

# Superradiance and quantum states on black hole space-times

Elizabeth Winstanley

Balakumar, Bernar, Crispino & EW *PLB* **811** 135904 (2020)

Balakumar, Bernar & EW *PRD* **106** 125013 (2022)

Balakumar, Bernar & EW *J. Phys. Conf. Ser.* **2531** 012011 (2023)

Consortium for Fundamental Physics  
School of Mathematics and Statistics  
The University of Sheffield



University of  
Sheffield

# Outline

## 1 Canonical quantization

- Neutral scalar field

## 2 QFT on Schwarzschild black holes

## 3 QFT on Kerr black holes

## 4 Charged scalar QFT on RN

- B state on RN
- HH state on RN

## 5 Conclusions

# Canonical quantization of a neutral scalar field

# Canonical quantization of a neutral scalar field

## Klein-Gordon equation

$$\square\Phi = 0$$

# Canonical quantization of a neutral scalar field

## Klein-Gordon equation

$$\square\Phi = 0$$

## Klein-Gordon scalar product

$$(\Phi_1, \Phi_2)_{KG} = i \int_{\Sigma} [\Phi_2^* \nabla_{\mu} \Phi_1 - \Phi_1 \nabla_{\mu} \Phi_2^*] d\Sigma^{\mu}$$

Involves **time derivative** of  $\Phi$

# Canonical quantization of a neutral scalar field $\Phi$

# Canonical quantization of a neutral scalar field $\Phi$

Expand classical field in terms of orthonormal basis of field modes

$$\Phi = \sum_j a_j \phi_j^+ + a_j^\dagger \phi_j^-$$

# Canonical quantization of a neutral scalar field $\Phi$

Expand classical field in terms of orthonormal basis of field modes

$$\Phi = \sum_j a_j \phi_j^+ + a_j^\dagger \phi_j^-$$

## Positive frequency modes

$$\phi_j^+ \propto e^{-i\omega(t+x)} \quad \omega > 0$$

Positive KG “norm”

$$(\phi_j^+, \phi_k^+)_{KG} \propto \delta_{jk},$$

# Canonical quantization of a neutral scalar field $\Phi$

Expand classical field in terms of orthonormal basis of field modes

$$\Phi = \sum_j a_j \phi_j^+ + a_j^\dagger \phi_j^-$$

## Negative frequency modes

$$\phi_j^- \propto e^{-i\omega(t \pm x)} \quad \omega < 0$$

## Negative KG “norm”

$$(\phi_j^-, \phi_k^-)_{KG} \propto -\delta_{jk},$$

# Canonical quantization of a neutral scalar field $\Phi$

Expand classical field in terms of orthonormal basis of field modes

$$\Phi = \sum_j a_j \phi_j^+ + a_j^\dagger \phi_j^-$$

# Canonical quantization of a neutral scalar field $\Phi$

Expand classical field in terms of orthonormal basis of field modes

$$\hat{\Phi} = \sum_j \hat{a}_j \phi_j^+ + \hat{a}_j^\dagger \phi_j^-$$

Promote expansion coefficients to operators  $\hat{a}_j, \hat{a}_j^\dagger$  with

$$[\hat{a}_j, \hat{a}_k^\dagger] = \delta_{jk} \quad [\hat{a}_j, \hat{a}_k] = 0 \quad [\hat{a}_j^\dagger, \hat{a}_k^\dagger] = 0$$

# Canonical quantization of a neutral scalar field $\Phi$

Expand classical field in terms of orthonormal basis of field modes

$$\hat{\Phi} = \sum_j \hat{a}_j \phi_j^+ + \hat{a}_j^\dagger \phi_j^-$$

Promote expansion coefficients to operators  $\hat{a}_j, \hat{a}_j^\dagger$  with

$$[\hat{a}_j, \hat{a}_k^\dagger] = \delta_{jk} \quad [\hat{a}_j, \hat{a}_k] = 0 \quad [\hat{a}_j^\dagger, \hat{a}_k^\dagger] = 0$$

$\hat{a}_j$  - particle annihilation operators

# Canonical quantization of a neutral scalar field $\Phi$

Expand classical field in terms of orthonormal basis of field modes

$$\hat{\Phi} = \sum_j \hat{a}_j \phi_j^+ + \hat{a}_j^\dagger \phi_j^-$$

Promote expansion coefficients to operators  $\hat{a}_j, \hat{a}_j^\dagger$  with

$$[\hat{a}_j, \hat{a}_k^\dagger] = \delta_{jk} \quad [\hat{a}_j, \hat{a}_k] = 0 \quad [\hat{a}_j^\dagger, \hat{a}_k^\dagger] = 0$$

$\hat{a}_j$  - particle annihilation operators

$\hat{a}_j^\dagger$  - particle creation operators

# Canonical quantization of a neutral scalar field $\Phi$

Expand classical field in terms of orthonormal basis of field modes

$$\hat{\Phi} = \sum_j \hat{a}_j \phi_j^+ + \hat{a}_j^\dagger \phi_j^-$$

Promote expansion coefficients to operators  $\hat{a}_j, \hat{a}_j^\dagger$  with

$$[\hat{a}_j, \hat{a}_k^\dagger] = \delta_{jk} \quad [\hat{a}_j, \hat{a}_k] = 0 \quad [\hat{a}_j^\dagger, \hat{a}_k^\dagger] = 0$$

$\hat{a}_j$  - particle annihilation operators

$\hat{a}_j^\dagger$  - particle creation operators

Define the **vacuum** state  $|0\rangle$

$$\hat{a}_j |0\rangle = 0$$

# Schwarzschild black hole

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2$$

# Schwarzschild black hole

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2$$

## Neutral scalar field modes

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \frac{1}{r\mathcal{N}} e^{-i\omega t} e^{im\varphi} Y_{\ell m}(\theta) R_{\omega\ell}(r)$$

# Schwarzschild black hole

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2$$

## Neutral scalar field modes

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \frac{1}{r\mathcal{N}} e^{-i\omega t} e^{im\varphi} Y_{\ell m}(\theta) R_{\omega\ell}(r)$$

$Y_{\ell m}(\theta)$ : spherical harmonics

# Schwarzschild black hole

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2$$

## Neutral scalar field modes

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \frac{1}{r\mathcal{N}} e^{-i\omega t} e^{im\varphi} Y_{\ell m}(\theta) R_{\omega\ell}(r)$$

$Y_{\ell m}(\theta)$ : spherical harmonics

$\mathcal{N}$ : normalization constant

# Schwarzschild black hole

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2$$

## Neutral scalar field modes

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \frac{1}{r\mathcal{N}} e^{-i\omega t} e^{im\varphi} Y_{\ell m}(\theta) R_{\omega\ell}(r)$$

$Y_{\ell m}(\theta)$ : spherical harmonics

$\mathcal{N}$ : normalization constant

Positive frequency with respect to Schwarzschild time  $t$ :  $\omega > 0$

# Schwarzschild black hole

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2$$

## Neutral scalar field modes

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \frac{1}{r\mathcal{N}} e^{-i\omega t} e^{im\varphi} Y_{\ell m}(\theta) R_{\omega\ell}(r)$$

$Y_{\ell m}(\theta)$ : spherical harmonics

$\mathcal{N}$ : normalization constant

Positive frequency with respect to Schwarzschild time  $t$ :  $\omega > 0$

$$0 = \left[ \frac{d^2}{dr_*^2} + V_{\omega\ell}(r) \right] R_{\omega\ell}(r) \quad \frac{dr_*}{dr} = \left(1 - \frac{2M}{r}\right)^{-1}$$

# Schwarzschild black hole

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2$$

## Neutral scalar field modes

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \frac{1}{r\mathcal{N}} e^{-i\omega t} e^{im\varphi} Y_{\ell m}(\theta) R_{\omega\ell}(r)$$

$Y_{\ell m}(\theta)$ : spherical harmonics

$\mathcal{N}$ : normalization constant

Positive frequency with respect to Schwarzschild time  $t$ :  $\omega > 0$

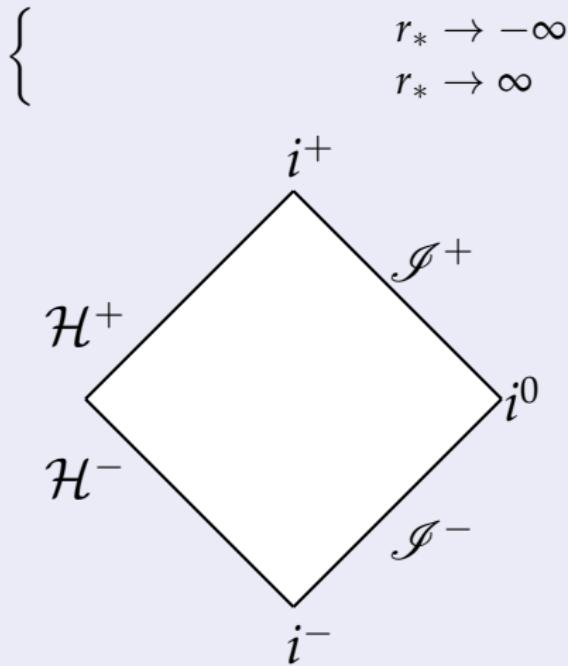
$$0 = \left[ \frac{d^2}{dr_*^2} + V_{\omega\ell}(r) \right] R_{\omega\ell}(r) \quad \frac{dr_*}{dr} = \left(1 - \frac{2M}{r}\right)^{-1}$$

$$V_{\omega\ell}(r) \rightarrow \omega^2 \quad r_* \rightarrow \pm\infty$$

# “In” and “Up” modes

# “In” and “Up” modes

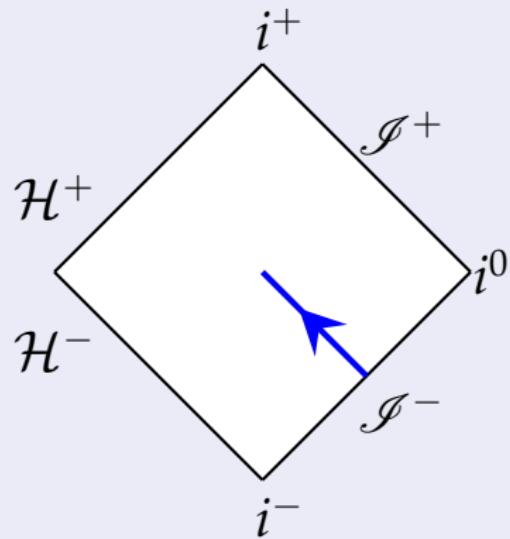
“In” modes  $R_{\omega\ell}^{\text{in}}$



# “In” and “Up” modes

“In” modes  $R_{\omega\ell}^{\text{in}}$

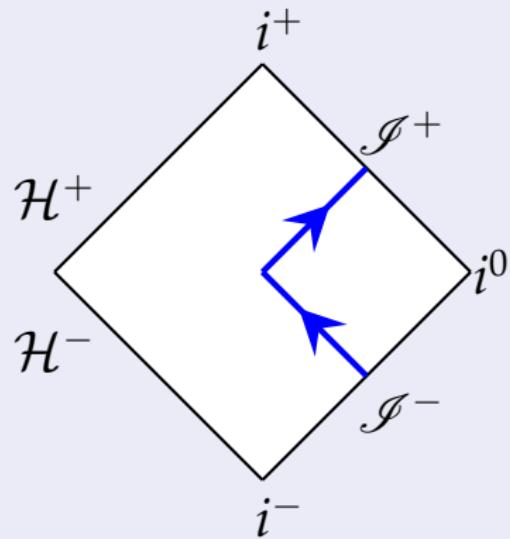
$$\left\{ \begin{array}{ll} & r_* \rightarrow -\infty \\ e^{-i\omega r_*} & r_* \rightarrow \infty \end{array} \right.$$



