UNIVERSITÄT LEIPZIG

Charged scalar field in (near-extremal) Reissner-Nordström spacetime

19 Sept 2023

Maria Alberti

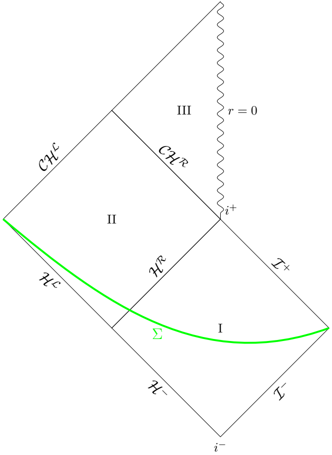
Physik-Combo MPI MiS

Outline

Overview and motivation

Strong cosmic censorship

– BH interiors → determinism?



Strong cosmic censorship

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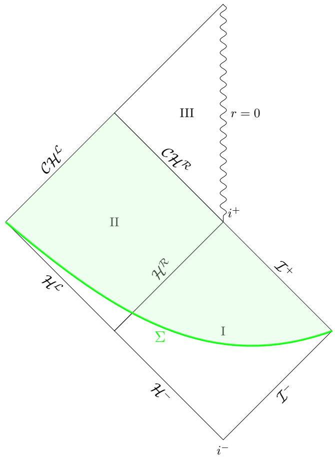
  - Cauchy hypersurface Σ
  - Domain of dependence $D^+(\Sigma)$



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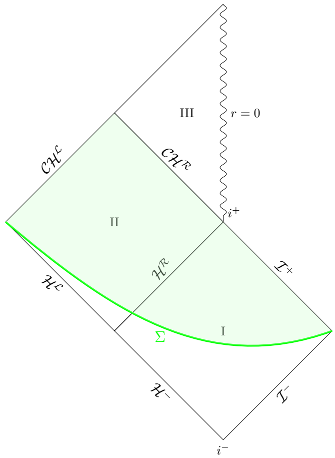

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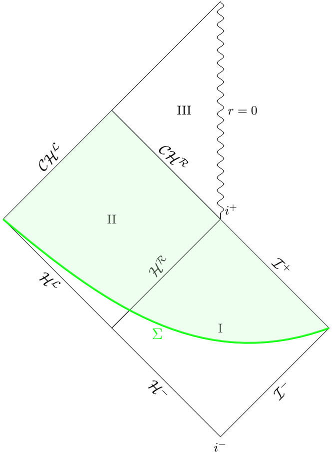

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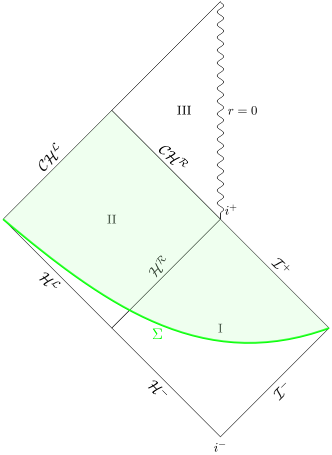

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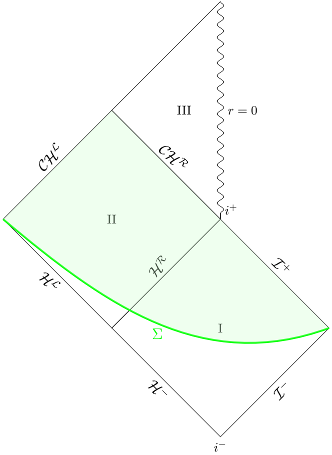

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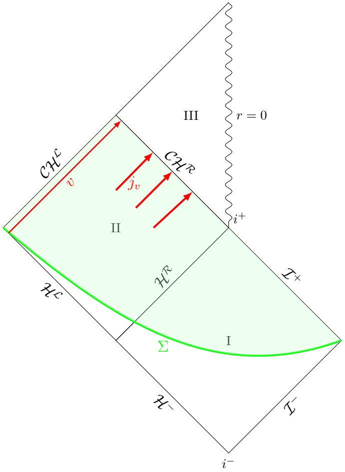
    (weak) backreaction: $\partial_v Q(u,v) = -4\pi r^2 \langle j_v \rangle_U$

