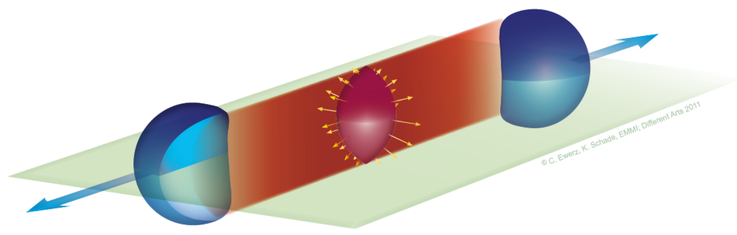
From spin chains to real-time thermal field theory using tensor networks

M.C. Bañuls, M.P. Heller, K. Jansen, J. Knaute, V. Svensson

Max-Planck-Institut für Gravitationsphysik
Albert-Einstein-Institut

Motivation

- The understanding of quantum many-body systems ab initio is one of the grand challenges in condensed matter and high-energy physics.
- On the experimental side, heavy-ion collisions study collective phases of QCD matter. The aim is to elucidate mechanisms which govern equilibration to the quark-gluon plasma (QGP) from a non-equilibrium initial state.
- In holography, quasinormal modes (characteristic complex frequencies at which black holes (BHs) absorb matter) set the time scale for dissipation in dual quantum field theories (QFTs). Such studies help to understand equilibration in several classes of models of the QGP.



Here, we use tensor network techniques to study real-time QFT dynamics at finite temperature. We explore thermal quantum quenches of the 1D Ising spin chain and its deformations, focusing on the IR regime in which the system is described by a (1+1)D QFT.

⇒ Our main interests are: (1) to evaluate numerically the structure of the retarded 2-point function from tensor network simulations, and (2) compare to integrable results and make predictions in the non-integrable QFT regime

Linear Response Theory

- Dynamics in linear response theory:

$$\delta\langle\mathcal{O}(t,x)\rangle = \int d\tilde{t}\, d\tilde{x}\, G_R(t-\tilde{t}, x-\tilde{x})\, \delta J(\tilde{t},\tilde{x})$$

local operator — retarded 2-point function at non-zero temperature $T = 1/\beta$ — source of Hamiltonian H

$$G_R(t-\tilde{t}, x-\tilde{x}) = i\,\theta(t-\tilde{t})\, \mathrm{Tr}\left(Z_\beta^{-1} e^{-\beta H}\, [\mathcal{O}(t,x), \mathcal{O}(\tilde{t},\tilde{x})]\right)$$

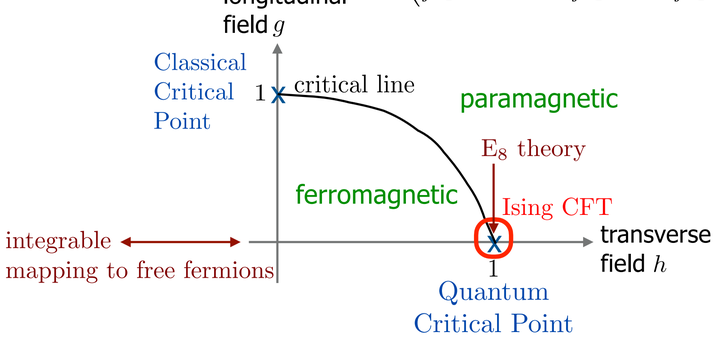
- In Fourier space, the time response is governed by the structure of $G_R(\omega, p)$ in the complex ω plane:

$$\delta\langle\mathcal{O}(t,p)\rangle = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega\, G_R(\omega,p)\, \delta J(-\omega,-p)\, e^{-i\omega t}$$

The Quantum Ising Model

- The quantum Ising Hamiltonian has the following phase diagram:

$$H = -J\left(\sum_{j=1}^{L-1} \sigma_z^j \sigma_z^{j+1} + h\sum_{j=1}^{L}\sigma_x^j + g\sum_{j=1}^{L}\sigma_z^j\right)$$