# "In" and "Up" modes

"In" modes  $R_{\omega\ell}^{\text{in}}$

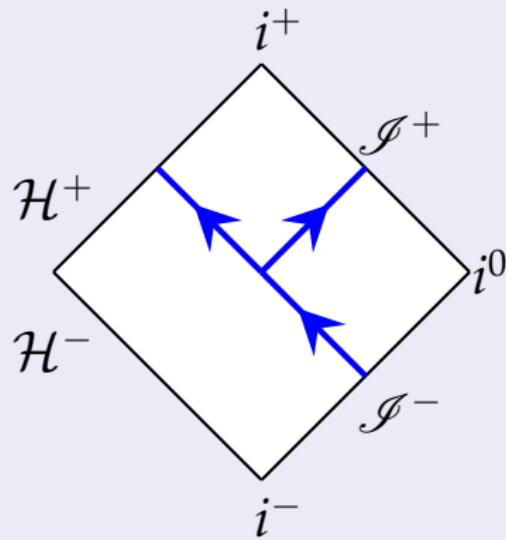
$$\left\{ \begin{array}{ll} & r_* \rightarrow -\infty \\ e^{-i\omega r_*} + A_{\omega\ell}^{\text{in}} e^{i\omega r_*} & r_* \rightarrow \infty \end{array} \right.$$



# “In” and “Up” modes

“In” modes  $R_{\omega\ell}^{\text{in}}$

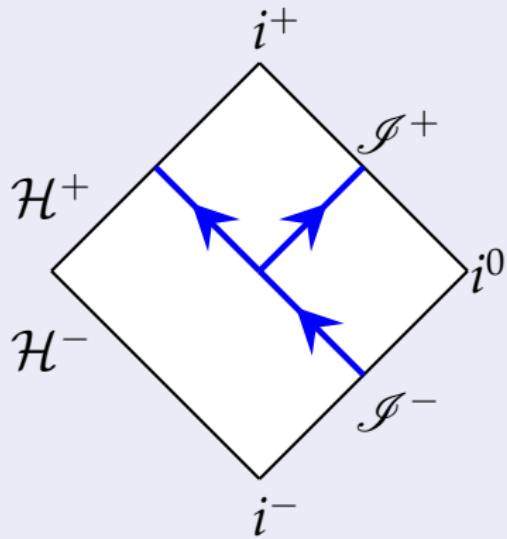
$$\begin{cases} B_{\omega\ell}^{\text{in}} e^{-i\omega r_*} & r_* \rightarrow -\infty \\ e^{-i\omega r_*} + A_{\omega\ell}^{\text{in}} e^{i\omega r_*} & r_* \rightarrow \infty \end{cases}$$



# “In” and “Up” modes

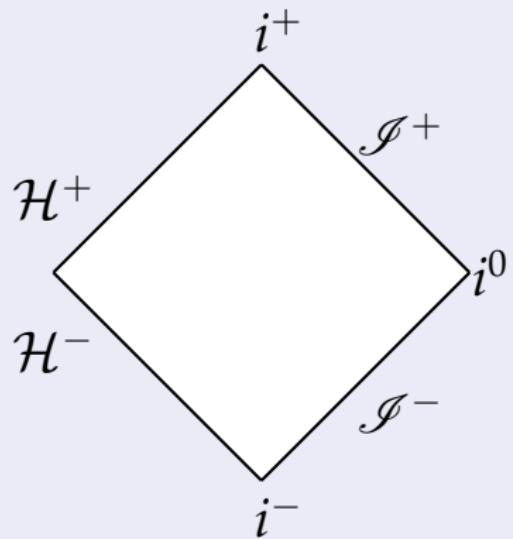
“In” modes  $R_{\omega\ell}^{\text{in}}$

$$\left\{ \begin{array}{ll} B_{\omega\ell}^{\text{in}} e^{-i\omega r_*} & r_* \rightarrow -\infty \\ e^{-i\omega r_*} + A_{\omega\ell}^{\text{in}} e^{i\omega r_*} & r_* \rightarrow \infty \end{array} \right.$$



“Up” modes  $R_{\omega\ell}^{\text{up}}$

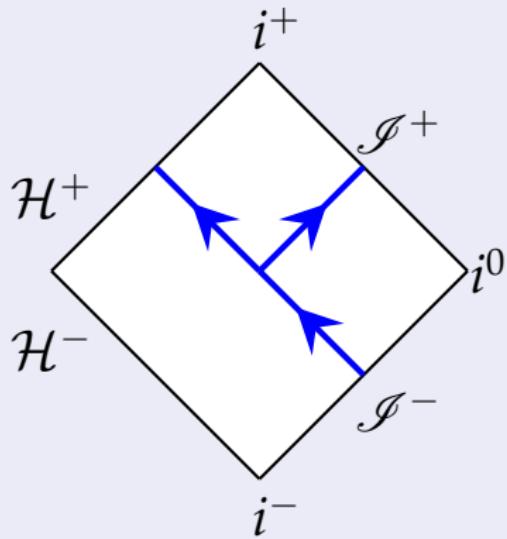
$$\left\{ \begin{array}{ll} & r_* \rightarrow -\infty \\ & r_* \rightarrow \infty \end{array} \right.$$



# “In” and “Up” modes

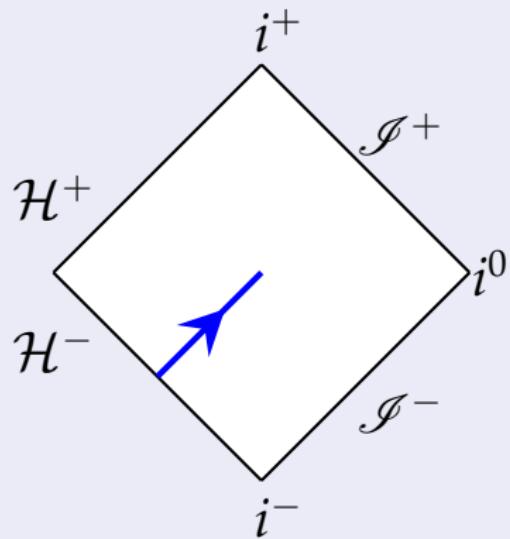
“In” modes  $R_{\omega\ell}^{\text{in}}$

$$\left\{ \begin{array}{ll} B_{\omega\ell}^{\text{in}} e^{-i\omega r_*} & r_* \rightarrow -\infty \\ e^{-i\omega r_*} + A_{\omega\ell}^{\text{in}} e^{i\omega r_*} & r_* \rightarrow \infty \end{array} \right.$$



“Up” modes  $R_{\omega\ell}^{\text{up}}$

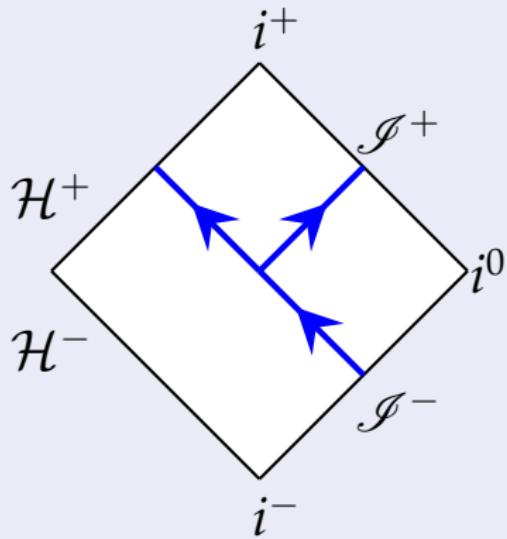
$$\left\{ \begin{array}{ll} e^{i\omega r_*} & r_* \rightarrow -\infty \\ & r_* \rightarrow \infty \end{array} \right.$$



# “In” and “Up” modes

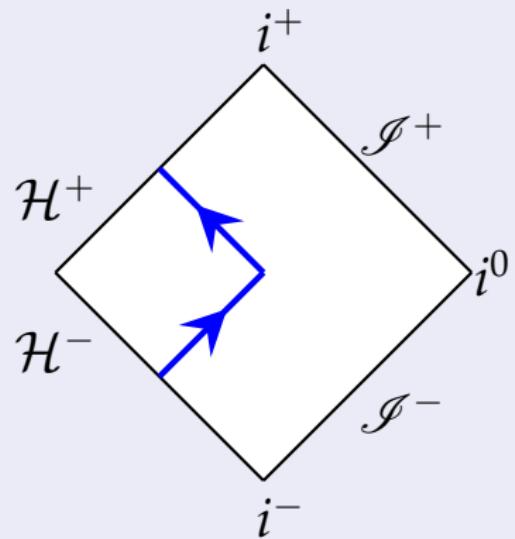
“In” modes  $R_{\omega\ell}^{\text{in}}$

$$\left\{ \begin{array}{ll} B_{\omega\ell}^{\text{in}} e^{-i\omega r_*} & r_* \rightarrow -\infty \\ e^{-i\omega r_*} + A_{\omega\ell}^{\text{in}} e^{i\omega r_*} & r_* \rightarrow \infty \end{array} \right.$$



“Up” modes  $R_{\omega\ell}^{\text{up}}$

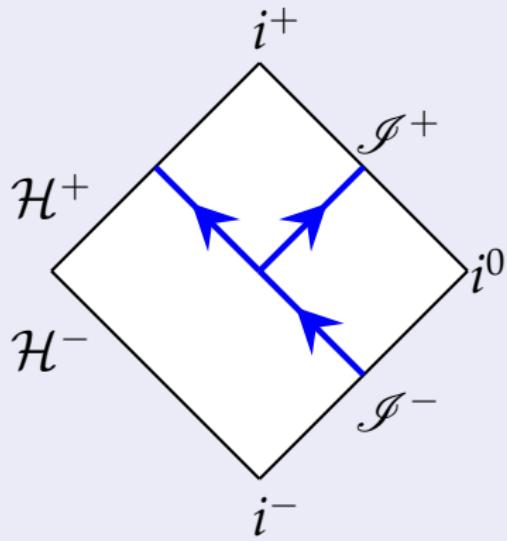
$$\left\{ \begin{array}{ll} e^{i\omega r_*} + A_{\omega\ell}^{\text{up}} e^{-i\omega r_*} & r_* \rightarrow -\infty \\ r_* \rightarrow \infty & \end{array} \right.$$



# “In” and “Up” modes

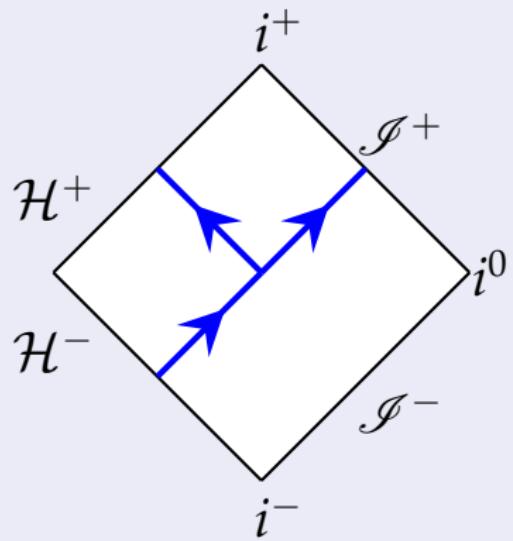
“In” modes  $R_{\omega\ell}^{\text{in}}$

$$\begin{cases} B_{\omega\ell}^{\text{in}} e^{-i\omega r_*} & r_* \rightarrow -\infty \\ e^{-i\omega r_*} + A_{\omega\ell}^{\text{in}} e^{i\omega r_*} & r_* \rightarrow \infty \end{cases}$$



“Up” modes  $R_{\omega\ell}^{\text{up}}$

$$\begin{cases} e^{i\omega r_*} + A_{\omega\ell}^{\text{up}} e^{-i\omega r_*} & r_* \rightarrow -\infty \\ B_{\omega\ell}^{\text{up}} e^{i\omega r_*} & r_* \rightarrow \infty \end{cases}$$



# Quantum states on Schwarzschild

# Quantum states on Schwarzschild

Boulware state  $|B\rangle$  [ Boulware *PRD* **11** 1404 (1975) ]

# Quantum states on Schwarzschild

Boulware state  $|B\rangle$  [ Boulware *PRD* **11** 1404 (1975) ]

Positive frequency with respect to Schwarzschild time

# Quantum states on Schwarzschild

Boulware state  $|B\rangle$  [ Boulware *PRD* **11** 1404 (1975) ]

Positive frequency with respect to Schwarzschild time

$$\langle B | \hat{O} | B \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}]$$

