

Significance of relativistic corrections in atoms interacting with gravity

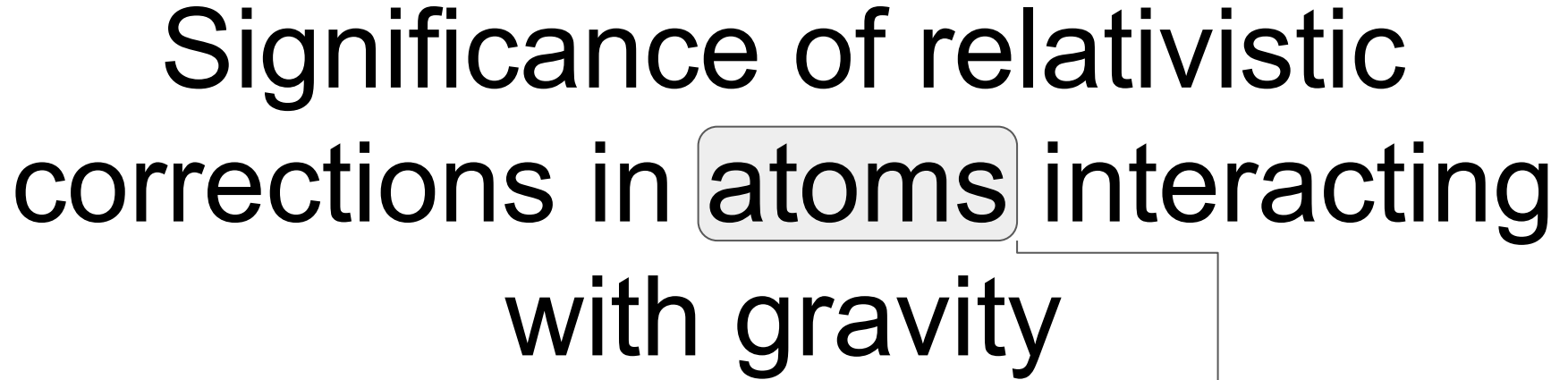
Linda van Manen

Physik-Combo 26-27 March 2024

Funded by
DFG Deutsche
Forschungsgemeinschaft
German Research Foundation


VolkswagenStiftung

Significance of relativistic corrections in **atoms** interacting with gravity



Two (non interacting)
point particle

Significance of relativistic corrections in atoms interacting with gravity

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graph TD; Title[Significance of relativistic corrections in atoms interacting with gravity]; Title --- A[atoms]; Title --- B[interacting with gravity]; A --- C[Open quantum system with gravity acting as environment]; B --- D[Two (non interacting) point particle];
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Open quantum system with gravity acting as environment

Two (non interacting) point particle

Significance of relativistic
corrections?

Significance of relativistic corrections?

PHYSICAL REVIEW A **98**, 042106 (2018)

Mass-energy and anomalous friction in quantum optics

Matthias Sonnleitner

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(Received 1 June 2018; published 4 October 2018)

The usual multipolar Hamiltonian for atom-light interaction features a nonrelativistic moving atom interacting with electromagnetic fields which inherently follow Lorentzian symmetry. This combination can lead to situations where atoms appear to experience a friction force, when in fact they only change their internal mass-energy due to the emission or absorption of a photon. Unfortunately, the simple Galilean description of the atom's motion is not sufficient to distinguish between a change in momentum due to acceleration and a change in momentum due to a change in internal mass-energy. In this work we show how a low-order relativistic correction can be included in the multipolar atom-light Hamiltonian. We also give examples how this affects the most basic mechanical interactions between atoms and photons.

DOI: [10.1103/PhysRevA.98.042106](https://doi.org/10.1103/PhysRevA.98.042106)

Significance of relativistic corrections?

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PHYSICAL REVIEW
LETTERS

VOLUME 46

26 JANUARY 1981

NUMBER 4

Influence of Dissipation on Quantum Tunneling in Macroscopic Systems

A. O. Caldeira and A. J. Leggett

School of Mathematical and Physical Sciences, University of Sussex, Brighton BN1 9QH, Sussex, United Kingdom

(Received 28 July 1980)

A quantum system which can tunnel, at $T = 0$, out of a metastable state and whose interaction with its environment is adequately described in the classically accessible region by a phenomenological friction coefficient η , is considered. By only assuming that the environment response is linear, it is found that dissipation multiplies the tunneling probability by the factor $\exp[-A\eta(\Delta q)^2/\hbar]$, where Δq is the "distance under the barrier" and A is a numerical factor which is generally of order unity.

