Landau-Ginzburg analysis of the causally complete Lorentzian Barrett-Crane model

based on 2112.00091, 2206.15442, 2404.04524, 2407.02325

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MCQST

Overview

- Background-independent gravity path integral approach
- consider $d = 4$ and $\mathrm{SL}(2, \mathbb{C})$
- Barrett-Crane model is a spin foam and group field theory model

  Barrett, Crane gr-qc/9904025; Perez, Rovelli gr-qc/0009021; Baratin, Oriti 1108.1178; AFJ, Oriti, Pithis 2112.00091
- “causally complete” means spacelike, lightlike, and timelike tetrahedra are included

  AFJ, Oriti, Pithis 2206.15442

microscopic quantum geometric degrees of freedom $\xrightarrow{\text{coarse-graining}}$ continuum spacetime geometries

- LG mean-field theory as approximation method and for coarse account of phase structure

Landau-Ginzburg analysis of the causally complete Lorentzian Barrett-Crane model

BF-theory quantization

$$S_{\mathrm{EH}}[g] \xrightarrow{1.} S_{\mathrm{P}}[e,\omega] \xrightarrow{2.} S_{BF}[B,\omega] + \text{constr.} \xrightarrow[\text{disc.}]{3.} \underbrace{S_{BF}^{\text{disc}} \quad + \quad \text{constr.}}$$

$$\begin{tikzcd} S_{BF}^{\text{disc}} \arrow[d, "4.\ \text{quantization}"'] & \text{constr.} \arrow[d, "?"] \\ Z_{BF}[\Delta^*] \arrow[r, "5."', "\text{qu. constr.}"] & Z_{\mathrm{SF}}[\Delta^*] \end{tikzcd}$$

1. First-order Palatini gravity $\quad S_{\mathrm{P}}[e,\omega] = \int_M \epsilon_{IJKL} e^I \wedge e^J \wedge F(\omega)^{KL}$

   no Holst-term $\quad \frac{1}{\gamma_{\mathrm{BI}}} \int_M e^I \wedge e^J \wedge F(\omega)_{IJ}$

   orthonormal frame $\quad \{e_I\}, \qquad g(e_I, e_J) = \eta_{IJ}$

   connection $\quad \omega \in \Omega^1(M) \times \mathfrak{sl}(2,\mathbb{C})$

   spin-connection $\quad \frac{\delta S_{\mathrm{P}}}{\delta \omega} = 0 \ \Rightarrow\ \omega(e)$

Baez gr-qc/9905087; Simão ‘24

BF-theory quantization

$$\begin{tikzcd} S_{\mathrm{EH}}[g] \arrow[r, "1."] & S_{\mathrm{P}}[e,\omega] \arrow[r, "2."] & S_{BF}[B,\omega]+\text{constr.} \arrow[r, "3.", "\text{disc.}"'] & S^{\text{disc}}_{BF} \arrow[d, "4."', "\text{quantization}"] & + \quad \text{constr.} \arrow[dr, "?"] & \\ & & & Z_{BF}[\Delta^*] \arrow[rr, "5.", "\text{qu. constr.}"'] & & Z_{\mathrm{SF}}[\Delta^*] \end{tikzcd}$$

no prop. d.o.f.'s

2. Constrained *BF*-theory $\quad S_{BF}[B,\omega] = \int B_{IJ} \wedge F(\omega)^{IJ}$

B-field 2-form $\quad B \in \Omega^2(M) \times \mathfrak{sl}(2,\mathbb{C})$

simplicity constraints $\quad n_I(*B)^{IJ} = 0 \;\Rightarrow\; B^{IJ} = \epsilon^{IJKL} e_K \wedge e_L$

- normal vector field n
- defining ingredient of BC model
- other possibilities with γ_{BI}

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BF-theory quantization

$$
\begin{tikzcd}
S_{\mathrm{EH}}[g] \arrow[r, "1."] & S_{\mathrm{P}}[e,\omega] \arrow[r, "2."] & S_{BF}[B,\omega] + \text{constr.} \arrow[r, "3."', "\text{disc.}"'] & \underbrace{S_{BF}^{\text{disc}} \quad + \quad \text{constr.}} \arrow[d, "4.\ \text{quantization}"] \arrow[dr, "?"] & \\
 & & & Z_{BF}[\Delta^*] \arrow[r, "5."', "\text{qu. constr.}"'] & Z_{\mathrm{SF}}[\Delta^*]
\end{tikzcd}
$$

