The Floquet Fermi liquid

Likun Shi, Physik-Combo, 18.09.2023

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PRB 107, 195135
PRB 107, 125151

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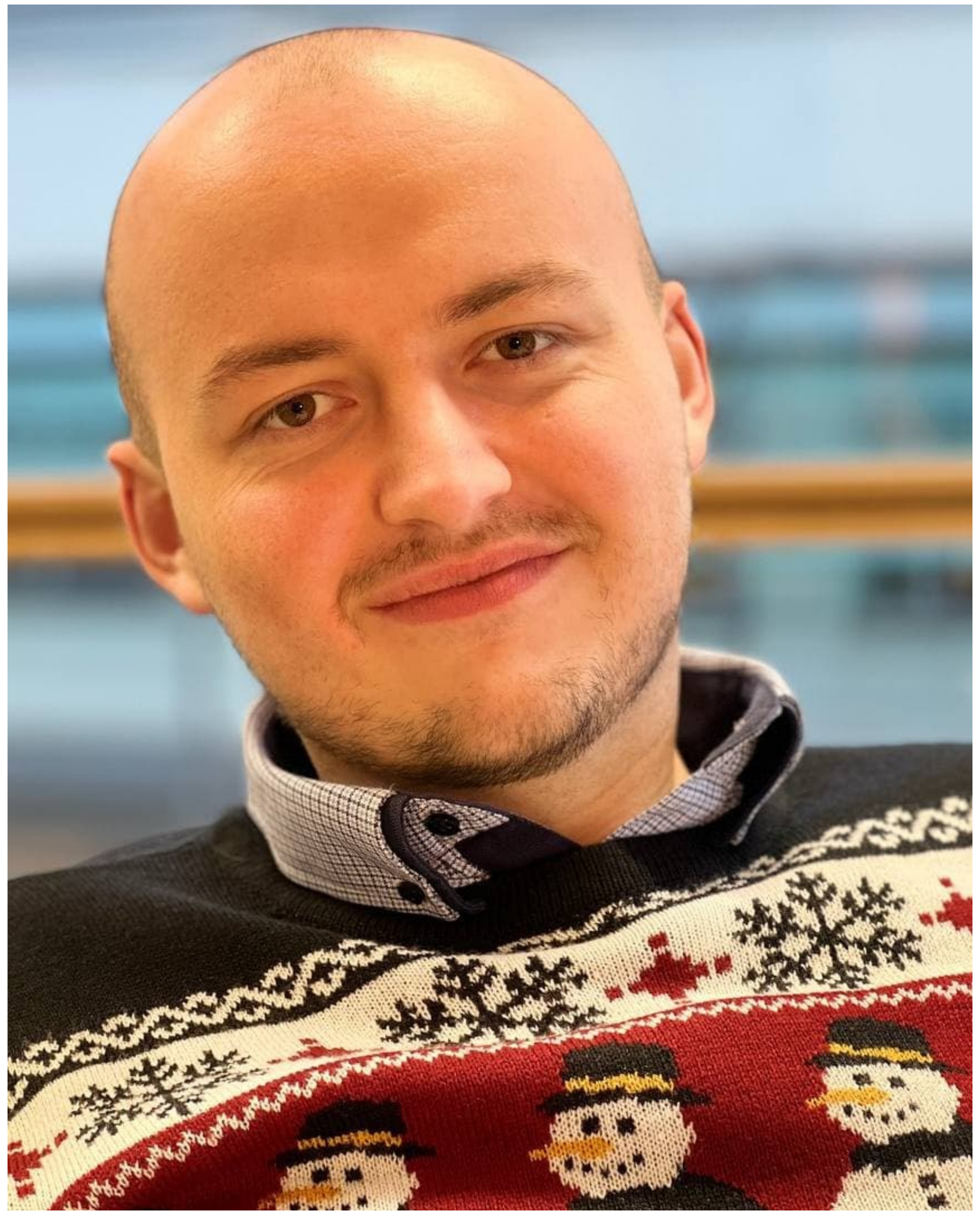

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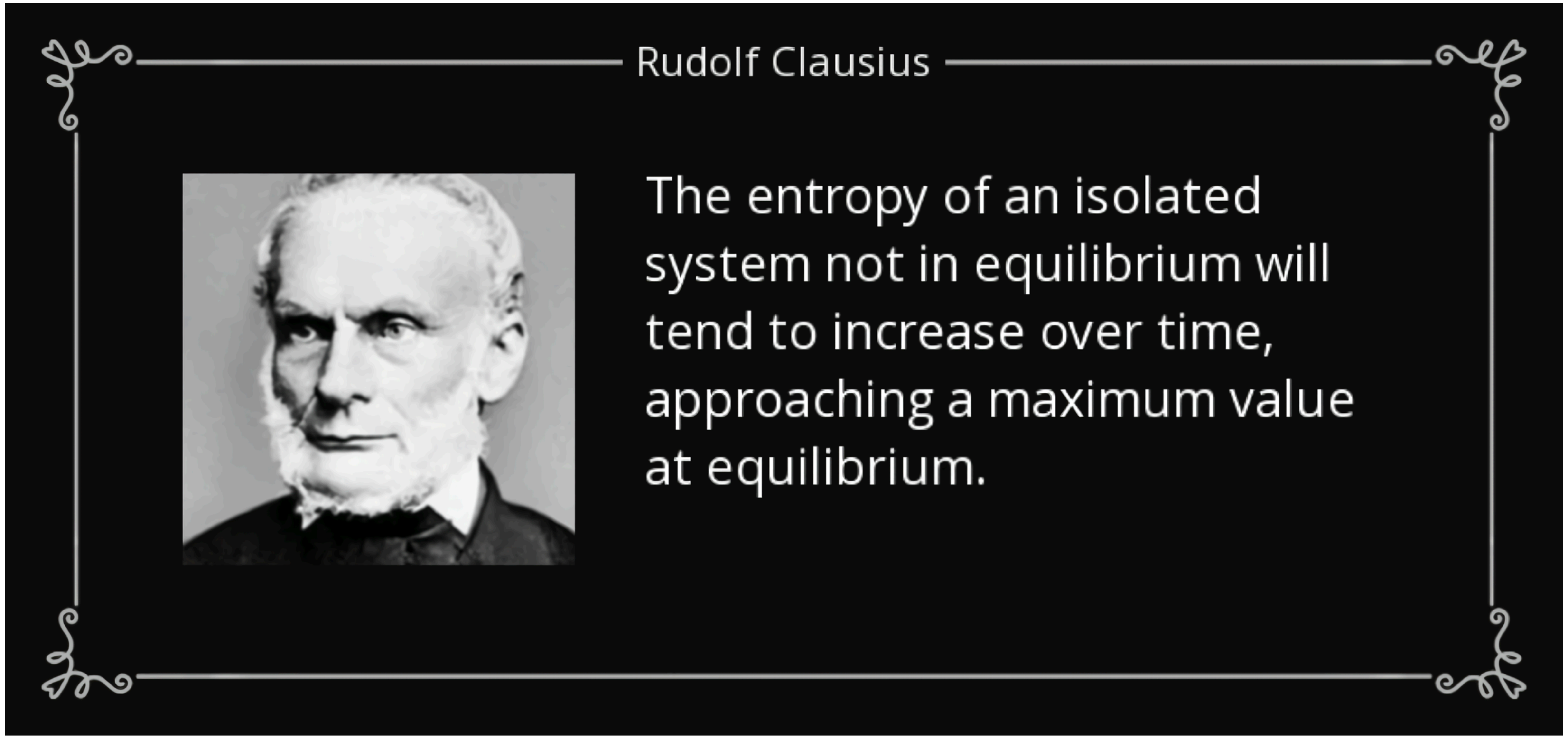
Prelude

The Fermi liquid

An equilibrium state of matter of fermions

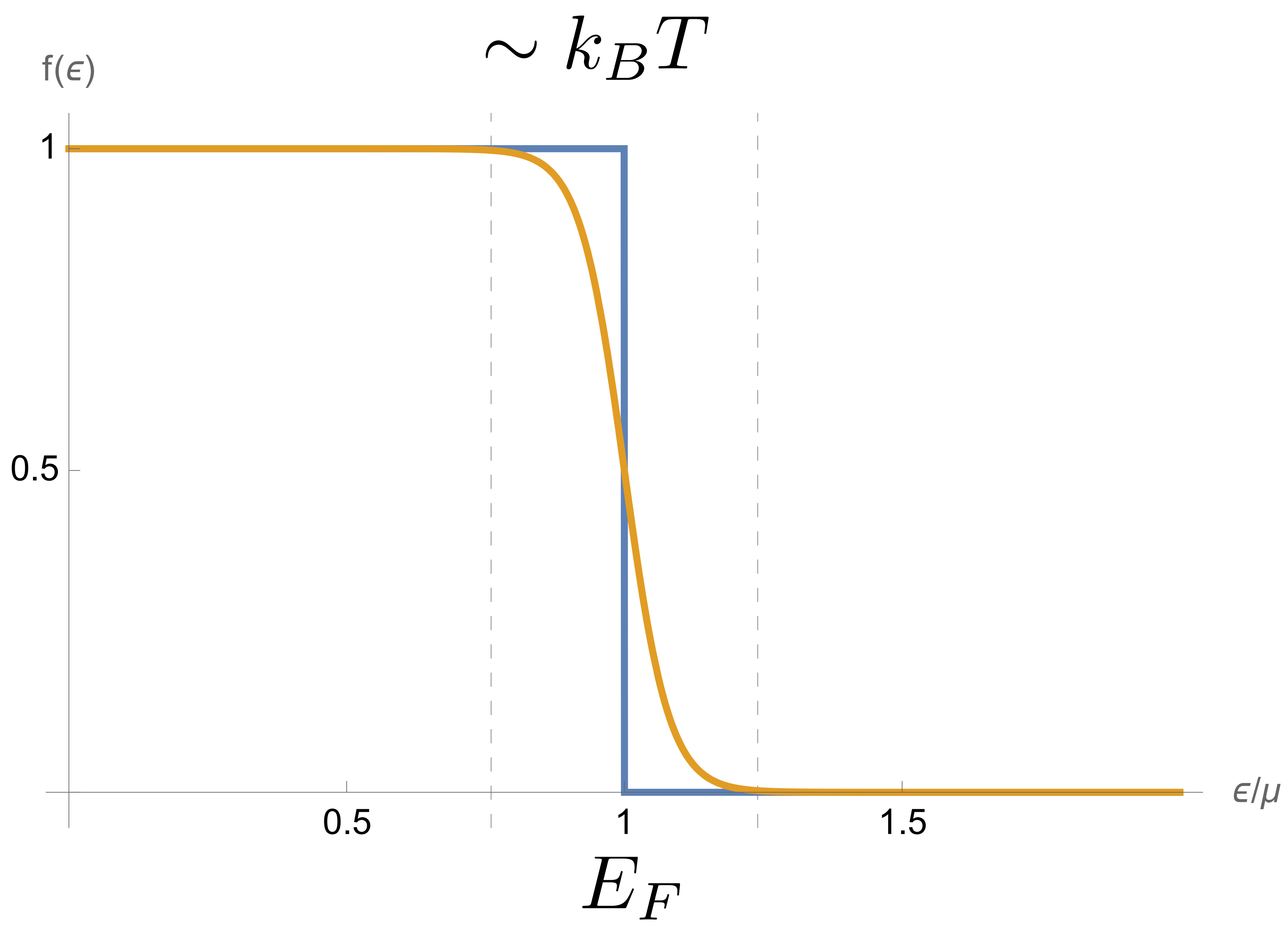
- Max entropy
- Pauli exclusion principle



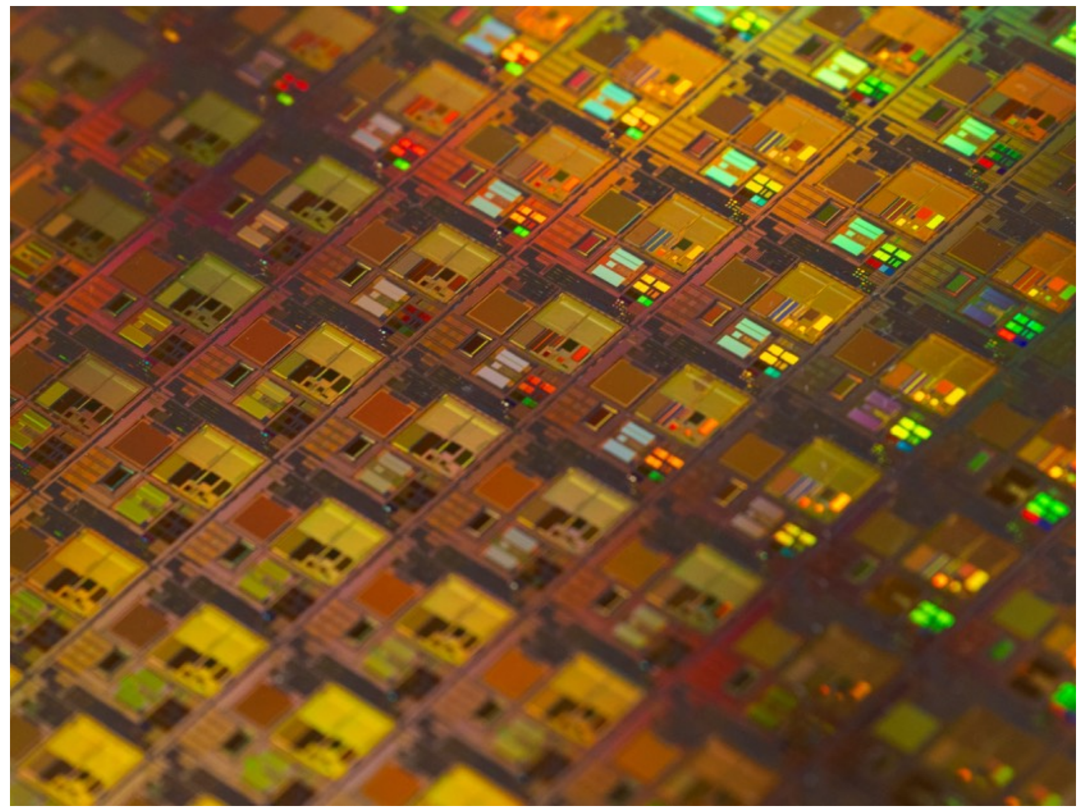
The Fermi liquid

Pauli exclusion principle + Maximizing the entropy → Fermi-Dirac distribution features a sharp Fermi surface at zero temperature

