FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

Effective Spin foam models

Constructing a 'quantum spacetime'

Seth Kurankyi Asante, FSU Jena

[Bianca Dittrich, Hal Haggard, José Padua-Argüelles]

Physik-Combo

TPI, FSU-Jena

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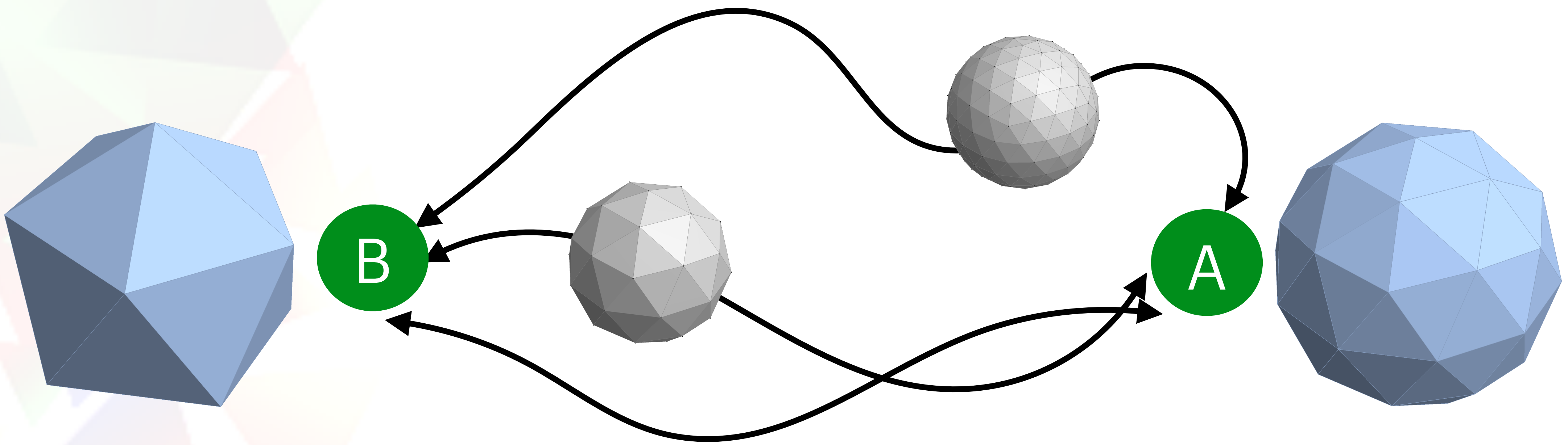
Alexander von HUMBOLDT STIFTUNG

Path Integrals

[Feynman, Schwinger, Dyson, ...]

A formalism adopted by many approaches:

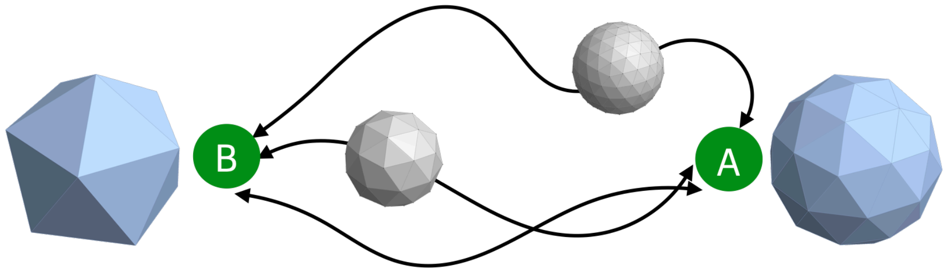


Transition amplitude between A and B: $Z[A;B] = \int D\mu(\text{geom}) \ \exp(\mathrm{i}\, S[\text{geom}])$

- Sum over histories of geometries

What are the fundamental geometries ?

Gravitational Path Integrals



Non-perturbative quantum gravity

Many approaches-

$$\mathscr{Z} = \int_{M/\mathrm{Diff}(M)} [\mathscr{D}\mu(\mathrm{geom})]\, e^{-\mathrm{i}\, S[\mathrm{geom}]}$$

Path Integral approaches

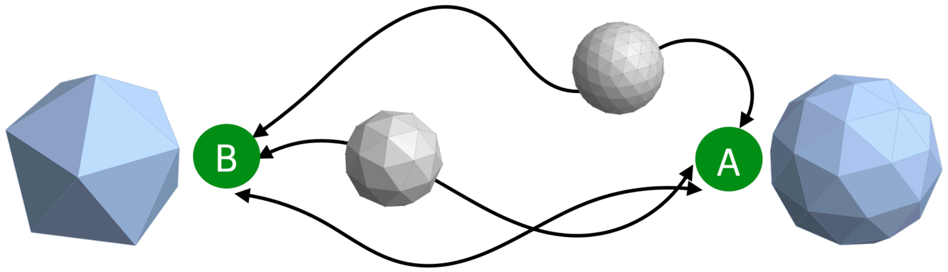
❖ Computation: Lorentzian oscillatory path integrals

❖ Control: What configurations should be summed over in path integral ?

Discrete formulations

❖ Continuum limit: Refinement or coarse graining techniques necessary

Gravitational Path Integrals



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Discrete formulations

Effective spin foam models

provide insights into many of these interesting questions

Outline

Spin foam models

- Path integrals for gravity
- Quantum geometry from area variables

Effective models

- Area Regge calculus
- Weak implementation of constraints

Testing the model

- Discrete Regge dynamics
- Refinement limit **

Spin foam models

In a nutshell

Defined as path integral formulation over discrete geometries:

Based on Plebanski gauge formulation for gravity

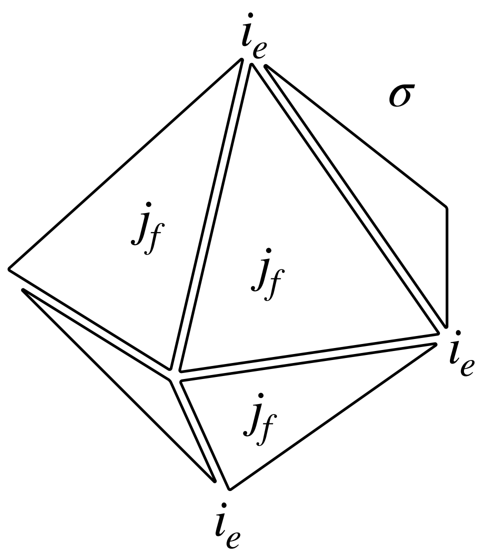
$$\mathcal{Z} = \int_{\mathcal{G}} dA dB \, e^{i \int_M B \wedge F(A) + \phi B \wedge B}$$

Discretization a priori scale free regulators- have to take refinement limit

[Long list of names , … , Steinhaus et.al ,…]

Dynamics: as transition amplitudes between LQG states

$$\mathcal{Z} = \sum_{\{j_f, i_e\}} \prod_f \mathcal{A}_f \prod_e \mathcal{A}_e \prod_v \mathcal{A}_v$$



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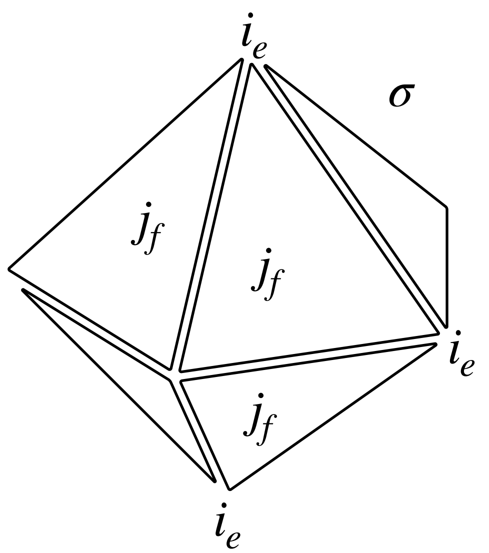


Does this lead to GR?

Asymptotic analysis:

$$\mathscr{A}_v \sim \sum_{\{j_f\}} \cos\Big(\sum_f j_f \, \theta_f \Big)$$

one-simplex

Regge action

[*Barrett et al, … *]

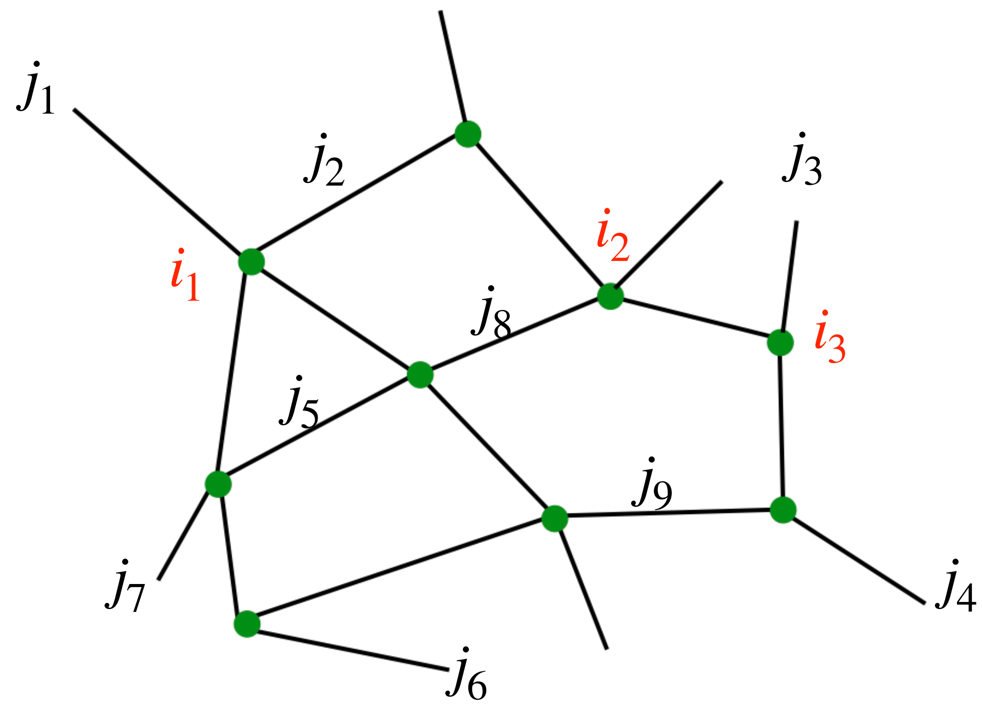
Quantum geometries (Discrete)

