CEL-like scaling solutions in a Yukawa-QCD system

Physik Combo 2024 - Leipzig

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September 16, 2024

Introduction - CEL scaling solutions in a Higgs-QCD model

Study UV properties of the recently investigated model combining symmetry breaking mechanisms coming from the Higgs sector and QCD

These models, where an asymptotically free gauge sector is present, allow for special asymptotically free trajectories in the marginal couplings, so called Cheng-Eichten-Li solutions .

- Investigate existence of CEL solutions in the Higgs-QCD model
- Analyse properties of the scalar potential in the UV

CEL scaling solutions

Gauged Yukawa systems can, for a suitable field content, exhibit asymptotic freedom in all marginal couplings already in the perturbative analysis [Cheng,Eichten,Lee ’74].

In these cases, the marginal couplings are “controlled” by the running of the AF gauge coupling.

These solutions allow for enhanced predictivity, since all couplings run $\propto g^2$ and the values of the ratios are fixed by demanding UV completeness.

Controlled asymptotic freedom

A simple gauged Yukawa model exhibiting asymptotic freedom perturbatively is a $\mathbb{Z}_2$-Yukawa-QCD model [Gies et al ’18]

$$S = \int_x \frac{1}{2}\partial_\mu\phi\partial_\mu\phi + \frac{m^2}{2}\phi^2 + \frac{\lambda}{8}\phi^4 + \bar{\psi}\mathrm{i}\not{D}\psi + \frac{\mathrm{i}h}{\sqrt{2}}\phi\bar{\psi}\psi + \frac{1}{4}F^z_{\mu\nu}F^{\mu\nu}_z + \mathcal{L}_{\mathrm{gh+gf}},$$

with a real scalar field ϕ playing the role of the Higgs, a Dirac fermion ψ, substituting the top quark, and a $SU(N_\mathrm{c})$ gauge field A^μ.

For a suitable number of colors N_c and favors N_f, the gauge sector exhibits asymptotic freedom, i.e. $g^2 \to 0$ in the UV limit.

RG flow equations in the $\mathbb{Z}_2$-Yukawa-QCD model

In this model, the RG flow equation of g^2 reads

$$\partial_t g^2 = \eta_A g^2, \qquad \eta_A = -\frac{g^2}{8\pi^2}\left(\frac{11}{3}N_\mathrm{c} - \frac{2}{3}N_\mathrm{f}\right).$$

The flow equation for the Yukawa coupling is given by

$$\partial_t h^2 = \frac{h^2}{16\pi^2}\left[(3 + 2N_\mathrm{c})h^2 - 6\frac{N_\mathrm{c}^2 - 1}{N_\mathrm{c}}g^2\right].$$

These two equations entail that AF trajectories exist in the (g^2, h^2) plane. It can be best seen when looking at the flow of the composite coupling $\hat{h}^2 = h^2/g^2$.

Quasi fixed points in the model

The flow equation of the composite coupling

$$\partial_t \hat{h}^2 = \frac{1}{g^2}\partial_t h^2 - \frac{h^2}{g^4}\partial_t g^2$$

can be brought into the form

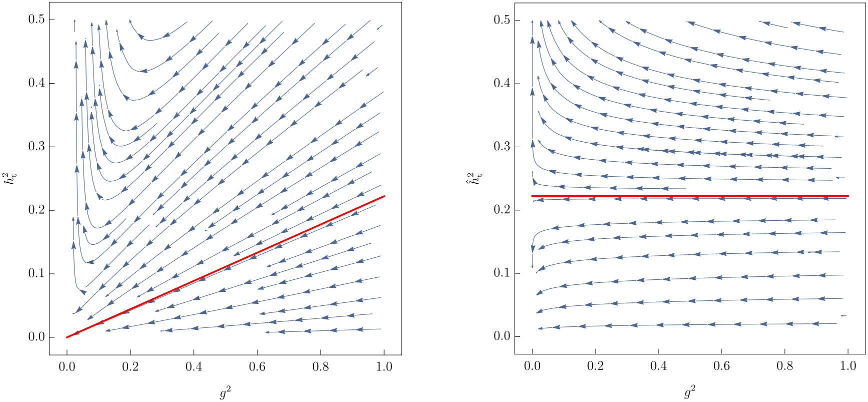
$$\partial_t \hat{h}^2 = \frac{3+2N_\mathrm{c}}{16\pi^2} g^2 \hat{h}^2 \left(\hat{h}^2 - \hat{h}_*^2\right),$$

showing that if the ratio h^2/g^2 takes the particular value

$$\hat{h}_*^2 = \frac{1}{3+2N_\mathrm{c}}\left[\frac{4}{3}(N_\mathrm{f} - N_\mathrm{c}) - \frac{6}{N_\mathrm{c}}\right],$$

it is frozen at any RG time. This solution exists for a finite window of N_f for fixed N_c.