3. Discretization

$$\omega \to g_e = \mathcal{P}\exp\left(\int_e \omega\right)$$

$$F \to g_f = \overrightarrow{\prod_{e \supset f}} g_e$$

$$B \to b_f = \int_{t(f)} B$$

$$n \to X$$

$$S_{BF}^{\text{disc}} = \sum_f \mathrm{Tr}[b_f g_f]$$

Δ	Δ^*
4-cell	vertex (v)
tetrahedron	edge (e)
triangle	face (f)
link	
point	

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BF-theory quantization

$$S_{\mathrm{EH}}[g] \xrightarrow{1.} S_{\mathrm{P}}[e,\omega] \xrightarrow{2.} S_{BF}[B,\omega] + \text{constr.} \xrightarrow[\text{disc.}]{3.} S^{\text{disc}}_{BF} + \text{constr.}$$

$$\begin{tikzcd}
\underbrace{S^{\text{disc}}_{BF} + \text{constr.}} \arrow[d, "4."', "\text{quantization}"] \arrow[dr, "?"] & \\
Z_{BF}[\Delta^*] \arrow[r, "5.", "\text{qu. constr.}"'] & Z_{\mathrm{SF}}[\Delta^*]
\end{tikzcd}$$

?

Baez gr-qc/9905087; Simão ‘24

BF-theory quantization

$$S_{\mathrm{EH}}[g] \xrightarrow{1.} S_{\mathrm{P}}[e,\omega] \xrightarrow{2.} S_{BF}[B,\omega] + \text{constr.} \xrightarrow[\text{disc.}]{3.} \underbrace{S_{BF}^{\text{disc}} \quad + \quad \text{constr.}}$$

$$\begin{tikzcd} S_{BF}^{\text{disc}} \arrow[d, "4."', "\text{quantization}"] & \text{constr.} \arrow[dr, "?"] & \\ Z_{BF}[\Delta^*] \arrow[rr, "5."', "\text{qu. constr.}"'] & & Z_{\mathrm{SF}}[\Delta^*] \end{tikzcd}$$

4. Path integral quantization $\quad Z_{BF}[\Delta^*] = \prod_f \int \mathrm{d}g_f \int \mathrm{d}b_f \; ^{iS_{BF}} = \prod_f \int \mathrm{d}g_f \prod_f \delta(\prod_{e\subset f} g_e)$

Plancherel-decomposition $\quad \delta(g) = \int_{\mathbb{R}} \mathrm{d}\rho \sum_{\nu\in\mathbb{Z}/2} (\rho^2+\nu^2) \sum_{j\in\mathbb{N}/2} \sum_{m=-j}^{j} D^{(\rho,\nu)}_{jmjm}(g)$

unitary irreps & can. basis $\quad (\rho,\nu)\in\mathbb{R}\times\mathbb{Z}/2, \qquad |(\rho,\nu);jm\rangle \in \mathcal{D}^{(\rho,\nu)}$

$$Z_{BF}[\Delta^*] = \prod_f \int \mathrm{d}g_f \prod_f \delta(\prod_{e\subset f} g_e) = \sum\!\!\!\!\!\!\int_{\{\rho_f,\nu_f,\iota_e\}} \prod_f (\rho_f^2+\nu_f^2) \prod_e \mathcal{A}_e \prod_v \mathcal{A}_v$$

Baez gr-qc/9905087; Simão '24; AFJ, Oriti, Pithis 2112.00091, 2206.15442

From BF-theory to spin foams

$$S_{\text{EH}}[g] \xrightarrow{1.} S_{\text{P}}[e,\omega] \xrightarrow{2.} S_{BF}[B,\omega] + \text{constr.} \xrightarrow[\text{disc.}]{3.} \underbrace{S^{\text{disc}}_{BF} \quad + \quad \text{constr.}}$$

4. quantization

?

$$Z_{BF}[\Delta^*] \xrightarrow[\text{qu. constr.}]{5.} Z_{\text{SF}}[\Delta^*]$$

5. Imposition of constraints on the quantum level

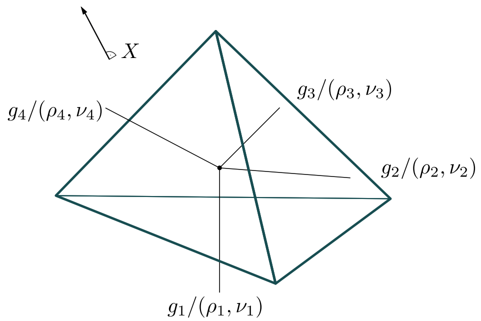
$$\left(\boldsymbol{L}^2 - \boldsymbol{K}^2\right)|(\rho,\nu), jm\rangle = \left(-\rho^2 + \nu^2 - 1\right)|(\rho,\nu), jm\rangle$$

$$\boldsymbol{L}\cdot\boldsymbol{K}\,|(\rho,\nu), jm\rangle = \rho\nu\,|(\rho,\nu), jm\rangle$$

