Emergent spacetime geometry for quantum matter

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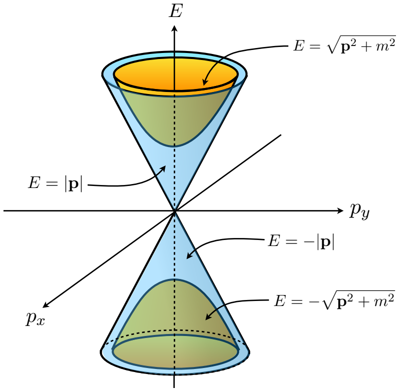
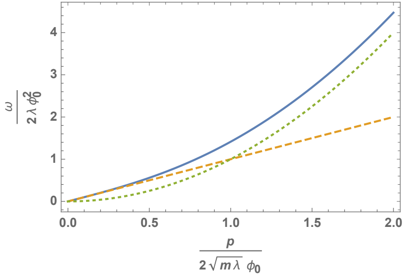
Quantum matter at low energy

- quasi-particles or collective excitations of matter
- can be *relativistic* at low energies



Bose-Einstein condensates with repulsive interaction

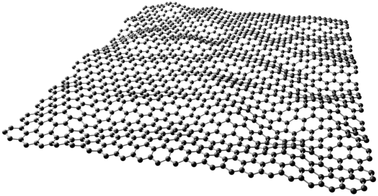
$$\omega = \sqrt{\left(\frac{\mathbf{p}^2}{2m} + 2\lambda\phi_0^2\right)\frac{\mathbf{p}^2}{2m}}$$

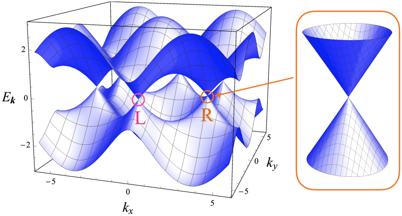


- Bogoliubov dispersion relation
- bosonic atoms with repulsive contact interaction strength λ
- condensate density ϕ_0^2
- low momentum: phonons $\omega \approx c|\mathbf{p}|$
- high momentum: particles $\omega \approx \mathbf{p}^2/2m$

Fermions

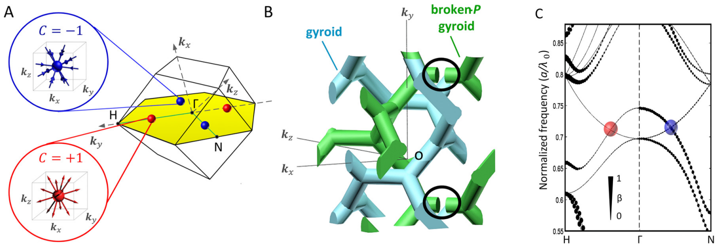
- two-dimensional materials like graphene
- Dirac points in electron dispersion relation
- low energy excitation behave relativistic
- gap can be opened by breaking parity or time reversal





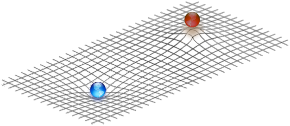
Photonic crystals

- materials with periodic structure of refractive index
- band structure for propagating light
- Weyl points have been realized [1][2]
- analog of massless Weyl fermions



[1][Lu, Wang, Ye, Ran, Fu, Joannopoulos & Soljačić, Science 349, 622 (2015)]
[2][Goi, Yue, Cumming & Gu, Laser Photonics Rev. 12, 1700271 (2018)]

Space and time-dependent geometry



- emergent spacetime geometry in quantum matter with metric $g_{\mu\nu}$
- depends on material properties like
  - velocity of sound
  - fluid velocity
  - Fermi velocity
  - band gap opening field
  - refractive index structure
- what happens if these depend on space and time?
- we get a spacetime metric $g_{\mu\nu}(t, \mathbf{x})$ with curvature!

Why are curved spacetimes interesting?