RG flows in the (g^2, h^2) plane



Scalar potential in AF scaling solutions

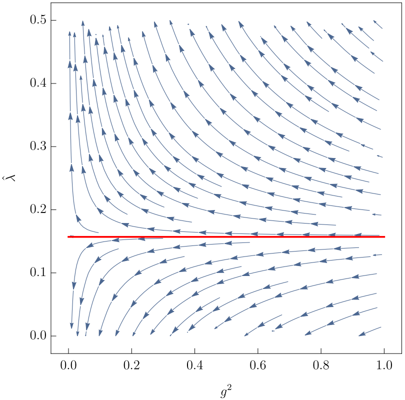
The last remaining marginal coupling to be analyzed is the scalar self interaction λ, which flow equation reads

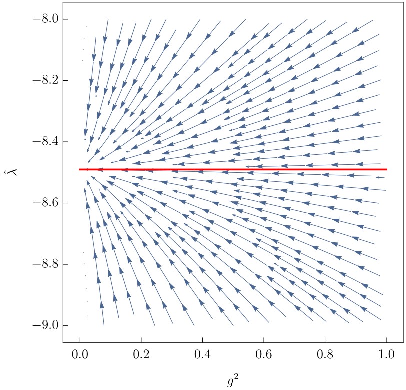
$$\partial_t \lambda = \frac{N_c}{4\pi^2} h^2 \lambda + \frac{9}{16\pi^2} \lambda^2 - \frac{N_c}{4\pi^2} h^4$$

We investigate the flow of the composite coupling $\hat{\lambda} = \lambda / h^2$ and find two possible quasi fixed points, one positive (UV repulsive) and one negative (UV attractive).

Together with $h^2 \propto g^2$ that implies that the Yukawa coupling, as well as the scalar self interaction, run proportional to the strong gauge coupling g^2.

Scalar potential in AF scaling solutions





Recap on the Higgs-QCD model

We will investigate CEL scaling solutions in the Higgs-QCD model

$$\Gamma_k = \int_x \; Z_\phi |\partial_\mu \phi|^2 + Z_{\tilde{\phi}} \left|\partial_\mu \tilde{\phi}\right|^2 + \bar{\psi}_i^a \mathrm{i} \not{D}_{ij} \psi_j^a + \frac{Z_\mathrm{F}}{4} F^z_{\mu\nu} F_z^{\mu\nu} + \mathcal{L}_{\mathrm{gf+gh}}$$
$$+ U(\rho, \tilde{\rho}) + \frac{\mathrm{i} h_t}{\sqrt{2}} \left(\bar{\psi}^a_{\mathrm{L},i} \phi^a_{\mathcal{C}} t_{\mathrm{R},i} + \mathrm{h.c.}\right) + \frac{\mathrm{i} h_b}{\sqrt{2}} \left(\bar{\psi}^a_{\mathrm{L},i} \phi^a b_{\mathrm{R},i} + \mathrm{h.c.}\right)$$
$$+ \frac{\mathrm{i} h}{\sqrt{2}} \left(\bar{\psi}^a_{\mathrm{R},i} \tilde{\phi}^a b_{\mathrm{L},i} + \bar{\psi}^a_{\mathrm{R},i} \tilde{\phi}^a_{\mathcal{C}} t_{\mathrm{L},i} + \mathrm{h.c.}\right) \;,$$

obtained from the QCD action after rebosonization and isolating the $\mathrm{SU}(2)_\mathrm{L}$ scalar doublet ϕ.

