Interplay of Chiral Transitions in the Standard Model

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Motivation - Mass Generation in the Standard Model

- Fundamental fermionic degrees of freedom in the Standard Model are massless while all matter particles possess mass

  Two symmetry breaking mechanisms:

  - Higgs mechanism, spontaneously breaks gauge and chiral symmetry
  - Chiral symmetry breaking in the strong interaction

- Investigate interplay of these two mechanisms in a suitable toy model

The Functional Renormalization Group

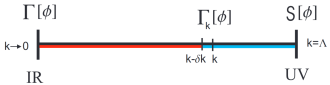
Modified generating functional

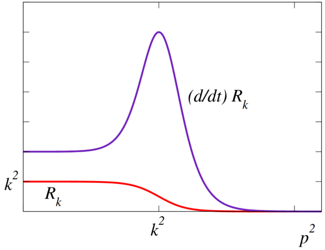
$$e^{W_k[J]} = \int \mathcal{D}\varphi e^{-S[\varphi]-\Delta S_k[\varphi]+\int_x J\varphi}$$

where:

$S[\varphi]$: action of the theory

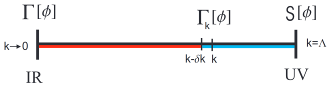
$\Delta S_k[\varphi] = \int_x \frac{1}{2}\varphi(x)R_k(x)\varphi(x)$:
momentum dependent mass term





Typical form of the regulator $R_k(p^2)$ and its derivative [Gies '06].

The Functional Renormalization Group

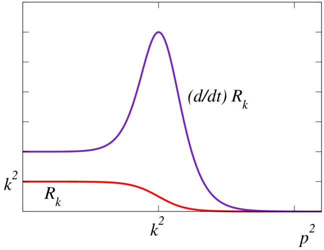


Effective average action

$$\Gamma_k[\phi] = \sup_J \left(\int \mathrm{d}^d x J\phi - W_k[J] \right) - \Delta S_k[\phi]$$

$\Rightarrow$ Wetterich equation [Wetterich '93]

$$\boxed{\partial_t \Gamma_k[\Phi] = \frac{1}{2}\, \mathrm{STr}\left[\partial_t R_k \left(\Gamma_k^{(2)}[\Phi] + R_k \right)^{-1} \right]}$$



Typical form of the regulator $R_k(p^2)$ and its derivative [Gies '06].

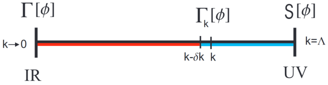
Flow Equations

Truncate the effective average action

$\rightarrow$restrict to manageable amount of operators

Extract flow equations of operators by suitable projection

Compare different truncation schemes as convergence checks



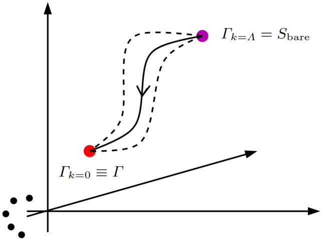


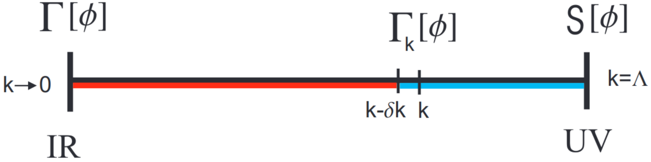
Figure: RG flow of the effective action in theory space [Gies '06].

Higgs mechanism

The Higgs mechanism induces masses for all Fermions as well as the weak bosons through the scalar potential acquiring a non-zero vacuum expectation value (vev)

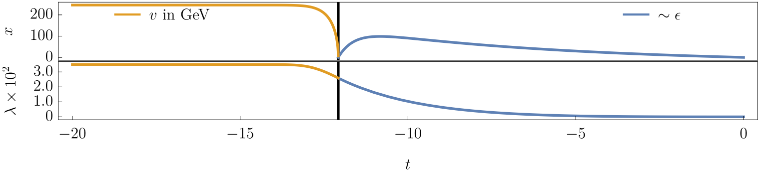
E.g. look at the flow of a scalar potential in a Higgs-Yukawa toy model

$$\Gamma_k = \int_x (\partial_\mu \phi)^2 + U(\phi^2) + \bar{\psi} \mathrm{i} \not{\partial} \psi + \mathrm{i} h \bar{\psi} \phi \psi$$