Motivation from general relativity

- the emergent metric has non-trivial curvature
- Einsteins field equations

$$R_{\mu\nu}(x) - \frac{1}{2}R(x)g_{\mu\nu}(x) = 8\pi G_N T_{\mu\nu}(x)$$

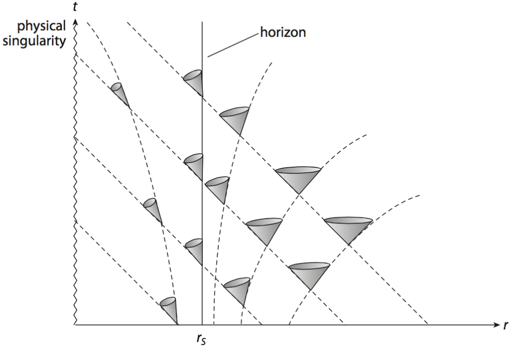
- *not* satisfied for emergent spacetime geometry

Quantum field theory in curved spacetime

- works with classical background geometry $g_{\mu\nu}(x)$
- considers quantum fields for matter and radiation propagating on this background
- particle concept
- particle production / Hawking radiation
- information theoretic questions

Black holes...

... or other spacetimes with horizons



- Hawking radiation and temperature $T = 1/(8\pi G_N M)$
- Bekenstein entropy $S = A/(4G_N)$
- and much more to be studied

Interplay of spacetime geometry and matter

- how is geometry influenced by matter field fluctuations?
- effective theory governing the geometry?
- origin of general relativity
- renormalization
- cosmological constant problem
- statistical and quantum fluctuation in the geometry?

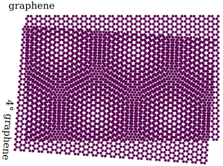
(Quantum) Information

- general relation between information theory and quantum field theory
- non-trivial space-times particularly interesting
- geometries with horizons
- relation between information theory and gravitation
- experimental insights

Beyond Riemannian geometry

- emergent spacetime geometry is not necessarily Riemannian
- spin connection
- Newton-Cartan
- torsion, non-metricity
- multi-metric
- Finsler geometry

Relativistic fermions in materials



- low energy theory of Dirac materials

$$\Gamma[\Psi] = \int dtd^2x \left\{-\bar{\Psi}\left[\gamma^0\partial_t + v_F(t)\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} + \Delta(t)\Gamma\right]\Psi\right\}$$

- time dependent Fermi velocity $v_F(t)$
  - change in twist angle for bilayer graphene
  - change in pressure
  - light pulses
- time-dependent gap or mass parameter $\Delta(t)\Gamma$ can be
  - breaking spatial inversion $\Gamma = \mathbb{1}$
  - Kekulé modulation of hoping $\Gamma = \gamma^3\cos(\alpha) + \gamma^5\sin(\alpha)$
  - Haldane mass breaking time parity $\Gamma = \gamma^{35}$
- can be manipulated with fast electronics

Fermions in curved spacetime

- action for Dirac fermions in general spacetime

$$\Gamma[\Psi] = \int dt d^2x \sqrt{g} \left\{ -\bar{\Psi} \left[\gamma^\alpha e_\alpha{}^\mu \partial_\mu (\partial_\mu + \Omega_\mu) + m\Gamma \right] \Psi \right\}$$

- tetrad field $e_\alpha{}^\mu$ inverse to $e^\alpha{}_\mu$ so that $g_{\mu\nu}(x) = e^\alpha{}_\mu(x) e^\beta{}_\nu(x) \eta_{\alpha\beta}$
- spin connection $\Omega_\mu = \omega_{\mu\alpha\beta}[\gamma^\alpha, \gamma^\beta]/8$ with

$$\omega_{\mu\alpha\beta} = -\eta_{\alpha\gamma} \left[\partial_\mu e^\gamma{}_\nu - \Gamma^\rho_{\mu\nu} e^\gamma{}_\rho \right] e_\beta{}^\nu$$

- local Lorentz transformations
- general coordinate transformations

Weyl scaling transformation

- transform Dirac fields (with conformal weight $\Delta_\Psi = (d-1)/2 = 1$)

$$\Psi(x) \to e^{-\zeta(x)}\Psi(x), \qquad \bar{\Psi}(x) \to e^{-\zeta(x)}\bar{\Psi}(x)$$