Spin Networks Mathematically well-defined structures [R. Penrose]

Decorated graphs

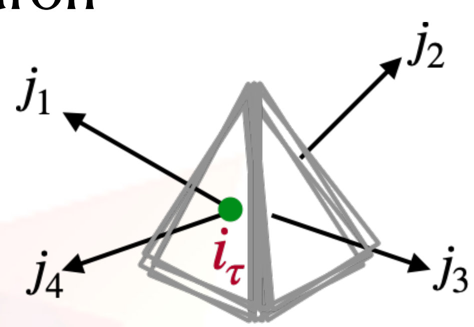
Encode quantum geometries

$$\mathscr{H}_\Gamma = L^2(\mathrm{SU}(2)^{\#\ell}/\mathrm{SU}(2)^{\#n})$$

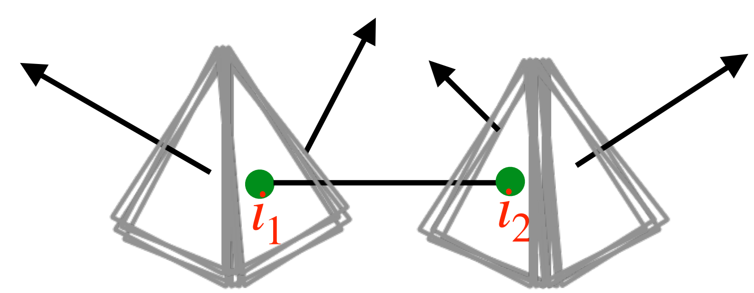


Labels:
quantum numbers

quantum tetrahedron



gluing geometries



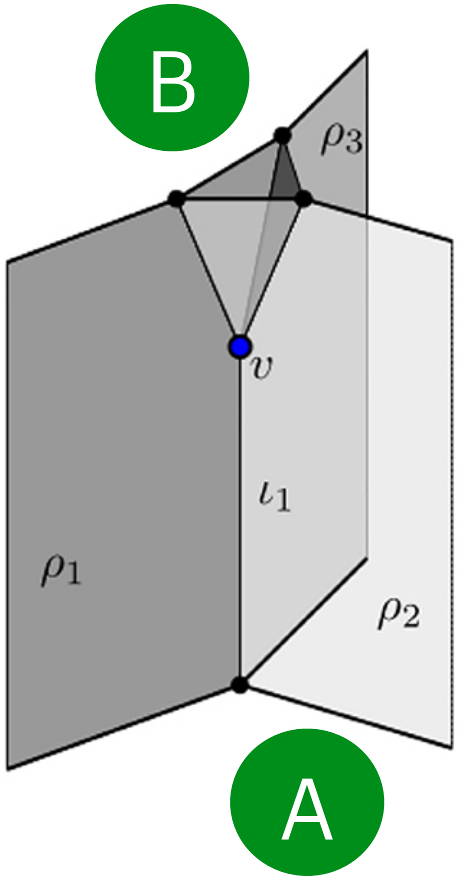
Functional of space of connections invariant under local gauge transformations

Spin foam models

Path integral over quantum geometries

- Sum over histories of quantum geometries for fixed boundaries
- Describe dynamics of quantum geometry
- Very complicated amplitudes
  - Difficult to compute for large discretization

$$Z[A;B] = \sum_{\{\iota_e,\rho_t\}}$$



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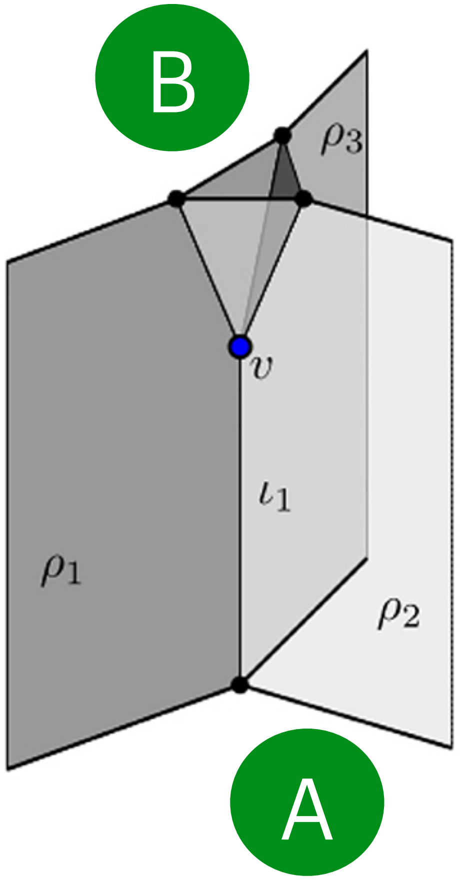

Need methods to: compute, have control and study its continuum limit..

Summary: Quantum Geometry

Key features in (3+1) D

[LQG: *Ashtekar, Rovelli, Smolin, Lewandowski, Isham…*]

Area variables fundamental

- support from BH physics, holography, generalized geometries, discrete symplectic geometry
- angles as auxiliary variables

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[Edge modes: *Wieland, Freidel-Pranzetti-Geiller*]

Area variables have discrete spectra

$$a = \gamma \ell_P \sqrt{j(j+1)} \sim \gamma \ell_P j, \quad j \in \mathbb{N}/2$$

asymptotically equidistant

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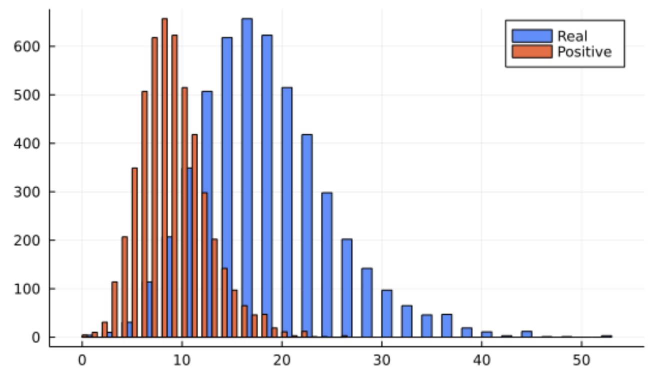
[*Freidel-Speziale, Dittrich-Ryan*]

[*Schuller, Wohlfahrt '06*]

Area configurations more general than length configurations



[*SKA, Brysiewicz 2024*]

Simple models

Need to find a way to access and isolate the geometric data from spin foam amplitudes

$$\mathcal{Z} = \sum_{\{j_f, i_e\}} \prod_f \mathcal{A}_f \prod_e \mathcal{A}_e \prod_v \mathcal{A}_v$$

Idea: Effective spin foam models

[SKA, Dittrich, Haggard]

Maintain dynamical principles of spin foams

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Replace with simple amplitude

- discrete areas + imposition of constraints
- effective description of quantum geometries $\mathcal{Z} \sim \sum_{\{j_f\}} \cos(S_R[j_f, \theta_f])$ from higher gauge theory

[Baratin, Freidel, Mikovic, Vojonovic, Girelli et.al]

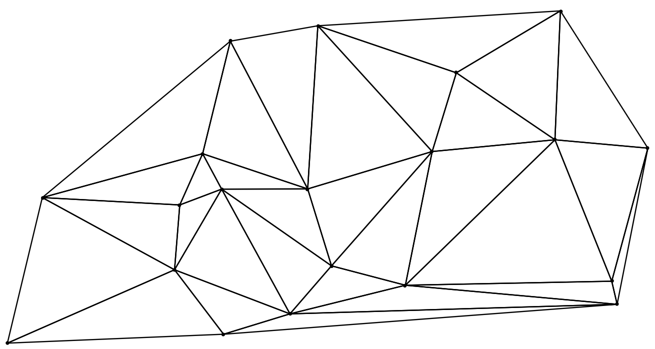
Simple Amplitudes

Triangulated spacetimes

Simple action (4D)

[Regge, Rovelli, Barrett, Rocek, Williams, SKA, Dittrich, Haggard…]