Identification of bi-vectors and generators $\quad b_f^{IJ} \longmapsto (L_f^i, K_f^j) \qquad X_I(*b)^{IJ} = 0 \;\Rightarrow\; \boldsymbol{L}\cdot\boldsymbol{K} = 0$

$$Z_{\text{SF}}[\Delta^*] = \left.\not\sum_{\{\rho_f,\nu_f,\iota_e\}}\right|_{\text{simpl.}} \prod_f (\rho_f^2 + \nu_f^2) \prod_e \mathcal{A}_e \prod_v \mathcal{A}_v$$

explicit discretization dependence



tet.	s.l.	l.l.	t.l.	
X_I	t.l.	l.l.	s.l.	
face	s.l.	s.l.	s.l.	t.l.
	$(\rho,0)$	$(\rho,0)$	$(\rho,0)$	$(0,\nu)$

Baez gr-qc/9905087; Simão '24; AFJ, Oriti, Pithis 2112.00091, 2206.15442

From spin foams to group field theories

Dual complex Δ^* can be understood as <u>stranded</u> graph and $Z_{\mathrm{SF}}[\Delta^*]$ as <u>Feynman amplitude</u>

Group field theories generate spin foam amplitudes in a perturbative expansion, i.e.

$$Z_{\mathrm{GFT}} = \int \mathcal{D}\Phi\, \mathrm{e}^{-S_{\mathrm{GFT}}[\Phi]} = \sum_{\Delta^*} \frac{\lambda^V}{\mathrm{sym}(\Delta^*)} Z_{\mathrm{SF}}[\Delta^*]$$

Other paths to the theory are:

- 3rd quantization of gravity [Giddings, Strominger '89]
- generalization of matrix and tensor models
- quantum cosmology as hydrodynamics on minisuperspace

$$\Phi : \mathrm{SL}(2,\mathbb{C})^4 \times \mathbb{R}^{d_{\mathrm{loc}}} \times \mathbb{H} \longrightarrow \mathbb{R}$$

$$(\boldsymbol{g}, \boldsymbol{\phi}, X) \longmapsto \Phi(\boldsymbol{g}, \boldsymbol{\phi}, X)$$

scalar field coupling

- closure $\quad \Phi(\boldsymbol{g}h^{-1}, \boldsymbol{\phi}, h \cdot X) = \Phi(\boldsymbol{g}, \boldsymbol{\phi}, X), \quad h \in \mathrm{SL}(2,\mathbb{C})$
- simplicity $\quad \Phi(\boldsymbol{g}\boldsymbol{u}, \boldsymbol{\phi}, X) = \Phi(\boldsymbol{g}, \boldsymbol{\phi}, X), \quad \boldsymbol{u} \in \mathrm{Stab}_X^4$

$$S_{\mathrm{GFT}} = K[\Phi] + V[\Phi]$$

$$K[\Phi] = \frac{1}{2} \int \mathrm{d}\boldsymbol{g}\, \mathrm{d}\boldsymbol{\phi}\, \mathrm{d}\boldsymbol{g}'\, \mathrm{d}\boldsymbol{\phi}'\, \mathrm{d}X\, \Phi(\boldsymbol{g}, \boldsymbol{\phi}, X) \mathcal{K}(\boldsymbol{g}^{-1}\boldsymbol{g}', |\boldsymbol{\phi} - \boldsymbol{\phi}'|) \Phi(\boldsymbol{g}', \boldsymbol{\phi}', X)$$

$\sim \mathcal{A}_e^{-1}$

kinetic: "propagation" of tetrahedra

$\sim \mathcal{A}_v$

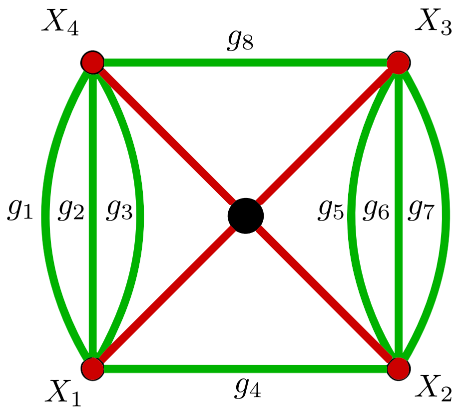
$$V[\Phi] = \lambda \int \mathrm{d}\boldsymbol{\phi}\, \mathrm{Tr}_\gamma \left[\prod_\tau \int \mathrm{d}X^\tau\, \Phi(\boldsymbol{g}^\tau, \boldsymbol{\phi}, X^\tau) \right]$$

