RTG 2522

DFG Research Training Group

From QFT to Hydrodynamic Transport Coefficients

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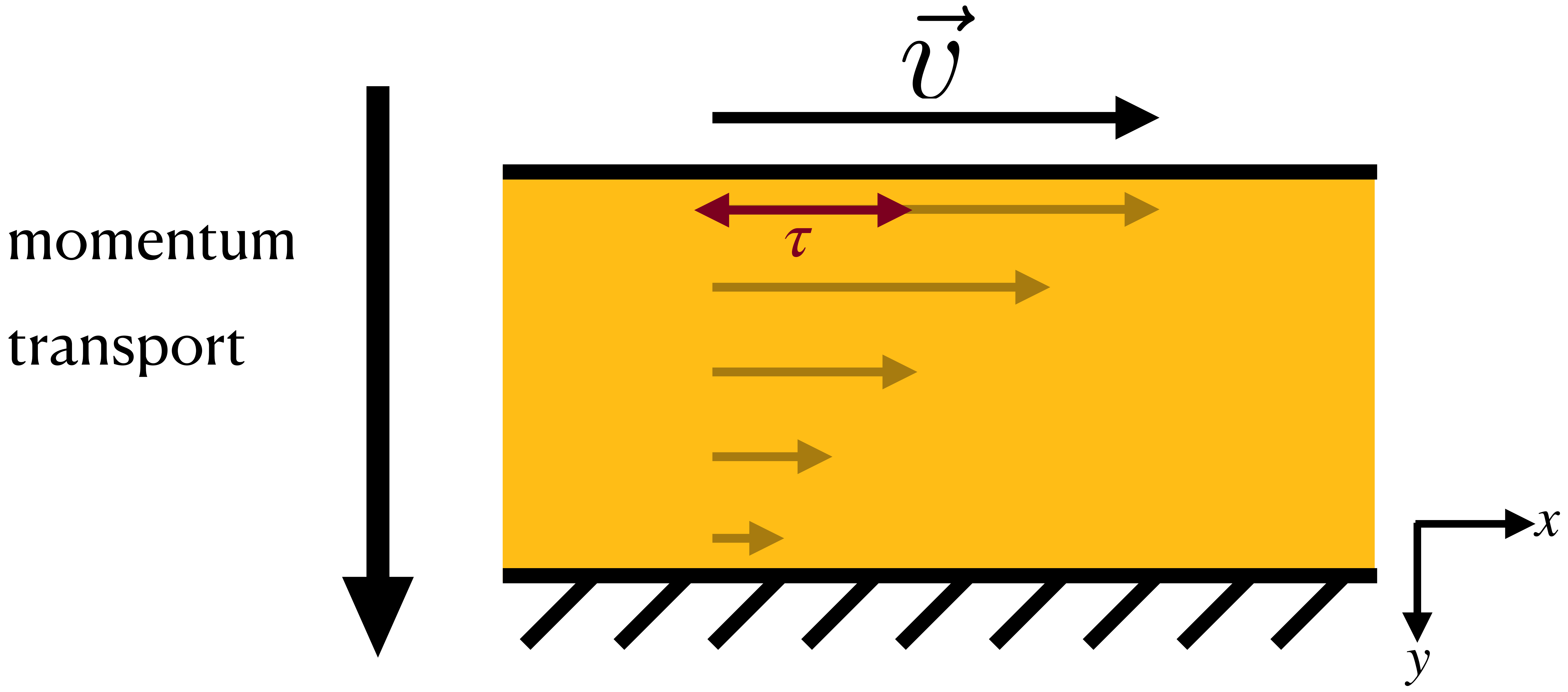


Newton's law of viscosity

$$\text{shear stress} \quad \tau = -\eta \frac{\mathrm{d}v_x}{\mathrm{d}y}$$

Motivation

- QGP, neutron stars or dark matter can use hydrodynamical description
- off-equilibrium requires knowledge of macroscopic properties like:
  - equation of state
  - viscosities, conductivities, etc.. $\sigma(T,\mu),\ \eta(T,\mu),\ D(T,\mu),\ldots$
- interactions determine coefficients

Microscopic Theory

Macroscopic Theory

$$\langle \varphi(x_1) \cdots \varphi(x_N) \rangle \longleftrightarrow \{\sigma, D, \eta, \zeta, \ldots\}$$

$$\mathcal{L} = \frac{1}{2}\varphi \left[\partial_\mu \partial_\mu + m^2\right] \varphi + \frac{\lambda}{4!}\varphi^4$$

$$\nabla_\mu T^{\mu\nu} = 0, \quad \nabla_\mu J^\mu = 0$$

Linear Response

Classical:

response kernel

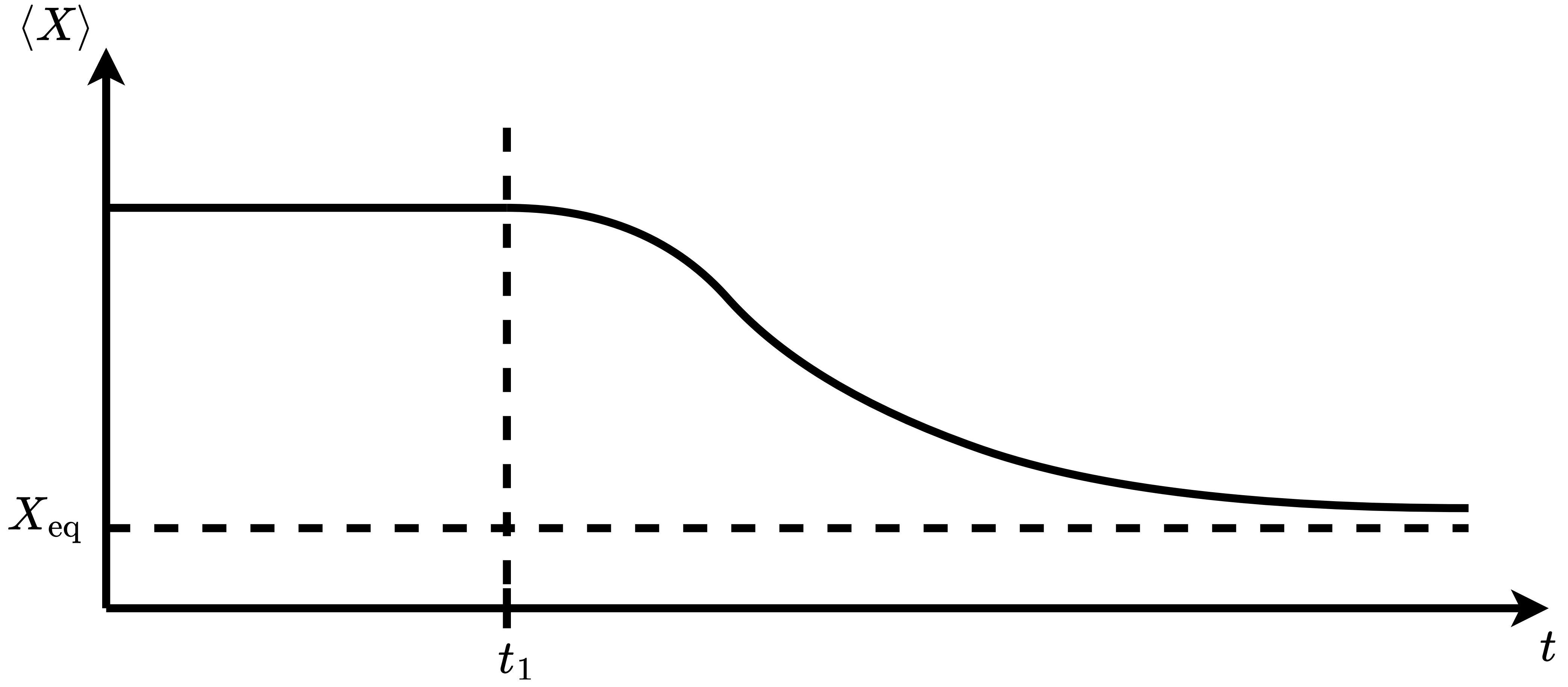
$$\langle X\rangle(t) = \int \mathrm{d}t' G_{\mathrm{R}}(t-t')F(t') + \mathcal{O}(F^2)$$