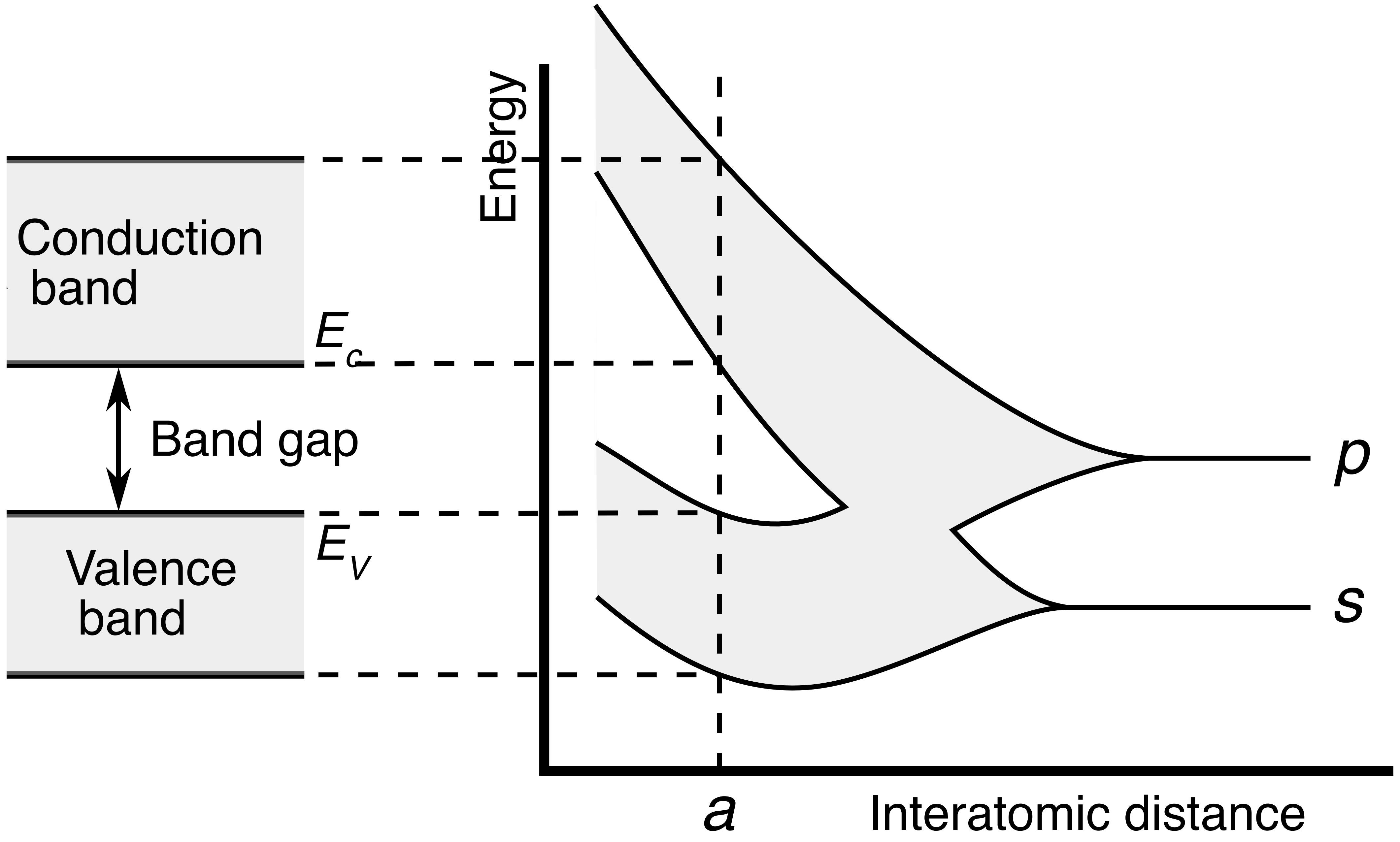
$$f(\epsilon) = \frac{1}{e^{(\epsilon-\mu)/k_B T} + 1}$$

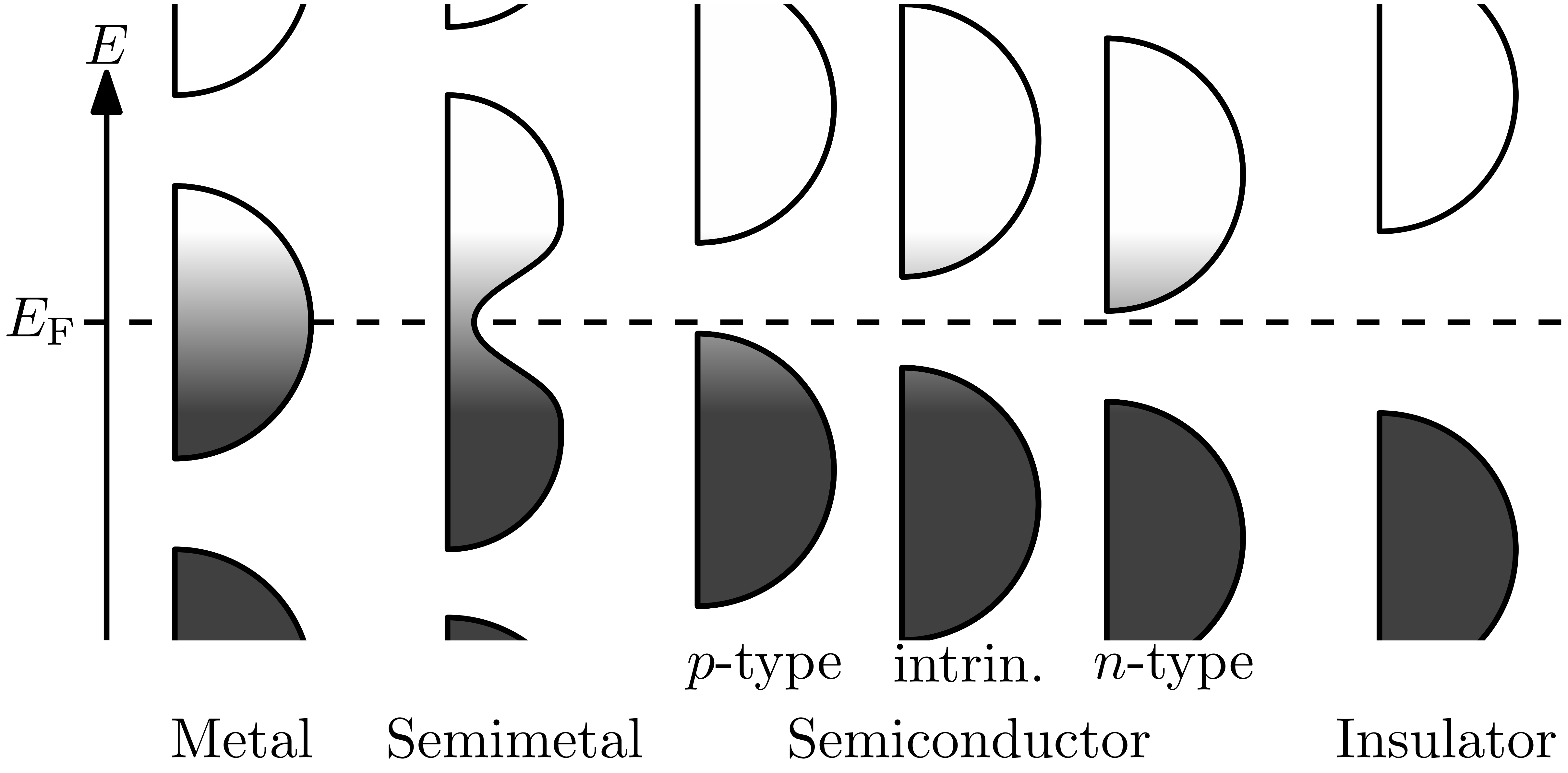


The Fermi liquid



Cornerstone for other theories and technologies





The Floquet Fermi liquid

A periodically driven system, out of equilibrium

$$H_S(t) = H_S(t+T)$$

$$i\partial_t |\psi_n^F(t)\rangle = H_S(t) |\psi_n^F(t)\rangle$$

$$|\psi_n^F(t)\rangle = \sum_l e^{-i(\epsilon_n^F + l\Omega)t} |\varphi_{n,l}\rangle \, , \quad \Omega = 2\pi/T$$

quasi energy

discretized Fourier series

The Floquet Fermi liquid

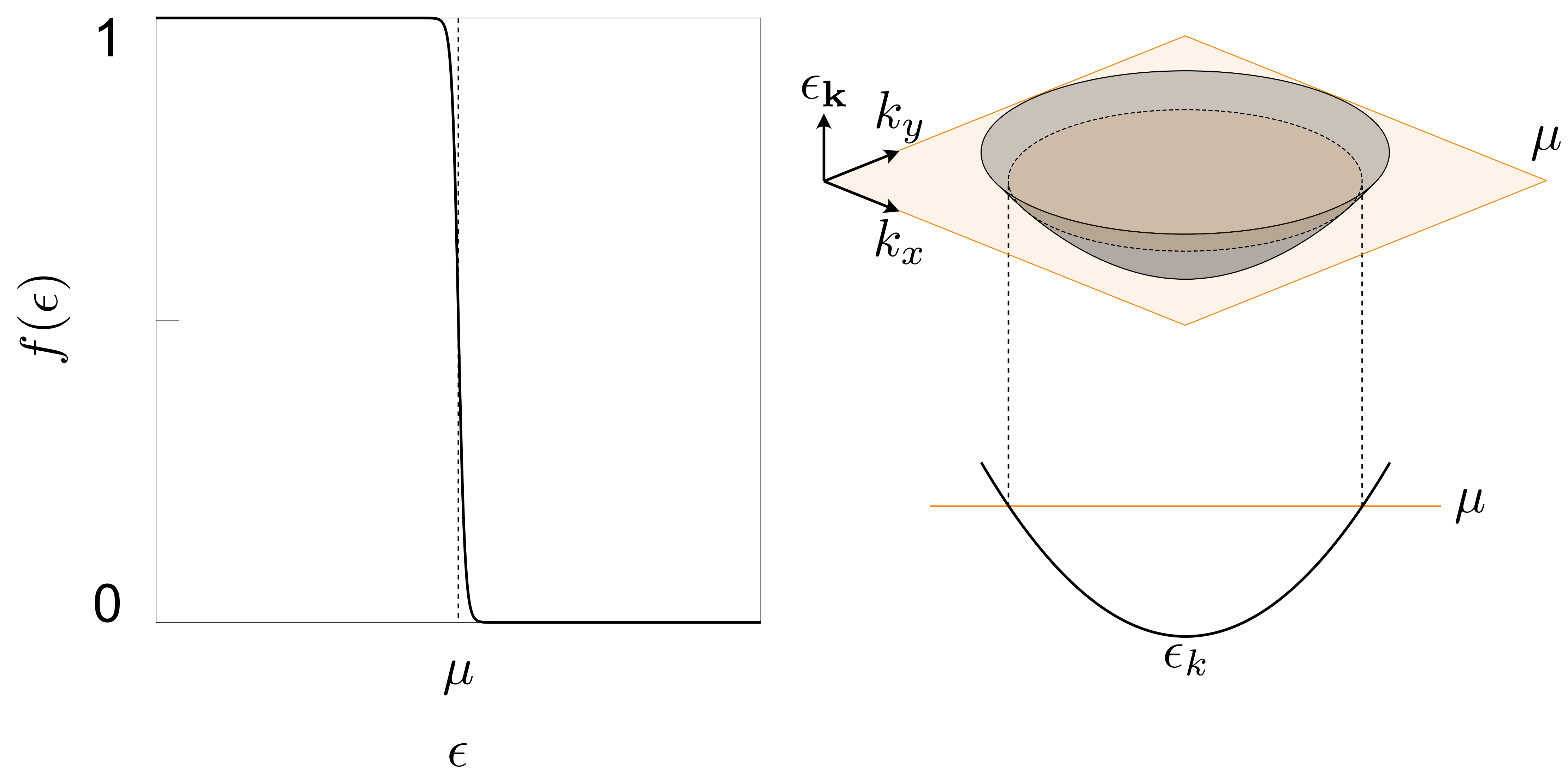
- Dissipate energy
- Erase memory
- Unique steady state

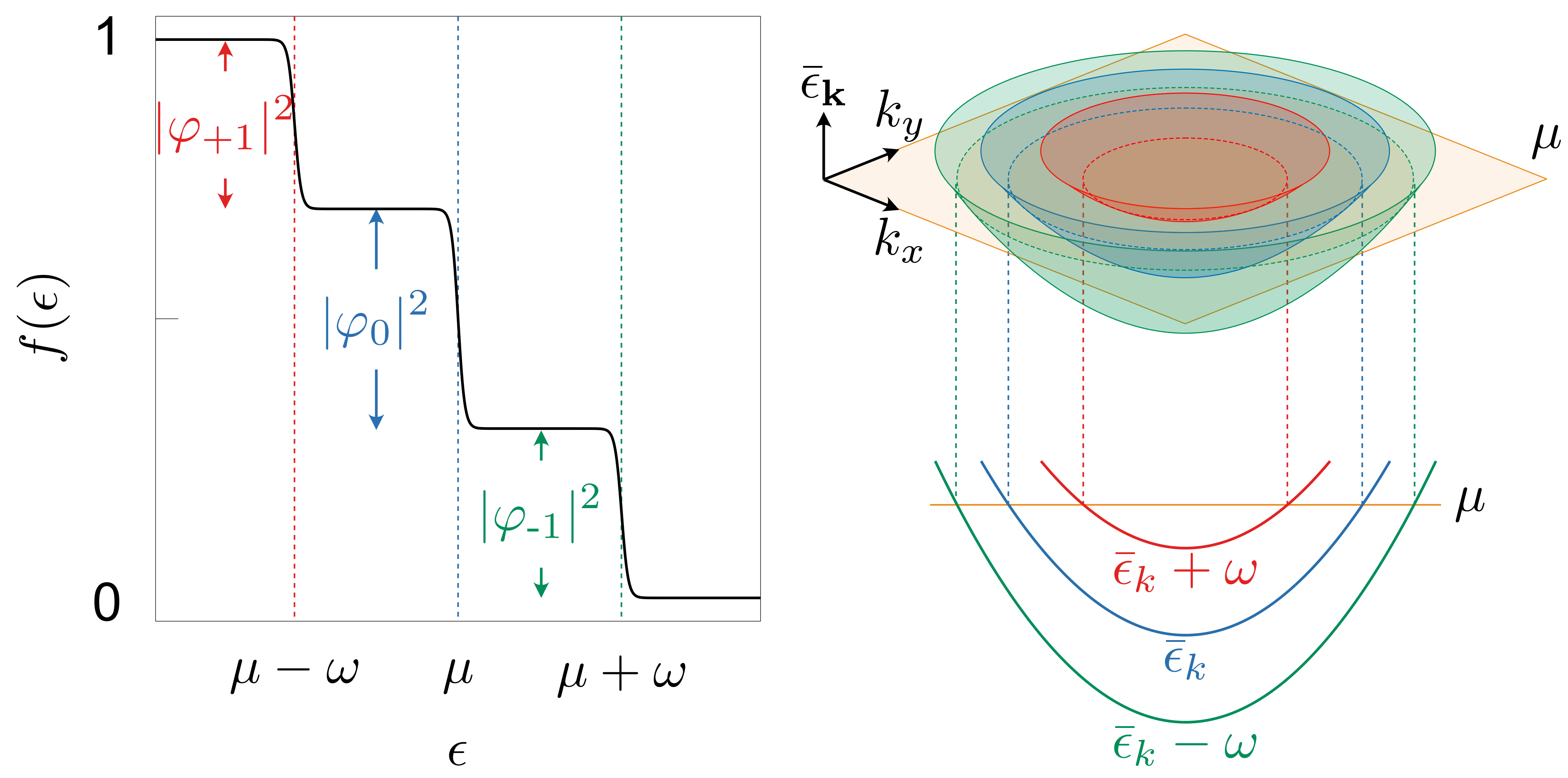
To reach a steady state under periodically driven, a bath is crucial !