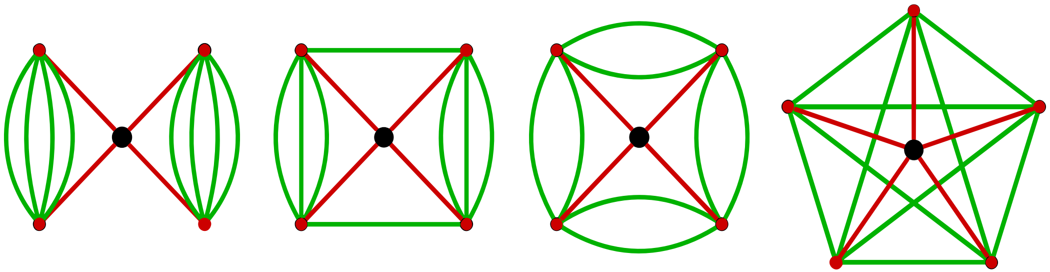
vertex: glueing of tetrahedra, <u>non-local</u> in g and <u>local</u> in ϕ

Oriti gr-qc/0607032; Li, Oriti, Zhang 1701.08719

Example: $\gamma =$ quartic melon

$$V[\Phi] = \int \mathrm{d}^8 g\, \mathrm{d}\boldsymbol{\phi}\, \mathrm{d}\boldsymbol{X}\; \Phi(g_1, g_2, g_3, g_4, \boldsymbol{\phi}, X_1)\Phi(g_5, g_6, g_7, g_4, \boldsymbol{\phi}, X_2)\Phi(g_5, g_6, g_7, g_8, \boldsymbol{\phi}, X_3)\Phi(g_1, g_2, g_3, g_8, \boldsymbol{\phi}, X_4)$$





Oriti, Ryan, Thürigen 1409.3150

The causally complete Barrett-Crane GFT model

Resolving restriction to spacelike tetrahedra (X_+ timelike) to include timelike tetrahedra (X_- spacelike) and lightlike tetrahedra (X_0 lightlike)

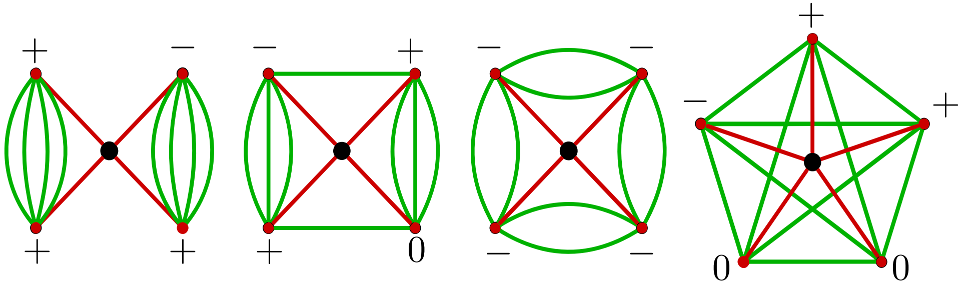
$$\Phi_\alpha : \mathrm{SL}(2,\mathbb{C})^4 \times \mathbb{R}^{d_{\mathrm{loc}}} \times \mathbb{H}_\alpha \longrightarrow \mathbb{R}$$
$$(\boldsymbol{g}, \boldsymbol{\phi}, X_\alpha) \longmapsto \Phi_\alpha(\boldsymbol{g}, \boldsymbol{\phi}, X_\alpha)$$

- closure $\quad \Phi_\alpha(\boldsymbol{g}h^{-1}, \boldsymbol{\phi}, h \cdot X_\alpha) = \Phi_\alpha(\boldsymbol{g}, \boldsymbol{\phi}, X_\alpha), \quad h \in \mathrm{SL}(2,\mathbb{C})$
- simplicity $\quad \Phi_\alpha(\boldsymbol{g}\boldsymbol{u}_\alpha, \boldsymbol{\phi}, X_\alpha) = \Phi_\alpha(\boldsymbol{g}, \boldsymbol{\phi}, X_\alpha), \quad \boldsymbol{u}_\alpha \in \mathrm{Stab}^4_{X_\alpha}$

$$K[\Phi_+, \Phi_0, \Phi_-] = \sum_\alpha (\Phi_\alpha, \mathcal{K}_\alpha \Phi_\alpha)$$

diagonal in causal characters

$$V[\Phi_+, \Phi_0, \Phi_-] = \lambda \int \mathrm{d}\boldsymbol{\phi}\, \mathrm{Tr}_\gamma \left[\Phi_+^{n_+} \Phi_0^{n_0} \Phi_-^{n_-}\right]$$



Landau-Ginzburg analysis of the causally complete Lorentzian Barrett-Crane model

Landau-Ginzburg mean-field theory

microscopic quantum geometric degrees of freedom $\xrightarrow{\text{coarse-graining}}$ continuum spacetime geometries