external force

QFT:

$$\langle \hat{T}^{\mu\nu}\rangle(x) = \int_y G_{\mathrm{R}}^{\mu\nu\alpha\beta}(x-y)h_{\alpha\beta}(y) + \mathcal{O}(h^2)$$

$$h_{\mu\nu}(x) = g_{\mu\nu}(x) - \eta_{\mu\nu}$$



$$\hat{T}^{\mu\nu}(x) = \frac{1}{2}\frac{\delta S[\varphi; g_{\mu\nu}]}{\sqrt{g}\delta g_{\mu\nu}(x)}$$

Response Kernel:

$$G_{\mathrm{R}}^{\mu\nu\alpha\beta}(x-y) = \mathrm{i}\theta(x^0-y^0)\langle[\hat{T}^{\mu\nu}(x), \hat{T}^{\alpha\beta}(y)]\rangle$$

Hydrodynamics & Transport

- Energy momentum tensor in *Landau frame* (no heat current)

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + (p + \boxed{\Pi})\Delta^{\mu\nu} + \boxed{\pi^{\mu\nu}}$$

bulk pressure shear stress

- constituent equations at 1st order

$$\boxed{\Pi = \zeta(T,\mu)\nabla_\mu u^\mu}$$

$$\boxed{\pi^{\mu\nu} = -\eta\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\nabla_\alpha u_\beta + \nabla_\alpha u_\beta - \frac{1}{2}g_{\alpha\beta}\nabla_\mu u^\mu\right)}$$

- next order: relaxation times $\tau_\Pi, \tau_\pi, \ldots$

Hydrodynamics & Transport

- use EoM, constituent equation
- Expand around external source $h_{\mu\nu}$
- e.g. variational approach [Kovtun]:

$$G_{\mathrm{R}}^{\mu\nu\alpha\beta} = -2\frac{\delta\sqrt{g}T^{\mu\nu}}{g_{\alpha\beta}}\bigg|^{g_{\mu\nu}=\eta_{\mu\nu}}$$

Kubo formula

$$\boxed{G_{\mathrm{R}}^{xyxy}(\omega, \vec{p} = 0) = \mathrm{const} - \mathrm{i}\omega\eta(T,\mu) + \mathcal{O}(\omega^2)}$$

QFT

- temperature dependence: Imaginary time formalism

$$\int_{-\infty}^{\infty} \mathrm{d}t \to \int_0^\beta \mathrm{d}\tau, \quad \int_{-\infty}^{\infty} \mathrm{d}\omega f(\omega) \to T \sum_{\omega_n} f(\mathrm{i}\omega_n)$$

- retarded Greensfunction:

$$G_{\mathrm{R}}^{\mu\nu\alpha\beta}(\omega, \vec{p}) \sim \frac{1}{2} \frac{\delta^2 \log Z}{\delta^2 g_{\mu\nu} g_{\alpha\beta}}\Bigg|_{g_{\mu\nu}=\eta_{\mu\nu}}$$

Analytic continuation $\mathrm{i}\omega_m \to \omega + \mathrm{i}0^+$

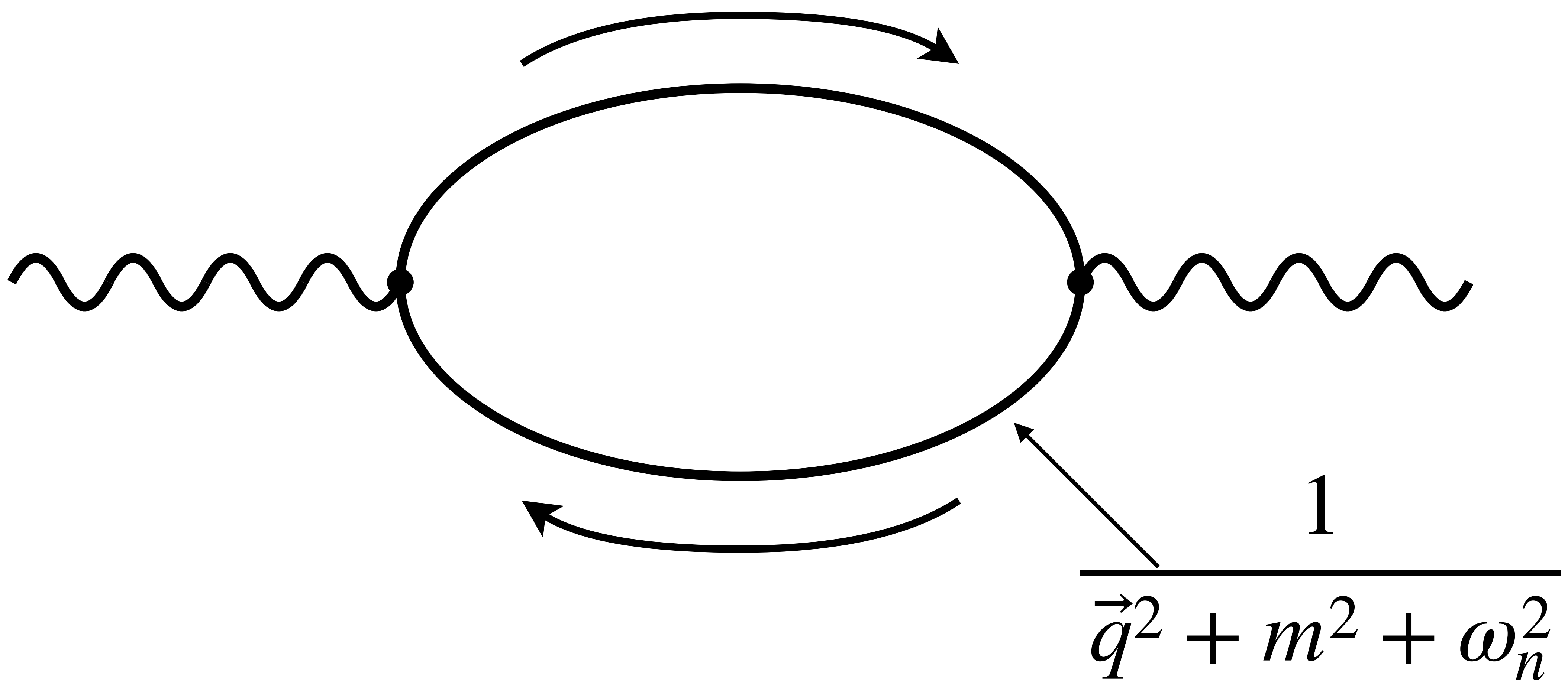
- viscosity in terms of effective action