Area Regge action: $$S_{\text{ARC}} = -\sum_{t \in \text{bulk}} a_t\, \epsilon_t(a_{t'}) - \sum_{t \in \text{bdry}} a_t\, \psi_t(a_{t'})$$



discrete GR action for area configurations

State sum model: $$Z = \sum_{\{a\}} \mu(a)\, \exp(\, i\, S_{\text{ARC}}(a))$$

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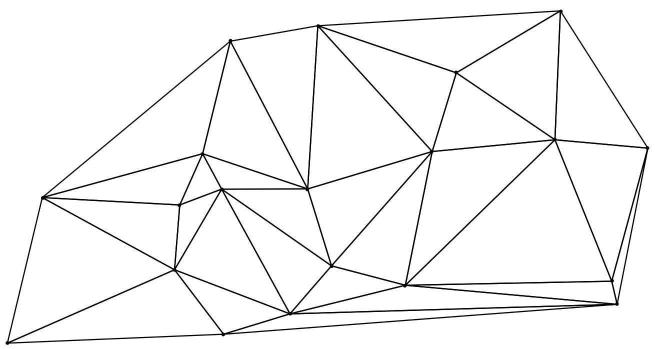

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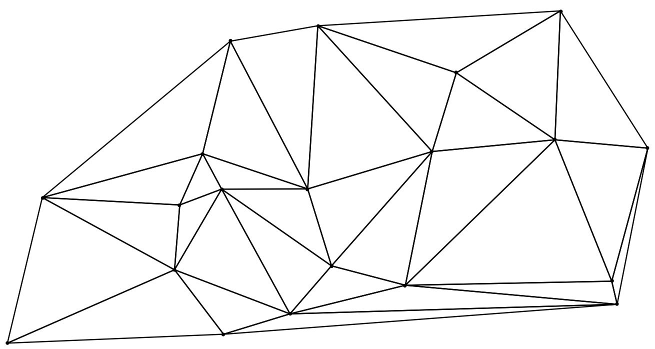

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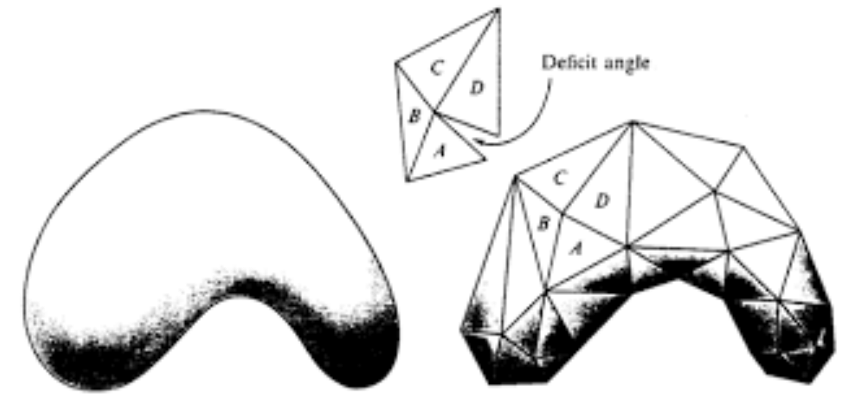
Discrete dynamics: Add constraints to reproduce discrete GR Yes?

Continuum limit: possibly Yes *[Dittrich-Kogios]*

Regge calculus

[Regge '61]

Discrete gravity



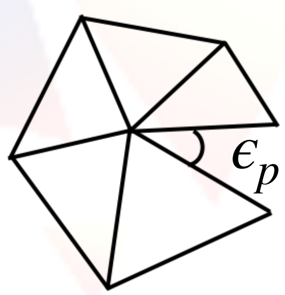
Based on a simplicial decomposition

Assign length to edges : defines piecewise-flat geometry

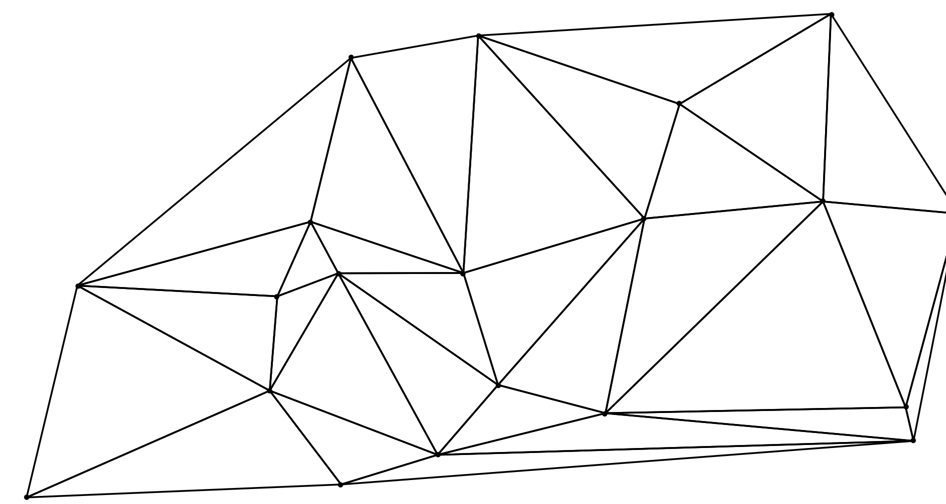
$(\mathscr{M} \to \mathscr{T})$

Curvature as deficit angles distributed on co-dimension 2 simplices

2D: curvature around a point



$$\epsilon_p = 2\pi - \sum_{p \subset t} \theta_p^t$$



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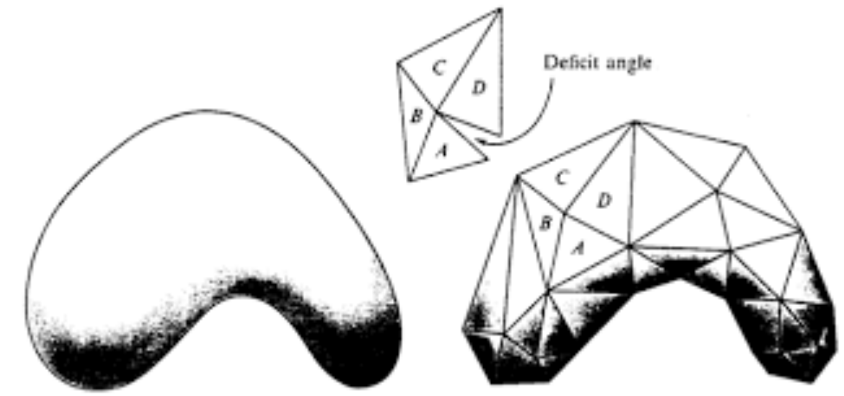

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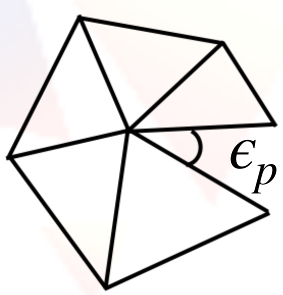

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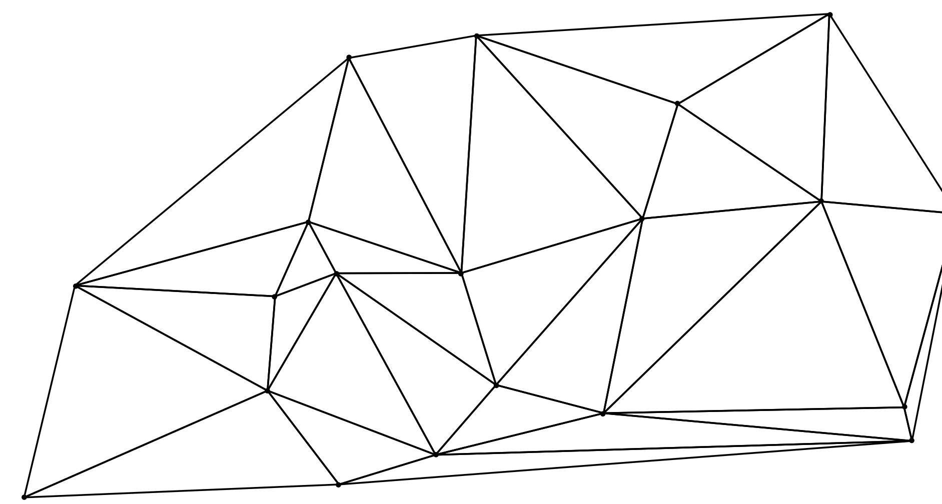
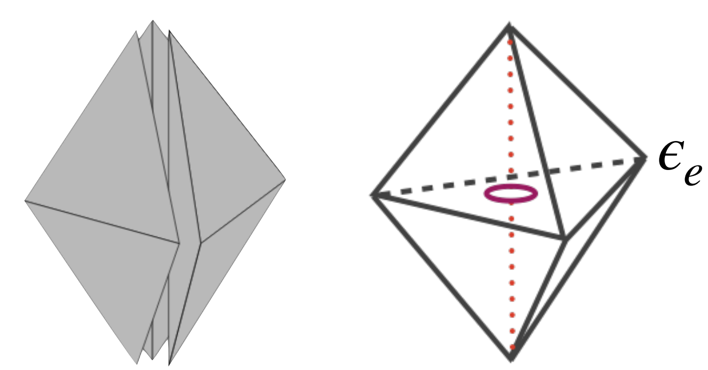


2D

3D: curvature around 1 bones (edges)

Edge lengths conjugate to (compact) angles



Regge Calculus (4D)

(Euclidean and Lorentzian)

