Thermalization and hadronization of SU(N) gauge theories

Lukas Ebner, Joseph Lap, Berndt Müller, AS, Leonhard Schmotzer, Clemens Seidl, Xiaojun Yao

- "It from Qubit" and real experiments (just two examples):
  1.) High energy heavy ion collisions at LHC
  2.) Ultracold atoms
- AdS/CFT duality
- Question: Does 1+2 dim SU(2) gauge theory show ETH behavior? arXiv:2308.16202, 2401.15184
- Question: Can the 1+1 dim double split (Takayanagi et al.) be generalized? tbp

These are two important non-perturbative dynamical QCD questions which cannot be answered by pQCD or LQCD.

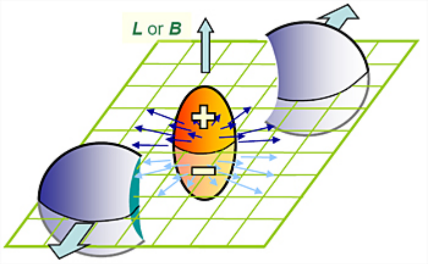
NISQ quantum computing could prove its power by answering such questions.

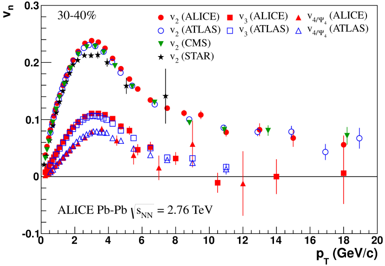
Simulating real time QCD processes on a quantum computer became the goal of many ongoing efforts. We would also love to do this.

However, there exist also holographic approaches and in addition quantum computations can be simulated on classical computers

We try to help clarifying what these approaches can do for real world, experimentally motivated questions in high-energy heavy-ion collisions.

Key questions of relativistic heavy ion physics: Does the quark gluon plasma realy thermalize? Is “hydrodynamization” equivalent to thermalization? Does thermal Lattice QCD describe experiment?





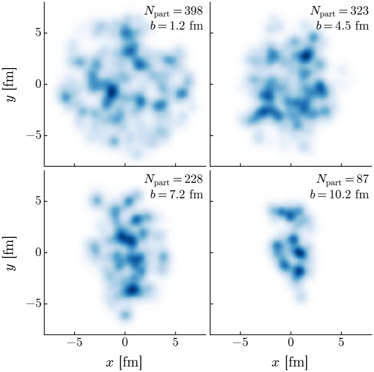
Observable: Elliptic flow $v_n \sim \cos(n\phi)$ with $n = 2$

How can transverse communication happen in less than 1fm/c?

$\gamma(Pb) > 2500$ giving it a width of $11\mathrm{fm}/2500 = 0.004\mathrm{fm}$

In QCD the transverse color coherence length is of order $1/Q_s < 0.2$ fm which is much smaller than the transverse size.

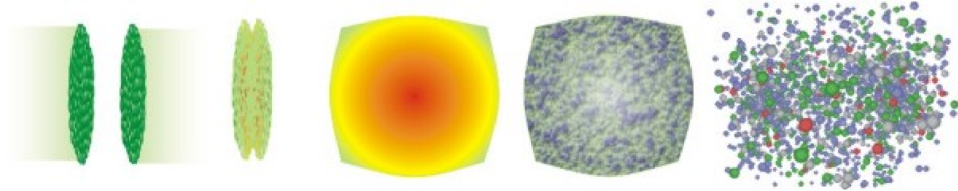
Nuclear fluctuations are large. arXiv:1605.03954



Also: Entropy cannot be produced because time dependence is unitary! The apparent thermalisation must be observable dependent.⇒ ETH “Eigenstate Thermalization Hypothesis”

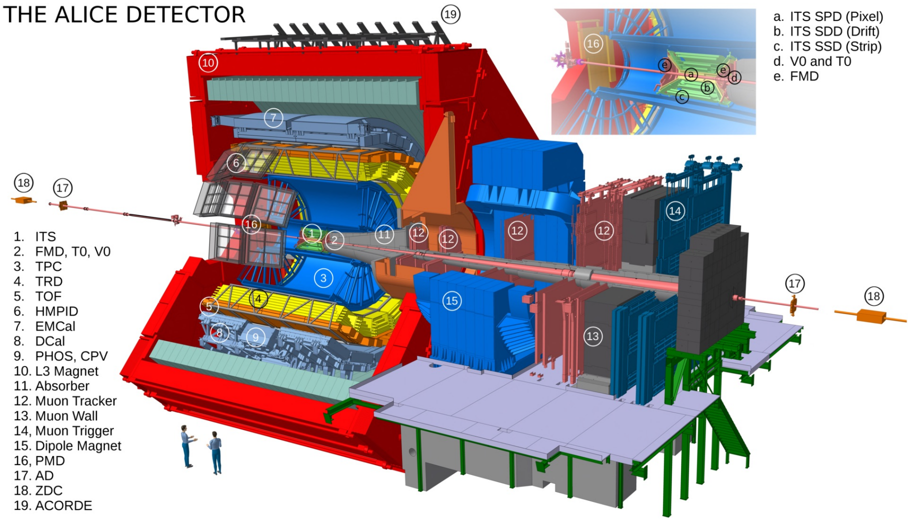
⇒ Focus on anomalies

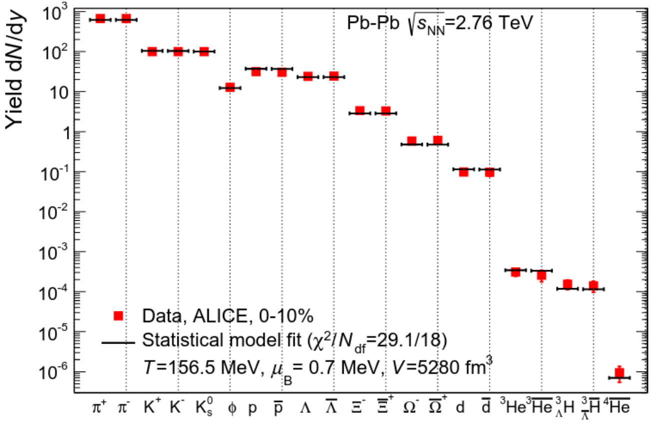
Just one example, the hadron yields: arXiv:1809.04681, ALICE, CERN



AdS/CFT clarified that hydrodynamization (local obervables) is fast.
ETH requires much longer to apply, see below (system wide correlations).