$$\boxed{\eta = -\lim_{\omega\to 0} \partial_\omega \mathrm{Im} \frac{\delta^2 \Gamma[\Phi; g_{\mu\nu}]}{\delta g_{\mu\nu} \delta g_{\alpha\beta}}\Bigg|^{g_{\mu\nu}=\eta_{\mu\nu}}}$$

$\langle T^{\mu\nu} T^{\alpha\beta} \rangle$

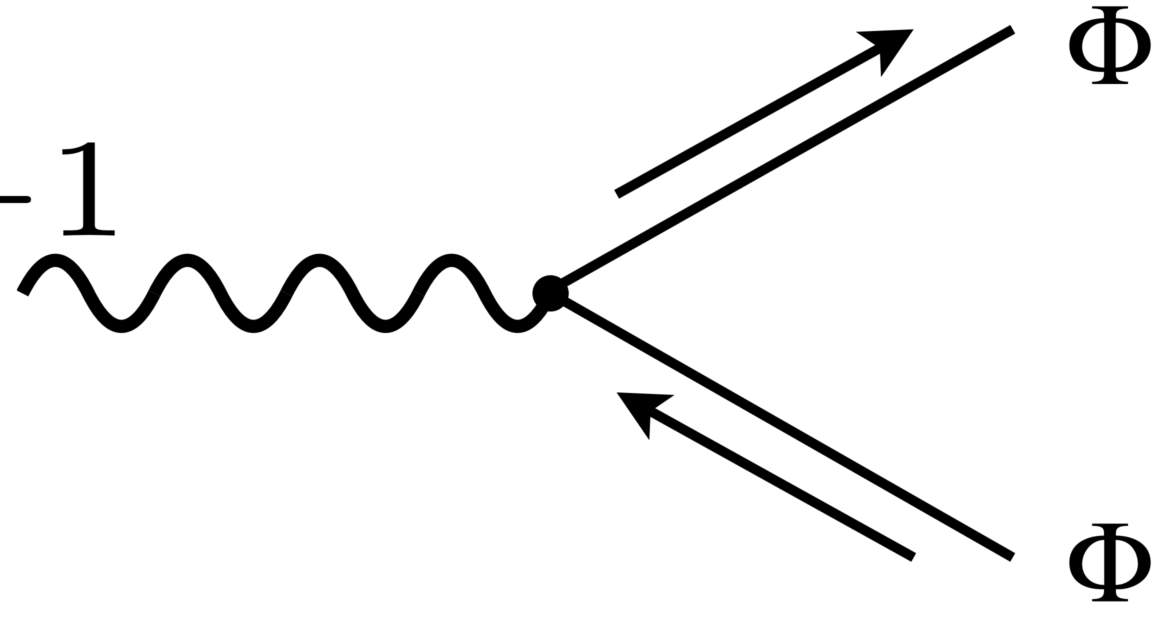
Viscosity Diagram

$$\eta \sim \lim_{\omega \to 0} \omega^{-1} \mathrm{Im}$$



Using Cutting rules:

$$\eta \sim \lim_{\omega \to 0} \omega^{-1}$$



FRG Flow Equation

- Introduce regulator and RG-scale

$$S[\varphi] \to S[\varphi] + \int \frac{1}{2}\varphi R_k \varphi$$

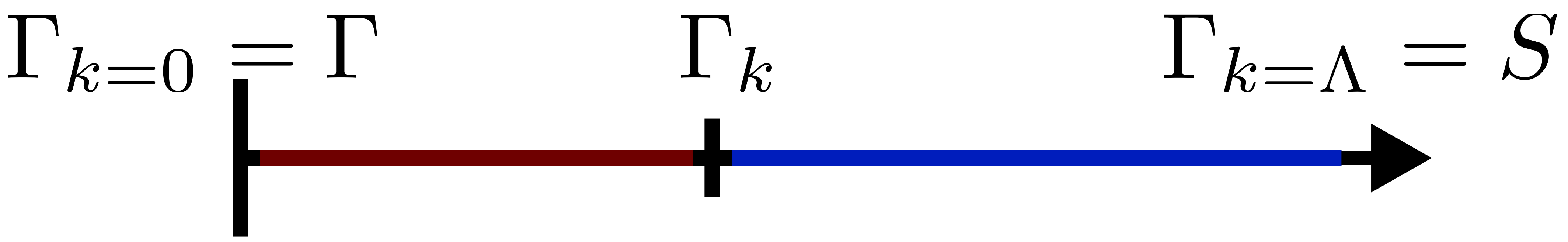
- Exact evolution equation [Wetterich '93]