Origin of the action

We obtain the action under consideration by starting with QCD, supplemented with a four fermion interaction

$$\Gamma_k = \int_x \bar{\psi}_i^a (\mathrm{i} Z_\psi \partial\!\!\!/ \delta_{ij} + \bar{g} A\!\!\!/_{ij}) \psi_j^a + \frac{Z_F}{4} F_{\mu\nu} F^{\mu\nu} + \frac{(\partial_\mu A^\mu)^2}{2\xi} + \frac{1}{2} \bar{\lambda}_\sigma (\mathrm{S} - \mathrm{P}),$$

$$\text{where } (\mathrm{S} - \mathrm{P}) = \left(\bar{\psi}_i^a \psi_i^b\right)^2 - \left(\bar{\psi}_i^a \gamma_5 \psi_i^b\right)^2$$

Flow of four fermion coupling $\partial_t \lambda_\sigma \sim g^4$

Towards IR: $g \nearrow$ due to asymptotic freedom, hence $\lambda_\sigma \nearrow$

$\rightarrow$ Analysis of the model on all energy scales requires an effective field theory description

Dynamical Bosonization

Translate microscopic degrees of freedom to macroscopic ones (quarks, gluons $\to$ mesons)

$$\bar{\psi}_i^a \psi_i^b \quad \to \quad \varphi^{ab}$$

Encode four-fermion interaction in Yukawa interaction

$$\lambda_\sigma \left(\left(\bar{\psi}_i^a \psi_i^b\right)^2 - \left(\bar{\psi}_i^a \gamma_5 \psi_i^b\right) \right) \qquad h\bar{\psi}_i^a \left(P_{\mathrm{R}} \varphi^{ab} + P_{\mathrm{L}} \varphi^{\dagger ab}\right) \psi_i^b$$

On all scales k: Additional contributions to the flow eqs. of the Yukawa model to compensate the 4-Fermi coupling

Consequences of dynamical bosonization

Bosonization introduces a new parameter into the model, namely the Yukawa coupling and the scalar potential for the new auxiliary scalar field. The initial values of these couplings at the high energy scale Λ can be chosen such that QCD physics are unchanged, namely

$$\begin{aligned} h_\Lambda^2 &\to 0 \\ m^2 &\to \Lambda^2 \\ \lambda_{\phi^4} &\to 0. \end{aligned}$$

When promoting the scalar field to play the role of the Higgs, the condition on the scalar potential however can be relaxed.
To compensate for the effective four fermion vertex, the flows of these couplings however receive additional contributions.

Flow equations in the Higgs-QCD model

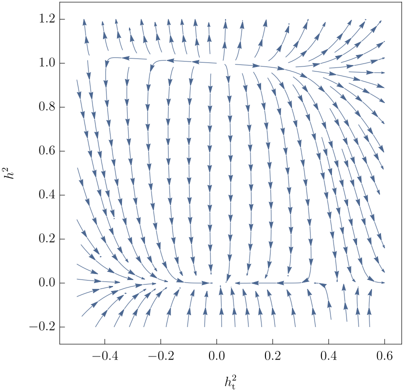
We use the Wetterich equation [Wetterich '93] to obtain the flow equation of the model. Assuming validity of the deep Euclidean region, we extract the perturbative beta functions.

We look for quasi fixed points in the Yukawa sector by looking for solutions of

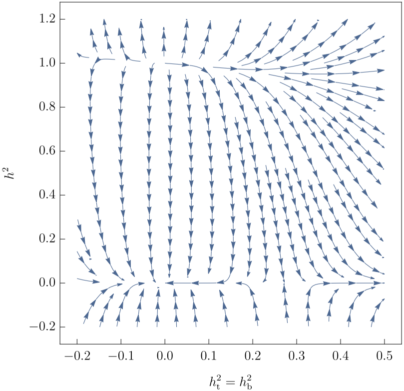
$$\partial_t \left(\frac{h_{\mathrm{t}}^2}{g^2} \right) = 0 \qquad \partial_t \left(\frac{h_{\mathrm{b}}^2}{g^2} \right) = 0 \qquad \partial_t \left(\frac{h^2}{g^2} \right) = 0.$$

Of the found solutions, some admit physical trajectories ($h_*^2 \geq 0$).