The Floquet Fermi liquid

How does it look like?

- multiple surfaces
- spacing between the surfaces
- size of the jump





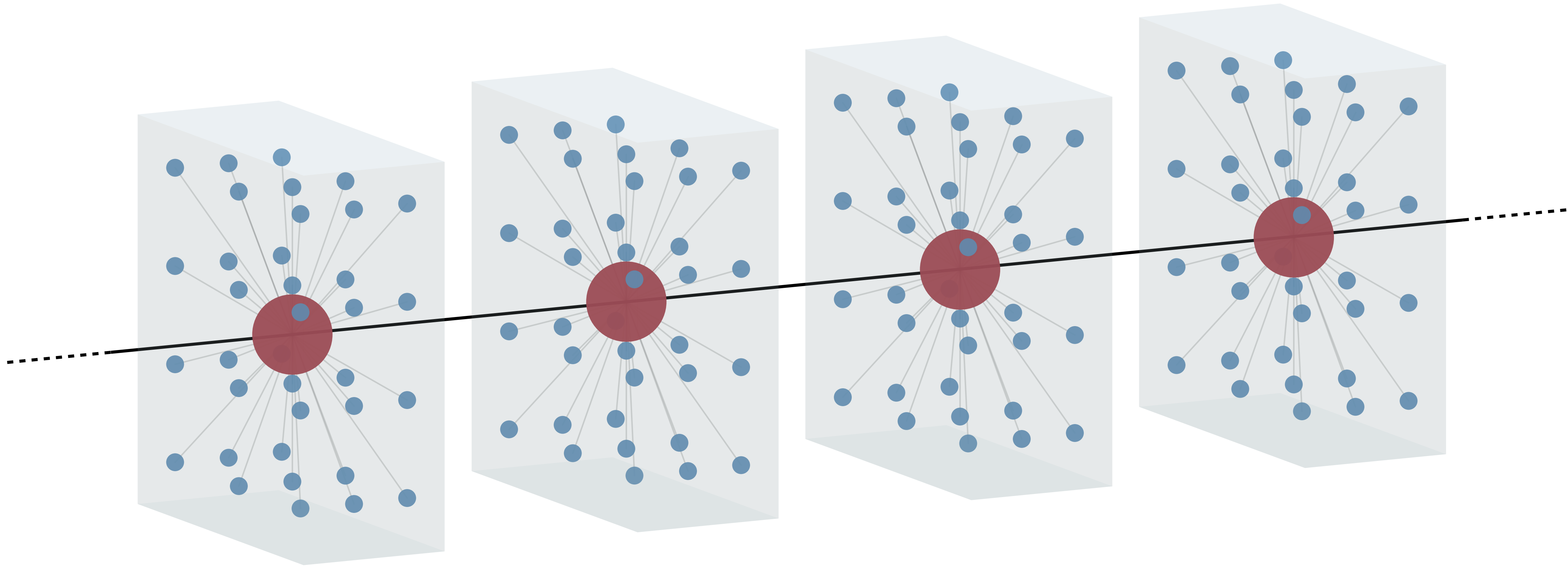
Derivations

Open system Schrödinger equation

non-interacting, single particle

separate system and bath as a direct sum of their individual Hilbert spaces

$$H(t) = \begin{bmatrix} H_S(t) & H_{SB}(t) \\ H_{BS}(t) & H_B(t) \end{bmatrix}, \quad |\psi(t)\rangle = \begin{bmatrix} |\psi_S(t)\rangle \\ |\psi_B(t)\rangle \end{bmatrix}$$



$$i\partial_t |\psi_S(t)\rangle = H_S(t)|\psi_S(t)\rangle + H_{SB}(t)|\psi_B(t)\rangle$$
$$i\partial_t |\psi_B(t)\rangle = H_{BS}(t)|\psi_S(t)\rangle + H_B(t)|\psi_B(t)\rangle$$

Open system Schrödinger equation

reduce the bath state dynamics

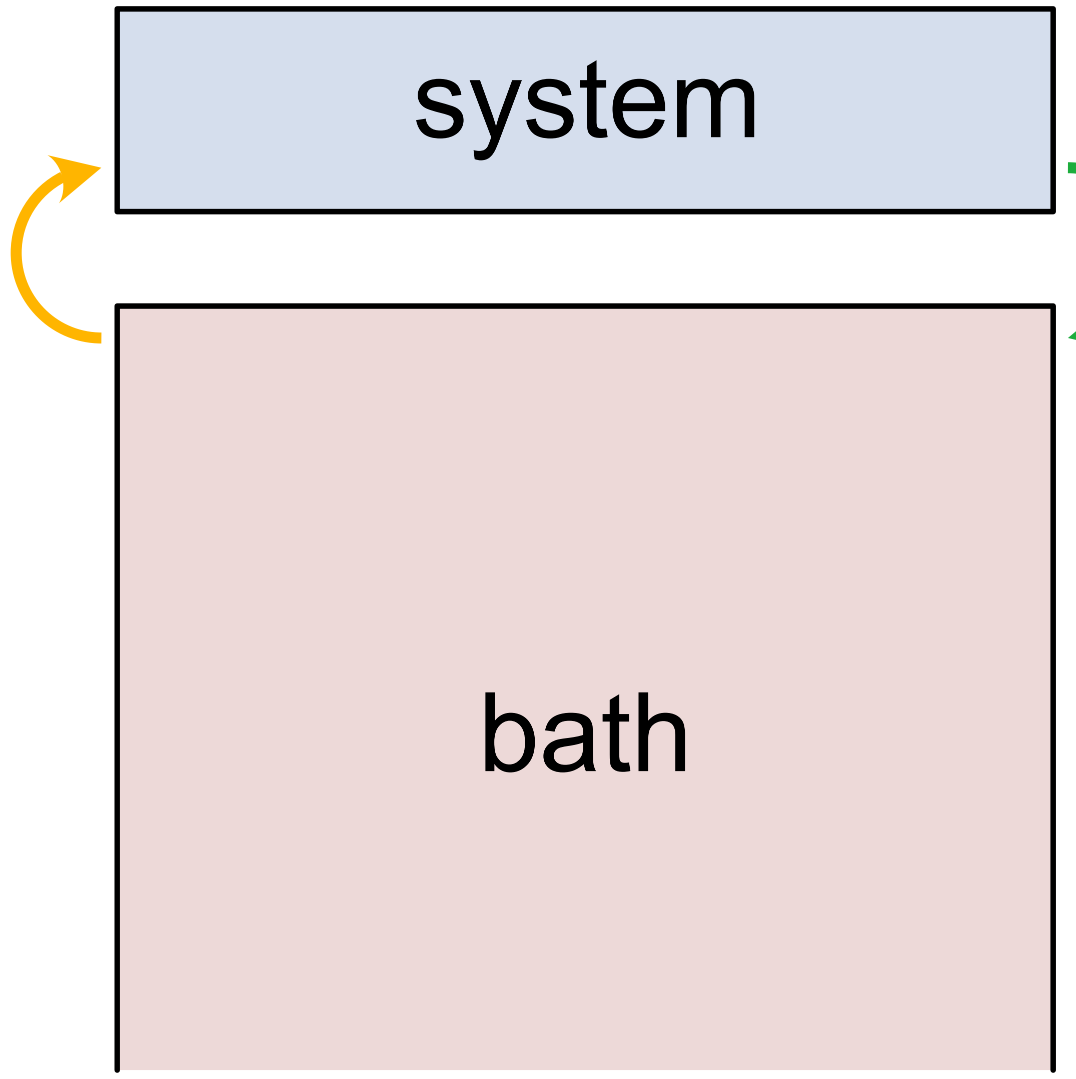
$$i\partial_t |\psi_S(t)\rangle = H_S(t)|\psi_S(t)\rangle + H_{SB}(t)|\psi_B(t)\rangle$$
$$i\partial_t |\psi_B(t)\rangle = H_{BS}(t)|\psi_S(t)\rangle + H_B(t)|\psi_B(t)\rangle$$

$$i\partial_t U_B(t,t_0) = H_B(t)U_B(t,t_0)$$

Bath feedback effect

$$i\partial_t |\psi_S(t)\rangle = H_S(t)|\psi_S(t)\rangle + H_{SB}(t)U_B(t,t_0)|\psi_B(t_0)\rangle$$
$$- iH_{SB}(t)\int_{t_0}^{t} \mathrm{d}t'\; U_B(t,t')H_{BS}(t')|\psi_S(t')\rangle\,,$$

Decay, memory and energy renormalization effects

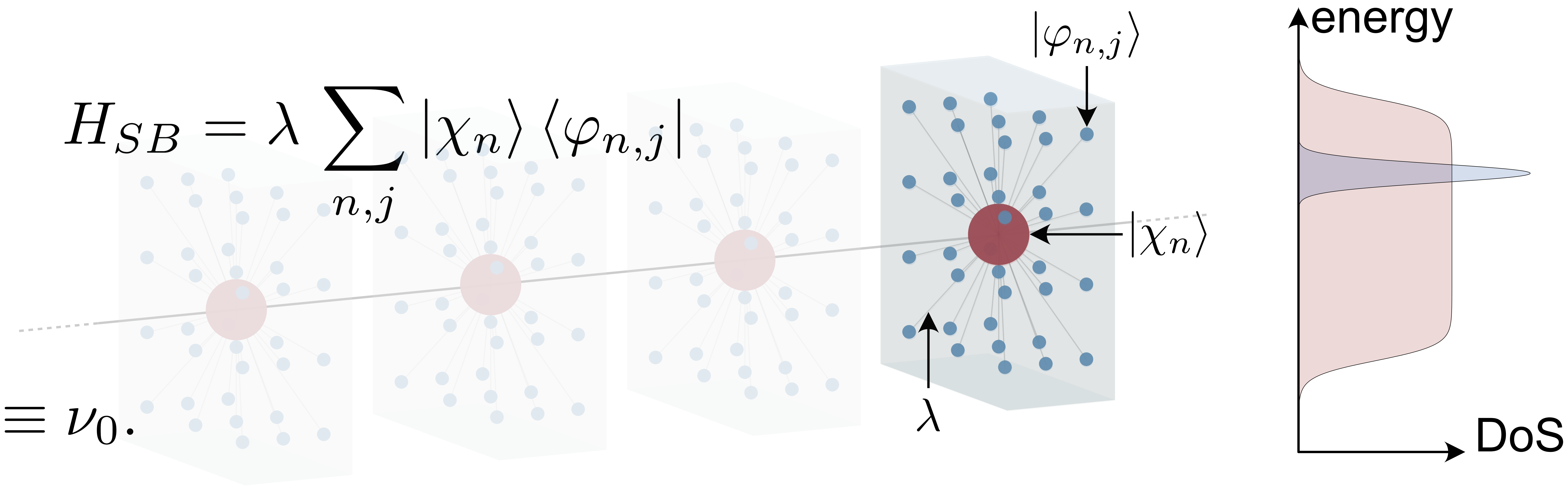


Open system Schrödinger equation

restrict to a featureless fermionic bath

$$H_B = \sum_{n,j} \varepsilon_j \left|\varphi_{n,j}\right\rangle \left\langle\varphi_{n,j}\right|, \quad H_{SB} = \lambda \sum_{n,j} \left|\chi_n\right\rangle \left\langle\varphi_{n,j}\right|$$

$$\nu_B(\omega) = 2\pi \sum_j \delta(\omega - \varepsilon_j) \equiv \nu_0.$$

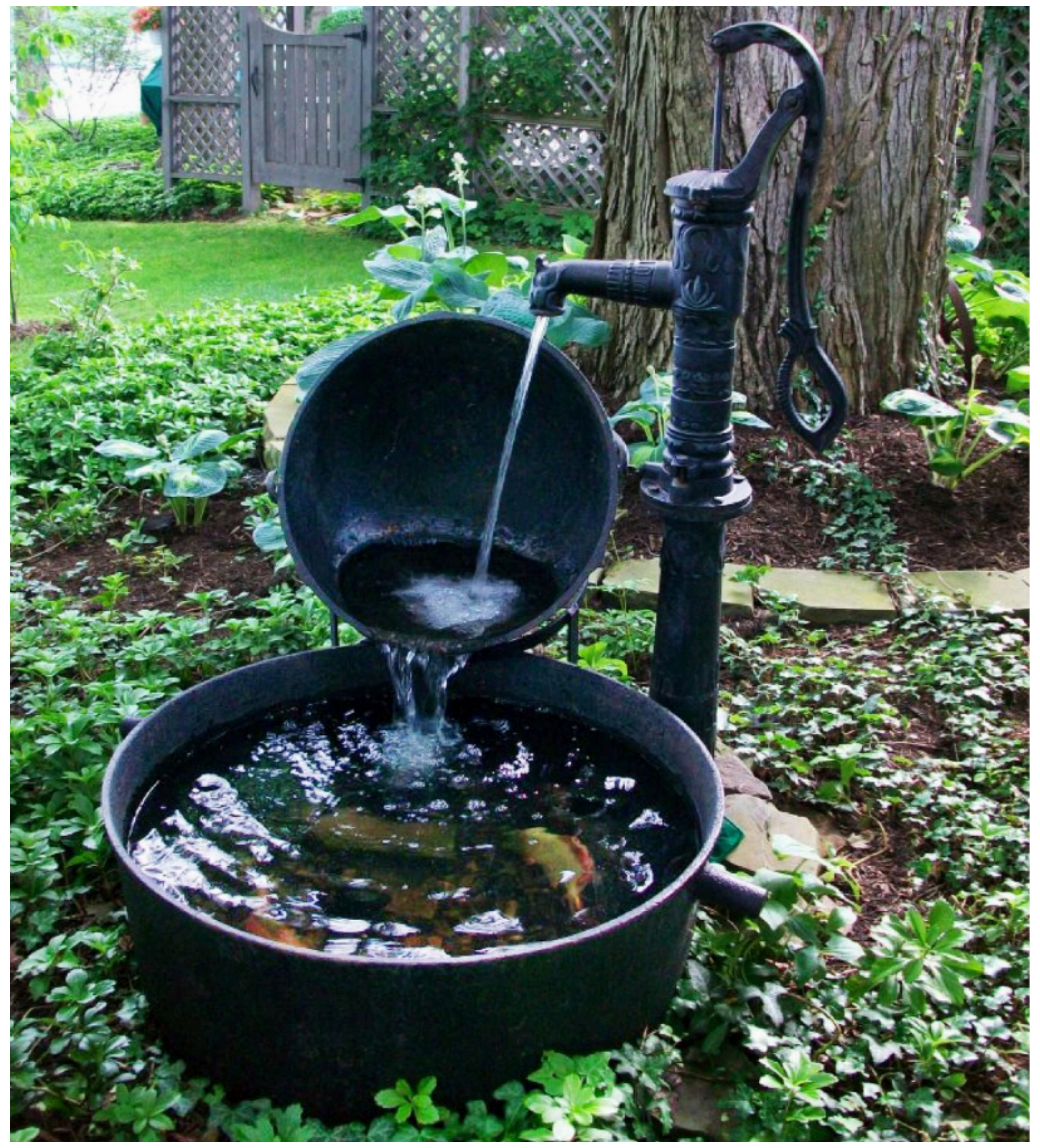


Open system Schrödinger equation

Bath feedback effect

$$i\partial_t |\psi_S(t)\rangle = H_S(t)|\psi_S(t)\rangle + H_{SB}(t)U_B(t,t_0)|\psi_B(t_0)\rangle$$
$$- iH_{SB}(t)\int_{t_0}^{t} \mathrm{d}t'\; U_B(t,t')H_{BS}(t')|\psi_S(t')\rangle\,,$$