# Quantum states on Schwarzschild

Boulware state  $|B\rangle$  [ Boulware *PRD* **11** 1404 (1975) ]

Positive frequency with respect to Schwarzschild time

$$\langle B | \hat{O} | B \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}]$$

Hartle-Hawking state  $|H\rangle$  [ Hartle & Hawking *PRD* **13** 2188 (1976) ]

# Quantum states on Schwarzschild

Boulware state  $|B\rangle$  [ Boulware *PRD* **11** 1404 (1975) ]

Positive frequency with respect to Schwarzschild time

$$\langle B | \hat{O} | B \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}]$$

Hartle-Hawking state  $|H\rangle$  [ Hartle & Hawking *PRD* **13** 2188 (1976) ]

Positive frequency with respect to Kruskal time

# Quantum states on Schwarzschild

Boulware state  $|B\rangle$  [ Boulware *PRD* **11** 1404 (1975) ]

Positive frequency with respect to Schwarzschild time

$$\langle B | \hat{O} | B \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}]$$

Hartle-Hawking state  $|H\rangle$  [ Hartle & Hawking *PRD* **13** 2188 (1976) ]

Positive frequency with respect to Kruskal time

$$\langle H | \hat{O} | H \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}] \coth \left( \frac{\omega}{2T_H} \right)$$

# Quantum states on Schwarzschild

Boulware state  $|B\rangle$  [ Boulware *PRD* **11** 1404 (1975) ]

Positive frequency with respect to Schwarzschild time

$$\langle B | \hat{O} | B \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}]$$

Hartle-Hawking state  $|H\rangle$  [ Hartle & Hawking *PRD* **13** 2188 (1976) ]

Positive frequency with respect to Kruskal time

$$\langle H | \hat{O} | H \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}] \coth \left( \frac{\omega}{2T_H} \right)$$

Unruh state  $|U\rangle$  [ Unruh *PRD* **14** 870 (1976) ]

# Quantum states on Schwarzschild

Boulware state  $|B\rangle$  [ Boulware *PRD* **11** 1404 (1975) ]

Positive frequency with respect to Schwarzschild time

$$\langle B | \hat{O} | B \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}]$$

Hartle-Hawking state  $|H\rangle$  [ Hartle & Hawking *PRD* **13** 2188 (1976) ]

Positive frequency with respect to Kruskal time

$$\langle H | \hat{O} | H \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}] \coth\left(\frac{\omega}{2T_H}\right)$$

Unruh state  $|U\rangle$  [ Unruh *PRD* **14** 870 (1976) ]

$$\langle U | \hat{O} | U \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega \left[ o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}} \coth\left(\frac{\omega}{2T_H}\right) \right]$$

# Properties of standard quantum states

# Properties of standard quantum states

Boulware state  $|B\rangle$  [ Boulware *PRD* **11** 1404 (1975) ]

- State which is as empty as possible at infinity
- Diverges on the event horizon

# Properties of standard quantum states

Boulware state  $|B\rangle$  [ Boulware *PRD* **11** 1404 (1975) ]

- State which is as empty as possible at infinity
- Diverges on the event horizon

Hartle-Hawking state  $|H\rangle$  [ Hartle & Hawking *PRD* **13** 2188 (1976) ]

- Represents a black hole in thermal equilibrium with a heat bath
- Regular on and outside event horizon

# Properties of standard quantum states

Boulware state  $|B\rangle$  [ Boulware *PRD* **11** 1404 (1975) ]

- State which is as empty as possible at infinity
- Diverges on the event horizon

Hartle-Hawking state  $|H\rangle$  [ Hartle & Hawking *PRD* **13** 2188 (1976) ]

- Represents a black hole in thermal equilibrium with a heat bath
- Regular on and outside event horizon

Unruh state  $|U\rangle$  [ Unruh *PRD* **14** 870 (1976) ]

- Hawking radiation at  $\mathcal{I}^+$
- Regular at  $\mathcal{H}^+$

# Properties of standard quantum states

Boulware state  $|B\rangle$  [ Boulware *PRD* **11** 1404 (1975) ]

- State which is as empty as possible at infinity
- Diverges on the event horizon

Hartle-Hawking state  $|H\rangle$  [ Hartle & Hawking *PRD* **13** 2188 (1976) ]

- Represents a black hole in thermal equilibrium with a heat bath
- Regular on and outside event horizon

[ Sanders *LMP* **105** 575 (2015) ]

Unruh state  $|U\rangle$  [ Unruh *PRD* **14** 870 (1976) ]

- Hawking radiation at  $\mathcal{I}^+$
- Regular at  $\mathcal{H}^+$

[ Dappiaggi, Moretti & Pinamonti *ATMP* **15** 355 (2011) ]

# Kerr black hole

# Kerr black hole

$$ds^2 = -\frac{\Delta}{\Sigma} [dt - a \sin^2 \theta d\varphi]^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{\sin^2 \theta}{\Sigma} [(r^2 + a^2) d\varphi - a dt]^2$$
$$\Delta = r^2 - 2Mr + a^2 \quad \Sigma = r^2 + a^2 \cos^2 \theta$$

# Kerr black hole

$$ds^2 = -\frac{\Delta}{\Sigma} [dt - a \sin^2 \theta d\varphi]^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{\sin^2 \theta}{\Sigma} [(r^2 + a^2) d\varphi - a dt]^2$$

## Neutral scalar field modes

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \frac{1}{\mathcal{N}} \frac{1}{(r^2 + a^2)^{\frac{1}{2}}} e^{-i\omega t} e^{im\varphi} S_{\omega\ell m}(\cos \theta) R_{\omega\ell m}(r)$$

# Kerr black hole

$$ds^2 = -\frac{\Delta}{\Sigma} [dt - a \sin^2 \theta d\varphi]^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{\sin^2 \theta}{\Sigma} [(r^2 + a^2) d\varphi - a dt]^2$$

## Neutral scalar field modes

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \frac{1}{\mathcal{N}} \frac{1}{(r^2 + a^2)^{\frac{1}{2}}} e^{-i\omega t} e^{im\varphi} S_{\omega\ell m}(\cos \theta) R_{\omega\ell m}(r)$$

$S_{\omega\ell m}(\cos \theta)$ : spheroidal harmonics

# Kerr black hole

$$ds^2 = -\frac{\Delta}{\Sigma} [dt - a \sin^2 \theta d\varphi]^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{\sin^2 \theta}{\Sigma} [(r^2 + a^2) d\varphi - a dt]^2$$

## Neutral scalar field modes

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \frac{1}{\mathcal{N}} \frac{1}{(r^2 + a^2)^{\frac{1}{2}}} e^{-i\omega t} e^{im\varphi} S_{\omega\ell m}(\cos \theta) R_{\omega\ell m}(r)$$

## Radial mode equation

$$0 = \left[ \frac{d^2}{dr_*^2} + V_{\omega\ell m}(r) \right] R_{\omega\ell m}(r) \quad \frac{dr_*}{dr} = \frac{r^2 + a^2}{\Delta}$$

# Kerr black hole

$$ds^2 = -\frac{\Delta}{\Sigma} [dt - a \sin^2 \theta d\varphi]^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{\sin^2 \theta}{\Sigma} [(r^2 + a^2) d\varphi - a dt]^2$$

## Neutral scalar field modes

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \frac{1}{\mathcal{N}} \frac{1}{(r^2 + a^2)^{\frac{1}{2}}} e^{-i\omega t} e^{im\varphi} S_{\omega\ell m}(\cos \theta) R_{\omega\ell m}(r)$$

## Radial mode equation

$$0 = \left[ \frac{d^2}{dr_*^2} + V_{\omega\ell m}(r) \right] R_{\omega\ell m}(r) \quad \frac{dr_*}{dr} = \frac{r^2 + a^2}{\Delta}$$

$$V_{\omega\ell m}(r) = \left\{ \begin{array}{l} \end{array} \right.$$

# Kerr black hole

$$ds^2 = -\frac{\Delta}{\Sigma} [dt - a \sin^2 \theta d\varphi]^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{\sin^2 \theta}{\Sigma} [(r^2 + a^2) d\varphi - a dt]^2$$

## Neutral scalar field modes

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \frac{1}{\mathcal{N}} \frac{1}{(r^2 + a^2)^{\frac{1}{2}}} e^{-i\omega t} e^{im\varphi} S_{\omega\ell m}(\cos \theta) R_{\omega\ell m}(r)$$

## Radial mode equation

$$0 = \left[ \frac{d^2}{dr_*^2} + V_{\omega\ell m}(r) \right] R_{\omega\ell m}(r) \quad \frac{dr_*}{dr} = \frac{r^2 + a^2}{\Delta}$$

$$V_{\omega\ell m}(r) = \begin{cases} \omega^2 & \text{as } r_* \rightarrow \infty \end{cases}$$

# Kerr black hole

$$ds^2 = -\frac{\Delta}{\Sigma} [dt - a \sin^2 \theta d\varphi]^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{\sin^2 \theta}{\Sigma} [(r^2 + a^2) d\varphi - a dt]^2$$

## Neutral scalar field modes

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \frac{1}{\mathcal{N}} \frac{1}{(r^2 + a^2)^{\frac{1}{2}}} e^{-i\omega t} e^{im\varphi} S_{\omega\ell m}(\cos \theta) R_{\omega\ell m}(r)$$

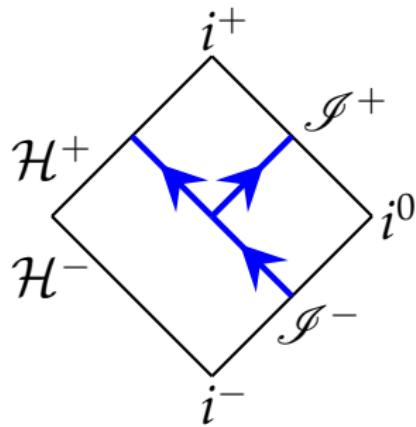
## Radial mode equation

$$0 = \left[ \frac{d^2}{dr_*^2} + V_{\omega\ell m}(r) \right] R_{\omega\ell m}(r) \quad \frac{dr_*}{dr} = \frac{r^2 + a^2}{\Delta}$$

$$V_{\omega\ell m}(r) = \begin{cases} \tilde{\omega}^2 = (\omega - m\Omega_H)^2 & \text{as } r_* \rightarrow -\infty \\ \omega^2 & \text{as } r_* \rightarrow \infty \end{cases}$$

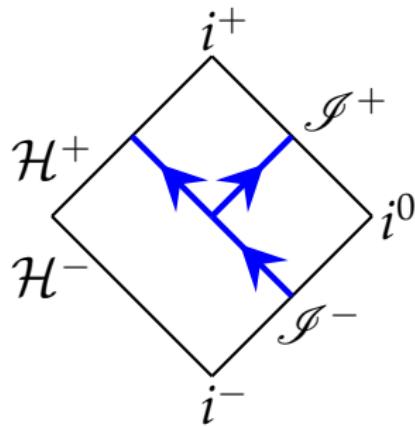
# Classical superradiance

$$R_{\omega\ell m}^{\text{in}}(r) = \begin{cases} B_{\omega\ell m}^{\text{in}} e^{-i\tilde{\omega}r_*} & r_* \rightarrow -\infty \\ e^{-i\omega r_*} + A_{\omega\ell m}^{\text{in}} e^{i\omega r_*} & r_* \rightarrow \infty \end{cases}$$



# Classical superradiance

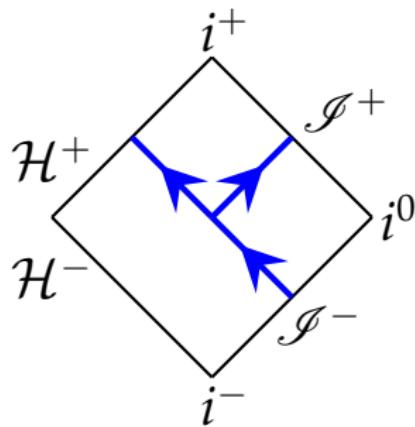
$$R_{\omega\ell m}^{\text{in}}(r) = \begin{cases} B_{\omega\ell m}^{\text{in}} e^{-i\tilde{\omega}r_*} & r_* \rightarrow -\infty \\ e^{-i\omega r_*} + A_{\omega\ell m}^{\text{in}} e^{i\omega r_*} & r_* \rightarrow \infty \end{cases}$$



$$\omega \left[ 1 - |A_{\omega\ell m}^{\text{in}}|^2 \right] = \tilde{\omega} |B_{\omega\ell m}^{\text{in}}|^2$$