PACS numbers: 03.65.Bz, 05.30.-d, 05.40.+j, 73.40.Gk

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Outline

Caldeira Leggett model: the basics

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Emphasize on emerging relativistic corrections

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Emphasize on emerging relativistic corrections

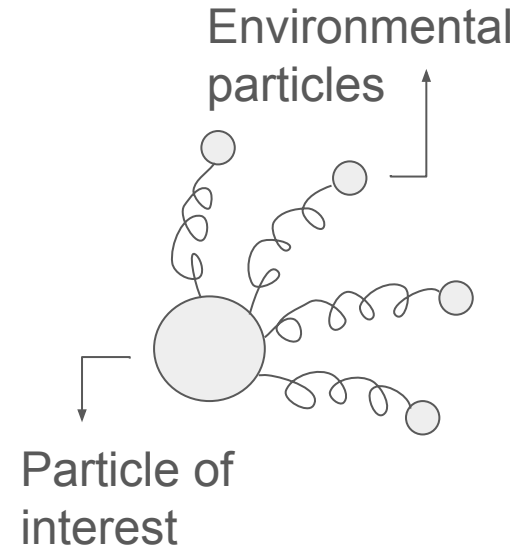
Relativistic corrections correlate internal coordinates with center-of-mass coordinates.
Decoherence!

Caldeira-Leggett model [A. Caldeira, A. J. Leggett, Phys. Rev. Lett. 46 (1981) 211]

(In a nutshell)

Hamiltonian

$$H = \frac{p^2}{2M} + V_0(x) + \sum_{j=1}^N \left(\frac{p_j^2}{2m_j} + \frac{1}{2} m_j \omega_j^2 x_j^2 \right) - \sum_{j=1}^N C_j x_j x.$$



Caldeira-Leggett model [A. Caldeira, A. J. Leggett, Phys. Rev. Lett. 46 (1981) 211]

(In a nutshell)

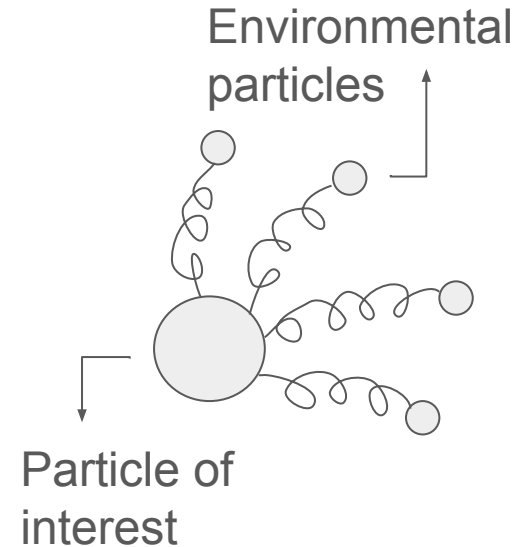
Hamiltonian

$$H = \frac{p^2}{2M} + V_0(x) + \sum_{j=1}^N \left(\frac{p_j^2}{2m_j} + \frac{1}{2} m_j \omega_j^2 x_j^2 \right) - \sum_{j=1}^N C_j x_j x.$$

Langevin equation

$$\frac{d\hat{p}(t)}{dt} = \sum_{j=1}^N \frac{C_j^2}{2m_j\omega_j} \left[i \int_{t_0}^t \hat{x}(t') e^{-i\omega_j(t-t')} dt' + \text{h.c.} \right] + \hat{F}_L(t)$$

$$\hat{F}_L(t) \equiv \sum_{j=1}^N C_j \sqrt{\frac{\hbar}{2m_j\omega_j}} \left[\hat{a}_j(t_0) e^{-i\omega_j(t-t_0)} + \hat{a}_j^\dagger(t_0) e^{i\omega_j(t-t_0)} \right].$$



Langevin equation

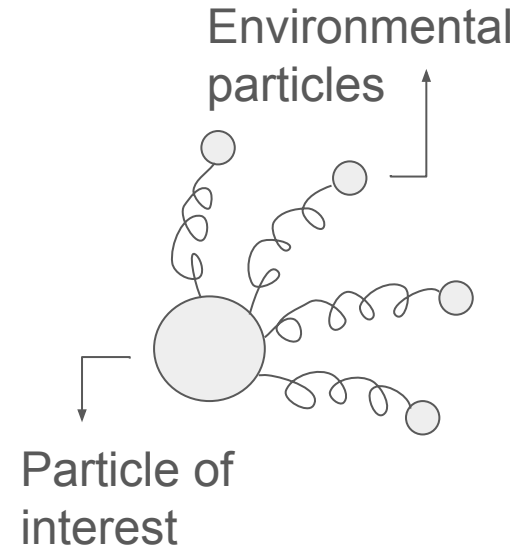
$$\frac{d\hat{p}(t)}{dt} = \sum_{j=1}^N \frac{C_j^2}{2m_j\omega_j} \left[i \int_{t_0}^t \hat{x}(t') e^{-i\omega_j(t-t')} dt' + \text{h.c.} \right] + \hat{F}_L(t)$$



