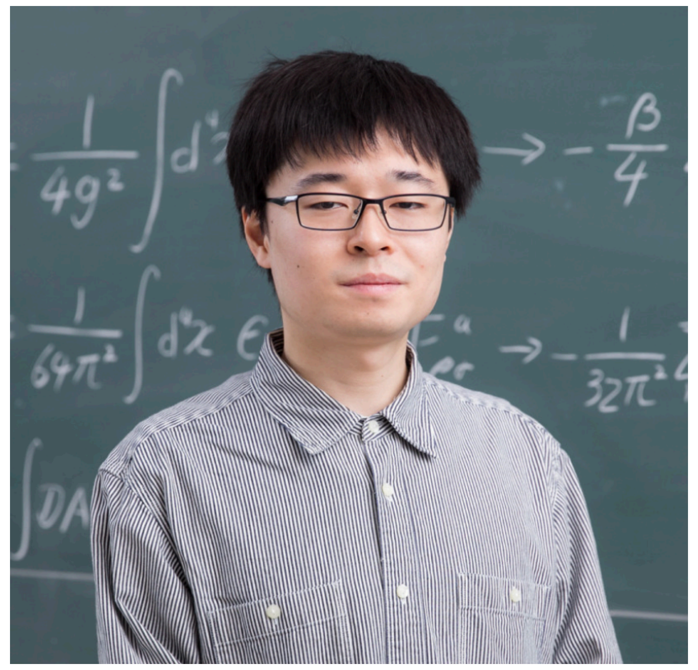
Three ways of calculating mass spectra in the Hamiltonian formalism



Etsuko Itou
(YITP, Kyoto University / RIKEN iTHEMS)

based on JHEP11(2023)231 w/ A.Matsumoto and Y.Tanizaki

work in progress

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

iTHEMS Interdisciplinary Theoretical & Mathematical Sciences

PRESTO

科研費 KAKENHI

SQAI サステイナブル量子AI研究拠点

Towards quantum simulation of gauge/gravity duality and lattice gauge theory, 2024/03/06

Outline

1. Introduction

2. 2-flavor Schwinger model

3. Our proposal for calculating "Hadron" spectra ($\theta = 0$)

   Correlation-function scheme

   One-point function scheme

   Dispersion-relation scheme

4. "Hadron" spectra ($\theta \neq 0$, preliminary)

   Correlation-fn. + one-point-fn. scheme

   Dispersion-relation shceme

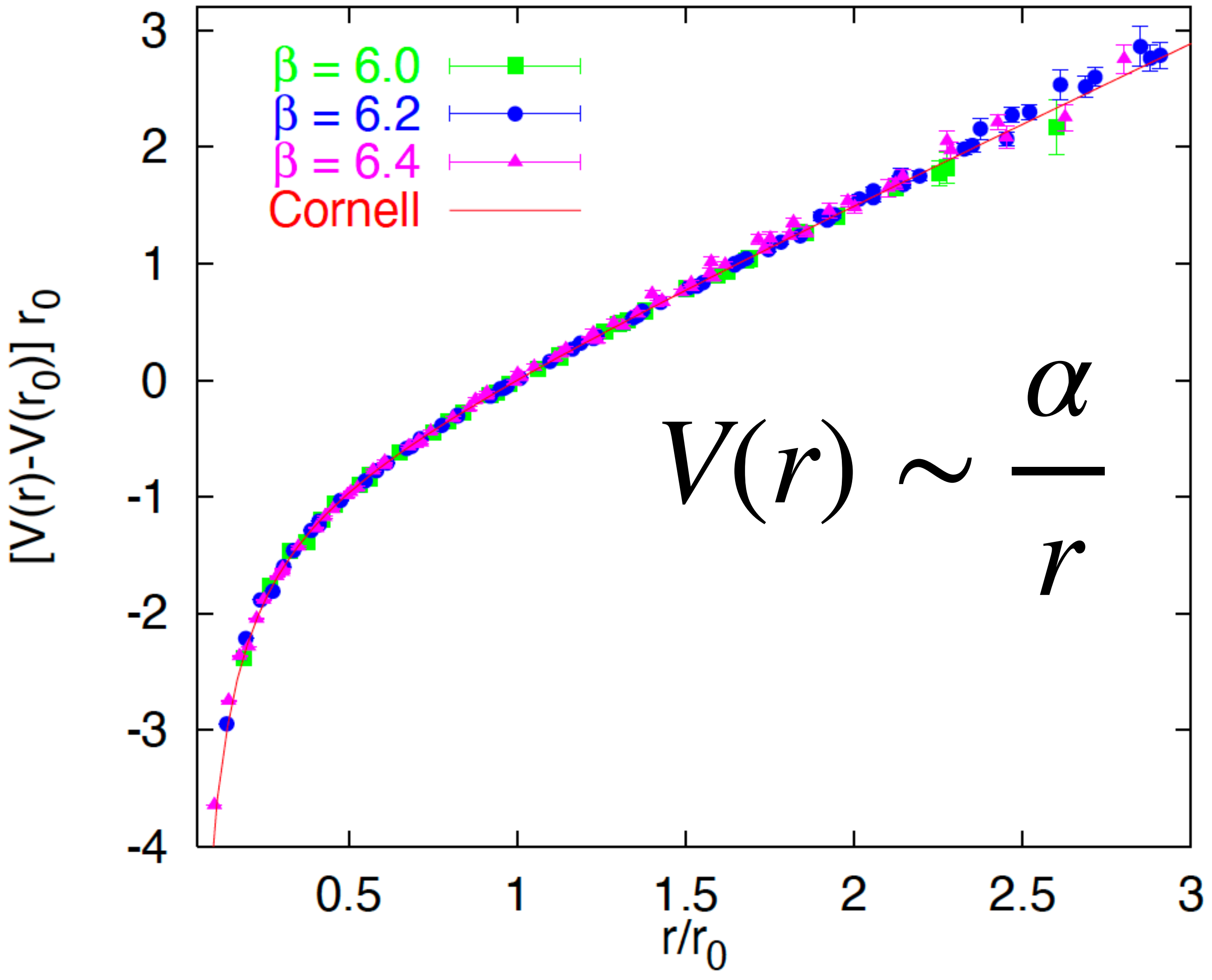
5. Summary

1.Introduction

New calculation method for QCD observables

Introduction：Sucesses of Lattice MC QCD

G.Bali, Phys.Rept.343:1 (2000)



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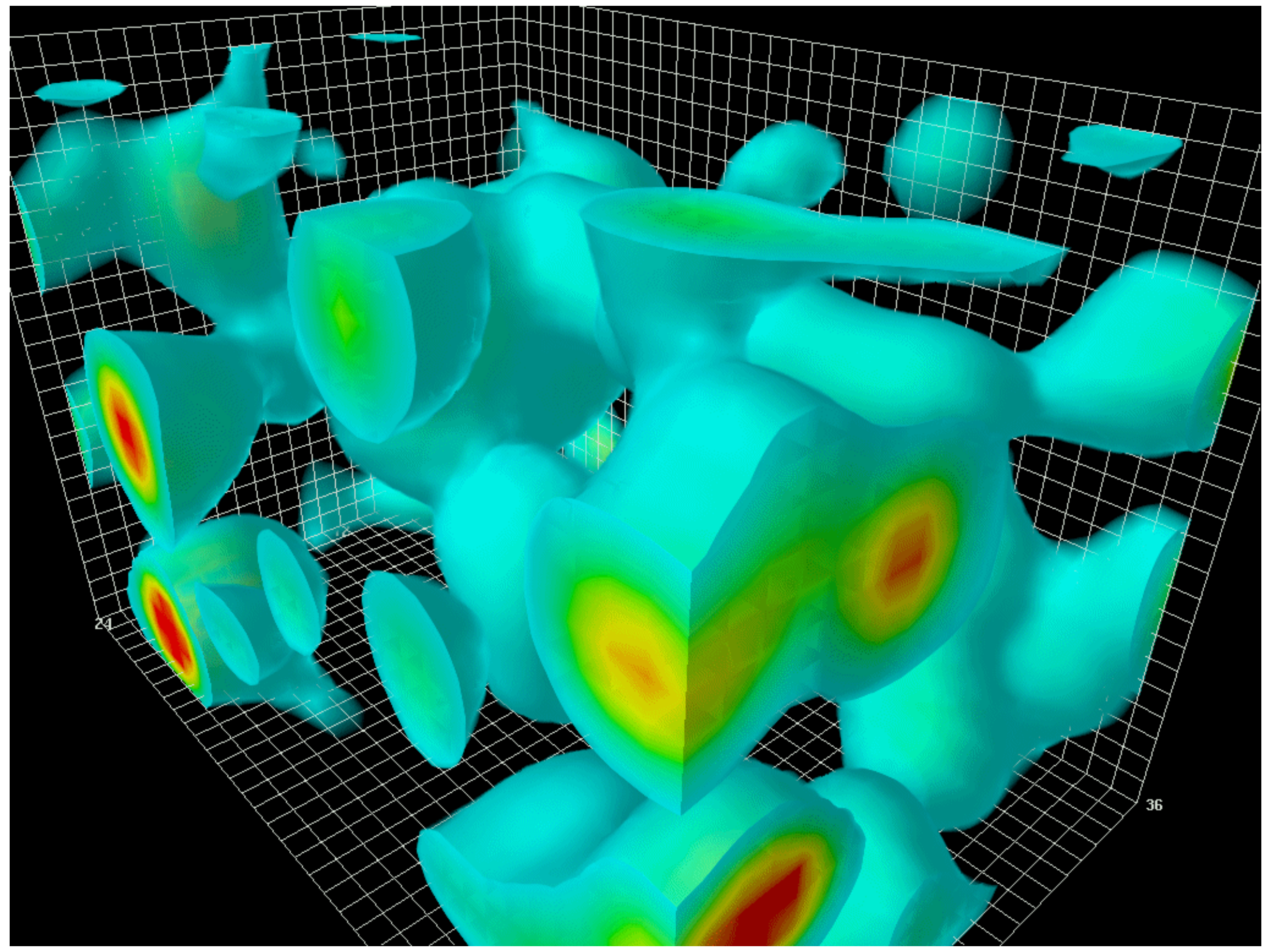
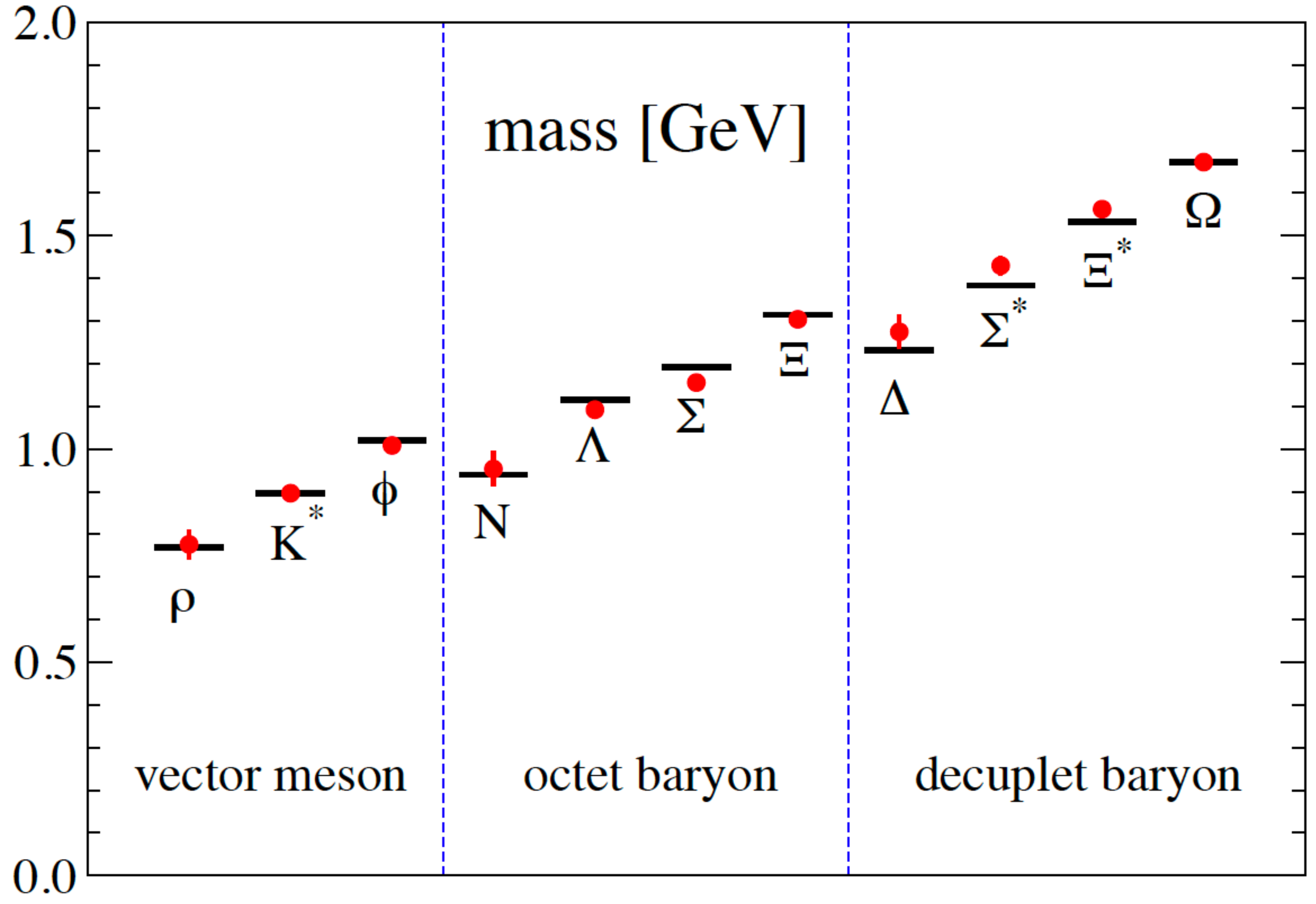
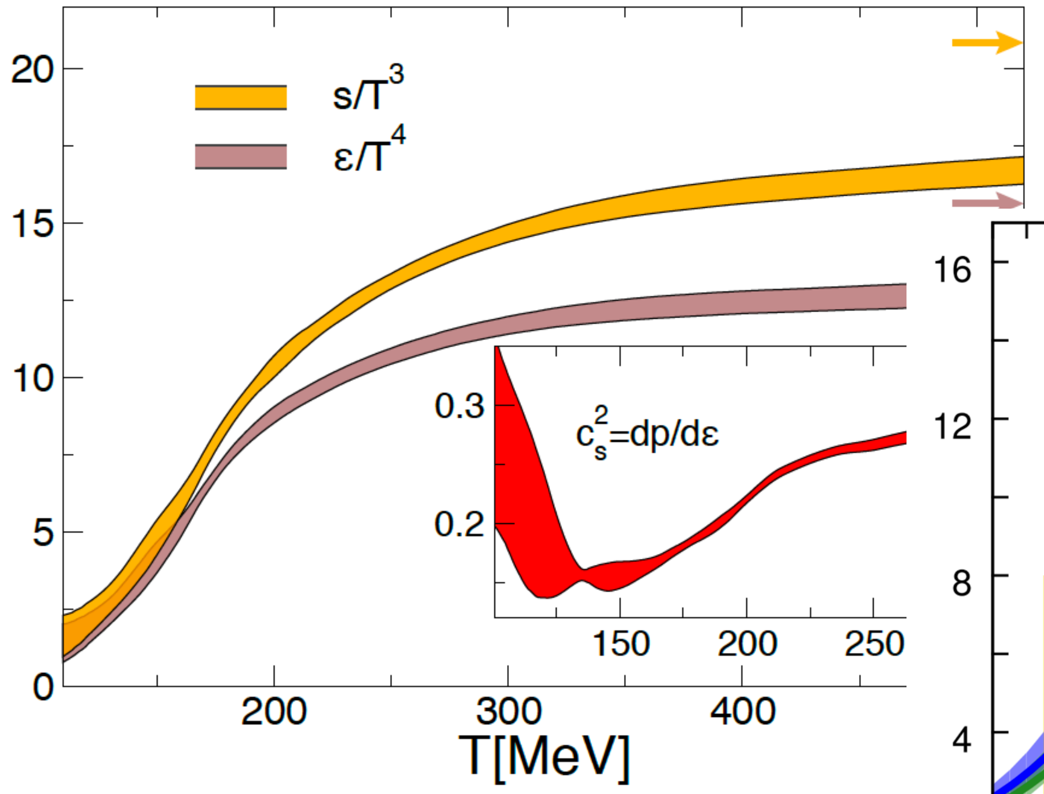


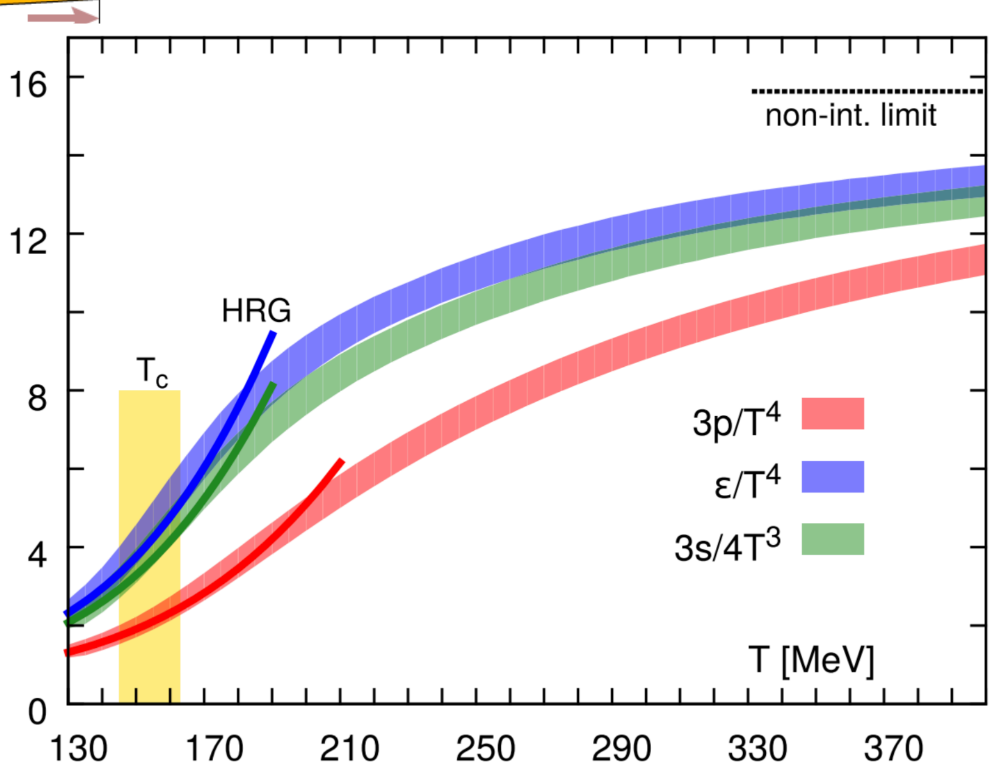
Z.Fodor and C.Hoelbling
arXiv:1203.4789



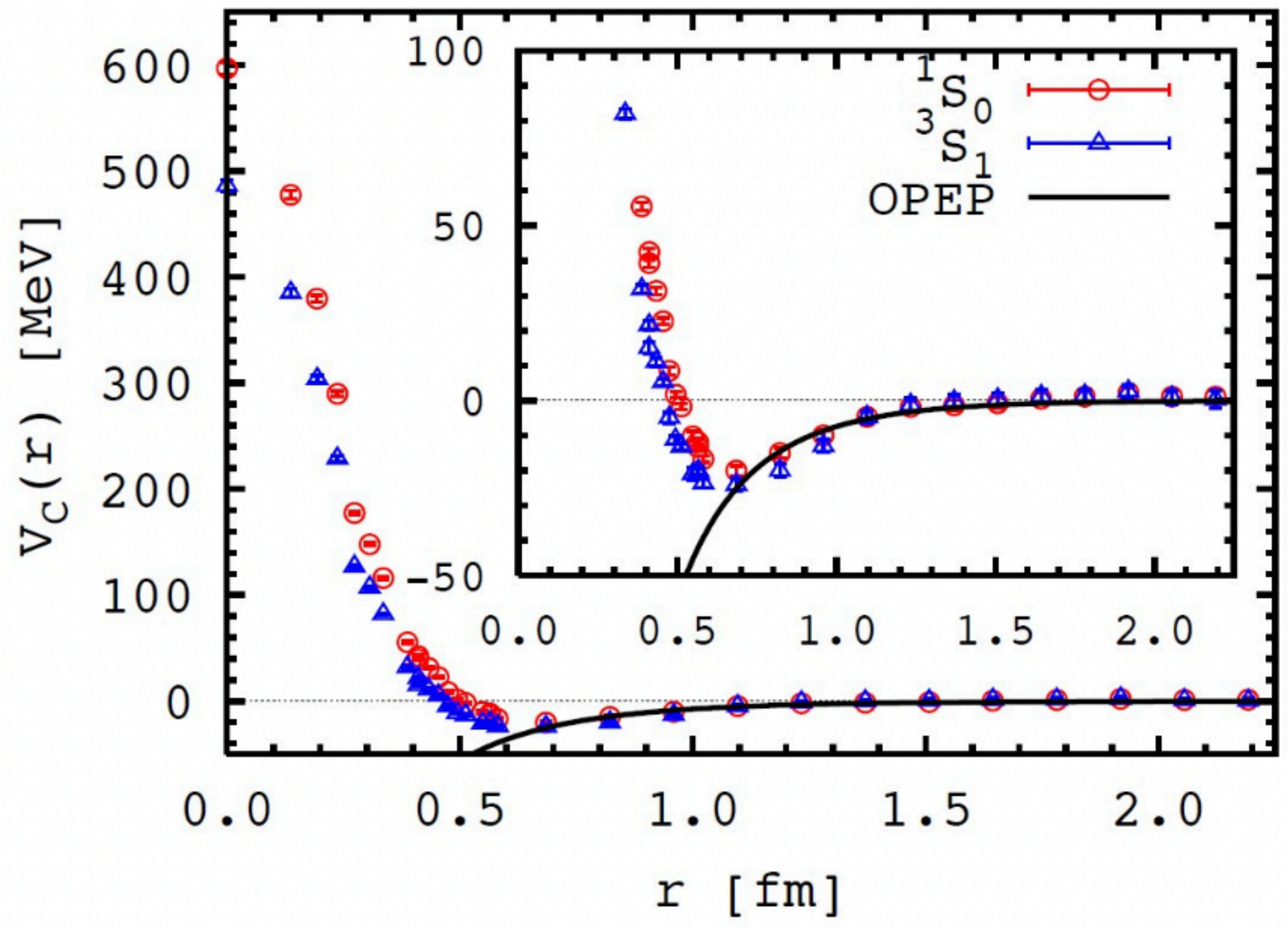
ıt hadron spectrum
Experimental data
is reproduced from
on of the PACS-CS



Borsanyi et al. (2013)



HotQCD (2014)



Aoki, Ishii, Hatsuda
HAL QCD coll.
(2007 -)

QCD (gauge theories) in Hamiltonian formalism

- How to deal with gauge d.o.f.?

  (In LQCD, introducing link variable, e^{iA_μ}, is compact reps. instead of $-\infty \le A_\mu \le \infty$)

- How to generate state? (In LQCD: PHB, HMC, RHMC)

  quantum algorithm (adiabatic state preparation, variational..)

  tensor network (DMRG, PEPs..)

- How to measure physical observables? conceptually and technically

  confinement (In LQCD: Polyakov loop, Wilson loop, smearing tech.)

  hadron spectrum (In LQCD: 2pt. fn, several source improvement)

  hadron scattering (In LQCD: Luscher method, HAL QCD method…)

  thermodynamic quantities (In LQCD: integration method, fixed scale, gradient flow)

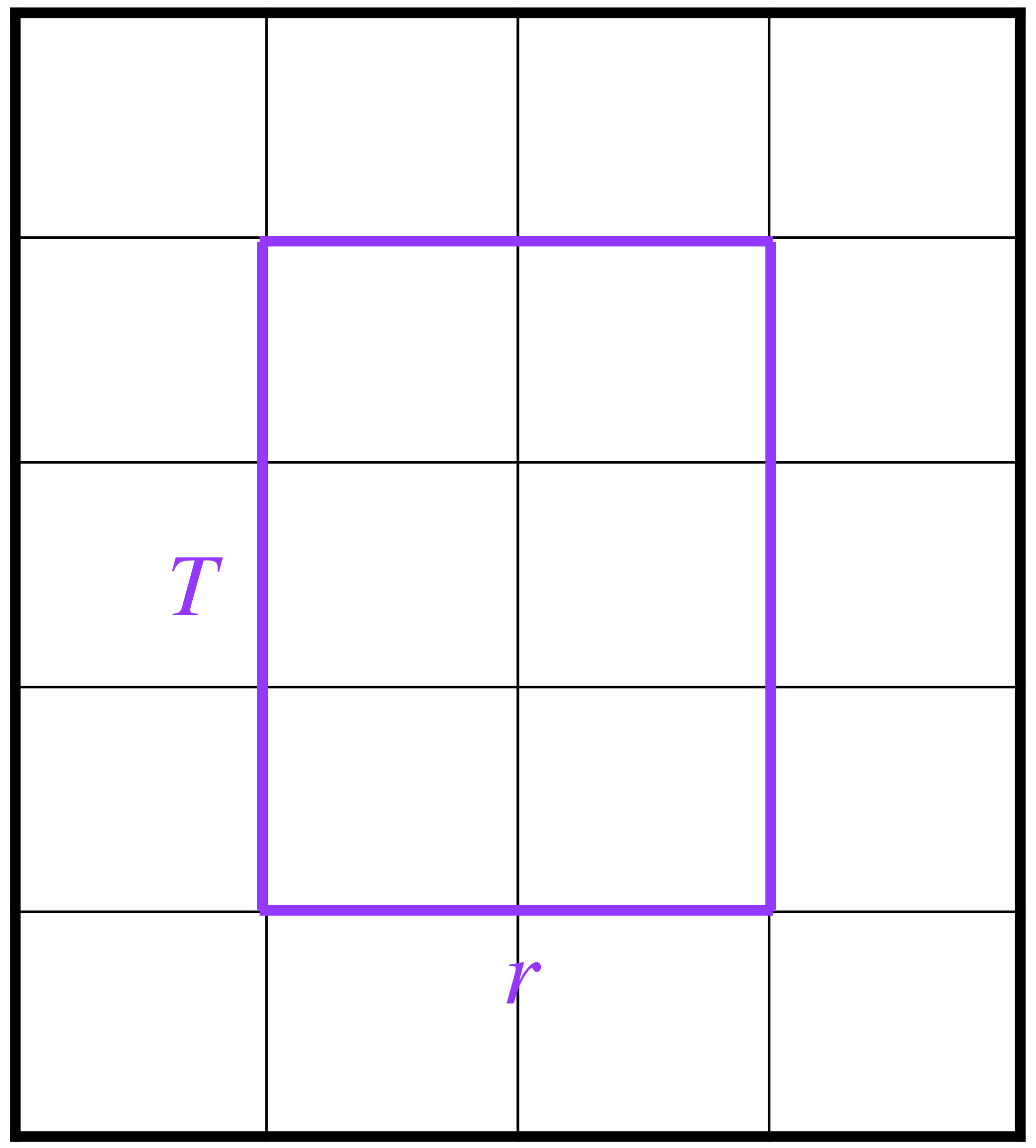
New calculation method for QCD observables

- In Hamiltonian formalism, different calculation method is available

Ex.) $q - \bar{q}$ potential

- Lagrangian formalism: Wilson loop

$$\langle W(C) \rangle \underset{T\to\infty}{\approx} e^{-TV(r)} = \mathrm{tr}\left[\prod_{i\in C} U_i\right]$$



Measure the product of link variables
and see its exponent

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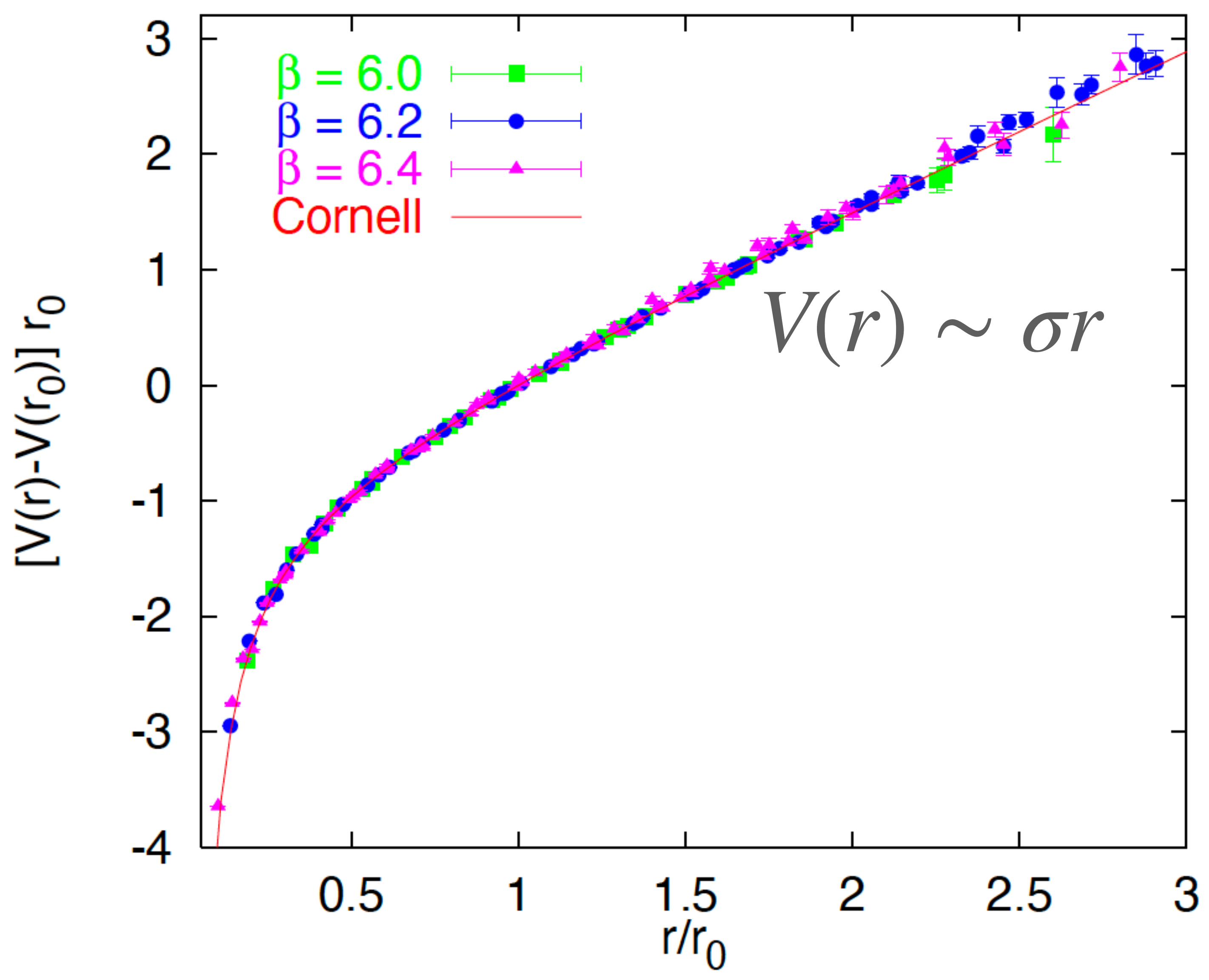