    $\Longrightarrow$ (dis)charge BH interior RNdS



QFTCS: an overview

- Quantum fields on curved background (classical) spacetimes
- Semiclassical Einstein-Maxwell equations

$$G_{\mu\nu} = 8\pi(\langle T_{\mu\nu}\rangle_U + T^{EM}_{\mu\nu})$$
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1. Fix the background geometry $\Longrightarrow$ Reissner-Nordström ST
2. Compute EVs of observables of a certain field in a (Hadamard) state of choice $\Longrightarrow$ Charged scalar field in the Unruh state
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   $\implies$ Charged scalar field in the Unruh state
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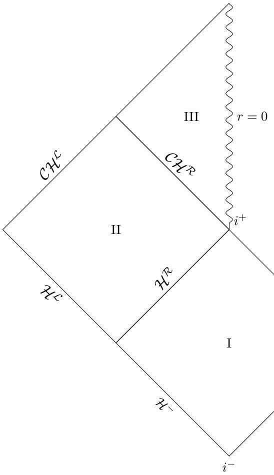
- Analytical analysis of real scalar in (near-extremal) RN [Zilberman and Ori 2021]
- My MSc. project: can this be generalized to the charged scalar?

Setup

Setup: background geometry



- Reissner-Nordström ST

$$g_{\mu\nu} = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$$
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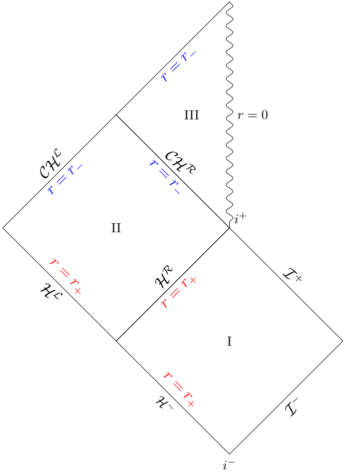

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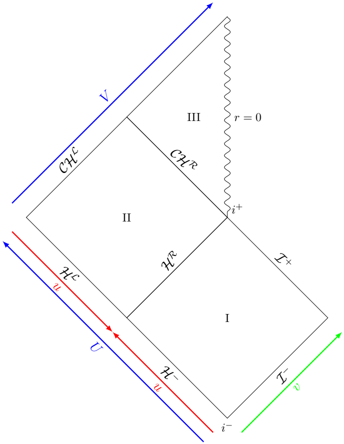

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- Tortoise coordinate $dr_* = f(r)^{-1}dr$
- Lightlike coordinates $u = t - r_*, \quad v = t + r_*$
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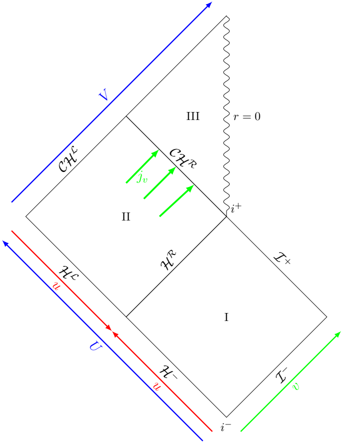

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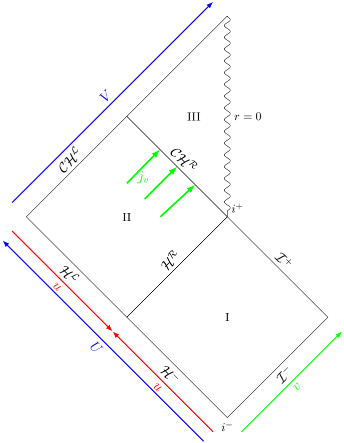

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- Near-extremality $\Delta = \sqrt{1 - Q^2/M^2} \ll 1$

Setup: Quantum theory

- Charged scalar field: $D_\mu D^\mu \Phi = 0$

$$D_\mu = \nabla_\mu - iqA_\mu, \qquad A = -\frac{Q}{r}dt.$$

- Mode ansatz $\Phi(t, r_*, \theta, \phi) = (4\pi)^{-1/2} r^{-1} Y_{lm}(\theta, \phi) H_{\omega\ell}(t, r_*)$

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$$\chi(t, r_*, \theta, \phi) = \frac{qQ}{r_0} t, \quad r_0 \in \{r_+, r_-\}$$

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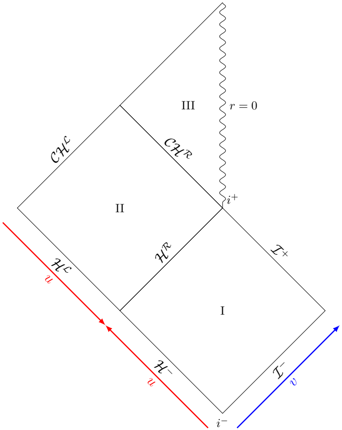