$$\sum_{j=1}^N \frac{C_j^2}{2m_j\omega_j} \left[i \int_{t_0}^t \hat{x}(t') e^{-i\omega_j(t-t')} dt' + \text{h.c.} \right] = \frac{1}{\pi} \int J(\omega) \left[i \int_{t_0}^t \hat{x}(t') e^{-i\omega(t-t')} dt' + \text{h.c.} \right] d\omega$$

Spectral density of the coupling to the bath

$$J(\omega) \equiv \frac{\pi}{2} \sum_j \frac{C_j^2}{m_j\omega_j} \delta(\omega - \omega_j),$$



Langevin equation

$$\frac{d\hat{p}(t)}{dt} = \sum_{j=1}^N \frac{C_j^2}{2m_j\omega_j} \left[i \int_{t_0}^t \hat{x}(t') e^{-i\omega_j(t-t')} dt' + \text{h.c.} \right] + \hat{F}_L(t)$$



$$\sum_{j=1}^N \frac{C_j^2}{2m_j\omega_j} \left[i \int_{t_0}^t \hat{x}(t') e^{-i\omega_j(t-t')} dt' + \text{h.c.} \right] = \frac{1}{\pi} \int J(\omega) \left[i \int_{t_0}^t \hat{x}(t') e^{-i\omega(t-t')} dt' + \text{h.c.} \right] d\omega$$

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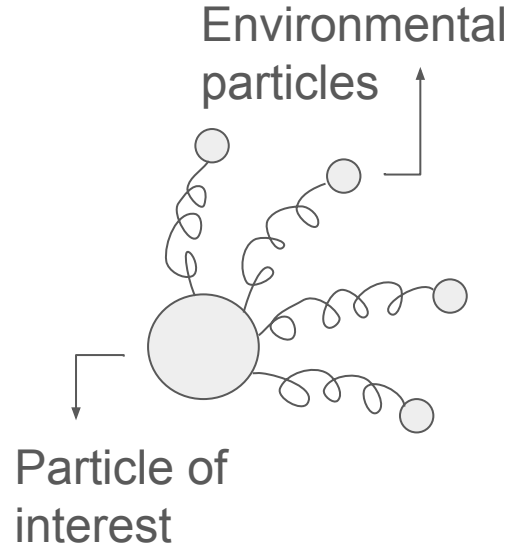
$$J(\omega) \equiv \frac{\pi}{2} \sum_j \frac{C_j^2}{m_j\omega_j} \delta(\omega - \omega_j),$$



$$\frac{d\hat{p}(t)}{dt} = -M \int_0^{t-t_0} \hat{x}(t-\tau) \gamma'(\tau) d\tau + \hat{F}_L(t)$$

with the memory

$$\text{kernel } \gamma(t) = \frac{2}{\pi} \int \frac{J(\omega)}{M\omega} \cos \omega t d\omega.$$



Equation of motion

$$\frac{d\hat{p}(t)}{dt} = -M \int_0^{t-t_0} \hat{x}(t-\tau) \gamma'(\tau) d\tau + \hat{F}_L(t)$$

Memory kernel

$$\gamma(t) = \frac{2}{\pi} \int \frac{J(\omega)}{M\omega} \cos \omega t d\omega$$

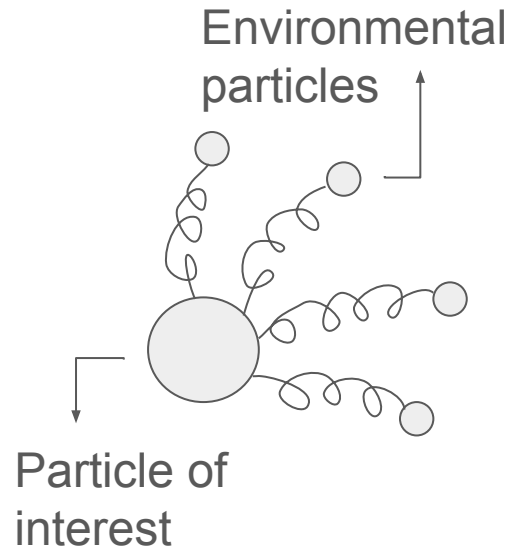
Spectral density

$$J(\omega) \equiv \frac{\pi}{2} \sum_j \frac{C_j^2}{m_j \omega_j} \delta(\omega - \omega_j),$$

By choosing $J(\omega) = \eta\omega \left[\frac{\Lambda^2}{\Lambda^2 + \omega^2} \right]$ we can match the underlying microscopic equation of motion with the phenomenological Langevin equation

$$M \frac{dv(t)}{dt} = -M\eta v(t) + F_L(t)$$

Warning: Infinities ahead!