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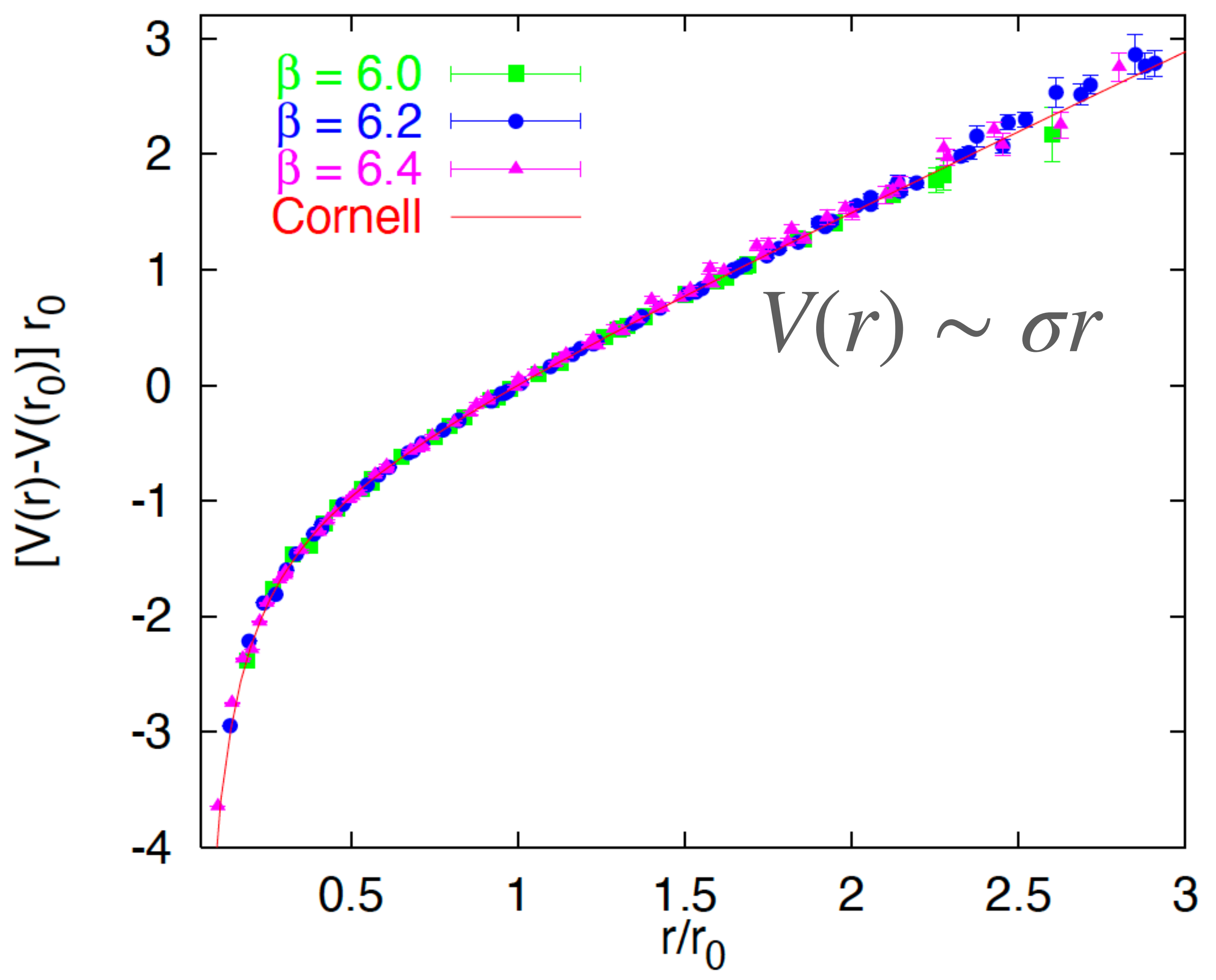
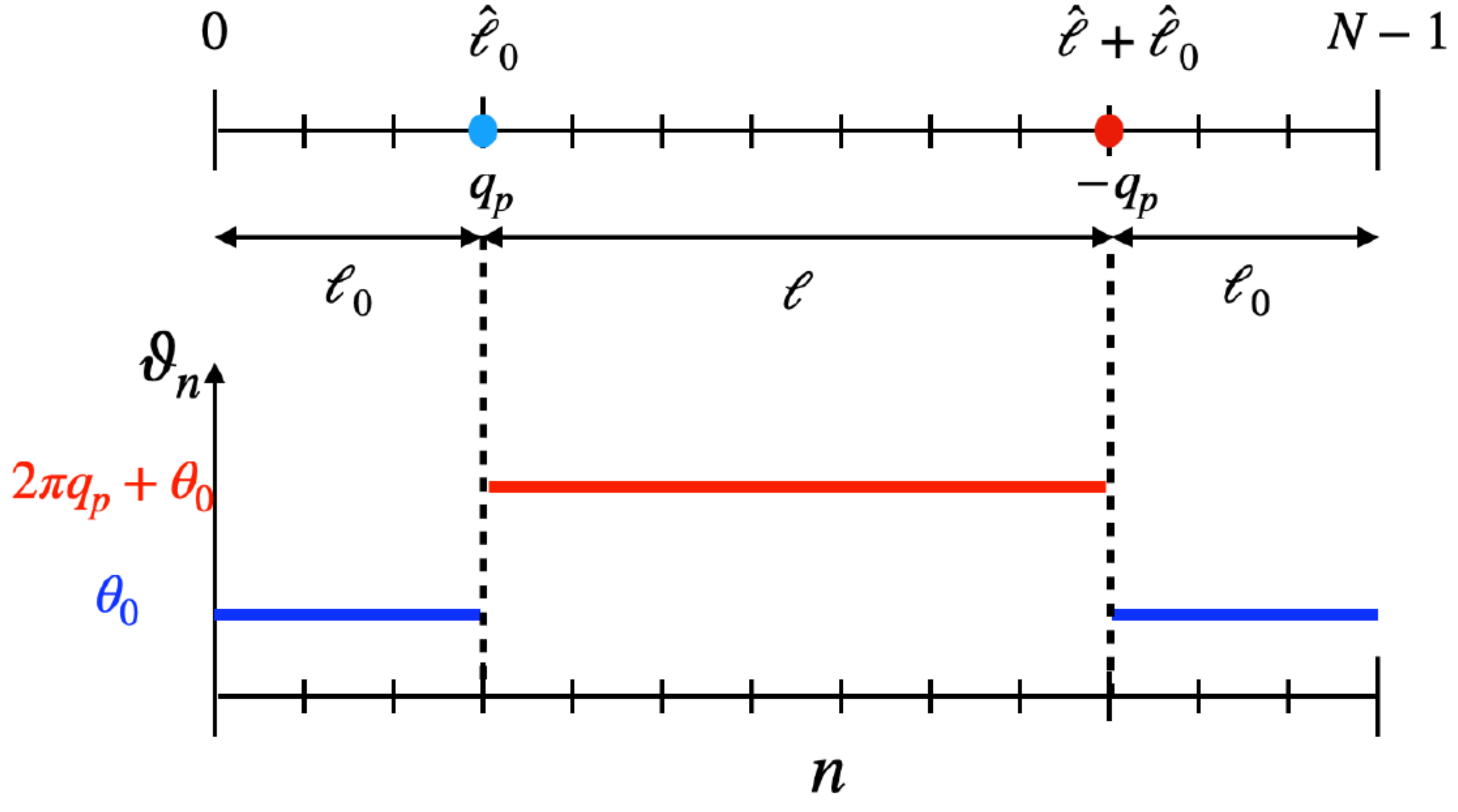


- Hamiltonian formalism (for Schwinger model) ground state energy w/probe charges system

Measure $E(\ell) = \langle \Omega | H(\ell) | \Omega \rangle$ with several ℓ

potential $V(\ell) = E(\ell) - E(0)$



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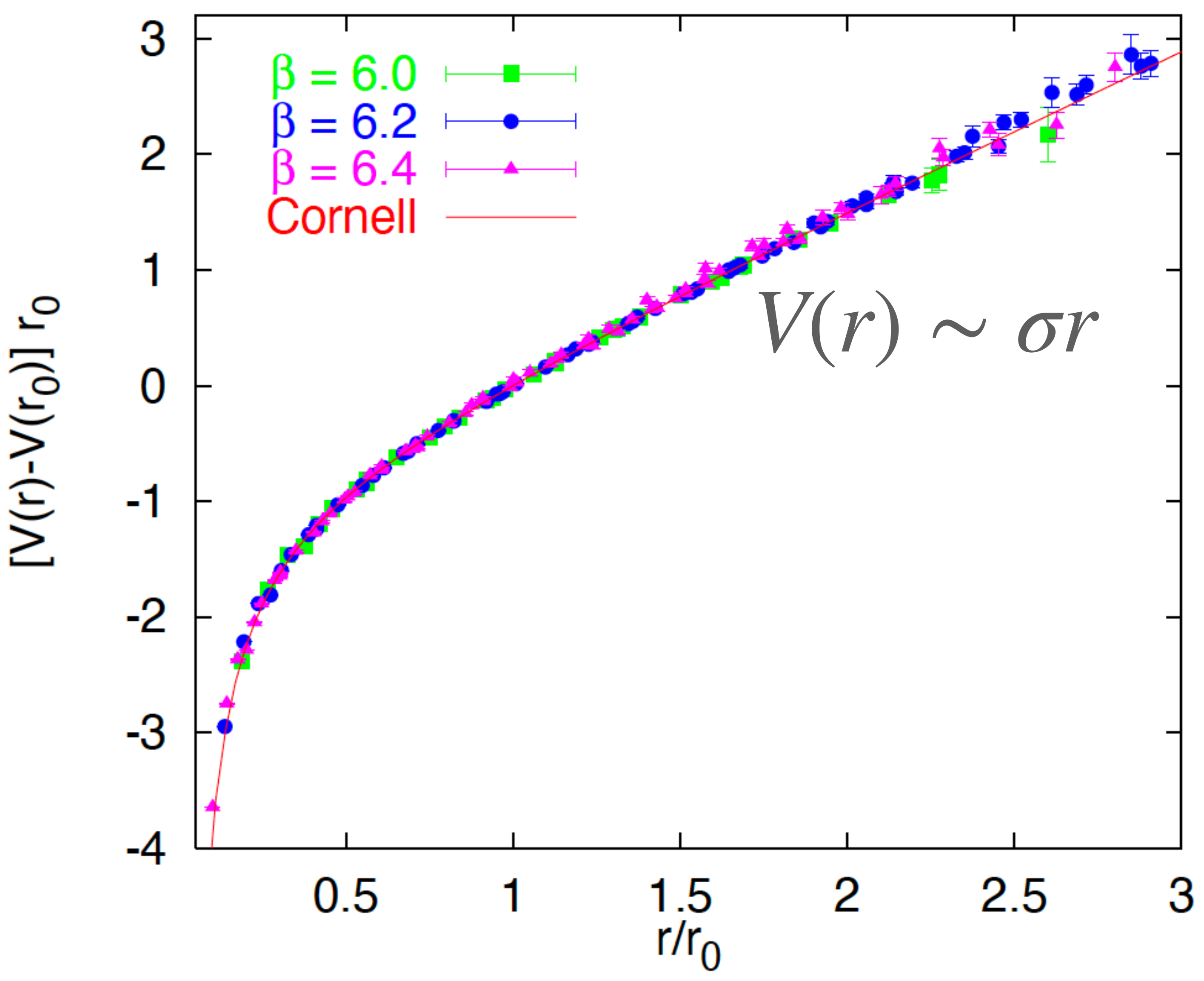

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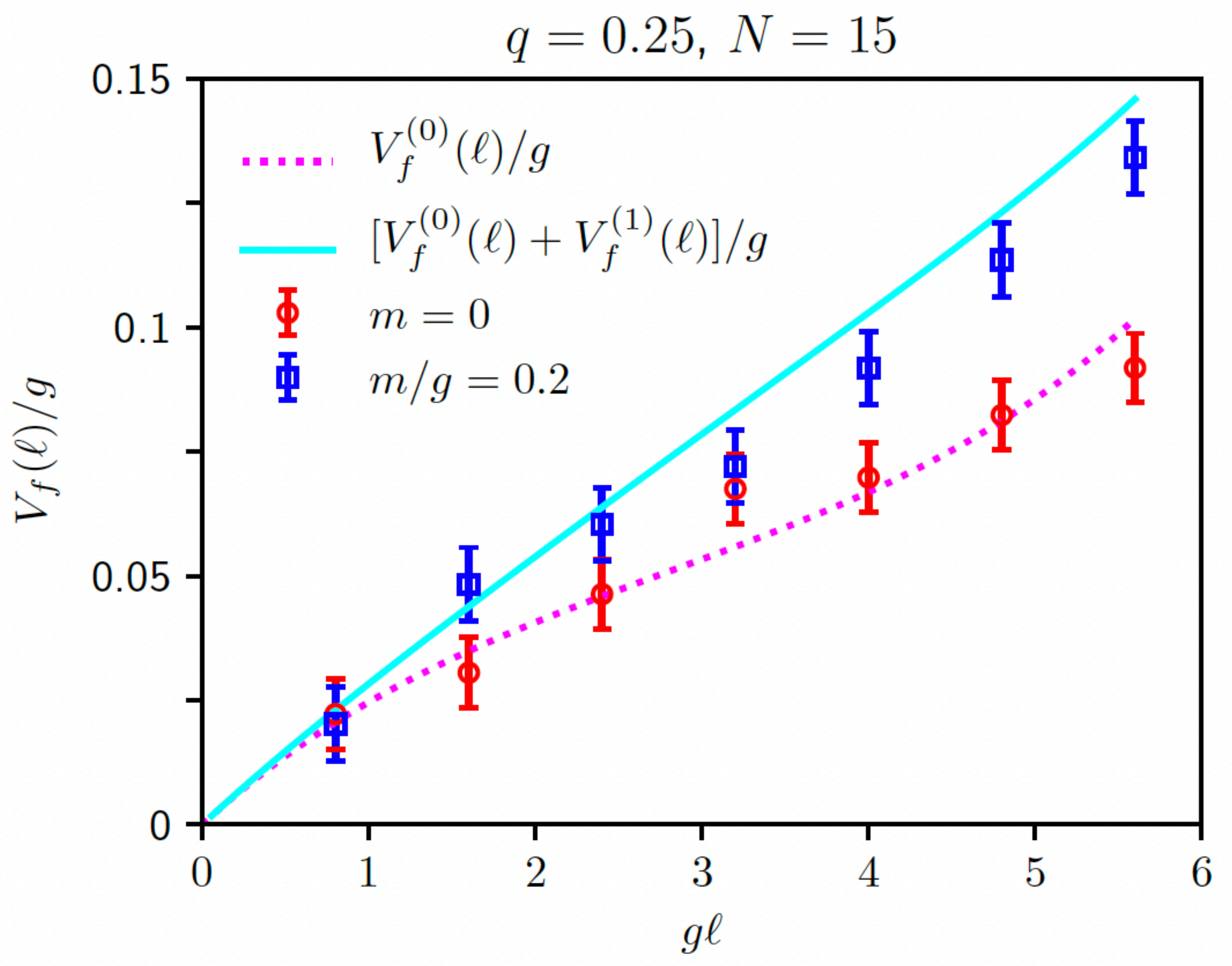
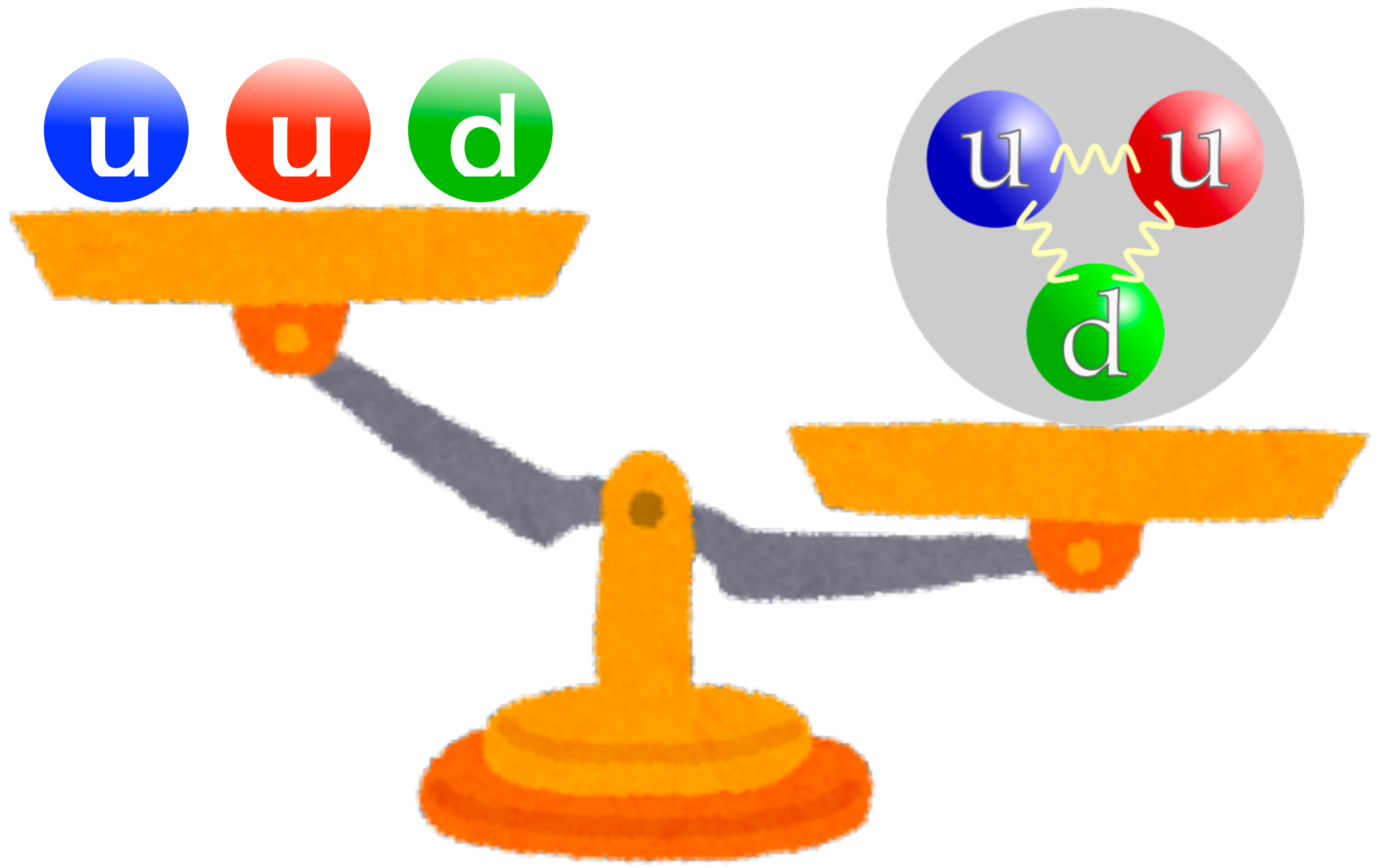
M.Honda, E.I., Y.Kikuchi, L.Nagano, T.Okuda, Phys.Rev.D 105 (2022) 1, 01450



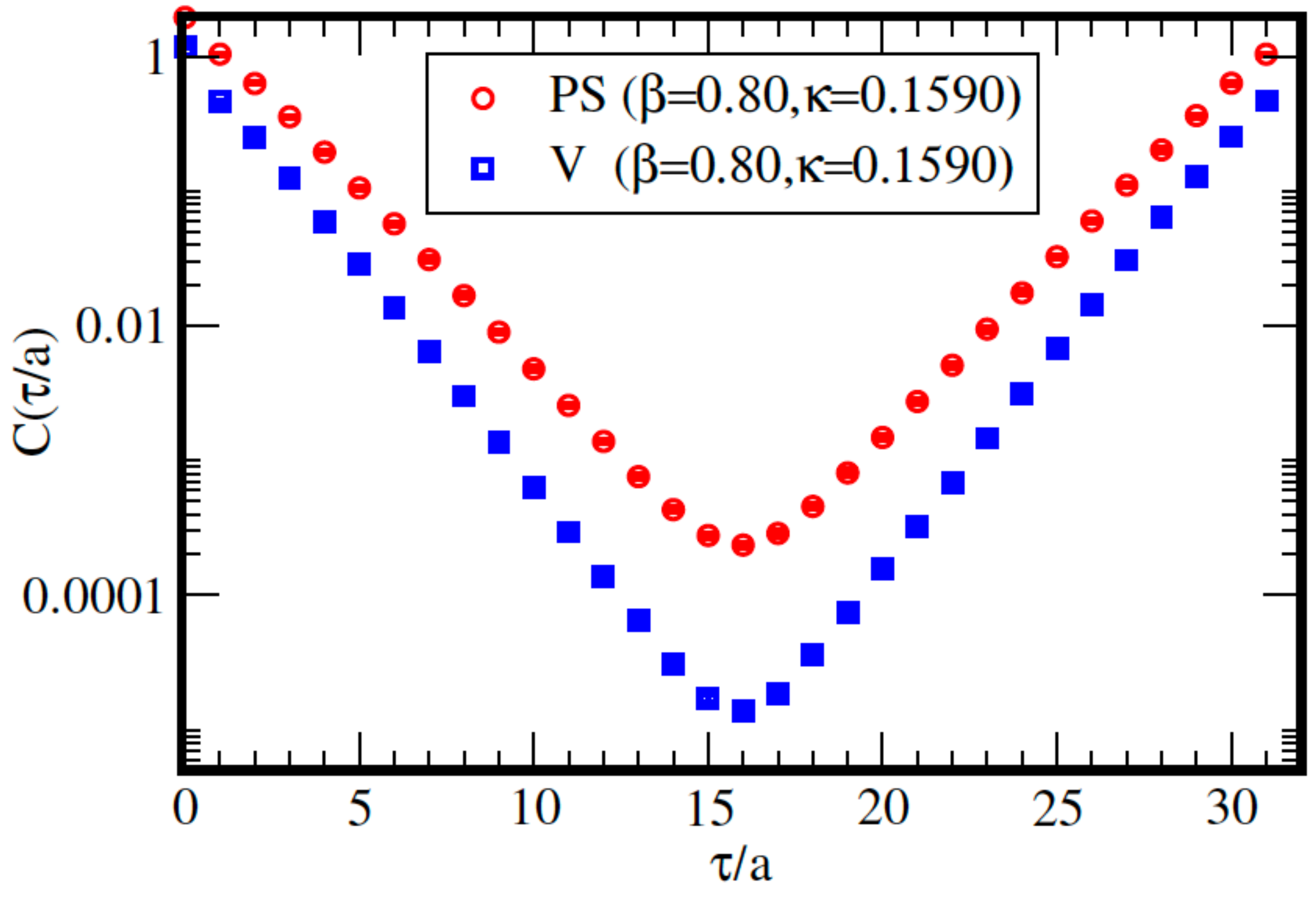
Today's main subject

- How to calculate "hadron" spectrum in Hamiltonian formalism

- Hadron spectrum calc. in conventional Lattice MC



u,d quark mass ~ 2-5MeV
proton mass ~ 938MeV



$$C(\tau) = \langle O(\tau) O(0) \rangle$$

$$\lim_{\tau\to\infty} C(\tau) \sim e^{-m\tau}$$

pion: $O = \bar{\psi}\gamma_5\psi$

rho meson: $O = \bar{\psi}\gamma_1\psi$

Z.Fodor and C.Hoelbling
arXiv:1203.4789

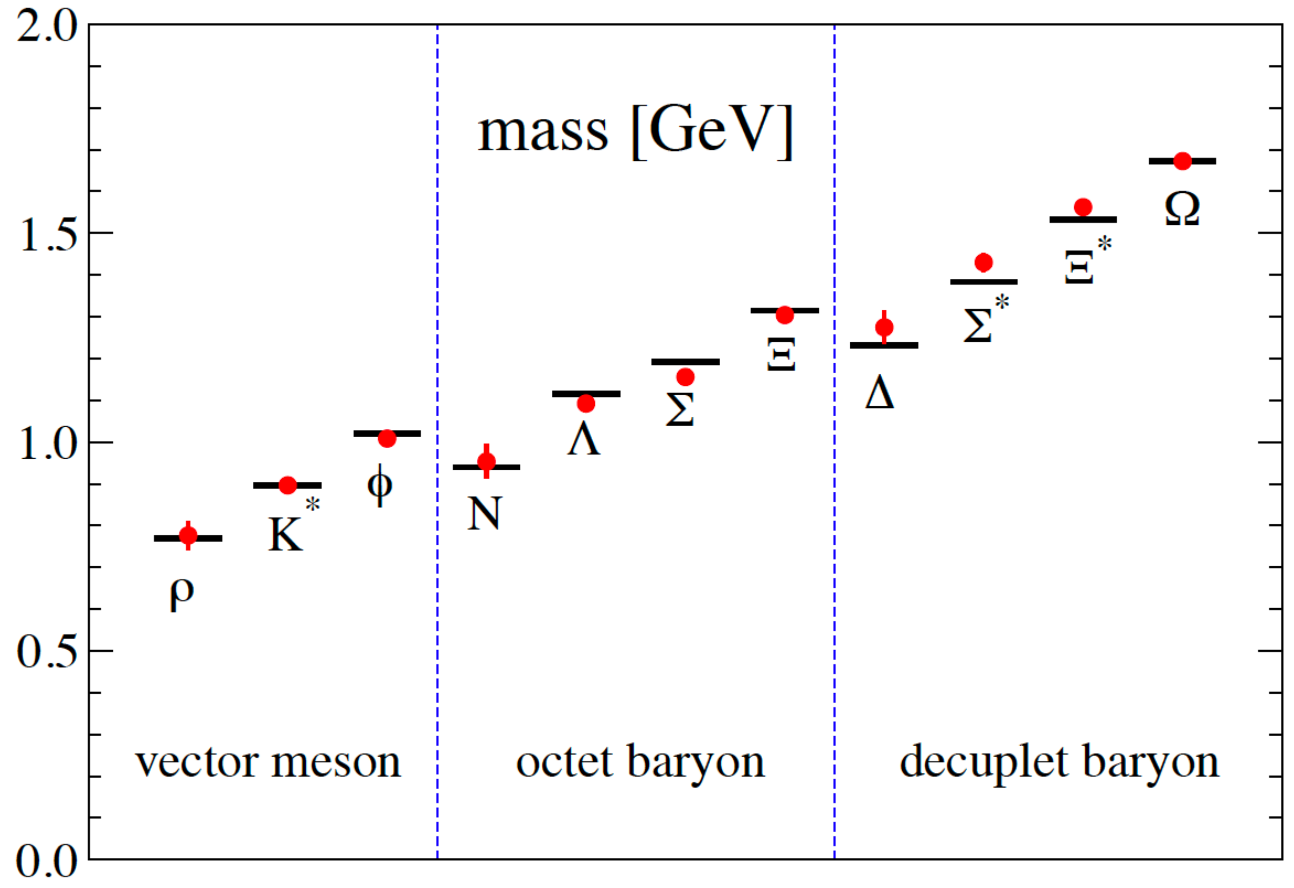
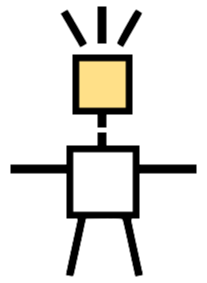


FIG. 20 The extrapolated $N_f = 2+1$ light hadron spectrum results from the PACS-CS collaboration. Experimental data are from (Amsler *et al.*, 2008). The plot is reproduced from (Aoki *et al.*, 2009a) with friendly permission of the PACS-CS collaboration.

Our work

- To test Hamiltonian formalism, tensor network method is also useful
  Density Matrix Renormalization Group (DMRG) method
  (N~1000 is doable)
  White (1992)
  ITensor , Fishman et al.(2022)
  
  Find ground state (Matrix Product State, MPS) w/ variational algorithm: cost fn. is ${}_{try}\langle\Psi|H|\Psi\rangle_{try}$
  Also obtain excited states by modified cost fn. $H \to H + \lambda \sum_{k=0}^{\ell-1} |\psi_k\rangle\langle\psi_k|$

- Non-abelian gauge and/or higher dim. QFT suffers from several problems
  Nf=2 Schwinger model, namely 1+1d. QED is a good testing ground

2. Schwinger model

Schwinger model

- Toy model of QCD
  (discrete) chiral symmetry breaking
  confinement / screening potential
  composite states

- 1+1d U(1) gauge theory (QED)

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta_0}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}\psi$$

- w/ θ term
  sign problem in conventional MC method

Schwinger model + θ term (Nf=1)

- Lagrangian

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \boxed{\frac{g\theta_0}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu}} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}\psi$$

non-zero θ_0: Sign problem in conventional method

Gauge fixing
Gauss law
Open BC
Jourdan-Winger trans.