Decay, memory and energy renormalization effects



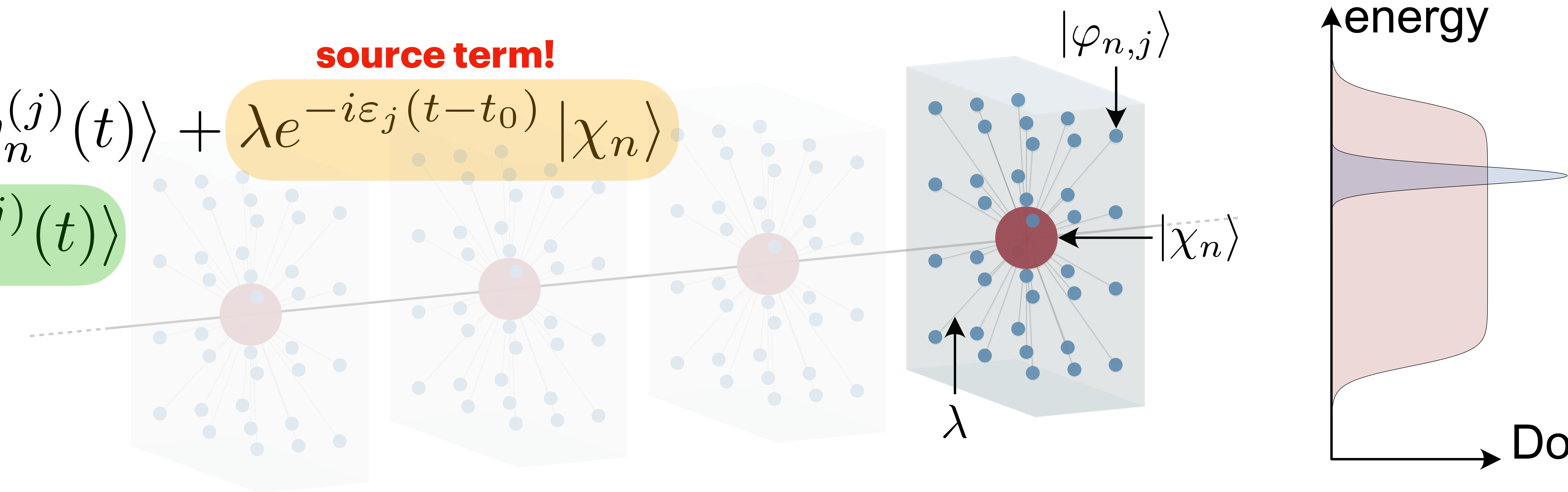
system state $\psi_n^{(j)}$ evolved

out of a pure bath state $\varphi_{n,j}$

source term!

$$i\partial_t |\psi_n^{(j)}(t)\rangle = H_S(t)|\psi_n^{(j)}(t)\rangle + \lambda e^{-i\varepsilon_j(t-t_0)}|\chi_n\rangle$$
$$- i\Gamma|\psi_n^{(j)}(t)\rangle$$

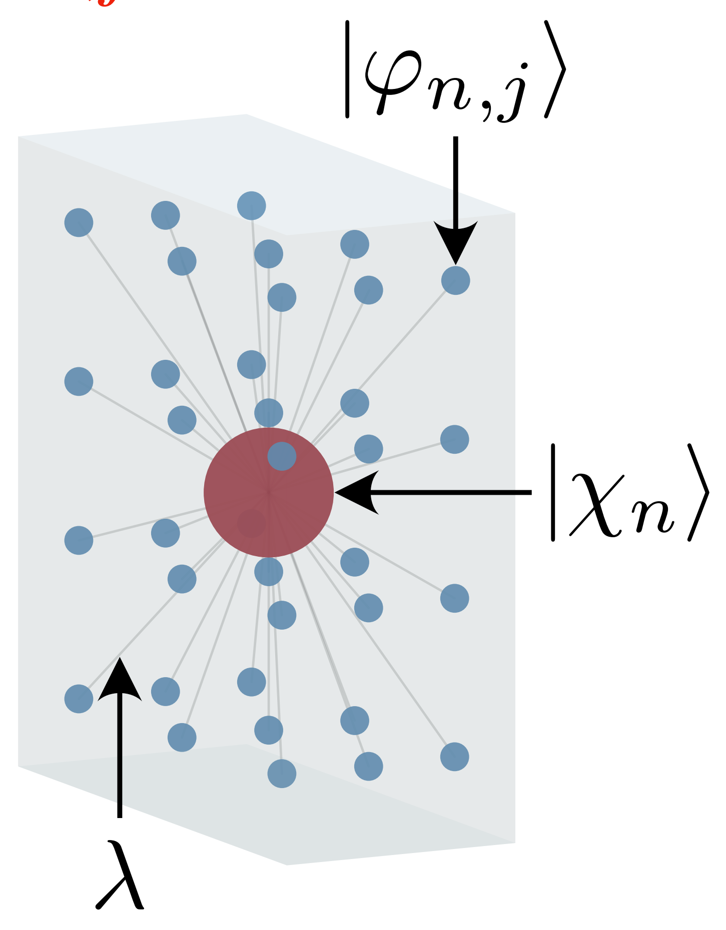
$$\Gamma \equiv \lambda^2\nu_0/2$$



Steady state occupation

$$i\partial_t |\psi_n^{(j)}(t)\rangle = H_S(t) |\psi_n^{(j)}(t)\rangle + \lambda e^{-i\varepsilon_j(t-t_0)} |\chi_n\rangle - i\Gamma |\psi_n^{(j)}(t)\rangle$$

system state $\psi_n^{(j)}$ evolved out of a pure bath state $\varphi_{n,j}$



$$\rho_B(t_0) = \sum_{n,j} f_0(\varepsilon_j) |\varphi_{n,j}\rangle \langle\varphi_{n,j}|, \quad f_0(\varepsilon_j) = \frac{1}{\exp[\beta_0(\varepsilon_j - \mu_0)] + 1}$$

$$\rho_S(t) = \sum_{n,j} f_0(\varepsilon_j) |\psi_n^{(j)}(t)\rangle \langle\psi_n^{(j)}(t)|$$

ensemble (weighted) summation of system states evolved out of all initial bath states

Steady state occupation

$$\rho_S(t) = \Gamma \int_{-\infty}^{+\infty} \frac{\mathrm{d}\epsilon}{\pi} f_0(\epsilon) U_\Gamma(t,\epsilon) U_\Gamma^\dagger(t,\epsilon), \quad U_\Gamma(t,\epsilon) = \int_{-\infty}^{t} \mathrm{d}t' e^{\Gamma(t'-t) - i\epsilon t'} U_S(t,t')$$

$$i\partial_t U_S(t,t') = H_S(t) U_S(t,t')$$

$$\rho_S(t+T) = \rho_S(t)$$ **synchronized with the drive**

In the clean limit ..

Steady state occupation

$$\lim_{\Gamma\to 0} \rho_S(t) = \sum_n p_n \left|\psi_n^F(t)\right\rangle \left\langle\psi_n^F(t)\right|, \quad p_n = \sum_l \langle\varphi_{n,l}|\varphi_{n,l}\rangle\, f_0(\epsilon_n^F + l\Omega).$$

occupation in the Floquet basis becomes time independent

$$i\partial_t \left|\psi_n^F(t)\right\rangle = H_S(t) \left|\psi_n^F(t)\right\rangle \qquad \left|\psi_n^F(t)\right\rangle = \sum_l e^{-i(\epsilon_n^F + l\Omega)t} \left|\varphi_{n,l}\right\rangle, \quad \Omega = 2\pi/T$$

a sum of Fermi-Dirac functions, each shifted by integer multiples of the driving frequency !

Demonstrations

Staircase occupation

Consider a diagonal Hamitonian with a harmonic drive

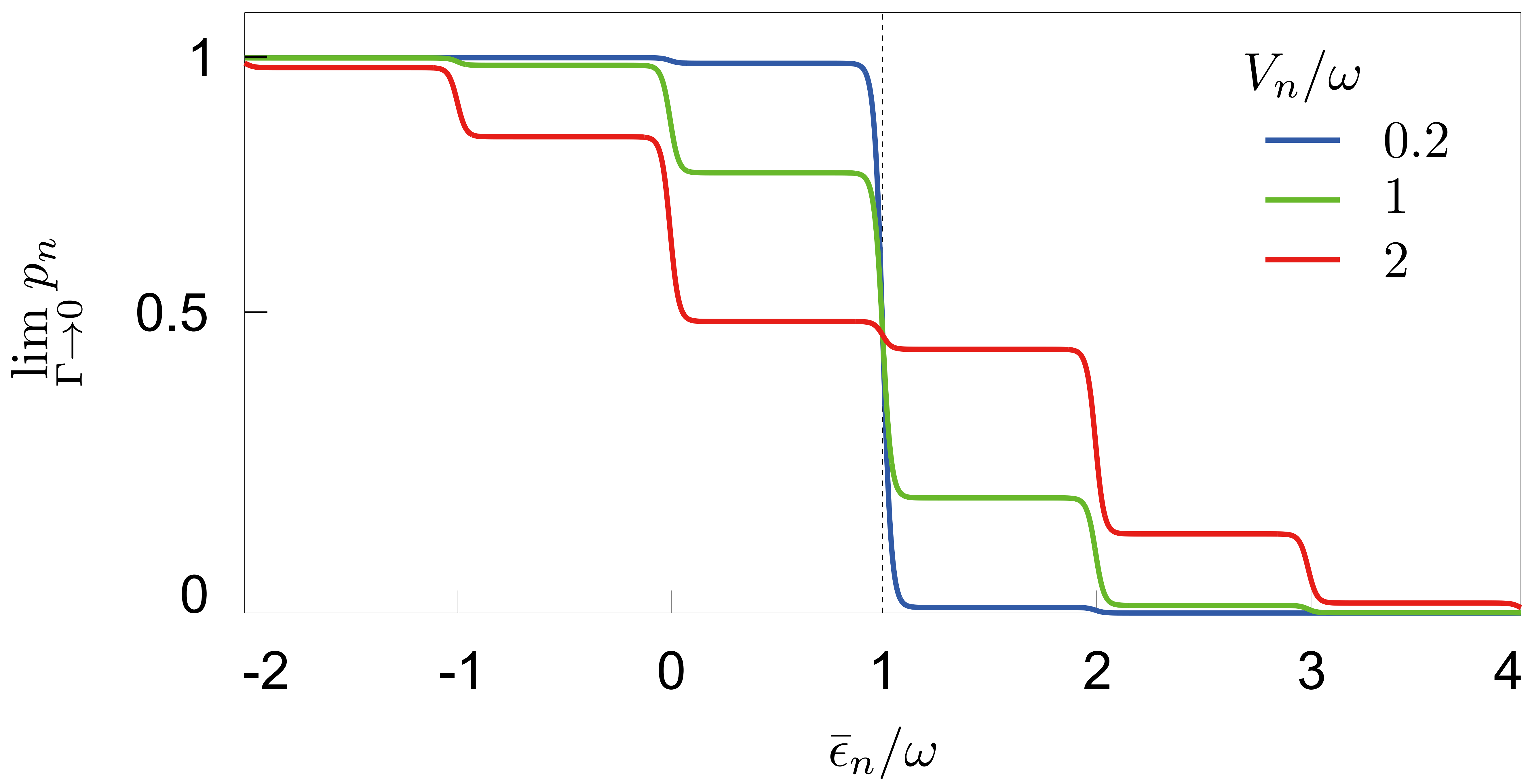
$$\langle \chi_n | H_S(t) | \chi_m \rangle = \delta_{nm}[\epsilon_n + V_n(t)] = \delta_{nm}\epsilon_n(t), \quad V_n(t) = V_n \cos[\omega(t - t_0)]$$

$$\lim_{\Gamma \to 0} p_n = \sum_{l=-\infty}^{+\infty} J_l^2 \big(V_n/\omega \big) f_0(\bar{\epsilon}_n + l\omega)$$

Bessel functions

Staircase occupation

$$\lim_{\Gamma\to 0} p_n = \sum_{l=-\infty}^{+\infty} J_l^2\left(V_n/\omega\right) f_0(\bar{\epsilon}_n + l\omega)$$