# Classical superradiance

$$R_{\omega\ell m}^{\text{in}}(r) = \begin{cases} B_{\omega\ell m}^{\text{in}} e^{-i\tilde{\omega}r_*} & r_* \rightarrow -\infty \\ e^{-i\omega r_*} + A_{\omega\ell m}^{\text{in}} e^{i\omega r_*} & r_* \rightarrow \infty \end{cases}$$

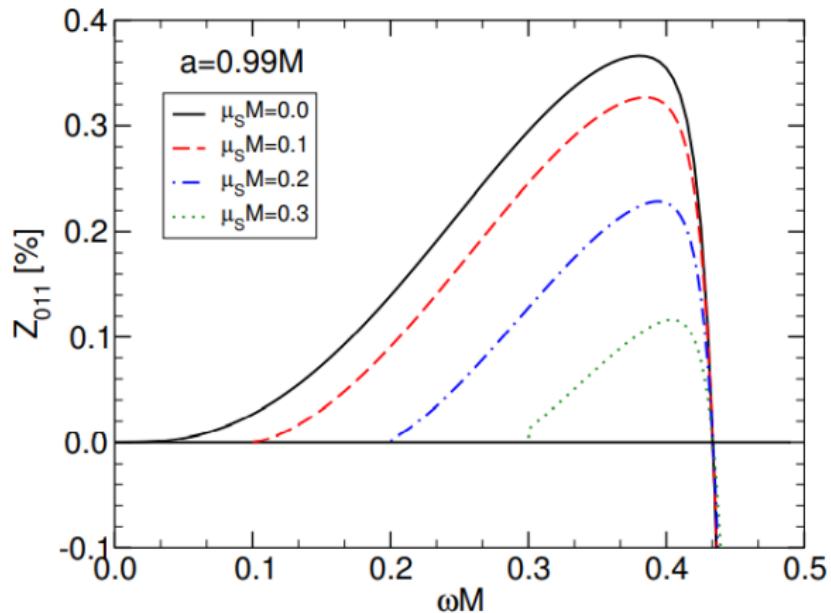
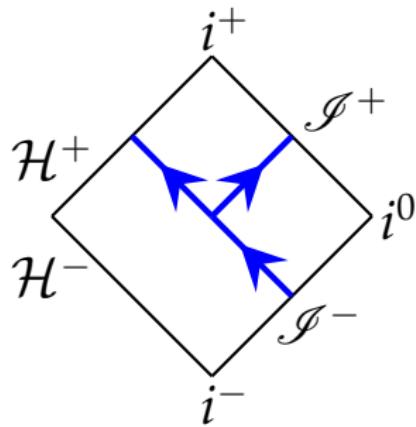


$$\omega \left[ 1 - |A_{\omega\ell m}^{\text{in}}|^2 \right] = \tilde{\omega} |B_{\omega\ell m}^{\text{in}}|^2$$

$$|A_{\omega\ell m}^{\text{in}}|^2 > 1 \text{ if } \omega\tilde{\omega} < 0$$

# Classical superradiance

$$R_{\omega\ell m}^{\text{in}}(r) = \begin{cases} B_{\omega\ell m}^{\text{in}} e^{-i\tilde{\omega}r_*} & r_* \rightarrow -\infty \\ e^{-i\omega r_*} + A_{\omega\ell m}^{\text{in}} e^{i\omega r_*} & r_* \rightarrow \infty \end{cases}$$



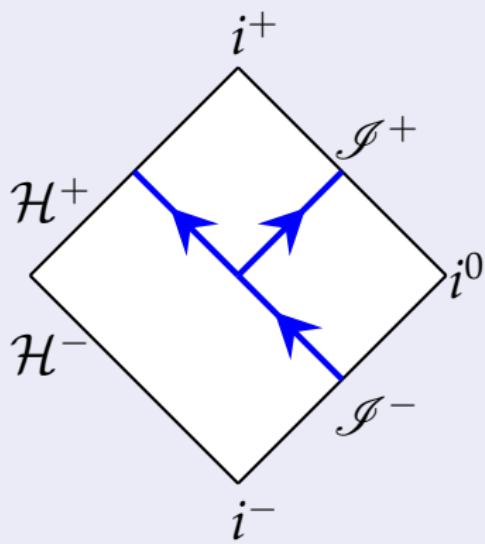
$$|A_{\omega\ell m}^{\text{in}}|^2 > 1 \text{ if } \omega\tilde{\omega} < 0$$

[ Brito, Cardoso & Pani *LNP 905* (2015) ]

# “In” and “Up” modes

# “In” and “Up” modes

## “In” modes

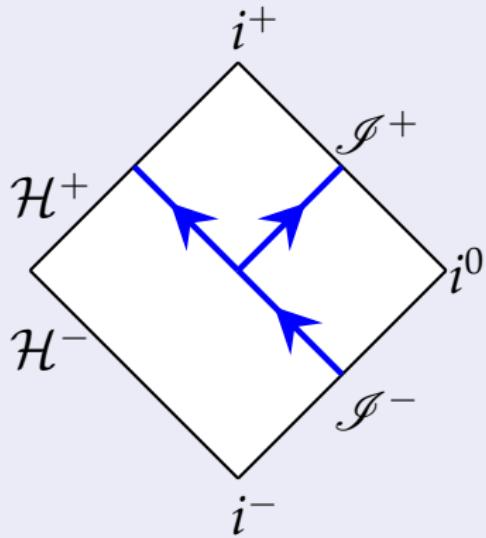


# “In” and “Up” modes

## “In” modes

Positive frequency/“norm” for

$$\omega > 0$$

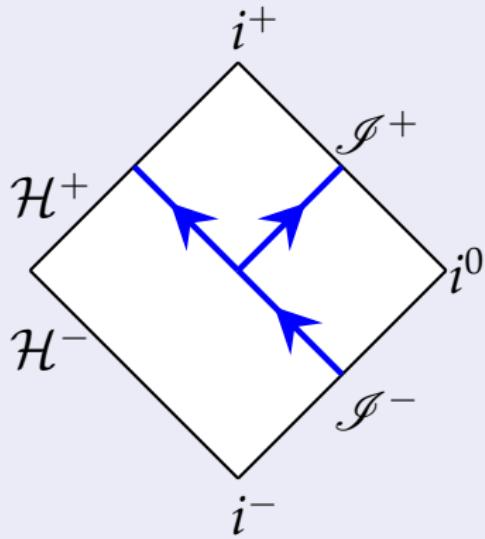


# “In” and “Up” modes

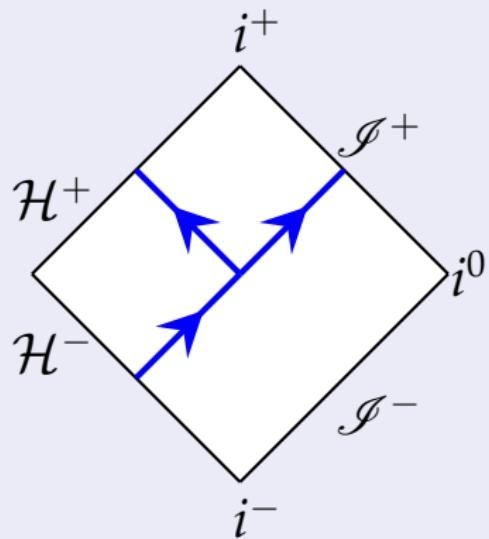
## “In” modes

Positive frequency/“norm” for

$$\omega > 0$$



## “Up” modes

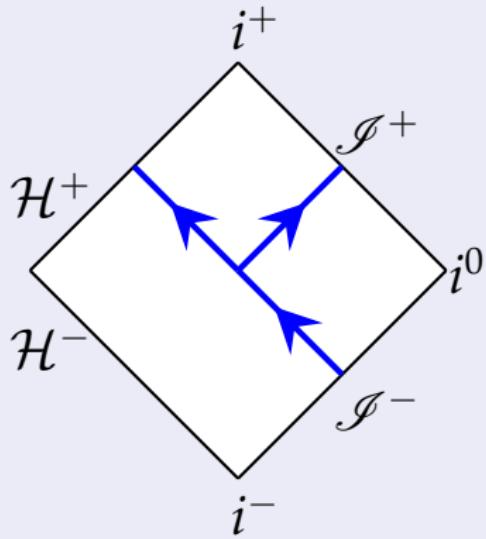


# “In” and “Up” modes

## “In” modes

Positive frequency/“norm” for

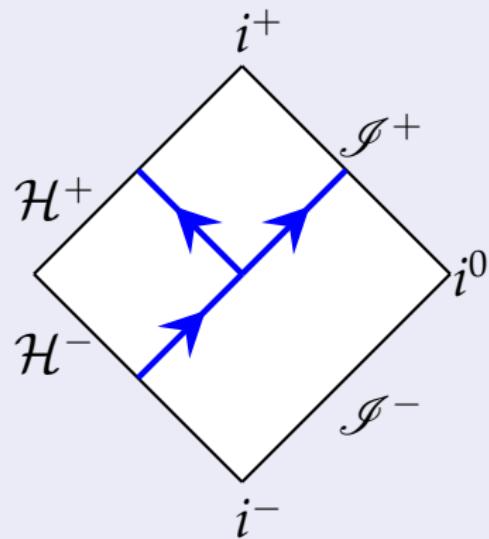
$$\omega > 0$$



## “Up” modes

Positive frequency/“norm” for

$$\tilde{\omega} = \omega - m\Omega_H > 0$$



# Quantum states on Kerr

# Quantum states on Kerr

## Unruh state

# Quantum states on Kerr

## Unruh state

$$\langle U | \hat{O} | U \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left\{ \int_0^{\infty} d\omega o_{\omega\ell m}^{\text{in}} + \int_0^{\infty} d\tilde{\omega} o_{\omega\ell m}^{\text{up}} \coth\left(\frac{\tilde{\omega}}{2T_H}\right) \right\}$$

# Quantum states on Kerr

Unruh state

$$\langle U | \hat{O} | U \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left\{ \int_0^{\infty} d\omega o_{\omega\ell m}^{\text{in}} + \int_0^{\infty} d\tilde{\omega} o_{\omega\ell m}^{\text{up}} \coth\left(\frac{\tilde{\omega}}{2T_H}\right) \right\}$$

Hartle-Hawking state

# Quantum states on Kerr

## Unruh state

$$\langle U | \hat{O} | U \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left\{ \int_0^{\infty} d\omega o_{\omega\ell m}^{\text{in}} + \int_0^{\infty} d\tilde{\omega} o_{\omega\ell m}^{\text{up}} \coth\left(\frac{\tilde{\omega}}{2T_H}\right) \right\}$$

## Hartle-Hawking state

No Hartle-Hawking state exists on Kerr

[ Kay & Wald *Phys. Rept.* **207** 49 (1991) ]

# Quantum states on Kerr

Unruh state

$$\langle U | \hat{O} | U \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left\{ \int_0^{\infty} d\omega o_{\omega\ell m}^{\text{in}} + \int_0^{\infty} d\tilde{\omega} o_{\omega\ell m}^{\text{up}} \coth\left(\frac{\tilde{\omega}}{2T_H}\right) \right\}$$

$|CCH\rangle$  [ Candelas, Chrzanowski & Howard *PRD* **24** 297 (1981) ]

# Quantum states on Kerr

## Unruh state

$$\langle U | \hat{O} | U \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left\{ \int_0^{\infty} d\omega o_{\omega\ell m}^{\text{in}} + \int_0^{\infty} d\tilde{\omega} o_{\omega\ell m}^{\text{up}} \coth\left(\frac{\tilde{\omega}}{2T_H}\right) \right\}$$

$|CCH\rangle$  [ Candelas, Chrzanowski & Howard *PRD* **24** 297 (1981) ]

$$\begin{aligned} \langle CCH | \hat{O} | CCH \rangle = & \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega \coth\left(\frac{\omega}{2T_H}\right) o_{\omega\ell m}^{\text{in}} \\ & + \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\tilde{\omega} \coth\left(\frac{\tilde{\omega}}{2T_H}\right) o_{\omega\ell m}^{\text{up}} \end{aligned}$$