- Hamiltonian by spin variables

$$H = J\sum_{n=0}^{N-2}\left[\sum_{i=0}^{n}\frac{Z_i + (-1)^i}{2} + \boxed{\frac{\theta_0}{2\pi}}\right]^2 + \frac{w}{2}\sum_{n=0}^{N-2}\left[X_nX_{n+1} + Y_nY_{n+1}\right] + \frac{m_{\mathrm{lat.}}}{2}\sum_{n=0}^{N-1}(-1)^nZ_n$$

kinetic term of electric field — kinetic/mass terms of electron

$$m_{\mathrm{lat}} := m - \frac{N_f g^2 a}{8}$$

Kogut and Susskind (1975)
Shaw et al. Quantum 4, 306 (2020)
R.Dempsey et al. PRR 4 (2022) 043133

- θ_0 is constant shift of electric field

- all-to-all interaction of Z
  Gaped system (even in massless case for Nf=1)

Recent study on Schwinger model w/ Hamiltonian formalism

Nf=1 Schwinger model

- Real-time evolution

Schwinger effect, C.Muschik et al. NJ of Physics 19 103020
Martinez et al.,Nature 534, 516–519 (2016)
L.Nagano et al., arXiv:2302.10933

- Finite-density

Variational algorithm, A. Yamamoto Phys. Rev. D 104, 014506 (2021), Tomiya arXiv:2205.08860
Entangelement entropy, K.Ikeda et al. arXiv:2305.00996

- Topological theta term

Phase structure (DMRG): M.C.Banuls et al, PRD 93,094512 (2016)
L.Funcke et al. PRD 101, 054507 (2020)
Adiabatic state preparation: B.Chakraborty et al., PRD 105, 094503 (2022)

chiral condensate

potential between probe charges

charge-q Schwinger model ('t Hooft anomaly matching)

Mass spectrum M.C.Banuls et al., JHEP 11 (2013)158

M.Honda, EI, et al. PRD105, 014504 (2022)
M.Honda, EI, Y.Kikuchi,Y.Tanizaki, PTEP (2022)
M.Honda, EI, Y.Tanizaki, JHEP (2022)

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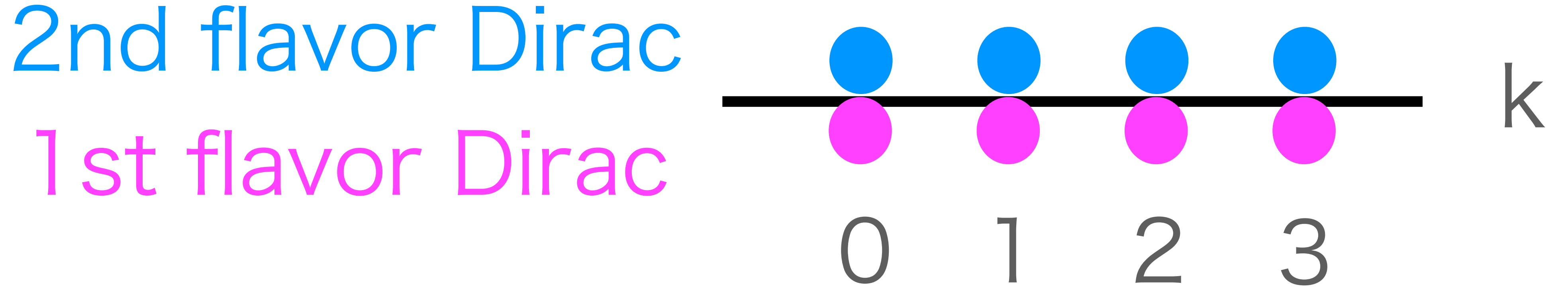
Multi-flavor Schwinger model: ordering

- Dirac fermion -> lattice fermion (staggered fermion)



M.C. Banuls et al, PRL 118, 071601 (2017)
R.Dempsey et al., arXiv:2305.00437
M.Rigobello et al., arXiv:2308.04488

- lattice fermion -> spin variable (Jordan-Wigner trans.)

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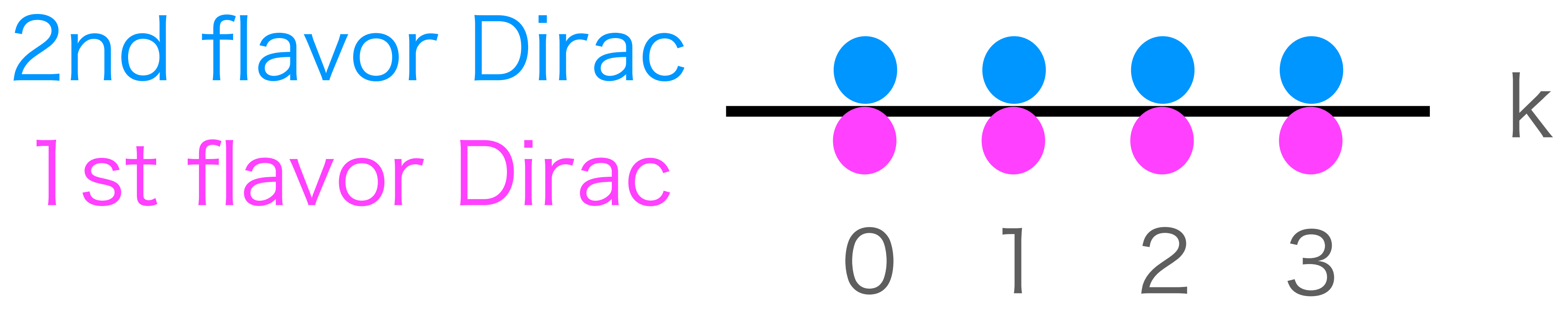

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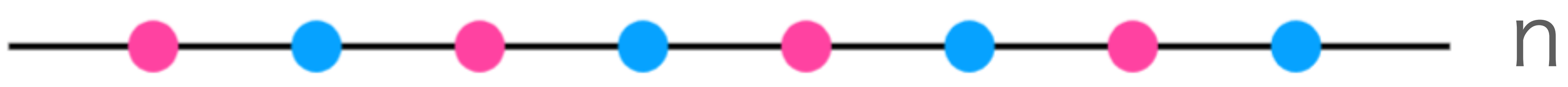
Flavor ordering (n=k+N(f-1))

Staggered ordering (n=2k+(f-1))



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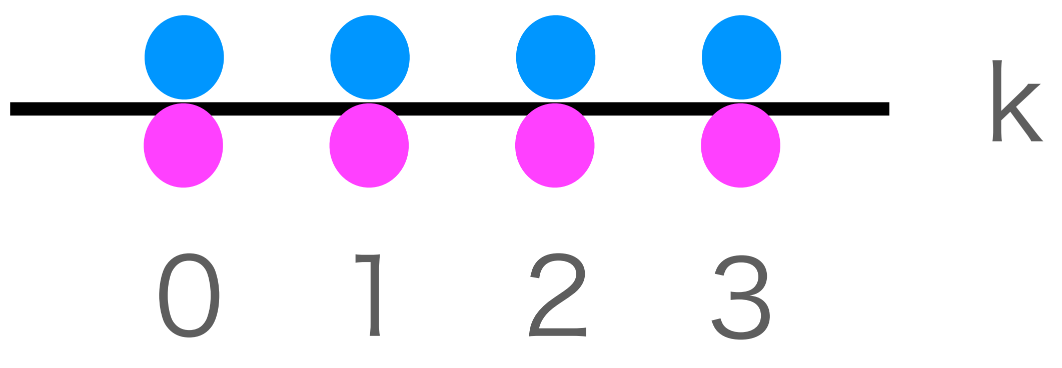
2nd flavor Dirac

1st flavor Dirac



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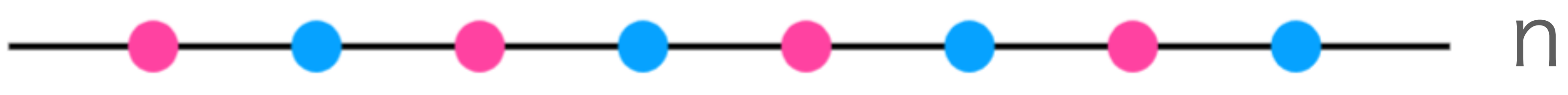

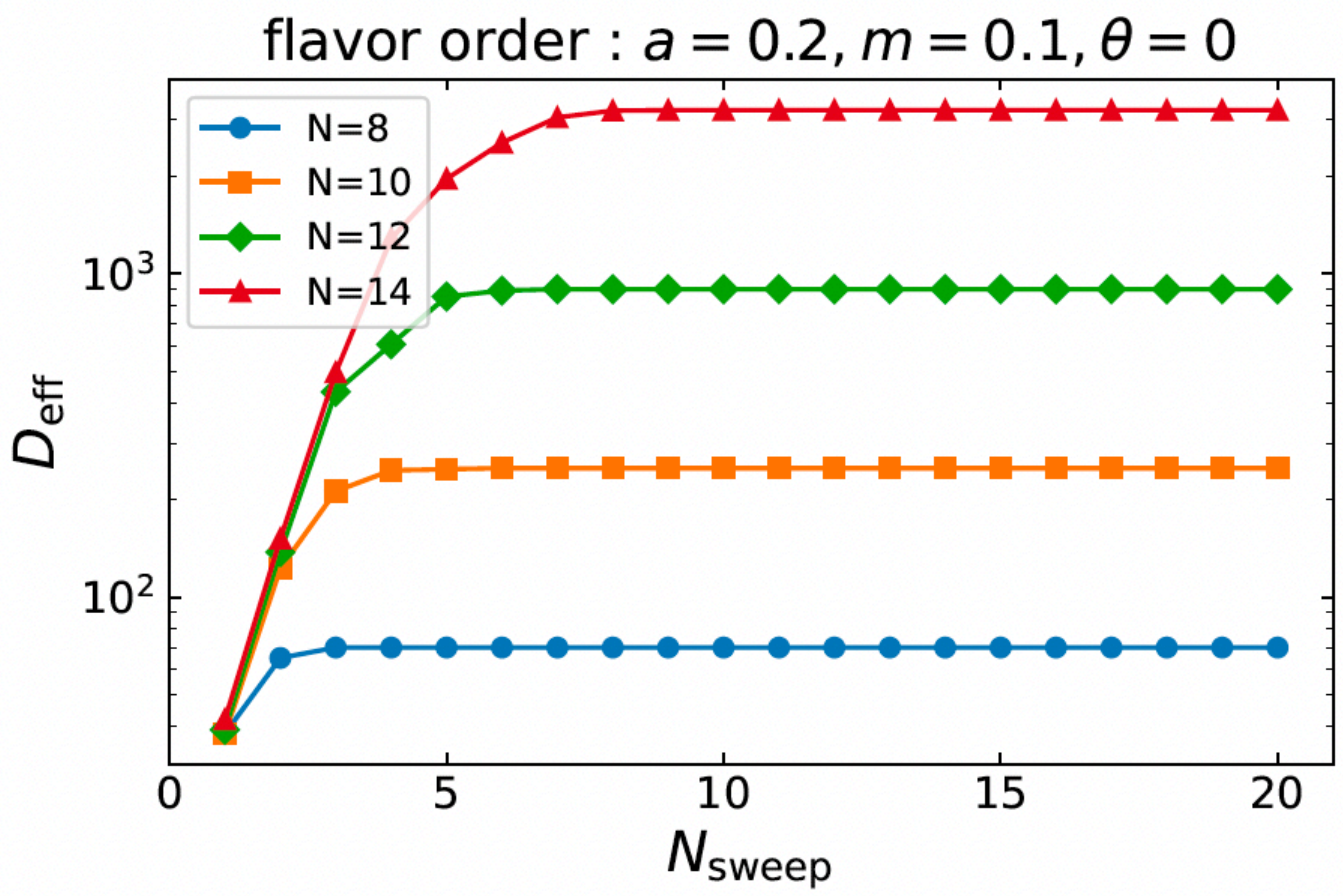

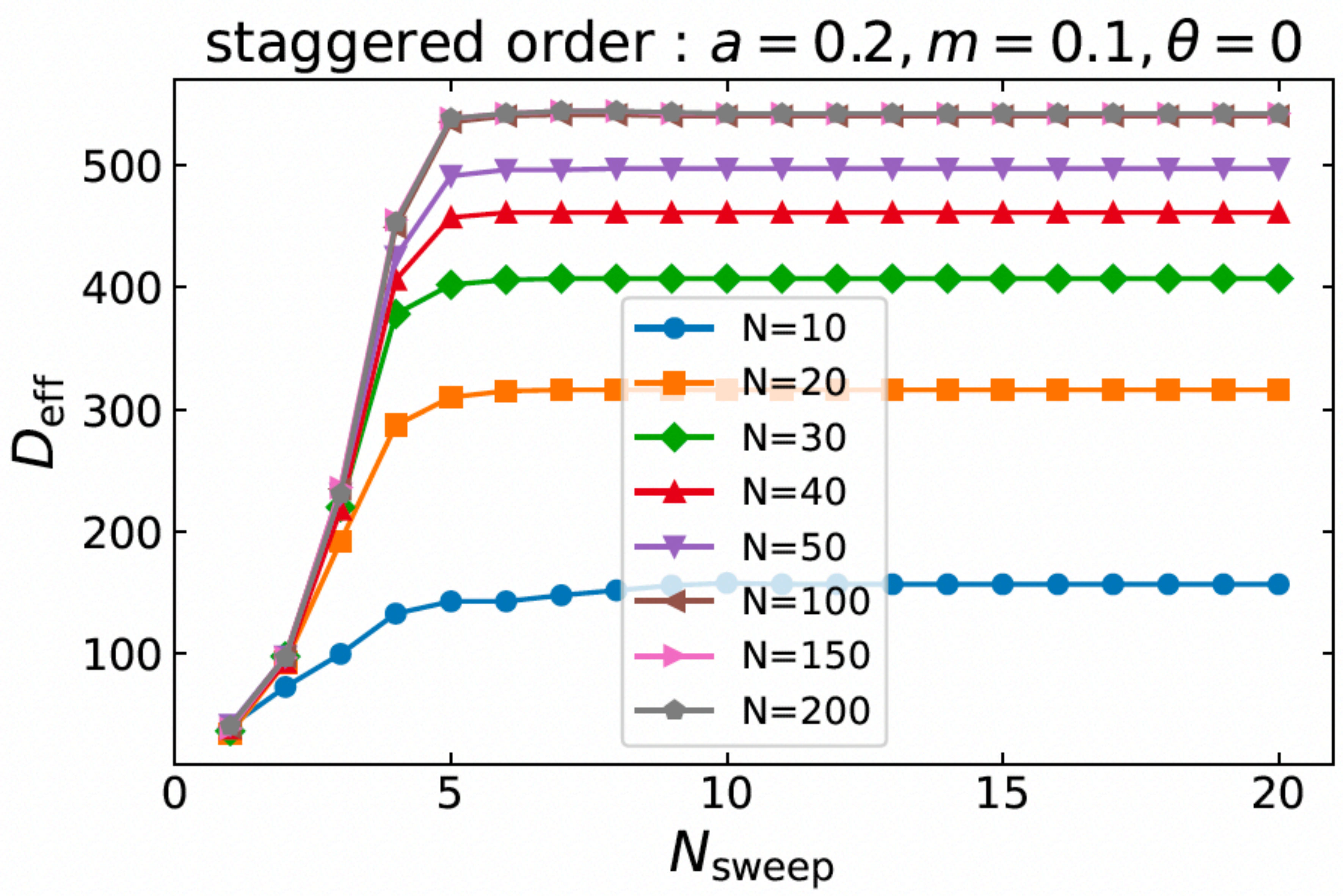

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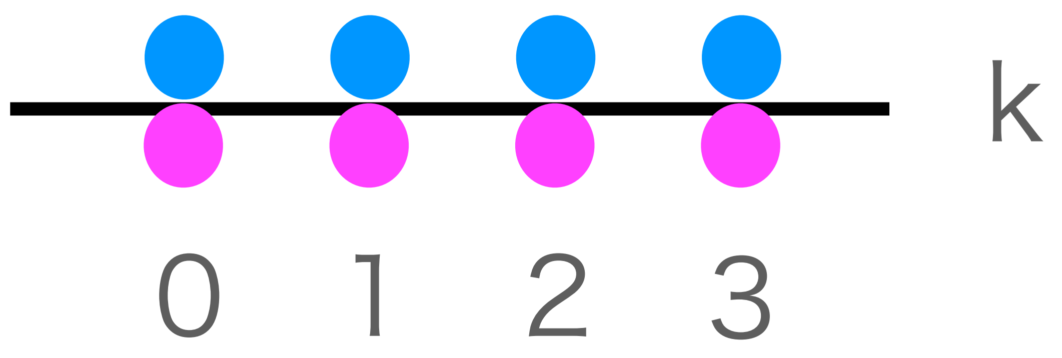

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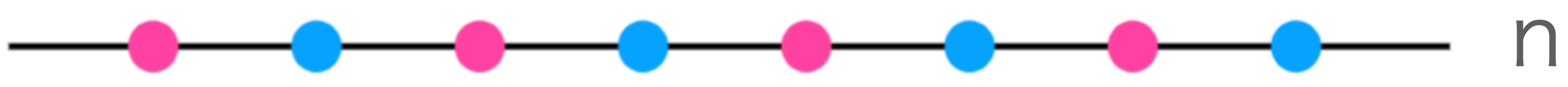


- lattice fermion -> spin variable (Jordan-Wigner trans.)

Conditions for Nf -fermion

$$\{\chi^\dagger_{f,n}, \chi_{\tilde{f},m}\} = \delta_{f,\tilde{f}}\delta_{n,m}$$
$$\{\chi_{f,n}, \chi_{\tilde{f},m}\} = \{\chi^\dagger_{f,n}, \chi^\dagger_{\tilde{f},m}\} = 0$$

our choice

$$\chi_{1,n} = \frac{\sigma^x_{1,n} - \sigma^y_{1,n}}{2}\prod_{j=0}^{n-1}(-\sigma^z_{2,j}\sigma^z_{1,j})$$
$$\chi_{2,n} = \frac{\sigma^x_{2,n} - \sigma^y_{2,n}}{2}(-i\sigma^z_{1,n})\prod_{j=0}^{n-1}(-\sigma^z_{2,j}\sigma^z_{1,j})$$

local op. (isospin and so on)
becomes only a few # of Pauli matrices

3. Mass spectra in the Hamiltonian formalism

JHEP11(2023)231

"Hadron" state in Nf=2 Schwinger model

- Prediction by analytical study (Coleman, 1976) at $\theta = 0$

(1)pion (Iso-triplet pseudo-scalar meson)

$$\pi = -i\left(\bar{\psi}_1\gamma^5\psi_1 - \bar{\psi}_2\gamma^5\psi_2\right)$$

$$J^{PG} = 1^{-+} \ (J_z = -1,0,1)$$

(2)sigma(Iso-singlet scalar meson)

$$\sigma = \bar{\psi}_1\psi_1 + \bar{\psi}_2\psi_2, \ J^{PG} = 0^{++}$$

(3)eta(Iso-singlet pseudo-scalar meson)

$$\eta = -i\left(\bar{\psi}_1\gamma^5\psi_1 + \bar{\psi}_2\gamma^5\psi_2\right), \ J^{PG} = 0^{--}$$

Quantum numbers:

$\mathbf{J}^2, J_z$ Isospin

associate with SU(2) flavor sym.

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \to \mathcal{U}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

P: Parity

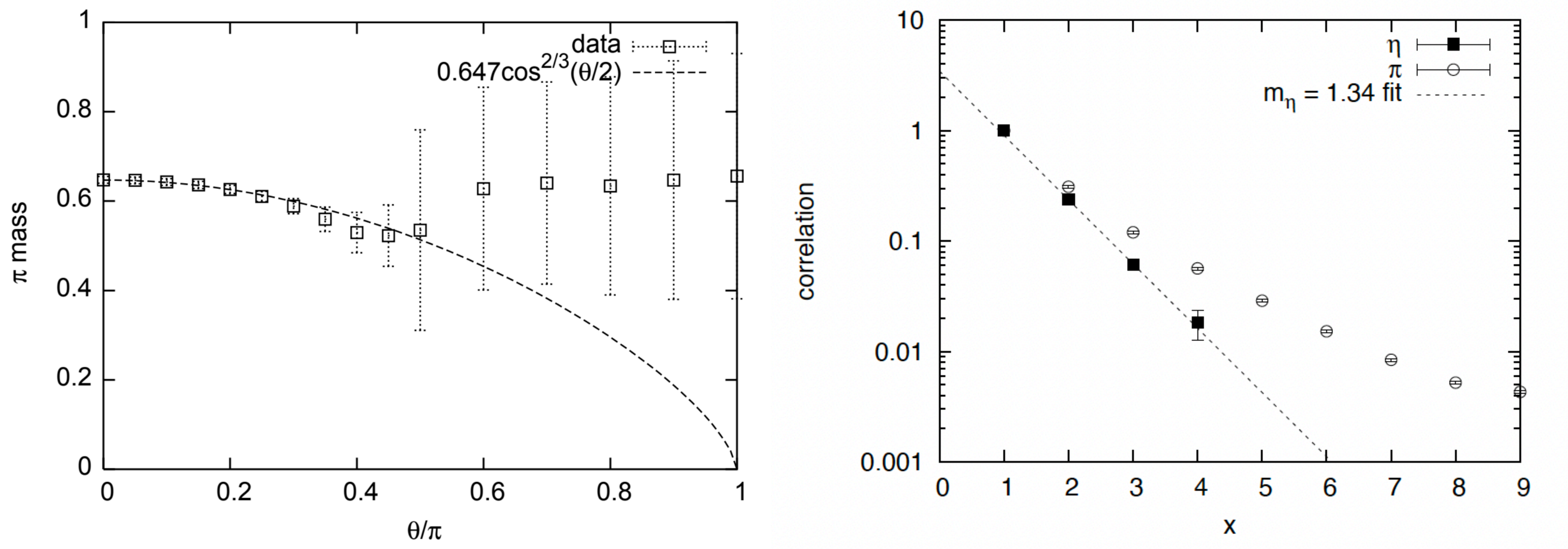
G-parity (generalized C.C.)

Monte Carlo result: Schwinger model + θ term

- Lagrangian

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta_0}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}\psi$$

non-zero θ_0: Sign problem in conventional method



Fukaya and Onogi
Phys.Rev. D68 (2003) 074503

- In large θ, the signal is very noisy because of the sign problem
- Difficult to find a heavy η-meson and σ-meson

Three calculation methods (at $\theta = 0$)

(1) (Spatial) correlation-function scheme (conventional method)

$$C(\tau) = \langle O(\tau)O(0)\rangle, \ \lim_{\tau\to\infty} C(\tau) \sim e^{-m\tau}$$

(In $a \to 0$ and $N \to \infty$ since H formula breaks Lorentz sym.)

(2) One-point-function scheme

OBC = Wall source

one-point fn. = correlation fn. with source state

(SPT phase, at $\theta = 0$ iso-singlet state / at $\theta = 2\pi$ iso-triplet state)

(3) Dispersion-relation scheme

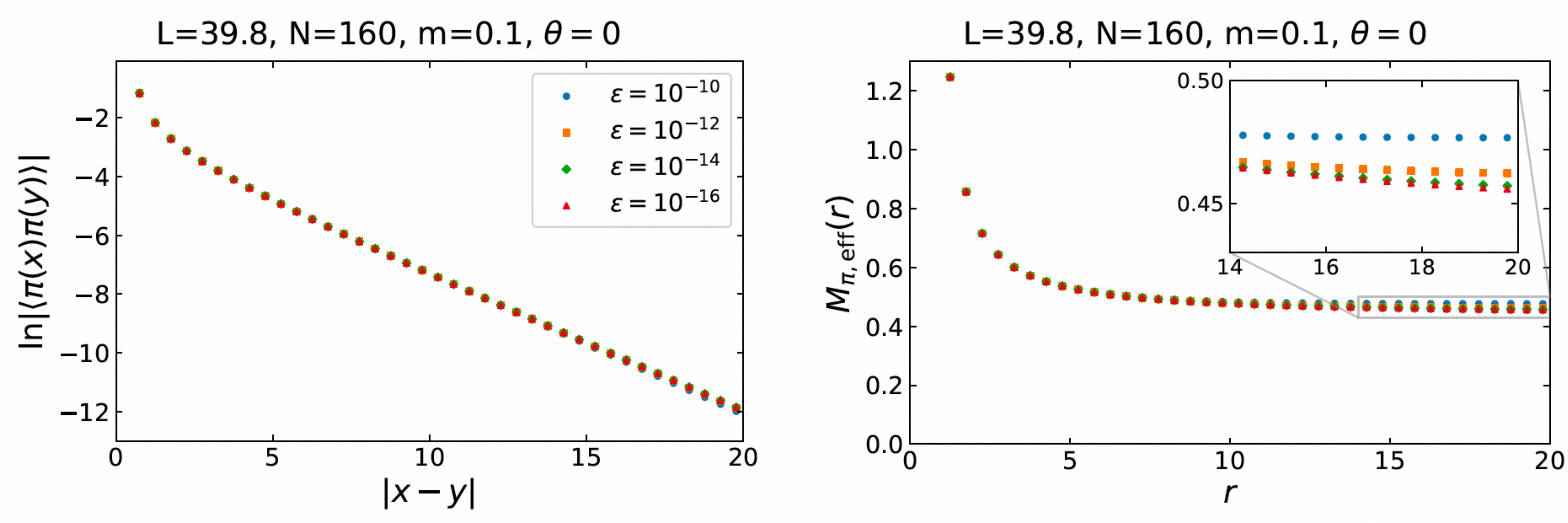
Construct excited states and measure energy, momentum and quantum numbers

(1) (Spatial) correlation-function scheme

Effective mass

log plot of $C_\pi(r) = \langle \pi(r)\pi(0) \rangle$

$$\tilde{M}_{\pi,\mathrm{eff}}(r) = -\frac{1}{2a} \log \frac{C_\pi(r+2a)}{C_\pi(r)}$$



Plateau of effective mass = pion mass ??

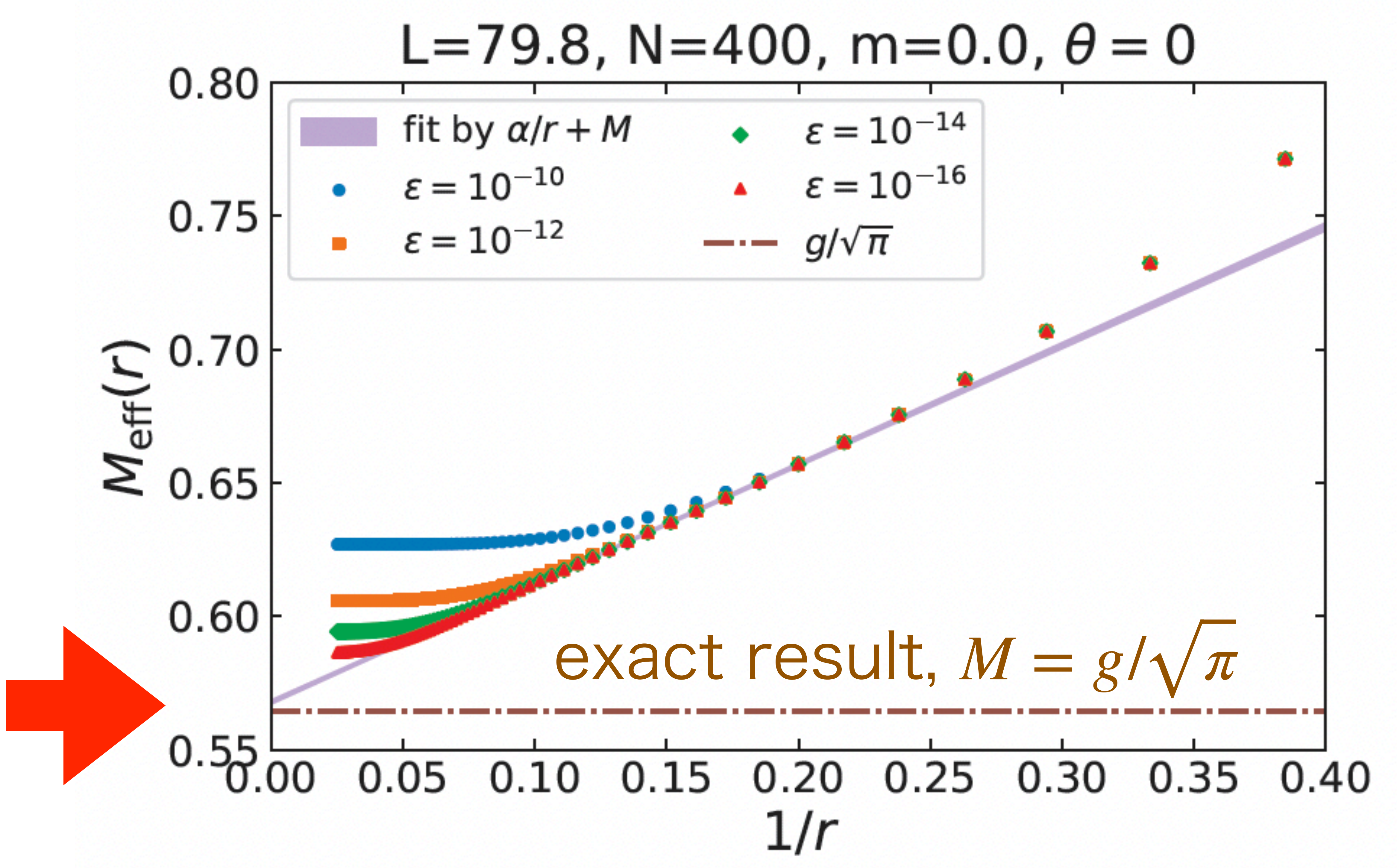
High precision calculation shows a slope....

What's happen??