Action as distribution of curvature

Discrete action: $S_{\text{Regge}}[l_e] = \sum_{t \in T} A_t(l_e)\, \epsilon_t(l_e)$ ⟷ $S_{\text{EH}}(g) = \int_M d^4x \sqrt{|g|}R$

equations of motion:

$$\delta S_{\text{Regge}} = \sum_t \delta A_t\, \epsilon_t$$

Euclidean $\epsilon_t = 2\pi - \sum_{\sigma \supset t} \theta_t^\sigma(l_e)$

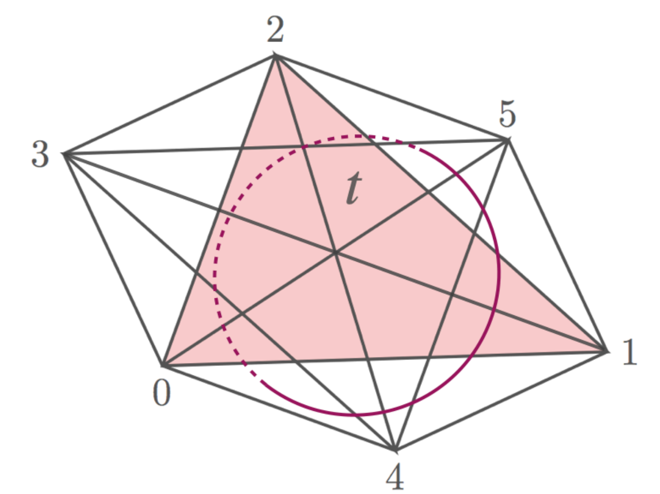
Einsteins equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 0$$

Schläfli identity: encodes symplectic structure

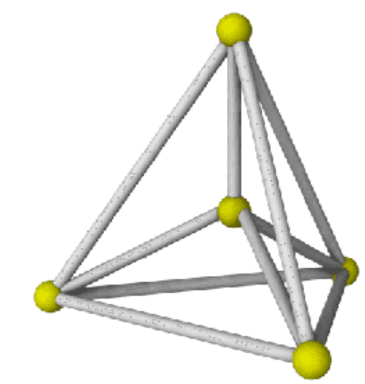
$$\sum_t A_t\, \delta\theta_t^\sigma = 0$$

4D: curvature around 2 bones (triangles)



Area Regge calculus (ARC)

Building blocks are 4-simplices: each simplex has 10 triangles and 10 edges



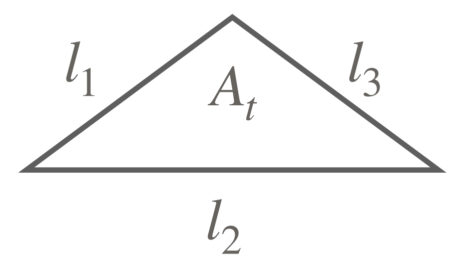
Locally invert areas and lengths

[Hero of Alexandria AD 60]

Heron's formula $A_t(l_e) = a_t$

$l_e = L_e^\sigma(a_t)$

$$A_t^2(l_1, l_2, l_3) = \frac{1}{16}(l_1 + l_2 + l_3)(l_1 + l_2 - l_3)(l_1 - l_2 + l_3)(-l_1 + l_2 + l_3)$$



area Regge action: $S_{\text{ARC}}[a_t] = \sum_t a_t\, \epsilon_t(a_{t'})$

Euclidean/Lorentzian versions

[Williams, Barett, ...] $\delta S_{\text{ARC}} \implies \epsilon_t(a_{t'}) = 0$ flatness?

[Bonzom]

Flatness problem

$$Z = \sum_{\{a\}} \mu(a)\, \exp(\, i\, S_{\text{ARC}}(a))$$

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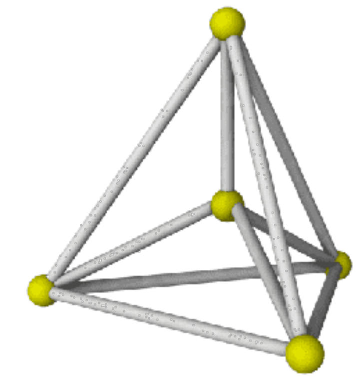

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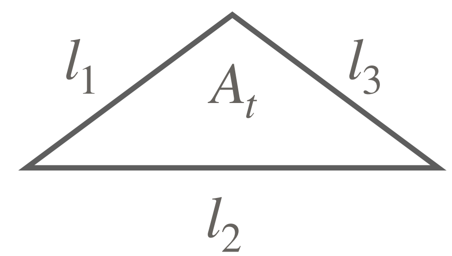

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[*Williams, Barett, …*] $\qquad \delta S_{\text{ARC}} \implies \epsilon_t(a_{t'}) = 0$ $\qquad$ flatness? $\qquad$ [*Bonzom*]

Flatness problem

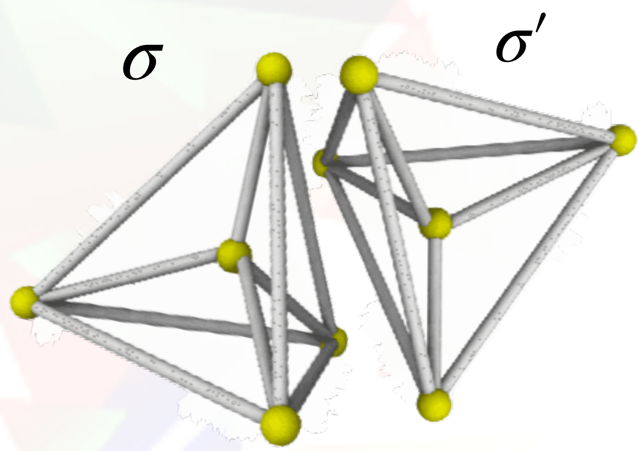
Does ARC lead to a discretization of general relativity? $\qquad Z = \sum_{\{a\}} \mu(a)\, \exp(\, i\, S_{\text{ARC}}(a))$

Impose area constraints between geometries

Constraints

General triangulation has mismatch between data on shared tetrahedra

Gluing simplices



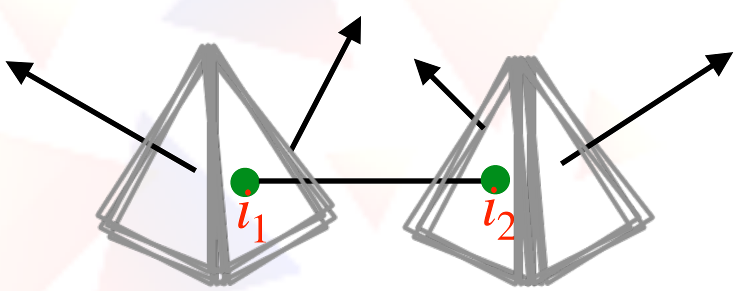
(localized geometric constraints)

$$\mathscr{C}_i^\tau := \phi_{e_i}^\tau - \Phi_{e_i}^{\tau,\sigma}(a_t) = 0$$

[Dittrich, Speziale, Ryan,…]

match two 3d dihedral angles $\Phi_{e_i}^{\tau,\sigma}(a_t) = \Phi_{e_i}^{\tau,\sigma'}(a_t)$

[Kapovich-Milson]



$$\{\mathscr{C}_i^\tau, \mathscr{C}_j^\tau\} = \gamma\,(9/2)\mathrm{Vol}_\tau$$

Barbero-Immrizi
Anomaly parameter

(second-class constraints)

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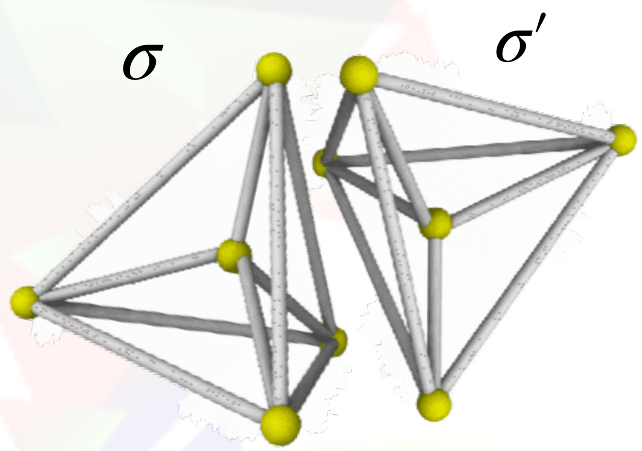

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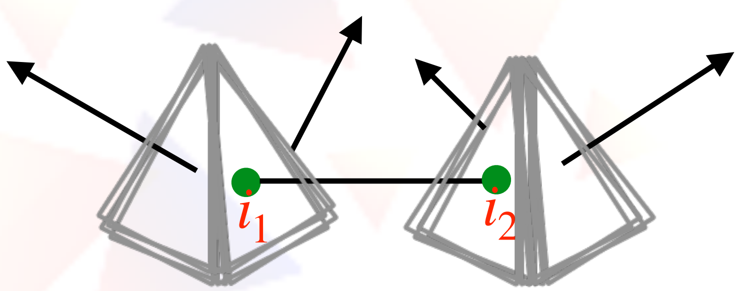

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[*Dirac, Gupta-Bleuler*]

Impose constraints ‘weakly’ in quantum theory: as allowed by uncertainty relation.