- Quantization: expansion in modes $\Phi_{\omega\ell}$

$$\Phi(x) = \sum_{\lambda,\ell,m} \int_0^\infty d\omega \left(\Phi^\lambda_{\omega\ell m}(x) a^\lambda_{\omega\ell m} + \Phi^\lambda_{-\omega\ell m}(x) b^{\lambda\dagger}_{\omega\ell m} \right)$$

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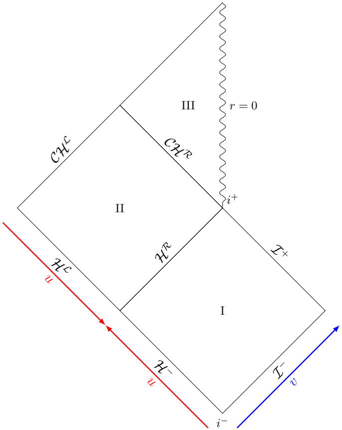

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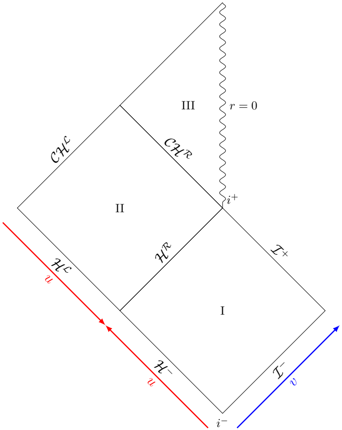

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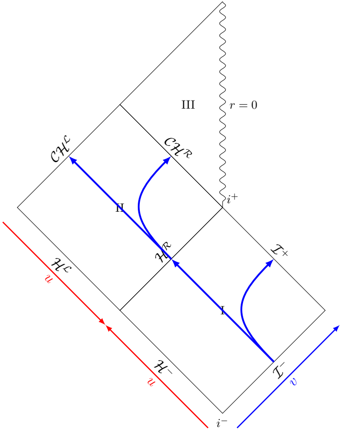

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$$h^{\text{in,I}}_{\omega\ell} \sim e^{-i\omega v} \quad \text{on } \mathcal{I}^-$$
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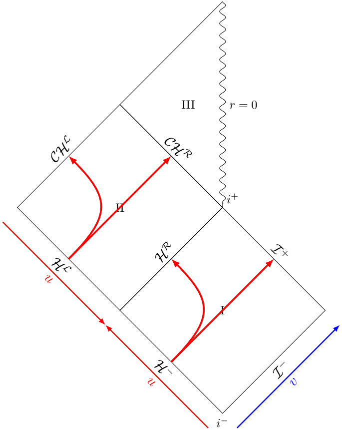

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$h^{\text{in,I}}_{\omega\ell} \sim e^{-i\omega v}$ on $\mathcal{I}^-$ $\quad$ $h^{(+)\text{up,I}}_{\omega\ell} \sim e^{-i\omega u}$ on $\mathcal{H}^-$

$h^{(+)\text{in,II}}_{\omega\ell} \sim e^{-i\omega v}$ on $\mathcal{H}^{\mathcal{R}}$ $\quad$ $h^{(+)\text{up,I}}_{\omega\ell} \sim e^{-i\omega u}$ on $\mathcal{H}^{\mathcal{L}}$

Setup: Quantum theory

– Wronskian relation in region I:

$$|R_{\omega\ell}^{\mathrm{up},I}|^2 + \frac{\omega - \omega_I}{\omega}|T_{\omega\ell}^{\mathrm{up},I}|^2 = 1, \qquad \omega_I = \frac{qQ}{r_+}.$$

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– Wronskian relation in region I:

$$|R^{\mathrm{up},I}_{\omega\ell}|^2 + \underbrace{\frac{\omega - \omega_I}{\omega}}_{<0} |T^{\mathrm{up},I}_{\omega\ell}|^2 = 1, \qquad \omega_I = \frac{qQ}{r_+}$$

$\Longrightarrow$ superradiance $\Longrightarrow$ $|R^{\mathrm{up},I}_{\omega\ell}|^2 > 1$ $\Longrightarrow$ $\langle j_v \rangle_U^{\mathcal{H}^R,\mathrm{ren}} \neq 0$ even at extremality

Scattering problem

The current j_μ and the SET $T_{\mu\nu}$

- Classically $$j_\nu = iq\Big(\Phi(D_\nu\Phi)^* - \Phi^* D_\nu \Phi\Big)$$

$$T_{\mu\nu} = \frac{1}{2}\Big((D_\mu\Phi)^* D_\nu\Phi + D_\mu\Phi(D_\nu\Phi)^*\Big) - \frac{1}{4} g_{\mu\nu} g^{\rho\lambda}\Big((D_\rho\Phi)^* D_\lambda\Phi + D_\rho\Phi(D_\lambda\Phi)^*\Big)$$