(1) Test calc. for Nf=1 massless fermion case

(1+1)d. point-point correlation fn. has Yukawa-shape

$$\langle \pi(x,t)\pi(y,t)\rangle \propto K_0(Mr) \sim \frac{1}{\sqrt{Mr}}e^{-Mr}$$ here $\pi = -i\bar{\psi}\gamma^5\psi$ for Nf=1



Effective mass has power correction:

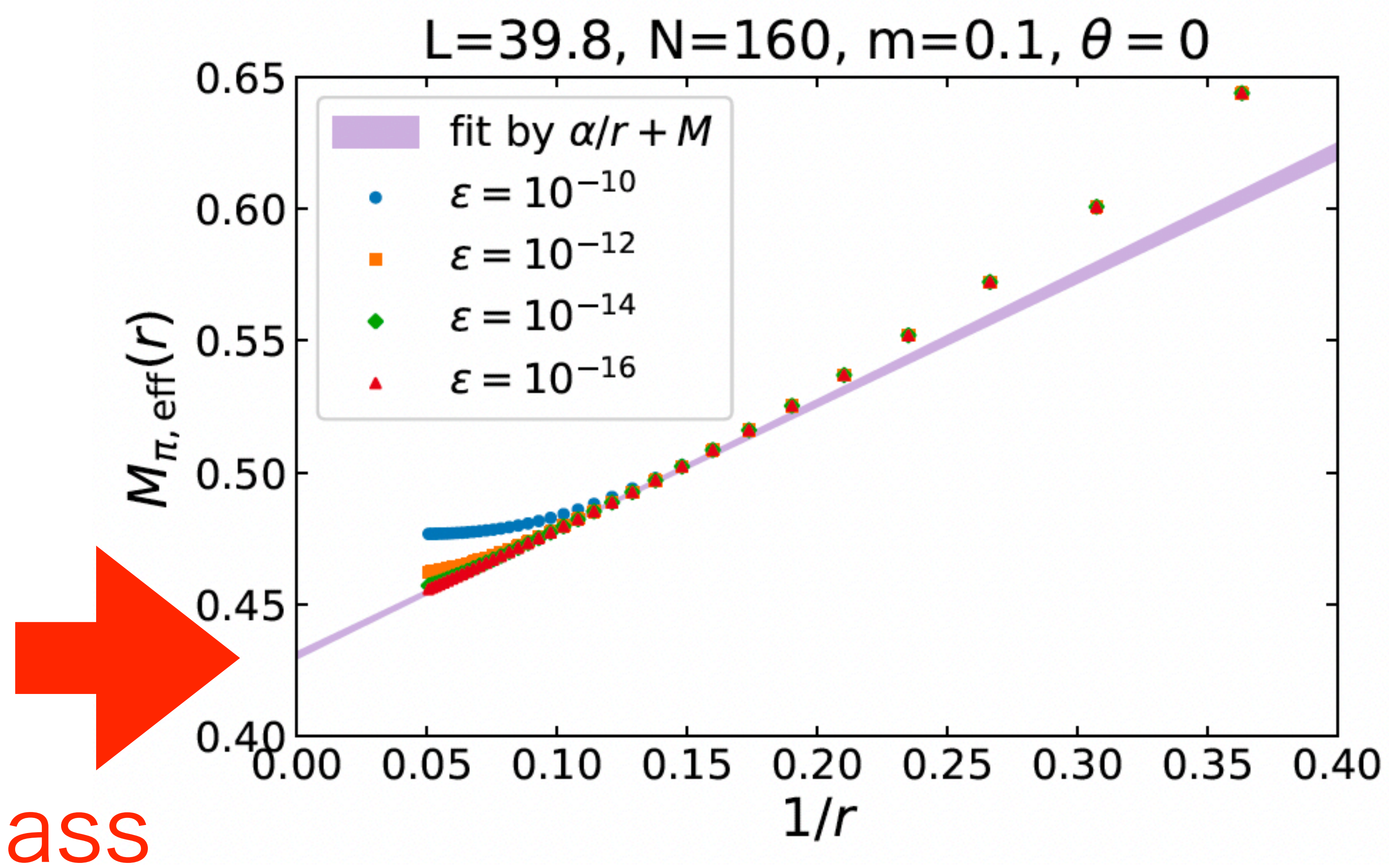
$$M_{\rm eff}(r) = -\frac{d}{dr}\log K_0(Mr) \sim \frac{1}{2r} + M$$

In $r \to \infty$ limit, obtained M is almost consistent with the exact result

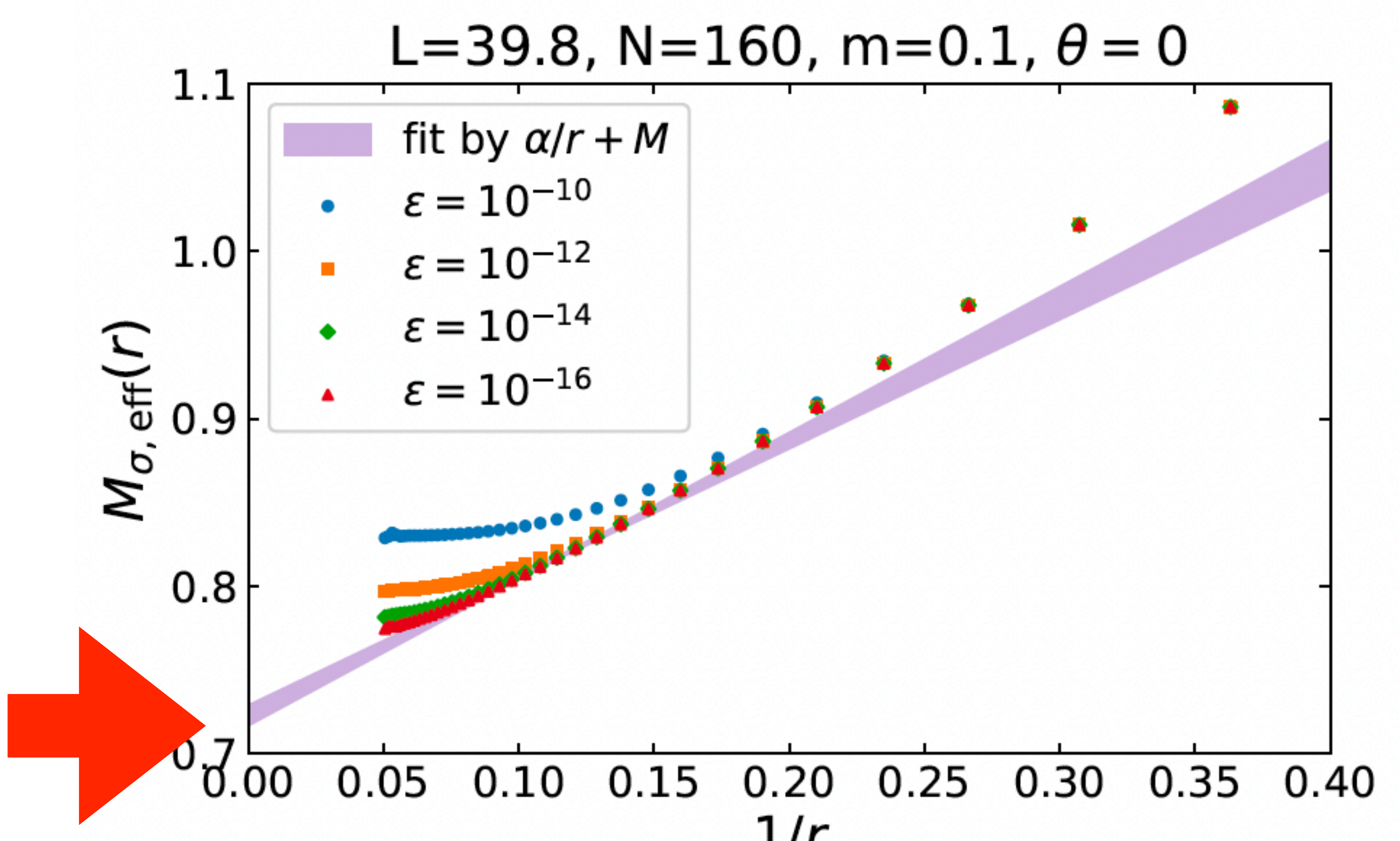
Why the convergence is slow?

=> DMRG can calculate exponential correlations and difficult to reproduce 1/r

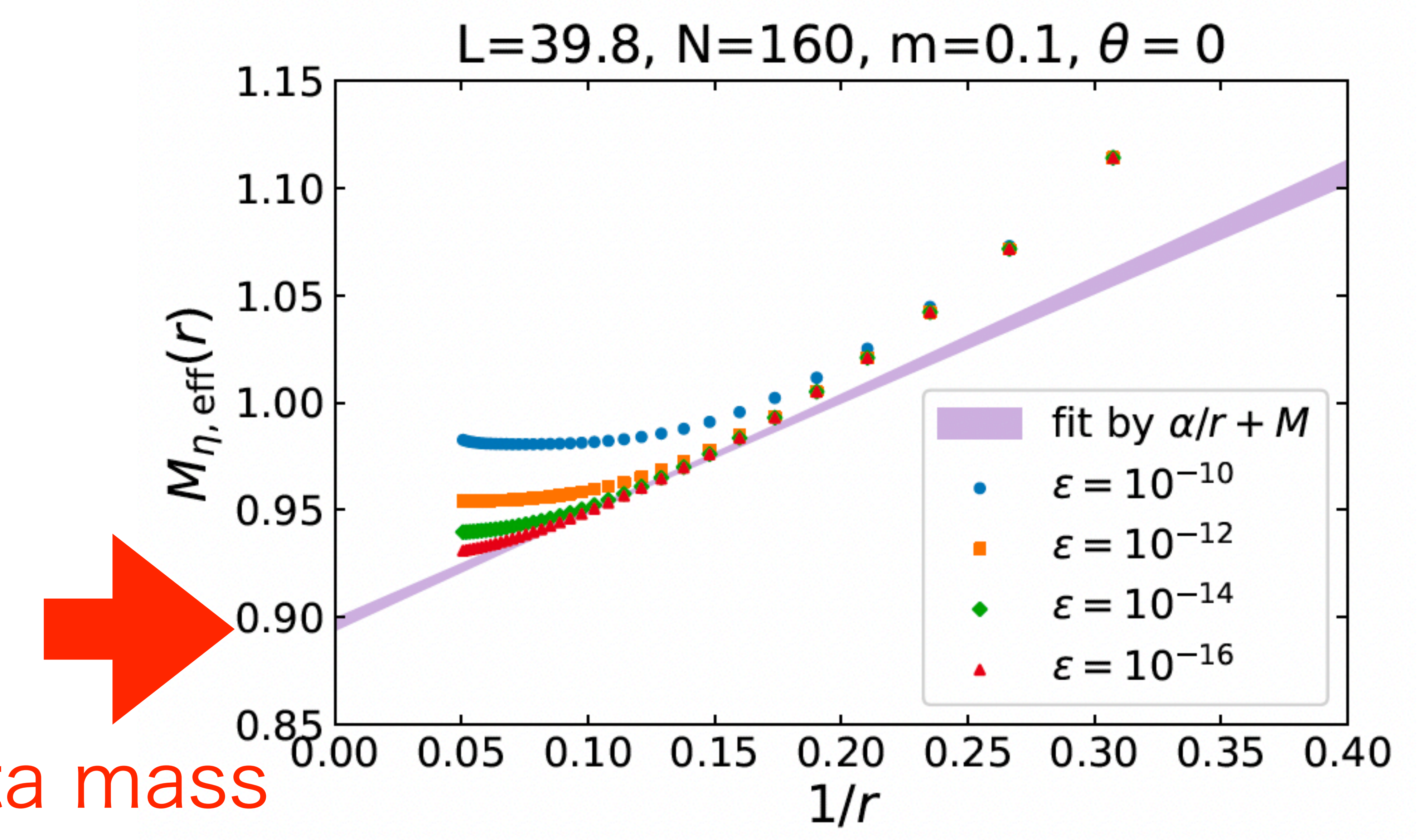
(1) Effective mass with a 1/r correction



pion mass



sigma mass



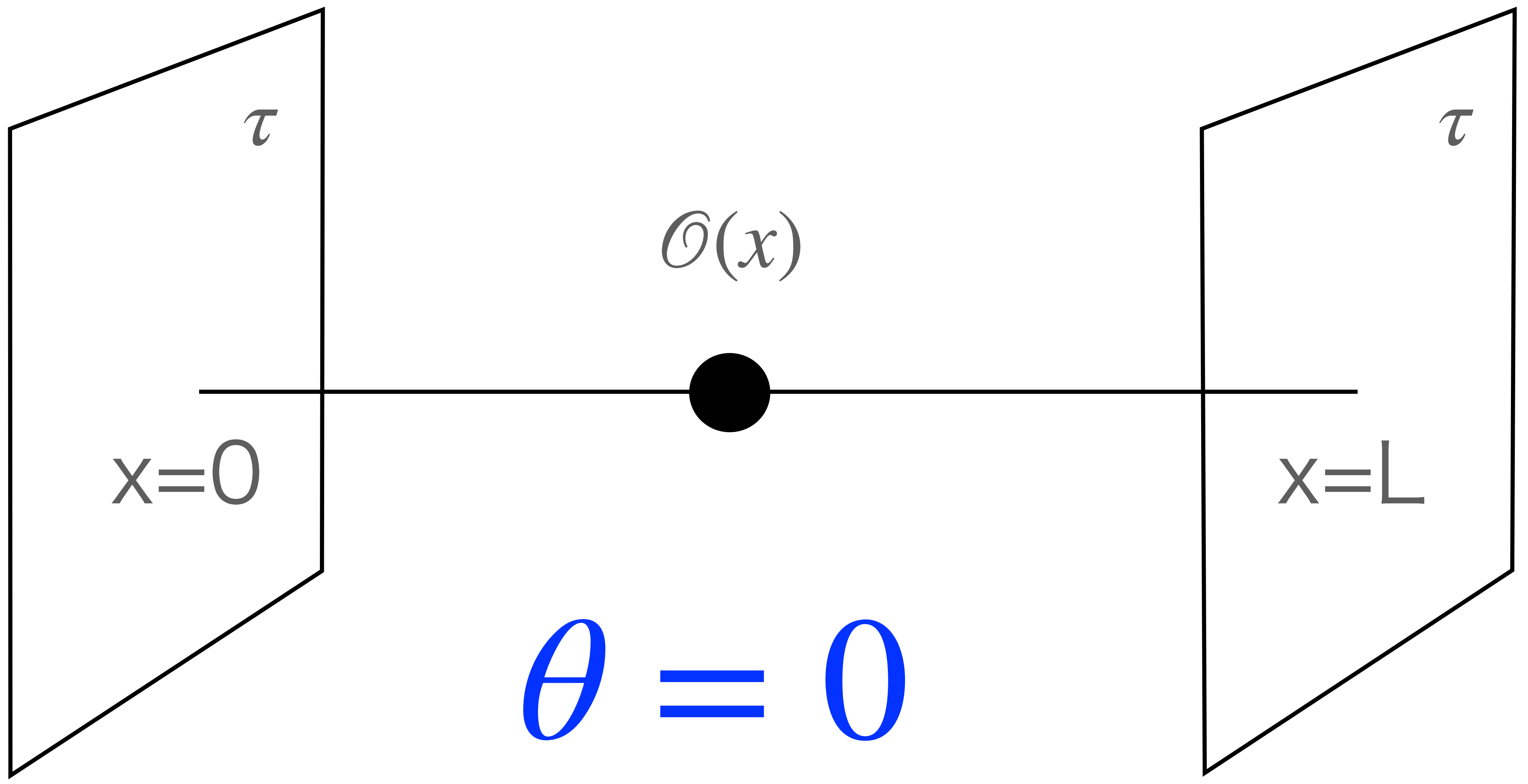
eta mass

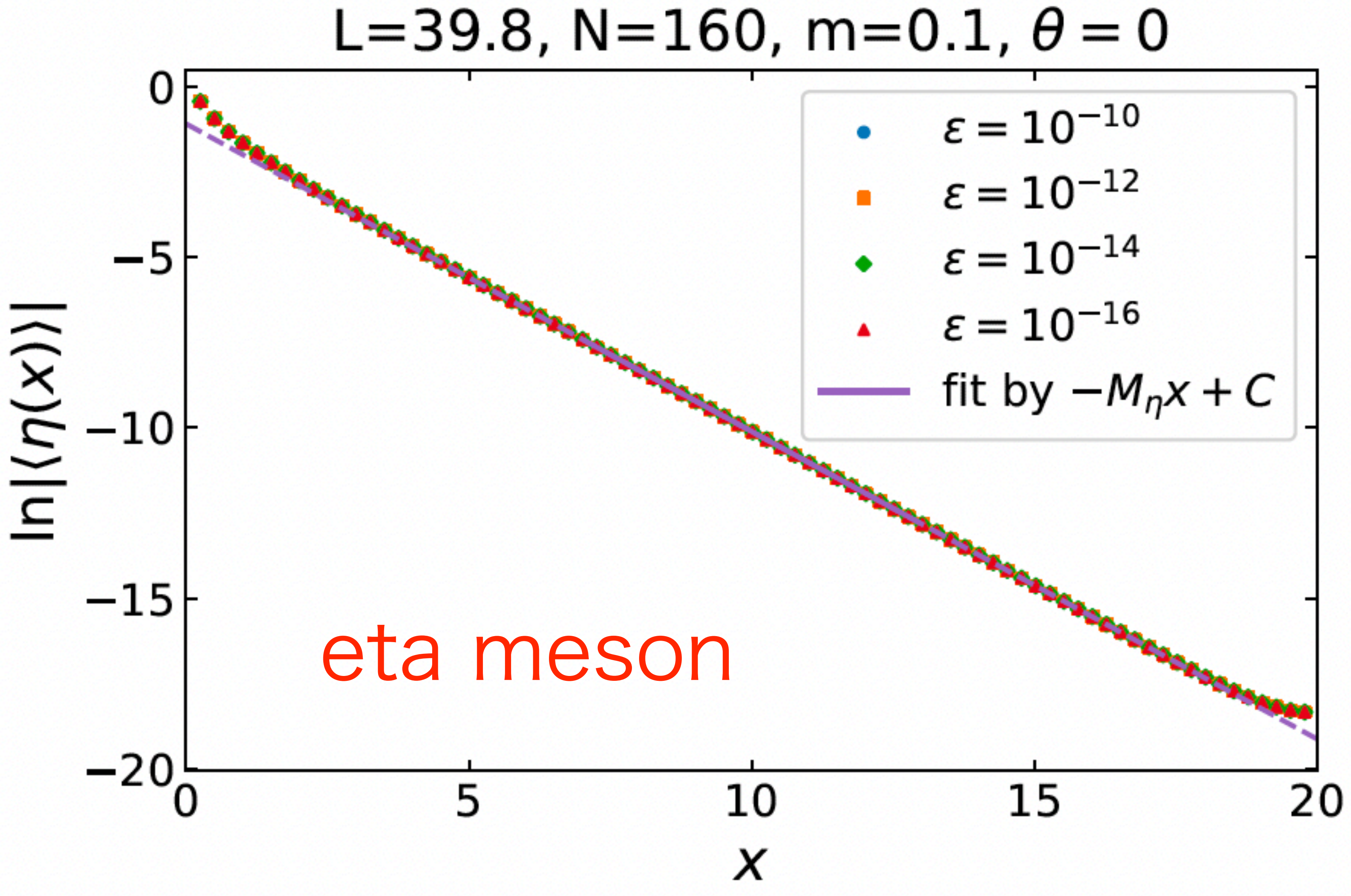
(2) One-point-function scheme

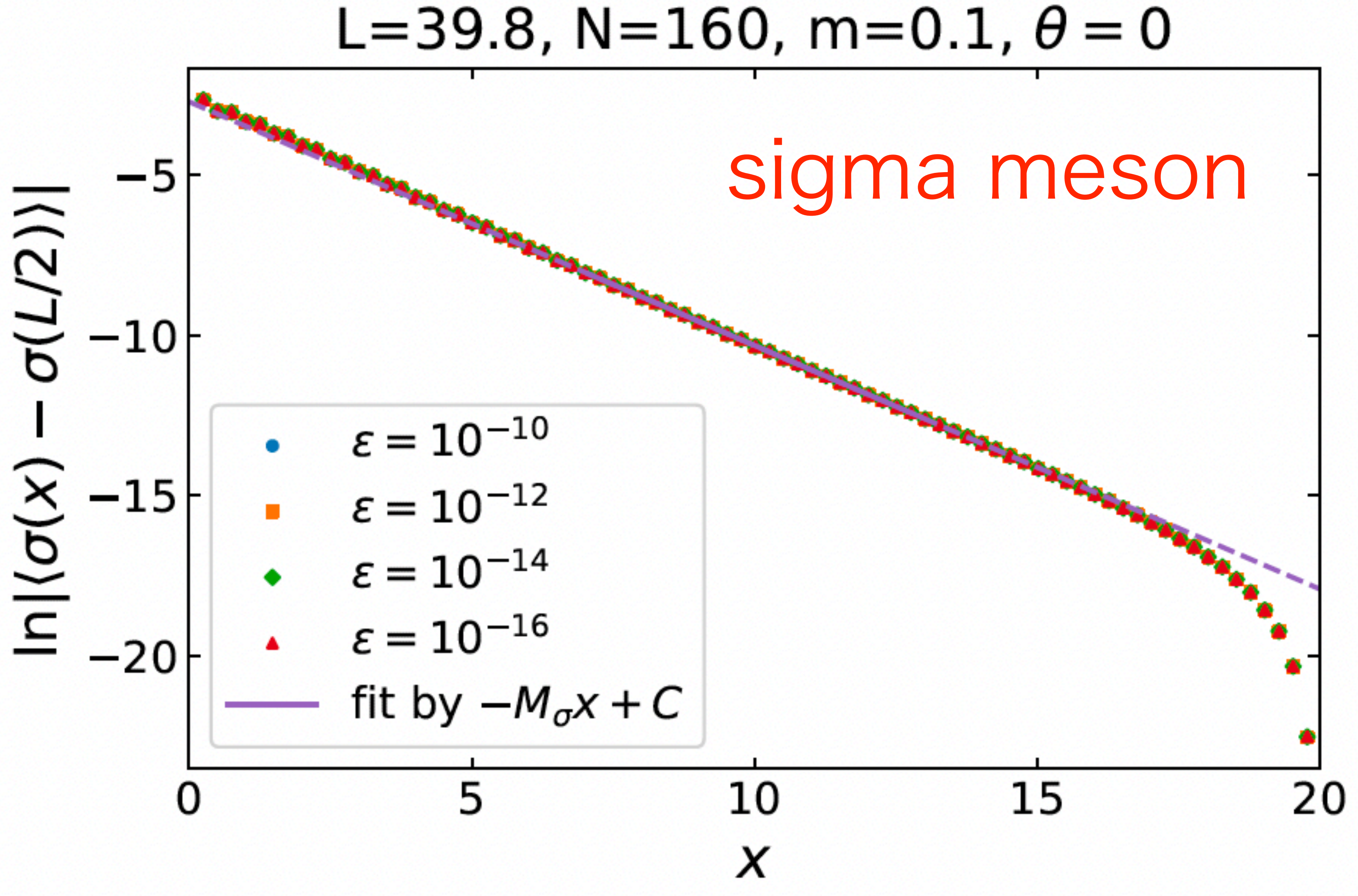
Calculate $\langle \mathcal{O}(x) \rangle$

$$\sum_{\tau} \langle \mathcal{O}(x,\tau) \mathcal{O}_{wall}(x=0) \rangle \equiv \langle \text{Vac.} \,|\, \mathcal{O}(x) \,|\, \text{Bdry} \rangle \sim e^{-Mx}$$

Wall-point correlation function



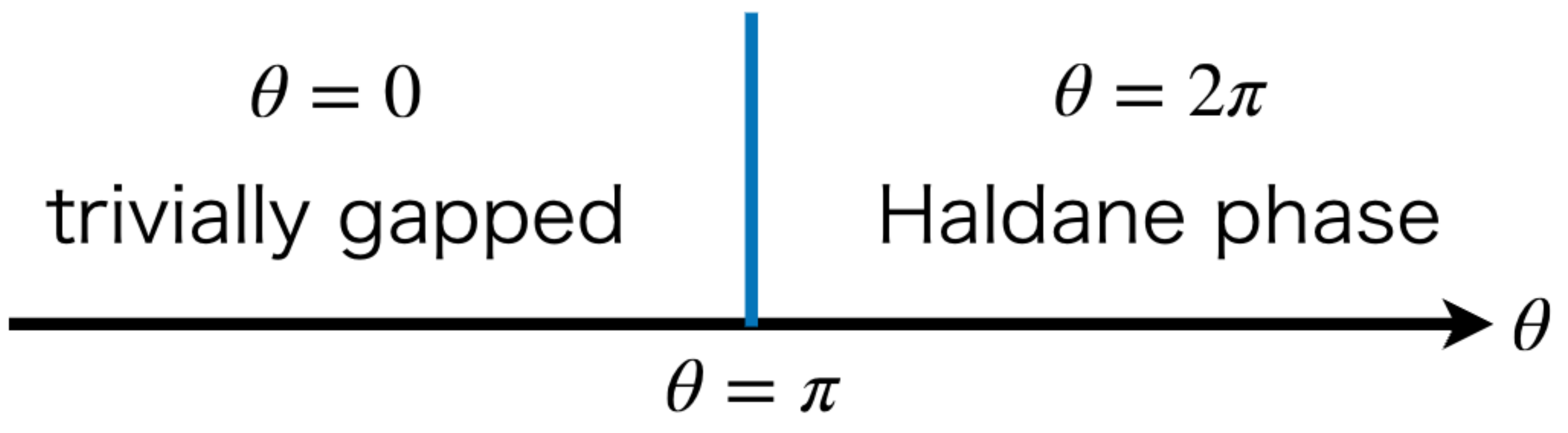




precision-dependence is not observed

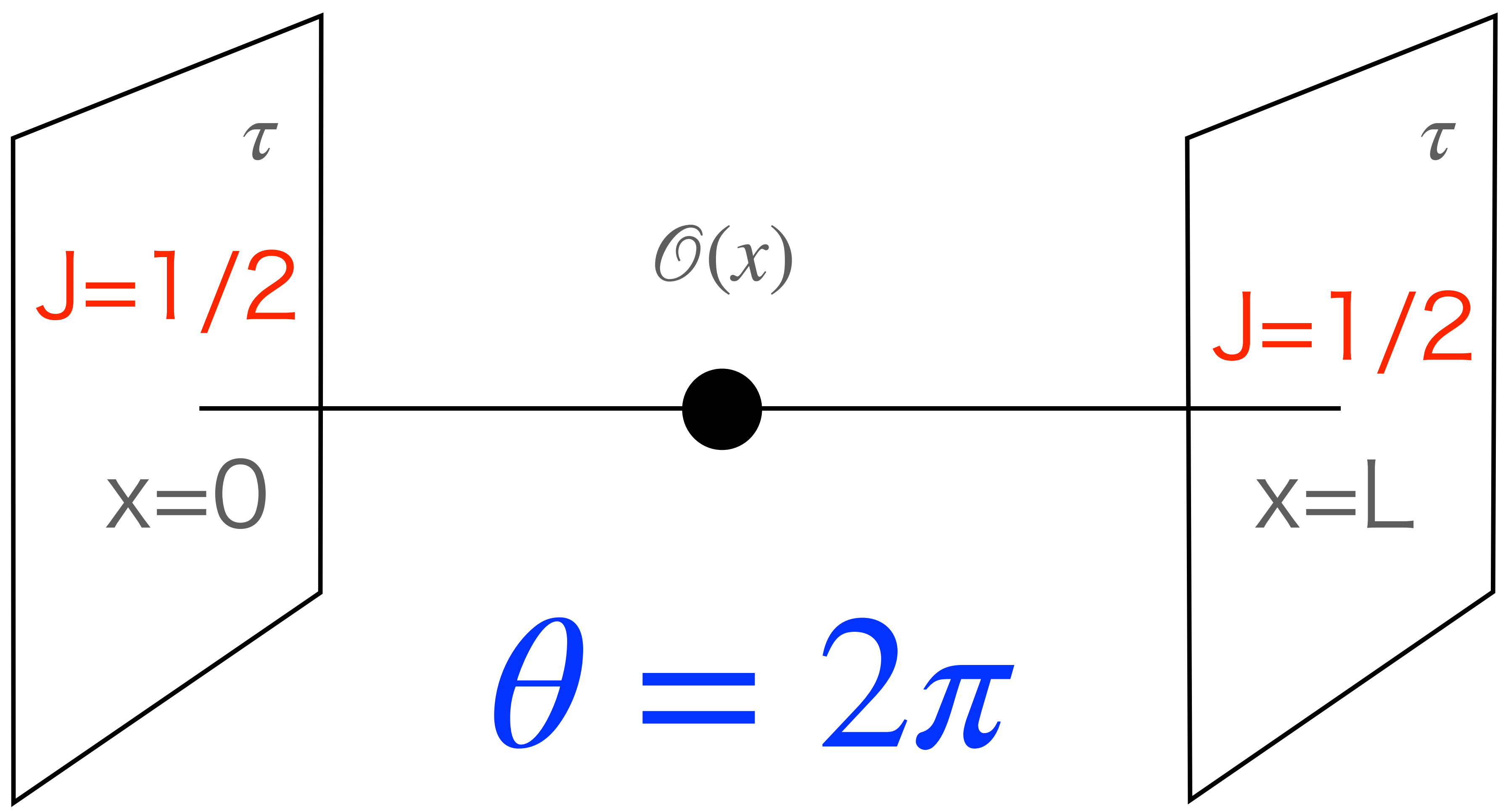
(2) One-point-function scheme：pion

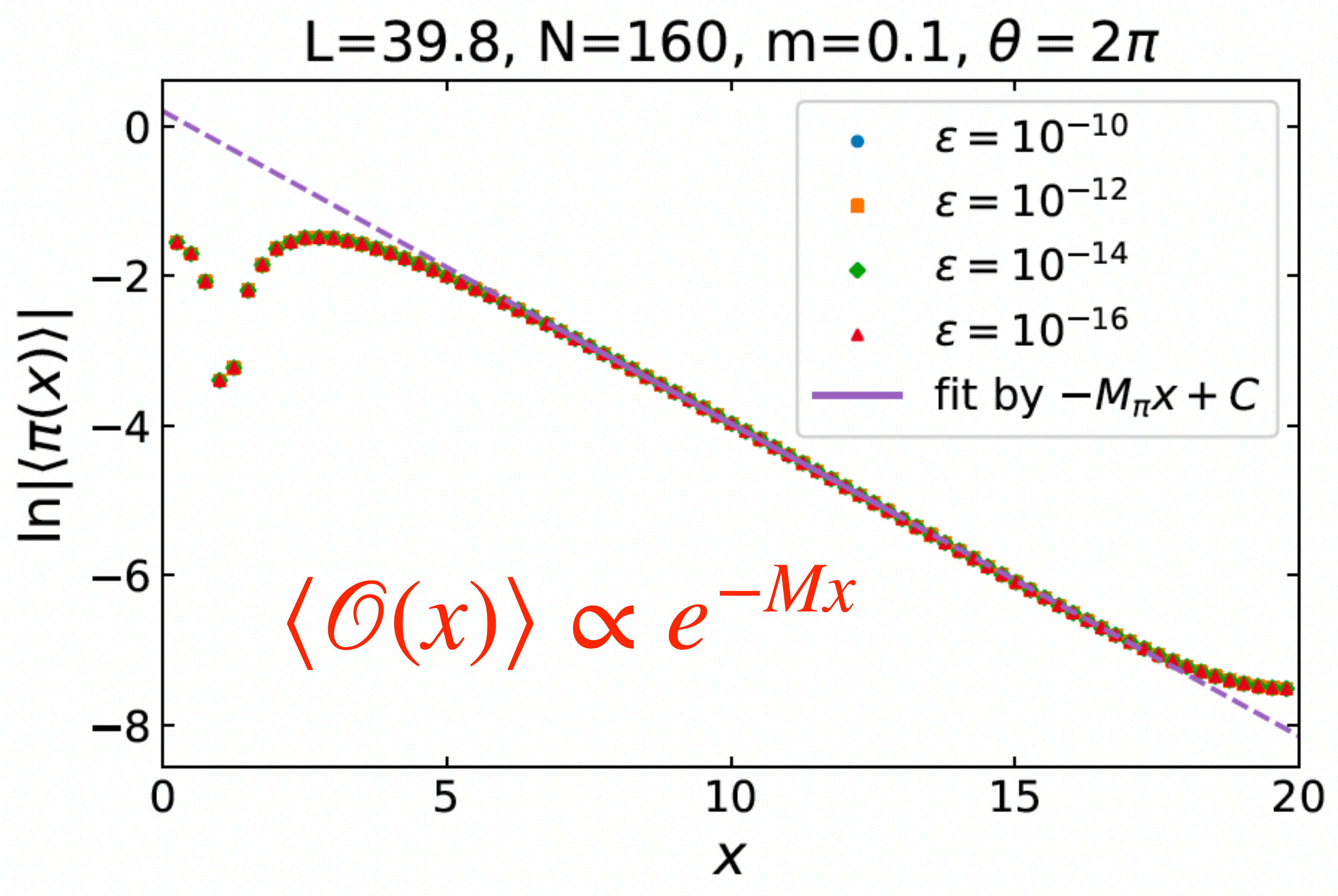
$\langle \pi(x) \rangle = 0$ everywhere, since the ground state is iso-singlet at $\theta = 0$



Haldane phase -> edge mode in OBC

isospin =1/2 at both edges = source of iso-triplet

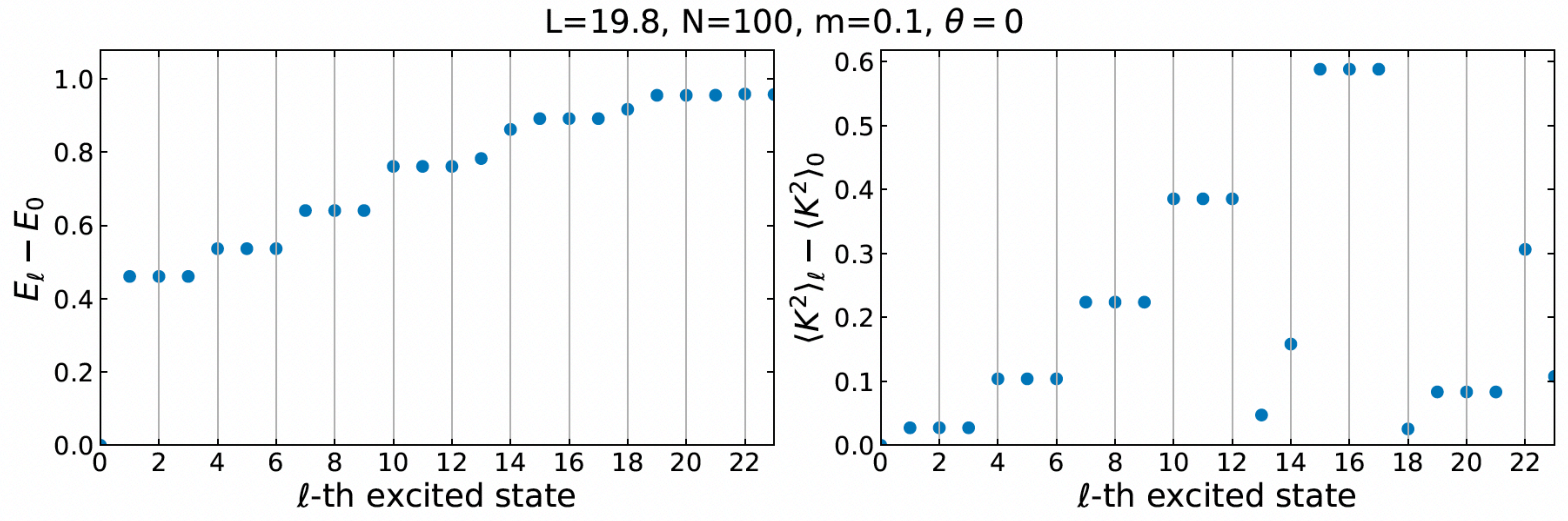




(3) dispersion-relation scheme

MPS for ℓ-th excited state is given by the modified cost fn.: $H_{eff} = H + \lambda \sum_{k=0}^{\ell-1} |\psi_k\rangle\langle\psi_k|$

Upto 20-th excited state



Measure the quantum number (Iso-spin, G-parity, Parity) of generated MPS to identify each meson

(3) Momentum op. and Quantum number op.