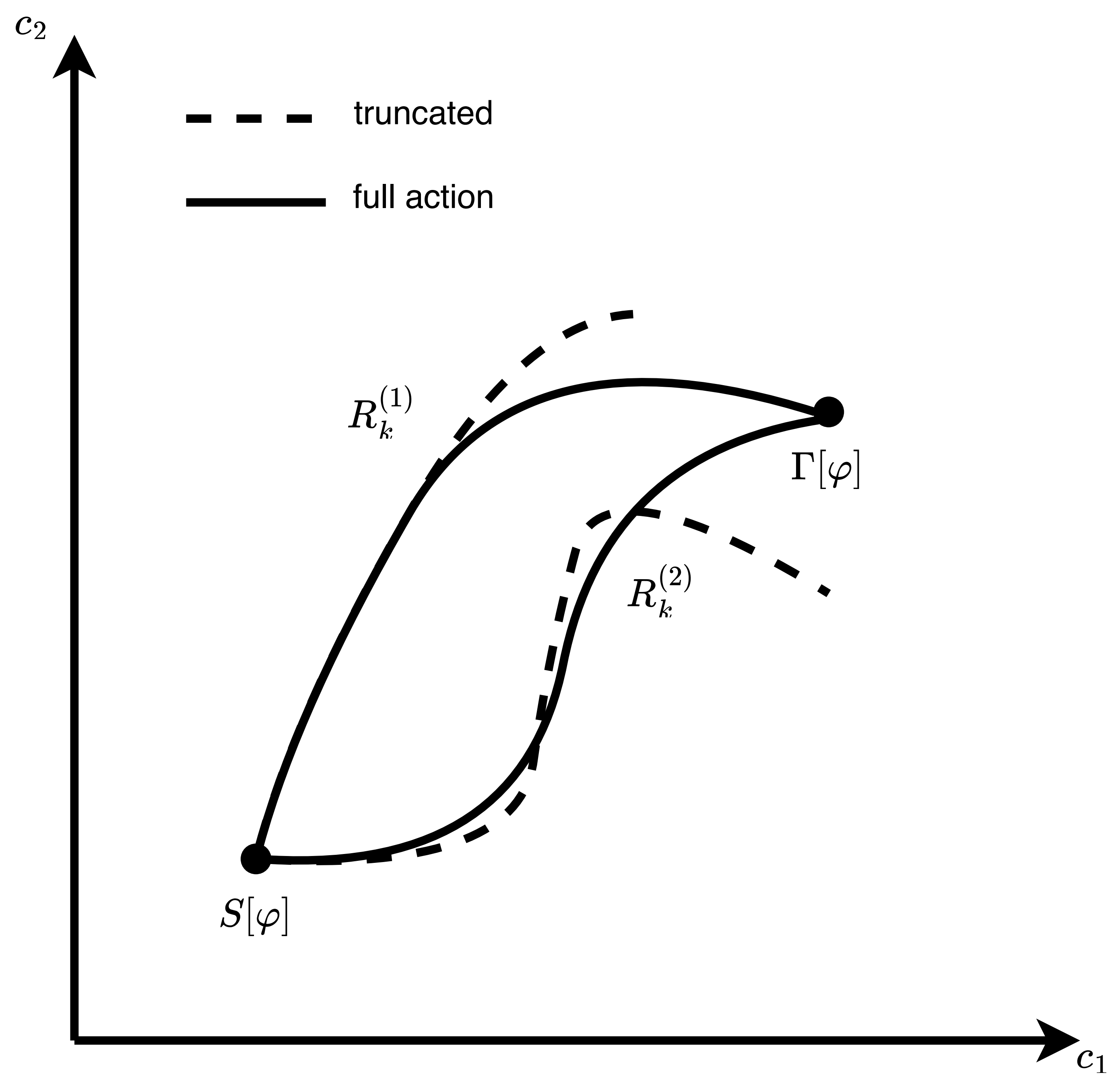
$$k\partial_k \Gamma_k[\Phi; g_{\mu\nu}] = \frac{1}{2}\,\mathrm{tr}\left[\frac{k\partial_k R_k}{\Gamma_k^{(2)}[\Phi] + R_k}\right]$$

- Truncation needed, e.g.:

$$\Gamma_k[\Phi] \approx \int_x \frac{1}{2}\Phi\left[\partial_\mu\partial^\mu + m_k^2\right]\Phi + \frac{\lambda_k}{4!}\Phi^4$$

- with some UV-condition $m_\Lambda^2, \lambda_\Lambda^2, \ldots$
- Kubo formula: $\eta(T) \to \eta_k(T), \quad \eta_{\mathrm{UV}} = 0$





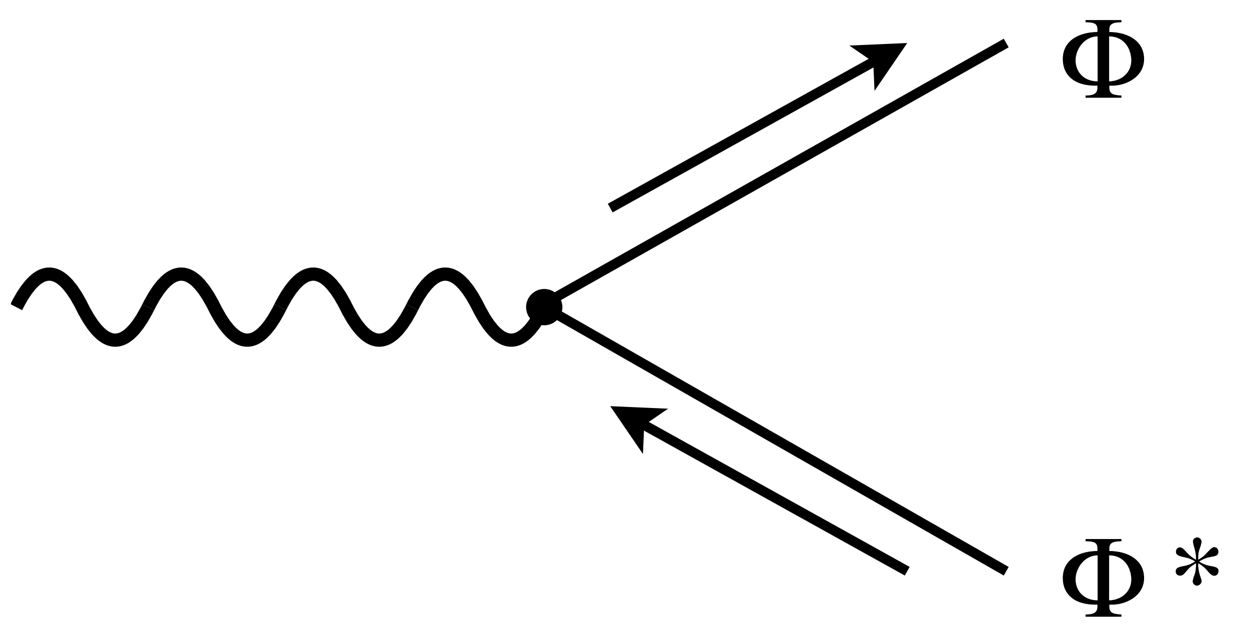
Truncation

- Introduce external gauge field [Rosé,Dupuis '16]
- interactions, thermal mass

$$\Gamma_k[\Phi; g_{\mu\nu}] = \int \mathrm{d}^4x\sqrt{g}\left[\frac{1}{2}\phi\left(\nabla_\mu\nabla^\mu + m_k^2\right)\phi + \frac{\lambda}{4!}\right]$$

- Damping coefficient [Floerchinger '16]:

$$G(z,\vec{p}) = \frac{1}{m_k^2 + \vec{p}^2 - z^2 - \mathrm{i}\gamma_k \mathrm{sgn}(\mathrm{Im}(z))}$$



- In d=1+0 dimensions, forward time evolution:

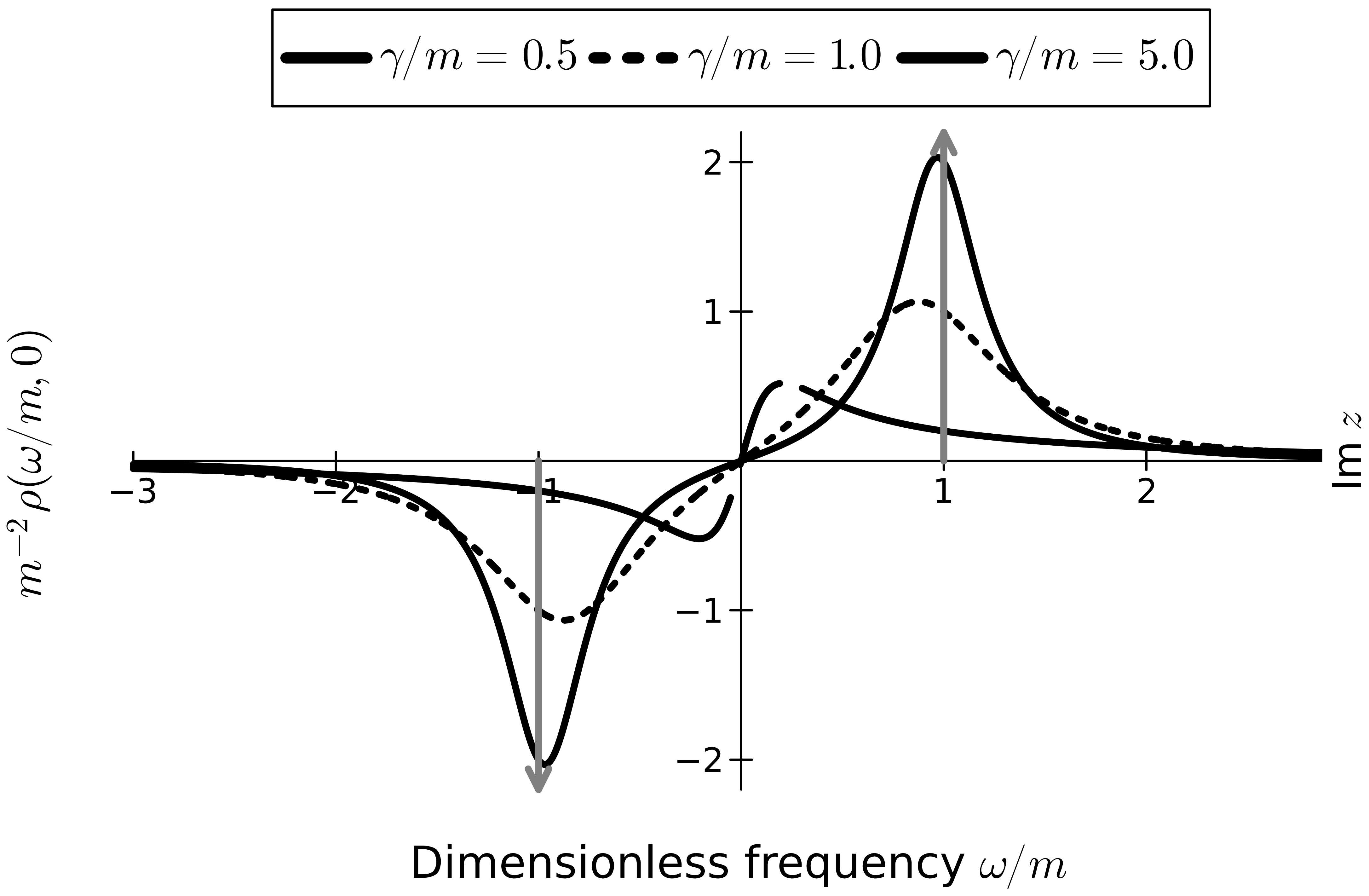
$$\left[\omega^2 + \mathrm{i}\gamma_k\omega - m_k^2\right]\tilde{\Phi}(\omega) = 0$$