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- Quantum theory $\Longrightarrow$ renormalization
- Literature:
  - Hadamard point-split [Klein and Zahn 2021] $\rightarrow \langle j_\mu(x)\rangle_U$ ✓
  - Comparison state [Klein, Zahn, and Hollands 2021] $\rightarrow \langle T_{vv}\rangle^{IH}_{U-C}$ ✓
  - Particular components do not require renormalization [Balakumar, Bernar, and Winstanley 2022] $\rightarrow \langle j_{r_*}(x)\rangle_\Psi$, $\langle T_{tr_*}(x)\rangle_\Psi$ ✓

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  - Particular components do not require renormalization [Balakumar, Bernar, and Winstanley 2022] $\rightarrow \langle j_{r_*}(x)\rangle_\Psi$, $\langle T_{tr_*}(x)\rangle_\Psi$ ✓
- Essential ingredient: scattering coefficients of Boulware modes in regions I, II

$$\frac{d^2 h_{\omega\ell}}{d{r_*}^2} = \Big\{ f(r)\Big(\frac{l(l+1)}{r^2} + \frac{\partial_r f(r)}{r}\Big) - \Big(\omega - \frac{qQ}{r}\Big)^2 \Big\} h_{\omega\ell}. \tag{1}$$

Interior region $r \in (r_-, r_+) \iff r_* \in (+\infty, -\infty)$

- (Leading order) radial equation of motion

$$\frac{d^2 h^{II(+)}_{\omega\ell}}{d\tilde{r}_*^2} = \Big\{ - l(l+1)\,\mathrm{sech}^2\,\tilde{r}_* - \Big(\tilde{\omega} - qQ(1+\tanh\tilde{r}_*)\Big)^2 + \mathcal{O}(\Delta)\Big\} h^{II(+)}_{\omega\ell}. \quad (2)$$

- BCs: $h^{in,II}_{\omega\ell} = \begin{cases} e^{-i\omega r_*} & r_* \to -\infty \\ T^{in,II}_{\omega\ell} e^{-i(\omega-\omega_{II})r_*} + R^{in,II}_{\omega\ell} e^{i(\omega-\omega_{II})r_*} & r_* \to \infty \end{cases}$

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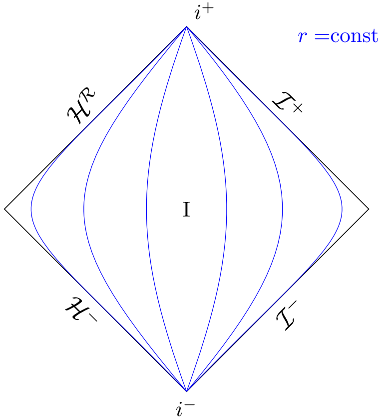
- Analytic solution to leading order in $\Delta \implies T^{in,II}_{\omega\ell},\ R^{in,II}_{\omega\ell}$ ✓

Exterior region $r \in (r_+, \infty) \iff r_* \in (-\infty, \infty)$ $\omega M \ll 1, qQ \ll 1$

– Radial KG:

$$\frac{d^2 h^{in,l}_{\omega\ell}}{dr_*^2} = \left\{ f(r)\left(\frac{l(l+1)}{r^2} + \frac{\partial_r f(r)}{r}\right) - \left(\omega - \frac{qQ}{r}\right)^2 \right\} h^{in,l}_{\omega\ell}$$



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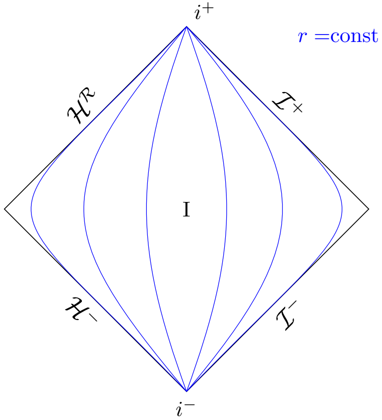
- Real scalar [Zilberman and Ori 2021]