γ controls how sharply we can implement the constraints

Weak Constraints

Implementing constraints weakly

[*Livine, Speziale*]

Use coherent states $|\, K(\phi^\tau, \Phi_{e_i}^{\tau,\sigma})\, \rangle$

[SF: *Engle-Perriera-Rovelli-Livine*]

Inner product between coherent states peaked on classical 3d angles

'Integrate out' ϕ^τ variables

$$G_\tau = \langle K_{\Phi_{e_i}^{\tau,\sigma}} \,|\, K_{\Phi_{e_i}^{\tau,\sigma'}} \rangle$$

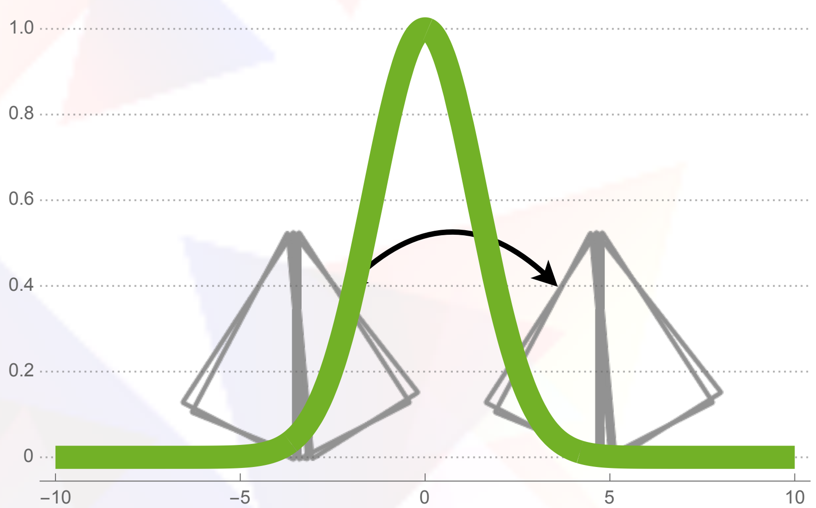
ansatz $\sim \mathcal{N}_k \exp\left(-\frac{\mathscr{C}^2}{4\Sigma^2(j)}\right)$

$\mathscr{C}$- constraints $\Phi_{e_i}^{\tau,\sigma}(a_t) = \Phi_{e_i}^{\tau,\sigma'}(a_t)$

$\Sigma^2(j)$- deviation determined by commutator of constraints

$$\{\mathscr{C}_i^\tau, \mathscr{C}_j^\tau\} = \gamma\,(9/2)\mathrm{Vol}_\tau$$

Gluing terms



Effective spin foams

Combine simple amplitude and gluing constraints

[Dittrich, Haggard, Padua-Argüelles, SKA]

Effective spin foam models are discrete geometrical path integrals for quantum gravity.

$$Z_{\mathrm{ESF}} = \sum_{\{a_t\}} \mu(a) \, \exp\left(\mathrm{i}\, S_{\mathrm{ARC}}(a)\right) \prod_{\tau} G_{\tau}^{\sigma,\sigma'}(a) \prod_{\sigma} \Theta_{\sigma}^{\mathrm{tr}}(a)$$

[Steinhaus, Simão, SKA '22]

Spin foam amplitudes may be cast into similar form

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Does this lead to GR?

Discrete dynamics results: Yes! for small γ

Due to anomaly of constraints or its weak implementation **But how small?**

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- Path integrals for gravity
- Quantum geometry from area variables

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Lorentzian spacetimes

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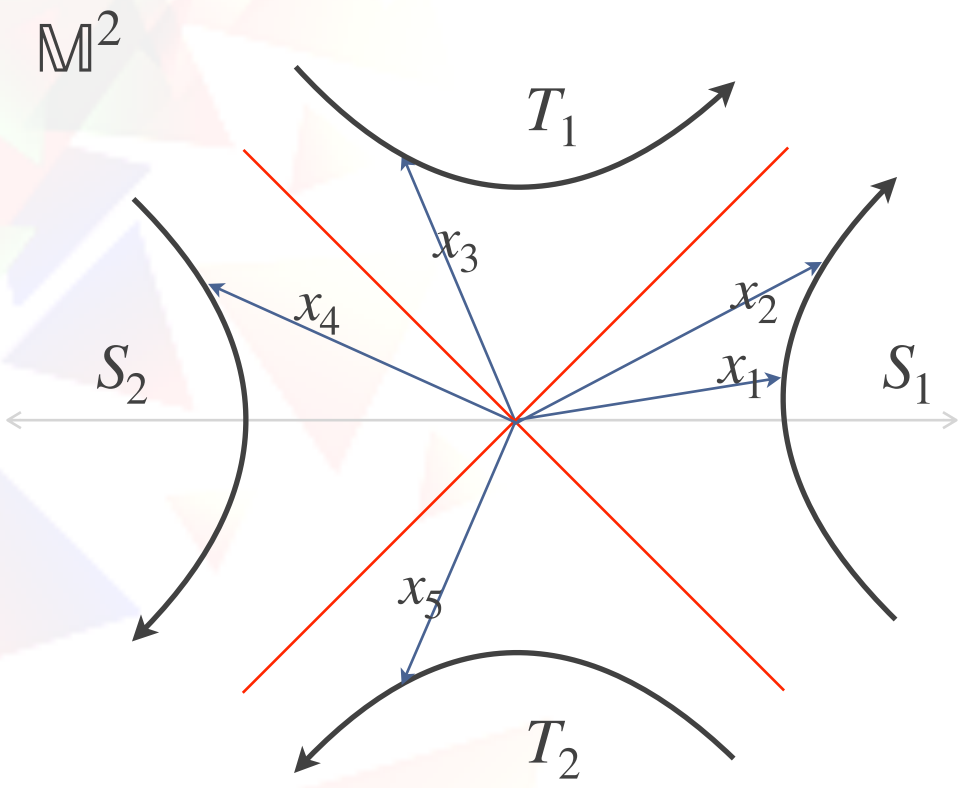
Lorentzian Angles

[Alexandrov '01, Sorkin '19, Jia '21, SKA, Dittrich, Padua-Argüelles '21]



Choice of $\mp \mathrm{i}\pi/2$ for light ray crossings

$$\theta_{12} = \cosh^{-1}(x_1 \cdot x_2)$$

$$\theta_{13} = \sinh^{-1}(x_1 \cdot x_3) \mp \frac{\pi \mathrm{i}}{2}$$

$$\theta_{14} = -\cosh^{-1}(-x_1 \cdot x_4) \mp \pi \mathrm{i}$$

$$\theta_{35} = \cosh^{-1}(x_3 \cdot x_5) \mp \pi \mathrm{i}$$

action: S_{ARC} is complex for causally irregular configurations

Two choices $L_{\mp}$: either enhance or suppress irregular configurations

Lorentzian geometries

Plethora of interesting configurations

Configurations can be grouped into two sets: Regular and Irregular

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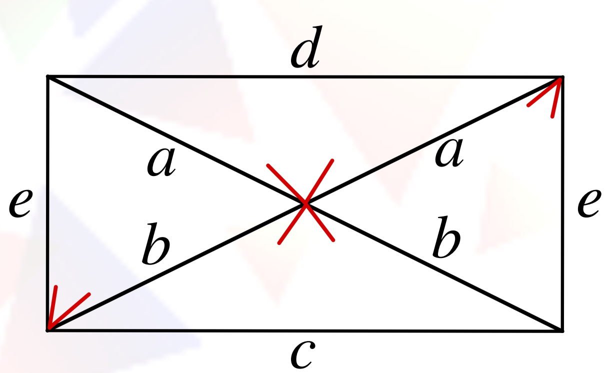
2D Examples

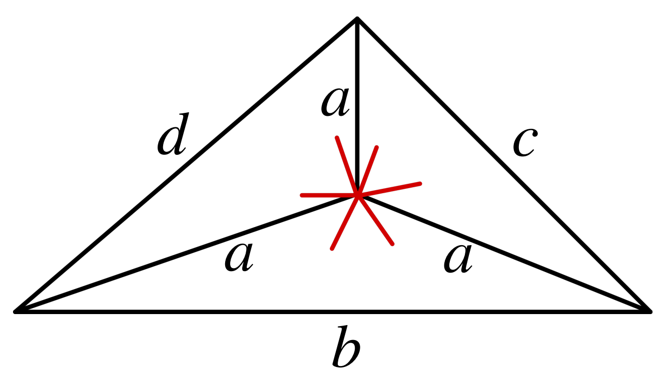
[*Luoko-Sorkin*]

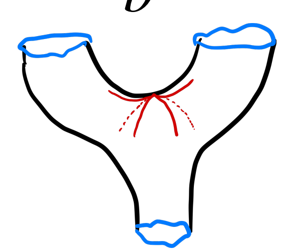
Regular configuration



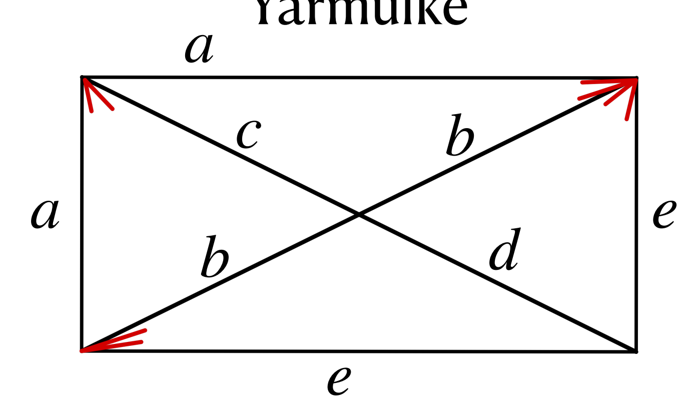
Irregular configurations

Trouser-like

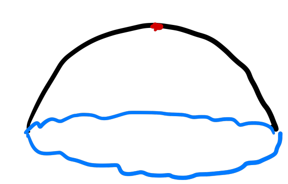




Yarmulke



Topology change



Lorentzian geometries

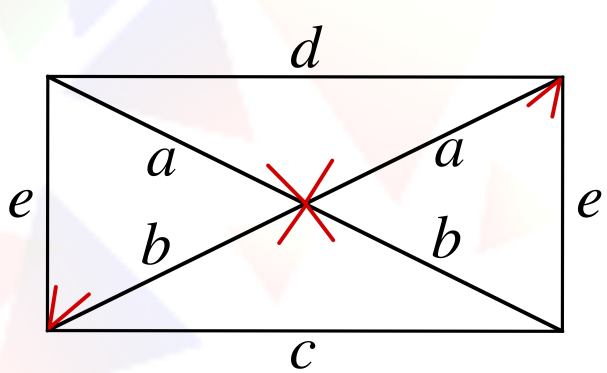
Plethora of interesting configurations

Configurations can be grouped into two sets: Regular and Irregular

2D Examples

[Luoko-Sorkin]