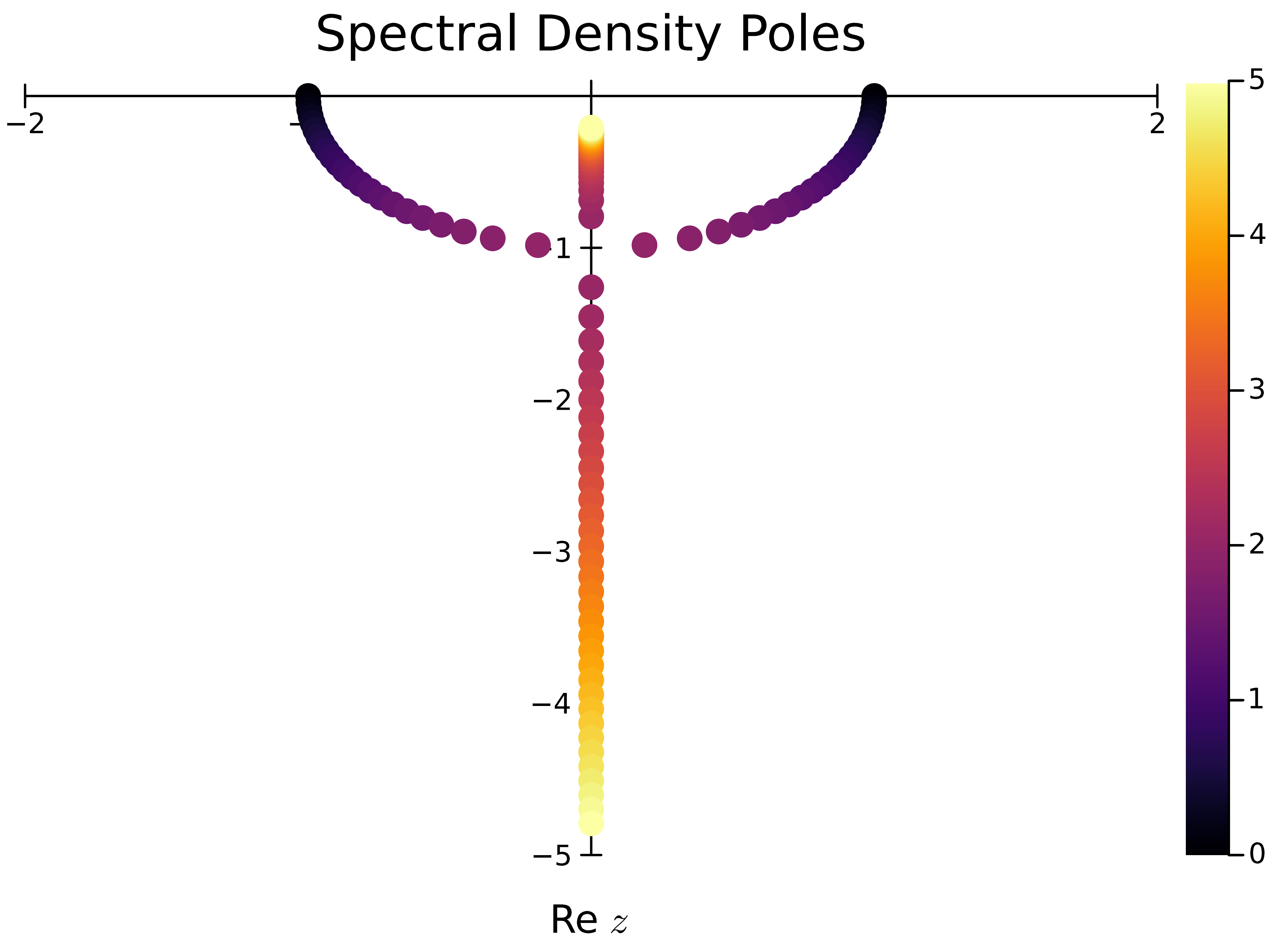
dissipation

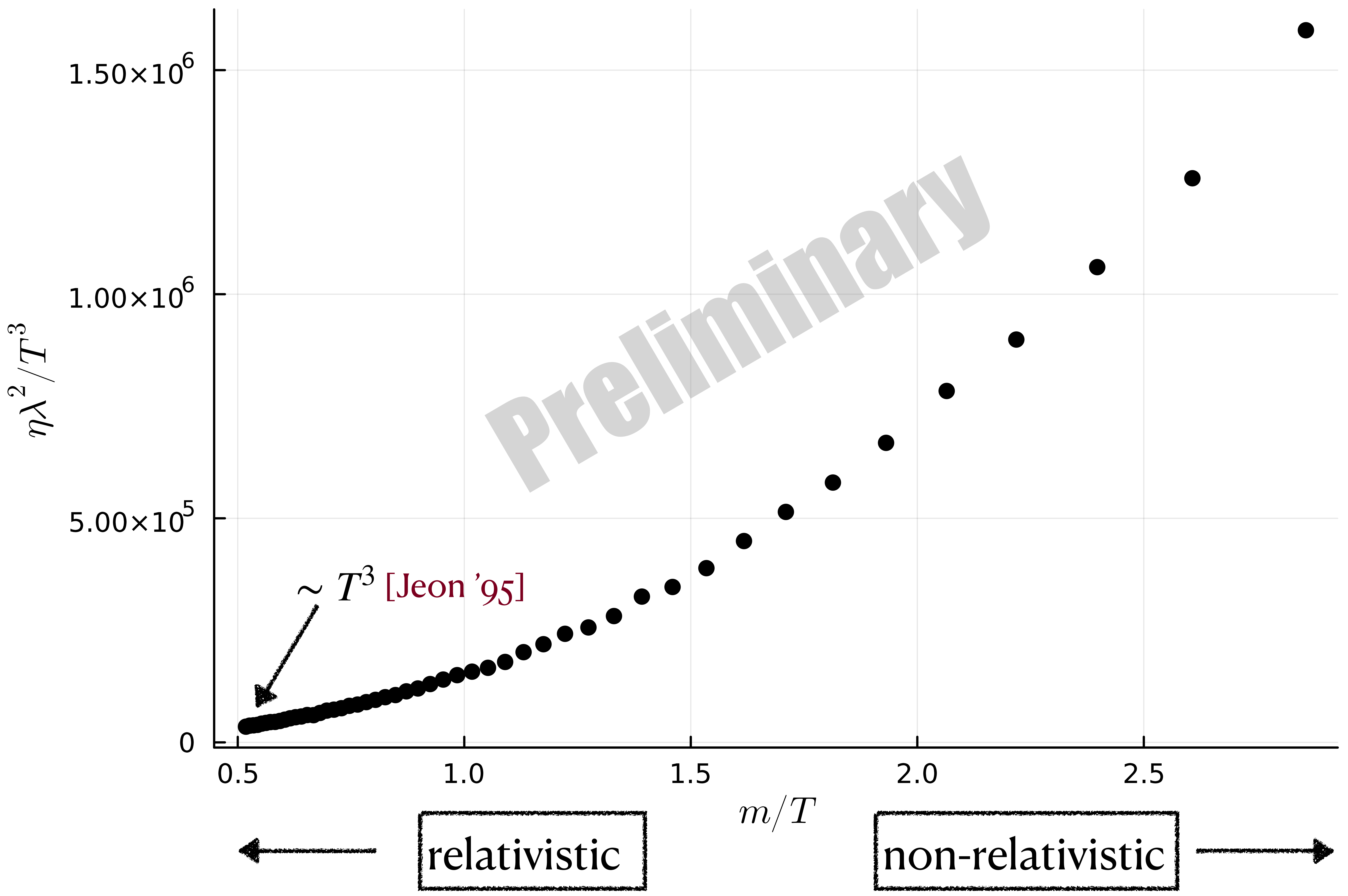
Analytic Structure

- mass pole shifted, decay width generated

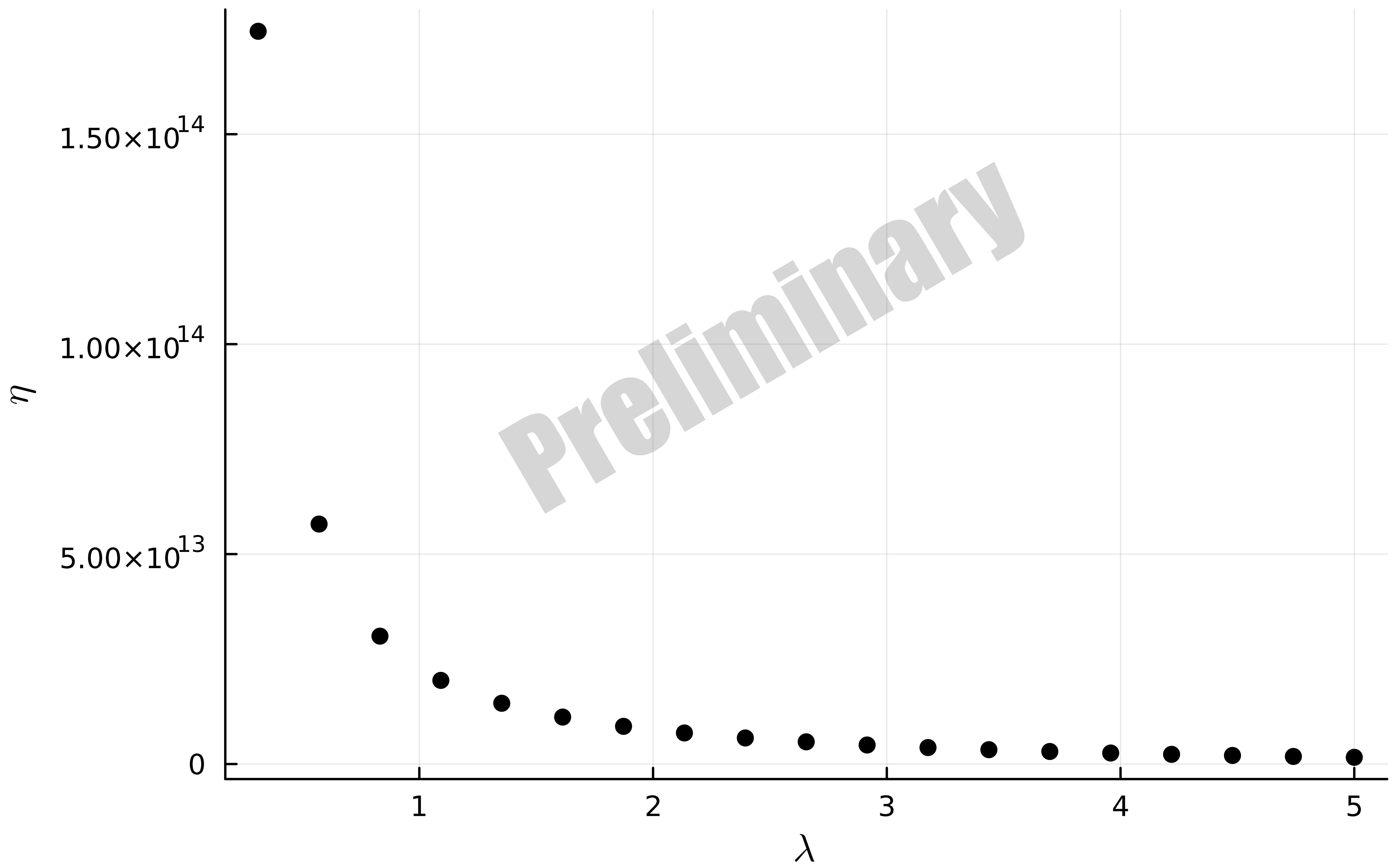
$$\rho(z,\vec{p}) = -\frac{1}{2}\mathrm{Im}G(\omega + \mathrm{i}0^0, \vec{p})$$