Equation of motion

$$\frac{d\hat{p}(t)}{dt} = -M \int_0^{t-t_0} \hat{x}(t-\tau) \gamma'(\tau) d\tau + \hat{F}_L(t)$$



$$= -\eta\Lambda x(t) + \eta\dot{x}(t) - \frac{\eta}{2\Lambda}\ddot{x}(t) \quad \text{with } \Lambda \rightarrow \infty$$



Unphysical divergences
a.k.a problematic!

Matching with

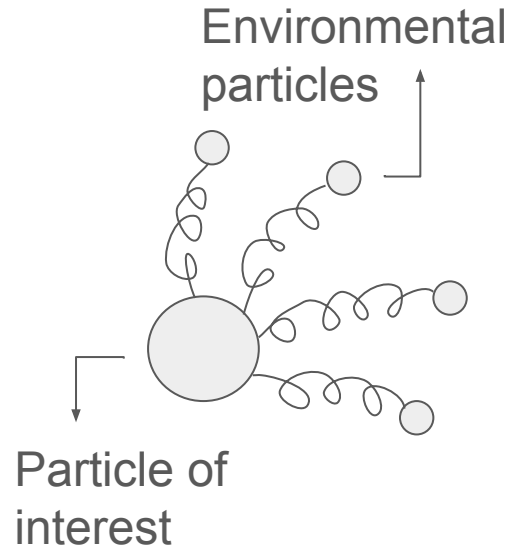
$$M \frac{dv(t)}{dt} = -M\eta v(t) + F_L(t)$$

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Spectral density

$$J(\omega) = \eta\omega \left[\frac{\Lambda^2}{\Lambda^2 + \omega^2} \right]$$



What's going wrong?

Hamiltonian

$$H = \frac{p^2}{2M} + V_0(x) + \sum_{j=1}^N \left(\frac{p_j^2}{2m_j} + \frac{1}{2} m_j \omega_j^2 x_j^2 \right) - \sum_{j=1}^N C_j x_j x.$$

$$V_0(x) = V(x) + \left(\sum_{j=1}^N \frac{C_j^2}{2m_j \omega_j^2} \right) x^2. \quad (\text{Lamb shift})$$



Measurements of what is now known as the Lamb shift by Willis Lamb (pictured) and Robert Retherford

Picture from National Archives and Records Admin., courtesy AIP Emilio Segrè Visual Archives

Traveling to 2024

Research to gravitational decoherence (decoherence rate from different models) [See overview paper by Bose et. al., ArXiv 2311.09218]

Lab tests decoherence regardless of its origin

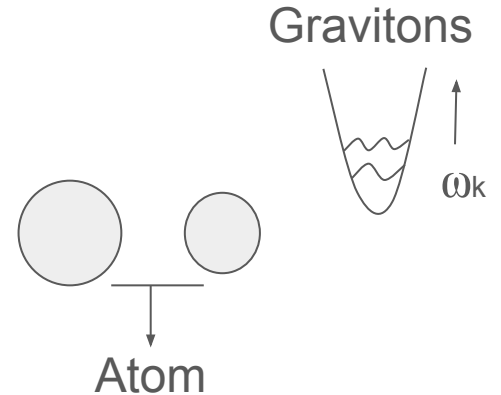
What does our theories not predict? (What either can't be tested in a lab yet.)

“Atom” interacting with gravity

Environment:

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with transverse- traceless (TT) gauge applied \longrightarrow Gravitational waves

with source far away \longrightarrow Vacuum solution



“Atom” interacting with gravity

Environment:

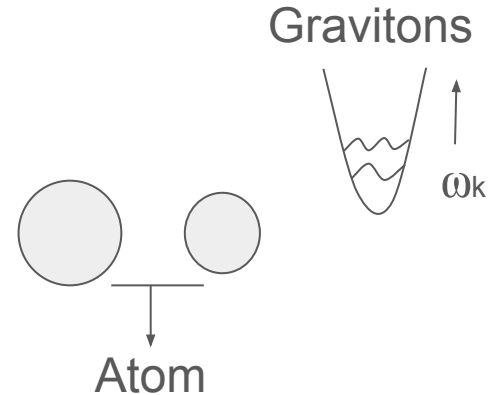
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with transverse- traceless (TT) gauge applied \longrightarrow Gravitational waves

with source far away \longrightarrow Vacuum solution

Expand the perturbation as a sum of discrete modes

$$h_{ij} = \sum_{k,s=+,\times} q_{k,s}(t) e^{ik \cdot x} \epsilon_{ij}^s(k)$$

Considered $k^\mu = (\omega_k, 0, 0, 0)$
at scale of an atom



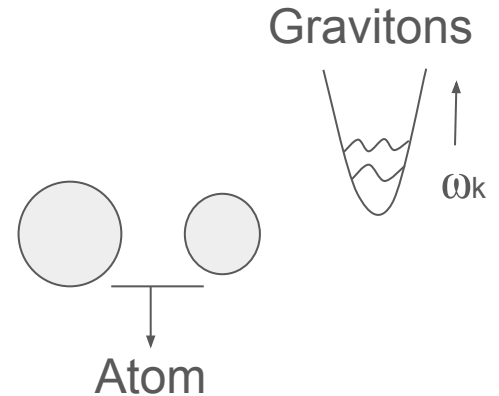
“Atom” interacting with gravity

System

N (for now) non-interacting particles

Hamiltonian

$$\hat{H} = \sum_{k,n} \frac{\hat{p}_n^2}{2m} - \frac{\hat{q}_k \hat{p}_n^i \hat{p}_n^j \epsilon_{ij}}{m} + \frac{\hat{\pi}_k^2}{2\mu} + \frac{\mu}{2} \omega_k^2 \hat{q}_k^2$$



Comment: Not force but momentum

$$\frac{d\hat{x}_n^j}{dt} = \frac{1}{i\hbar} [\hat{x}_n^j, \hat{H}] = \sum_k \frac{\hat{p}_n^j}{m} - 2 \frac{\hat{q}_k (\hat{p}_i \epsilon^{ij})_n}{m} \quad \longleftarrow$$

Interaction appears in the equation for velocity

$$\frac{d\hat{p}_n}{dt} = \frac{1}{i\hbar} [\hat{p}_n, \hat{H}] = \frac{dU(x_n)}{dx_n}$$

$$\frac{d\hat{q}_k}{dt} = \frac{1}{i\hbar} [\hat{q}_k, \hat{H}] = \frac{\hat{\pi}_k}{\mu}$$

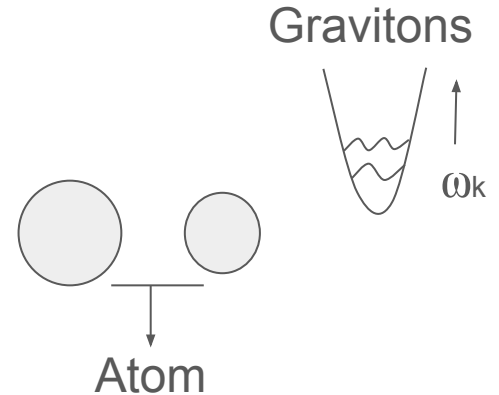
$$\frac{d\hat{\pi}_k}{dt} = \frac{1}{i\hbar} [\hat{\pi}_k, \hat{H}] = - \sum_n \frac{(\hat{p}^i \hat{p}^j \epsilon_{ij})_n}{m} - \mu \omega_k^2 \hat{q}_k$$

Equation of motion

$$\frac{d\hat{x}^j}{dt} = \frac{\hat{p}_n^j}{m} - 2(\hat{p}_i)_n \epsilon^{ij} \sum_m \int_0^t \gamma(t') \frac{d}{dt'} [(p_a(t') p_b(t'))_m] \epsilon^{ab} dt' + 2(p_i \epsilon^{ij})_n \sum_m (p_a(t) p_b(t) \epsilon^{ab})_m \gamma(0) - \hat{P}_L(t)$$

$$\hat{P}_L = -\frac{\hat{p}_i \epsilon^{ij}}{m} \sqrt{\frac{2\hbar}{\mu\omega}} (\hat{a}^\dagger(0) e^{i\omega t} + \hat{a}(0) e^{-i\omega t})$$

“Langevin momentum”