Spin foam renormalization via refinement	FRG methods applied to tensor models in large-N expansion	Homogeneous and inhomogeneous cosmology from GFT condensates	Landau-Ginzburg mean-field analysis applied to non-local field theories
Asante, Dittrich, Steinhaus 2211.09578; Bahr, Steinhaus 1605.07649; Dittrich, Mizera, Steinhaus 1409.2407	Gurau 1011.2726; 1105.3122,Bonzon, Riello, Gurau, Rivasseau; Carrozza 2404.07834	Oriti, Sindoni, Wilson-Ewing 1602.05881; Pithis, Sakellariadou 1904.00598 ; AFJ, Marchetti, Pithis 2308.13261	Marchetti, Oriti, Pithis, Thürigen 2110.15336, 2211.12768; Dekhil, AFJ, Oriti, Pithis 2404.04524, 2407.02325

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Landau-Ginzburg mean-field theory

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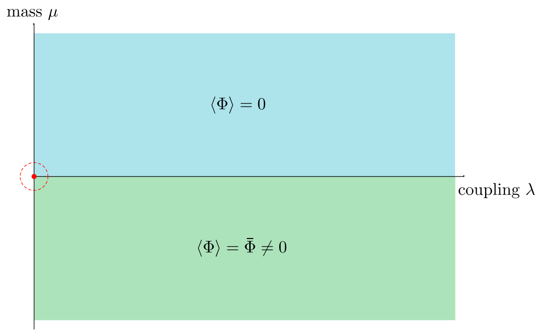
- coarse account for the phase structure
- self-consistency check near phase-transition
- condensate phase tentatively connected to continuum geometries



phenom. models for 2nd-order phase transitions via SSB

↓

approximate quantum gravity models via MF near the point $\mu \to 0$

Asante, Dittrich, Steinhaus 2211.09578; Carrozza 2404.07834; Oriti, Sindoni, Wilson-Ewing 1602.05881; Marchetti, Oriti, Pithis, Thürigen 2110.15336, 2211.12768

LG-theory applied to spacelike BC model

$$S[\Phi] = (\Phi, \mathcal{K}\Phi) + \lambda \int \mathrm{d}\boldsymbol{\phi}\, \mathrm{Tr}_\gamma \left[\prod_\tau \int \mathrm{d}X_+^\tau\, \Phi(\boldsymbol{g}^\tau, \boldsymbol{\phi}, X_+^\tau)\right], \qquad \mathcal{K} = \mu\delta(\boldsymbol{g}^{-1}\boldsymbol{g}') - Z(\boldsymbol{g}^{-1}\boldsymbol{g}')\Delta_{\boldsymbol{\phi}} - \sum_{c=1}^{4}\Delta_c$$

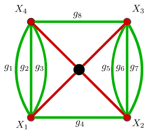
- spacelike tetrahedra
- spacelike faces $(\rho, 0)$

uniform sol. of equations of motion → linearization and effective action → effective mass → correlation function $\langle\delta\Phi(\boldsymbol{e},\boldsymbol{0})\delta\Phi(\boldsymbol{g},\boldsymbol{\phi})\rangle$ → correlation length → Ginzburg-Q

$$\bar{\Phi} = \left(\frac{|\mu|}{\lambda n_\gamma}\right)^{\frac{1}{n_\gamma - 2}} \mathrm{V}_L^{-\frac{r}{2}} \mathrm{V}_L^{-\frac{n_\gamma - 1}{n_\gamma - 2}}$$

$$\Phi(\boldsymbol{g}, \boldsymbol{\phi}, X_+) = \bar{\Phi} + \delta\Phi(\boldsymbol{g}, \boldsymbol{\phi}, X_+)$$

$$S_{\text{eff}}[\delta\Phi] = (\delta\Phi, G\delta\Phi)$$



$$b^{\boldsymbol{\rho}} = |\mu|(\chi^{\boldsymbol{\rho}} - 1)$$

$$\chi^{\boldsymbol{\rho}} = \prod_{c=1}^{4}\delta_{\rho_c, i} + \prod_{c\neq 4}\delta_{\rho_c, i} + \delta_{\rho_4, i}$$

- effective mass momentum dep.
- $\chi^{\boldsymbol{\rho}}$ encodes non-locality
- projection onto zero-modes
- local result $b = 2|\mu|$ for 4 zero modes otherwise, $b^{\boldsymbol{\rho}} \leq 0$ [Dekhil, AFJ, Pithis 2404.04524]

$$C_{\text{loc}}(\boldsymbol{\phi}) = \int \mathrm{d}\boldsymbol{g}\, C(\boldsymbol{g}, \boldsymbol{\phi})$$

$$C_{\text{nloc}}(\boldsymbol{g}) = \int \mathrm{d}\boldsymbol{\phi}\, C(\boldsymbol{g}, \boldsymbol{\phi})$$