Quantum oscillations

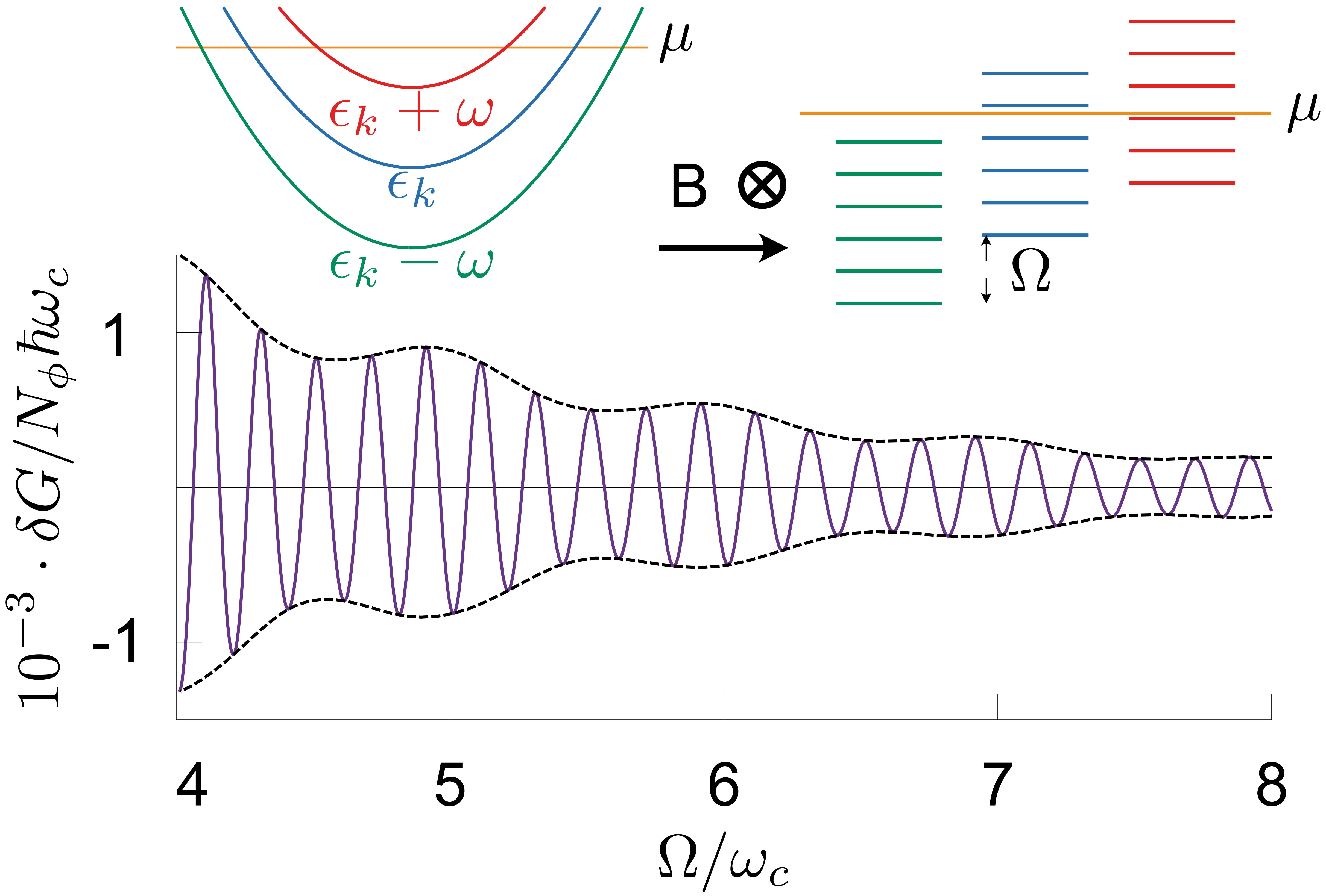
is a powerful way measure the size of the Fermi surface.

$$S = 2m\pi\mu$$

Quantum oscillations

when we have multiple Fermi surfaces, beating appears

$$S_l \approx 2m\pi(\mu - l\Omega)$$



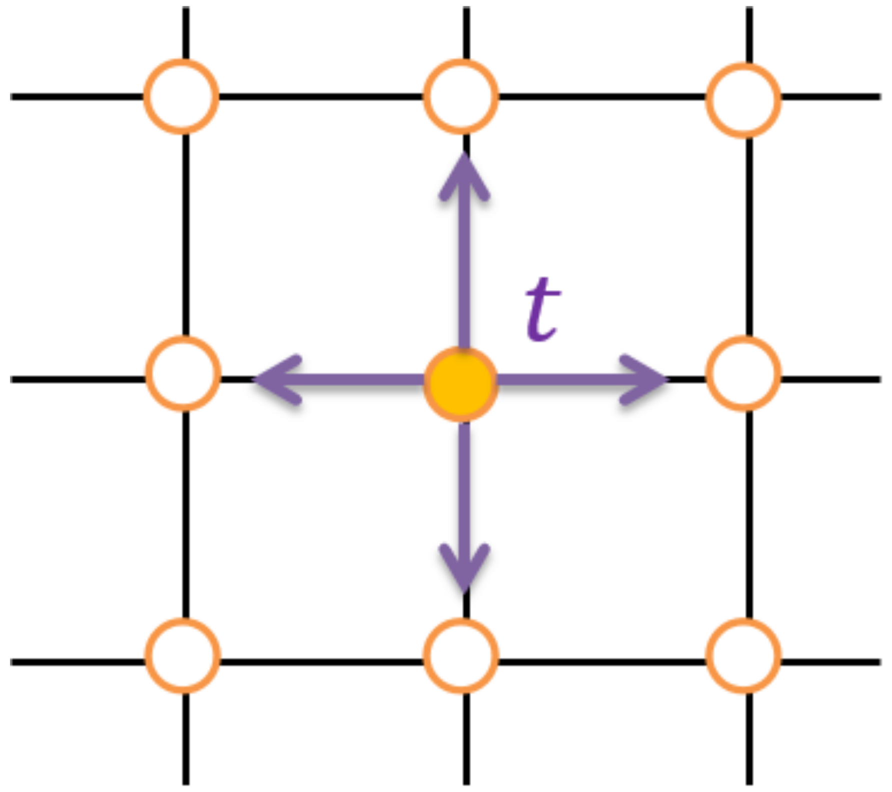
$$\text{Quantum oscillation} \sim \cos\left(\frac{2m\pi\mu}{\omega_c}\right)$$

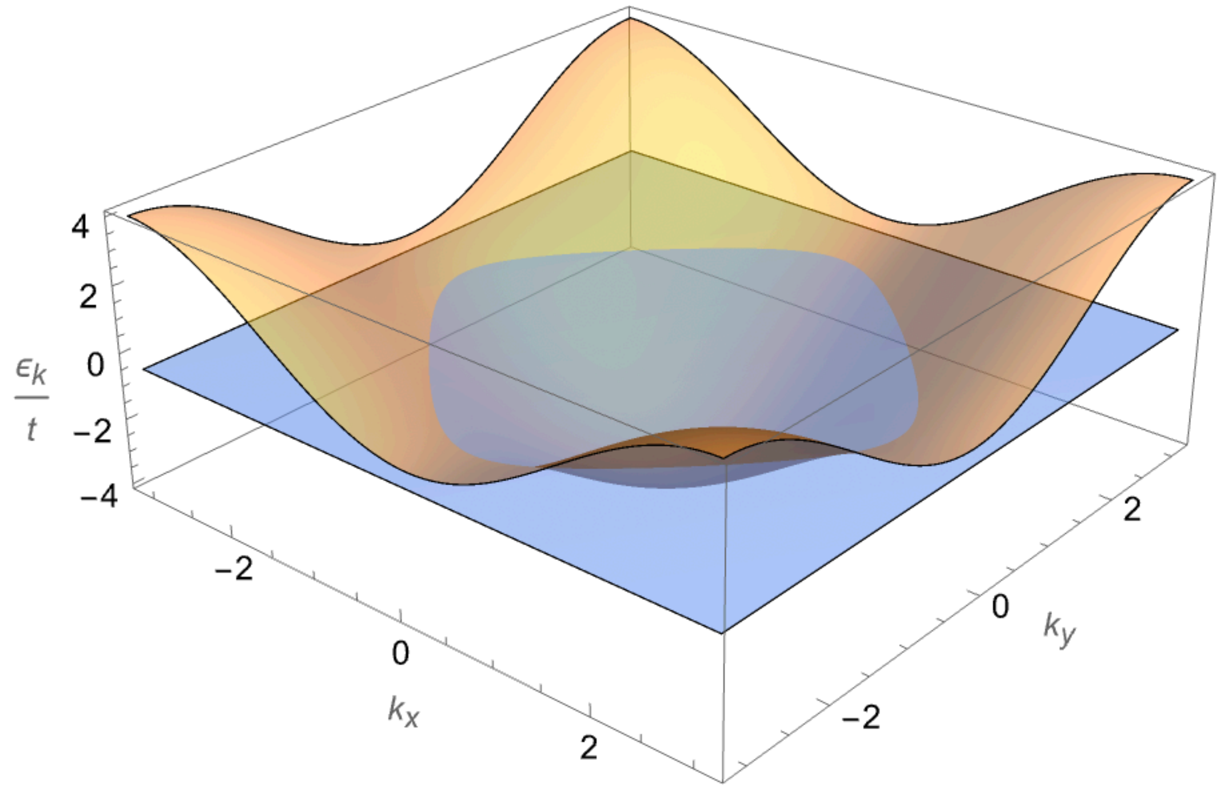
$$\text{Beating} \sim \cos\left(\frac{2m\pi\Omega}{\omega_c}\right)$$

$$\text{Cyclotron frequency } \omega_c \sim B$$

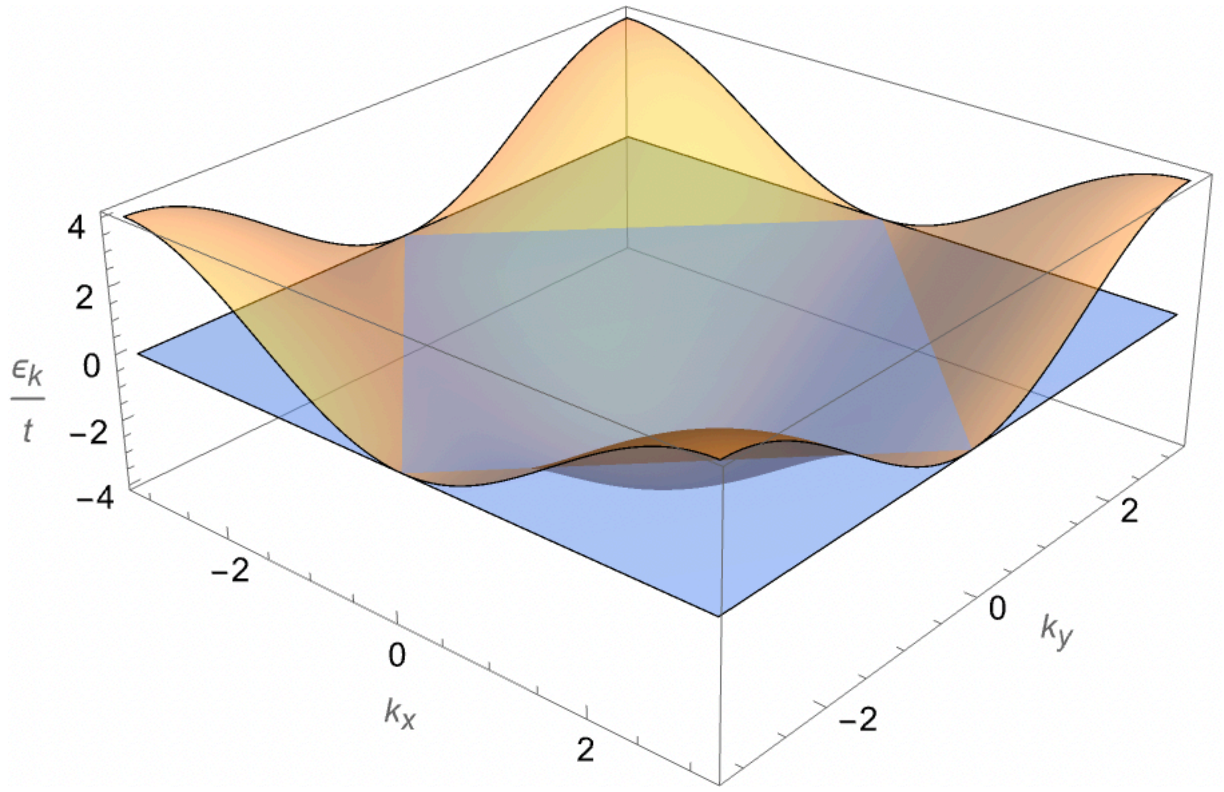
non-equilibrium van-Hove singularities

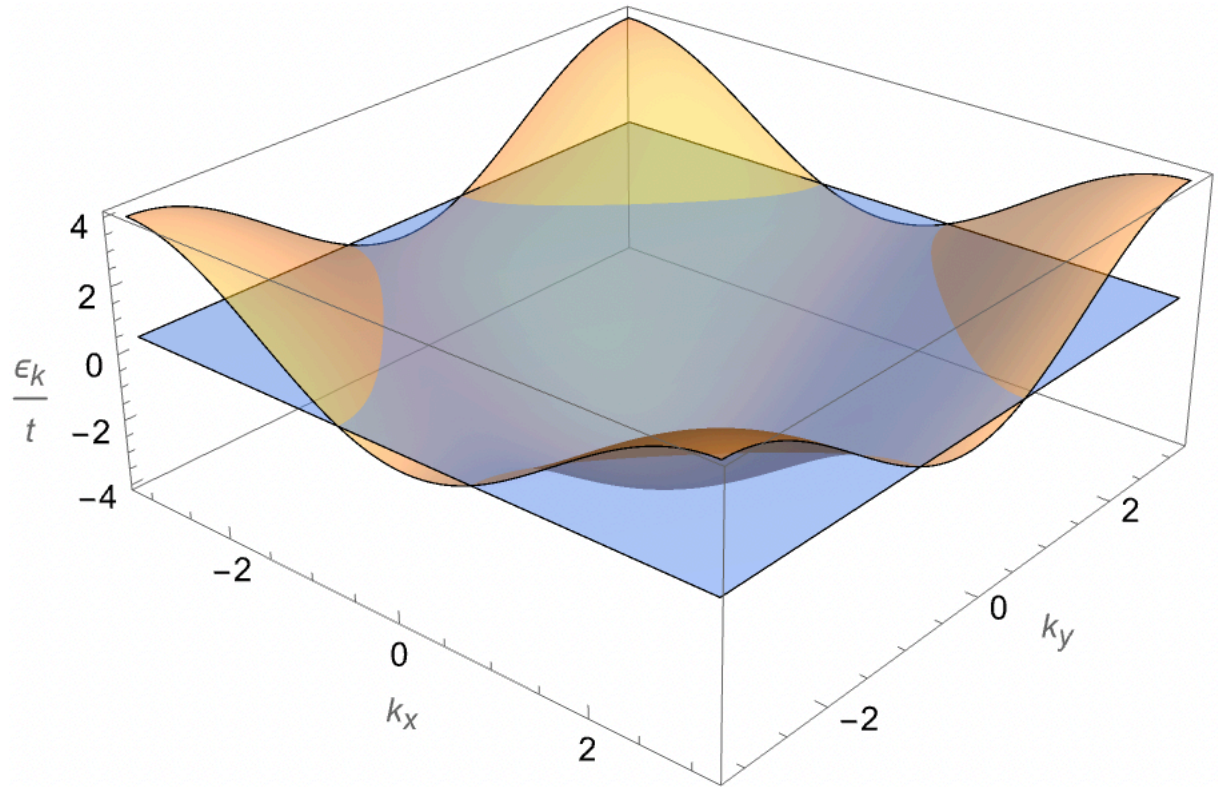
van-Hove singularities usually appear when the Fermi surface has “kinks”