There is very much high precision data, e.g. from ALICE.





But: R(rms , ${}^{3}_{\Lambda}H$)=10.6 fm $\sim 2R_{\rm Pb}$;
$-B = 0.4 MeV << 156$ MeV the yield should be suppressed

One has two convincingly motivated interpretations which seem to be contradictory

- Hundreds of detailed measurements support the fireball interpretation, i.e. entropy production, hydrodynamics etc.
- General T-invariance suggest a microcanonical picture with highly entangled many particle quark-gluon and hadronic states.

One needs two standard elements of quantum information theory: Page curve plus ETH.

All of this concerns time dependence but are these really ideal problems for NISQ quantum computing or can they be answered without?

ETH

ETH: D'Alesio, Kafri, Polkovnikov, Rigol 1509.06411

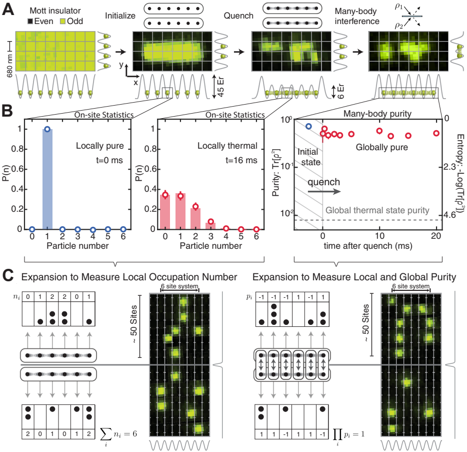
$$O_{mn} = \langle m|\hat{O}|n\rangle = O(\bar{E})\delta_{mn} + e^{-S(\bar{E})/2} f_O(\bar{E},\omega) R_{mn}$$

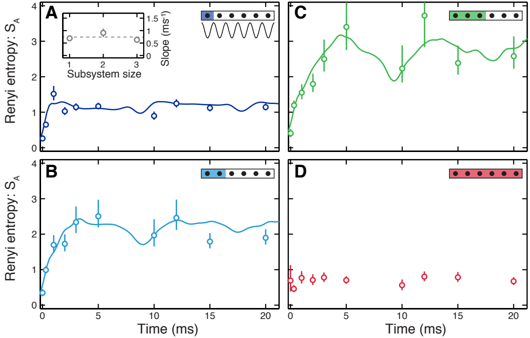
$\bar{E} = (E_m + E_n)/2$, $\omega = E_m - E_m)$, $S(\bar{E})$ thermodynamic entropy at energy $\bar{E}$, $O(\bar{E})$ and $f_O(\bar{E},\omega)$ are smooth functions, $O(\bar{E})$ is identical to the expectation value of the microcanonical ensemble at energy $\bar{E}$, and R_{mn} is a strongly fluctuating matrix (in the sense of RMT?)

Questions: For which operators does ETH apply? Does it apply to QCD?

A HIC in the ultra vacuum of the LHC is a prime example for an isolated system.

The experiment arXiv:1603.04409 “Quantum thermalization through entanglement in an isolated many-body system”

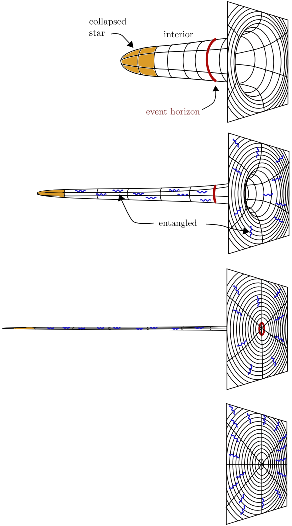


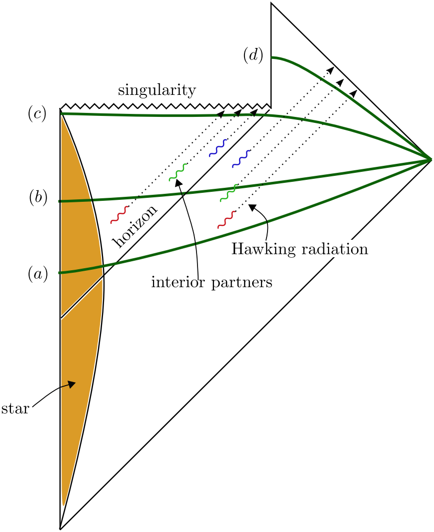


Subsystem entropy

$$S_A = -\log(Tr[\rho_A^2])$$

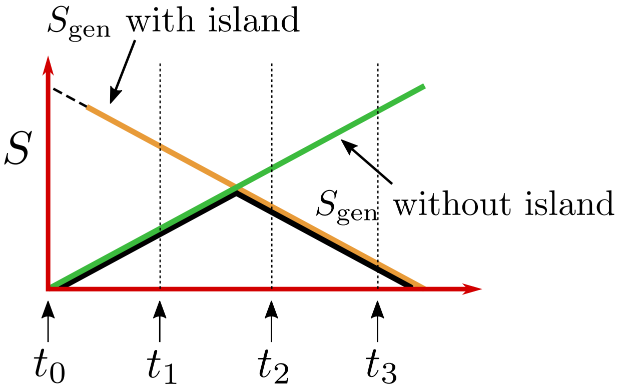
Island mechanism of BH evaportion Almheiri et al. 2006.06872





green: spatial slices

The Hawking radiation is entangled with an “island”.
This results in the Page curve