$$\xi_{\text{loc}} = \frac{1}{\sqrt{b^i}} \sim |\mu|^{-\frac{1}{2}}$$

$$\xi_{\text{nloc}} = \frac{2}{a b^{c_1 \cdots c_s}} \sim (a|\mu|)^{-1}$$

$$Q = \frac{\int_{\Omega_\xi} \mathrm{d}\boldsymbol{g}\, \mathrm{d}\boldsymbol{\phi}\, C(\boldsymbol{g}, \boldsymbol{\phi})}{\int_{\Omega_\xi} \mathrm{d}\boldsymbol{g}\, \mathrm{d}\boldsymbol{\phi}\, \bar{\Phi}^2}$$

$$Q \sim \tilde{\lambda}^{\frac{2}{n_\gamma - 2}} \xi_{\text{loc}}^{-\frac{1}{2}\left(d_{\text{loc}} - 2\frac{n_\gamma}{n_\gamma - 2}\right)} \mathrm{e}^{-2(4 - s_0)\xi_{\text{nloc}}/a}$$

$$\lim_{\xi_{\text{nloc}} \to \infty} Q = 0$$

Marchetti, Oriti, Pithis, Thürigen 2110.15336, 2211.12768

LG-theory applied to spacelike BC model

Key results

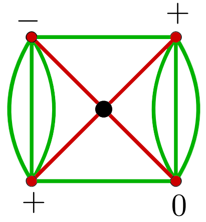
- exponential suppression of fluctuations
- hyperbolic part of $\mathrm{SL}(2,\mathbb{C})$ crucial
- holds for vanishing effective mass
- generalizes to multiple interactions
- supports the mean-field hypothesis of GFT condensate cosmology

Marchetti, Oriti, Pithis, Thürigen 2110.15336, 2211.12768

Extension to causally complete BC model

$$S[\Phi_+,\Phi_0,\Phi_-] = \sum_\alpha (\Phi_\alpha, \mathcal{K}_\alpha \Phi_\alpha) + \lambda \int \mathrm{d}\boldsymbol{\phi}\, \mathrm{Tr}_\gamma \left[\Phi_+^{n_+}\Phi_0^{n_0}\Phi_-^{n_-}\right],$$

$$\mathcal{K}_\alpha = \mu_\alpha \delta(\boldsymbol{g}^{-1}\boldsymbol{g}') - Z_\alpha^\phi(\boldsymbol{g}^{-1}\boldsymbol{g}')\Delta_\phi - Z_\alpha^g(|\boldsymbol{\phi}-\boldsymbol{\phi}'|)\sum_{c=1}^{4}\Delta_c$$



- multi-field theory (Φ_+,Φ_0,Φ_-)
- no $\mathrm{O}(3)$ only discrete $\mathbb{Z}_2$
- matrix-valued correlator $C_{\alpha\beta}$
- timelike tetrahedra with timelike faces $(0,\nu)$

uniform sol. of equations of motion → linearization and effective action → effective mass → correlation function $\langle \delta\Phi_\alpha(\boldsymbol{e},\boldsymbol{0})\delta\Phi_\beta(\boldsymbol{g},\boldsymbol{\phi})\rangle$ → correlation length → Ginzburg-Q

$$\mathrm{EOM}(\bar{\Phi}_\alpha) = 0$$

$$\Phi_\alpha(\boldsymbol{g},\boldsymbol{\phi},X_\alpha) = \bar{\Phi}_\alpha + \delta\Phi_\alpha(\boldsymbol{g},\boldsymbol{\phi},X_\alpha)$$

$$S_{\mathrm{eff}}[\delta\Phi] = \sum_{\alpha,\beta}(\delta\Phi_\alpha, G_{\alpha\beta}\delta\Phi_\beta)$$

$$b_m^{\boldsymbol{\rho}}(\{\mu_\alpha\},\{Z_\alpha^\phi\},\{Z_\alpha^g\})$$

$$b_m^{\boldsymbol{\rho}}(\mu\mu_+,\mu\mu_0,\mu\mu_-) = \mu\, b_m^{\boldsymbol{\rho}}(\mu_+,\mu_0,\mu_-)$$

$$C_{\alpha\beta}(\boldsymbol{g},\boldsymbol{\phi}) = \left(G^{-1}(\boldsymbol{g},\boldsymbol{\phi})\right)_{\alpha\beta}$$

restr. range of $\{Z_\alpha^\phi, Z_\alpha^g\}$

$$\xi_{\mathrm{loc}} = \frac{1}{\sqrt{b^{\boldsymbol{i}}_{m_*}}} \sim |\mu|^{-\frac{1}{2}}$$

$$\xi_{\mathrm{nloc}} = \frac{2}{a b^{c_1\cdots c_s}_{m_*}} \sim (a|\mu|)^{-1}$$

$$Q_{\alpha\beta} = \frac{\int_{\Omega_\xi}\mathrm{d}\boldsymbol{g}\,\mathrm{d}\boldsymbol{\phi}\, C_{\alpha\beta}(\boldsymbol{g},\boldsymbol{\phi})}{\int_{\Omega_\xi}\mathrm{d}\boldsymbol{g}\,\mathrm{d}\boldsymbol{\phi}\,\bar{\Phi}_\alpha\bar{\Phi}_\beta}$$

