrared regime: Fix $\zeta = \frac{d}{4}$ [Gross, Rosenhaus '16] in $d = 1$

- large $N \to$ non-perturbative flow
- λ has a finite flow: $g = \lambda / Z^2$
- for the two other directions (g_1, g_2): $\beta_{g_i} = \alpha_2^{(i)}(g)\, g_i^2 - 2\alpha_1^{(i)}(g)\, g_i + \alpha_0^{(i)}(g)$
- lines of fixed points parametrized by g.

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Euclidean bosonic tensor field theory in $d = 3$:

$$\mathcal{L} = \frac{1}{2}\varphi_{abc}(-\Delta)^{\zeta}\varphi_{abc} + \frac{m^{2\zeta}}{2}\varphi_{abc}\varphi_{abc}$$

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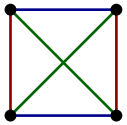

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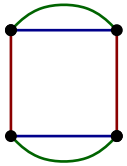

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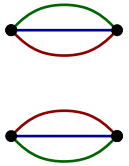

- $\lambda \in \mathrm{i}\mathbb{R} \Rightarrow \exists$ one infrared attractive fixed point.
- Assuming conformal invariance, the melonic limit allows to compute the conformal data. [Benedetti, Gurau, Harribey, Suzuki '19]
  - Spectrum and OPE coefficients consistent with unitarity.
  - No local stress-tensor.

Glimpse of a new class of large N CFTs?

Outlook and summary

Melonic theories and glassy dynamics

[Facoetti, Biroli, Kurchan, Reichman '19]

$$E = \sum_{i_1 < \ldots < i_p} J_{i_1 \ldots i_p} \, q_{i_1} \ldots q_{i_p} \, , \qquad \sum_i q_i^2 = N$$

Glassy dynamics		SYK-like quantum model
Classical Langevin process	$\longleftrightarrow$	Quantum Hamiltonian
Dynamics of metastable states	$\longleftrightarrow$	Equilibrium partition function
Dynamical heterogeneity	$\longleftrightarrow$	Almost reparametrization invariance

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Melonic flows for inference problems in theoretical machine learning:

spiked tensor model or *tensor PCA*

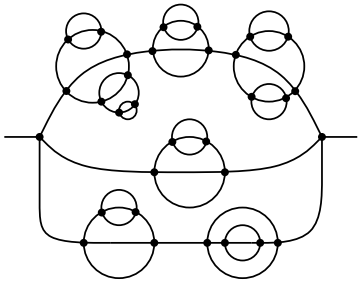
[Ben Arous, Montanari,... Biroli, Cammarota, Ros,... Krzakala, Zdeborova...]

Summary

Tensor T_{abc}

$$\frac{\lambda}{N^{3/2}} T_{aeb} T_{bfc} T_{ced} T_{dfa}$$

Melon diagrams



Tractable

- Third universal class of large N methods.
- Melon diagrams lie in a sweet spot: both tractable and rich!
- Research initially motivated by random geometry and quantum gravity...

  ... but led to a new and very active interface with AdS/CFT and strongly-coupled physics.
- Robust methods, domain of validity significantly enlarged recently:

  colored $\rightarrow$ irreducible tensor models