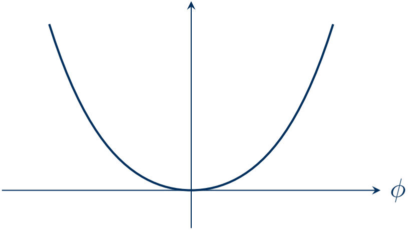


vev $\neq 0$ $\Leftarrow$ vev $= 0$

Mass Generation



$$U = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4, \quad U(\phi) = U(-\phi)$$



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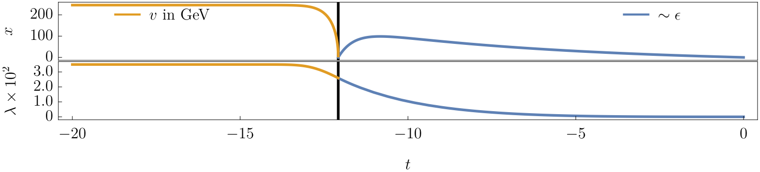

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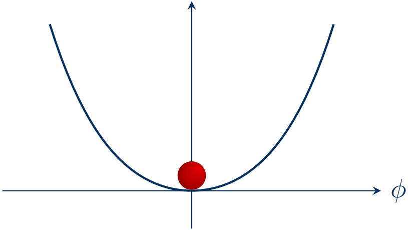
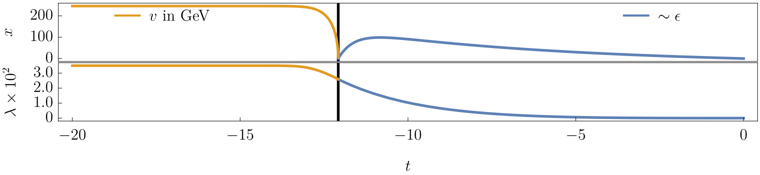
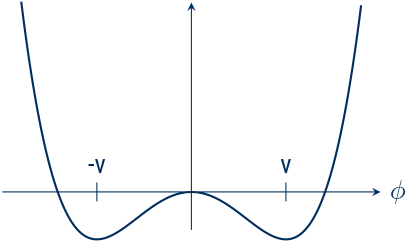


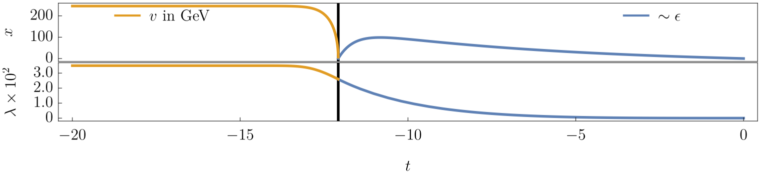
Mass Generation



$$U = \frac{\lambda}{4!}(\phi^2 - v^2)^2, \quad U(\phi) = U(-\phi)$$



Mass Generation



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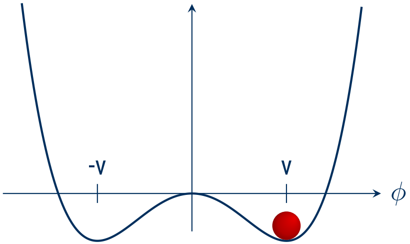
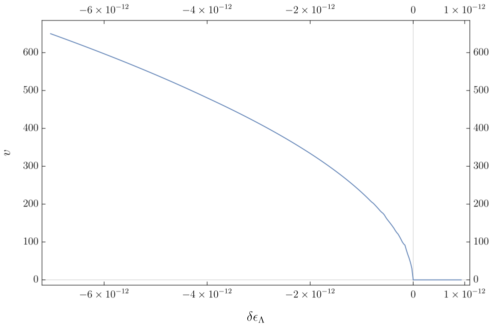


Higgs mechanism - second order phase transition

Varying the couplings in the UV changes the obtained vev



$$\epsilon_\Lambda = \frac{m^2}{\Lambda^2}$$

Larger values of ϵ_Λ: Potential stays symmetric

Smaller values of ϵ_Λ: Potential develops minimum at non-vanishing field.