$$Q_{\alpha\beta} \sim \tilde{\lambda}^{\frac{2}{n_\gamma-2}}\xi_{\mathrm{loc}}^{-\frac{1}{2}\left(d_{\mathrm{loc}}-2\frac{n_\gamma}{n_\gamma-2}\right)} \mathrm{e}^{-2(4-s_0)\xi_{\mathrm{nloc}}/a}$$

$$\lim_{\xi_{\mathrm{nloc}}\to\infty} Q_{\alpha\beta} = 0$$

Dekhil, AFJ, Pithis 2407.02325

Extension to causally complete BC model

Key results

- exponential suppression of fluctuations in restricted regime of $\{Z_\alpha^\phi, Z_\alpha^g\}$
- $$\mathrm{SL}(2,\mathbb{C}) \sim \underbrace{\mathbb{H}^3}_{(\rho,0)} \times \underbrace{\mathbb{S}^3}_{(0,\nu)}$$

  $(\rho, 0)$: spacelike faces drive criticality

  $(0, \nu)$: timelike faces projected out
- generalizes: multiple interactions, CDT-like model, colorized model
- supports studies on cosmological perturbations from GFT condensates

Dekhil, AFJ, Pithis 2407.02325

Summary

- GFTs are combinatorially non-local field theories
- causally complete BC model
- exponential suppression of fluctuations
- hyperbolic structure of $\mathrm{SL}(2,\mathbb{C})$
- spacelike faces drive criticality
- timelike faces projected out

Open Questions

- extension to other causally complete models
- geometrical characterization of mean-field
- limited range of parameters
- FRG-methods applied to QG models
- is classical GFT already capturing QG?

Back-up

LG-theory applied to spacelike BC model

$$S[\Phi] = (\Phi, \mathcal{K}\Phi) + \lambda \int \mathrm{d}\boldsymbol{\phi}\, \mathrm{Tr}_\gamma \left[\prod_v \int \mathrm{d}X_+^v\, \Phi(\boldsymbol{g}^v, \boldsymbol{\phi}, X_+^v) \right], \qquad \mathcal{K} = \mu\delta(\boldsymbol{g}^{-1}\boldsymbol{g}') - Z(\boldsymbol{g}^{-1}\boldsymbol{g}')\Delta_{\boldsymbol{\phi}} - \sum_{c=1}^{4} \Delta_c$$

- spacelike tetrahedra
- spacelike faces $(\rho, 0)$

uniform sol. of equations of motion → linearization and effective action → effective mass → correlation function $\langle \delta\Phi(\boldsymbol{e}, \boldsymbol{0})\delta\Phi(\boldsymbol{g}, \boldsymbol{\phi})\rangle$ → correlation length → Ginzburg-Q

$$0 = \mu\bar{\Phi} + \lambda n_\gamma V_L^{\frac{r}{2}(n_\gamma - 2)} V_L^{n_\gamma - 1} \bar{\Phi}^{n_\gamma - 1}$$

$$\bar{\Phi} = \left(\frac{|\mu|}{\lambda n_\gamma}\right)^{\frac{1}{n_\gamma - 2}} \mathrm{V}_L^{-\frac{r}{2}} \mathrm{V}_L^{-\frac{n_\gamma - 1}{n_\gamma - 2}}$$

- volume factors $V_L = \mathrm{e}^{2L/a}$ from group and normal integration
- cf. quartic scalar field theory

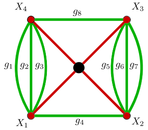
$$\bar{\phi} = \pm\sqrt{\frac{|\mu|}{4\lambda}}$$

$$\Phi(\boldsymbol{g}, \boldsymbol{\phi}, X_+) = \bar{\Phi} + \delta\Phi(\boldsymbol{g}, \boldsymbol{\phi}, X_+)$$

$$S_{\text{eff}}[\delta\Phi] = (\delta\Phi, G\delta\Phi)$$

$$G^{\boldsymbol{\rho}}(\boldsymbol{k}) = Z^{\boldsymbol{\rho}}\boldsymbol{k}^2 + \frac{1}{a^2}\sum_{c=1}^{4}(\rho_c^2 + 1) + b^{\boldsymbol{\rho}}$$

$$b^{\boldsymbol{\rho}} = |\mu|(\chi^{\boldsymbol{\rho}} - 1)$$



$$\chi^{\boldsymbol{\rho}} = \prod_{c=1}^{4} \delta_{\rho_c, i} + \prod_{c \neq 4} \delta_{\rho_c, i} + \delta_{\rho_4, i}$$