Regular configuration



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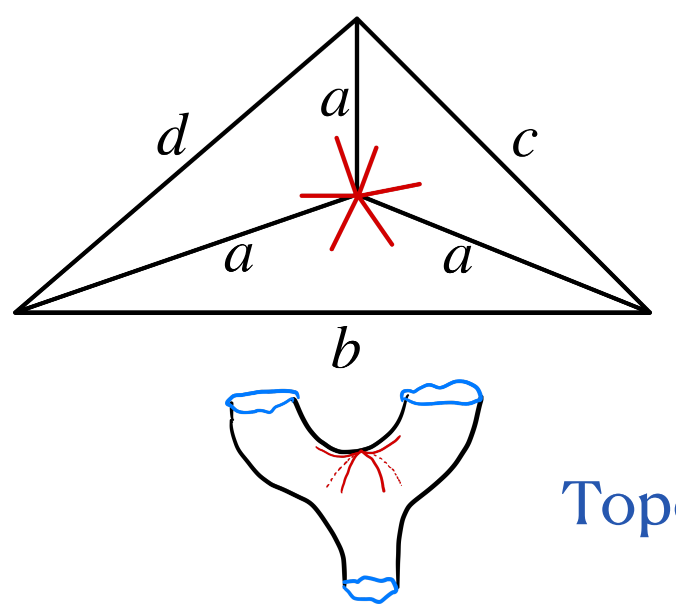

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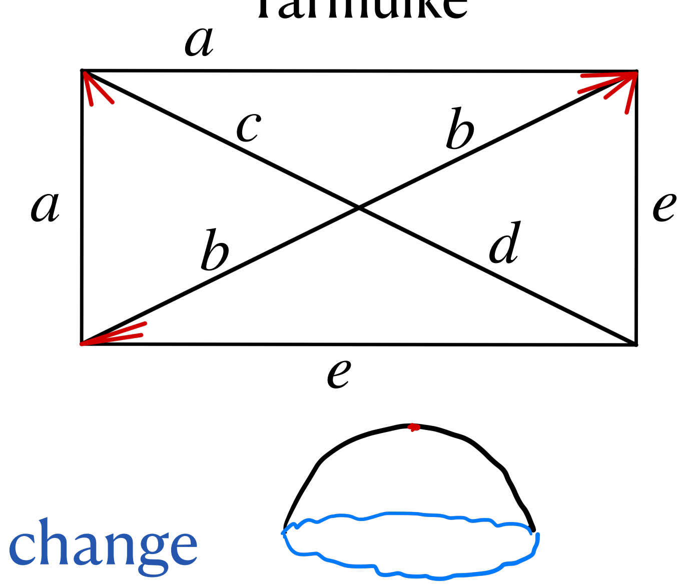


Topology change

[Jordan, Loll '13]

Higher Dimensions: Other causality conditions Edge causality, Vertex Causality

[Borgolte, SKA wip]

Advantages

Regge calculus   Gluing terms

$$Z_{\mathrm{ESF}} = \sum_{\{a_t\}} \mu(a) \exp\left(\mathrm{i}\, S_{\mathrm{ARC}}(a)\right) \prod_{\tau} G_{\tau}^{\sigma,\sigma'}(a) \prod_{\sigma} \Theta_{\sigma}^{\mathrm{tr}}(a)$$

✦ 'Effective' dynamics of quantum geometries

keep dynamic principles of LQG and spin foam models

✦ Computationally efficient

[Marseille CNRS group, Florida FAU group, London Western group, Bahr, Steinhaus..]

Fast numerical computations compared to BF and EPRL/FK numerics

✦ Control: can test many features

[Steinhaus, Simão]

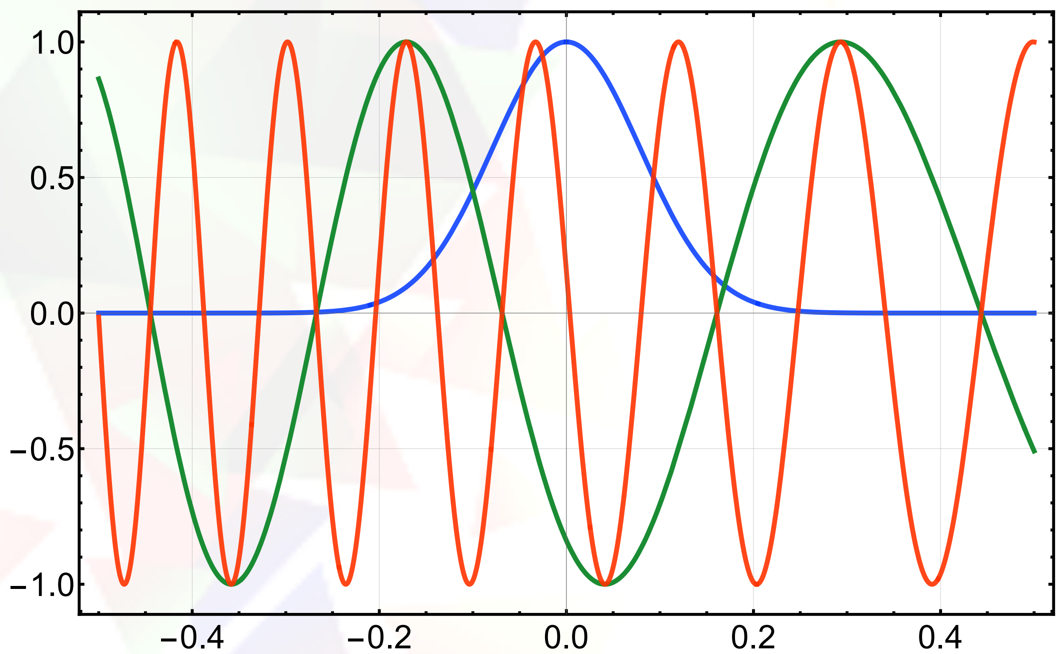
Easy construction of Lorentzian model   allows spacelike and timelike areas

SF Cosmology applications *[Dittrich, Gielen, Schander, Padua-Argüelles]* *[Jercher, Marchetti, Pithis]*

[Steinhaus, Jercher]

ESF model

Weakly imposed constraints



But how small?

γ anomaly parameter

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Oscillations

Gaussians
peaked on constraints

Semi-classical limit:

Few oscillations over Gaussian needed

$$\gamma \sqrt{a_t}\, \mathrm{curv}_t \lesssim \mathcal{O}(1)$$

[*SKA, Dittrich, Haggard*]

[SF: *Han 13*]

ESF model

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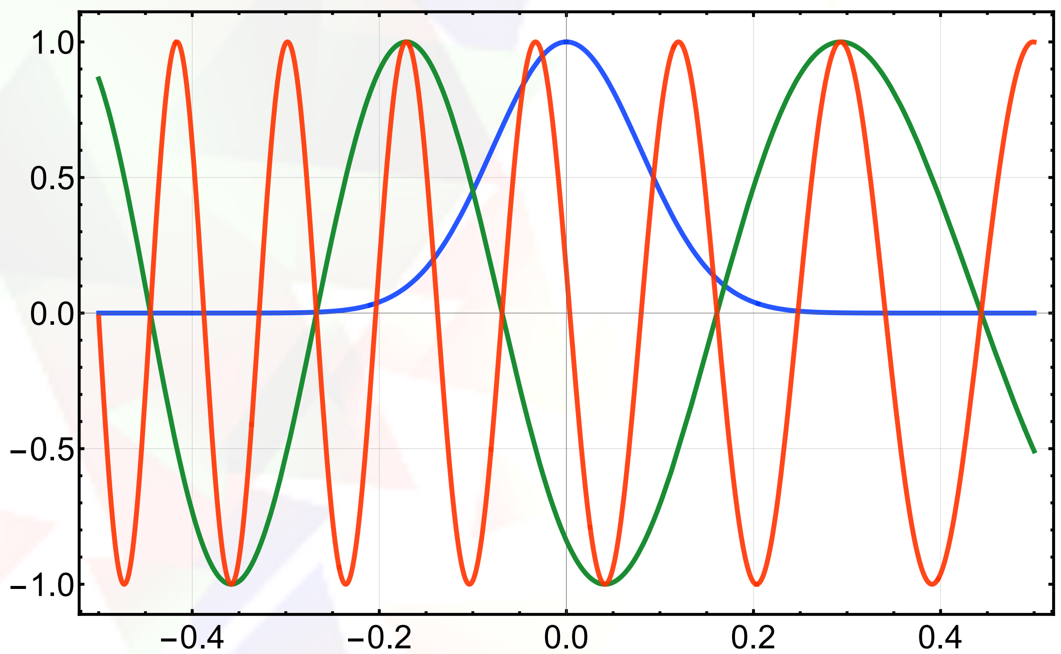