- Momentum op.(flavor-dependent, $[\hat{k}_f, H] \neq 0$)

1st flavor
$$\hat{k}_{1,n} = \frac{i}{4a}(S^-_{1,n-1}Z_{2,n-1}Z_{1,n}Z_{2,n}S^+_{1,n+1} - S^+_{1,n-1}Z_{2,n-1}Z_{1,n}Z_{2,n}S^-_{1,n+1}),$$

2nd flavor
$$\hat{k}_{2,n} = \frac{i}{4a}(S^-_{2,n-1}Z_{1,n}Z_{2,n}Z_{1,n+1}S^+_{2,n+1} - S^+_{2,n-1}Z_{1,n}Z_{2,n}Z_{1,n+1}S^-_{2,n+1}).$$

- Isospin operator (flavor SU(2) sym.), $\mathbf{J}^2, J_z$

$$[H,\, J_z] = 0$$

$$J_z = \sum_{n=0}^{N-1} j_z(n) = \frac{1}{2}\sum_{n=0}^{N-1}\left(\chi^\dagger_{1,n}\chi_{1,n} - \chi^\dagger_{2,n}\chi_{2,n}\right) = \frac{1}{4}\sum_{n=0}^{N-1}(Z_{1,n} - Z_{2,n})$$

$$[H,\, \mathbf{J}^2] = \left[H,\, \left(\frac{1}{2}J_+J_- + \frac{1}{2}J_+J_- + J_z^2\right)\right] = 0$$

(3) Quantum number op.

- Charge conjugation (broken due to OBC and finite lattice spacing)

$$C := \prod_{f=1}^{N_f} \left(\prod_{n=0}^{N-1} \sigma^x_{f,n} \right) \left(\prod_{n=0}^{N-2} (\mathrm{SWAP})_{f;N-2-n,N-1-n} \right)$$

- Parity (broken due to OBC, N=even)

$$P := \prod_{f=1}^{N_f} \left(\prod_{j=0}^{N/2-1} \sigma^z_{f,2j+1} \right)$$
$$\times \left(\prod_{n=0}^{N-2} (\mathrm{SWAP})_{f;N-2-n,N-1-n} \right) \left(\prod_{n=0}^{N/2-1} (\mathrm{SWAP})_{f;n,N-1-n} \right)$$

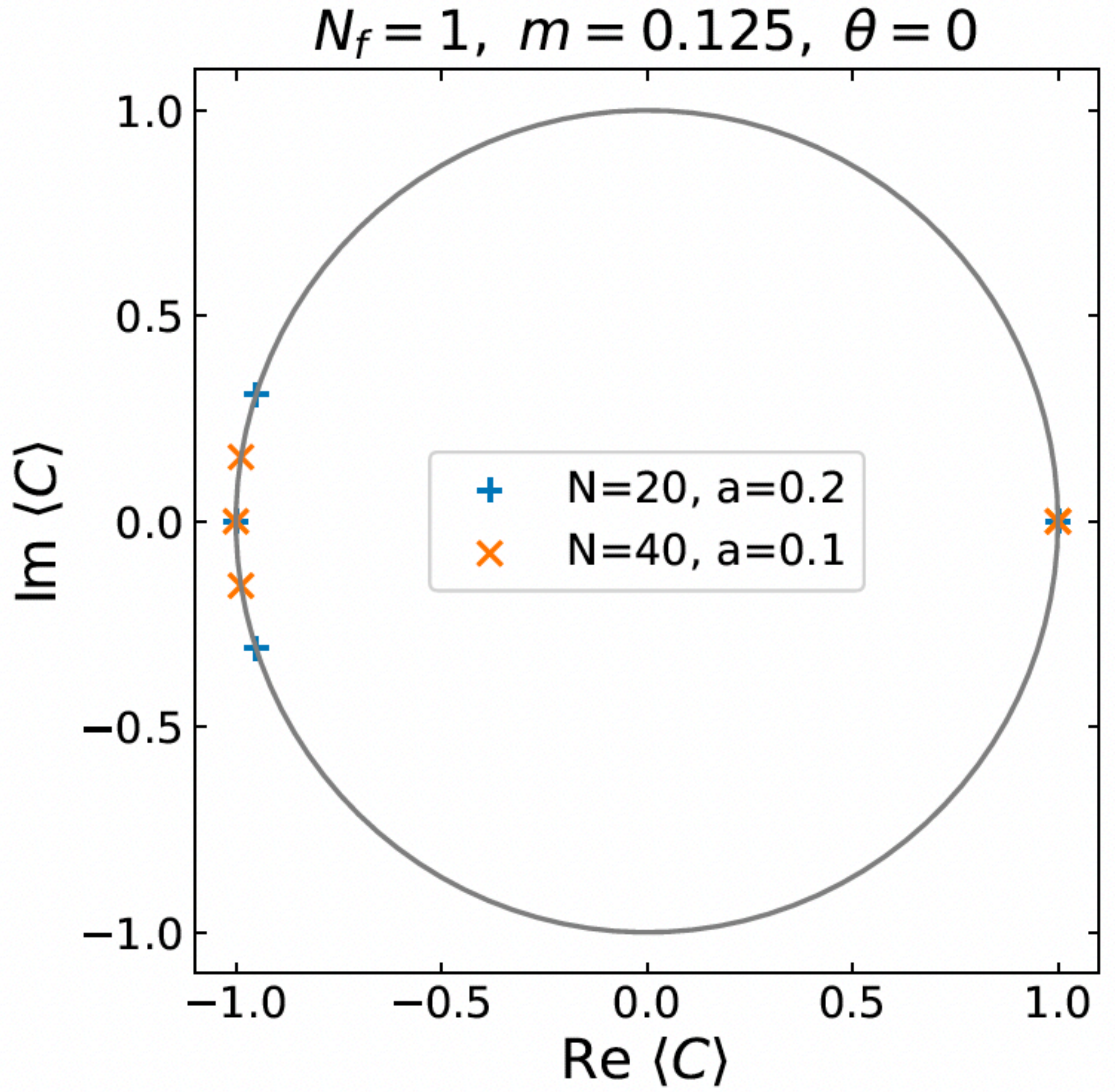
1 site translation

x <->L-x

p <-> ap flip

- G-Parity (commute with iso-spin)

$$G := Ce^{i\pi J_y},$$



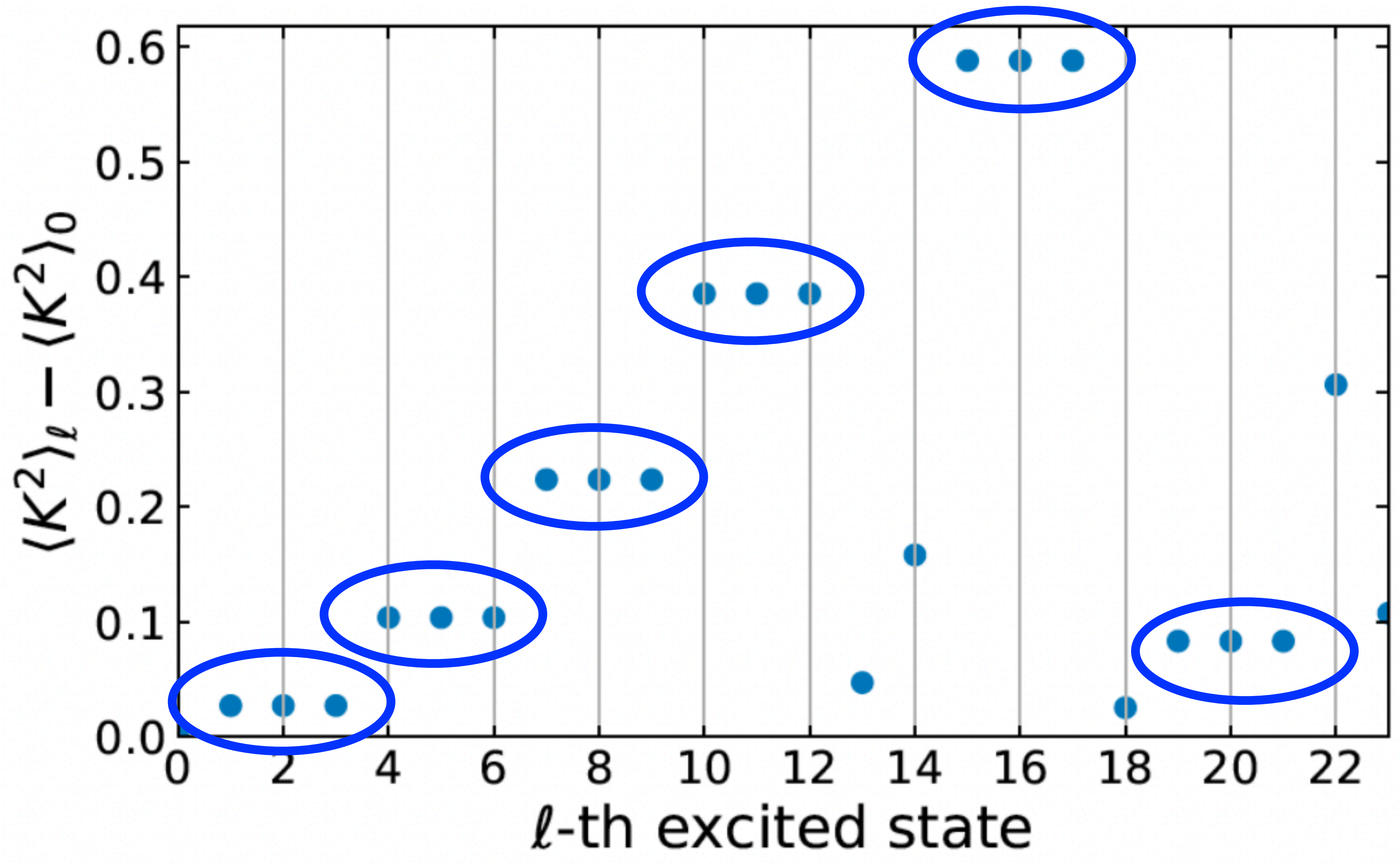
Free theory w/ PBC

In cont. lim., $\langle C \rangle = \pm 1$

the sign of Re$\langle C \rangle$ is

a remnant of exact C

(3) Results: iso-triplet channel



ℓ	$\boldsymbol{J}^2$	J_z	G	P
1	2.00000004	0.99999997	0.27872443	-6.819×10^{-8}
2	2.00000012	-0.00000000	0.27872416	-6.819×10^{-8}
3	2.00000004	-0.99999996	0.27872443	-6.819×10^{-8}
4	2.00000007	0.99999999	0.27736066	7.850×10^{-8}
5	2.00000006	0.00000000	0.27736104	7.850×10^{-8}
6	2.00000009	-0.99999998	0.27736066	7.850×10^{-8}
7	2.00000010	1.00000000	0.27536687	-8.838×10^{-8}
8	2.00000002	0.00000000	0.27536702	-8.837×10^{-8}
9	2.00000007	-0.99999998	0.27536687	-8.838×10^{-8}
10	2.00000007	0.99999998	0.27356274	9.856×10^{-8}
11	2.00000005	0.00000001	0.27356277	9.856×10^{-8}
12	2.00000007	-0.99999999	0.27356274	9.856×10^{-8}
15	1.99999942	0.99999966	0.27173470	-1.077×10^{-7}
16	2.00000052	0.00000000	0.27173482	-1.077×10^{-7}
17	2.00000015	-1.00000003	0.27173470	-1.077×10^{-7}
19	2.00009067	1.00004377	0.27717104	-3.022×10^{-8}
20	2.00002578	-0.00000004	0.27717020	-3.023×10^{-8}
21	2.00003465	-1.00001622	0.27717104	-3.023×10^{-8}

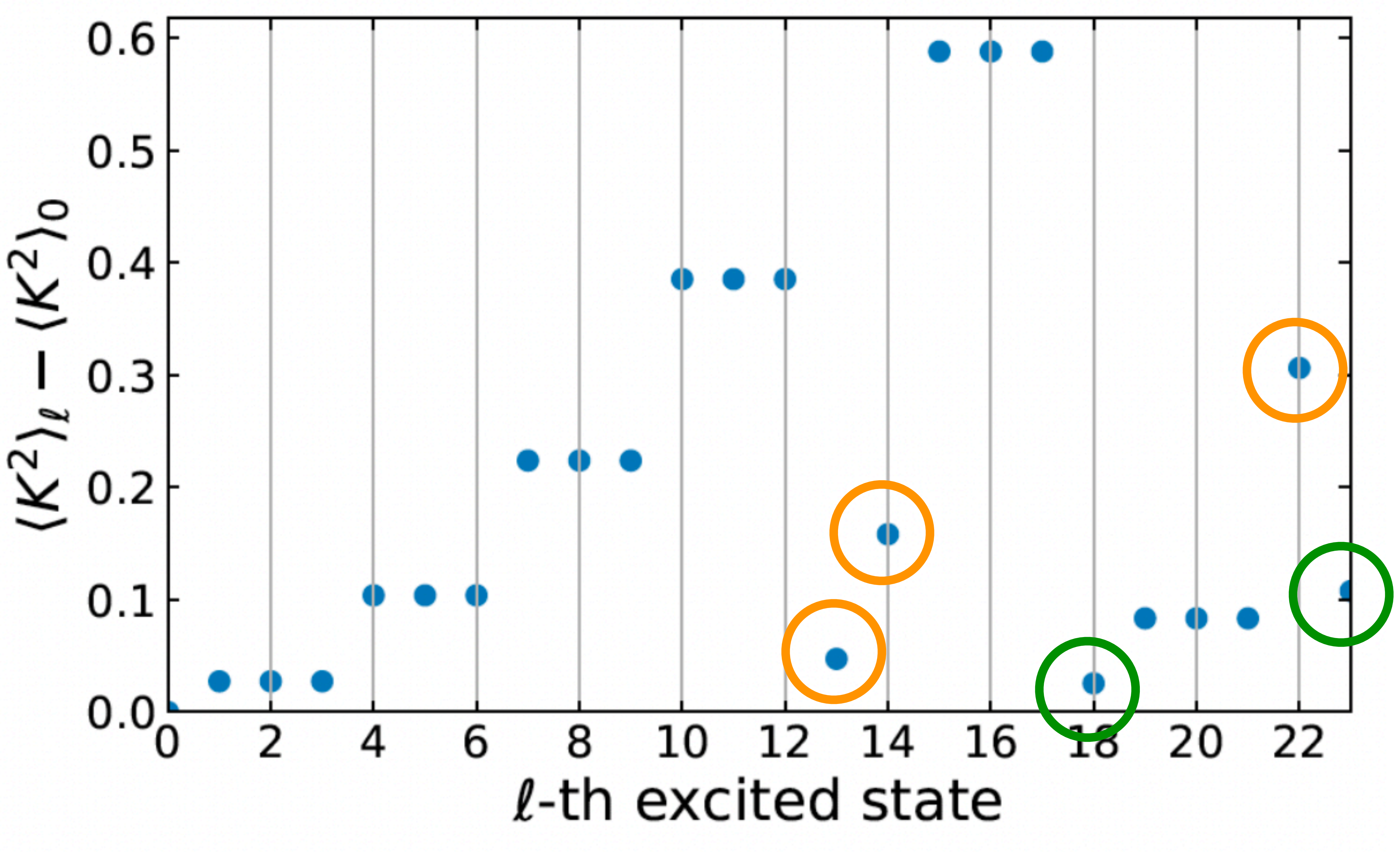
zero-mode

$P < 0$

J=1 $\quad J_z = \pm 1 \quad G > 0 \quad P < 0$

pion : $J^{PG} = 1^{-+}$

(3) Results: iso-singlet channel



ℓ	$\boldsymbol{J}^2$	J_z	G	P
0	0.00000003	-0.00000000	0.27984227	3.896×10^{-7}
13	0.00000003	0.00000000	0.27865844	1.273×10^{-7}
14	0.00000003	0.00000000	0.27508176	-2.765×10^{-8}
18	0.00000028	0.00000006	-0.27390909	-6.372×10^{-7}
22	0.00001537	0.00000115	0.26678987	7.990×10^{-8}
23	0.00003607	-0.00000482	-0.27664779	5.715×10^{-7}

zero-mode

$P > 0$

zero-mode

$P < 0$

J=0 $J_z = 0$ $G > 0$ $P > 0$

sigma meson : $J^{PG} = 0^{++}$

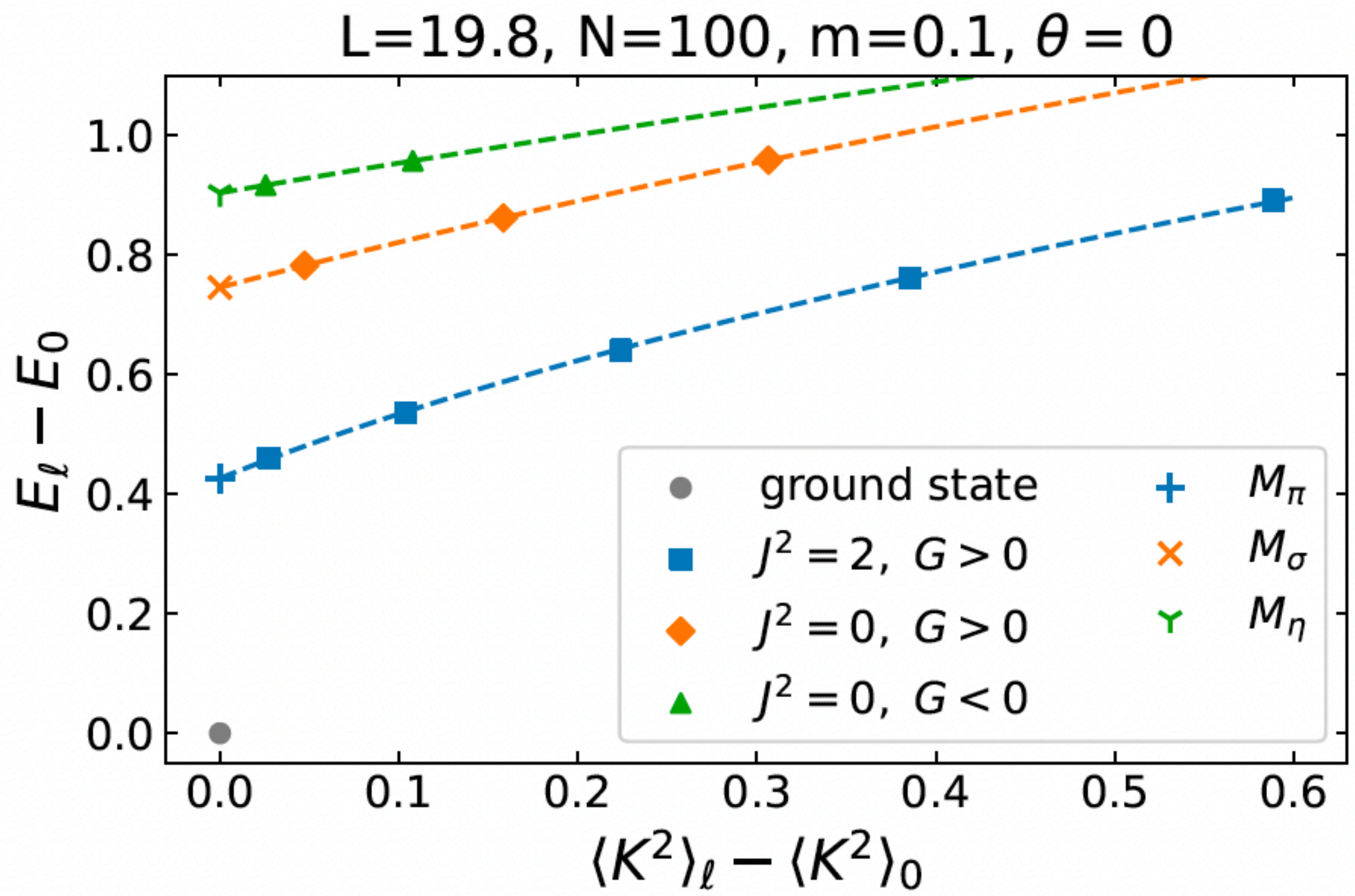
J=0 $J_z = 0$ $G < 0$ $P < 0$

eta meson : $J^{PG} = 0^{--}$

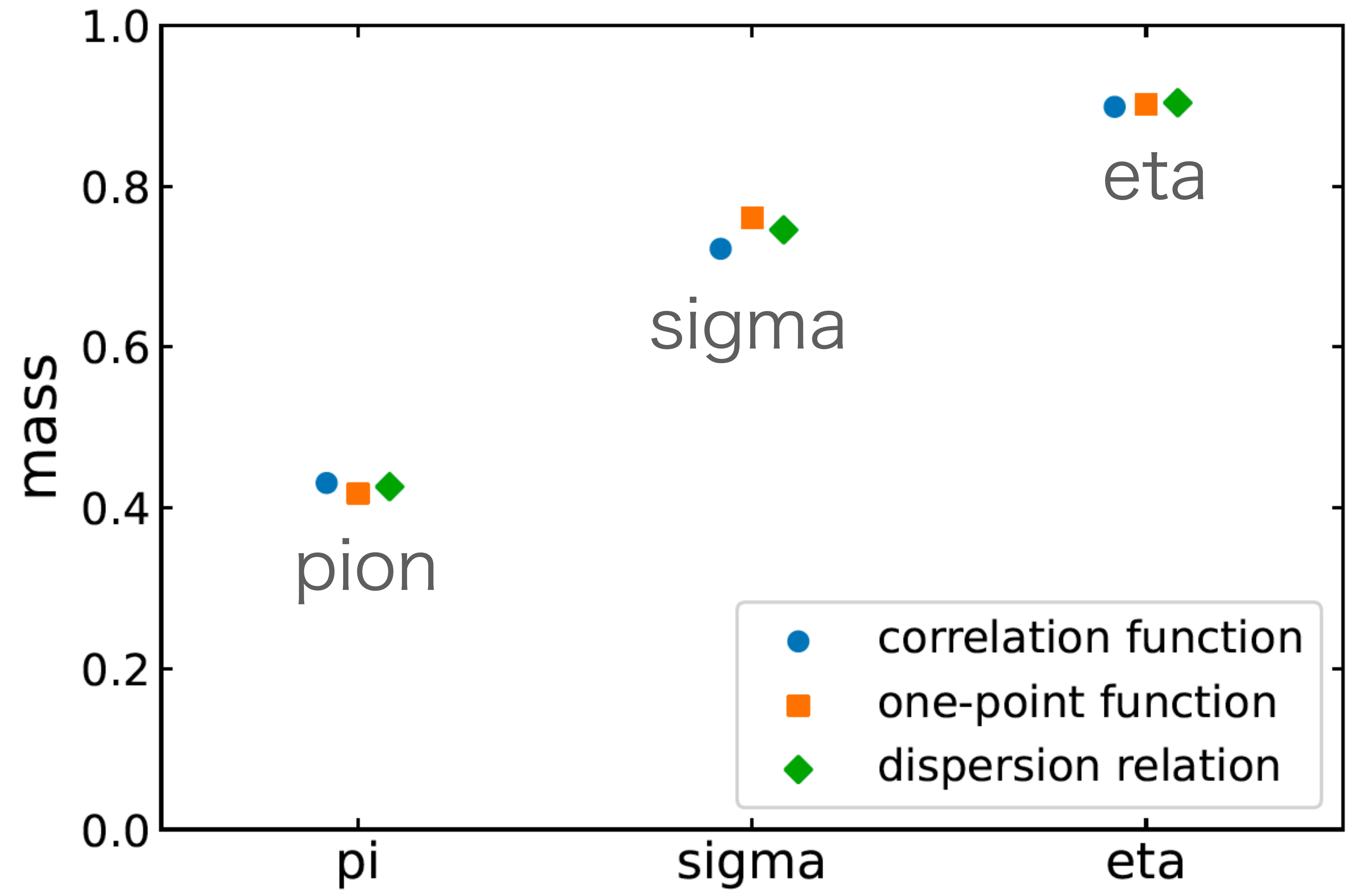
(3) Results of dispersion-relation scheme

Plot ΔE_ℓ against ΔK_ℓ^2 for each meson

Fit the data using $\Delta E = \sqrt{M^2 + b^2 \Delta K^2}$



Three meson masses obtained by three methods



Theoretical predictions

Coleman(1976), Dashen et al. (1975)

✓ $M_\pi < M_\sigma < M_\eta$: U(1) problem

✓ $M_\eta = \mu + O(m)$ $\left(\mu = g\sqrt{N_f/\pi} \sim 0.8, \quad m = 0.1\right)$

✓ $M_\sigma / M_\pi = \sqrt{3}$ (within 5% deviation)

	correlation fn.	one-point fn.	dispersion
M_σ/M_π	1.68(2)	1.821(6)	1.75(1)

Short summary of scheme

- Three calculation methods for hadron spectra in Hamiltonian formalism

(1)correlation-function scheme

👍 applicability to broad class of theories

😥 sensitive to the bond dimension (DMRG) —> 😊 quantum computation

(2)one-point-function scheme

👍 need to increase neither the bond dimension nor the system size L

😥 only the lowest state having given quantum numbers of Bdry state

(3)dispersion-relation scheme

👍 obtain various states heuristically / directly see wave functions (s/p-wave)

😥 computational cost to generate excited states/ how to implement to QC?