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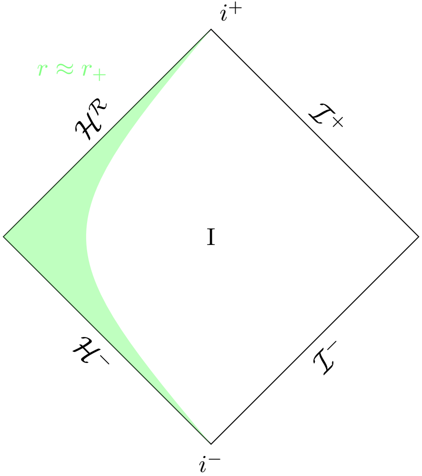
– Radial KG:

$$\frac{d^2 h^{in,l}_{\omega\ell}}{dr_*^2} = \left\{ f(r)\left(\frac{l(l+1)}{r^2} + \frac{\partial_r f(r)}{r}\right) - \left(\omega - \frac{qQ}{r}\right)^2 \right\} h^{in,l}_{\omega\ell} \qquad r \approx r_+$$

– BCs: $h^{in,l}_{\omega\ell} = \begin{cases} e^{-i(\omega-\omega_\mathrm{l})r_*} & r_* \to -\infty \\ A_{\omega\ell} e^{-i\omega r_*} + B_{\omega\ell} e^{i\omega r_*} & r_* \to \infty \end{cases}$

– Real scalar [Zilberman and Ori 2021]

– Subregions $\{$ Vicinity of EH

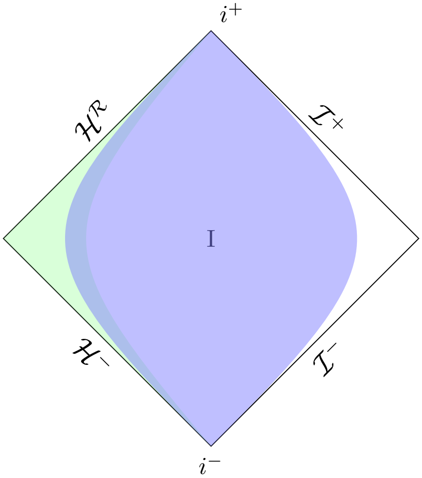


Exterior region $r \in (r_+, \infty) \iff r_* \in (-\infty, \infty)$ $\omega M \ll 1, qQ \ll 1$

- Radial KG:

$$\frac{d^2 h^{in,l}_{\omega\ell}}{dr_*^2} = \left\{ f(r)\left(\frac{l(l+1)}{r^2} + \frac{\partial_r f(r)}{r}\right) - \left(\omega - \frac{qQ}{r}\right)^2 \right\} h^{in,l}_{\omega\ell}$$

- BCs: $h^{in,l}_{\omega\ell} = \begin{cases} e^{-i(\omega-\omega_l)r_*} & r_* \to -\infty \\ A_{\omega\ell} e^{-i\omega r_*} + B_{\omega\ell} e^{i\omega r_*} & r_* \to \infty \end{cases}$
- Real scalar [Zilberman and Ori 2021]
- Subregions $\begin{cases} \text{Vicinity of EH} \\ \text{Intermediate region} \end{cases}$

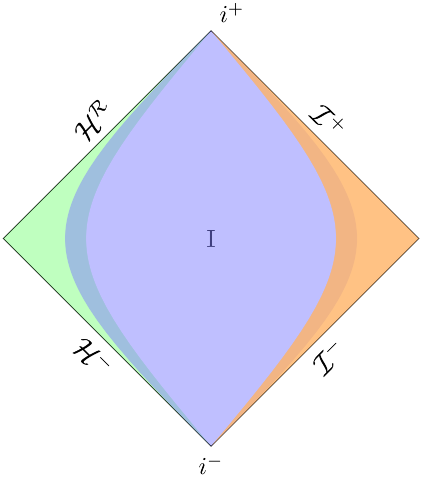


Exterior region $r \in (r_+, \infty) \iff r_* \in (-\infty, \infty)$ $\omega M \ll 1, qQ \ll 1$

- Radial KG:

$$\frac{d^2 h^{in,l}_{\omega\ell}}{dr_*^2} = \left\{ f(r) \left(\frac{l(l+1)}{r^2} + \frac{\partial_r f(r)}{r} \right) - \left(\omega - \frac{qQ}{r} \right)^2 \right\} h^{in,l}_{\omega\ell}$$

- BCs: $h^{in,l}_{\omega\ell} = \begin{cases} e^{-i(\omega-\omega_I)r_*} & r_* \to -\infty \\ A_{\omega\ell} e^{-i\omega r_*} + B_{\omega\ell} e^{i\omega r_*} & r_* \to \infty \end{cases}$
- Real scalar [Zilberman and Ori 2021]
- Subregions: Vicinity of EH; Intermediate region; Asymptotically flat region