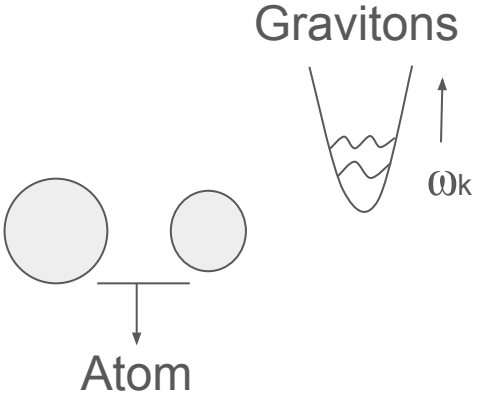


Equation of motion

$$\frac{d\hat{x}^j}{dt} = \frac{\hat{p}_n^j}{m} - 2(\hat{p}_i)_n \epsilon^{ij} \sum_m \int_0^t \gamma(t') \frac{d}{dt'} [(p_a(t') p_b(t'))_m] \epsilon^{ab} dt' + 2(p_i \epsilon^{ij})_n \sum_m (p_a(t) p_b(t) \epsilon^{ab})_m \gamma(0) - \hat{P}_L(t)$$

Diffusion term

Analogous to divergent term in the Caldeira Leggett model



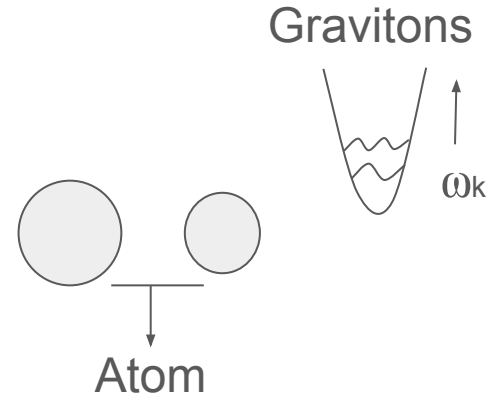
Equation of motion

$$\frac{d\hat{x}^j}{dt} = \frac{\hat{p}_n^j}{m} - 2(\hat{p}_i)_n \epsilon^{ij} \sum_m \int_0^t \gamma(t') \frac{d}{dt'} [(p_a(t') p_b(t'))_m] \epsilon^{ab} dt' + 2(p_i \epsilon^{ij})_n \sum_m (p_a(t) p_b(t) \epsilon^{ab})_m \gamma(0) - \hat{P}_L(t)$$

Diffusion term

Analogous to divergent term in the Caldeira Leggett model

Matching with macroscopic Brownian motion?

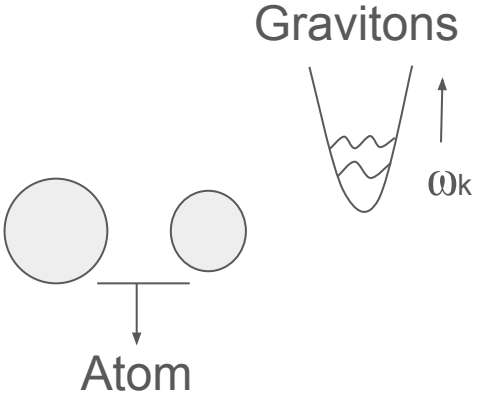


On closer inspection

$$\frac{d\hat{x}^j}{dt} = \frac{\hat{p}_n^j}{m} - 2(\hat{p}_i)_n \epsilon^{ij} \sum_m \int_0^t \gamma(t') \frac{d}{dt'} [(p_a(t')p_b(t'))_m] \epsilon^{ab} dt' + 2(p_i \epsilon^{ij})_n \sum_m (p_a(t)p_b(t) \epsilon^{ab})_m \gamma(0) - \hat{P}_L(t)$$

This term is cancelled with a correction to the Hamiltonian:

$$H_c = \sum_{m,n} \frac{(p_i p_j)_n (p_a p_b)_m \epsilon^{ij} \epsilon^{ab}}{2m^2 \mu \omega_k^2}$$



On closer inspection

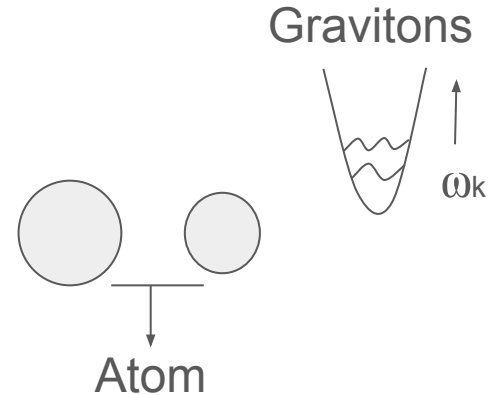
$$\frac{d\hat{x}^j}{dt} = \frac{\hat{p}_n^j}{m} - 2(\hat{p}_i)_n \epsilon^{ij} \sum_m \int_0^t \gamma(t') \frac{d}{dt'} [(p_a(t') p_b(t'))_m] \epsilon^{ab} dt' + 2(p_i \epsilon^{ij})_n \sum_m (p_a(t) p_b(t) \epsilon^{ab})_m \gamma(0) - \hat{P}_L(t)$$

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$$H_c = \sum_{m,n} \frac{(p_i p_j)_n (p_a p_b)_m \epsilon^{ij} \epsilon^{ab}}{2m^2 \mu \omega_k^2}$$

This is just a

- relativistic correction
- correlation between the particles momentum as a consequence of interacting with the same environment.