4. $\theta \neq 0$

Preliminary

What is different from $\theta = 0$ (theoretical predictions)

- Sign problem appears in Lattice Monte Carlo

- operator mixing between Scalar and Pseudo-Scalar ops. occurs,
  $\mathcal{O} = C_S S + C_{PS} PS$

- loss of quantum numbers (G-parity is broken, η-decay is no longer prohibited)

- decay mode: η meson -> 2 pions
  η meson is not a stable particle

- (almost) conformal theory at $\theta = \pi$ (level-1, SU(2) WZW theory)
  DMRG is hard, shape of correlation fn. is changed

Two calculation methods (at $\theta \neq 0$)

(1) 2-pt. correlation-function for mixed op. and find the mixing angle

$$C(\tau) = \langle O(\tau)O(0)\rangle, \text{ for } O = C_S S + C_{PS} PS$$

+ (1') One-point-function scheme

one-point fn. = correlation fn. with source state

(SPT phase, at θ iso-singlet state / at $\theta + 2\pi$ iso-triplet state)

near $\theta = \pi$, a shape of corr. fn. change to CFT-like

(2) Dispersion-relation scheme

Construct excited states and measure energy, momentum and (approximate) quantum numbers

exact sym. is only isospin, e.g. iso-singlet and iso-triplet

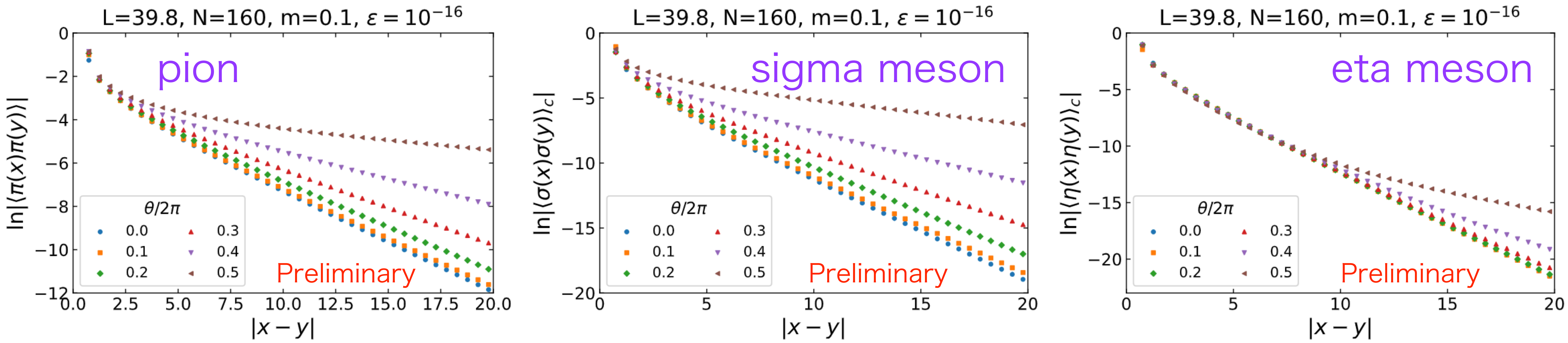
(1) correlation fn. scheme

Operator mixing between Scalar and Psuedo-Scalar ops. occurs, $\mathcal{O} = C_S S + C_{PS} PS$

Diagonalise 2pt. correlation matrix: $C_\pm(x,y) = \begin{pmatrix} \langle S_\pm(x) S_\pm(y) \rangle_c & \langle S_\pm(x) PS_\pm(y) \rangle_c \\ \langle PS_\pm(x) S_\pm(y) \rangle_c & \langle PS_\pm(x) PS_\pm(y) \rangle_c \end{pmatrix}$

-------> $C_+(x,y) = R_+^{\mathrm{T}} \begin{pmatrix} \langle \sigma(x)\sigma(y) \rangle_c & 0 \\ 0 & \langle \eta(x)\eta(y) \rangle_c \end{pmatrix} R_+$ for iso-singlet mesons

-------> $C_-(x,y) = R_-^{\mathrm{T}} \begin{pmatrix} ** & 0 \\ 0 & \langle \pi(x)\pi(y) \rangle_c \end{pmatrix} R_-$ for iso-triplet mesons

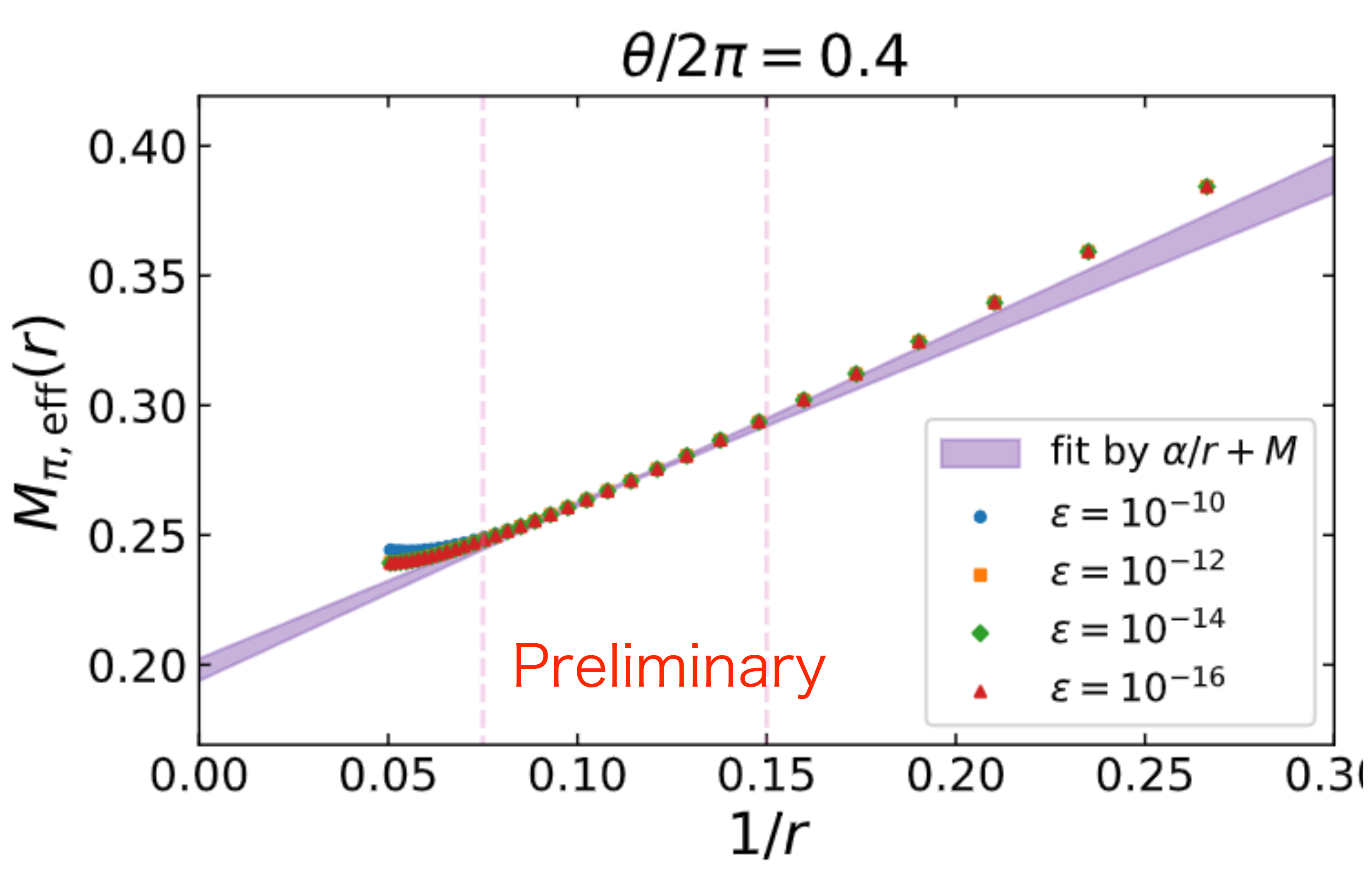


The slope is slower in the larger θ.

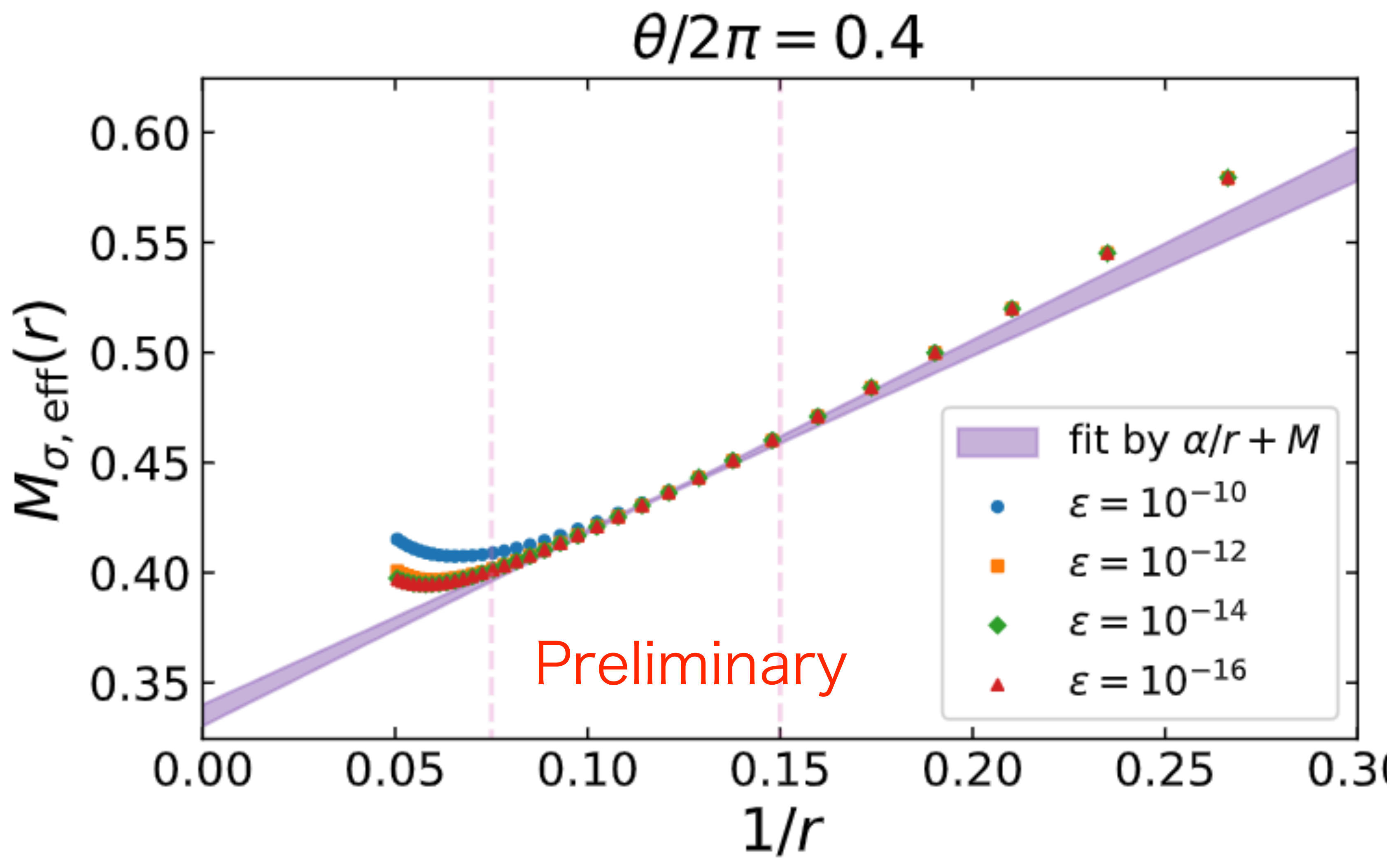
(1) correlation fn. scheme

- Effective mass as a function of 1/r at large θ (large mixing angle, near conformal)

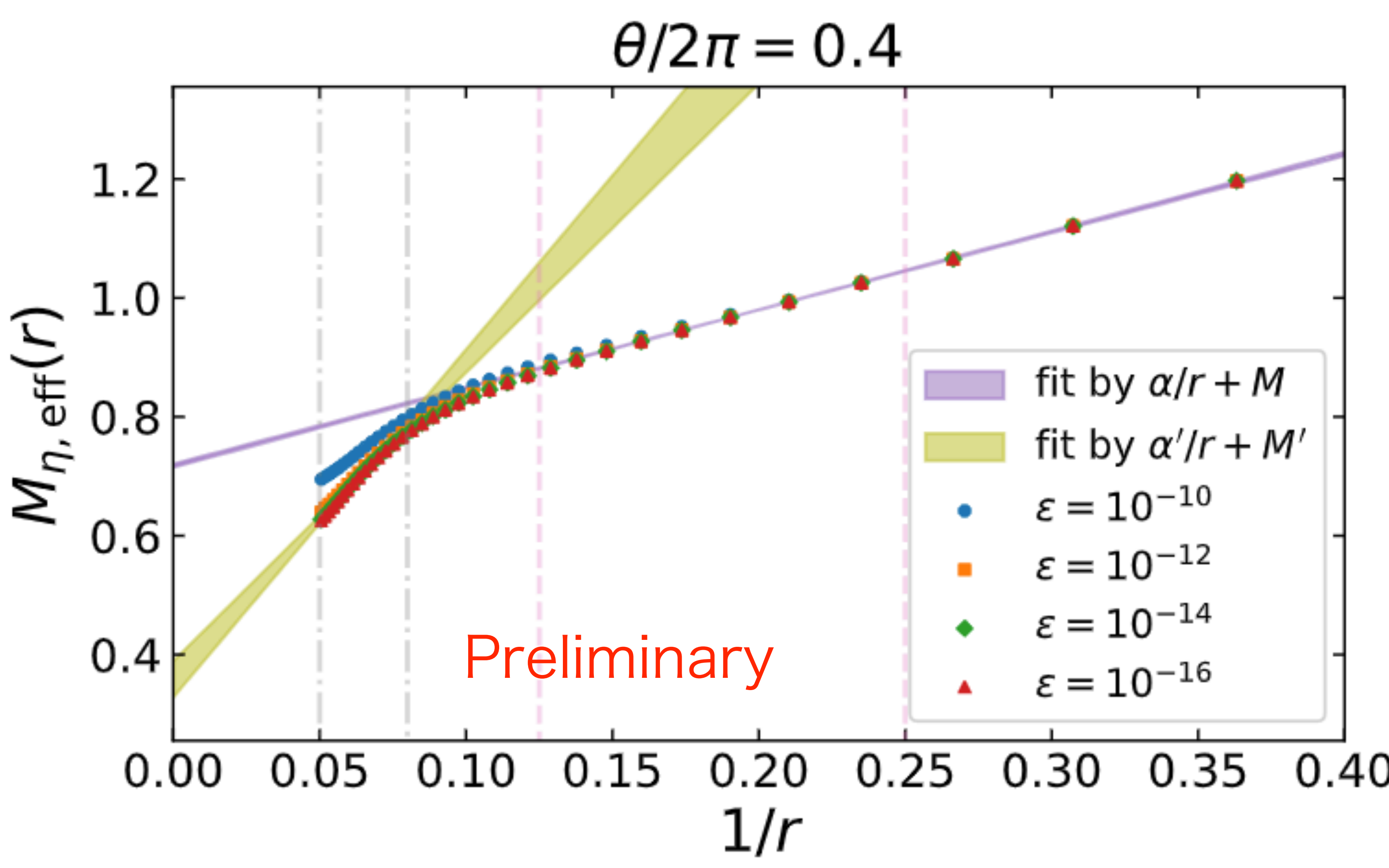
pion



sigma meson



eta meson



The mass becomes smaller (pion and sigma)

Eta meson decays into a lighter mode over long distances.

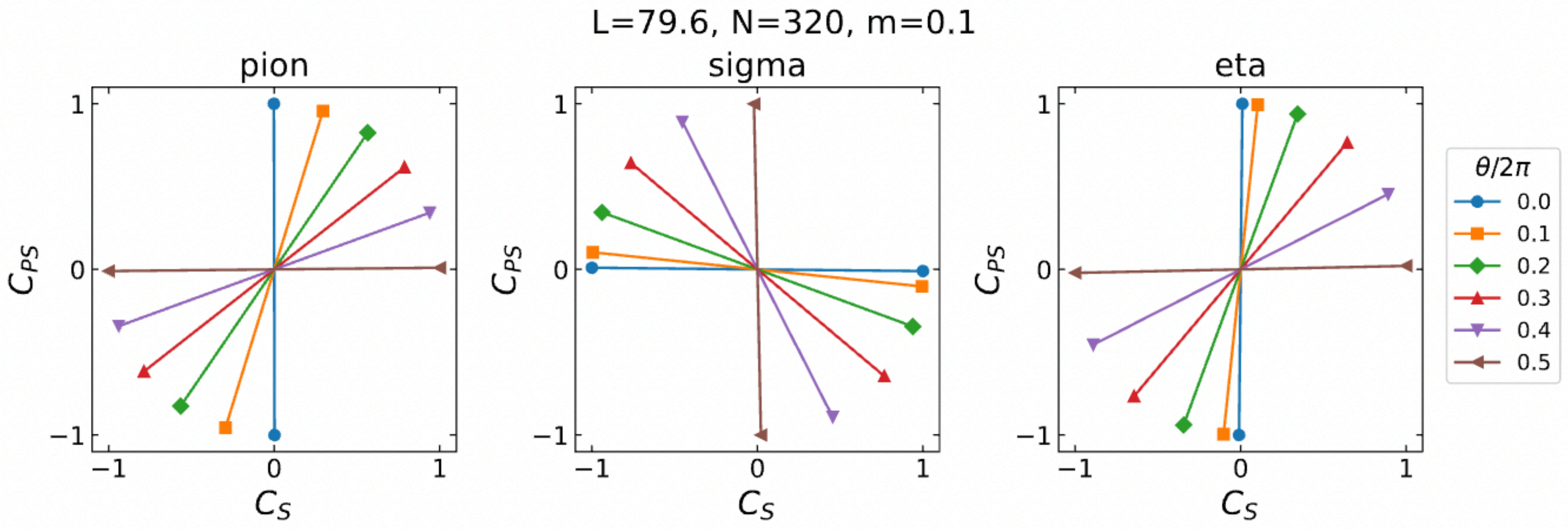
(1') one-point fn. scheme in $\theta < \pi$

- Need to increase neither the bond dimension nor the system size L

- To find the mixing of ops., $\mathcal{O} = C_S S + C_{PS} PS$, we use the rotation matrices by the 2-pt. fn. scheme : $\langle \mathcal{O}(x) \rangle \propto e^{-Mx}$ for $\theta < \pi$

$$\begin{pmatrix} * \\ \pi(x) \end{pmatrix} = R_- \begin{pmatrix} S_-(x) \\ PS_-(x) \end{pmatrix}$$

$$\begin{pmatrix} \sigma(x) \\ \eta(x) \end{pmatrix} = R_+ \begin{pmatrix} S_+(x) \\ PS_+(x) \end{pmatrix}$$

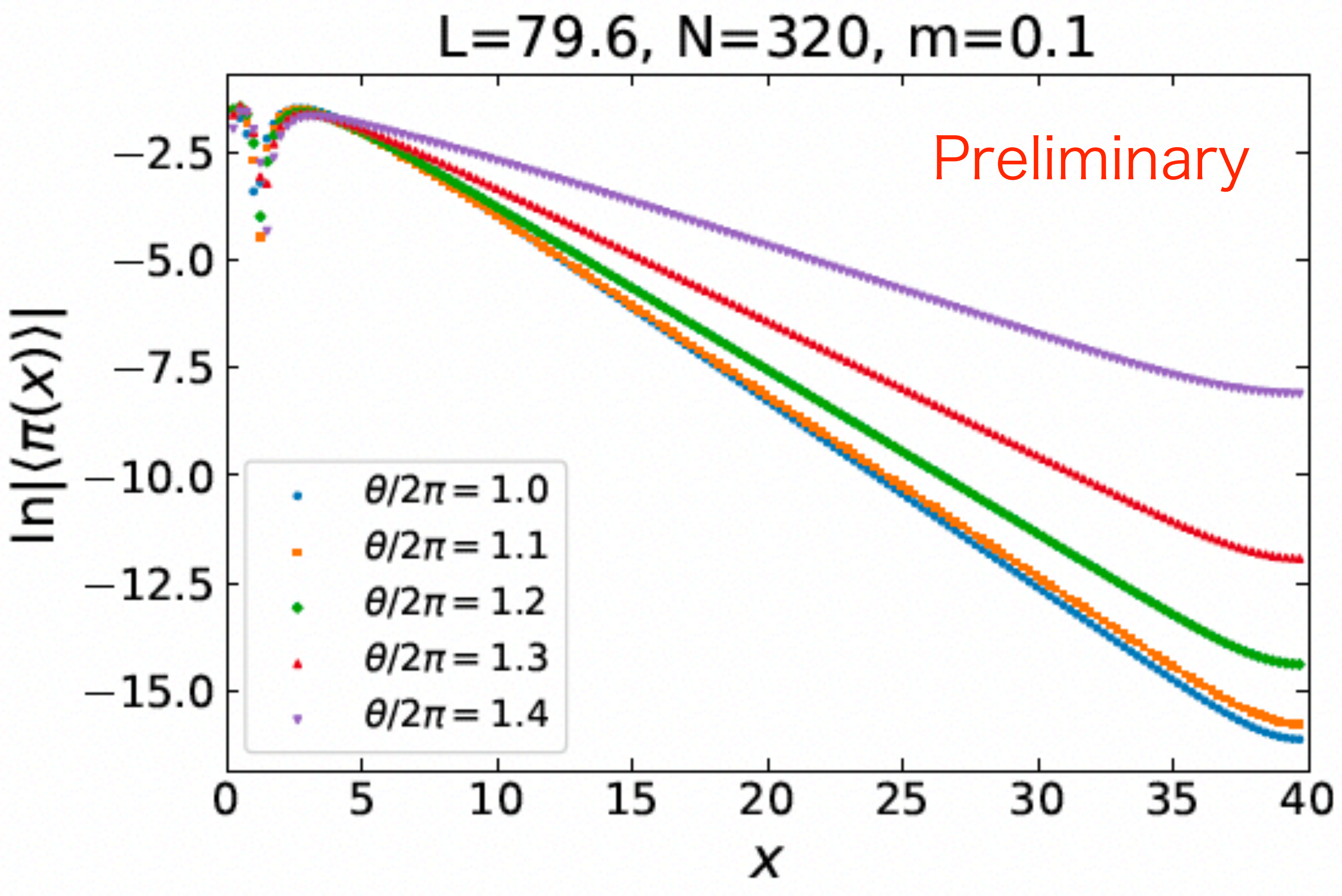


(1') one-point fn. scheme in $\theta < \pi$

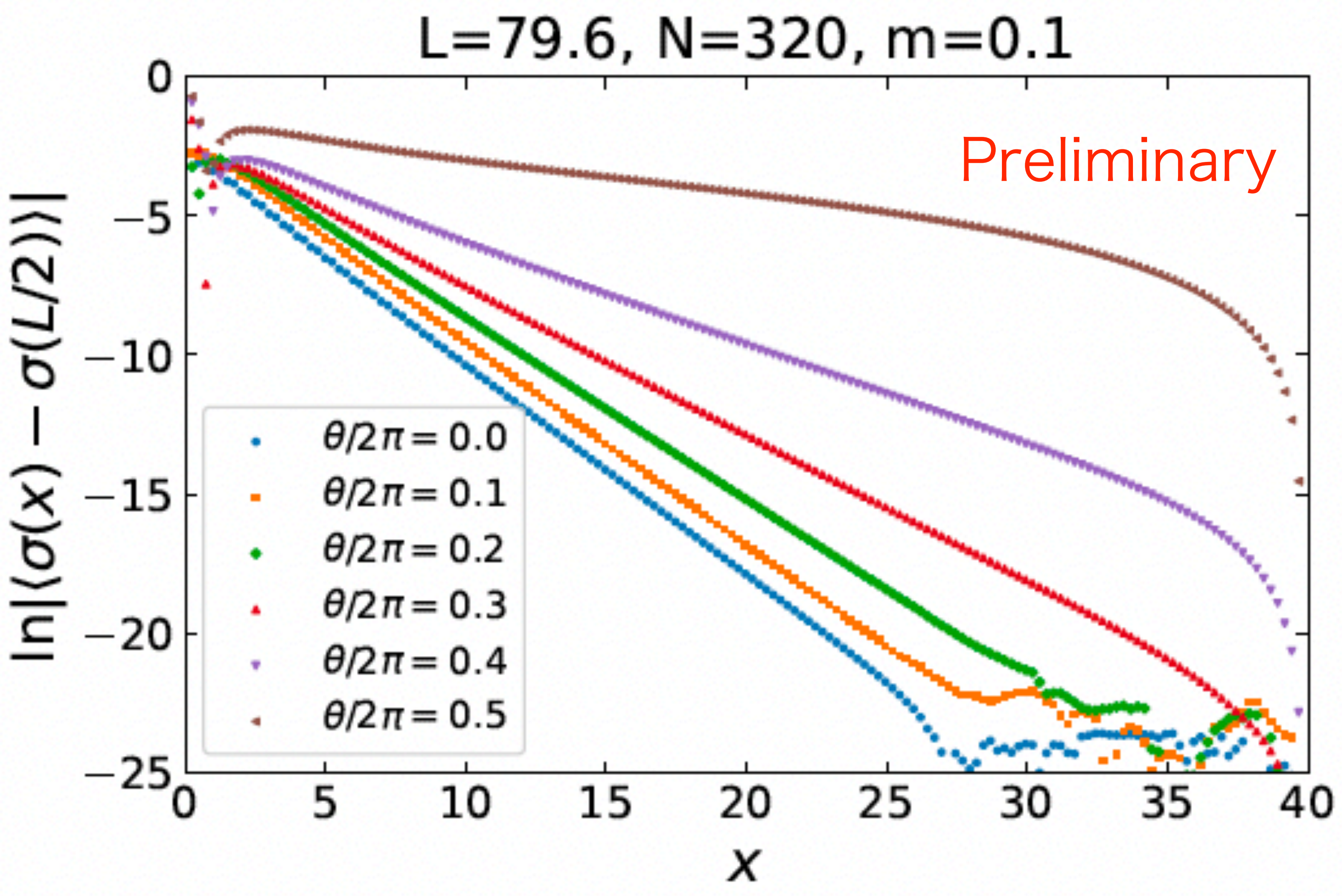
- Need to increase neither the bond dimension nor the system size L

- No longer an independent scheme
  To find the mixing of ops., we use the mixing matrix by the 2-pt. fn.
  scheme : $\langle \mathcal{O}(x) \rangle \propto e^{-Mx}$ for $\theta < \pi$

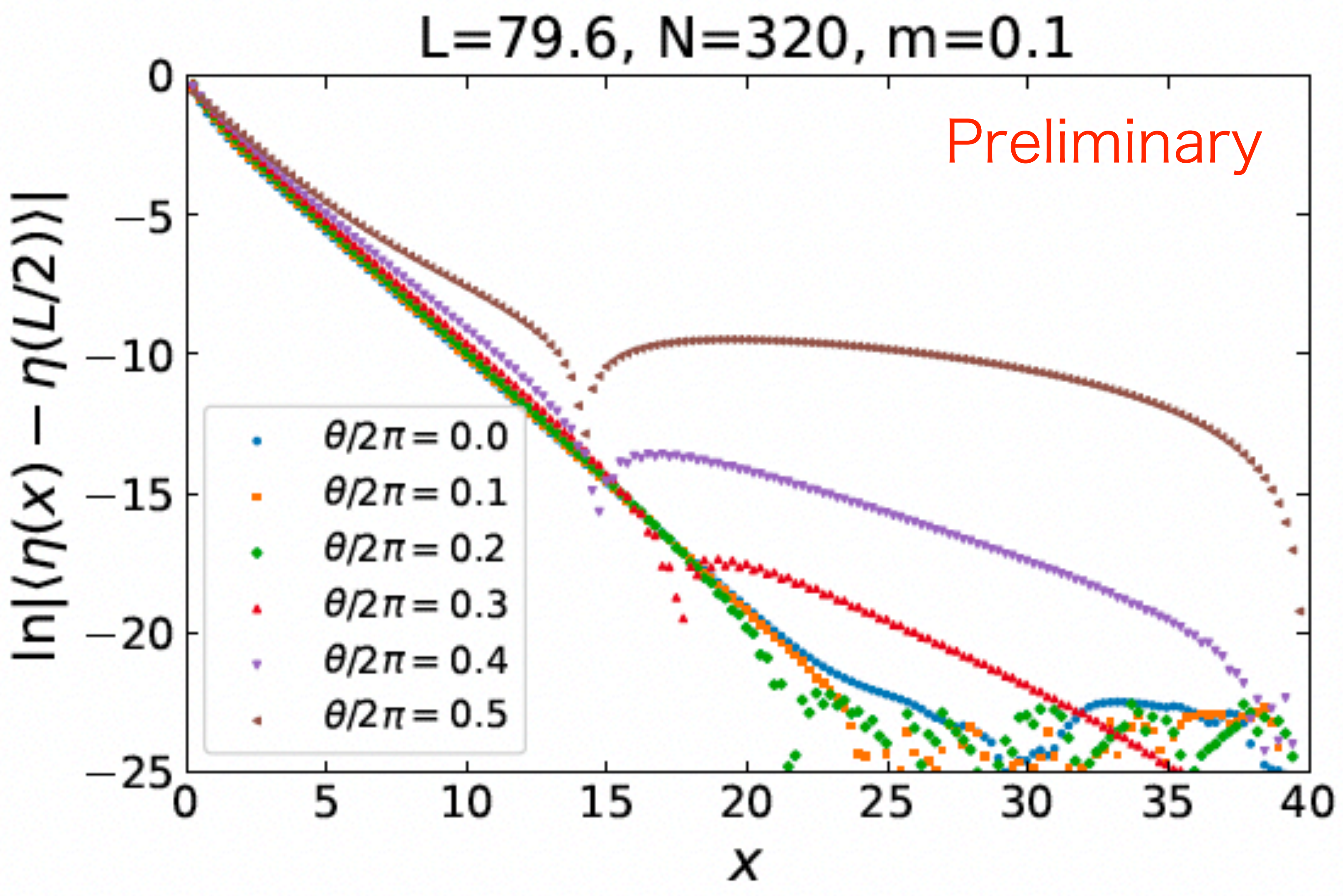
pion



sigma meson



eta meson

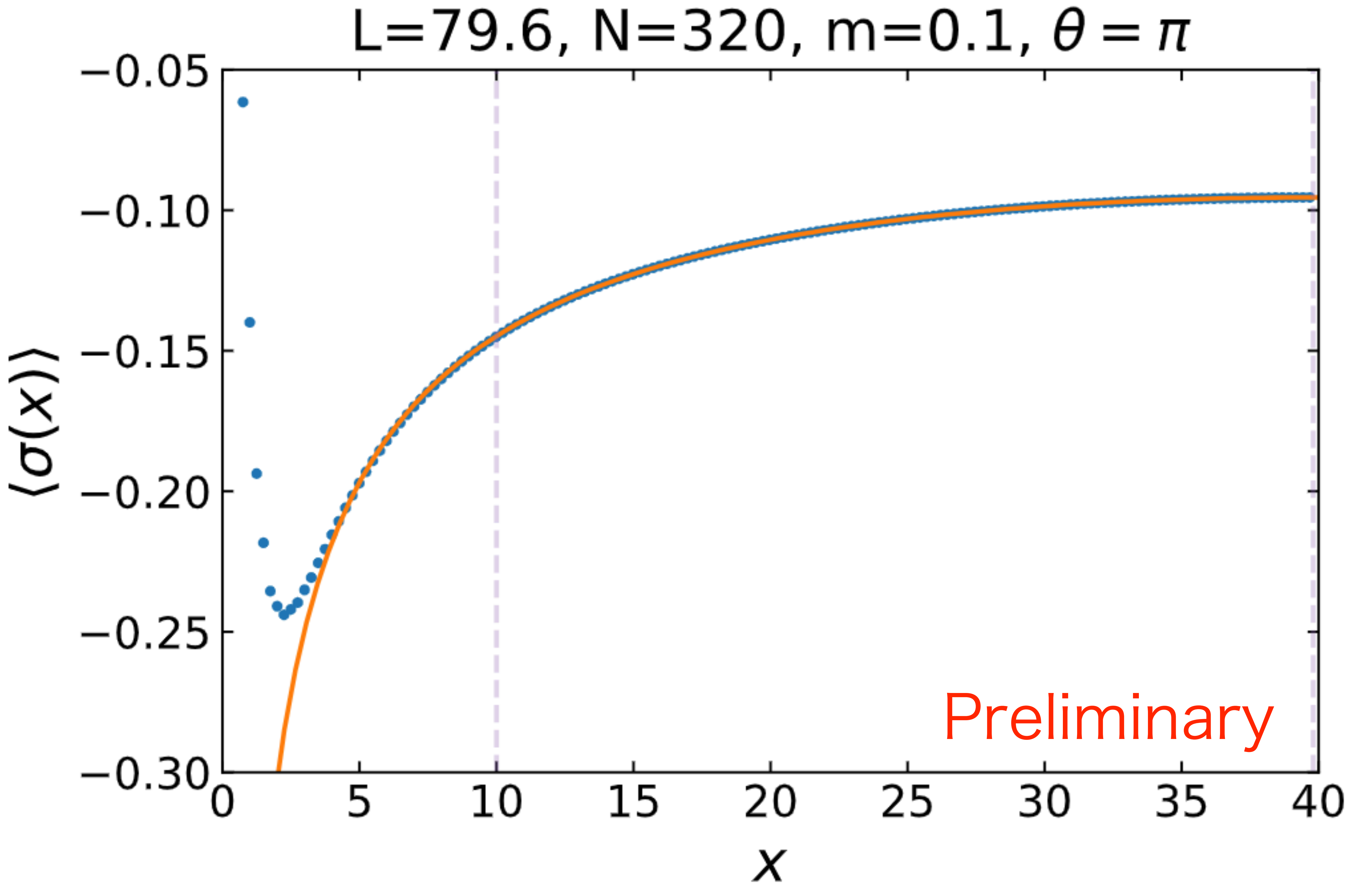


$\theta = \pi$ is difficult

In middle x regime, there is a cusp

(1') one-point fn. scheme at $\theta = \pi$

- Analytic form of one-point fn. with OBC $\langle \sigma(x) \rangle \sim \frac{1}{\sqrt{\sin(\pi x/L)}}$



The data is well-fitted !!