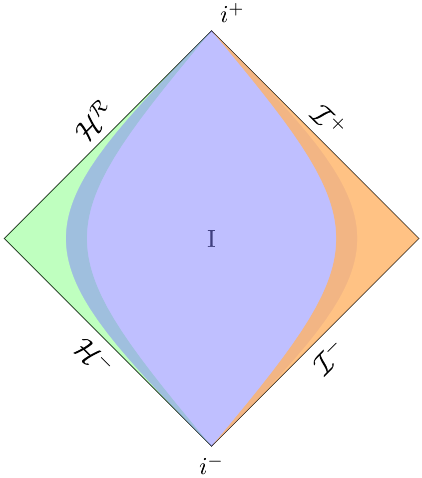


Exterior region $r \in (r_+, \infty) \iff r_* \in (-\infty, \infty)$ $\omega M \ll 1, qQ \ll 1$

- Radial KG:

$$\frac{d^2 h^{in,l}_{\omega\ell}}{dr_*^2} = \left\{ f(r) \left(\frac{l(l+1)}{r^2} + \frac{\partial_r f(r)}{r} \right) - \left(\omega - \frac{qQ}{r} \right)^2 \right\} h^{in,l}_{\omega\ell}$$

- BCs: $h^{in,l}_{\omega\ell} = \begin{cases} e^{-i(\omega-\omega_I)r_*} & r_* \to -\infty \\ A_{\omega\ell} e^{-i\omega r_*} + B_{\omega\ell} e^{i\omega r_*} & r_* \to \infty \end{cases}$
- Real scalar [Zilberman and Ori 2021]
- Subregions $\begin{cases} \text{Vicinity of EH} \\ \text{Intermediate region} \\ \text{Asymptotically flat region} \end{cases}$
- Obtain $A_{\omega\ell}, B_{\omega\ell}$ ✓



Exterior region $r \in (r_+, \infty) \iff r_* \in (-\infty, \infty)$ $\omega M \ll 1, qQ \ll 1$

$$\left|T^{in,l}_{\omega,l=0}\right|^2 = 4r_+^2\omega^2, \qquad \left|R^{in,l}_{\omega,l=0}\right|^2 = 1 - 2\omega(\omega - \omega_I)r_+^2$$

$$\left|T^{in,l}_{\omega,l>0}\right|^2 \sim \Delta^{4l+2}, \qquad \left|R^{in,l}_{\omega,l>0}\right|^2 = 1$$

Results

At the event horizon (EH)

– Explicitly

$$\langle j_v \rangle_U^{EH} = \frac{1}{12\pi^2 r_+^2}\left(\frac{q^4 Q^3}{r_+} + \frac{q^2 Q M^2 \Delta^2}{2 r_+^3}\right) + \mathcal{O}(q^4 \Delta, q^2 M^{-2} \Delta^3)$$

$$\langle T_{vv} \rangle_U^{EH} = \frac{-M^4 \Delta^4}{480\pi^2 r_+^8} + \frac{q^4 Q^4}{48\pi^2 r_+^4} + \mathcal{O}(\Delta^5 M^{-4}, q^4 \Delta, q^2 M^{-2} \Delta^3)$$

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- Explicitly

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- Conservation laws & semiclassical E-M equations [Balakumar, Bernar, and Winstanley 2022]

$$\langle j_{r_*} \rangle_\Psi = \frac{\mathcal{K}}{r^2} \quad (3)$$

$$\langle T_{tr_*} \rangle_\Psi = -\frac{\mathcal{L}}{r^2} + \frac{\mathcal{K}Q}{r^3} \quad (4)$$

At the event horizon (EH)

- Explicitly

$$\langle j_v \rangle_U^{EH} = \frac{1}{12\pi^2 r_+^2}\left(\frac{q^4 Q^3}{r_+} + \frac{q^2 Q M^2 \Delta^2}{2r_+^3}\right) + \mathcal{O}(q^4\Delta, q^2 M^{-2}\Delta^3)$$

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- Conservation laws & semiclassical E-M equations [Balakumar, Bernar, and Winstanley 2022]

$$\langle j_v \rangle_\Psi - \langle j_u \rangle_\Psi = \langle j_{r_*} \rangle_\Psi = \frac{\mathcal{K}}{r^2} \tag{3}$$

$$\langle T_{vv} \rangle_\Psi - \langle T_{uu} \rangle_\Psi = \langle T_{tr_*} \rangle_\Psi = -\frac{\mathcal{L}}{r^2} + \frac{\mathcal{K}Q}{r^3} \tag{4}$$

At the event horizon (EH)