# Quantum states on Kerr

## Unruh state

$$\langle U | \hat{O} | U \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left\{ \int_0^{\infty} d\omega o_{\omega\ell m}^{\text{in}} + \int_0^{\infty} d\tilde{\omega} o_{\omega\ell m}^{\text{up}} \coth\left(\frac{\tilde{\omega}}{2T_H}\right) \right\}$$

$|CCH\rangle$  [ Candelas, Chrzanowski & Howard *PRD* **24** 297 (1981) ]

$$\begin{aligned} \langle CCH | \hat{O} | CCH \rangle = & \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega \coth\left(\frac{\omega}{2T_H}\right) o_{\omega\ell m}^{\text{in}} \\ & + \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\tilde{\omega} \coth\left(\frac{\tilde{\omega}}{2T_H}\right) o_{\omega\ell m}^{\text{up}} \end{aligned}$$

# Quantum states on Kerr

## Unruh state

$$\langle U | \hat{O} | U \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left\{ \int_0^{\infty} d\omega o_{\omega\ell m}^{\text{in}} + \int_0^{\infty} d\tilde{\omega} o_{\omega\ell m}^{\text{up}} \coth\left(\frac{\tilde{\omega}}{2T_H}\right) \right\}$$

$|CCH\rangle$  [ Candelas, Chrzanowski & Howard *PRD* **24** 297 (1981) ]

$$\begin{aligned} \langle CCH | \hat{O} | CCH \rangle = & \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega \coth\left(\frac{\omega}{2T_H}\right) o_{\omega\ell m}^{\text{in}} \\ & + \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\tilde{\omega} \coth\left(\frac{\tilde{\omega}}{2T_H}\right) o_{\omega\ell m}^{\text{up}} \end{aligned}$$

- Does not represent an equilibrium state

[ Ottewill & Winstanley *PRD* **62** 084018 (2000) ]

# Quantum states on Kerr

## Unruh state

$$\langle U | \hat{O} | U \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left\{ \int_0^{\infty} d\omega o_{\omega\ell m}^{\text{in}} + \int_0^{\infty} d\tilde{\omega} o_{\omega\ell m}^{\text{up}} \coth\left(\frac{\tilde{\omega}}{2T_H}\right) \right\}$$

$|CCH\rangle$  [ Candelas, Chrzanowski & Howard *PRD* **24** 297 (1981) ]

$$\begin{aligned} \langle CCH | \hat{O} | CCH \rangle = & \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega \coth\left(\frac{\omega}{2T_H}\right) o_{\omega\ell m}^{\text{in}} \\ & + \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\tilde{\omega} \coth\left(\frac{\tilde{\omega}}{2T_H}\right) o_{\omega\ell m}^{\text{up}} \end{aligned}$$

- Does not represent an equilibrium state
- Regular outside the event horizon

[ Ottewill & Winstanley *PRD* **62** 084018 (2000) ]

# Quantum states on Kerr

$|\text{CCH}\rangle$  [ Candelas, Chrzanowski & Howard *PRD* **24** 297 (1981) ]

$$\begin{aligned} \langle \text{CCH} | \hat{O} | \text{CCH} \rangle = & \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega \coth \left( \frac{\omega}{2T_H} \right) o_{\omega\ell m}^{\text{in}} \\ & + \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\tilde{\omega} \coth \left( \frac{\tilde{\omega}}{2T_H} \right) o_{\omega\ell m}^{\text{up}} \end{aligned}$$

$|\text{FT}\rangle$  [ Frolov & Thorne *PRD* **39** 2125 (1989) ]

$$\begin{aligned} \langle \text{FT} | \hat{O} | \text{FT} \rangle = & \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega \coth \left( \frac{\tilde{\omega}}{2T_H} \right) o_{\omega\ell m}^{\text{in}} \\ & + \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\tilde{\omega} \coth \left( \frac{\tilde{\omega}}{2T_H} \right) o_{\omega\ell m}^{\text{up}} \end{aligned}$$

# Quantum states on Kerr

$|\text{CCH}\rangle$  [ Candelas, Chrzanowski & Howard *PRD* **24** 297 (1981) ]

$$\begin{aligned} \langle \text{CCH} | \hat{O} | \text{CCH} \rangle = & \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega \coth \left( \frac{\omega}{2T_H} \right) o_{\omega\ell m}^{\text{in}} \\ & + \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\tilde{\omega} \coth \left( \frac{\tilde{\omega}}{2T_H} \right) o_{\omega\ell m}^{\text{up}} \end{aligned}$$

$|\text{FT}\rangle$  [ Frolov & Thorne *PRD* **39** 2125 (1989) ]

$$\begin{aligned} \langle \text{FT} | \hat{O} | \text{FT} \rangle = & \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\textcolor{red}{\omega} \coth \left( \frac{\textcolor{red}{\tilde{\omega}}}{2T_H} \right) o_{\omega\ell m}^{\text{in}} \\ & + \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\tilde{\omega} \coth \left( \frac{\tilde{\omega}}{2T_H} \right) o_{\omega\ell m}^{\text{up}} \end{aligned}$$

# Quantum states on Kerr

$|\text{CCH}\rangle$  [ Candelas, Chrzanowski & Howard *PRD* **24** 297 (1981) ]

$$\begin{aligned} \langle \text{CCH} | \hat{O} | \text{CCH} \rangle = & \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega \coth \left( \frac{\omega}{2T_H} \right) o_{\omega\ell m}^{\text{in}} \\ & + \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\tilde{\omega} \coth \left( \frac{\tilde{\omega}}{2T_H} \right) o_{\omega\ell m}^{\text{up}} \end{aligned}$$

$|\text{FT}\rangle$  [ Frolov & Thorne *PRD* **39** 2125 (1989) ]

$$\begin{aligned} \langle \text{FT} | \hat{O} | \text{FT} \rangle = & \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega \coth \left( \frac{\tilde{\omega}}{2T_H} \right) o_{\omega\ell m}^{\text{in}} \\ & + \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\tilde{\omega} \coth \left( \frac{\tilde{\omega}}{2T_H} \right) o_{\omega\ell m}^{\text{up}} \end{aligned}$$

- Divergent everywhere except on the axis of rotation

[ Ottewill & Winstanley *PRD* **62** 084018 (2000) ]

# Charged scalar field on RN space-time

# Charged scalar field on RN space-time

Reissner-Nordström black hole

$$ds^2 = - \left( 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right) dt^2 + \left( 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right)^{-1} dr^2 + r^2 d\Omega^2$$

# Charged scalar field on RN space-time

Reissner-Nordström black hole

$$ds^2 = - \left( 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right) dt^2 + \left( 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right)^{-1} dr^2 + r^2 d\Omega^2$$

Charged scalar field equation

$$D_\mu D^\mu \Phi = 0 \quad D_\mu = \nabla_\mu - iqA_\mu \quad A_0 = -\frac{Q}{r}$$

# Charged scalar field on RN space-time

Reissner-Nordström black hole

$$ds^2 = - \left( 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right) dt^2 + \left( 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right)^{-1} dr^2 + r^2 d\Omega^2$$

Charged scalar field equation

$$D_\mu D^\mu \Phi = 0 \quad D_\mu = \nabla_\mu - iqA_\mu \quad A_0 = -\frac{Q}{r}$$

Scalar product

$$\langle \Phi_1, \Phi_2 \rangle = i \int_{\Sigma} \left[ (D_\mu \Phi_1)^* \Phi_2 - \Phi_1^* D_\mu \Phi_2 \right] \sqrt{-g} d\Sigma^\mu$$

# Charged scalar field on RN space-time

# Charged scalar field on RN space-time

## Charged scalar field modes

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \frac{1}{\mathcal{N}r} e^{-i\omega t} e^{im\varphi} Y_{\ell m}(\theta) R_{\omega\ell}(r)$$

# Charged scalar field on RN space-time

## Charged scalar field modes

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \frac{1}{\mathcal{N}r} e^{-i\omega t} e^{im\varphi} Y_{\ell m}(\theta) R_{\omega\ell}(r)$$

## Radial mode equation

$$0 = \left[ \frac{d^2}{dr_*^2} + V_{\omega\ell}(r) \right] R_{\omega\ell}(r) \quad \frac{dr_*}{dr} = \frac{1}{1 - \frac{2M}{r} + \frac{Q^2}{r^2}}$$

# Charged scalar field on RN space-time

## Charged scalar field modes

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \frac{1}{\mathcal{N}r} e^{-i\omega t} e^{im\varphi} Y_{\ell m}(\theta) R_{\omega\ell}(r)$$

## Radial mode equation

$$0 = \left[ \frac{d^2}{dr_*^2} + V_{\omega\ell}(r) \right] R_{\omega\ell}(r) \quad \frac{dr_*}{dr} = \frac{1}{1 - \frac{2M}{r} + \frac{Q^2}{r^2}}$$

$$V_{\omega\ell}(r) = \left\{ \begin{array}{l} \end{array} \right.$$

# Charged scalar field on RN space-time

## Charged scalar field modes

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \frac{1}{\mathcal{N}r} e^{-i\omega t} e^{im\varphi} Y_{\ell m}(\theta) R_{\omega\ell}(r)$$

## Radial mode equation

$$0 = \left[ \frac{d^2}{dr_*^2} + V_{\omega\ell}(r) \right] R_{\omega\ell}(r) \quad \frac{dr_*}{dr} = \frac{1}{1 - \frac{2M}{r} + \frac{Q^2}{r^2}}$$

$$V_{\omega\ell}(r) = \begin{cases} \omega^2 & \text{as } r_* \rightarrow \infty \end{cases}$$

# Charged scalar field on RN space-time

## Charged scalar field modes

$$\phi_{\omega\ell m}(t, r, \theta, \varphi) = \frac{1}{\mathcal{N}r} e^{-i\omega t} e^{im\varphi} Y_{\ell m}(\theta) R_{\omega\ell}(r)$$

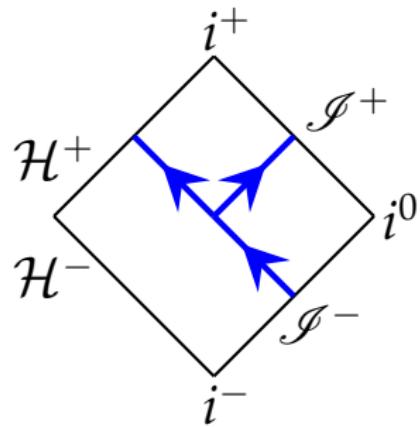
## Radial mode equation

$$0 = \left[ \frac{d^2}{dr_*^2} + V_{\omega\ell}(r) \right] R_{\omega\ell}(r) \quad \frac{dr_*}{dr} = \frac{1}{1 - \frac{2M}{r} + \frac{Q^2}{r^2}}$$

$$V_{\omega\ell}(r) = \begin{cases} \tilde{\omega}^2 = \left( \omega - \frac{qQ}{r_+} \right)^2 & \text{as } r_* \rightarrow -\infty \\ \omega^2 & \text{as } r_* \rightarrow \infty \end{cases}$$

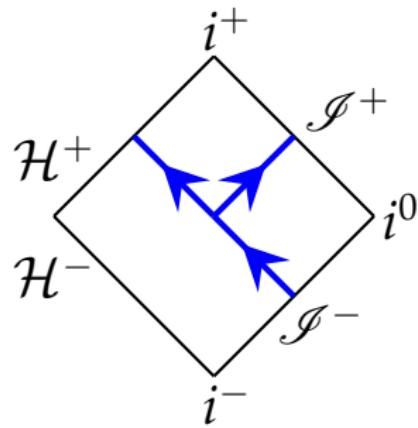
# Charge superradiance

$$R_{\omega\ell}^{\text{in}}(r) = \begin{cases} B_{\omega\ell}^{\text{in}} e^{-i\tilde{\omega}r_*} & r_* \rightarrow -\infty \\ e^{-i\omega r_*} + A_{\omega\ell}^{\text{in}} e^{i\omega r_*} & r_* \rightarrow \infty \end{cases}$$