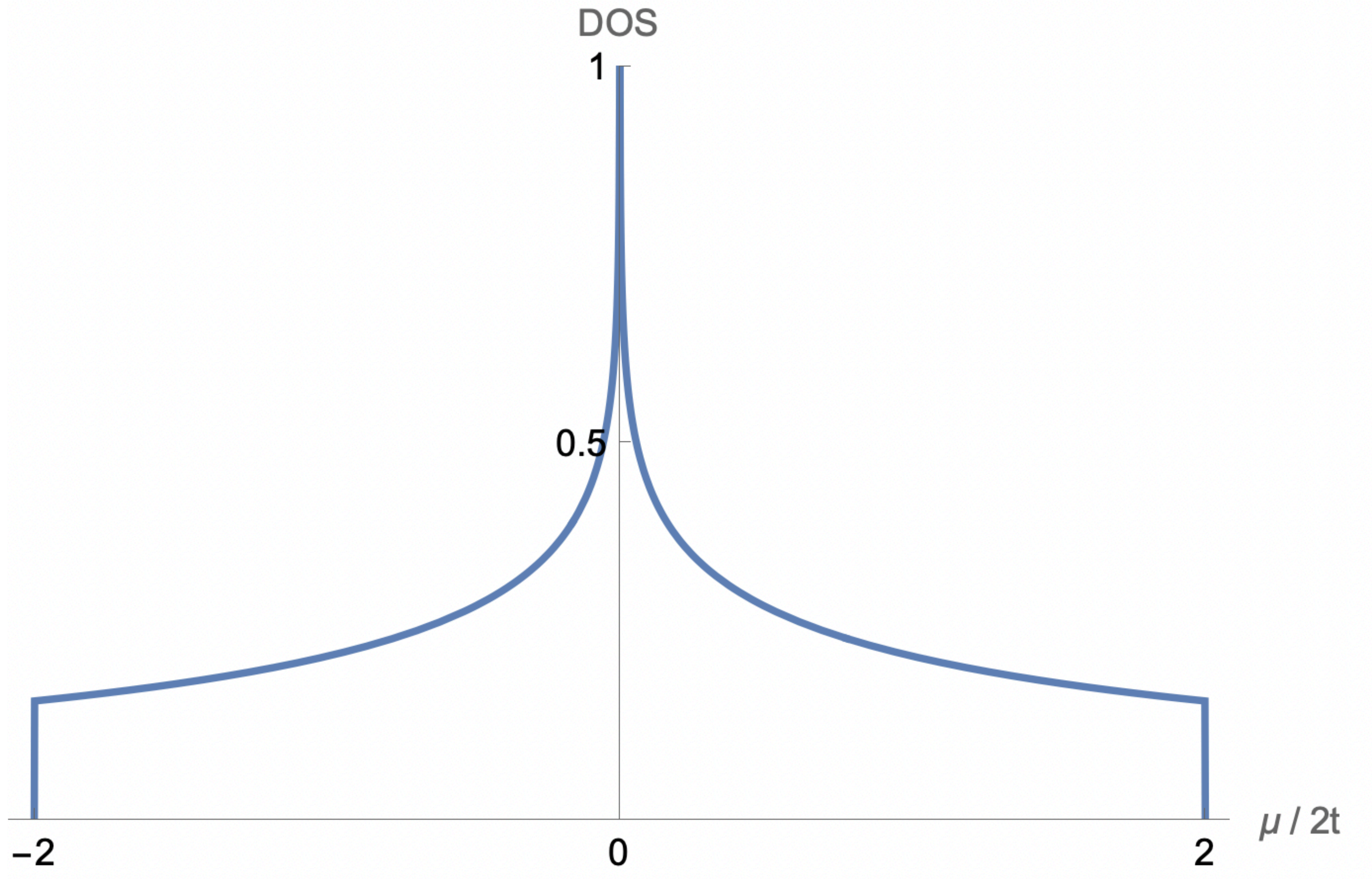




$$\epsilon_{\mathbf{k}} = -2t\cos(k_x) - 2t\cos(k_y)$$

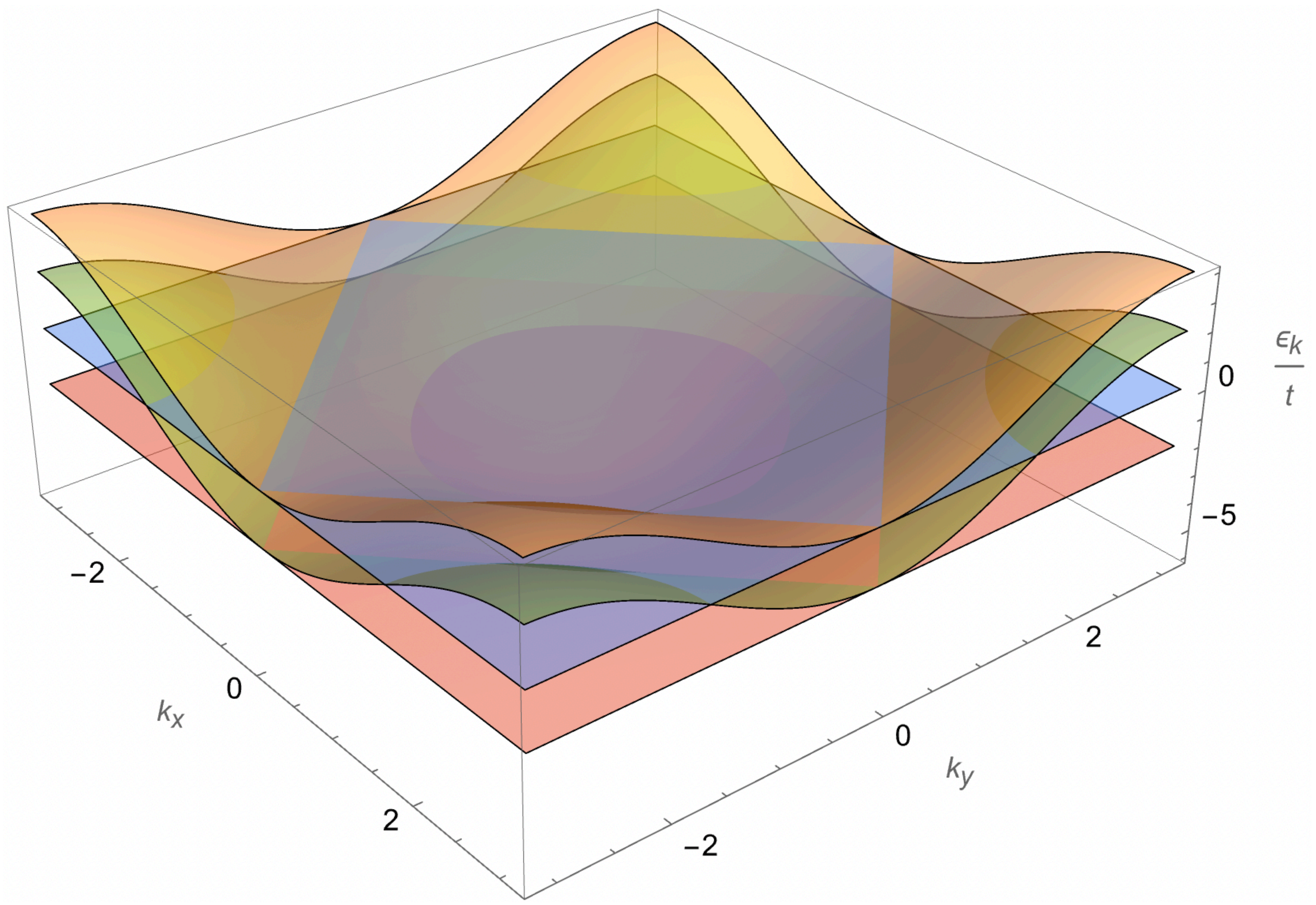


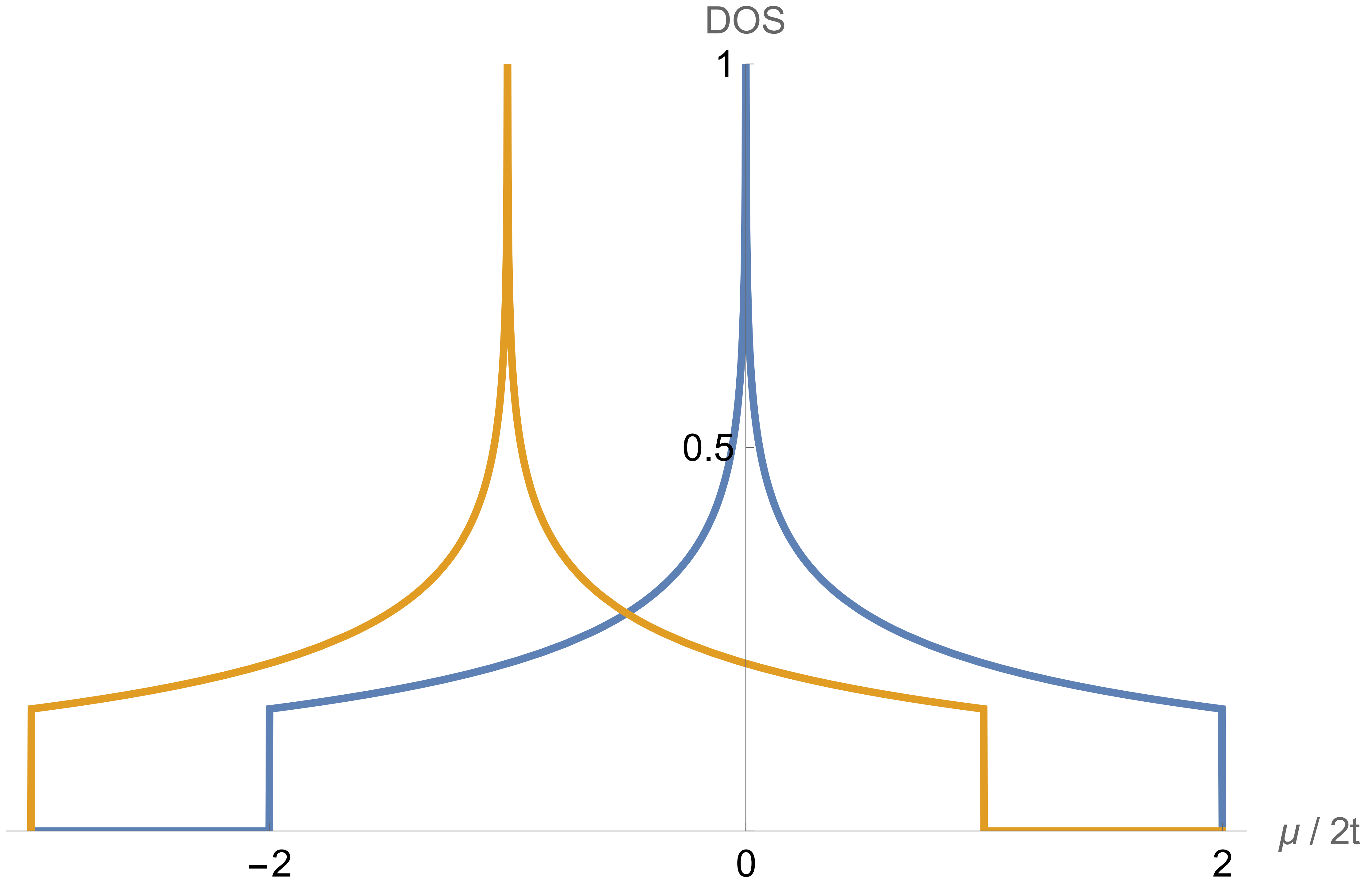




non-equilibrium van-Hove singularities

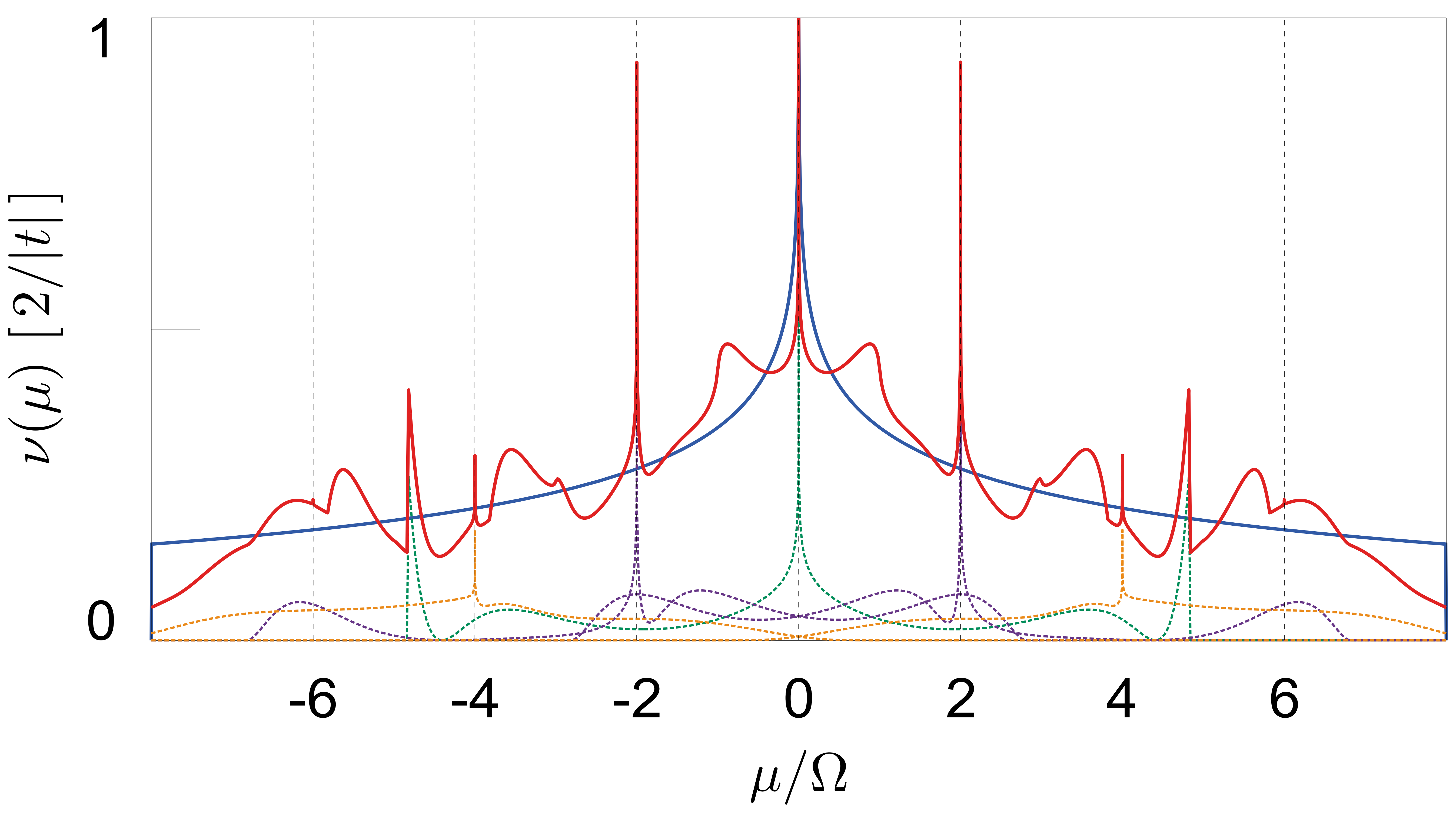
if we have multiple bands, we could have more van-Hove singularities