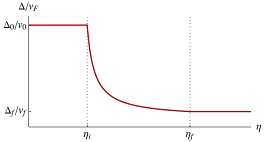
- transform tetrad field $e^{\alpha}{}_{\mu}(x) \to e^{\zeta}(x) e^{\alpha}{}_{\mu}(x)$
- and accordingly metric like $g_{\mu\nu}(x) \to e^{2\zeta(x)} g_{\mu\nu}(x)$
- spin connection transforms like

$$\omega_{\mu\alpha\beta} \to \omega_{\mu\alpha\beta} + \left[e_{\alpha\mu} e_{\beta}{}^{\nu} - e_{\beta\mu} e_{\alpha}{}^{\nu} \right] \partial_\nu \zeta$$

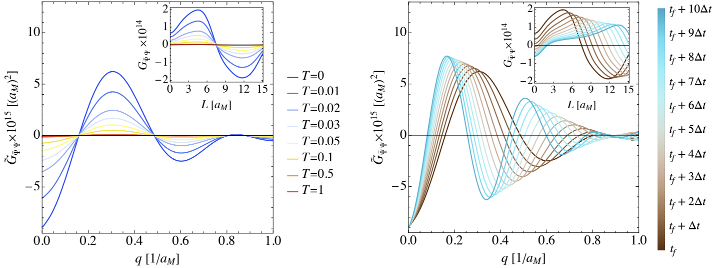
- gap term is **not** invariant $m\Gamma = e^{\zeta(x)} m\Gamma$
- allows to transform a time-dependent mass term into a constant mass term
- only ratio $\Delta(t)/v_F(t)$ matters for particle production

Fermionic particle production

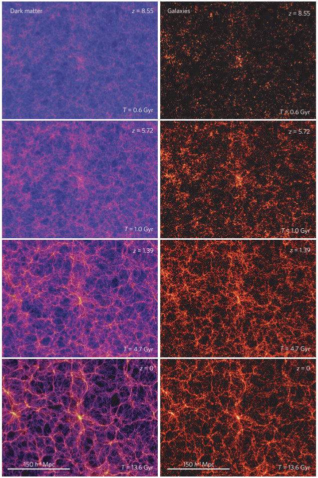
- time dependence of ratio Δ/v_F



- leads to particle production



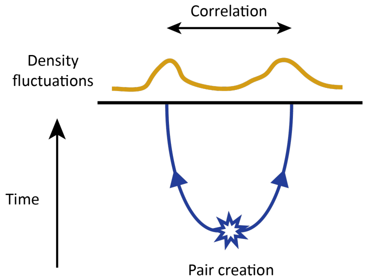
Evolution of cosmic large-scale structure



[Springel, Frenk & White, Nature 440, 1137 (2006)]

Quantum origin of fluctuations

- Universe was almost homogeneous at early times
- small fluctuations magnified by gravitational attraction
- primordial quantum fluctuations from inflation
  [Mukhanov & Chibisov (1981), Hawking (1982), Starobinsky (1982), Guth & Pi (1982), Bardeen, Steinhardt & Turner (1983), Fischler, Ratra & Susskind (1985)]



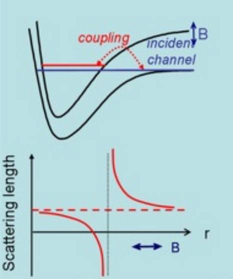
Non-relativistic quantum fields

- Bose-Einstein condensate in two dimensions
  [Gross (1961), Pitaevskii (1961)]

$$\Gamma[\Phi] = \int \mathrm{d}t\, \mathrm{d}^2x \left\{ \hbar\Phi^*(t,\mathbf{x}) \left[i\frac{\partial}{\partial t} - V(t,\mathbf{x}) \right] \Phi(t,\mathbf{x}) \right.$$
$$\left. - \frac{\hbar^2}{2m} \boldsymbol{\nabla}\Phi^*(t,\mathbf{x}) \boldsymbol{\nabla}\Phi(t,\mathbf{x}) - \frac{\lambda(t)}{2} \Phi^*(t,\mathbf{x})^2 \Phi(t,\mathbf{x})^2 \right\}$$