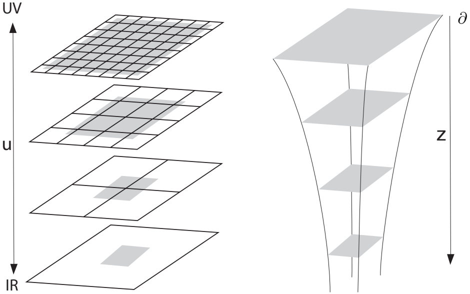


The ideas behind AdS/CFT nice review: Ramallo 1310.4319

renormalization flow of a SU(N) vertex function on ever coarser lattices

$$
\begin{aligned}
V(x,a) \quad \rightarrow \quad V(x,2a) \quad \rightarrow \quad V(x,4a) \quad \rightarrow \quad \ldots \\
u \quad = \quad a,\ \ 2a,\ \ 4a \\
\frac{\partial}{\partial \log u}\, g(u) \quad = \quad \beta(u)
\end{aligned}
$$

$$J|_{\mathrm{UV}} \quad = \quad \Phi|_{\partial}$$



geometric interpretation of new coordinate called z

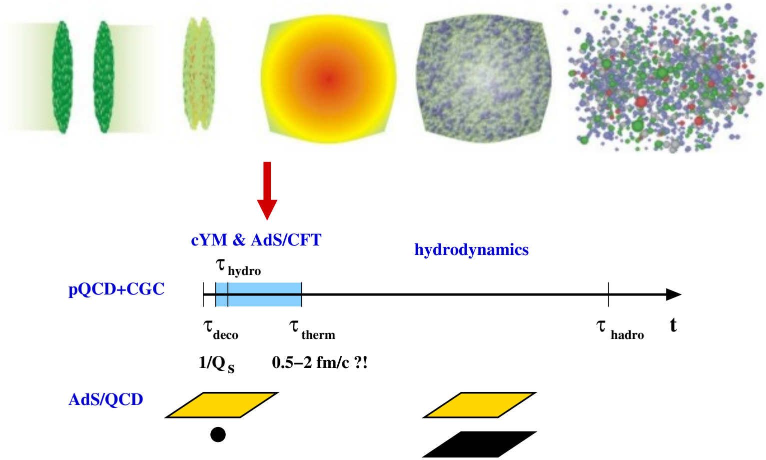
$$ds^2 = \Omega^2(z)\,[dt^2 - dx^i dx^i - dz^2]$$

The properties of the renormalization flow is only simple for conformal theories.

$$\begin{aligned} z &\rightarrow \lambda z \\ \Omega(z) &= \frac{L}{z} \rightarrow \lambda^{-1}\Omega(z) \\ ds^2 &= \frac{L^2}{z^2}\,[dt^2 - dx^i dx^i - dz^2] \qquad \text{AdS} - \text{metric} \end{aligned}$$

$SU(N)$, $\mathcal{N} = 4$ is conformal

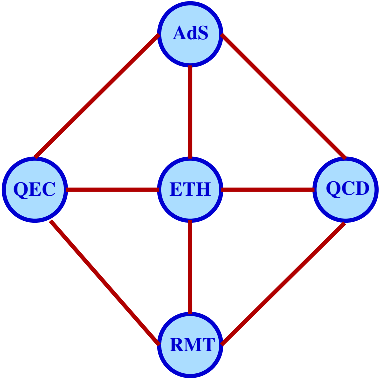
The AdS/CFT picture of HICs



entropy production = information loss

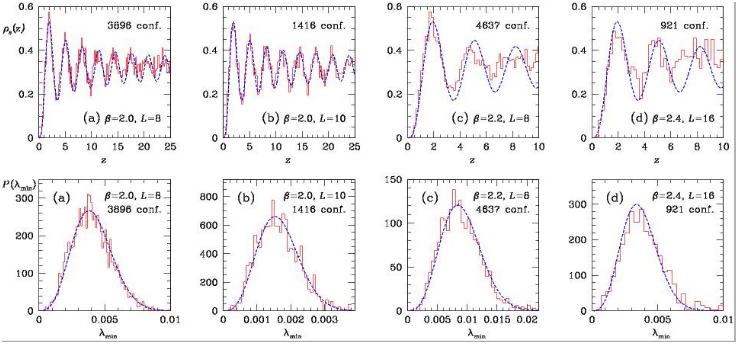
ETH could, e.g., explain the ${}^{3}_{\Lambda}H$ puzzle.

ETH predicts that small probes thermalize fast, large probes thermalize slowly and probes of > half the system do not thermalize completely.



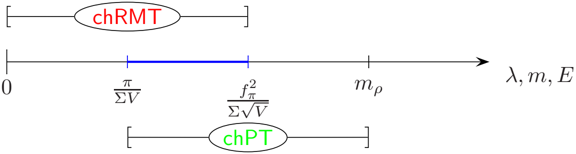
We performed many tests, e.g.: Berbenni-Bitsch, Meyer, AS, Verbaarschot and Wettig, “Microscopic universality in the spectrum of the lattice Dirac operator,” hep-lat/9704018

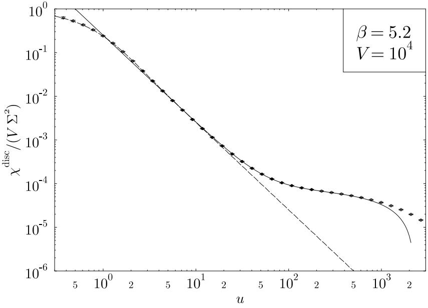
Comparison of microscopic level spacing for LQCD (red) and RMT(blue)