# Charge superradiance

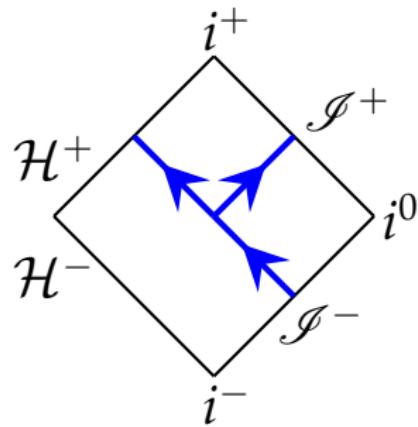
$$R_{\omega\ell}^{\text{in}}(r) = \begin{cases} B_{\omega\ell}^{\text{in}} e^{-i\tilde{\omega}r_*} & r_* \rightarrow -\infty \\ e^{-i\omega r_*} + A_{\omega\ell}^{\text{in}} e^{i\omega r_*} & r_* \rightarrow \infty \end{cases}$$



$$\omega \left[ 1 - |A_{\omega\ell}^{\text{in}}|^2 \right] = \tilde{\omega} |B_{\omega\ell}^{\text{in}}|^2$$

# Charge superradiance

$$R_{\omega\ell}^{\text{in}}(r) = \begin{cases} B_{\omega\ell}^{\text{in}} e^{-i\tilde{\omega}r_*} & r_* \rightarrow -\infty \\ e^{-i\omega r_*} + A_{\omega\ell}^{\text{in}} e^{i\omega r_*} & r_* \rightarrow \infty \end{cases}$$

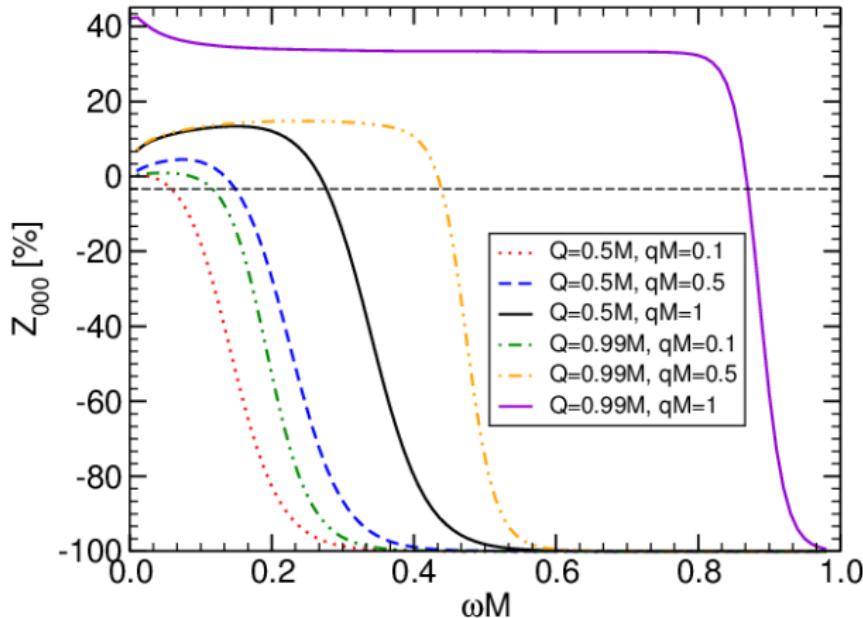
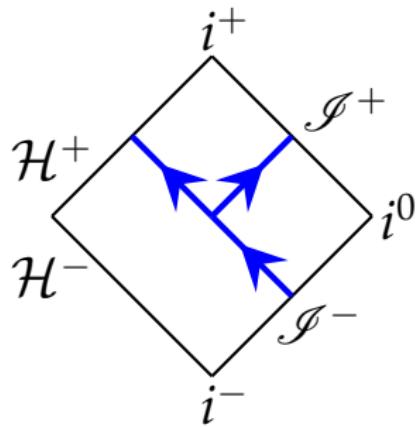


$$\omega \left[ 1 - |A_{\omega\ell}^{\text{in}}|^2 \right] = \tilde{\omega} |B_{\omega\ell}^{\text{in}}|^2$$

$$|A_{\omega\ell}^{\text{in}}|^2 > 1 \text{ if } \omega\tilde{\omega} < 0$$

# Charge superradiance

$$R_{\omega\ell}^{\text{in}}(r) = \begin{cases} B_{\omega\ell}^{\text{in}} e^{-i\tilde{\omega}r_*} & r_* \rightarrow -\infty \\ e^{-i\omega r_*} + A_{\omega\ell}^{\text{in}} e^{i\omega r_*} & r_* \rightarrow \infty \end{cases}$$



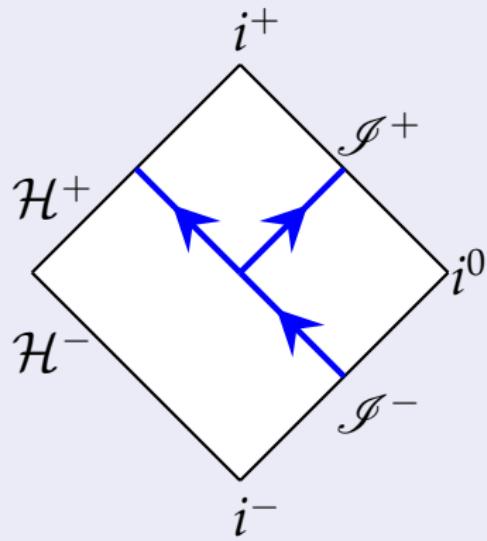
$$|A_{\omega\ell}^{\text{in}}|^2 > 1 \text{ if } \omega\tilde{\omega} < 0$$

[ Brito, Cardoso & Pani LNP 905 (2015) ]

# “In” and “Up” modes

# “In” and “Up” modes

## “In” modes

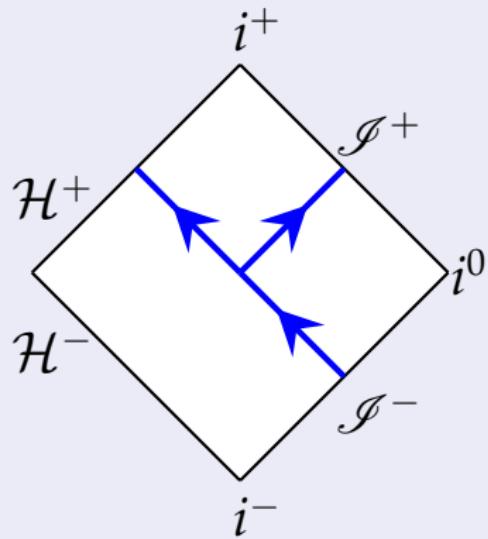


# “In” and “Up” modes

## “In” modes

Positive frequency/“norm” for

$$\omega > 0$$

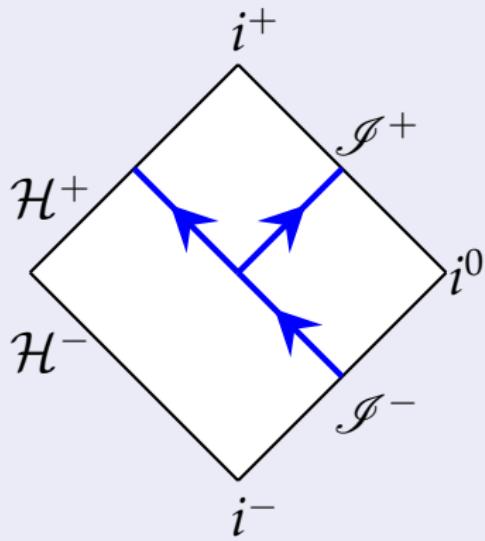


# “In” and “Up” modes

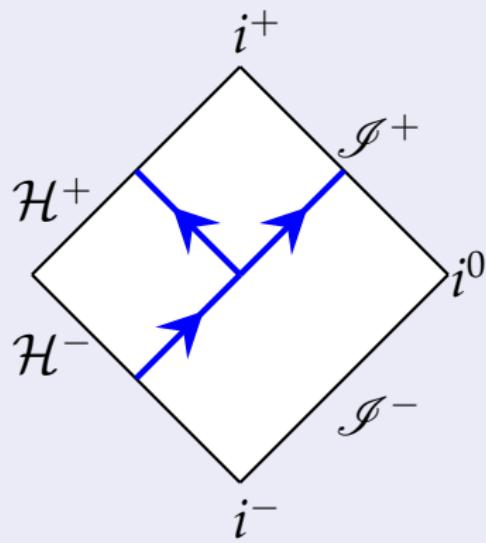
## “In” modes

Positive frequency/“norm” for

$$\omega > 0$$



## “Up” modes

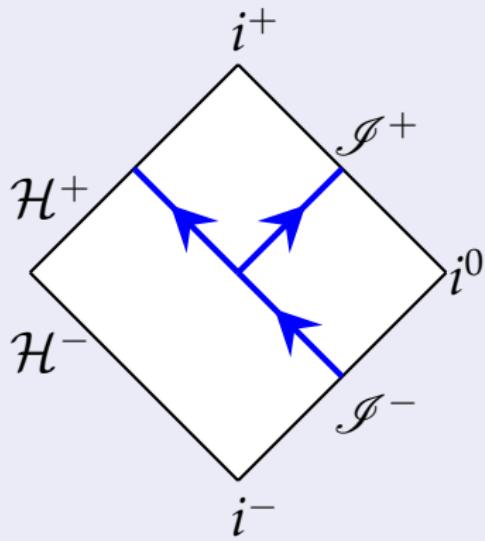


# “In” and “Up” modes

## “In” modes

Positive frequency/“norm” for

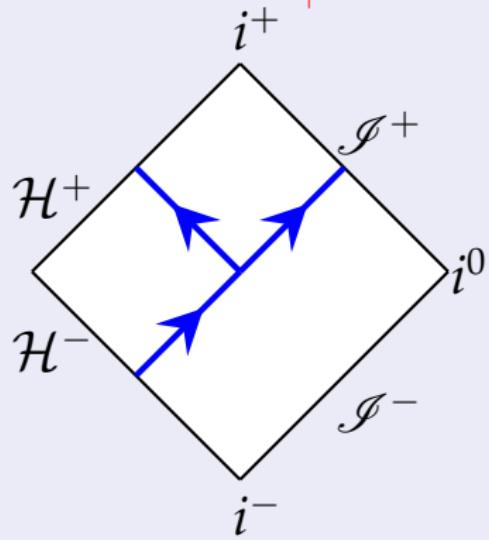
$$\omega > 0$$



## “Up” modes

Positive frequency/“norm” for

$$\tilde{\omega} = \omega - \frac{qQ}{r_+} > 0$$



# Observables

# Observables

- Scalar condensate  $\widehat{SC} = \frac{1}{2} [\hat{\Phi}\hat{\Phi}^\dagger + \text{h.c.}]$

# Observables

- Scalar condensate  $\widehat{SC} = \frac{1}{2} [\hat{\Phi}\hat{\Phi}^\dagger + \text{h.c.}]$
- Current  $\hat{J}^\mu = \frac{iq}{16\pi} [\hat{\Phi}^\dagger (D^\mu \hat{\Phi}) + (D^\mu \hat{\Phi}) \hat{\Phi}^\dagger - \text{h.c.}]$

# Observables

- Scalar condensate  $\widehat{SC} = \frac{1}{2} [\hat{\Phi}\hat{\Phi}^\dagger + \text{h.c.}]$
- Current  $\hat{J}^\mu = \frac{iq}{16\pi} [\hat{\Phi}^\dagger (D^\mu \hat{\Phi}) + (D^\mu \hat{\Phi}) \hat{\Phi}^\dagger - \text{h.c.}]$
- Stress-energy tensor  $\hat{T}_{\mu\nu}$

# Observables

- Scalar condensate  $\widehat{SC} = \frac{1}{2} [\hat{\Phi}\hat{\Phi}^\dagger + \text{h.c.}]$
- Current  $\hat{J}^\mu = \frac{iq}{16\pi} [\hat{\Phi}^\dagger (D^\mu \hat{\Phi}) + (D^\mu \hat{\Phi}) \hat{\Phi}^\dagger - \text{h.c.}]$
- Stress-energy tensor  $\hat{T}_{\mu\nu}$