- low energy theory for bosonic atoms
- optical trap potential $V(t,\mathbf{x})$
- coupling strength $\lambda(t)$

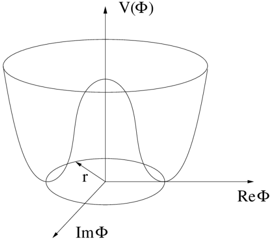
Feshbach resonance



- allow to control scattering length or effective s-wave interaction strength through magnetic field B
- can be made time-dependent by varying magnetic field

$$\frac{\lambda(t)}{2}\Phi^*(t,\mathbf{x})^2\Phi(t,\mathbf{x})^2$$

Superfluid and small excitations



- complex non-relativistic field can be decomposed

$$\Phi = e^{iS_0}\left(\sqrt{n_0} + \frac{1}{\sqrt{2}}\left[\phi_1 + i\phi_2\right]\right)$$

- real fields ϕ_1 and ϕ_2 describe excitations on top of the superfluid
- low energy field $\phi_2(t, \mathbf{x})$
- stationary superfluid density $n_0(\mathbf{x})$ and vanishing superfluid velocity

$$\mathbf{v} = \frac{\hbar}{m}\boldsymbol{\nabla} S_0 = 0$$

Sound waves / phonons

- small energy excitations are sound waves or phonons
- propagate with finite velocity, similar to light
- local speed of sound

$$c_S(t,\mathbf{x}) = \sqrt{\frac{\lambda(t)\, n_0(\mathbf{x})}{m}}$$

- sound waves propagate along

$$ds^2 = -dt^2 + \frac{1}{c_S(t,\mathbf{x})^2}(d\mathbf{x} - \mathbf{v}dt)^2 = 0$$

- acoustic metric for vanishing fluid velocity $\mathbf{v} = 0$

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{c_S(t,\mathbf{x})^2} & 0 \\ 0 & 0 & \frac{1}{c_S(t,\mathbf{x})^2} \end{pmatrix}$$

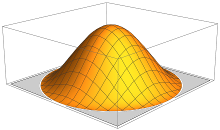
Relativistic scalar field

- Low energy theory for phonons (with $\phi = \phi_2/\sqrt{2m}$)

$$\Gamma[\phi] = \int \mathrm{d}t\, \mathrm{d}^2x\, \sqrt{g} \left\{ -\frac{1}{2} g^{\mu\nu}\, \partial_\mu \phi\, \partial_\nu \phi \right\}$$

- metric determinant $\sqrt{g} = \sqrt{-\det(g_{\mu\nu})}$
- acoustic metric depends on space and time like the space-time metric in general relativity
- phonons behave like a real, massless, relativistic scalar field in a curved spacetime !
- quantum simulator for QFT in curved space

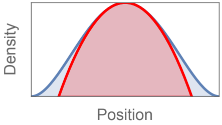
Density profiles



- assume specifically for $r = |\mathbf{x}| < R$

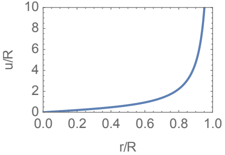
$$n_0(r) = \bar{n}_0 \times \left[1 - \frac{r^2}{R^2}\right]^2$$

- experimental realization with optical trap and digital micromirror device
- approximate realization in harmonic trap



- variable transform to $0 \le u < \infty$

$$u(r) = \frac{r}{1 - \frac{r^2}{R^2}}$$



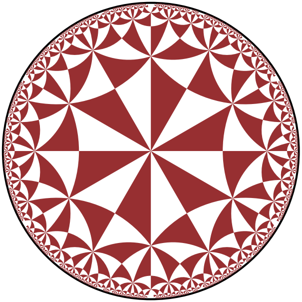
- leads to Friedmann-Lemaitre-Robertson-Walker metric

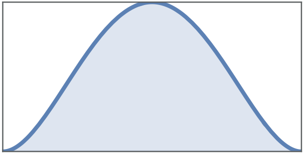
$$ds^2 = -dt^2 + a^2(t)\left(\frac{du^2}{1 - \kappa u^2} + u^2 d\varphi^2\right)$$