Simulations with quenched SU(3) Kogut-Susskind fermions

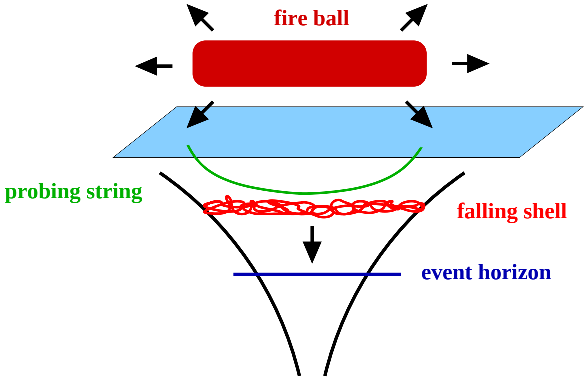
M. Göckeler, H. Hehl, P. Rakow, AS, T. Wettig hep-lat/0105011





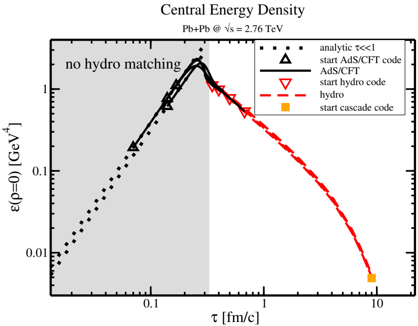
Equilibration times from AdS/CFT

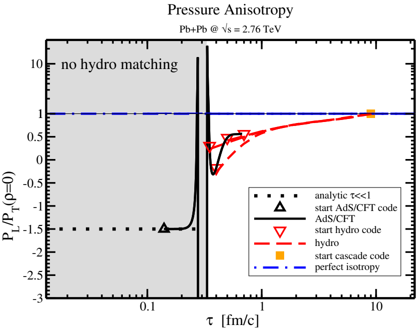
Idea: Probe black brane formation with a string or membrane, breaking conformal invariance by a “quench”;
V. Balasubramanian, J. de Boer, B. Craps, ..., B.Müller, AS
arXiv:1012.4753



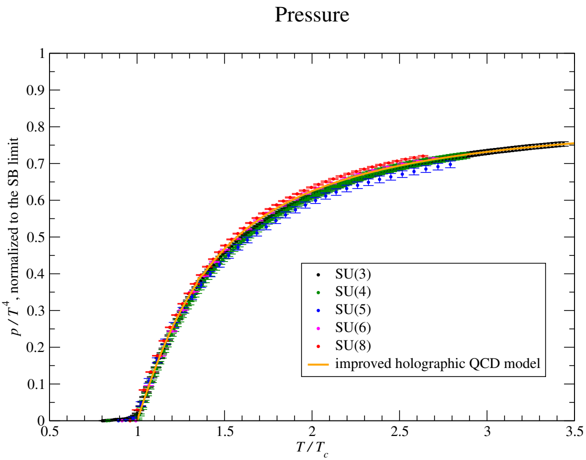
The AdS gravity equations result in a smooth transition to hydrodynamics. Viscous relativistic hydrodynamics is a gradient expansion which fails at early times. The late time behavior seems to be very stable and confirms perfect thermal and hydrodynamic behavior from 1fm/c on.

Hydrodynamics must, in fact, already apply at 1 fm/c to describe v_2 etc. This can be explained by AdS/CFT: Schee, Romatschke, Pratt 1307.2539



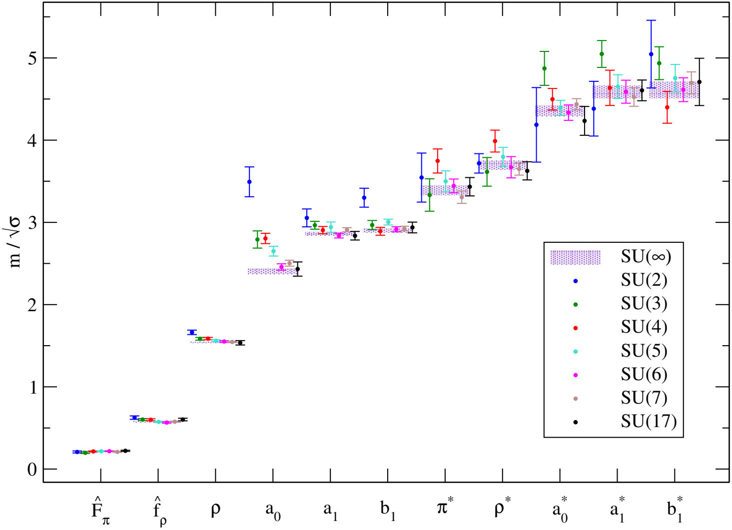


Does holography tell anything about SU(3) non-supersymmetric gauge theory? Various lattice tests



SU(*N*) pure gauge theory in 1+3 dimensions

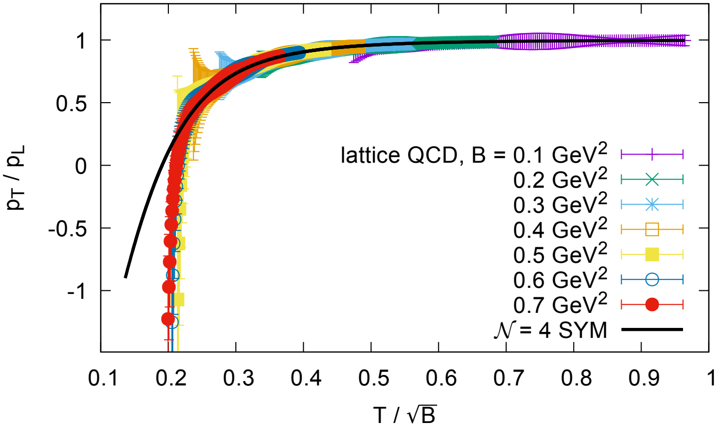
M. Panero, 0907.3719



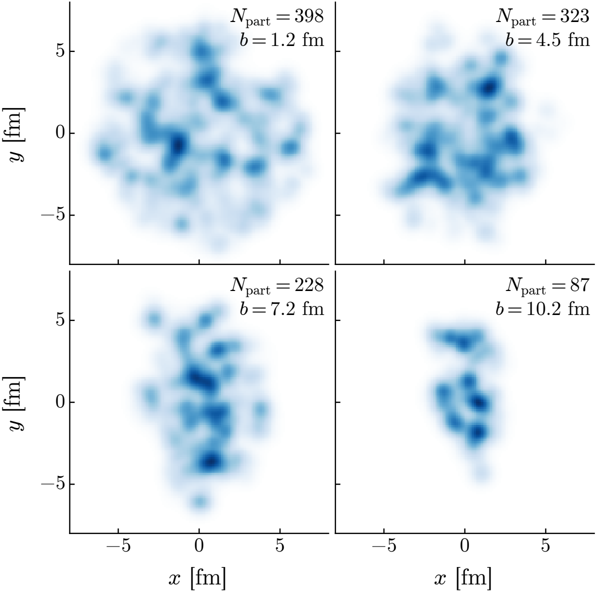
T = 0 meson spectrum and decay constants