Final Hamiltonian

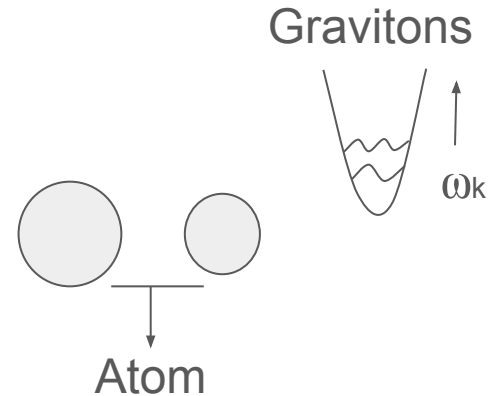
$$\hat{H} = \sum_{k,n} \frac{\hat{p}_n^2}{2m} + \frac{\hat{\pi}_k^2}{2\mu} + \frac{\mu}{2} \omega_k^2 \left(\hat{q}_k - \frac{(\hat{p}_i \hat{p}_j)_n \epsilon^{ij}}{m \mu \omega_k^2} \right)^2 + \sum_{m \neq n} \frac{(p_i p_j)_n (p_a p_b)_m \epsilon^{ij} \epsilon^{ab}}{2m^2 \mu \omega_k^2}$$

Then with linear spectral density

$$J(\omega) = \eta \omega \left[\frac{\Lambda^2}{\Lambda^2 + \omega^2} \right]$$

The “Langevin equation” becomes

$$\frac{d\hat{x}_n^j}{dt} = \frac{\hat{p}_n^j}{m} - 2\eta (p_i \epsilon^{ij})_n \sum_m [\hat{p}_m^a p_m^b + p_m^a \hat{p}_m^b] \epsilon_{ab} - \hat{P}_L(t)$$



Recap

Different ways to model Brownian motion from underlying interactions with gravitons leads to different (testable?) predictions.

Implausible physics can be used to restrict possible spectral density of the environment.

So, why are incredible small relativistic correction interesting?

Decoherence in an unexpected way

ARTICLES

PUBLISHED ONLINE: 15 JUNE 2015 | DOI: 10.1038/NPHYS3366

nature
physics

Universal decoherence due to gravitational time dilation

Igor Pikovski^{1,2,3,4*}, Magdalena Zych^{1,2,5}, Fabio Costa^{1,2,5} and Časlav Brukner^{1,2}

The physics of low-energy quantum systems is usually studied without explicit consideration of the background spacetime. Phenomena inherent to quantum theory in curved spacetime, such as Hawking radiation, are typically assumed to be relevant only for extreme physical conditions: at high energies and in strong gravitational fields. Here we consider low-energy quantum mechanics in the presence of gravitational time dilation and show that the latter leads to the decoherence of quantum superpositions. Time dilation induces a universal coupling between the internal degrees of freedom and the centre of mass of a composite particle. The resulting correlations lead to decoherence in the particle position, even without any external environment. We also show that the weak time dilation on Earth is already sufficient to affect micrometre-scale objects. Gravity can therefore account for the emergence of classicality and this effect could in principle be tested in future matter-wave experiments.



Gravitational time dilation couples internal degrees of freedom (d.o.f.) with center of mass d.o.f.

Decoherence in an unexpected way

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nature
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No it doesn't!



Universal decoherence due to gravitational time dilation

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The physics of low-energy quantum systems is usually studied without explicit consideration of the phenomena inherent to quantum theory in curved spacetime, such as Hawking radiation, are typical only for extreme physical conditions: at high energies and in strong gravitational fields. Here we consider mechanics in the presence of gravitational time dilation and show that the latter leads to the superpositions. Time dilation induces a universal coupling between the internal degrees of freedom of a composite particle. The resulting correlations lead to decoherence in the particle position, an environment. We also show that the weak time dilation on Earth is already sufficient to affect it. Gravity can therefore account for the emergence of classicality and this effect could in principle be tested in wave experiments.



Gravitational time dilation couples internal degrees of freedom (d.o.f.) with center of mass d.o.f.