- negative spatial curvature $\kappa = -4/R^2$
- time-dependent scale factor from time dependent scattering length $\sim \lambda(t)$

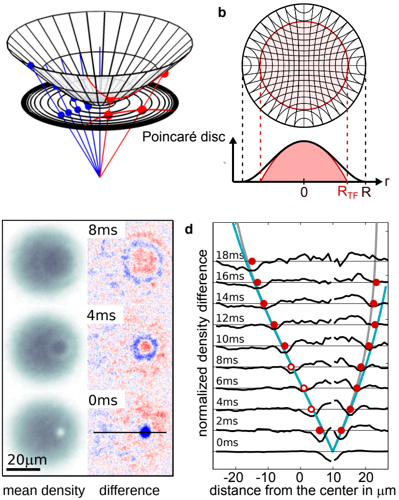
$$a(t) = \sqrt{\frac{m}{\bar{n}_0} \frac{1}{\lambda(t)}}$$

Hyperbolic geometry

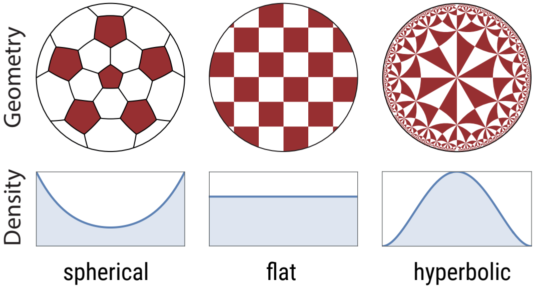


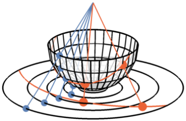


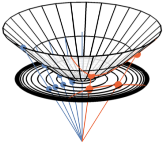
Experimental realization in a Bose-Einstein condensate



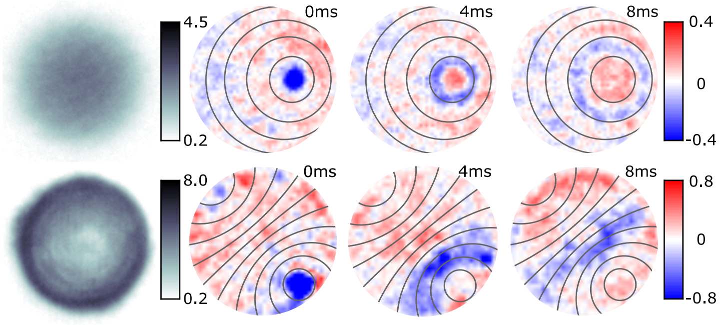
Geometries with constant spatial curvature







Propagating sound waves



Mode functions and Bogoliubov transforms

- field gets expanded in modes

$$\phi(t, u, \varphi) = \int_{k,m} \left[\hat{a}_{km} \mathcal{H}_{km}(u, \varphi) v_k(t) + \hat{a}^\dagger_{km} \mathcal{H}^*_{km}(u, \varphi) v^*_k(t) \right]$$

- temporal mode functions satisfy

$$\ddot{v}_k(t) + 2\frac{\dot{a}(t)}{a(t)}\dot{v}_k(t) + \frac{k^2 + |\kappa|/4}{a^2(t)} v_k(t) = 0$$

- vacuum state only unique for $\dot{a}(t) = 0$ where one has positive frequency soliutions

$$v_k(t) \sim \exp(-i\omega_k t)$$

Laplace operator

$$\Delta = \begin{cases} |\kappa|\left[\frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\,\partial_\theta\right) + \frac{1}{\sin^2\theta}\partial_\varphi^2\right] & \text{for} \quad \kappa > 0 \\ \partial_u^2 + \frac{1}{u}\partial_u + \frac{1}{u^2}\partial_\varphi^2 & \text{for} \quad \kappa = 0 \\ |\kappa|\left[\frac{1}{\sinh\sigma}\partial_\sigma\left(\sinh\sigma\,\partial_\sigma\right) + \frac{1}{\sinh^2\sigma}\partial_\varphi^2\right] & \text{for} \quad \kappa < 0 \end{cases}$$