G. Bali et. al, 1304.4437

Another test: QCD has no conformal symmetry (e.g scale anomaly, Λ_{QCD}) AdS is $\Rightarrow$ What happens if you break conformal symmetry explicitly by a background magnetic field? Endrodi, Kaminski, A.S, Wu and Yaffe, [arXiv:1806.09632].

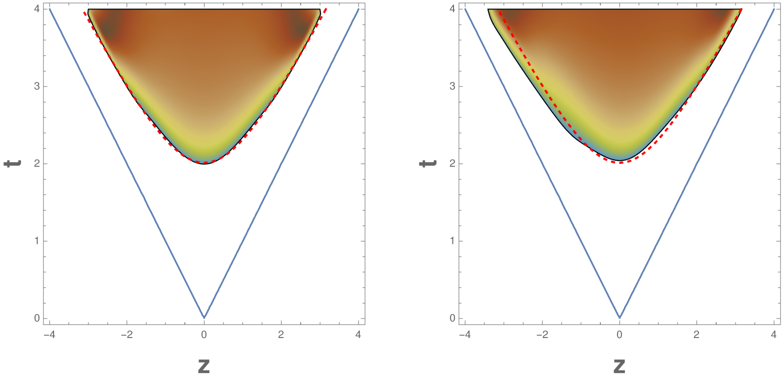


remember



$N_{\text{part}} = 398$
$b = 1.2$ fm
$N_{\text{part}} = 323$
$b = 4.5$ fm
$N_{\text{part}} = 228$
$b = 7.2$ fm
$N_{\text{part}} = 87$
$b = 10.2$ fm
y [fm]
x [fm]
5
0
−5

Also this can be described by AdS/CFT 1906.05086 Waeber, Yaffe et al.



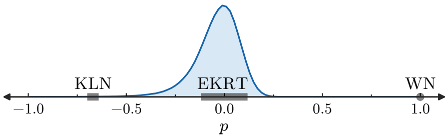
answer: Hydrodynamization occurs at **fixed eigenzeit** ⇒ basically not boost dependent, geometric mean

criterium: $\Delta = \frac{1}{p}\sqrt{\delta T^{\mu\nu}\,\delta T_{\mu\nu}} < 0.15$ with $\delta T^{\mu\nu} = T^{\mu\nu} - T^{\mu\nu}_{\rm hydro}$

Bernhard, Moreland, Bass Liu, Heinz arXiv:1605.03954 Fit result: parameterization of combined entropy density:

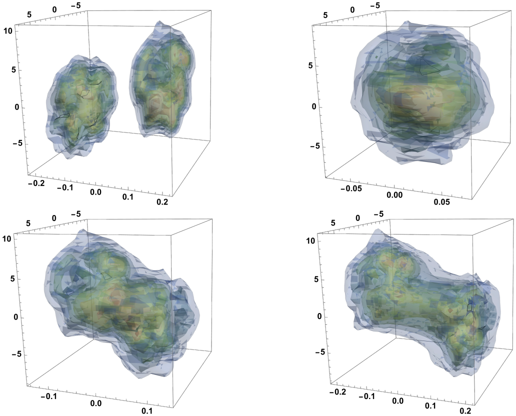
$$s \sim \left(\frac{\tilde{T}_A^p + \tilde{T}_B^p}{2} \right)^{1/p} \quad , \quad -\infty \le p \le \infty$$

$$\tilde{T}(x_{perp}) = \sum_{i=1}^{N_{part}} \frac{\gamma_i}{2\pi w^2} \exp\left(-\frac{(x_\perp - x_{i,\perp})^2}{2w^2} \right)$$



By construction the hydro initialization time must be identical for each transverse pixel. Both features are reproduced by AdS/CFT 1906.05086

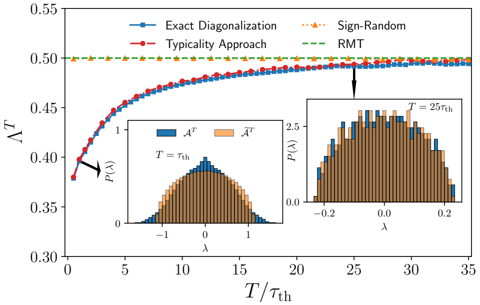
S. Waeber and L. Yaffe have tremendously improved the numerics in the meantime arXiv:2211.09190



energy density

Wang, Lamann, Richter, Steinigeweg, Dymarsky 2110.04085

The time needed to establish ETH behavior depends on the observable. Here for an Ising spin chain. It can take much longer than a HIC.



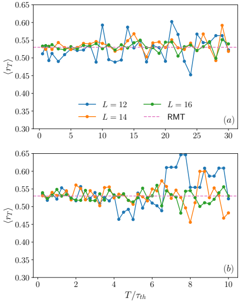
$$\Lambda^T = \frac{\mathcal{M}_2^2}{\mathcal{M}_4}; \qquad \mathcal{M}_k = \mathrm{Tr}[(\mathcal{O}_c^T)^k]/d; \qquad \mathcal{O}_c^T = \mathcal{O}^T - \mathrm{Tr}(\mathcal{O}^T)/d$$

energy window $\left[-\frac{\pi}{T}, \frac{\pi}{T}\right]$

the mean ratio of adjacent level spacings

$$\langle r_T \rangle = \frac{1}{d} \sum_{\alpha} \frac{\min(\Delta_\alpha, \Delta_{\alpha+1})}{\max(\Delta_\alpha, \Delta_{\alpha+1})}$$

gap between two adjacent eigenvalues $\Delta_\alpha = |\lambda^T_{\alpha+1} - \lambda^T_\alpha|$ of $\mathcal{O}^T$



We do the same for SU(2).