CEL solutions in the Yukawa sector

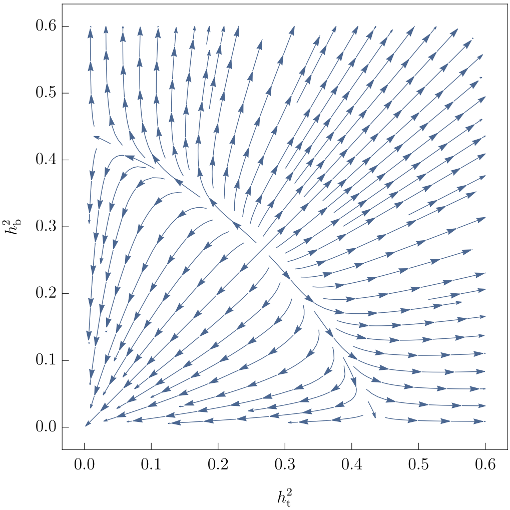


QFPs in the $h_b^2 = 0$ plane



QFPs in the $h_b^2 = h_t^2$ plane

CEL solutions in the Yukawa sector



QFPs in the scalar sector

We then look for QFPs in the scalar sector, where we have as a first step assumed that the scalar potential is given by

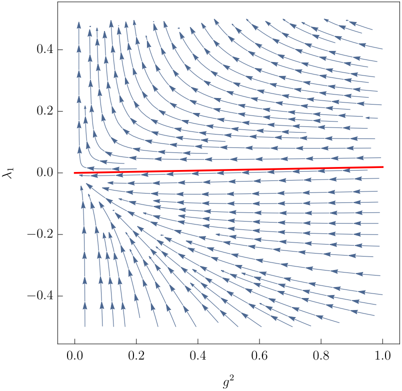
$$U(\rho, \tilde{\rho}) = \frac{\lambda_1}{2}(\rho + \tilde{\rho})^2 + \lambda_2 \rho \tilde{\rho},$$

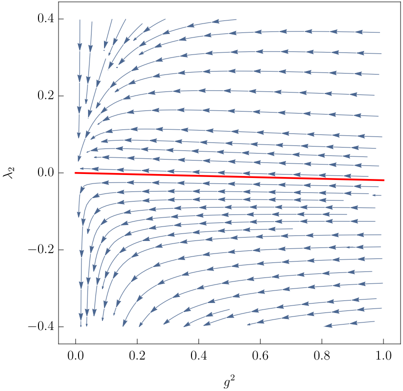
by solving the system of equations

$$\partial_t \left(\frac{\lambda_1}{g^2}\right)\bigg|_{h_\chi^2 = h_*^2} = 0 \qquad \partial_t \left(\frac{\lambda_2}{g^2}\right)\bigg|_{h_\chi^2 = h_*^2} = 0.$$

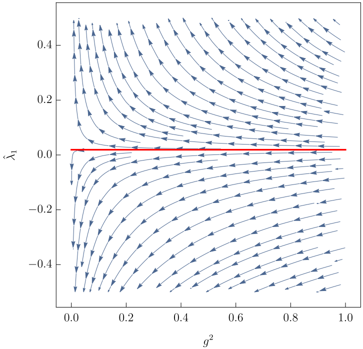
Physically admissible trajectories are those with $\lambda_1 \geq 0$ and $\lambda_2 \geq -\lambda_1$, making the potential bounded from below at the high energy scale Λ

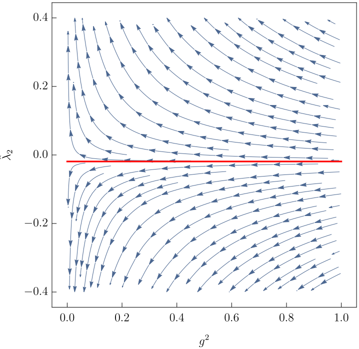
CEL solution for the Scalar potential





CEL solution for the Scalar potential





These solutions are exactly the edge case $\hat{\lambda}_{1*} = -\hat{\lambda}_{2*}$, making the UV potential $U(\rho, \tilde{\rho}) = \frac{\lambda_1}{2}\left(\rho^2 + \tilde{\rho}^2\right)$.

Conclusions

CEL scaling solutions exist in the Higgs-QCD model

- All marginal couplings are controlled by the running of the strong gauge coupling, making them asymptotically free
- Scalar fixed-point potential bounded from below, approaches flatness in the UV

Further points of interest:

$\Rightarrow$ Study the fate of the rebosonization on the QFPs

$\Rightarrow$ Check reliability of the found CEL solutions using the full set of equations

$\Rightarrow$ Construct full flows emanating from the QFPs

Thank you!

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