- eigenfunctions

$$\mathcal{H}_{km}(u,\varphi) = \begin{cases} Y_{lm}(\theta,\varphi) & \text{for} \quad \kappa > 0 \quad \text{with} \quad l \in \mathbb{N}_0, m \in \{-l, \ldots, l\} \\ X_{km}(u,\varphi) & \text{for} \quad \kappa = 0 \quad \text{with} \quad k \in \mathbb{R}_0^+, m \in \mathbb{Z} \\ W_{lm}(\sigma,\varphi) & \text{for} \quad \kappa < 0 \quad \text{with} \quad l \in \mathbb{R}_0^+, m \in \mathbb{Z} \end{cases}$$

- eigenvalues with $k = |\kappa| l$

$$h(k) = \begin{cases} -k(k + \sqrt{|\kappa|}) & \text{for} \quad \kappa > 0 \\ -k^2 & \text{for} \quad \kappa = 0 \\ -\left(k^2 + \frac{1}{4}|\kappa|\right) & \text{for} \quad \kappa < 0 \end{cases}$$

Eigenfunctions

- positive spatial curvature $\kappa > 0$: spherical harmonics

$$Y_{lm}(\theta, \varphi) = \sqrt{\frac{(l-m)!}{(l+m)!}}\, e^{im\varphi}\, P_{lm}(\cos\theta),$$

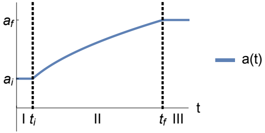
- vanishing spatial curvature $\kappa = 0$: Bessel functions

$$X_{km}(u, \varphi) = e^{im\varphi}\, J_m(ku),$$

- negative spatial curvature $\kappa < 0$: sperical harmonics with complex angular momentum

$$W_{lm}(\sigma, \varphi) = (-i)^m \frac{\Gamma(il + 1/2)}{\Gamma(il + m + 1/2)} e^{im\varphi} P^m_{il-1/2}(\cosh\sigma)\,,$$

Bogoliubov transforms



- in region I: positive frequency modes v_k, vacuum so that $\hat{a}_{km}|\Omega\rangle = 0$
- in region III: positive frequency modes u_k, vacuum so that $\hat{b}_{km}|\Psi\rangle = 0$
- Bogoliubov transform

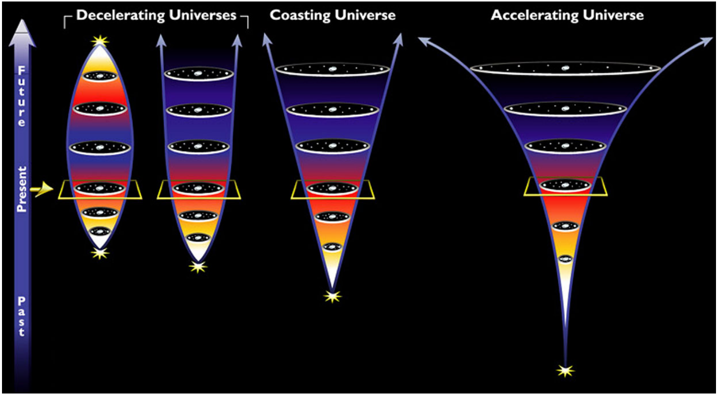
$$u_k = \alpha_k v_k + \beta_k v_k^*, \qquad v_k = \alpha_k^* u_k - \beta_k u_k^*$$

- results in excitation spectrum

$$S_k(t) = \frac{1}{2} + N_k + A_k \cos\left(\theta_k + 2\omega_k t\right)$$

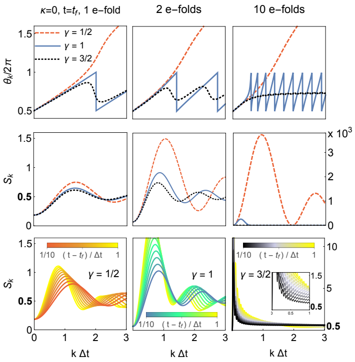
- constant term in spectrum $N_k = |\beta_k|^2$
- oscillating term $\mathrm{Re}[\alpha_k \beta_k e^{2i\omega_k t}]$