Testing ETH for SU(2): arXiv:2308.16202

L.Ebner, B. Müller, AS, C. Seidl, X. Yao

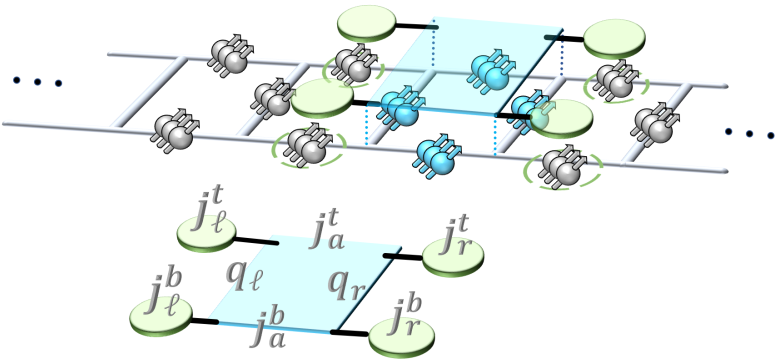
Time dependence from Hamiltonian lattice gauge theory would be the ideal tool but requires quantum computing.

$$\hat{H} = \frac{g^2}{2} \sum_{\text{links}} \hat{E}^2 - \frac{1}{2g^2} \sum_{\square} \left(\hat{\square} + \hat{\square}^\dagger \right)$$

$$\hat{\square} = \sum_{\alpha,\beta,\gamma,\delta=\frac{1}{2}}^{\frac{1}{2}} \hat{U}_{\alpha\beta} \hat{U}_{\beta\gamma} \hat{U}_{\gamma\delta} \hat{U}_{\delta\alpha} \quad .$$

Does SU(2) in e.g. 1+2 dimension show ETH behaviour? It can be simulated on classical computers, expressing it by spin couplings!!!

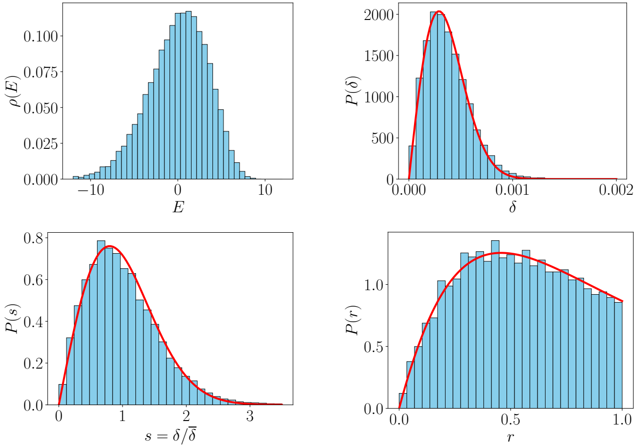
N. Klco, J. R. Stryker and M. J. Savage, arXiv:1803.03326



j_ℓ^t
j_a^t
j_r^t
q_ℓ
q_r
j_ℓ^b
j_a^b
j_r^b

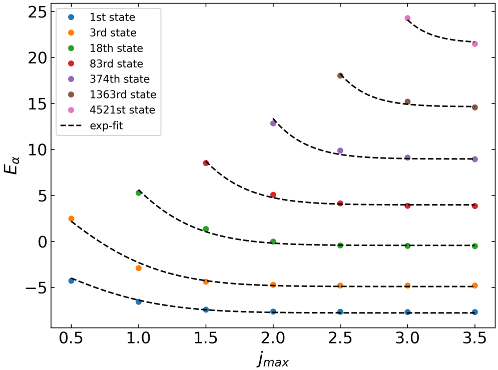
$$
\begin{aligned}
&\langle \chi_{\ldots, j_\ell^{t,b}, q_{\ell f}, j_{af}^{t,b}, q_{rf}, j_r^{t,b}, \ldots} | \hat{\Box} | \chi_{\ldots, j_\ell^{t,b}, q_{\ell i}, j_{ai}^{t,b}, q_{ri}, j_r^{t,b}, \ldots} \rangle = \\
&\sqrt{\dim(j_{ai}^t)\dim(j_{af}^t)\dim(j_{ai}^b)\dim(j_{af}^b)} \\
&\times \sqrt{\dim(q_{\ell i})\dim(q_{\ell f})\dim(q_{ri})\dim(q_{rf})} \\
&\times (-1)^{j_\ell^t + j_\ell^b + j_r^t + j_r^b + 2(j_{af}^t + j_{af}^b - q_{\ell i} - q_{ri})} \\
&\times \begin{Bmatrix} j_\ell^t & j_{ai}^t & q_{\ell i} \\ \frac{1}{2} & q_{\ell f} & j_{af}^t \end{Bmatrix} \begin{Bmatrix} j_\ell^b & j_{ai}^b & q_{\ell i} \\ \frac{1}{2} & q_{\ell f} & j_{af}^b \end{Bmatrix} \begin{Bmatrix} j_r^t & j_{ai}^t & q_{ri} \\ \frac{1}{2} & q_{rf} & j_{af}^t \end{Bmatrix} \begin{Bmatrix} j_r^b & j_{ai}^b & q_{ri} \\ \frac{1}{2} & q_{rf} & j_{af}^b \end{Bmatrix}
\end{aligned}
$$

Test of GOE predictions:

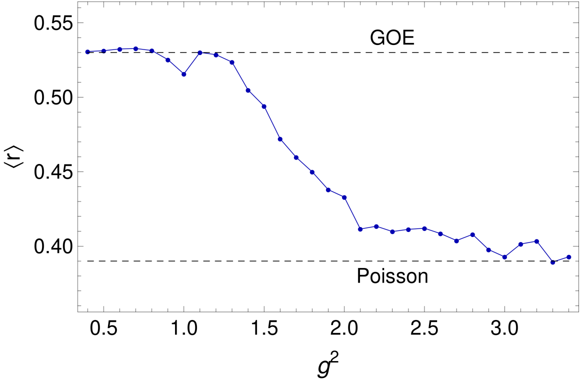


Density of eigenstates, distributions of gaps, rescaled gaps and gap ratios in the momentum $k_x = k_y = 1$ sector on the $N_x = 5, N_y = 4$ lattice for $g^2 = 0.75$.