cf.) 2-flavor Schwinger model at $\theta = \pi$

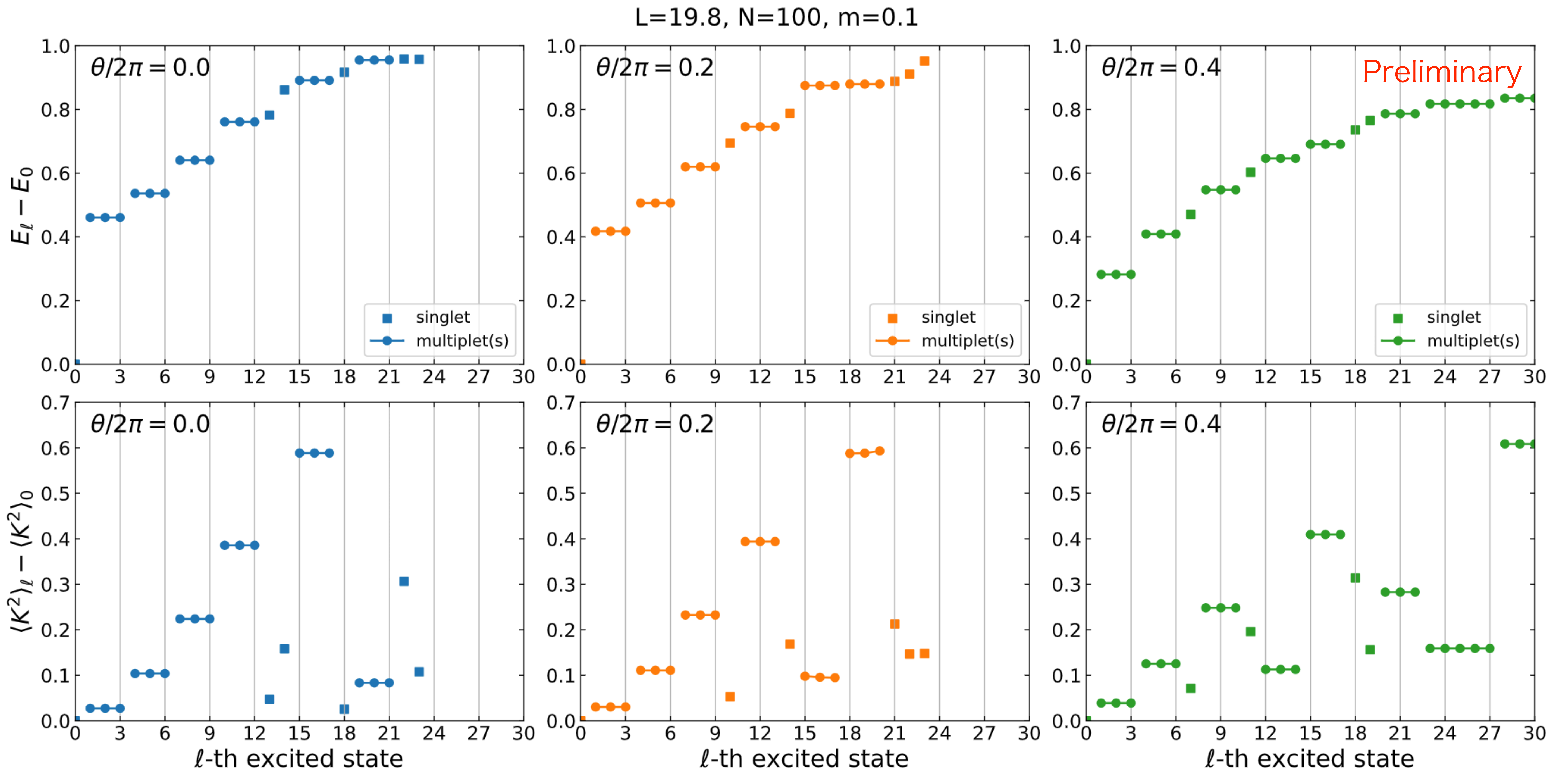
a small mass gap $\sim e^{-Ag^2/m^2}$ remains

(Not exact CFT if $m \neq 0$)

Dempsey et al., 2023

(2) dispersion-relation scheme

- G-parity is no longer exact quantum numbers

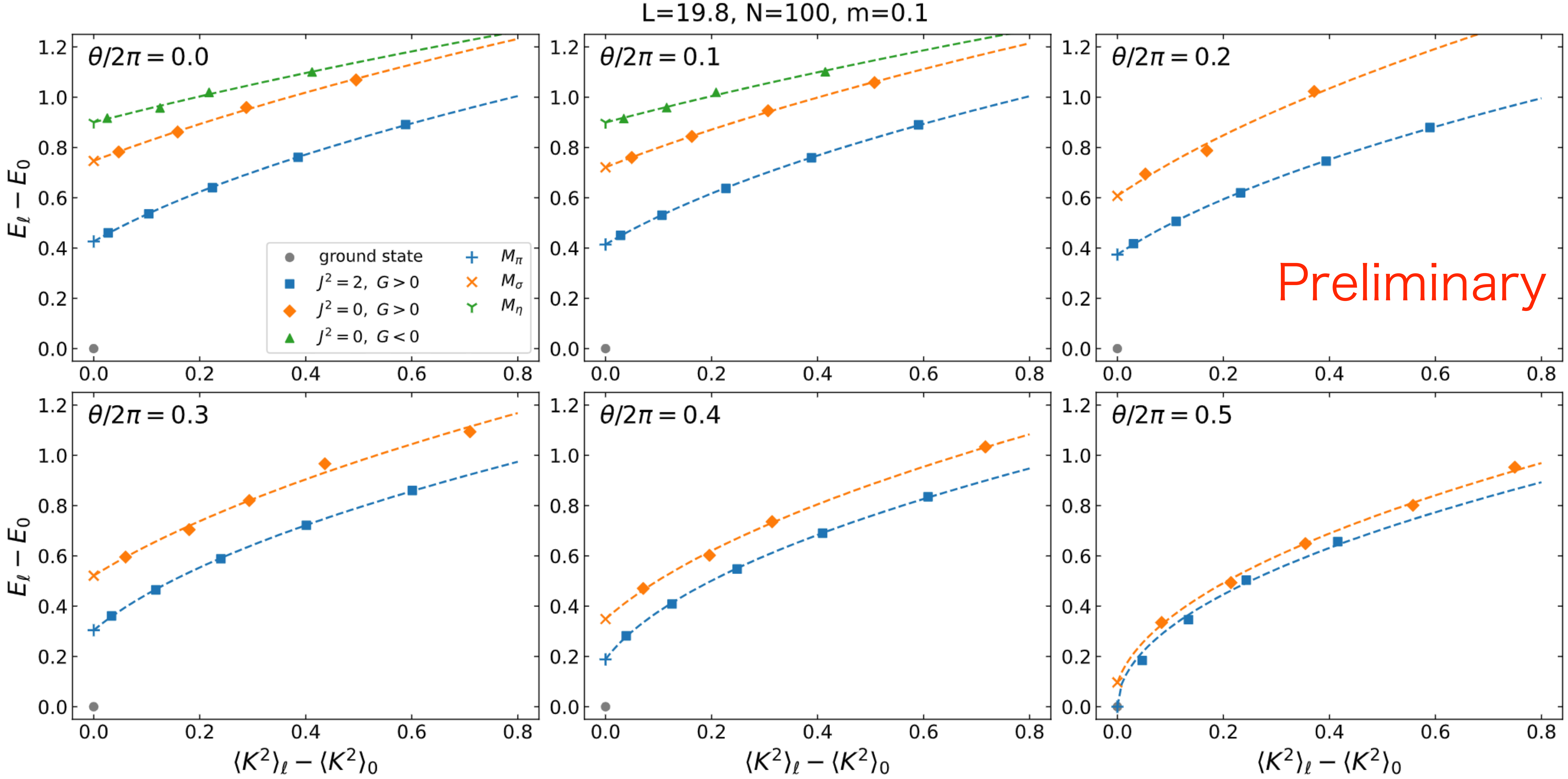


Iso-triplet must be pion

We cannot distinguish between eta and sigma

(2) dispersion-relation scheme

- fit the data for each meson using $\Delta E = \sqrt{M^2 + b^2 \Delta K^2}$

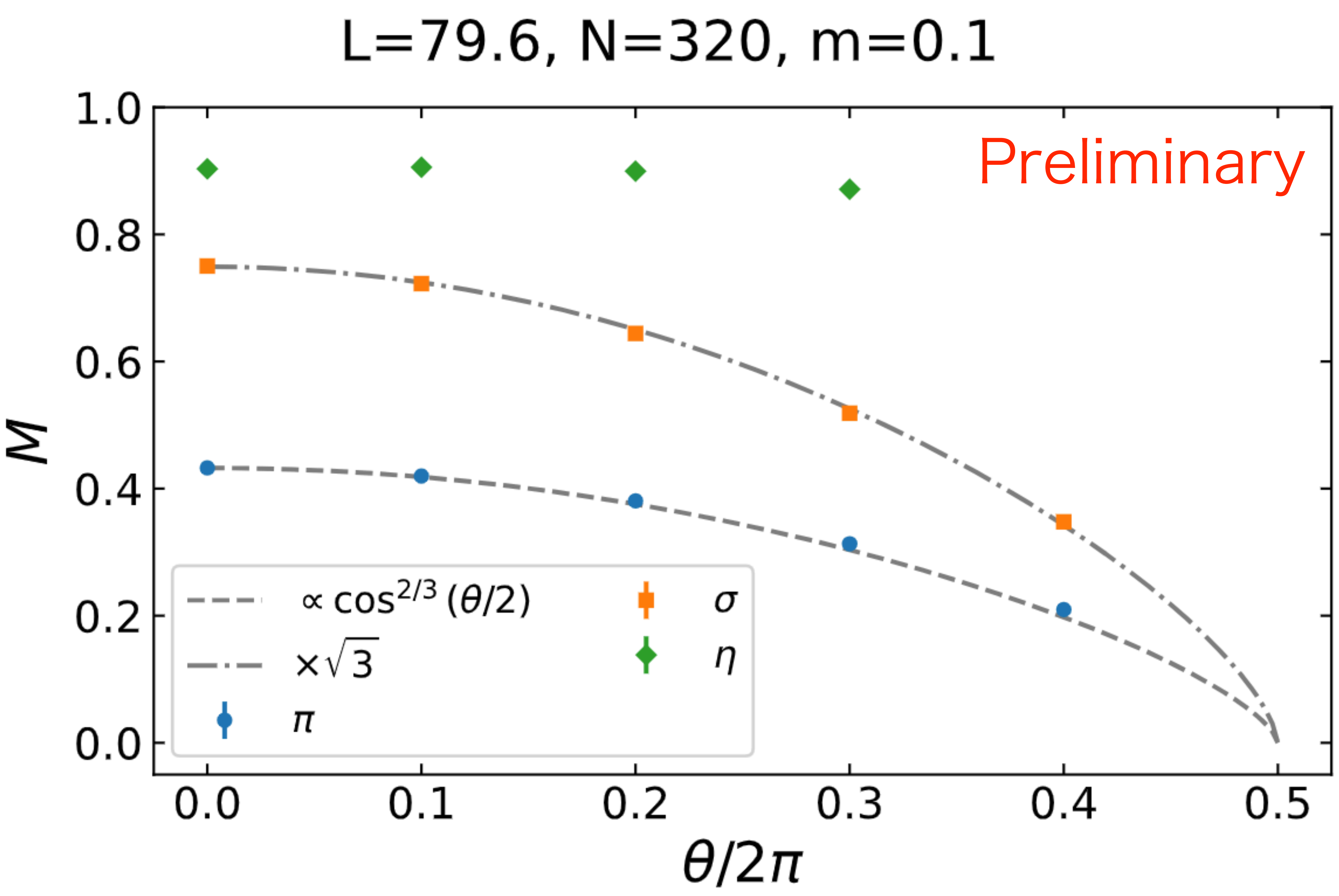


η disappear $\theta/2\pi > 0.2$

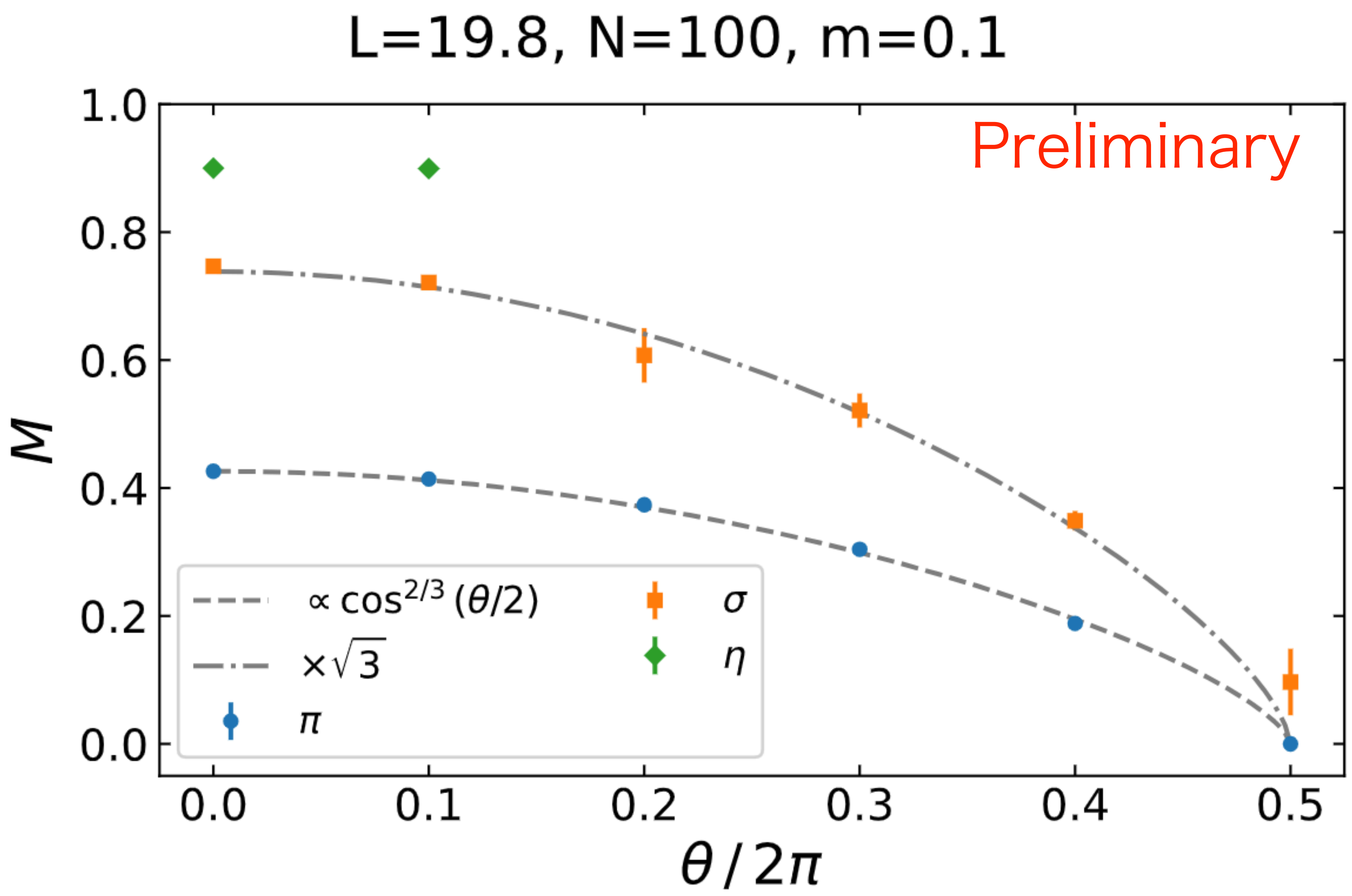
sigma (singlet) and pion (triplet) are degenerating at $\theta = \pi$

Summary plot

correlation-fn. scheme (2-pt. and 1-pt.)



dispersion-relation scheme

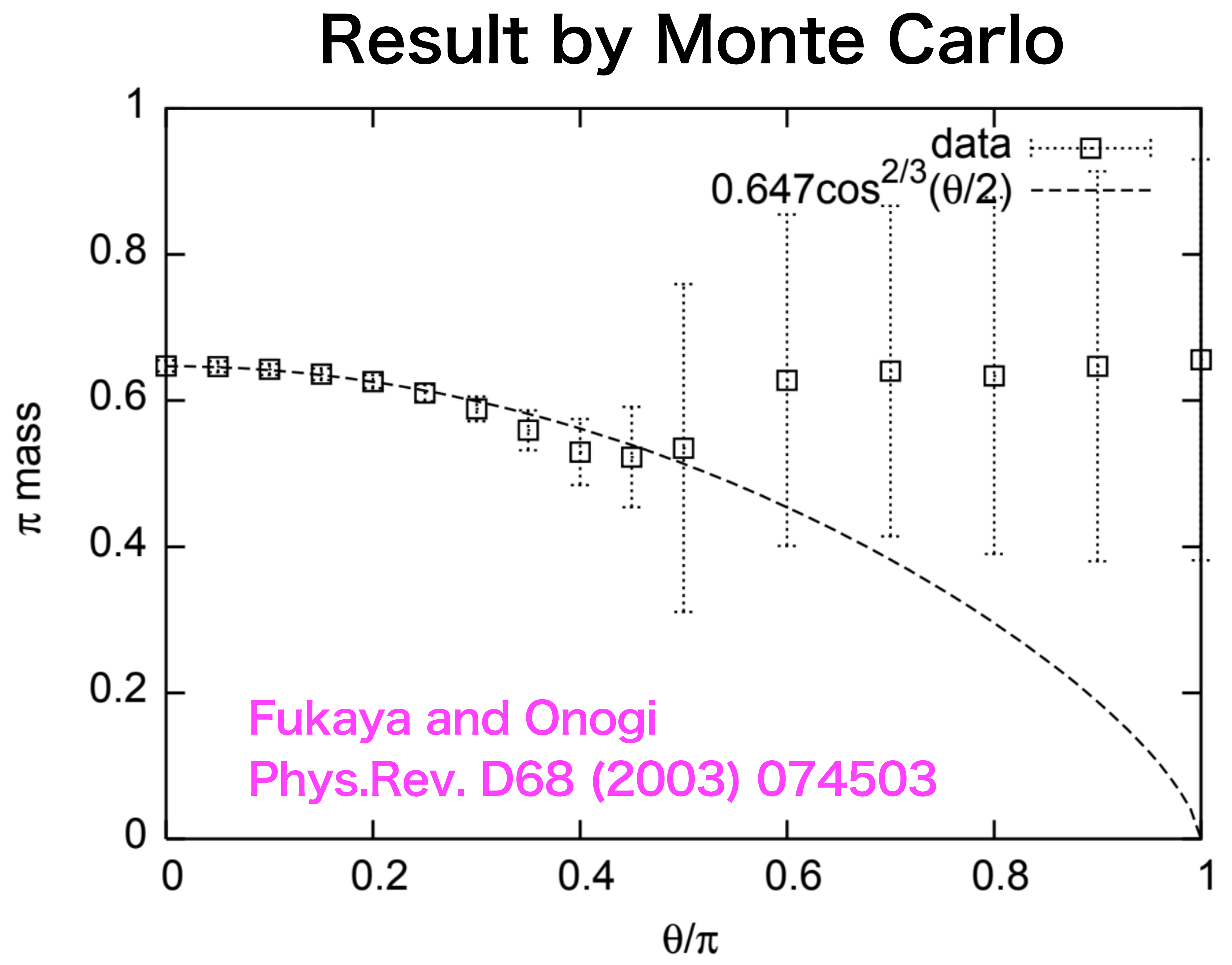


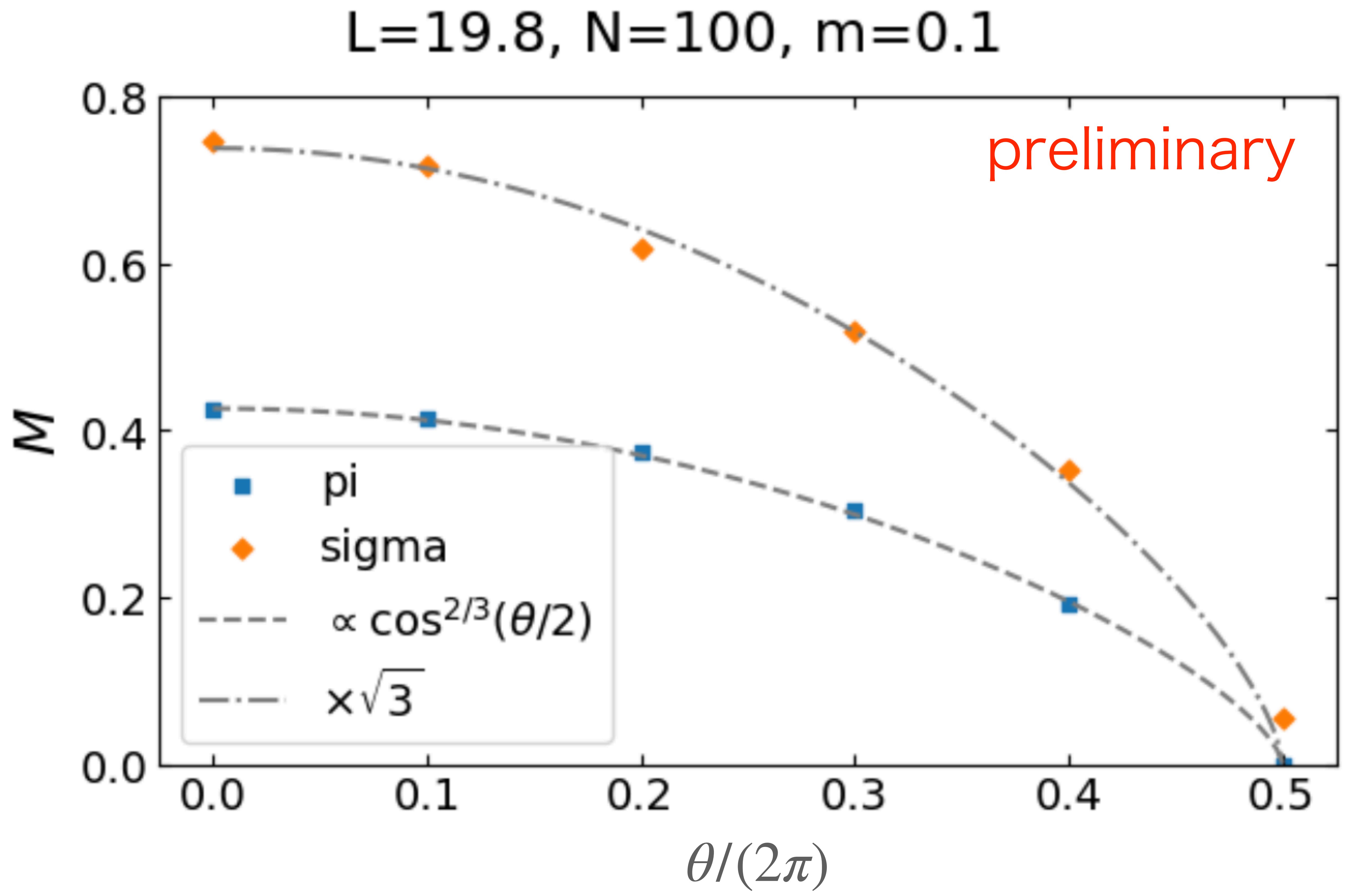
η gets an unstable particle in large θ

$m_\sigma = \sqrt{3} m_\pi$ is valid around $\theta = \pi$

Comparison with Monte Carlo

Nf=2 Schwinger model w/ θ-term





- In large θ, the signal is very noisy because of the sign problem
- Difficult to find a heavy η-meson and σ-meson

5. Summary

- Our calculation methods for hadron spectra in Hamiltonian formalism works well even at $\theta \neq 0$

  (1)correlation-function scheme (2pt + 1pt)

  resolve the op. mixing and obtain a precise result

  (2)dispersion-relation scheme

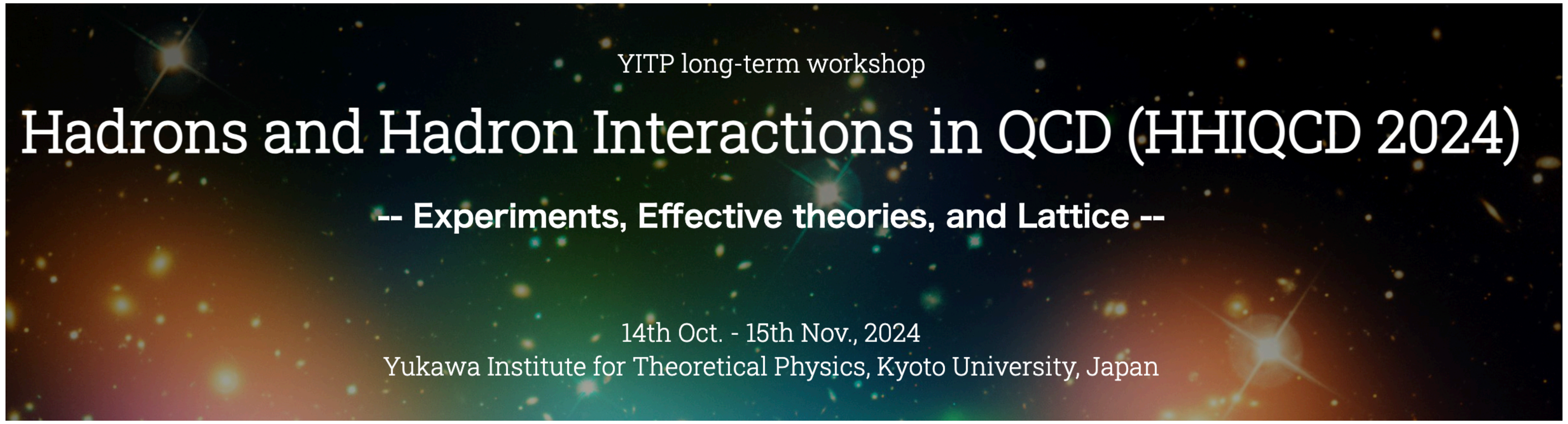
- Future direction

  - Apply QCD theory w/ finite-density

  (op. mixing and loss of quantum number occurs!)

  - Efficient quantum algorithm to generate excited state (for dispersion-relation scheme)

Please come to Kyoto this autumn!!



Registration opens! Here

1st and 2nd weeks: Hadron interactions, scattering

3rd week : symposium for all subjects

4th week：hot and dense QCD

5th week：Formal aspect and quantum computations

Invited speakers for 3rd and 5th weeks

- Zohreh Davoudi (Maryland U.)
- Erez Zohar (Hebrew U. of Jerusalem)
- Muhammad Asaduzzaman (U. of Iowa)
- Yahui Chai (DESY)
- Tomoya Hayata (Keio U.)
- Marc Illa (U. Washington)
- David B. Kaplan (Washington U.)
- Scott Lawrence(Los Alamos Natl. Lab.)
- Akira Matsumoto (YITP, Kyoto U.)
- Indrakshi Raychowdhury (BITS, Pilani)
- Pietro Silvi (Università di Padova)
- Judah Unmuth-Yockey (Fermilab)
- Uwe-Jens Wiese (Bern U.)
- Arata Yamamoto (U. Tokyo)
- Xiaojun Yao (U. Washington)
- Torsten V. Zache (Innsbruck U.)

… and more

backup

Introduction

May, 2023 @ U. of Minnesota

- QCD phenomena has been well understood for this 50 years

- Asymptotic freedom
  Topological objects
  Hadron mass
  Nuclear force
  Phase transition at finite-T
  Thermodynamics
  ….

From $\mathscr{L}$ to $\mathscr{H}$ for Quantum computer

Ex) Schwinger model with open b.c.

PBC: Shaw et al. Quantum 4, 306 (2020) arXiv:2002.11146

- Lagrangian in continuum

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta_0}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}\psi$$

- Hamiltonian in continuum

$$H_{\text{con}} = \int dx \left[\frac{1}{2}\left(\Pi - \frac{g\theta_0}{2\pi}\right)^2 - i\bar{\psi}\gamma^1(\partial_1 + igA_1)\psi + m\bar{\psi}\psi\right]$$

Canonical momentum: $\Pi = \partial_0 A^1 + \frac{g\theta}{2\pi}$

- Hamiltonian on lattice (staggered fermion, link variable)

$$H = J\sum_{n=0}^{N-2}\left(L_n + \frac{\vartheta_n}{2\pi}\right)^2 - iw\sum_{n=0}^{N-2}\left(\chi_n^{\dagger}U_n\chi_{n+1} - \chi_{n+1}^{\dagger}U_n^{\dagger}\chi_n\right) + m\sum_{n=0}^{N-1}(-1)^n\chi_n^{\dagger}\chi_n$$

Link variable: $L_n \leftrightarrow -\Pi(x)/g,\ \ U_n \leftrightarrow e^{-iagA^1(x)}$,

Staggered fermion: $\frac{\chi_n}{\sqrt{a}} \leftrightarrow \begin{cases}\psi_u(x) & n:\text{even}\\ \psi_d(x) & n:\text{odd}\end{cases}$

- Remove gauge d.o.f. (OBC and Gauss law constraint)

$$H = J\sum_{n=0}^{N-2}\left(\epsilon_{-1} + \sum_{i=0}^{n}\left(\chi_i^{\dagger}\chi_i - \frac{1-(-1)^i}{2}\right) + \frac{\vartheta_n}{2\pi}\right)^2 - iw\sum_{n=0}^{N-2}\left(\chi_n^{\dagger}\chi_{n+1} - \chi_{n+1}^{\dagger}\chi_n\right) + m\sum_{n=0}^{N-1}(-1)^n\chi_n^{\dagger}\chi_n$$

Gauss law:
$0 = \partial_1\Pi + g\psi^{\dagger}\psi \rightarrow L_n - L_{n-1} = \chi_n^{\dagger}\chi_n - \frac{1-(-1)^n}{2}$

- Spin Hamiltonian using Pauli matrices(Jordan-Wigner trans.)

$$H = J\sum_{n=0}^{N-2}\left[\sum_{i=0}^{n}\frac{Z_i + (-1)^i}{2} + \frac{\vartheta_n}{2\pi}\right]^2 + \frac{w}{2}\sum_{n=0}^{N-2}\left[X_nX_{n+1} + Y_nY_{n+1}\right] + \frac{m}{2}\sum_{n=0}^{N-1}(-1)^nZ_n$$

Jordan-Wigner trans.: $\chi_n = \frac{X_n - Y_n}{2}\prod_{i=0}^{n-1}(-iZ_i)$

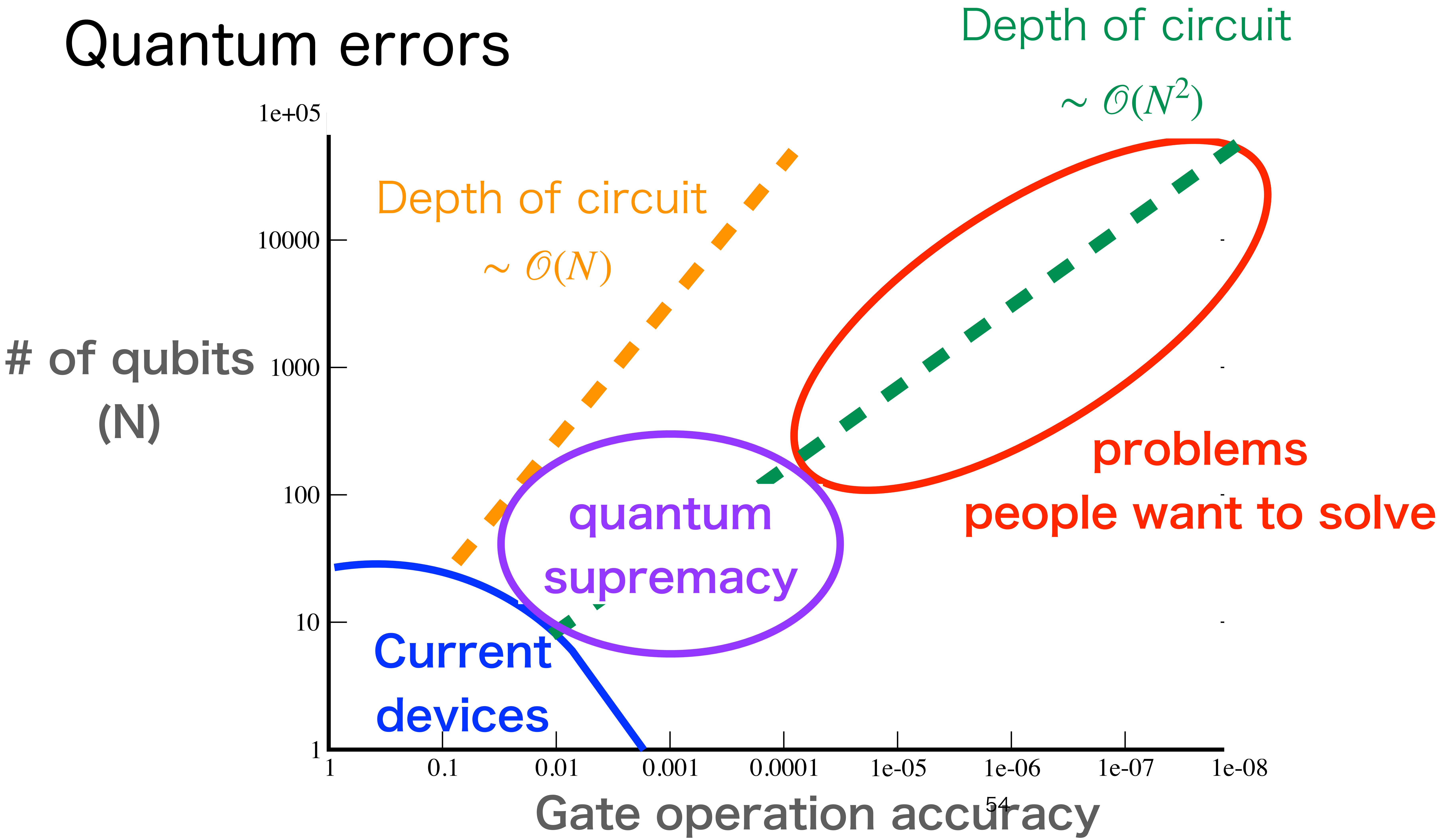
Apply quantum algorithms to this spin hamiltonian.