Expansion history

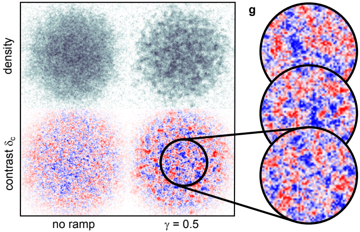


Cosmology in $d = 2 + 1$ spacetime dimensions

- analytic solutions for many choices of $a(t) = \text{const} \times t^{\gamma}$
- $\gamma < 1$ decelerating, $\gamma = 1$ coasting, $\gamma > 1$ accelerating
- depends on number of e-folds, exponent γ and time after expansion ceases



Observation of particle production

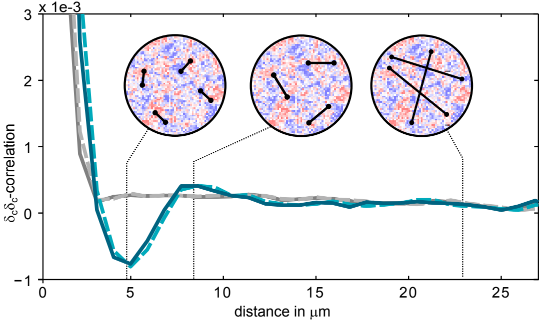


- rescaled density contrast

$$\delta_c(t,\mathbf{x}) = \sqrt{\frac{n_0(\mathbf{x})}{\bar{n}_0^3}}\,[n(t,\mathbf{x}) - n_0(\mathbf{x})] \sim \partial_t \phi(t,\mathbf{x})$$

- access correlation functions of relativistic scalar field through density fluctuations

Density contrast correlation function

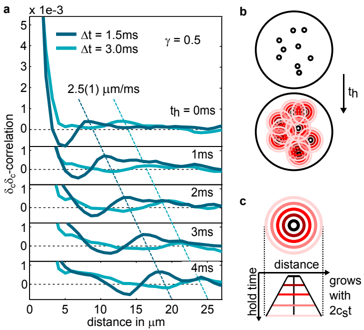


- correlation function

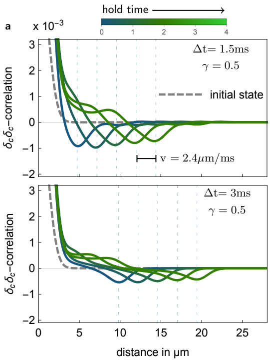
$$\langle \delta_c(\mathbf{x}) \delta_c(\mathbf{y}) \rangle$$

- before and after expansion

Time dependent correlation functions after expansion



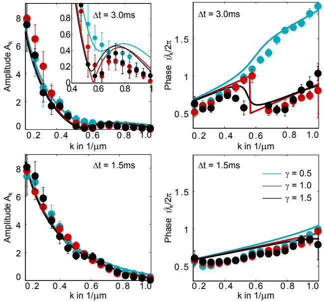
Experiment



Theory

- analogous to baryon acoustic or Sakharov oscillations in cosmology
- optical resolution important for detailed shape

Oscillations in Fourier space



- Fourier spectrum of excitations

$$S_k(t) = \frac{1}{2} + N_k + A_k \cos(2\omega_k(t - t_f) + \vartheta_k)$$

- decelerated, coasting and accelerated expansion
- good agreement with analytic theory (solid lines)

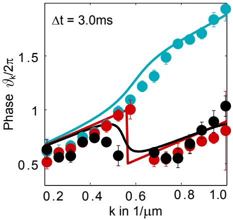
Quantum recurrences

- uniform expansion with $a(t) = Qt$ is special
- shows quantum recurrences of the incoming vacuum state at special values of wavenumber k

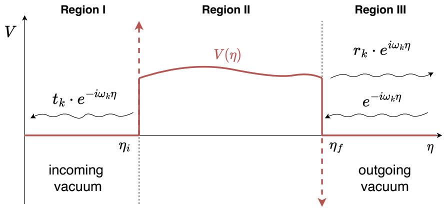
$$k_n = \frac{a_f - a_i}{\Delta t}\left[\left(\frac{n\pi}{\ln\left(a_f/a_i\right)}\right)^2 + \frac{1}{4}\right]^{\frac{1}{2}},$$

with integer $n = 1, 2, 3, \ldots$

- at these points one has trivial Bogoliubov coefficient $\beta_k = 0$
- can be seen experimentally as a discontinuity in the phase !