- The full scaling Ising field theory Hamiltonian in presence of transverse and longitudinal perturbations has the form [1]:

$$H = \int_{-\infty}^{\infty} dx \left\{\frac{1}{2\pi}\left[\frac{i}{2}\left(\psi(x)\partial_x\psi(x) - \bar{\psi}(x)\partial_x\bar{\psi}(x)\right) - iM_h\bar{\psi}(x)\psi(x)\right] + \bar{g}\sigma(x)\right\}$$

continuum limit: $L\to\infty,\ \beta J \gg 1$

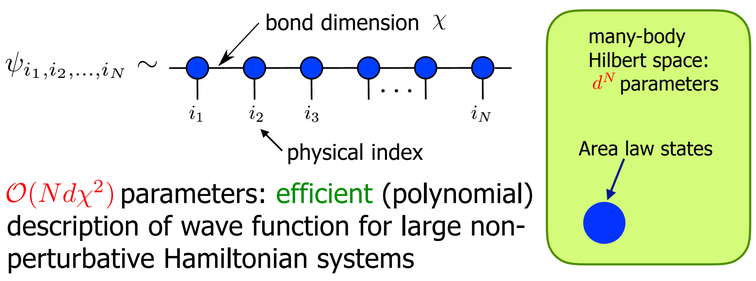
physical parameters: $M_h = 2J|1-h|, \quad M_{\bar{g}} = \bar{\eta}|\bar{g}|^{8/15}$

Matrix Product States

- Matrix product states (MPS) are ansätze for the wave function of a many-body quantum state [2] which satisfy an Area law [3] by construction:

$$|\Psi\rangle = \sum_{i_1,i_2,\ldots,i_N} A^1_{i_1} A^2_{i_2}\cdots A^N_{i_N}\, |i_1\rangle\, |i_2\rangle \cdots |i_N\rangle\,, \quad i_n = 1\ldots d$$

graphical representation:



$\mathcal{O}(Nd\chi^2)$ parameters: efficient (polynomial) description of wave function for large non-perturbative Hamiltonian systems

- We use the TEBD algorithm (time-evolving block decimation [4]) to construct thermal states and perform real-time evolution.

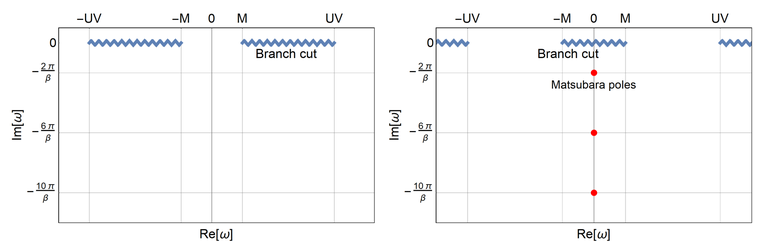
Expectation values are calculated as:

$$\langle O_2^{[n_2]}(t) O_1^{[n_1]}(0)\rangle_\beta = \mathrm{Tr}\left[U^\dagger(t) O_2^{[n_2]} U(t) O_1^{[n_1]} \rho(\beta)\right]$$

for Pauli matrices $O_j^{[n_j]} = \sigma_{x,z}^{n_j}$

CFT Results for Correlators

- Two equivalent representations (zero momentum):



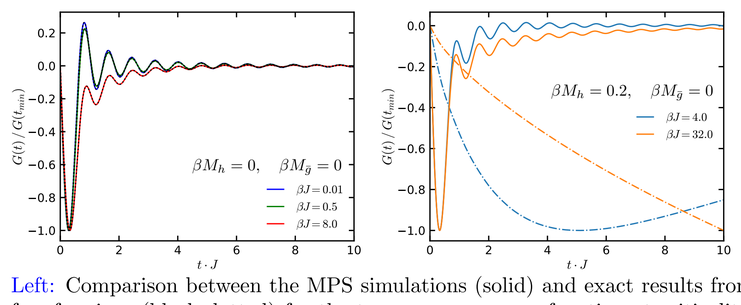
(1+1)D CFT: $\omega = \pm p - i\,2\pi T\,(\Delta + 2n) \quad \text{for} \quad n\in\{0,1,2,\ldots\}$