- Explicitly

$$\frac{\mathcal{K}}{r_+^2} = \langle j_v \rangle_U^{EH} = \frac{1}{12\pi^2 r_+^2}\left(\frac{q^4 Q^3}{r_+} + \frac{q^2 Q M^2 \Delta^2}{2r_+^3}\right) + \mathcal{O}(q^4\Delta, q^2 M^{-2}\Delta^3)$$

$$-\frac{\mathcal{L}}{r_+^2} + \frac{\mathcal{K}Q}{r_+^3} = \langle T_{vv} \rangle_U^{EH} = \frac{-M^4\Delta^4}{480\pi^2 r_+^8} + \frac{q^4 Q^4}{48\pi^2 r_+^4} + \mathcal{O}(\Delta^5 M^{-4}, q^4\Delta, q^2 M^{-2}\Delta^3)$$

- Conservation laws & semiclassical E-M equations [Balakumar, Bernar, and Winstanley 2022]

$$\langle j_v \rangle_\Psi - \langle j_u \rangle_\Psi = \langle j_{r_*} \rangle_\Psi = \frac{\mathcal{K}}{r^2} \tag{3}$$

$$\langle T_{vv} \rangle_\Psi - \langle T_{uu} \rangle_\Psi = \langle T_{tr_*} \rangle_\Psi = -\frac{\mathcal{L}}{r^2} + \frac{\mathcal{K}Q}{r^3} \tag{4}$$

At the event horizon (EH)

- Explicitly

$$\frac{\mathcal{K}}{r_+^2} = \langle j_v \rangle_U^{EH} = \frac{1}{12\pi^2 r_+^2}\left(\frac{q^4 Q^3}{r_+} + \frac{q^2 Q M^2 \Delta^2}{2r_+^3}\right) + \mathcal{O}(q^4\Delta, q^2 M^{-2}\Delta^3)$$

$$-\frac{\mathcal{L}}{r_+^2} + \frac{\mathcal{K}Q}{r_+^3} = \langle T_{vv} \rangle_U^{EH} = \frac{-M^4\Delta^4}{480\pi^2 r_+^8} + \frac{q^4 Q^4}{48\pi^2 r_+^4} + \mathcal{O}(\Delta^5 M^{-4}, q^4\Delta, q^2 M^{-2}\Delta^3)$$

- Conservation laws & semiclassical E-M equations [Balakumar, Bernar, and Winstanley 2022]

$$\langle j_v \rangle_\Psi - \langle j_u \rangle_\Psi = \langle j_{r_*} \rangle_\Psi = \frac{\mathcal{K}}{r^2} \quad \Longrightarrow \mathcal{K}Q > 0 \tag{3}$$

$$\langle T_{vv} \rangle_\Psi - \langle T_{uu} \rangle_\Psi = \langle T_{tr_*} \rangle_\Psi = -\frac{\mathcal{L}}{r^2} + \frac{\mathcal{K}Q}{r^3} \quad \Longrightarrow \mathcal{L} > 0 \tag{4}$$

At the event horizon (EH)

- Explicitly

$$\frac{\mathcal{K}}{r_+^2} = \langle j_v \rangle_U^{EH} = \frac{1}{12\pi^2 r_+^2}\left(\frac{q^4 Q^3}{r_+} + \frac{q^2 Q M^2 \Delta^2}{2r_+^3}\right) + \mathcal{O}(q^4\Delta, q^2 M^{-2}\Delta^3)$$

$$-\frac{\mathcal{L}}{r_+^2} + \frac{\mathcal{K}Q}{r_+^3} = \langle T_{vv} \rangle_U^{EH} = \frac{-M^4\Delta^4}{480\pi^2 r_+^8} + \frac{q^4 Q^4}{48\pi^2 r_+^4} + \mathcal{O}(\Delta^5 M^{-4}, q^4\Delta, q^2 M^{-2}\Delta^3)$$

- Conservation laws & semiclassical E-M equations [Balakumar, Bernar, and Winstanley 2022]

$$\langle j_v \rangle_\Psi - \langle j_u \rangle_\Psi = \langle j_{r_*} \rangle_\Psi = \frac{\mathcal{K}}{r^2} \implies \mathcal{K}Q > 0 \quad (3)$$

$$\langle T_{vv} \rangle_\Psi - \langle T_{uu} \rangle_\Psi = \langle T_{tr_*} \rangle_\Psi = -\frac{\mathcal{L}}{r^2} + \frac{\mathcal{K}Q}{r^3} \implies \mathcal{L} > 0 \quad (4)$$