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[SKA, *Dittrich, Haggard*]

[SF: *Han 13*]

Alternative point of view: Complex critical points

Imaginary part of saddle point controlled by γ needs to be small [SF: *Han, Huang, Liu, Qu*]

Outline

Spin foam models

- Path integrals for gravity
- Quantum geometry from area variables

Effective models

- Area Regge calculus
- Weak implementation of constraints

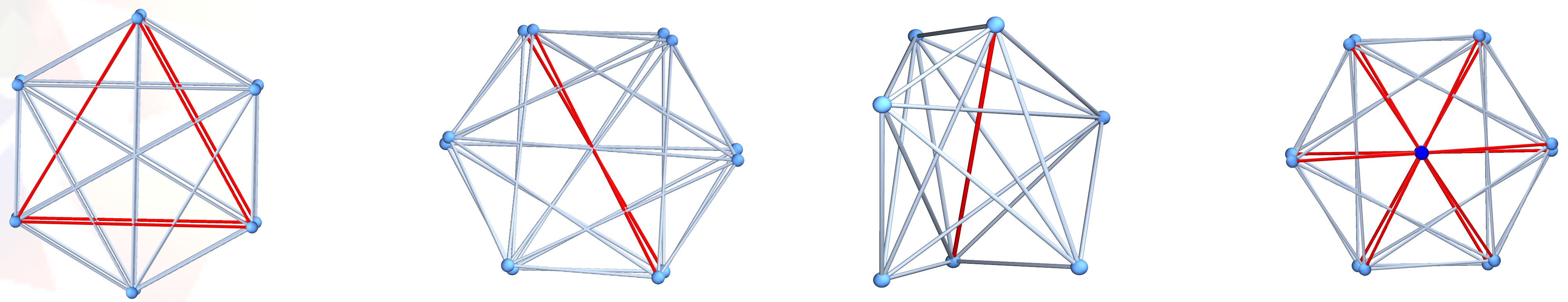
Testing the model

- Discrete Regge dynamics
- Refinement limit **

Testing ESF model

- Early non-trivial results

Several examples of discrete geometries with curvature



- Recover discrete gravity dynamics in certain range of parameters

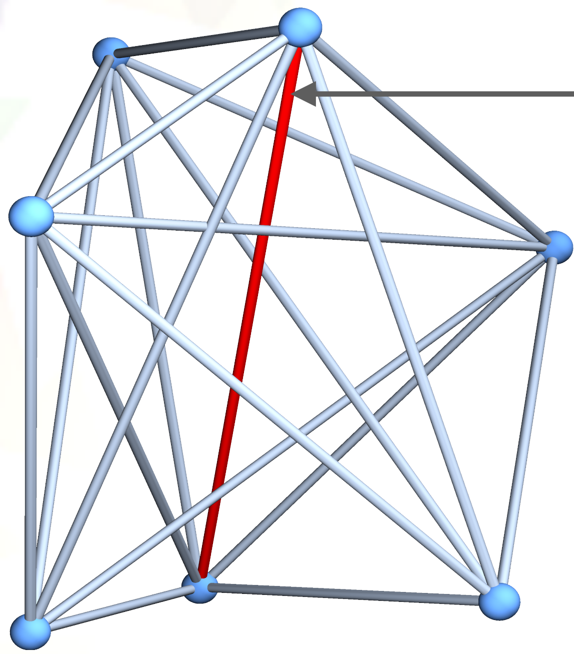
explicit path integral of expectation values, testing EOMs

interesting effects beyond saddle point evaluation

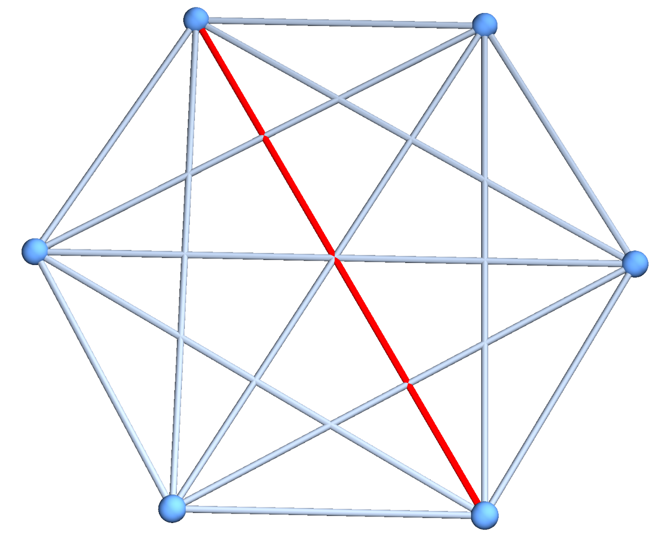
Testing the model

Triangulation with bulk edge

Example:



3D projection



2D projection

6 four-simplices
21 tetrahedra
29 triangles
20 edges

Symmetry reduction: 5 bulk areas → 3 bulk areas

Can test discrete classical equations of motion.

Compute expectation values of geometric objects

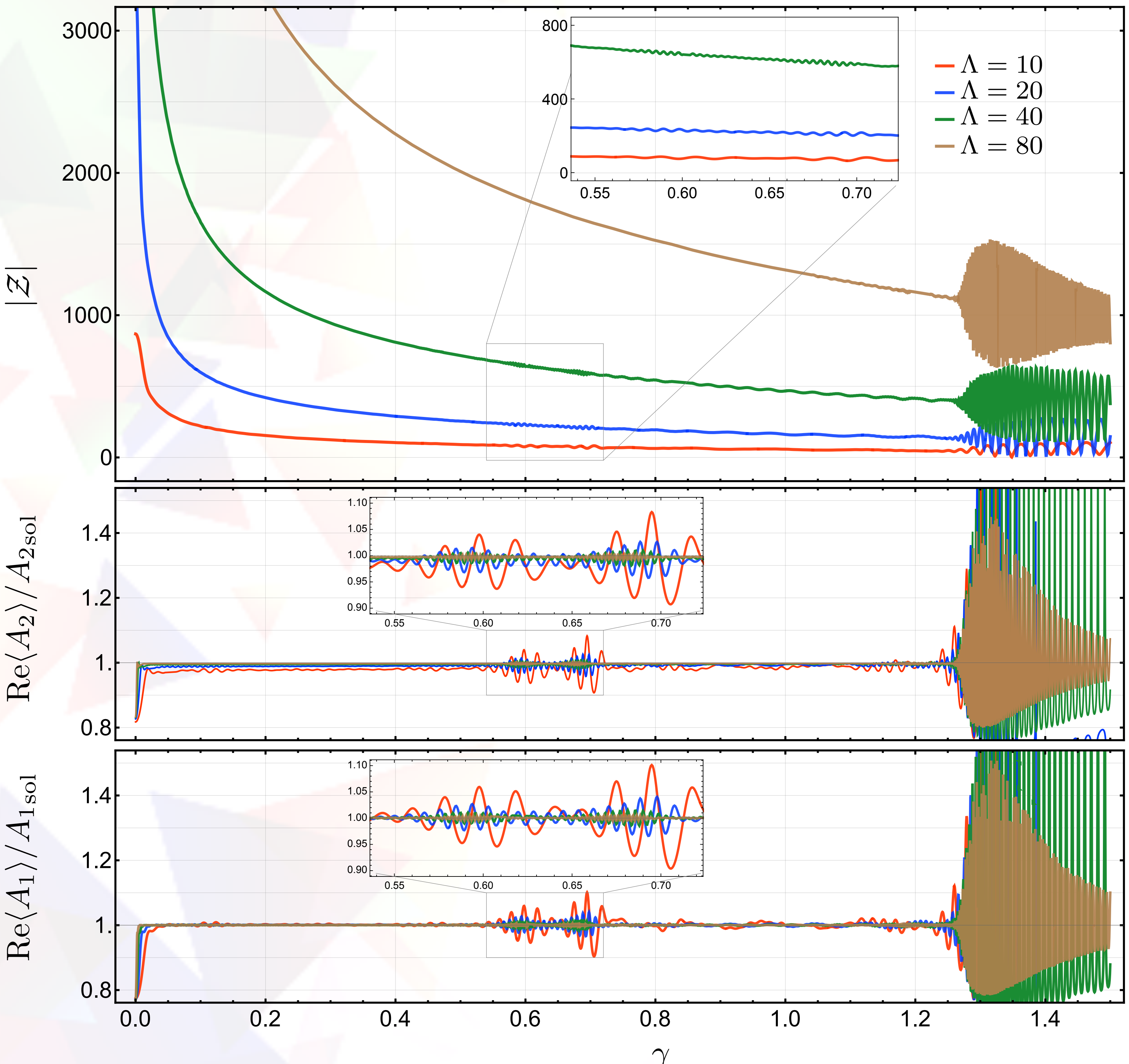
$$\langle \mathcal{O} \rangle(\gamma, j) = \frac{\sum_{j_t} \mathcal{O} \exp(i\, S_{\mathrm{ARC}}(a))\, G(\gamma, a)}{\sum_{j_t} \exp(i\, S_{\mathrm{ARC}}(a))\, G(\gamma, a)}$$