General problems of quantum computer

- Current quantum device

Small qubit size ($N = 10 - 30$)

Quantum errors



Lattice QCD: relevant users of supercomputer

- Confinement
- Hadron mass
  composite particles of quarks
- Nuclear force/structure
- Thermodynamics
- Nonperturbative calculation
  for the standard model

Slide of Lena Funcke @ Lattice2022

Supercomputer usage for different fields (INCITE 2019)

→ **Lattice QCD: ~ 40%**

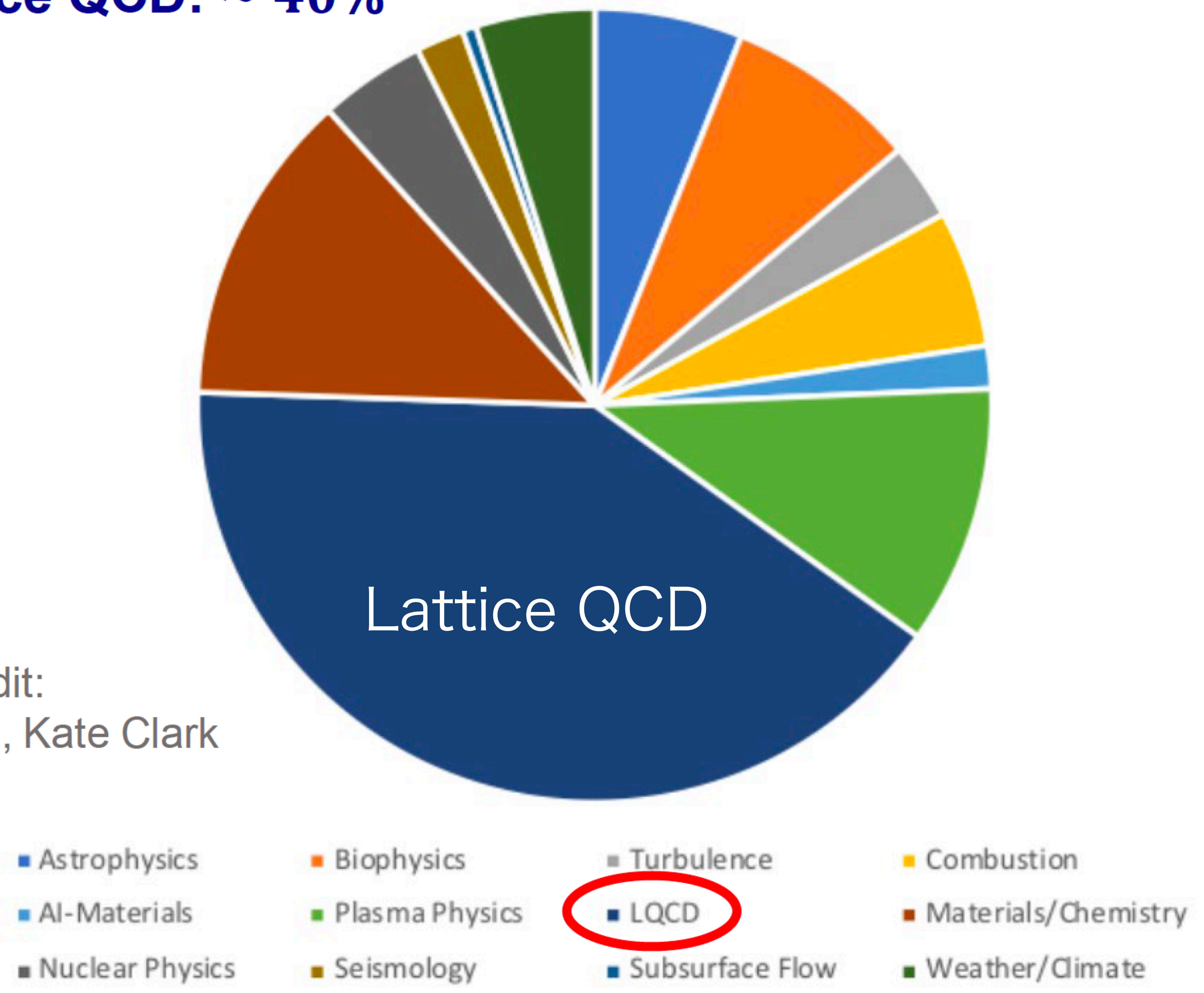


Figure credit:
Jack Wells, Kate Clark

Lattice Monte Carlo QCD

- we want to know: $\langle \mathcal{O} \rangle = \int D\phi \mathcal{O}[\phi] e^{-S_E[\phi]}$

  (dof=6×10^9 in current calc. on supercomputer)

- ex.) Area of fan shape: $S = \int_0^1 \sqrt{1-x^2} dx$

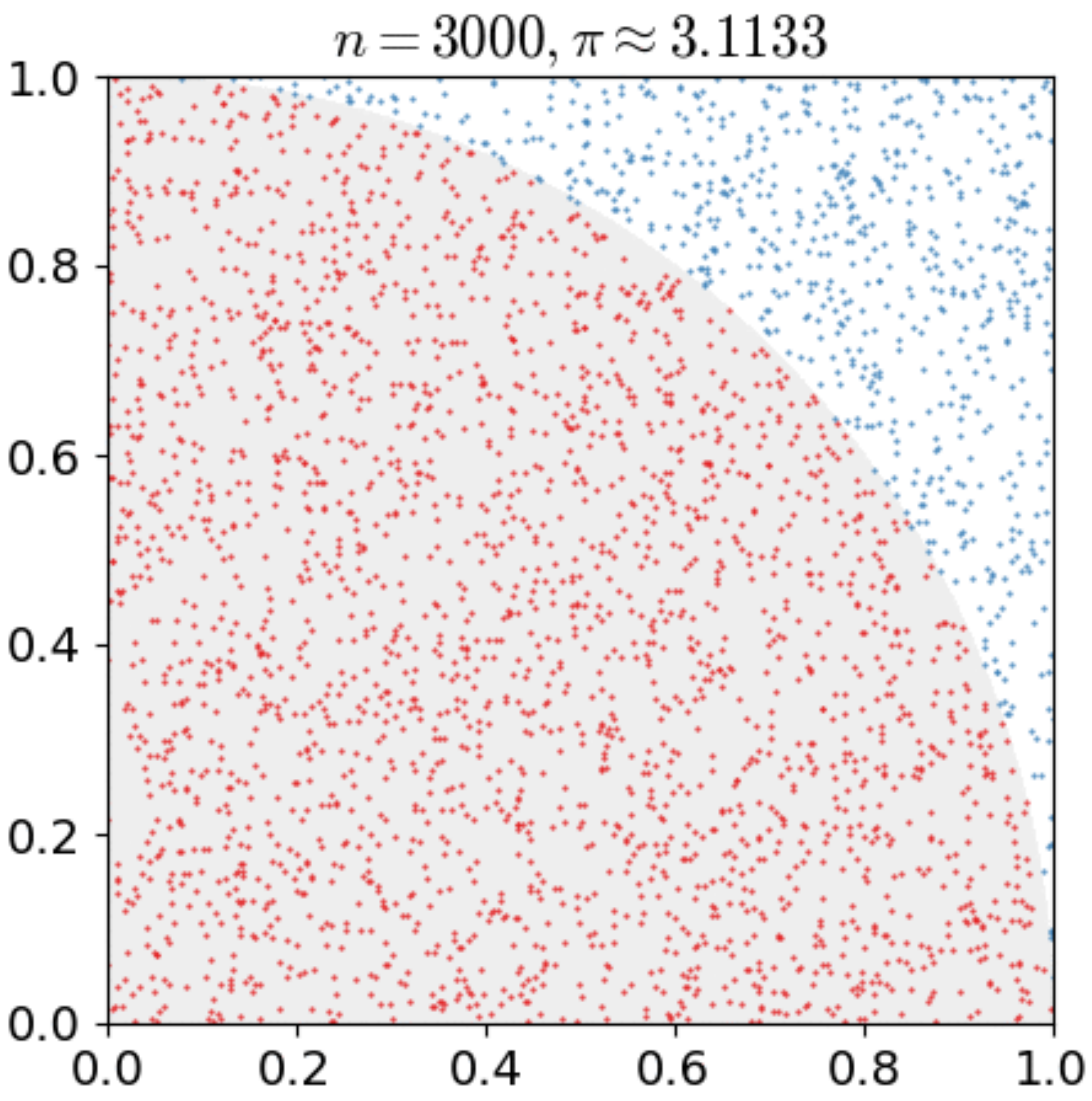
  (1) Generate two sets of random number (x,y)

  (2) Count # of dots inside fan shape = $s(N)$

  total # of trial = N

  (3) $S = \lim_{N \to \infty} s(N)/N$

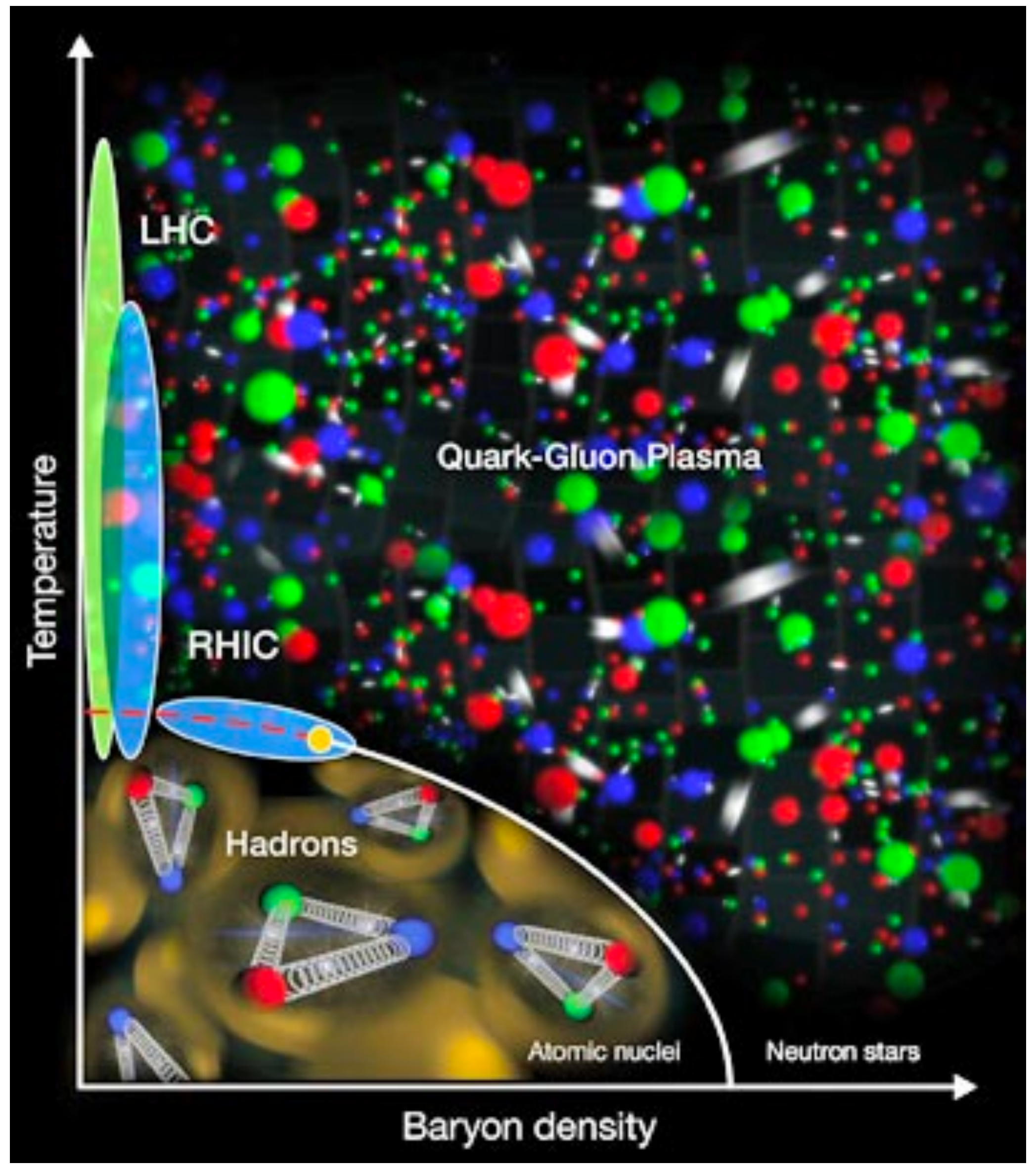
- MC is faster algorithm for multi-dim. integral than differentiation of product method (区分求積法)



$s(N)$ = # of red point

N = # of trial

Sign Problem in Lattice Monte Carlo



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- Sign problem if S_E becomes complex

  real-time evolution

  finite-density QCD

  topological theta-term

- Sign problem is NP-hard Troyer and Wiese, 2005

  Recent review: K.Nagata, Prog.Part.Nucl.Phys. 127 (2022) 103991,
  e-Print: 2108.12423 [hep-lat]

- Alternative methods....?

  => Simulation w/ Hamiltonian formalism

  Quantum computing

  Tensor network (DMRG, PEPs...)

Multi-flavor Schwinger model: ordering(2)

- staggered fermion -> spin variable

Nf=1 $\{\chi_n^\dagger, \chi_m\} = \delta_{n,m}$

$\{\chi_n, \chi_m\} = \{\chi_n^\dagger, \chi_m^\dagger\} = 0$

$$\chi_n = \frac{\sigma_n^x - \sigma_n^y}{2} \prod_{j=0}^{n-1} (-i\sigma_j^z)$$

Nf=2 $\{\chi_{f,n}^\dagger, \chi_{\tilde{f},m}\} = \delta_{f,\tilde{f}} \delta_{n,m}$

$\{\chi_{f,n}, \chi_{\tilde{f},m}\} = \{\chi_{f,n}^\dagger, \chi_{\tilde{f},m}^\dagger\} = 0$

$$\chi_{f,n} = \frac{\sigma_{f,n}^x - \sigma_{f,n}^y}{2} \prod_{j=0}^{n-1} (-i\sigma_{f,j}^z) \prod_{f'=1}^{f-1} \prod_{k=0}^{N-1} (-i\sigma_{f',k}^z)$$

$$\chi_{1,n} = \frac{\sigma_{1,n}^x - \sigma_{1,n}^y}{2} \prod_{j=0}^{n-1} (-\sigma_{2,j}^z \sigma_{1,j}^z)$$

$$\chi_{2,n} = \frac{\sigma_{2,n}^x - \sigma_{2,n}^y}{2} (-i\sigma_{1,n}^z) \prod_{j=0}^{n-1} (-\sigma_{2,j}^z \sigma_{1,j}^z)$$

⬇

local op. (isospin and so on)
expresses highly non-local Pauli matrices

⬇

local op. (isospin and so on)
becomes only a few # of Pauli matrices

Multi-flavor Schwinger model: ordering(2)

- staggered fermion -> spin variable

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local op. (isospin and so on)
becomes only a few # of Pauli matrices

Why QCD in Hamiltonian formalism?

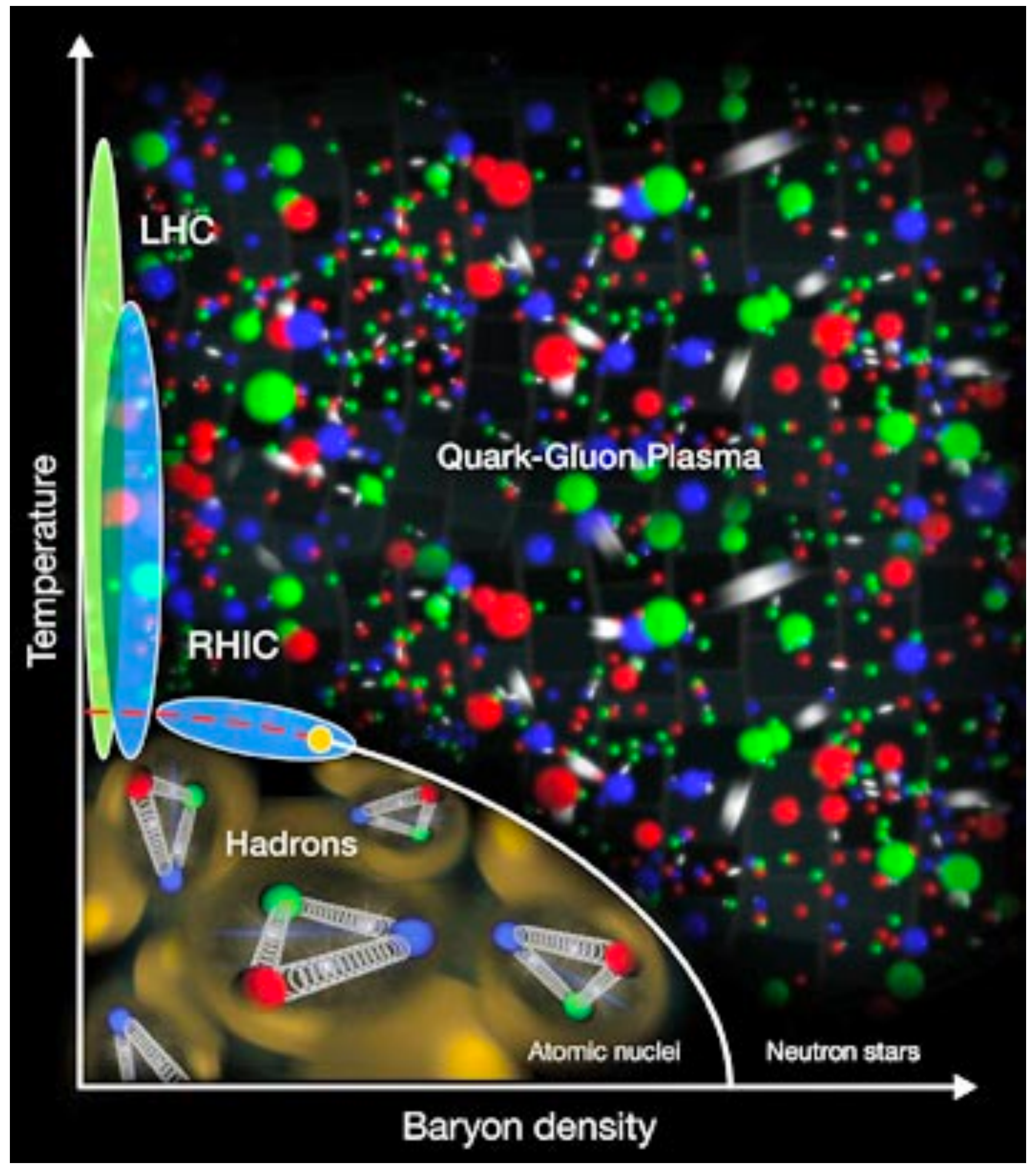
- So far, Lattice MC QCD is the most powerful tool $\langle \mathcal{O} \rangle = \int D\phi \mathcal{O}[\phi] e^{-S_E[\phi]}$

In importance sampling,
e^{-S_E} : Boltzmann weight
Should be real and positive

- Sign problem emerges
  real-time evolution
  finite-density QCD
  topological theta-term

- Sign problem is NP-hard Troyer and Wiese, 2005

  Recent review: K.Nagata, Prog.Part.Nucl.Phys. 127 (2022) 103991



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- Alternative methods....?
  => Simulation w/ Hamiltonian formalism
  Sign problem is absent from the beginning

Hamiltonian formalism for Gauge theory

Kogut-Susskind Hamiltonian of gauge theory (1975)

Review on Quantum Computing for Lattice Field Theory
Lena Funcke, arXiv:2302.00467

👍Natural formula to see real-time evolution : $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$

👍No additional difficulty emerges by finite-density and topological theta-term

👎 ∞-dim. of Hilbert space of gauge field

- truncate Hilbert space (naive truncation breaks gauge sym. / q-deformation Zache et al. arXiv:2304.02527)
- change the continuous gauge to finite gauge group $(Z_N, D_N, \cdots)$

💡Quantum computer will "solve" this problem?

N-qubit system describes 2^N-dim. Hilbert space

cf.) classical N-bit : O(N)-dim.

1-qubit: $|\psi_1\rangle = \begin{pmatrix} * \\ * \end{pmatrix}$

N-qubit: $|\Psi\rangle = |\psi_1\rangle \otimes \cdots \otimes |\psi_N\rangle = \begin{pmatrix} * \\ \vdots \\ * \end{pmatrix}$

Moore's law for quantum devices

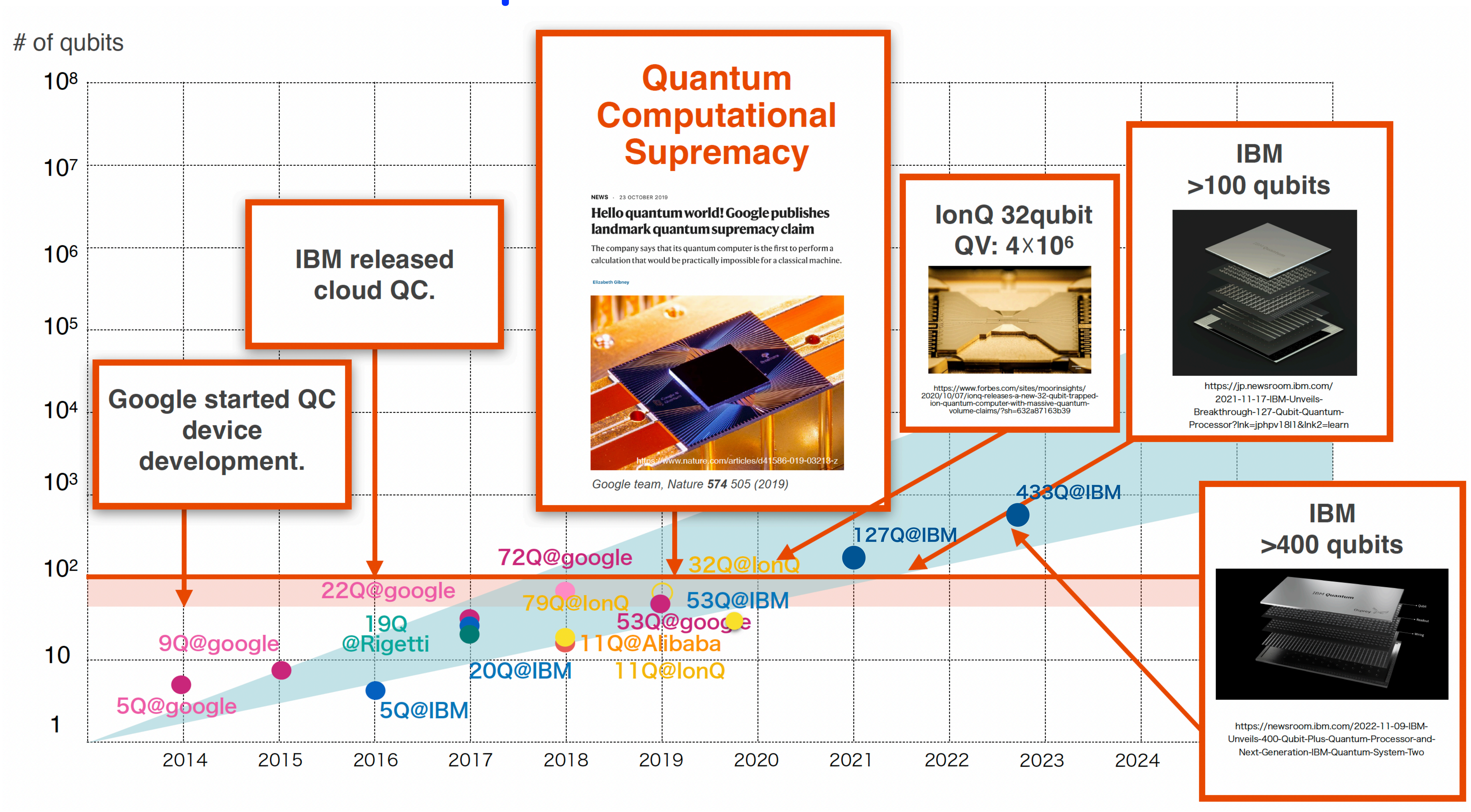
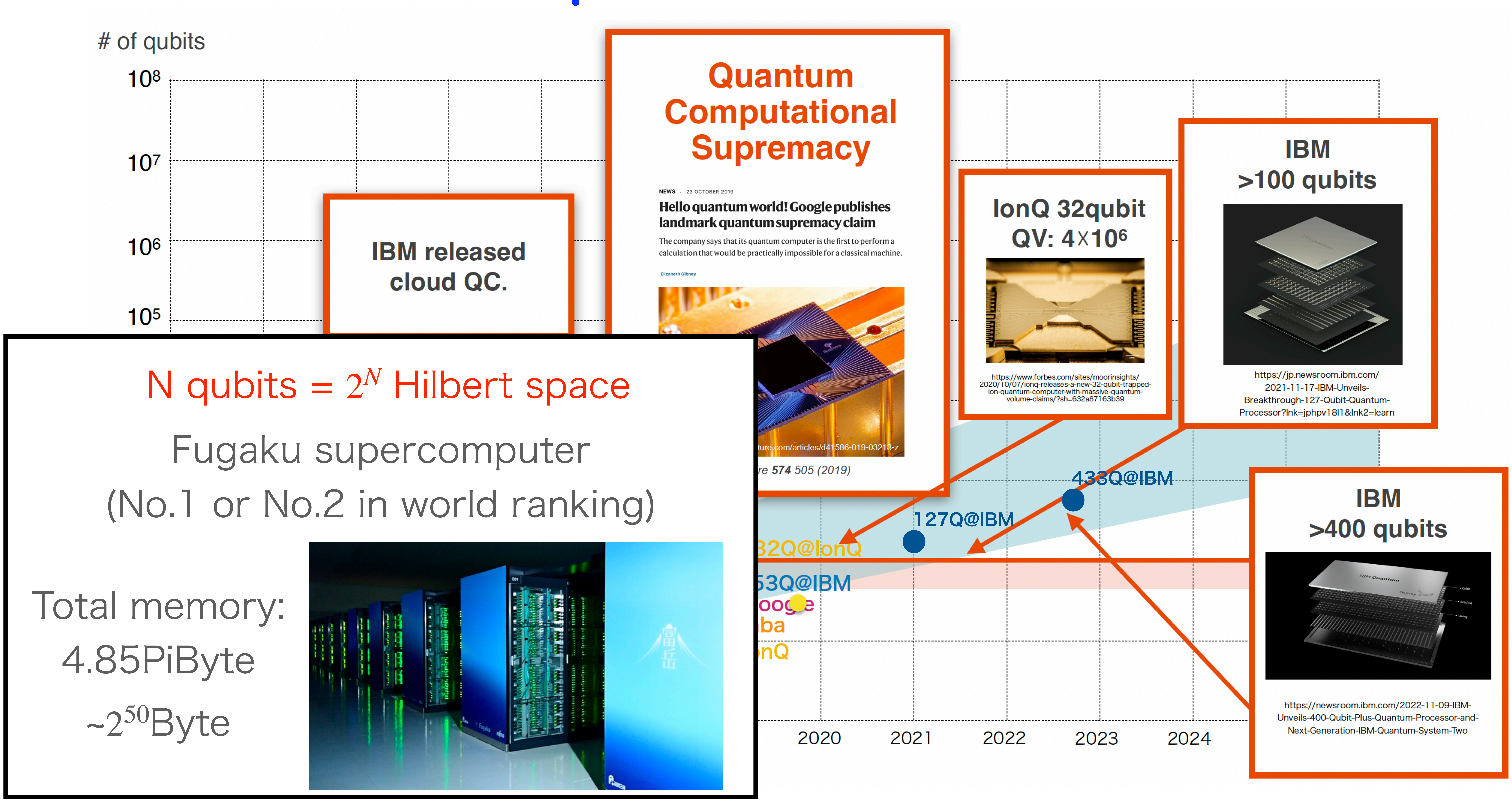


Figure given by Keisuke Fujii @QIQB, Osaka U.

Moore's law for quantum devices



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