- Weak backreaction: $\partial_v Q = -4\pi r^2 \langle j_v \rangle \implies$ BH discharge

At the inner horizon (IH)

– Explicitly

$$\langle j_v \rangle_U^{IH} = \frac{1}{12\pi^2 r_-^2}\left(\frac{q^4 Q^3}{r_+} + \frac{q^2 Q M^2 \Delta^2}{2 r_+^3}\right) + \mathcal{O}(q^4 \Delta, q^2 M^{-2} \Delta^3)$$

$$\langle T_{vv} \rangle_U^{IH} = \frac{-M^4 \Delta^4}{480\pi^2 r_-^2 r_+^6} + \frac{q^4 Q^4}{48\pi^2 r_-^2 r_+^2} + \mathcal{O}(\Delta^5 M^{-4}, q^4 \Delta, q^2 M^{-2} \Delta^3)$$

At the inner horizon (IH)

- Explicitly

$$\langle j_v \rangle_U^{IH} = \frac{1}{12\pi^2 r_-^2}\left(\frac{q^4 Q^3}{r_+} + \frac{q^2 Q M^2 \Delta^2}{2 r_+^3}\right) + \mathcal{O}(q^4 \Delta, q^2 M^{-2} \Delta^3)$$

$$\langle T_{vv} \rangle_U^{IH} = \frac{-M^4 \Delta^4}{480\pi^2 r_-^2 r_+^6} + \frac{q^4 Q^4}{48\pi^2 r_-^2 r_+^2} + \mathcal{O}(\Delta^5 M^{-4}, q^4 \Delta, q^2 M^{-2} \Delta^3)$$

- At the inner horizon,

$$\langle j_u \rangle_U^{IH} \ll \langle j_v \rangle_U^{IH}, \qquad \langle T_{uu} \rangle_U^{IH} \ll \langle T_{vv} \rangle_U^{IH}$$

At the inner horizon (IH)

– Explicitly

$$\langle j_v\rangle_U^{IH} = \frac{1}{12\pi^2 r_-^2}\left(\frac{q^4 Q^3}{r_+} + \frac{q^2 Q M^2 \Delta^2}{2r_+^3}\right) + \mathcal{O}(q^4\Delta, q^2 M^{-2}\Delta^3)$$

$$\langle T_{vv}\rangle_U^{IH} = \frac{-M^4\Delta^4}{480\pi^2 r_-^2 r_+^6} + \frac{q^4 Q^4}{48\pi^2 r_-^2 r_+^2} + \mathcal{O}(\Delta^5 M^{-4}, q^4\Delta, q^2 M^{-2}\Delta^3)$$

– At the inner horizon,

$$\langle j_u\rangle_U^{IH} \ll \langle j_v\rangle_U^{IH}\,, \qquad \langle T_{uu}\rangle_U^{IH} \ll \langle T_{vv}\rangle_U^{IH}$$

– Leading order contribution from $l = 0$ modes

At the inner horizon (IH)

- Explicitly

$$\langle j_v \rangle_U^{IH} = \frac{1}{12\pi^2 r_-^2}\left(\frac{q^4 Q^3}{r_+} + \frac{q^2 Q M^2 \Delta^2}{2 r_+^3}\right) + \mathcal{O}(q^4 \Delta, q^2 M^{-2} \Delta^3)$$

$$\langle T_{vv} \rangle_U^{IH} = \frac{-M^4 \Delta^4}{480\pi^2 r_-^2 r_+^6} + \frac{q^4 Q^4}{48\pi^2 r_-^2 r_+^2} + \mathcal{O}(\Delta^5 M^{-4}, q^4 \Delta, q^2 M^{-2} \Delta^3)$$

- At the inner horizon,

$$\langle j_u \rangle_U^{IH} \ll \langle j_v \rangle_U^{IH}, \qquad \langle T_{uu} \rangle_U^{IH} \ll \langle T_{vv} \rangle_U^{IH}$$

- Leading order contribution from $l = 0$ modes
- Weak backreaction $\quad \partial_v Q = -4\pi r^2 \langle j_v \rangle \implies$ discharge of BH interior

Conclusions and outlook

Conclusions

- Studied charged scalar field in Unruh state on RN spacetime
- Near-extremal domain $\Longrightarrow$ analytical treatment
- Quantum effects: charge superradiance, Hawking radiation
- Behaviour of $\langle j_v \rangle$, $\langle T_{vv} \rangle$ at the inner horizons $\rightarrow$ connection to sCC
- (Brief) backreaction analysis
- Internship with the RTG: numerical cross-check

Thank you!

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