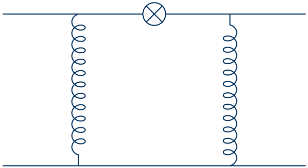
The vev as a function of the control parameter is described by a second order quantum phase transition (tied to the fine tuning problem)

Quantum Chromodynamics I

Fundamental QCD action

$$S_{\mathrm{QCD}} = \int_x \bar{\psi}_i^a (\mathrm{i}\partial\!\!\!/ \delta_{ij} + \bar{g} A\!\!\!/_{ij}) \psi_j^a + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{(\partial_\mu A^\mu)^2}{2\xi}$$

induces effective four-fermion vertices through quantum fluctuations



$\Rightarrow$ At intermediate scales: NJL-type interactions included

Quantum Chromodynamics II

$$\Gamma_k = \int_x \bar{\psi}_i^a (\mathrm{i} Z_\psi \partial\!\!\!/ \delta_{ij} + \bar{g} A\!\!\!/_{ij}) \psi_j^a + \frac{Z_F}{4} F_{\mu\nu} F^{\mu\nu} + \frac{(\partial_\mu A^\mu)^2}{2\xi} + \frac{1}{2} \bar{\lambda}_\sigma (\mathrm{S} - \mathrm{P}).$$

$$\text{where } (\mathrm{S} - \mathrm{P}) = \left(\bar{\psi}_i^a \psi_i^b\right)^2 - \left(\bar{\psi}_i^a \gamma_5 \psi_i^b\right)^2$$

Flow of four fermion coupling $\partial_t \lambda_\sigma \sim g^4$

Towards IR: $g \nearrow$ due to asymptotic freedom, hence $\lambda_\sigma \nearrow$

$\rightarrow$ Non-perturbative effects like χSB require effective low energy description

Dynamical Bosonization

Translate microscopic degrees of freedom to macroscopic ones (quarks, gluons $\rightarrow$ mesons)

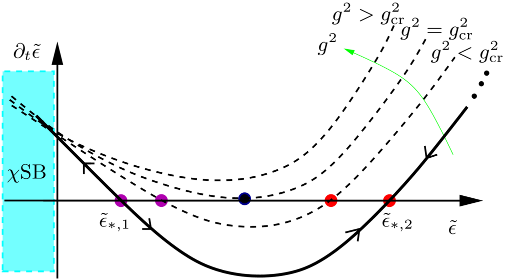
$$\bar{\psi}_i^a \psi_i^b \quad \rightarrow \quad \varphi^{ab}$$

Encode four-fermion interaction in Yukawa interaction

$$\lambda_\sigma \left(\left(\bar{\psi}_i^a \psi_i^b \right)^2 - \left(\bar{\psi}_i^a \gamma_5 \psi_i^b \right) \right) \qquad h \bar{\psi}_i^a \left(P_\mathrm{R} \varphi^{ab} + P_\mathrm{L} \varphi^{\dagger ab} \right) \psi_i^b$$

On all scales k: Additional contributions to the flow eqs. of the Yukawa model to compensate the 4-Fermi coupling

χSB: from quarks and gluons to mesons and bound states



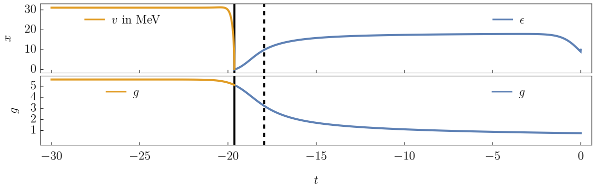
Flow equation for $\tilde{\epsilon} \sim m^2$ of the auxiliary field φ [Gies '06].

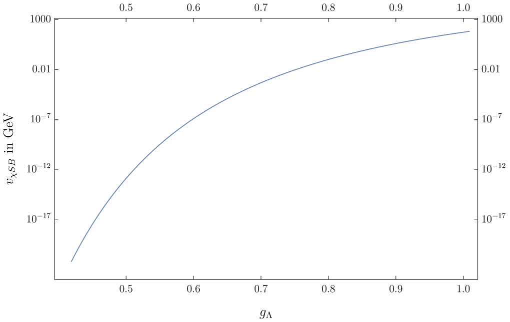
As long as $g < g_{\rm cr}$: $\tilde{\epsilon}$ controlled by fixed point structure

If $g > g_{\rm cr}$: Fixed points annihilate, $\tilde{\epsilon}$ runs fast towards negative values ($m^2 < 0$: χSB)

$\rightarrow$ Scale at which χSB is triggered ($\sim \Lambda_{\rm QCD}$) is set by the strong gauge coupling.

χSB: from quarks and gluons to mesons and bound states





Phase transition in QCD described as a crossover

$\rightarrow$ No fine tuning associated with χSB in QCD.