The scattering analogy



- evolution equation

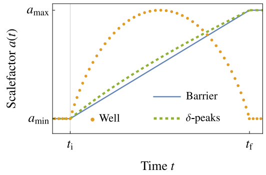
$$\ddot{v}_k(t) + 2\frac{\dot{a}(t)}{a(t)}\dot{v}_k(t) + \frac{k^2 + |\kappa|/4}{a^2(t)} v_k(t) = 0$$

- rescaled mode function $\psi_k(\eta) = \sqrt{a(t)}\, v_k(t)$ and conformal time $dt = a(t) d\eta$
- stationary Schrödinger equation ($V(\eta) = \dot{a}^2/4 + \ddot{a}a/2$ and $E = k^2$)

$$\frac{d^2}{d\eta^2}\psi_k(\eta) + [E - V(\eta)]\,\psi_k(\eta) = 0$$

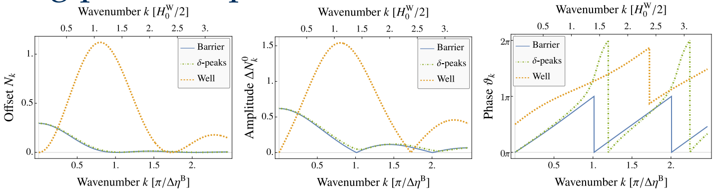
- $\psi_0(\eta) = \sqrt{a(t)}$ is solution for $k^2 + |\kappa|/4 = 0$

Some example potentials



- potential $V(\eta) = \dot{a}^2/4 + \ddot{a}a/2$ has Dirac peaks when $\dot{a}$ has discontinuity
- coasting universe $a \sim t$ leads to square barrier
- "radiation dominated" universe $a \sim t^{2/3}$ has only Dirac peaks
- particular anti-bounce leads to square well

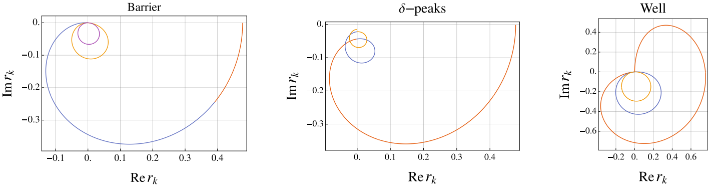
Resulting particle spectra



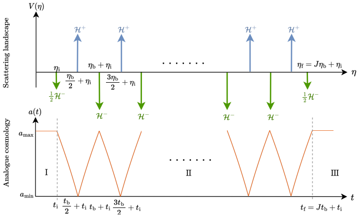
- resulting particle spectra

$$S_k(t) = \frac{1}{2} + N_k + \Delta N_k^0 \cos(2\omega_k(t - t_f) + \vartheta_k)$$

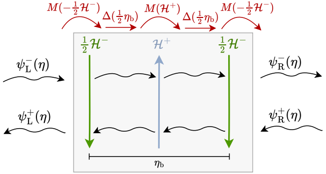
- reflection amplitude has zero crossings that explain phase jumps

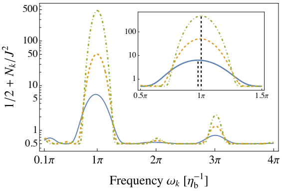


Periodic universes



- combination of expanding and contracting phases where $a \sim t^{2/3}$
- potential landscape with attractive and repulsive Dirac peaks
- can be solved with transfer matrix method





Conclusions

- Bose-Einstein condensates as quantum simulators for quantum fields in curved spacetime
- symmetric spaces with constant curvature can be realized with specific density profiles
- time-dependent coupling allows to simulate expansion or contraction
- particle production
- Sacherov oscillations after expansion allow detailed investigations
- scattering analogy allows to gain insights into many possible “cosmologies”
- fermion production in expanding geometry could be realized with Dirac materials
- extensions to three dimensions, other geometries, different field content, and more, to come