Preliminary
1.50×10^6
1.00×10^6
5.00×10^5
0
ηλ^2/T^3
~ T^3 [Jeon '95]
0.5
1.0
1.5
2.0
2.5
m/T
relativistic
non-relativistic



Hydro:

$\eta \sim l_{\mathrm{mfp}}$

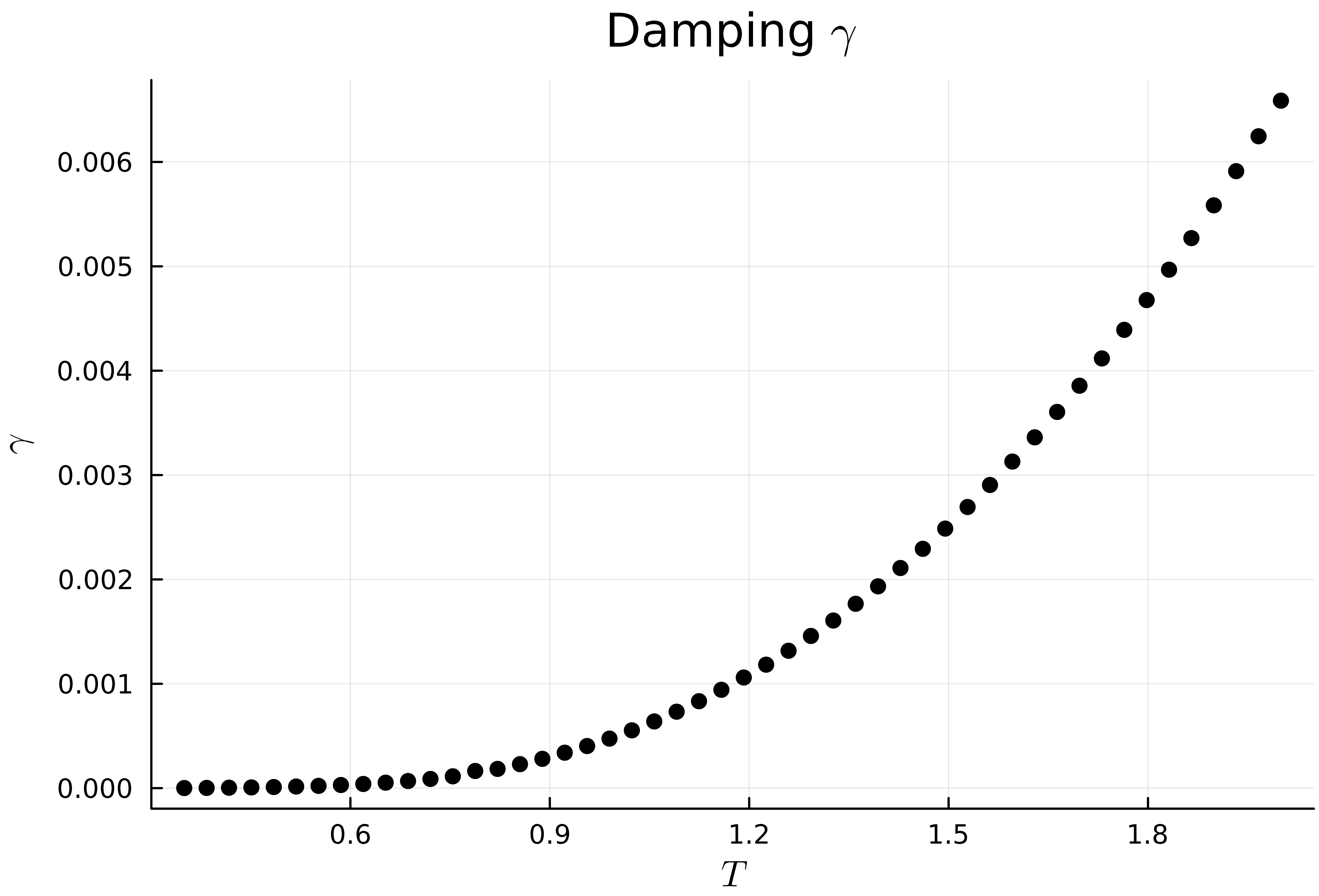
Micro:

$\frac{1}{\lambda} \sim l_{\mathrm{mfp}}$

Conclusion

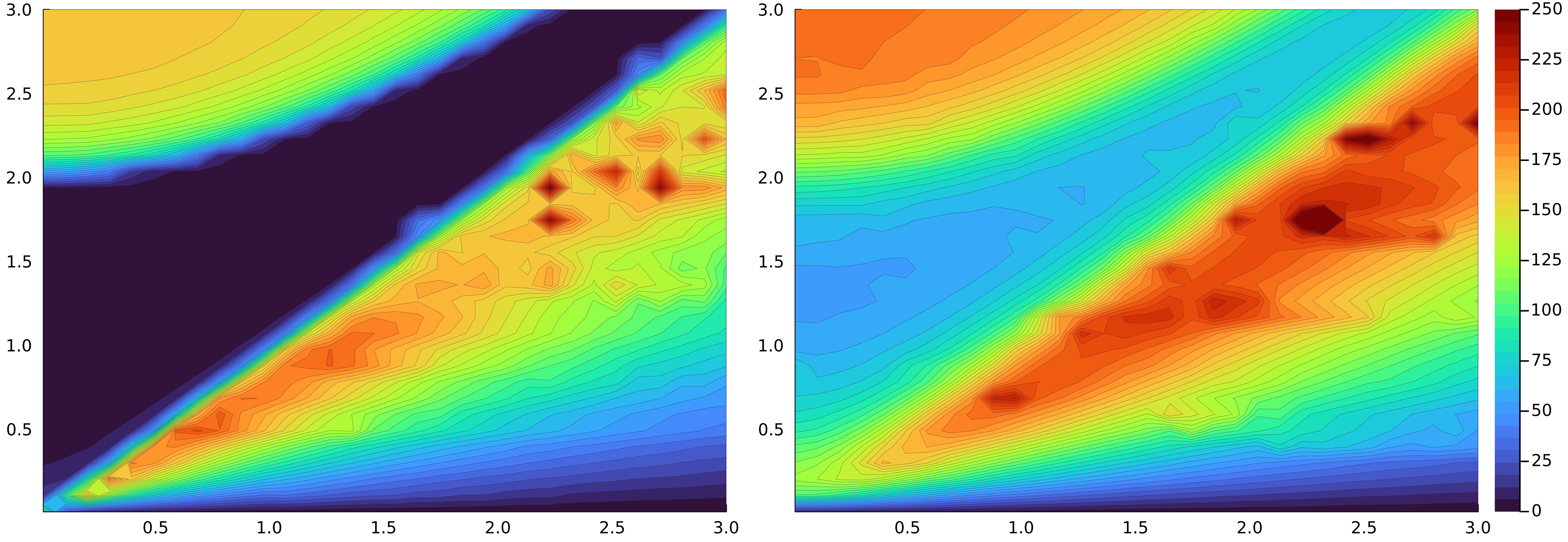
- Prediction of transport coefficients close to equilibrium
- Non-perturbative methods: coefficient flow equation
- Truncation introduces damping coefficients
- Viscosity scaling at small coupling: $\eta(T) \sim T^3$
- diverging at vanishing interaction $\eta \to \infty$

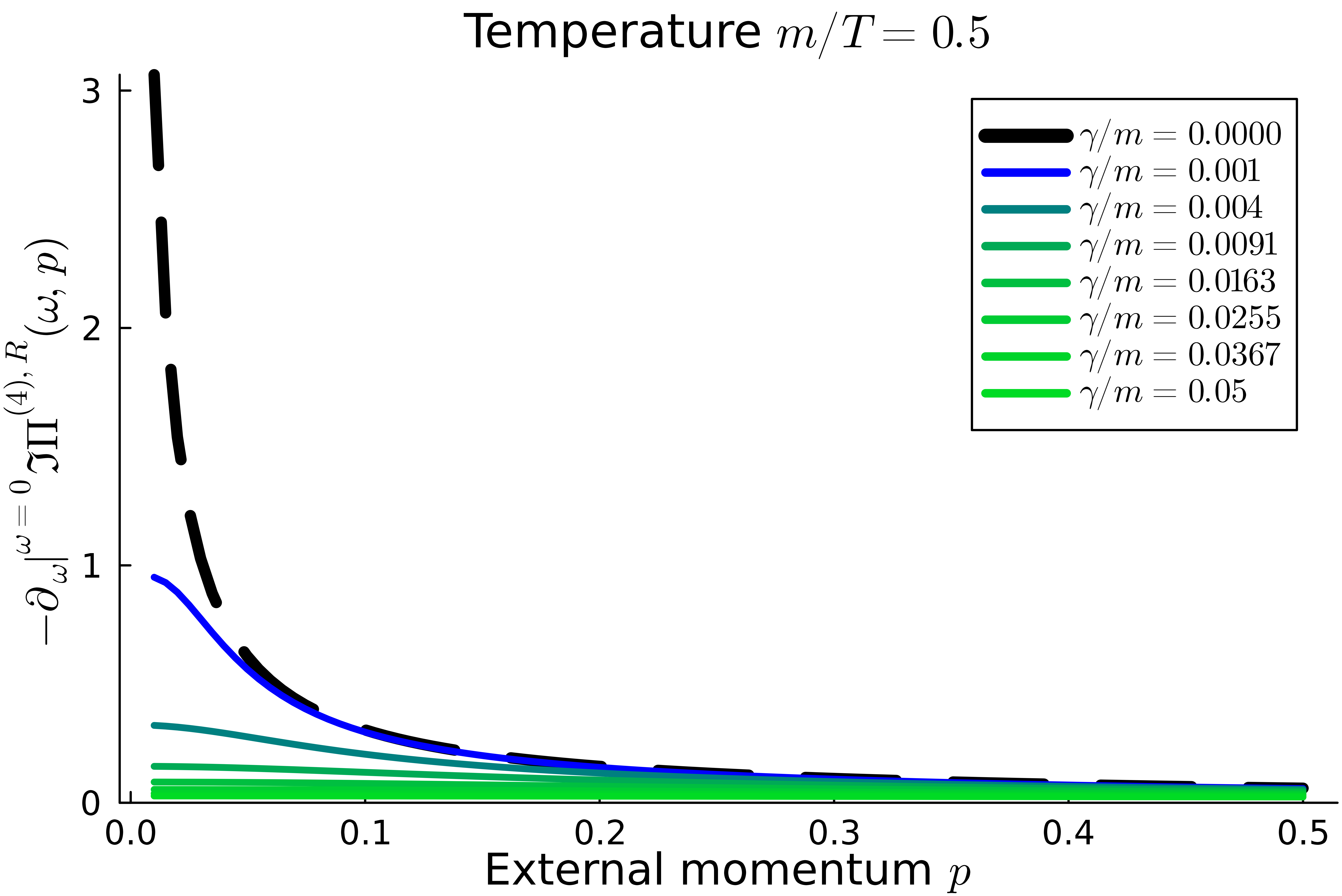
Thanks for your attention!



Damping γ
γ
T
0.000
0.001
0.002
0.003
0.004
0.005
0.006
0.6
0.9
1.2
1.5
1.8

Fourpoint Function - Production Rates





Temperature $m/T = 0.5$
$\gamma/m = 0.0000$
$\gamma/m = 0.001$
$\gamma/m = 0.004$
$\gamma/m = 0.0091$
$\gamma/m = 0.0163$
$\gamma/m = 0.0255$
$\gamma/m = 0.0367$
$\gamma/m = 0.05$
$-\partial_\omega|^{\omega=0}\Im\Pi^{(4),R}(\omega,p)$
External momentum p
0.0
0.1
0.2
0.3
0.4
0.5
0
1
2
3

Viscosity Flow Equation

$$\eta = -\lim_{\omega \to 0} \partial_\omega \mathrm{Im} \frac{\delta^2 \Gamma[\Phi; g_{\mu\nu}]}{\delta g_{\mu\nu} \delta g_{\alpha\beta}} \Bigg|^{g_{\mu\nu}=\eta_{\mu\nu}}$$

Pole Structure of Correlation Functions