Reparametrization of the scalar field

Reparametrization of the auxiliary scalar field φ as

$$\Phi^{ab} = \frac{1}{\sqrt{2}} \left(\varphi^{ab} + \epsilon^{ac} \epsilon^{bd} \varphi^{*\,cd} \right)$$

$$\tilde{\Phi}^{ab} = \frac{1}{\sqrt{2}} \left(\varphi^{*\,ab} - \epsilon^{ac} \epsilon^{bd} \varphi^{cd} \right)$$

and setting

$$\phi^a \equiv \Phi^{a2}$$

$$\tilde{\phi}^a \equiv \tilde{\Phi}^{2a}$$

allows us to identify the $\mathrm{SU}(2)_\mathsf{L}$ doublet (ϕ), which acquires its own dynamics through standard model interactions

$$\mathscr{L}_{\mathsf{Yuk}} = \frac{\mathrm{i} h_b}{\sqrt{2}} \left(\bar{\psi}^a_{\mathsf{L},i} \phi^a b_{\mathsf{R},i} + \text{h.c.} \right) + \frac{\mathrm{i} h_t}{\sqrt{2}} \left(\bar{\psi}^a_{\mathsf{L},i} \phi^a_{\mathcal{C}} t_{\mathsf{R},i} + \text{h.c.} \right)$$
$$+ \frac{\mathrm{i} h}{\sqrt{2}} \left(\bar{\psi}^a_{\mathsf{R},i} \tilde{\phi}^a b_{\mathsf{L},i} + \bar{\psi}^a_{\mathsf{R},i} \tilde{\phi}^a_{\mathcal{C}} t_{\mathsf{L},i} + \text{h.c.} \right)$$

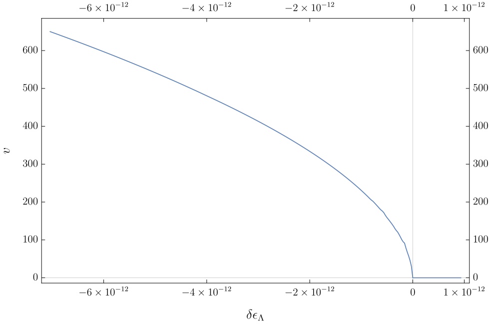
The Full Model

Reparametrization of the meson field to isolate the $\mathrm{SU}(2)_\mathrm{L}$ doublet (Φ) yields

$$\begin{aligned}\Gamma_k = \int_x \; & Z_\phi |\partial_\mu \phi|^2 + Z_{\tilde{\phi}} \left|\partial_\mu \tilde{\phi}\right|^2 + \bar{\psi}_i^a \mathrm{i} \not{D}_{ij} \psi_j^a + \frac{Z_\mathrm{F}}{4} F^z_{\mu\nu} F_z^{\mu\nu} \\ & + \frac{(\partial_\mu A^\mu)^2}{2\xi} + (\text{Ghosts}) + U(\rho, \tilde{\rho}) \\ & + \frac{\mathrm{i} h_t}{\sqrt{2}} \left(\bar{\psi}^a_{\mathrm{L},i} \phi^a_{\mathcal{C}} t_{\mathrm{R},i} + \text{h.c.}\right) + \frac{\mathrm{i} h_b}{\sqrt{2}} \left(\bar{\psi}^a_{\mathrm{L},i} \phi^a b_{\mathrm{R},i} + \text{h.c.}\right) \\ & + \frac{\mathrm{i} h}{\sqrt{2}} \left(\bar{\psi}^a_{\mathrm{R},i} \tilde{\phi}^a b_{\mathrm{L},i} + \bar{\psi}^a_{\mathrm{R},i} \tilde{\phi}^a_{\mathcal{C}} t_{\mathrm{L},i} + \text{h.c.}\right)\end{aligned}$$