Loss of coherence and coherence protection from a graviton bath

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¹*School of Physics and Astronomy, University of Glasgow, Glasgow, G12 8QQ, United Kingdom.*

²*Van Swinderen Institute, University of Groningen, 9747 AG Groningen, The Netherlands.*

³*University College London, Gower Street, WC1E 6BT London, United Kingdom.*

We consider a quantum harmonic oscillator coupled with a graviton bath and discuss the loss of coherence in the matter sector due to the matter-graviton vertex interaction. Working in the quantum-field-theory framework, we obtain a master equation by tracing away the gravitational field at the leading order $\sim \mathcal{O}(G)$ and $\sim \mathcal{O}(c^{-2})$. We find that the decoherence rate is proportional to the cube of the harmonic trapping frequency and vanishes for a free particle, as expected for a system without a mass quadrupole. Furthermore, our quantum model of graviton emission recovers the known classical formula for gravitational radiation from a classical harmonic oscillator for coherent states with a large occupation number. In addition, we find that the quantum harmonic oscillator eventually settles in a steady state with a *remnant coherence* of the ground and first excited states. While classical emission of gravitational waves would make the harmonic system lose all of its energy, our quantum field theory model does not allow the number states $|1\rangle$ and $|0\rangle$ to decay via graviton emission. In particular, the superposition of number states $\frac{1}{\sqrt{2}}[|0\rangle + |1\rangle]$ is a steady state and never decoheres.

Coordinate transformation

$$p_1 = p_r + \frac{m_1}{M} P$$

p_r is the relative momentum

$$p_2 = p_r - \frac{m_2}{M} P$$

P is the total momentum

Coordinate transformation

$$p_1 = p_r + \frac{m_1}{M}P \quad p_r \text{ is the relative momentum}$$

$$p_2 = p_r - \frac{m_2}{M}P \quad P \text{ is the total momentum}$$

When we ignore relativistic correction, we find the Hamiltonian for two particles:

$$H = \sum_k \frac{p_r^2}{2m_{eff}} + \frac{P^2}{2M} + \frac{\hat{\pi}_k^2}{2\mu} + \frac{\mu}{2}\omega_k^2 \hat{q}_k^2 - \frac{\hat{q}_k (p_r)_i (p_r)_j \epsilon^{ij}}{m_{eff}} - \frac{\hat{q}_k P_i P_j \epsilon^{ij}}{M} \quad \text{with } m_{eff} = \frac{m_1 m_2}{M}$$

Coordinate transformation

$$p_1 = p_r + \frac{m_1}{M} P \quad p_r \text{ is the relative momentum}$$

$$p_2 = p_r - \frac{m_2}{M} P \quad P \text{ is the total momentum}$$

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$$H = \sum_k \frac{p_r^2}{2m_{eff}} + \frac{P^2}{2M} + \frac{\hat{\pi}_k^2}{2\mu} + \frac{\mu}{2} \omega_k^2 \hat{q}_k^2 - \frac{\hat{q}_k (p_r)_i (p_r)_j \epsilon^{ij}}{m_{eff}} - \frac{\hat{q}_k P_i P_j \epsilon^{ij}}{M} \quad \text{with } m_{eff} = \frac{m_1 m_2}{M}$$

No interaction between internal d.o.f. and center of mass d.o.f.

Hamiltonian with correction term included

$$H = \sum_k \frac{p_r^2}{2m_{eff}} + \frac{P^2}{2M} + \frac{\hat{\pi}_k^2}{2\mu} + \frac{\mu}{2} \omega_k^2 \hat{q}_k^2 - \frac{\hat{q}_k (p_r)_i (p_r)_j \epsilon^{ij}}{m_{eff}} - \frac{\hat{q}_k P_i P_j \epsilon^{ij}}{M} + \frac{(p_r)_i (p_r)_j (p_r)_a (p_r)_b}{2m_{eff}^2} + \frac{(p_r)_i (p_r)_j P_a P_b}{m_{eff} M} + \frac{P_i P_j P_a P_b}{2M^2}$$



Interaction term!

and thus decoherence!

Take away

Small things might be more significant
then expected

Comments and outlook

A system interacting with gravity will behave as a more truthful Brownian motion when the center of mass coordinates are considered.

Coordinate dependent!

Summary

- Divergences in quantum physics were renormalized after experimentally discovering Lamb shift. This was not theoretical predicted!
- What does our theory about quantum systems interacting with gravity not predict? (what also can not be experimentally observed yet)
- Relativistic corrections and gravitationally induced correlation between particles renormalize equation of motion (with spectral density presented here)
- Relativistic corrections (and gravitationally induced correlation between particles) correlate internal d.o.f. and center of mass d.o.f., leading to decoherence.