## Fluxes

# Observables

- Scalar condensate  $\widehat{SC} = \frac{1}{2} [\hat{\Phi}\hat{\Phi}^\dagger + \text{h.c.}]$
- Current  $\hat{J}^\mu = \frac{iq}{16\pi} [\hat{\Phi}^\dagger (D^\mu \hat{\Phi}) + (D^\mu \hat{\Phi}) \hat{\Phi}^\dagger - \text{h.c.}]$
- Stress-energy tensor  $\hat{T}_{\mu\nu}$

## Fluxes

$$\langle \hat{J}^r \rangle = -\frac{\mathcal{K}}{r^2}$$

- Flux of charge  $\mathcal{K}$

# Observables

- Scalar condensate  $\widehat{SC} = \frac{1}{2} [\hat{\Phi}\hat{\Phi}^\dagger + \text{h.c.}]$
- Current  $\hat{J}^\mu = \frac{iq}{16\pi} [\hat{\Phi}^\dagger (D^\mu \hat{\Phi}) + (D^\mu \hat{\Phi}) \hat{\Phi}^\dagger - \text{h.c.}]$
- Stress-energy tensor  $\hat{T}_{\mu\nu}$

## Fluxes

$$\langle \hat{J}^r \rangle = -\frac{\mathcal{K}}{r^2} \quad \langle \hat{T}_t^r \rangle = -\frac{\mathcal{L}}{r^2} + \frac{4\pi Q \mathcal{K}}{r^3}$$

- Flux of charge  $\mathcal{K}$
- Flux of energy  $\mathcal{L}$

# Unruh state $|U\rangle$

# Unruh state $|U\rangle$

$$\langle U | \hat{O} | U \rangle = \frac{1}{2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left[ \int_{-\infty}^{\infty} d\omega o_{\omega\ell m}^{\text{in}} + \int_{-\infty}^{\infty} d\tilde{\omega} o_{\omega\ell m}^{\text{up}} \coth \left| \frac{\tilde{\omega}}{2T_H} \right| \right]$$

Unruh state  $|U\rangle$

$$\tilde{\omega} = \omega - \frac{qQ}{r_+}$$

$$\langle U | \hat{O} | U \rangle = \frac{1}{2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left[ \int_{-\infty}^{\infty} d\omega o_{\omega\ell m}^{\text{in}} + \int_{-\infty}^{\infty} d\tilde{\omega} o_{\omega\ell m}^{\text{up}} \coth \left| \frac{\tilde{\omega}}{2T_H} \right| \right]$$

Unruh state  $|U\rangle$

$$\tilde{\omega} = \omega - \frac{qQ}{r_+}$$

$$\langle U | \hat{O} | U \rangle = \frac{1}{2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left[ \int_{-\infty}^{\infty} d\omega o_{\omega\ell m}^{\text{in}} + \int_{-\infty}^{\infty} d\tilde{\omega} o_{\omega\ell m}^{\text{up}} \coth \left| \frac{\tilde{\omega}}{2T_H} \right| \right]$$

$$\mathcal{K}_U$$

$$\mathcal{L}_U$$

Unruh state  $|\text{U}\rangle$

$$\tilde{\omega} = \omega - \frac{qQ}{r_+}$$

$$\langle \text{U} | \hat{O} | \text{U} \rangle = \frac{1}{2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left[ \int_{-\infty}^{\infty} d\omega o_{\omega\ell m}^{\text{in}} + \int_{-\infty}^{\infty} d\tilde{\omega} o_{\omega\ell m}^{\text{up}} \coth \left| \frac{\tilde{\omega}}{2T_H} \right| \right]$$

$$\begin{aligned} \mathcal{K}_{\text{U}} &= \frac{q}{64\pi^3} \sum_{\ell=0}^{\ell} \int_0^{\infty} d\omega (2\ell+1) \omega \\ &\quad \times \left[ \frac{|B_{\omega\ell}^{\text{up}}|^2}{\tilde{\omega} \left( \exp \left[ \frac{\tilde{\omega}}{T_H} \right] - 1 \right)} \right] \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{U}} &= \frac{1}{16\pi^2} \sum_{\ell=0}^{\ell} \int_0^{\infty} d\omega (2\ell+1) \omega^2 \\ &\quad \times \left[ \frac{|B_{\omega\ell}^{\text{up}}|^2}{\tilde{\omega} \left( \exp \left[ \frac{\tilde{\omega}}{T_H} \right] - 1 \right)} \right] \end{aligned}$$

$$\text{Unruh state } |\mathbf{U}\rangle \quad \tilde{\omega} = \omega - \frac{qQ}{r_+} \quad \bar{\omega} = \omega + \frac{qQ}{r_+}$$

$$\langle \mathbf{U} | \hat{O} | \mathbf{U} \rangle = \frac{1}{2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left[ \int_{-\infty}^{\infty} d\omega o_{\omega\ell m}^{\text{in}} + \int_{-\infty}^{\infty} d\tilde{\omega} o_{\omega\ell m}^{\text{up}} \coth \left| \frac{\tilde{\omega}}{2T_H} \right| \right]$$

$$\begin{aligned} \mathcal{K}_{\mathbf{U}} &= \frac{q}{64\pi^3} \sum_{\ell=0}^{\ell} \int_0^{\infty} d\omega (2\ell+1) \omega \\ &\quad \times \left[ \frac{|B_{\omega\ell}^{\text{up}}|^2}{\tilde{\omega} \left( \exp \left[ \frac{\tilde{\omega}}{T_H} \right] - 1 \right)} - \frac{|B_{-\omega\ell}^{\text{up}}|^2}{\bar{\omega} \left( \exp \left[ \frac{\bar{\omega}}{T_H} \right] - 1 \right)} \right] \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\mathbf{U}} &= \frac{1}{16\pi^2} \sum_{\ell=0}^{\ell} \int_0^{\infty} d\omega (2\ell+1) \omega^2 \\ &\quad \times \left[ \frac{|B_{\omega\ell}^{\text{up}}|^2}{\tilde{\omega} \left( \exp \left[ \frac{\tilde{\omega}}{T_H} \right] - 1 \right)} + \frac{|B_{-\omega\ell}^{\text{up}}|^2}{\bar{\omega} \left( \exp \left[ \frac{\bar{\omega}}{T_H} \right] - 1 \right)} \right] \end{aligned}$$

# “Boulware”-like state on RN

[ Balakumar, Bernar, Crispino & EW *PLB* **811** 135904 (2020) ]

# “Boulware”-like state on RN

- Positive frequency with respect to Schwarzschild time  $t$

[ Balakumar, Bernar, Crispino & EW *PLB* **811** 135904 (2020) ]

# “Boulware”-like state on RN

- Positive frequency with respect to Schwarzschild time  $t$
- $\omega > 0$  for “in” modes

[ Balakumar, Bernar, Crispino & EW *PLB* **811** 135904 (2020) ]

# “Boulware”-like state on RN

- Positive frequency with respect to Schwarzschild time  $t$
- $\omega > 0$  for “in” modes
- $\tilde{\omega} > 0$  for “up” modes

[ Balakumar, Bernar, Crispino & EW *PLB* **811** 135904 (2020) ]

# “Boulware”-like state on RN

- Positive frequency with respect to Schwarzschild time  $t$
- $\omega > 0$  for “in” modes
- $\tilde{\omega} > 0$  for “up” modes

$$\langle B_1 | \hat{O} | B_1 \rangle = \frac{1}{2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left[ \int_{-\infty}^{\infty} d\omega o_{\omega\ell m}^{\text{in}} + \int_{-\infty}^{\infty} d\tilde{\omega} o_{\omega\ell m}^{\text{up}} \right]$$

[ Balakumar, Bernar, Crispino & EW *PLB* **811** 135904 (2020) ]

# “Boulware”-like state on RN

- Positive frequency with respect to Schwarzschild time  $t$
- $\omega > 0$  for “in” modes
- $\tilde{\omega} > 0$  for “up” modes

$$\langle B_1 | \hat{O} | B_1 \rangle = \frac{1}{2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left[ \int_{-\infty}^{\infty} d\omega o_{\omega\ell m}^{\text{in}} + \int_{-\infty}^{\infty} d\tilde{\omega} o_{\omega\ell m}^{\text{up}} \right]$$

$$\mathcal{K}_{B_1} = \frac{q}{64\pi^3} \sum_{\ell=0}^{\infty} \int_{\min\{\frac{qQ}{r_+}, 0\}}^{\max\{\frac{qQ}{r_+}, 0\}} d\omega \frac{\omega}{|\tilde{\omega}|} (2\ell + 1) |B_{\omega\ell}^{\text{up}}|^2$$

$$\mathcal{L}_{B_1} = \frac{1}{16\pi^2} \sum_{\ell=0}^{\infty} \int_{\min\{\frac{qQ}{r_+}, 0\}}^{\max\{\frac{qQ}{r_+}, 0\}} d\omega \frac{\omega^2}{|\tilde{\omega}|} (2\ell + 1) |B_{\omega\ell}^{\text{up}}|^2$$

[ Balakumar, Bernar, Crispino & EW *PLB* **811** 135904 (2020) ]

# “Boulware”-like state on RN

[ Bernar, Balakumar & EW *PRD* **106** 125013 (2022) ]

# “Boulware”-like state on RN

- Positive frequency with respect to Schwarzschild time  $t$

# “Boulware”-like state on RN

- Positive frequency with respect to Schwarzschild time  $t$
- $\omega > 0$  for *both* “in” and “up” modes

## “Boulware”-like state on RN

- Positive frequency with respect to Schwarzschild time  $t$
- $\omega > 0$  for both “in” and “up” modes
- Superradiant “up” modes relabelled

[ Bernar, Balakumar & EW PRD **106** 125013 (2022) ]

# “Boulware”-like state on RN

- Positive frequency with respect to Schwarzschild time  $t$
- $\omega > 0$  for both “in” and “up” modes
- Superradiant “up” modes relabelled

$$\langle \mathbf{B} | \hat{O} | \mathbf{B} \rangle = \frac{1}{2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left\{ \int_{-\infty}^{\infty} d\omega [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}] \right\}$$

# “Boulware”-like state on RN

- Positive frequency with respect to Schwarzschild time  $t$
- $\omega > 0$  for both “in” and “up” modes
- Superradiant “up” modes relabelled

$$\langle B | \hat{O} | B \rangle = \frac{1}{2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left\{ \int_{-\infty}^{\infty} d\omega [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}] - 2 \int_{\min\{0, \frac{qQ}{r_+}\}}^{\max\{0, \frac{qQ}{r_+}\}} d\omega o_{\omega\ell m}^{\text{up}} \right\}$$

# “Boulware”-like state on RN

- Positive frequency with respect to Schwarzschild time  $t$
- $\omega > 0$  for both “in” and “up” modes
- Superradiant “up” modes relabelled

$$\langle B | \hat{O} | B \rangle = \frac{1}{2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left\{ \int_{-\infty}^{\infty} d\omega [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}] - 2 \int_{\min\{0, \frac{qQ}{r_+}\}}^{\max\{0, \frac{qQ}{r_+}\}} d\omega o_{\omega\ell m}^{\text{up}} \right\}$$

## Properties of the state $|B\rangle$

[ Bernar, Balakumar & EW PRD **106** 125013 (2022) ]

# “Boulware”-like state on RN

- Positive frequency with respect to Schwarzschild time  $t$
- $\omega > 0$  for both “in” and “up” modes
- Superradiant “up” modes relabelled

$$\langle B | \hat{O} | B \rangle = \frac{1}{2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left\{ \int_{-\infty}^{\infty} d\omega [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}] - 2 \int_{\min\{0, \frac{qQ}{r_+}\}}^{\max\{0, \frac{qQ}{r_+}\}} d\omega o_{\omega\ell m}^{\text{up}} \right\}$$