- effective mass momentum dep.
- $\chi^{\boldsymbol{\rho}}$ encodes non-locality
- projection onto zero-modes
- local result $b = 2|\mu|$ for 4 zero modes otherwise, $b^{\boldsymbol{\rho}} \leq 0$ [Dekhil, AFJ, Pithis 2404.04524]

Marchetti, Oriti, Pithis, Thürigen 2110.15336, 2211.12768

LG-theory applied to spacelike BC model

$$S[\Phi] = (\Phi, \mathcal{K}\Phi) + \lambda \int \mathrm{d}\boldsymbol{\phi}\, \mathrm{Tr}_\gamma \left[\prod_v \int \mathrm{d}X_+^v\, \Phi(\boldsymbol{g}^v, \boldsymbol{\phi}, X_+^v) \right], \qquad \mathcal{K} = \mu\delta(\boldsymbol{g}^{-1}\boldsymbol{g}') - Z(\boldsymbol{g}^{-1}\boldsymbol{g}')\Delta_{\boldsymbol{\phi}} - \sum_{c=1}^{4} \Delta_c$$

- spacelike tetrahedra
- spacelike faces $(\rho, 0)$

uniform sol. of equations of motion → linearization and effective action → effective mass → correlation function $\langle \delta\Phi(\boldsymbol{e},\boldsymbol{0})\delta\Phi(\boldsymbol{g},\boldsymbol{\phi})\rangle$ → correlation length → Ginzburg-Q

$$C^{\boldsymbol{\rho}}(\boldsymbol{k}) = \left(Z^{\boldsymbol{\rho}}\boldsymbol{k}^2 + \frac{1}{a^2}\sum_{c=1}^{4}(\rho_c^2+1) + b^{\boldsymbol{\rho}}\right)^{-1}$$

$$C_{\mathrm{loc}}(\boldsymbol{\phi}) = \int \mathrm{d}\boldsymbol{g}\, C(\boldsymbol{g},\boldsymbol{\phi}) = \int \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^{d_{\mathrm{loc}}}} \frac{\mathrm{e}^{i\boldsymbol{k}\boldsymbol{\phi}}}{Z^{\boldsymbol{i}}\boldsymbol{k}^2 + b^{\boldsymbol{i}}}$$

$$C_{\mathrm{nloc}}(\boldsymbol{g}) = \int \mathrm{d}\boldsymbol{\phi}\, C(\boldsymbol{g},\boldsymbol{\phi}) = \prod_c \int \mathrm{d}\rho_c\, \rho_c^2 \frac{\prod_c \mathrm{Tr}\left[D^{(\rho_c,0)}(g_c)\right]}{\frac{1}{a^2}\sum_c(\rho_c^2+1) + b^{\boldsymbol{\rho}}}$$

$$C_{\mathrm{nloc}}(\boldsymbol{g}) \to \sum_{s=s_0(\gamma)}^{4} V_L^{-s} \sum_{(c_1\ldots c_s)} C^s_{\mathrm{nloc}}(\boldsymbol{g}_{4-s})$$

$$Z^{\boldsymbol{i}} > 0:\ C_{\mathrm{loc}}(\boldsymbol{\phi}) \xrightarrow[|\boldsymbol{\phi}|\gg 1]{} \mathrm{e}^{-|\boldsymbol{\phi}|/\xi_{\mathrm{loc}}}$$

$$\xi_{\mathrm{loc}} \sim |\mu|^{-\frac{1}{2}}$$

$$C^s_{\mathrm{nloc}}(\boldsymbol{\eta}_{4-s})\Big|_{\eta_{c_{s+1}}\ldots\eta_{c_{4-s}}\equiv\eta} \xrightarrow[\eta\gg 1]{} \mathrm{e}^{-\eta/\xi^s_{\mathrm{nloc}}}$$

$$\xi_{\mathrm{nloc}} = \frac{2}{ab_{c_1\ldots c_s}} \sim (a|\mu|)^{-1}$$

$$Q = \frac{\int_{\Omega_\xi} \mathrm{d}\boldsymbol{g}\,\mathrm{d}\boldsymbol{\phi}\, C(\boldsymbol{g},\boldsymbol{\phi})}{\int_{\Omega_\xi} \mathrm{d}\boldsymbol{g}\,\mathrm{d}\boldsymbol{\phi}\, \bar{\Phi}^2}$$

$$Q \sim \tilde{\lambda}^{\frac{2}{n_\gamma-2}} \xi_{\mathrm{loc}}^{-\frac{1}{2}\left(d_{\mathrm{loc}} - 2\frac{n_\gamma}{n_\gamma-2}\right)} \mathrm{e}^{-2(4-s_0)\xi_{\mathrm{nloc}}/a}$$

Hyperbolic part of Lorentz group renders MF-theory valid

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