$\Rightarrow$ analyze parametric dependence of phase transitions in this model

Phase Transitions

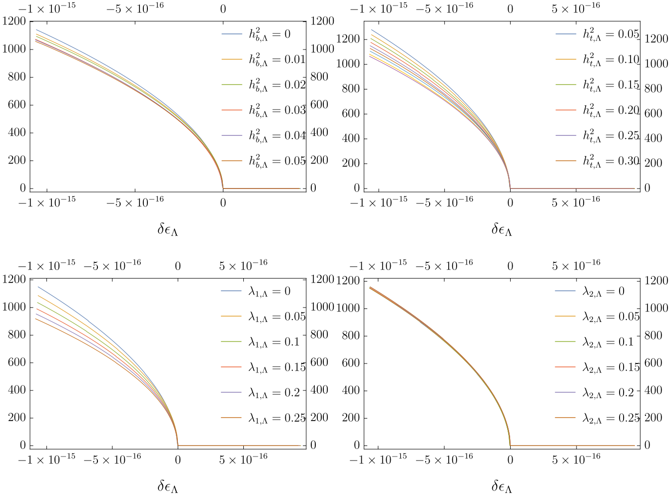


Scaling near quantum phase transition described by critical exponents

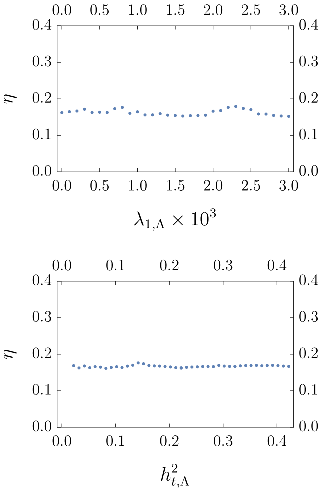
For the order parameter we have $v \propto |\delta\epsilon_\Lambda|^\beta$

β can be related to the critical exponent η which measures the deviation of the RG scaling exponent from its canonical value $\Theta = 2 - \eta$

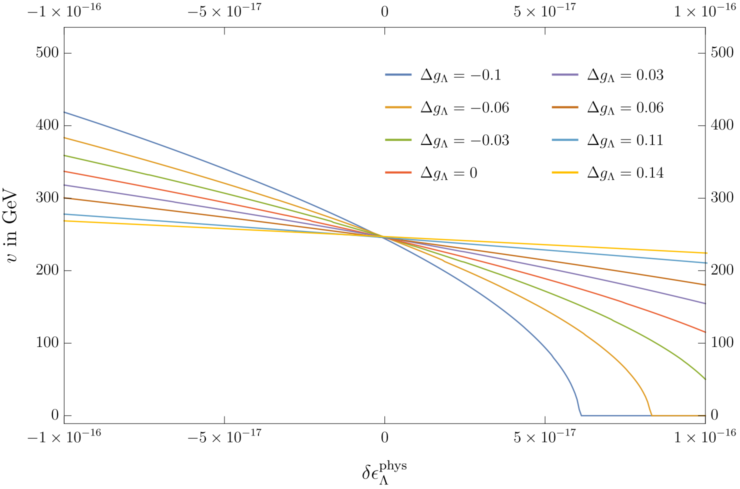
Phase Transitions for different Model Parameters



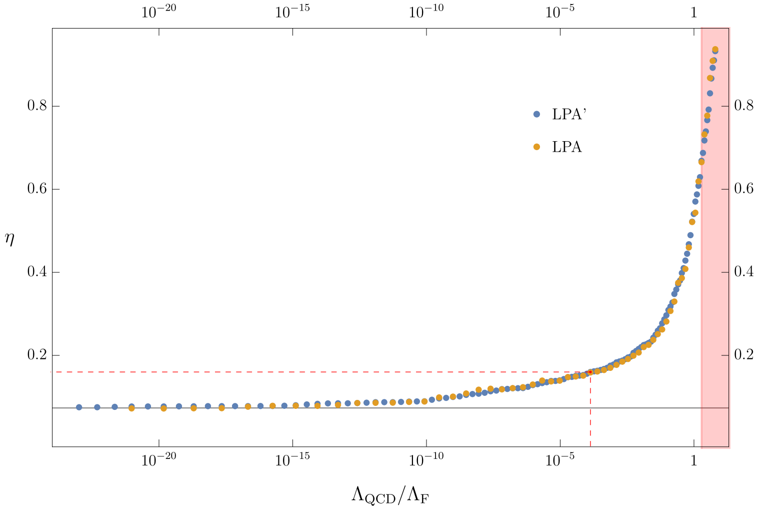
Critical Exponents for different Model Parameters



Phase transitions for increasing QCD coupling



Critical Exponents for increasing QCD coupling



Conclusions

Able to quantify interplay of the two breaking mechanisms through (pseudo-)critical exponents:

- Stronger gauge interactions lessen the fine-tuning necessary
- Other parameters have little influence on the phase transition

Further points of interest:

$\Rightarrow$ Study influence of the electroweak sector on the phase transition

$\Rightarrow$ Analyze fine tuning for asymptotically free solutions [Gies ’19]

Thank you for your attention

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Are there any questions?

Literature I

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