$\Delta_{\bar{\psi}\psi} = 1 \qquad \Delta_\sigma = \frac{1}{8}$

+ govern relaxation behavior
+ lowest decaying pole sets thermalization scale
+ holographic interpretation as BH quasi normal modes [5]

MPS Simulations

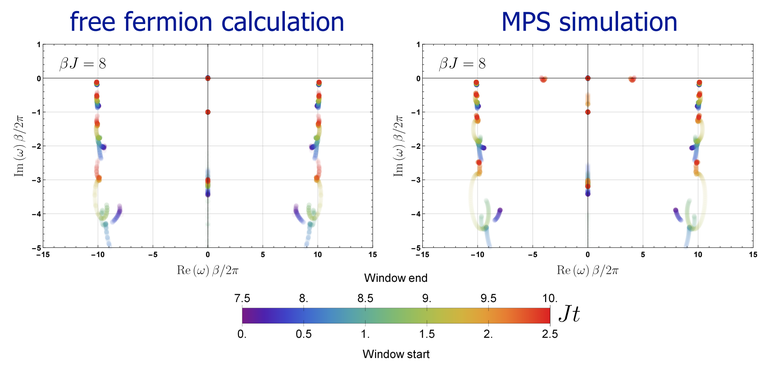
- We simulate spin chains of size $L\sim\mathcal{O}(100)$ at zero momentum (operators $\sigma_{x,z}^{L/2}(t)$ and $\sigma_{x,z}(0)$) and compare to integrable free fermion results:



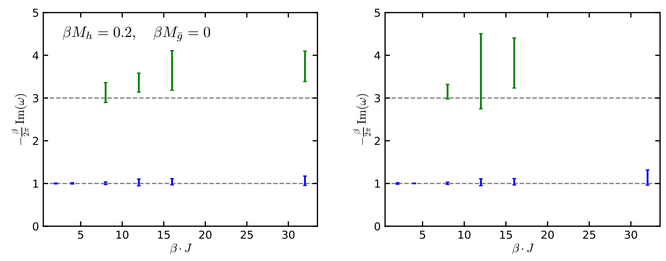
Left: Comparison between the MPS simulations (solid) and exact results from free fermions (black dotted) for the transverse response function at criticality. We observe an excellent agreement for all lattice spacing values βJ.
Right: Comparison of the transverse (solid) and longitudinal (dash-dotted) response function from MPS simulations. Note that the longitudinal response is oscillating on a much longer time scale.

The integrable QFT limit: MPS results

- We use Prony's method to represent the signal as a sum of complex exponentials, i.e. $G(t) = \sum_{k=1}^{M} c_k e^{\omega_k t}$



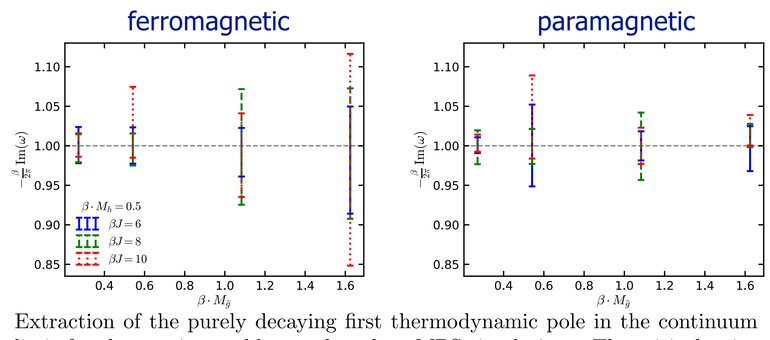
+ UV branch cut is approximated by vertical line of poles
+ good agreement of first decaying thermodynamic pole with analytical result $\omega = -i\,2\pi T$
+ second decaying pole $\omega = -i\,2\pi T\cdot 3$ partially identifiable
+ uncertainty estimation from parameter variation in Prony and time-shifted analysis window
+ residues agree with analytical result (free fermions) in QFT limit