Test of $j_{\max}$ convergence.

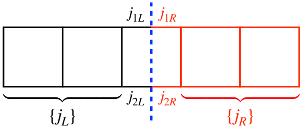


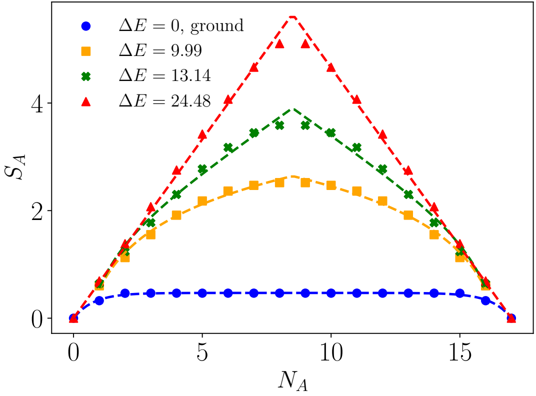
g^2 dependence of the restricted gap ratio $\langle r \rangle$. GOE predicts 0.53, Poisson predicts 0.39.



$$r = \frac{\min[\delta_\alpha, \delta_{\alpha-1}]}{\max[\delta_\alpha, \delta_{\alpha-1}]} \qquad \delta_\alpha = E_{\alpha+1} - E_\alpha$$

The Page curve for a chain of 17 plaquettes 2401.15184

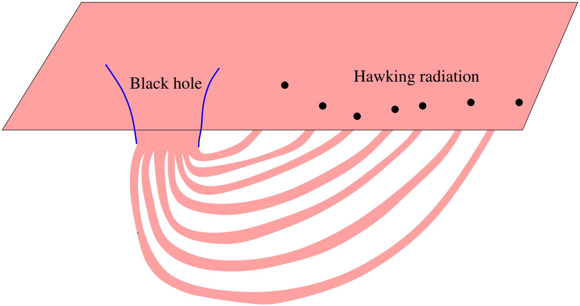




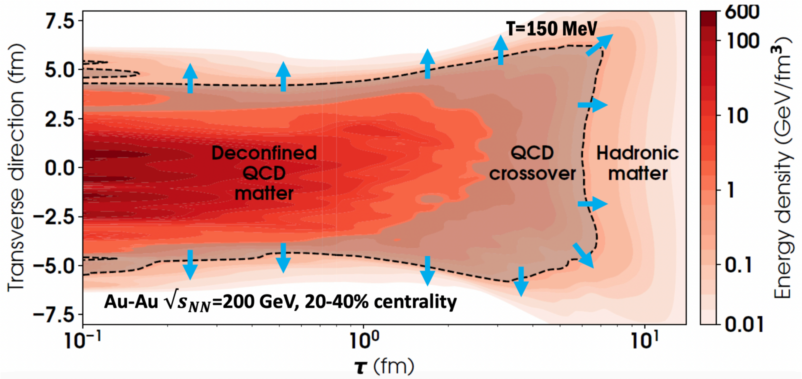
Duality links entanglement in the QFT to geometry in AdS/CFT, which is much easier to describe

Maldacena and Susskind 1306.0533

entangled CFT's in the boundary = Einstein-Rosen bridges in the holographical dual (EPR=ER).

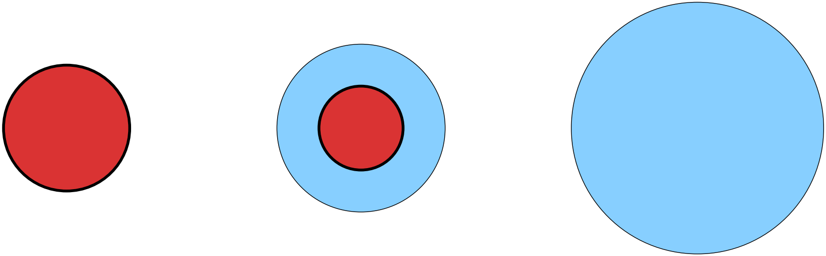


The long times dynamics of HICs is complicated. Gale et al., arXiv:2009.07841 (80% final freeze-out, 20 % hadron radiation)



Is there a holographic geometric describtion of hadronization?

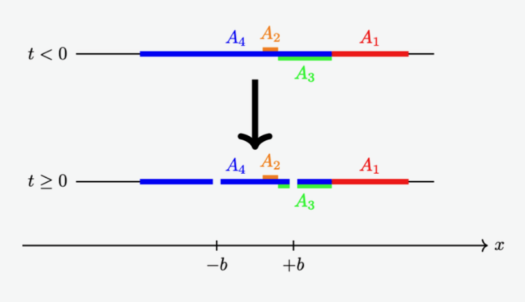
The analogy:



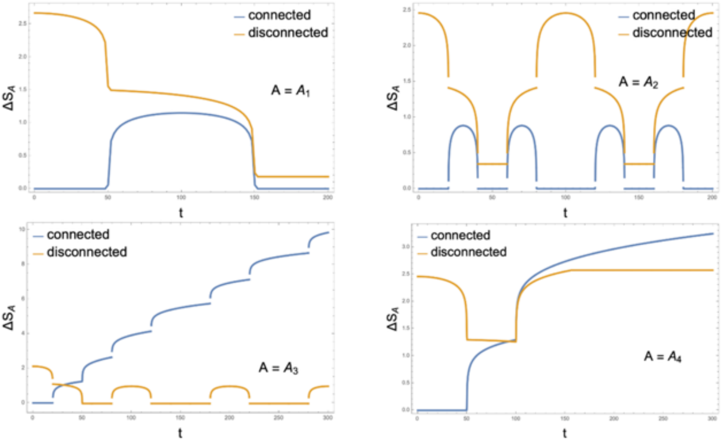
Fully entangled QGP | **Entangled QGP plus hadrons** | **Fully entangled hadron gas**

Can hadron-hole production at the boundary be treated in analogy to BH physics?
Does Monogamy of entanglement affect all of this?