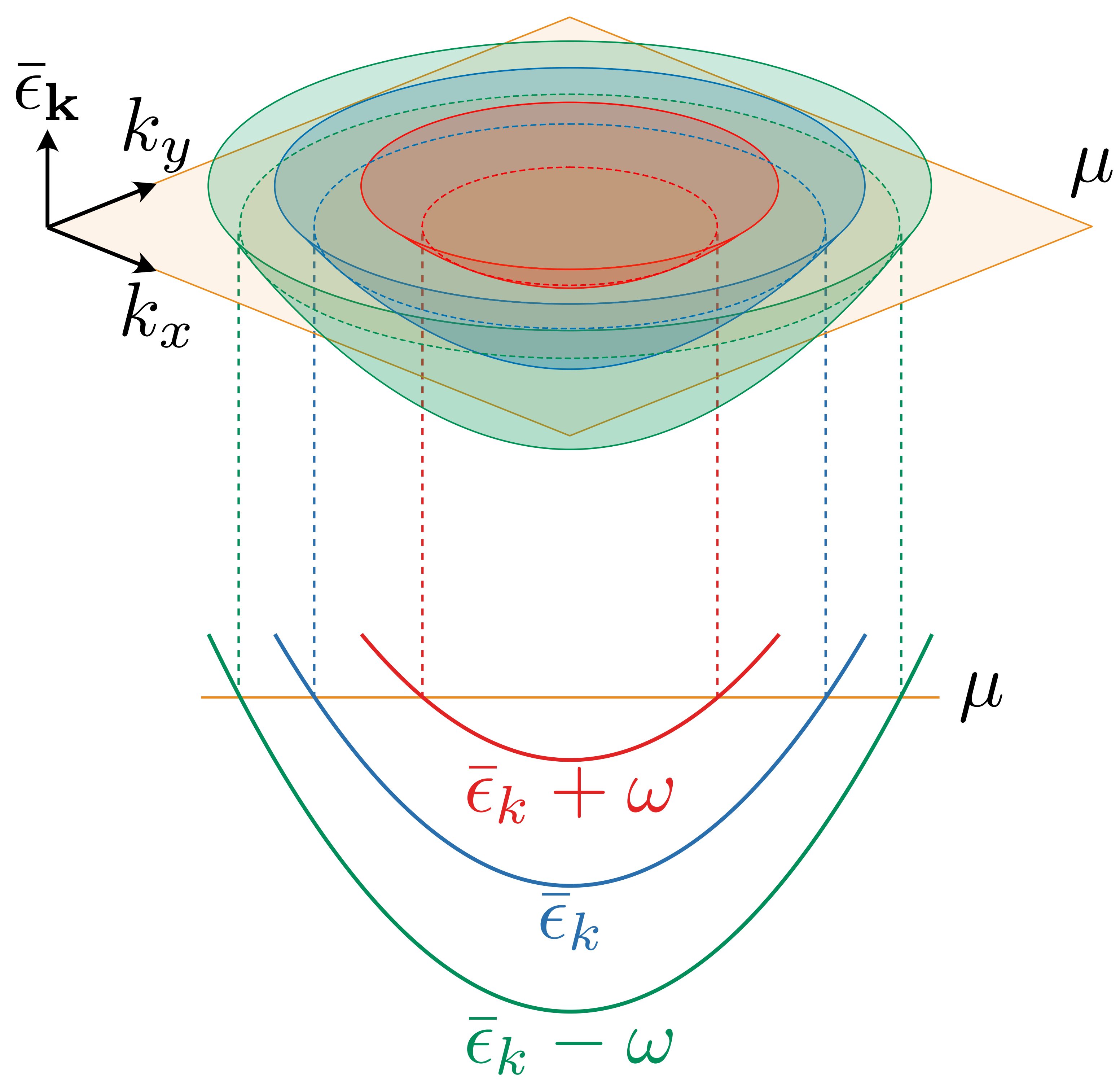




non-equilibrium van-Hove singularities

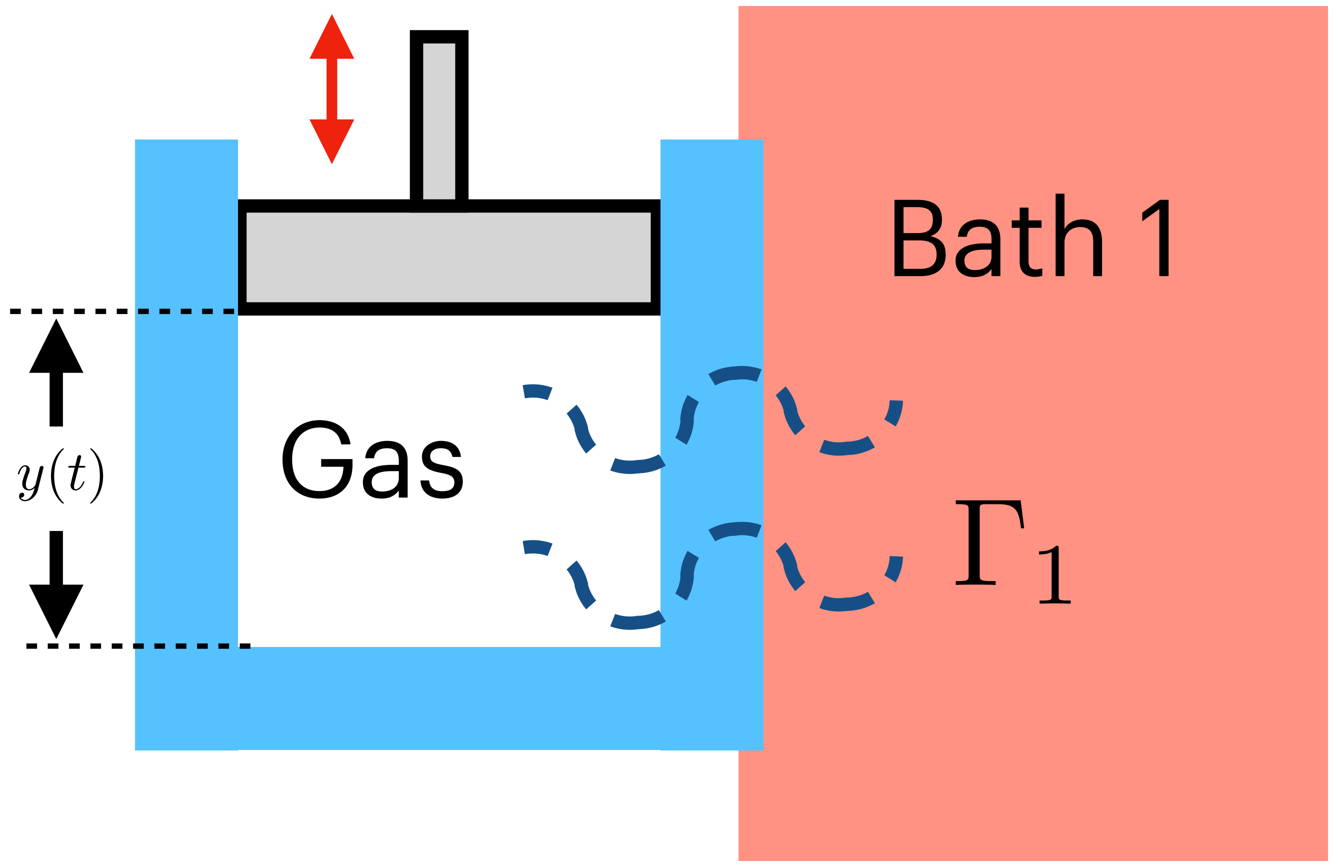
Additional van-Hove singularities appear when additional Fermi surfaces appear





Extra & ongoing

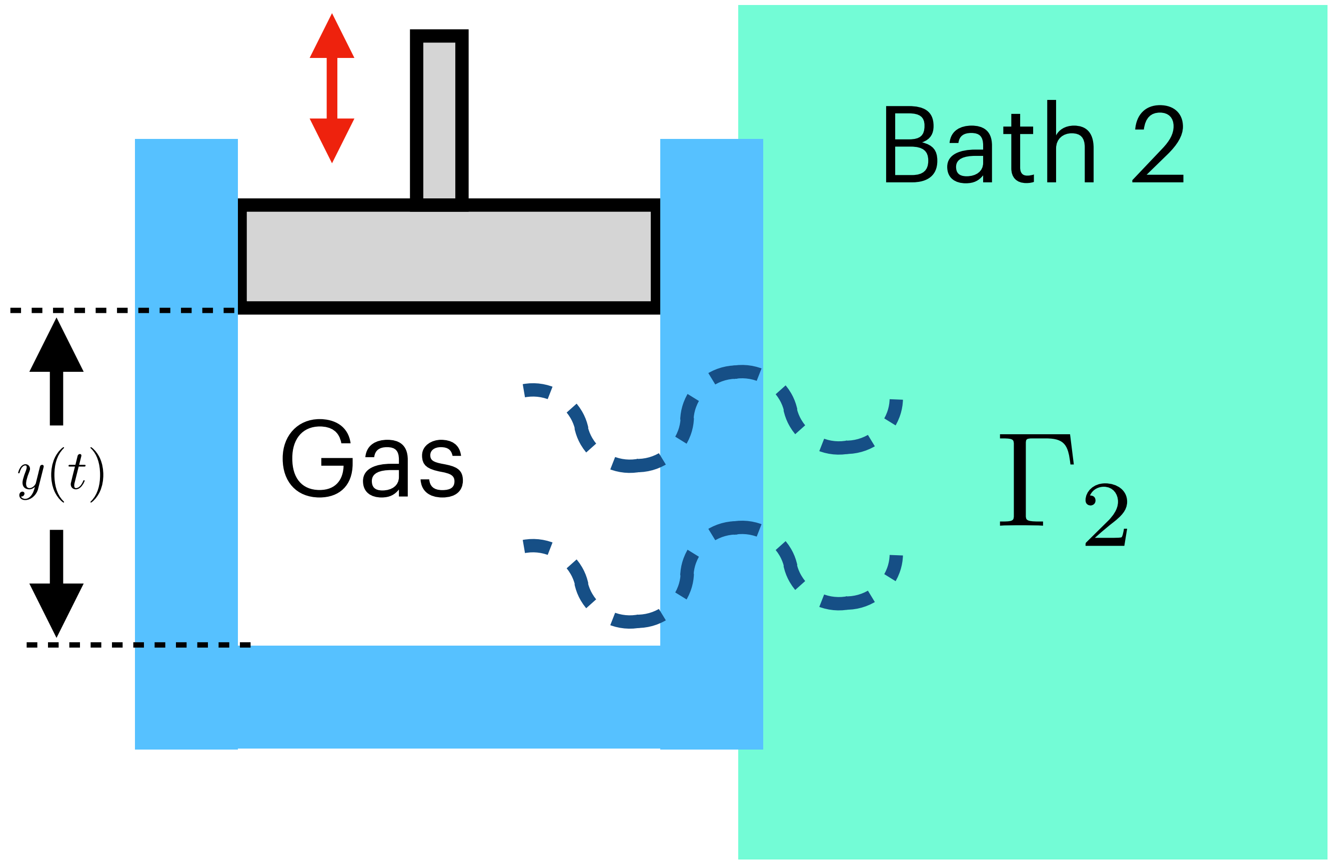
The nature of bath matters away from equilibrium



In equilibrium in limit of weak coupling to baths, nature of bath does not matter

$$\Gamma_1 \to 0 \qquad \Gamma_2 \to 0$$

$$\rho^{(1)}_{\text{equil}} = \rho^{(2)}_{\text{equil}} = \frac{e^{-\beta H}}{Z}$$