Extraction of the purely decaying thermodynamic poles in the continuum limit $\beta J\to\infty$, based on the analytical result (left) and MPS simulation (right). Blue/green errorbars: first/second pole, Grey lines: analytical result

The non-integrable QFT limit: MPS predictions

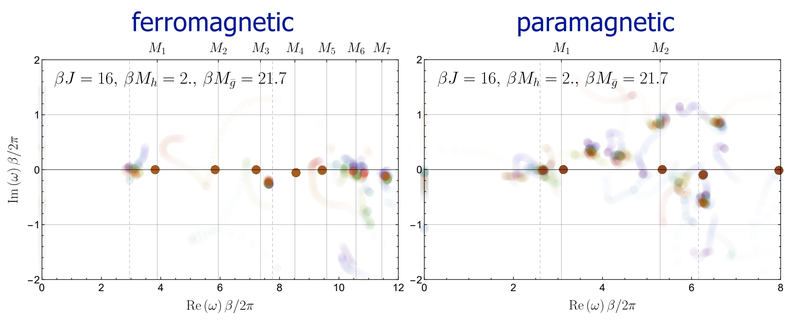
- For non-integrable transverse and longitudinal perturbations at zero momentum, no movement of the leading decaying thermodynamic pole is visible:



Extraction of the purely decaying first thermodynamic pole in the continuum limit for the non-integrable case based on MPS simulations. The critical point is approached from the ferromagnetic (left) and paramagnetic phase (right) for fixed $\beta M_h = 0.5$ and different values of the integrability breaking $\beta M_{\bar{g}}$.

Meson studies

- As a non-trivial cross-check in the non-integrable QFT regime, we identify meson / particle masses [6] of the perturbed Ising CFT and their decay rates [7] in different phases of the vacuum:



Frequency analysis from Prony for the non-integrable continuum limit: Solid vertical lines mark the predictions of particle masses in [6]. Dashed vertical lines mark a boundary excitation and the threshold $2M_1$ of the first multi-particle state.
Left: Ferromagnetic phase. The stable frequencies near the real line correspond to meson masses. The first 3 mesons are stable, while the finite imaginary parts (i.e. decay rates) of the 4th and 5th meson match the calculation in [7].
Right: Paramagnetic phase. There are 2 stable and 1 unstable particle.

Summary

- Tensor network techniques can be used to extract non-trivial real-time thermal field theory dynamics.
- The Prony method can be used to numerically evaluate the structure of the retarded 2-point function in the complex frequency plane:

⇒ agreement with CFT result / free fermions in integrable regime
⇒ no movement of first decaying thermodynamic pole for non-integrable perturbations
⇒ meson / particle masses match predictions from Ising CFT

- The effect of non-zero momentum is not yet studied.

References

[1] T. Rakovszky et al., Nucl. Phys. B 911, 805 (2016)
[2] U. Schollwöck, Ann. Phys. 326, 96 (2011)
[3] M.B. Hastings, J. Stat. Mech. 2007, P08024 (2007)
[4] G. Vidal, Phys. Rev. Lett. 91, 147902 (2003); Phys. Rev. Lett. 93, 040502 (2004)
[5] I. Sachs et al., Phys. Rev. Lett. 88, 151301 (2002)
[6] P. Fonseca, A. Zamolodchikov, hep-th/0612304 (2006); A. Zamolodchikov, arXiv:1310.4821 (2013)
[7] G. Delfino et al., Nucl. Phys. B 737, 291 (2006)