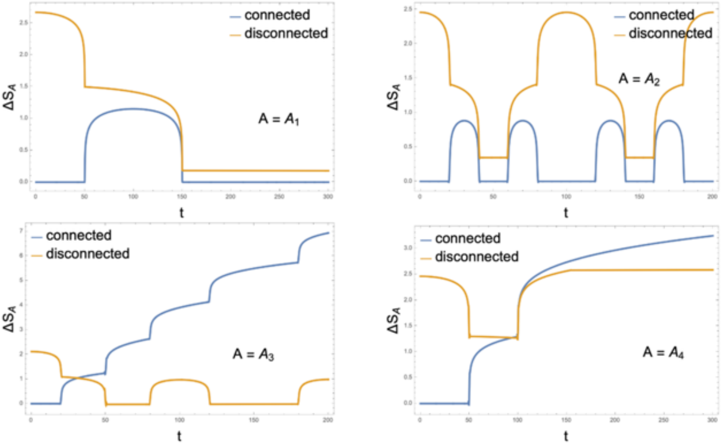
- Confinement: Quarks and gluons can only exist inside of the QCD fireball or hadrons. Hadronization can be regarded as multi-split of the quark-gluon universe.
- The double split was treated by Caputa, Numasawa, Shimaji, Takayanagi, and Wei for 1+1 d, 1905.08265
- We were not able to generalize their method to a multi-split
- But we (i.e. Clemens Seidl and Joseph Lap) found two alternative methods which might be generalizable (stay tuned).



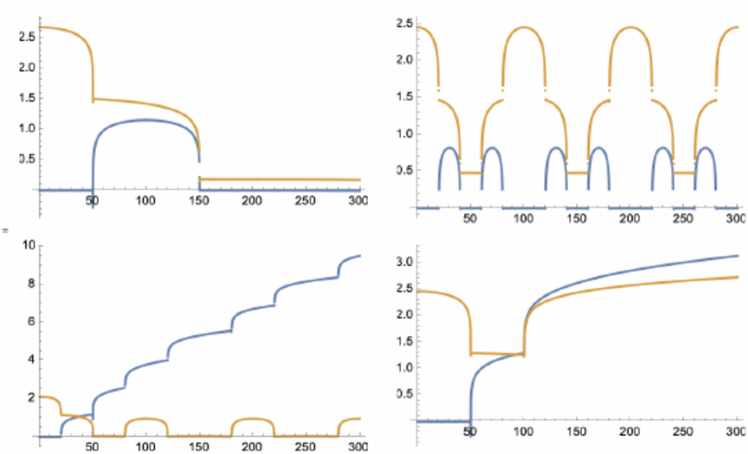
Different subregions are affected differently by the double split



entanglement entropy, inverse map: A_1 (top left), A_2 (top right),A_3 (bottom left), A_4 (bottom right)



entanglement entropy Abel-Jacobi map: A_1 (top left), A_2 (top right),A_3 (bottom left), A_4 (bottom right)



entanglement entropy Schottky uniformization: A_1 (top left), A_2 (top right),A_3 (bottom left), A_4 (bottom right)

As the entanglement entropy is calculated from the length of the geodate according to Ryu and Takayanagi [hep-th/063001]

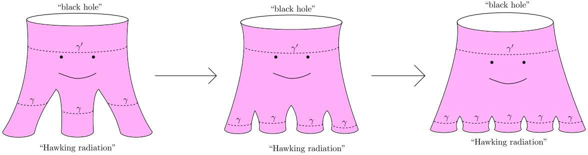
$$S_A = \frac{\text{length of } \gamma_A}{4G_N^{(3)}}$$

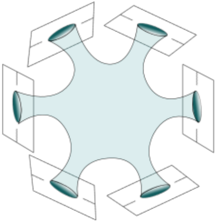
such that the results agree when the line elements agrees.

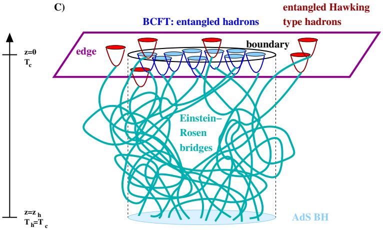
$$ds^2 = \frac{R^2}{\zeta^2}\left(2\left(1 - \frac{\pi^2\zeta^2}{2}\right)^2 d\Xi^2 + 2\left(1 + \frac{\pi^2\zeta^2}{2}\right)^2 d\Upsilon^2 + d\zeta^2\right)$$

However, one still has to show that Ξ and Υ depende in the same manner on the split parameters. Instead we show that the entanglement entropies agree.

How does the holographic dual of the multi-split look like?







Conclusions

- ETH, decoherence and thermalization of isolated quantum systems are topics of universal interest.
- Heavy Ion Collisions in the ultra-high vacuum of, e.g. the LHC, offer an ideal situation to study them. There are many Pbyte of data, the question is how to interpret them.
- There exist many technically different approaches (classical nonlinear dynamics, RMT and ETH, Lattice QCD, AdS/CFT, QCD phenomenology, pQCD, hydrodynamics, quantum computing ...) which are expected to provide compatible pieces of this puzzle.
- We have started to simulate quantum computing on classical computers.
- So far everything is compatible with SU(2) fulfilling ETH.