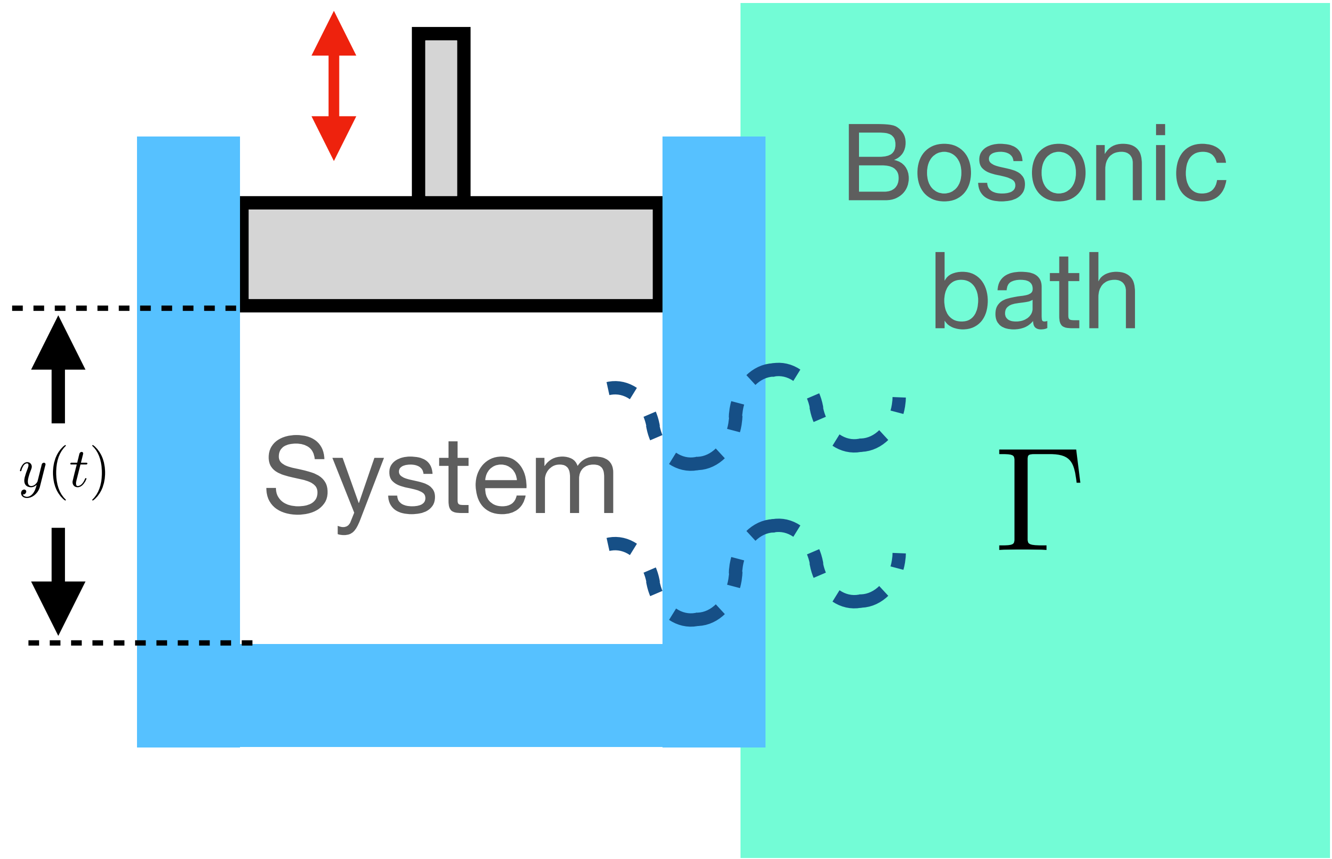


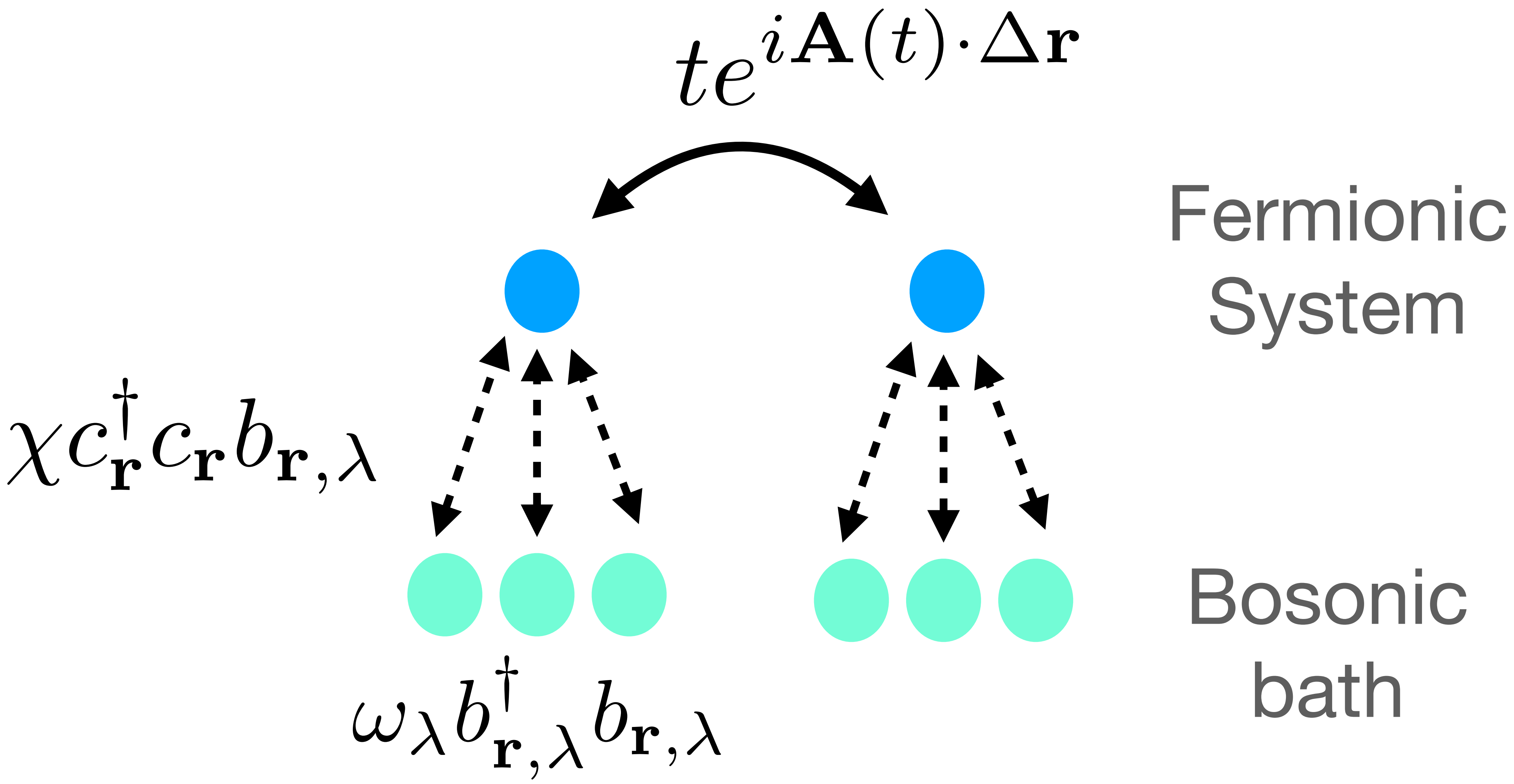
In **non-equilibrium** even in limit of weak coupling to baths, **nature of bath matters** for the steady state

$$\rho^{(1)}_{\text{non-eq.}}(t) \neq \rho^{(2)}_{\text{non-eq.}}(t)$$

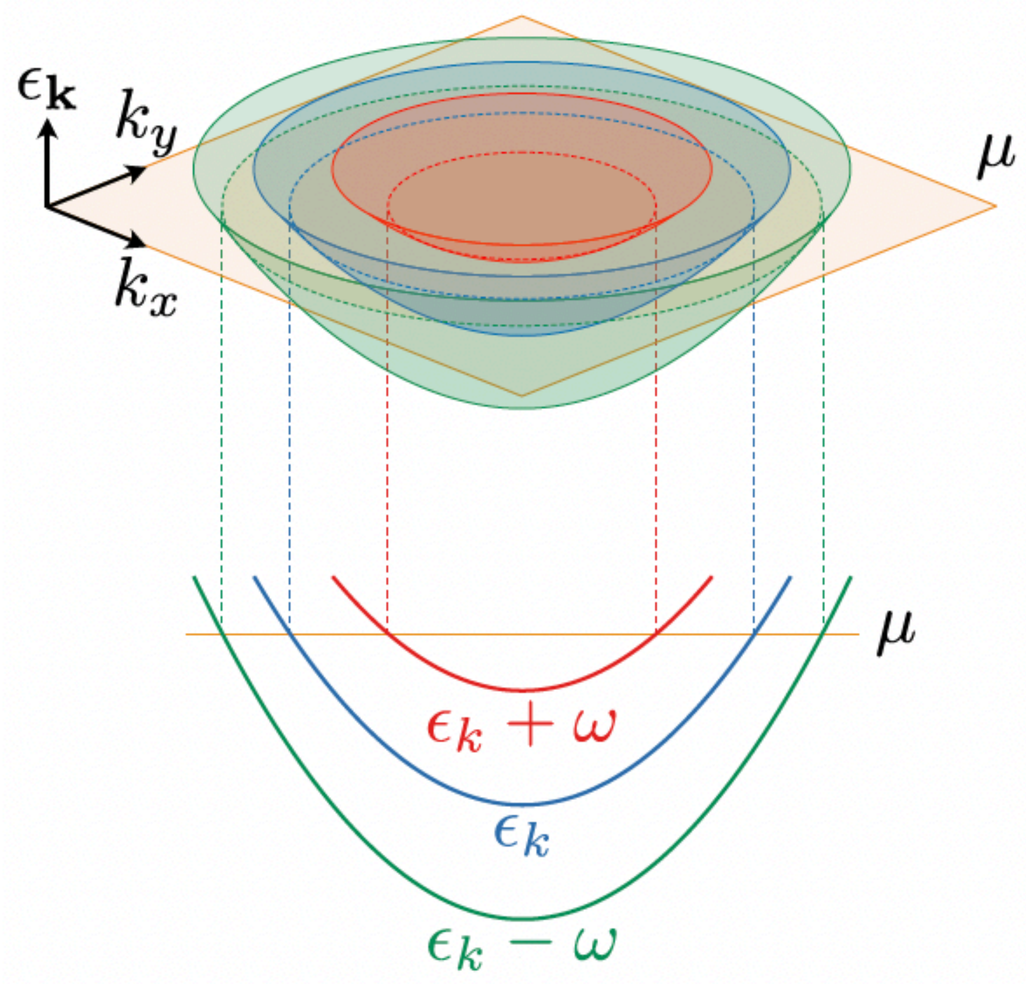
Floquet Non-Fermi Liquid for ideal bosonic baths

Shi, Matsyshyn, Song, Sodemann unpublished

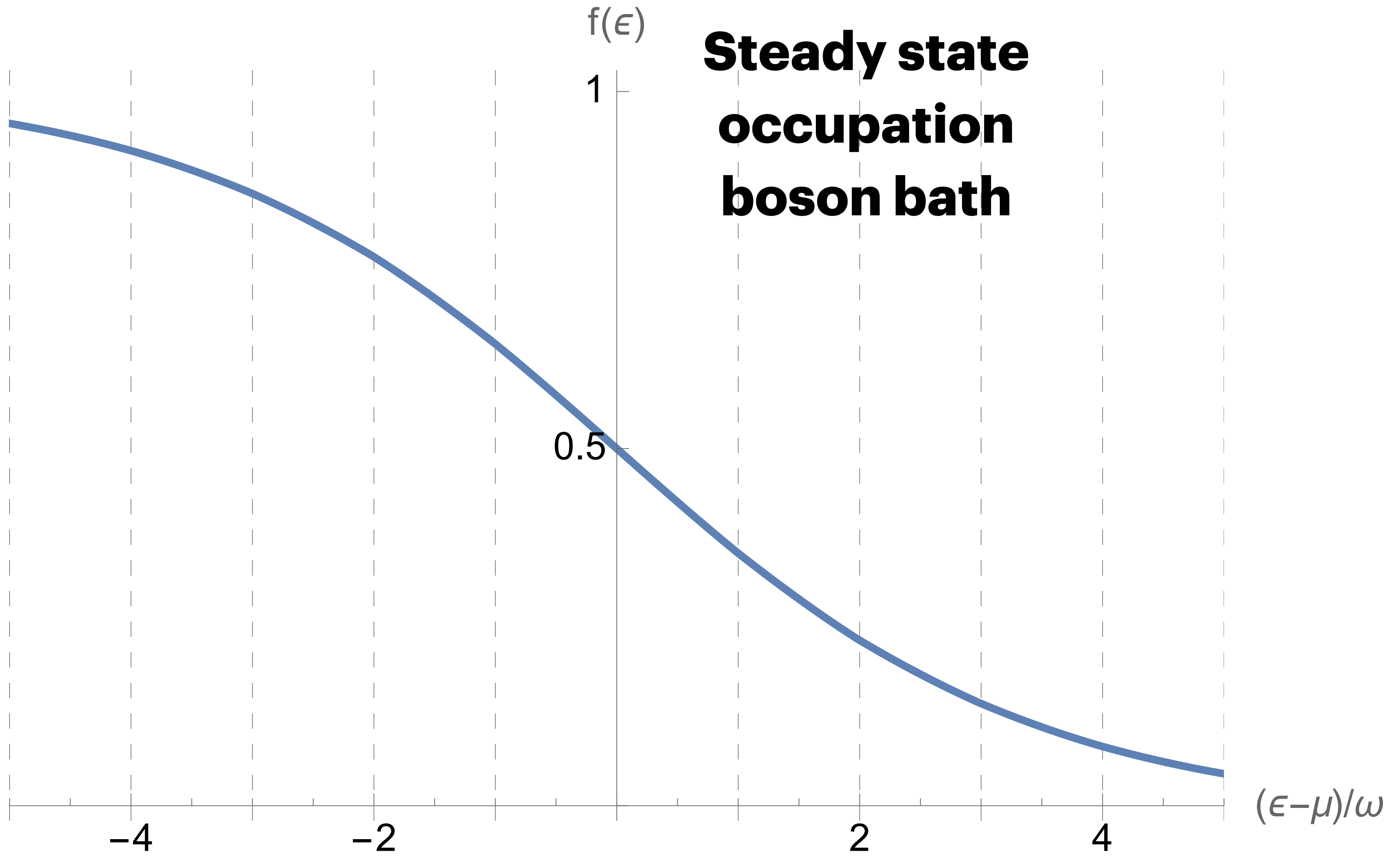




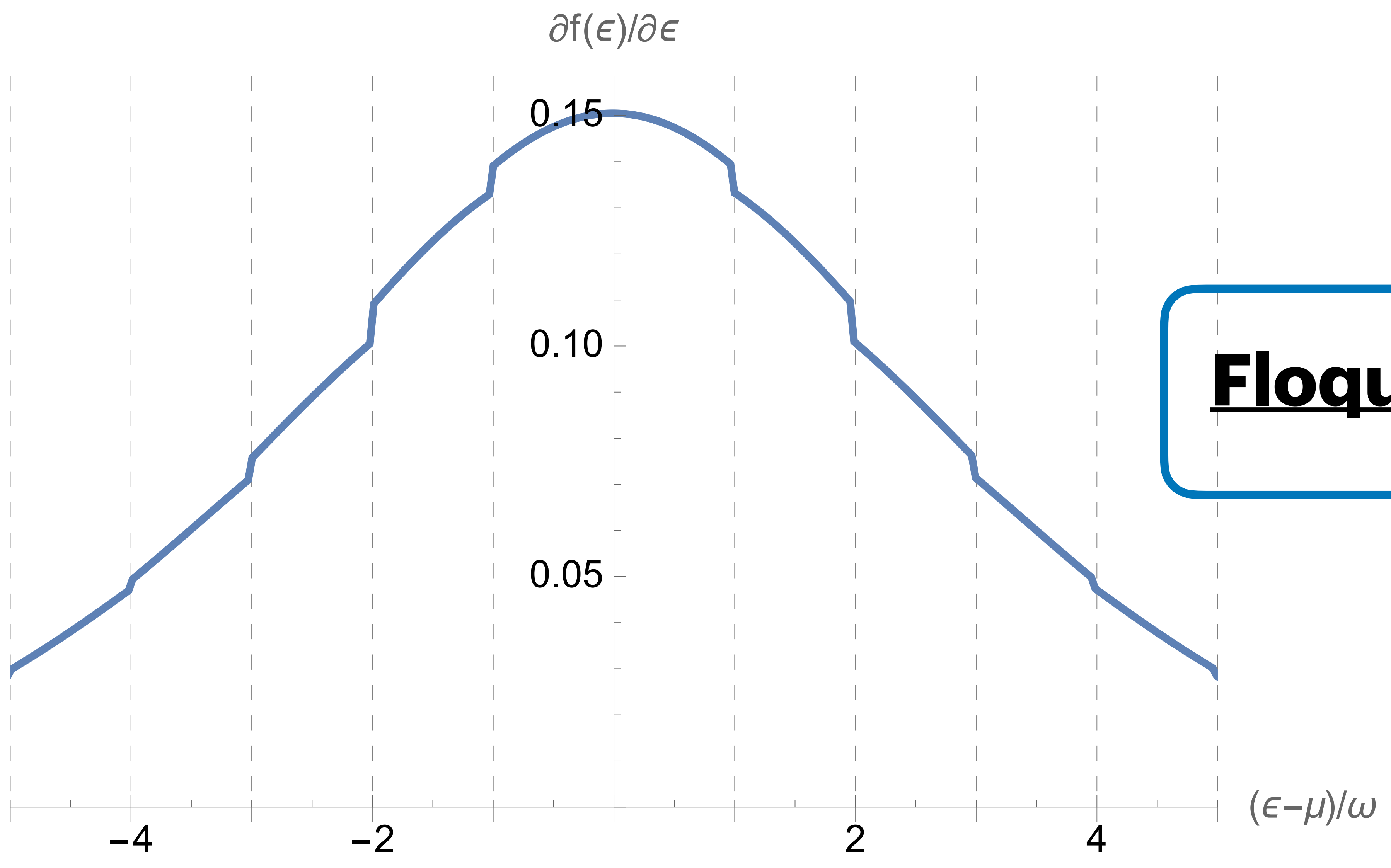
No quasiparticle jump, but sharp fermi surfaces !



Fermi Dirac staircase dissapears



Derivative of occupation has discontinuities



Floquet Non-Fermi liquid State

Summary

- the Floquet Fermi liquid is a periodic, non-equilibrium state for fermions
- the Floquet Fermi liquid has many fermi surfaces and thus exotic properties
- the nature of bath matters

Thank You for your attention!