Numerical results

Bulk-Edge

Small curvature

$$\epsilon_{1_{\rm cl}} = 0, \qquad \epsilon_{2_{\rm cl}} = \epsilon_{3_{\rm cl}} = 0.034$$



- Abs Z is a good indicator for oscillations.
- Threshold behaviour in gamma for oscillations.
- Matching to classical value gets better for larger j - no bound on j.
- acceptable γ range: <0.5 or <1.3 (depending on scale)

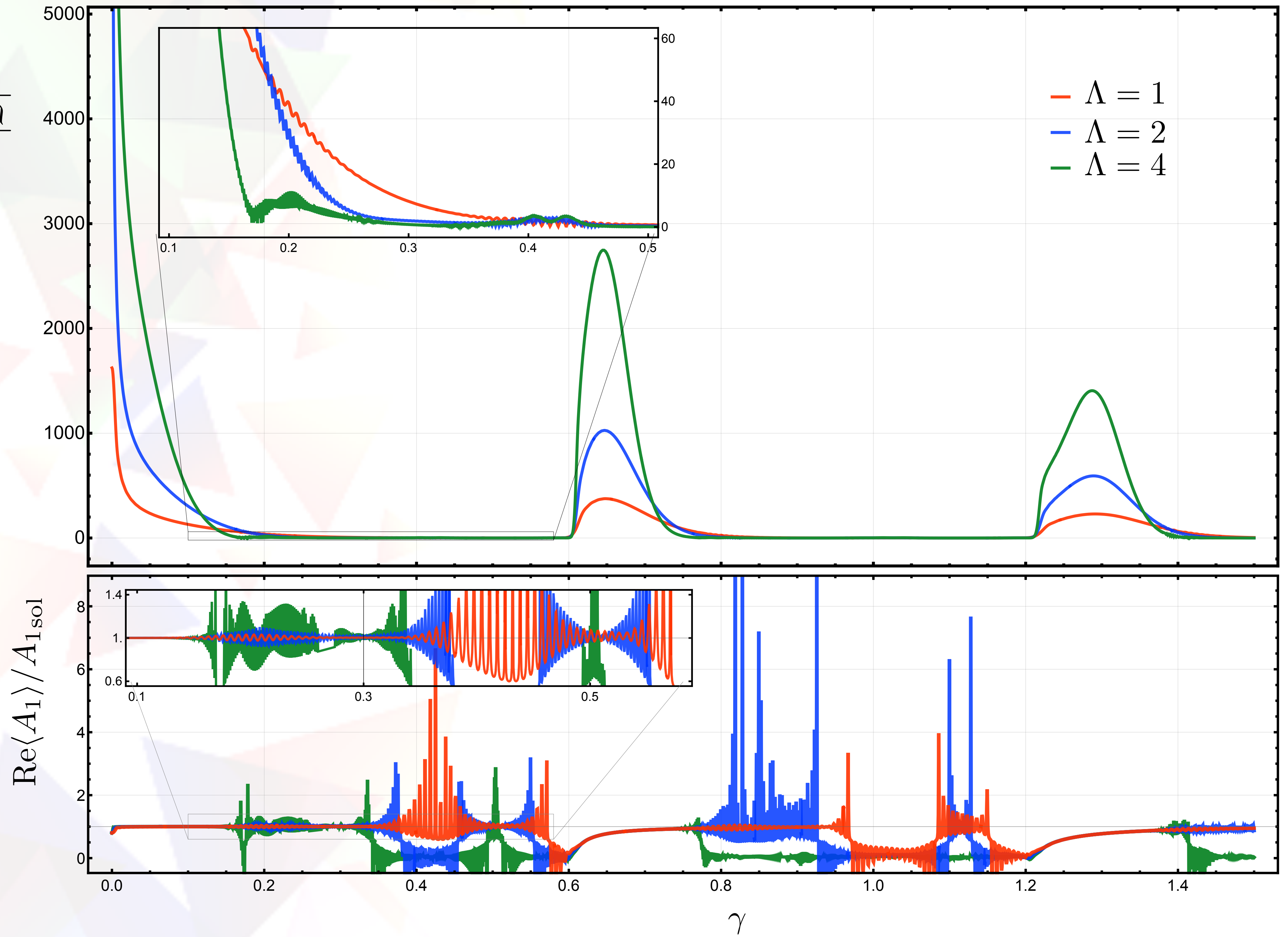
Surprises:
-threshold behaviour for oscillations
-threshold values independent of scale Λ

Improvement in semi-classical expectation values for large Λ

Bulk-Edge

$\epsilon_{1_{cl}} = 4.193, \quad \epsilon_{2_{cl}} = -1.790, \quad \epsilon_{3_{cl}} = -1.1432$

Large curvature



Discretization effects

Peaks explained by

pseudo saddle points

acceptable range

$\gamma < 0.16$

Remarks

- First test of spin foam implementing discrete equations of motion for gravity

  - small range of γ allows curved configurations   Resolves flatness problem

  - reproduce classical solutions for a regime where: $\gamma\sqrt{j_t}\epsilon_t \lesssim \mathcal{O}(1)$

  Suggests renormalization flow in γ

- Can easily check stability of these features, if we change certain details of model

  - different curvature and boundary scales

  - examples exist for Lorentzian model: allow irregular configurations

Fix diffeomorphism invariant measure from coarse graining and convergence   [*wip*]

(inner vertex configuration)

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Crucial question:

- Continuum limit - how do <u>weakly imposed constraints</u> behave under coarse graining/refinement ?

Refinement limit

Area Regge Calculus

possibly **Yes**

Summary

$$S_{\text{ARC}} = -\sum_{t\in\text{bulk}} a_t\,\epsilon_t(a_{t'}) - \sum_{t\in\text{bdry}} a_t\,\psi_t(a_{t'})$$

Linearize around flat background on hypercube lattice

[Dittrich et al, ...]

$$S^{(2)} = \frac{\partial \epsilon_t}{\partial a_{t'}}\,\delta a_t\,\delta a_{t'}$$

Scaling of Hessian block in lattice derivatives k

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Useful parametrization of area perturbation variables

20 area parameters per point

h + χ

trace part — trace-free part

10 — 10

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Effective action for metric variables

$$S_{\mathrm{eff}} = h\cdot\left(H_{hh} - H_{h\chi}H_{\chi\chi}^{-1}H_{\chi h}\right)\cdot h$$

$H_{h\chi}\cdot h \sim$ Weyl curvature

linearized GR $\sim k^2$ correction $\sim k^4$

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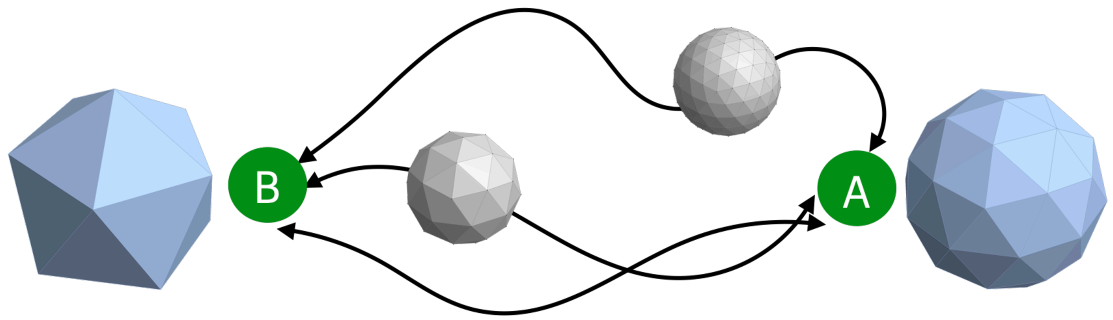
Linearized continuum limit

Area Regge Calculus ~ GR + Weyl^2

Summary

- Effective spin foam models: provides an effective description of quantum spacetime

- Simple model allows control over spin foam transition amplitudes
    - opportunity to test many features of spin foam models
- Computationally efficient models
    - study practical examples



Outlook:

- Go beyond discrete to continuous formulations
    - Continuum limit: Refinement or coarse graining

[Dittrich, Borissova, Krasnov]

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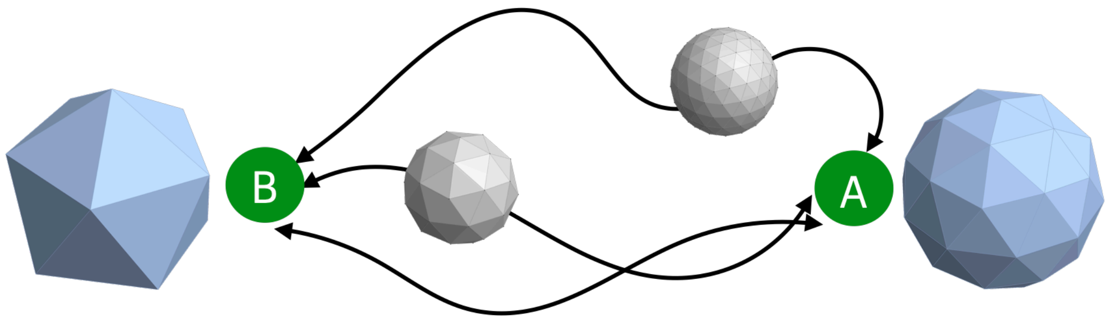

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THANK YOU !