## Properties of the state $|B\rangle$

- Equilibrium state

$$\mathcal{K}_B = 0 \quad \mathcal{L}_B = 0$$

# “Boulware”-like state on RN

- Positive frequency with respect to Schwarzschild time  $t$
- $\omega > 0$  for both “in” and “up” modes
- Superradiant “up” modes relabelled

$$\langle B | \hat{O} | B \rangle = \frac{1}{2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left\{ \int_{-\infty}^{\infty} d\omega [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}] - 2 \int_{\min\{0, \frac{qQ}{r_+}\}}^{\max\{0, \frac{qQ}{r_+}\}} d\omega o_{\omega\ell m}^{\text{up}} \right\}$$

## Properties of the state $|B\rangle$

- Equilibrium state

$$\mathcal{K}_B = 0 \quad \mathcal{L}_B = 0$$

- Empty as possible at infinity

[ Bernar, Balakumar & EW PRD **106** 125013 (2022) ]

# “Hartle-Hawking”-like state on RN

# "Hartle-Hawking"-like state on RN

$|H_1\rangle$

# “Hartle-Hawking”-like state on RN

$|H_1\rangle$

$$\langle H_1 | \hat{O} | H_1 \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_{-\infty}^{\infty} d\omega \coth \left| \frac{\omega}{2T_H} \right| o_{\omega\ell m}^{\text{in}} + \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_{-\infty}^{\infty} d\tilde{\omega} \coth \left| \frac{\tilde{\omega}}{2T_H} \right| o_{\omega\ell m}^{\text{up}}$$

# “Hartle-Hawking”-like state on RN

$|H_1\rangle$

$$\langle H_1 | \hat{O} | H_1 \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_{-\infty}^{\infty} d\omega \coth \left| \frac{\omega}{2T_H} \right| o_{\omega\ell m}^{\text{in}} + \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_{-\infty}^{\infty} d\tilde{\omega} \coth \left| \frac{\tilde{\omega}}{2T_H} \right| o_{\omega\ell m}^{\text{up}}$$

$$\mathcal{K}_{H_1} = \mathcal{K}_U$$

$$\mathcal{L}_{H_1} = \mathcal{L}_U$$

# “Hartle-Hawking”-like state on RN

$|H_1\rangle$

$$\langle H_1 | \hat{O} | H_1 \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_{-\infty}^{\infty} d\omega \coth \left| \frac{\omega}{2T_H} \right| o_{\omega\ell m}^{\text{in}} + \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_{-\infty}^{\infty} d\tilde{\omega} \coth \left| \frac{\tilde{\omega}}{2T_H} \right| o_{\omega\ell m}^{\text{up}}$$

$$\mathcal{K}_{H_1} = \mathcal{K}_U$$

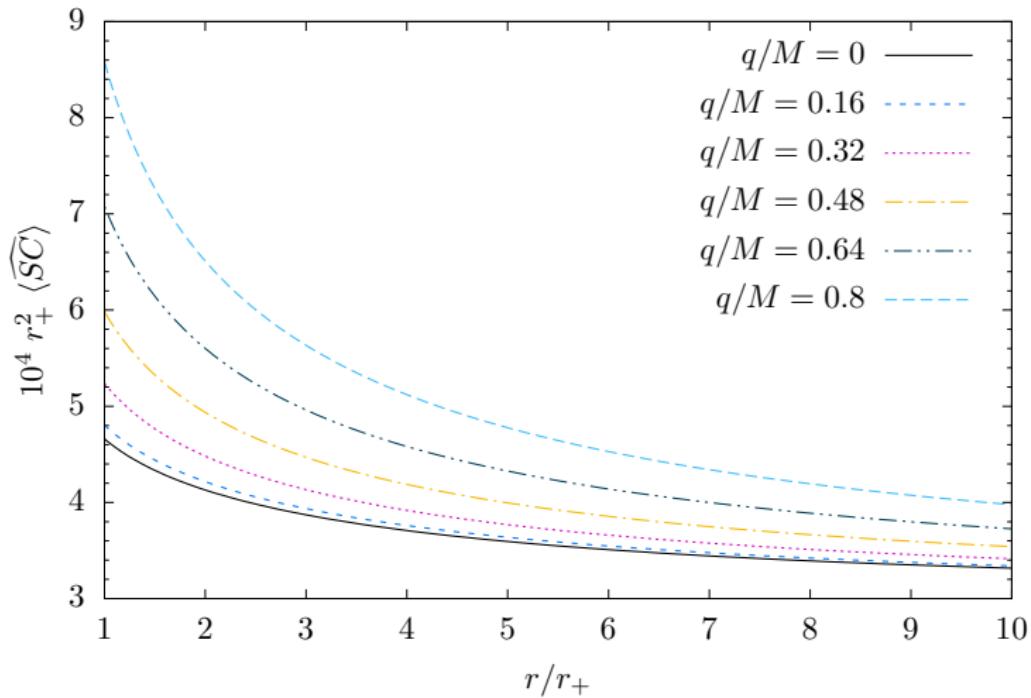
$$- \frac{q}{64\pi^3} \sum_{\ell=0}^{\ell} \int_0^{\infty} d\omega \frac{(2\ell+1) \tilde{\omega}^2}{\omega \left( \exp \left[ \frac{\omega}{T_H} \right] - 1 \right)} \left[ \frac{1}{\tilde{\omega}} |B_{\omega\ell}^{\text{in}}|^2 - \frac{1}{\bar{\omega}} |B_{-\omega\ell}^{\text{in}}|^2 \right]$$

$$\mathcal{L}_{H_1} = \mathcal{L}_U$$

$$- \frac{1}{16\pi^2} \sum_{\ell=0}^{\ell} \int_0^{\infty} d\omega \frac{(2\ell+1) \tilde{\omega}^2}{\left( \exp \left[ \frac{\omega}{T_H} \right] - 1 \right)} \left[ \frac{1}{\tilde{\omega}} |B_{\omega\ell}^{\text{in}}|^2 + \frac{1}{\bar{\omega}} |B_{-\omega\ell}^{\text{in}}|^2 \right]$$

# “Hartle-Hawking”-like state on RN

$$\langle H_1 | \widehat{SC} | H_1 \rangle - \langle U | \widehat{SC} | U \rangle$$



[ Bernar, Balakumar & EW PRD **106** 125013 (2022) ]

# “Hartle-Hawking”-like state on RN

# "Hartle-Hawking"-like state on RN

$|H_2\rangle$

# “Hartle-Hawking”-like state on RN

$|H_2\rangle$

$$\langle H_2 | \hat{O} | H_2 \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_{-\infty}^{\infty} d\tilde{\omega} \coth \left| \frac{\tilde{\omega}}{2T_H} \right| o_{\omega\ell m}^{\text{in}} + \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_{-\infty}^{\infty} d\tilde{\omega} \coth \left| \frac{\tilde{\omega}}{2T_H} \right| o_{\omega\ell m}^{\text{up}}$$

# “Hartle-Hawking”-like state on RN

$|H_2\rangle$

$$\langle H_2 | \hat{O} | H_2 \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_{-\infty}^{\infty} d\tilde{\omega} \coth \left| \frac{\tilde{\omega}}{2T_H} \right| o_{\omega\ell m}^{\text{in}} + \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_{-\infty}^{\infty} d\tilde{\omega} \coth \left| \frac{\tilde{\omega}}{2T_H} \right| o_{\omega\ell m}^{\text{up}}$$

$$\mathcal{K}_{H_2} = \frac{q}{64\pi^3} \sum_{\ell=0}^{\infty} \int_{\min\left\{\frac{qQ}{r_+}, 0\right\}}^{\max\left\{\frac{qQ}{r_+}, 0\right\}} d\omega \frac{|\tilde{\omega}|}{\omega} (2\ell + 1) \coth \left| \frac{\tilde{\omega}}{2T_H} \right| |B_{\omega\ell}^{\text{in}}|^2$$

$$\mathcal{L}_{H_2} = \frac{1}{16\pi^2} \sum_{\ell=0}^{\infty} \int_{\min\left\{\frac{qQ}{r_+}, 0\right\}}^{\max\left\{\frac{qQ}{r_+}, 0\right\}} d\omega |\tilde{\omega}| (2\ell + 1) \coth \left| \frac{\tilde{\omega}}{2T_H} \right| |B_{\omega\ell}^{\text{in}}|^2$$

# “Hartle-Hawking”-like state on RN

$|H_2\rangle$

$$\langle H_2 | \hat{O} | H_2 \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_{-\infty}^{\infty} d\tilde{\omega} \coth \left| \frac{\tilde{\omega}}{2T_H} \right| o_{\omega\ell m}^{\text{in}} \\ + \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_{-\infty}^{\infty} d\tilde{\omega} \coth \left| \frac{\tilde{\omega}}{2T_H} \right| o_{\omega\ell m}^{\text{up}}$$

$$\mathcal{K}_{H_2} = \frac{q}{64\pi^3} \sum_{\ell=0}^{\infty} \int_{\min\left\{\frac{qQ}{r_+}, 0\right\}}^{\max\left\{\frac{qQ}{r_+}, 0\right\}} d\omega \frac{|\tilde{\omega}|}{\omega} (2\ell+1) \coth \left| \frac{\tilde{\omega}}{2T_H} \right| |B_{\omega\ell}^{\text{in}}|^2$$

$$\mathcal{L}_{H_2} = \frac{1}{16\pi^2} \sum_{\ell=0}^{\infty} \int_{\min\left\{\frac{qQ}{r_+}, 0\right\}}^{\max\left\{\frac{qQ}{r_+}, 0\right\}} d\omega |\tilde{\omega}| (2\ell+1) \coth \left| \frac{\tilde{\omega}}{2T_H} \right| |B_{\omega\ell}^{\text{in}}|^2$$

$$\langle H_2 | \widehat{SC} | H_2 \rangle - \langle U | \widehat{SC} | U \rangle = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_{-\infty}^{\infty} d\omega \frac{1}{\exp \left| \frac{\tilde{\omega}}{T_H} \right| - 1} |\phi_{\omega\ell m}^{\text{in}}|^2$$

# “Hartle-Hawking”-like state on RN

# “Hartle-Hawking”-like state on RN

$|H\rangle$

# “Hartle-Hawking”-like state on RN

$|H\rangle$

$$\langle H | \hat{O} | H \rangle = \frac{1}{2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left\{ \int_{-\infty}^{\infty} d\tilde{\omega} [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}] \coth \left| \frac{\tilde{\omega}}{2T_H} \right| \right\}$$

# “Hartle-Hawking”-like state on RN

$|H\rangle$

$$\langle H | \hat{O} | H \rangle = \frac{1}{2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left\{ \int_{-\infty}^{\infty} d\tilde{\omega} [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}] \coth \left| \frac{\tilde{\omega}}{2T_H} \right| \right. \\ \left. - 2 \int_{\min\{0, \frac{qQ}{r_+}\}}^{\max\{0, \frac{qQ}{r_+}\}} d\omega o_{\omega\ell m}^{\text{in}} \coth \left| \frac{\tilde{\omega}}{2T_H} \right| \right\}$$

# “Hartle-Hawking”-like state on RN

$|H\rangle$

$$\langle H | \hat{O} | H \rangle = \frac{1}{2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left\{ \int_{-\infty}^{\infty} d\tilde{\omega} [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}] \coth \left| \frac{\tilde{\omega}}{2T_H} \right| \right. \\ \left. - 2 \int_{\min\{0, \frac{qQ}{r_+}\}}^{\max\{0, \frac{qQ}{r_+}\}} d\omega o_{\omega\ell m}^{\text{in}} \coth \left| \frac{\tilde{\omega}}{2T_H} \right| \right\}$$

$$\mathcal{K}_H = 0 \quad \mathcal{L}_H = 0$$

# “Hartle-Hawking”-like state on RN

$|H\rangle$

$$\langle H | \hat{O} | H \rangle = \frac{1}{2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \left\{ \int_{-\infty}^{\infty} d\tilde{\omega} [o_{\omega\ell m}^{\text{in}} + o_{\omega\ell m}^{\text{up}}] \coth \left| \frac{\tilde{\omega}}{2T_H} \right| \right. \\ \left. - 2 \int_{\min\{0, \frac{qQ}{r_+}\}}^{\max\{0, \frac{qQ}{r_+}\}} d\omega o_{\omega\ell m}^{\text{in}} \coth \left| \frac{\tilde{\omega}}{2T_H} \right| \right\}$$

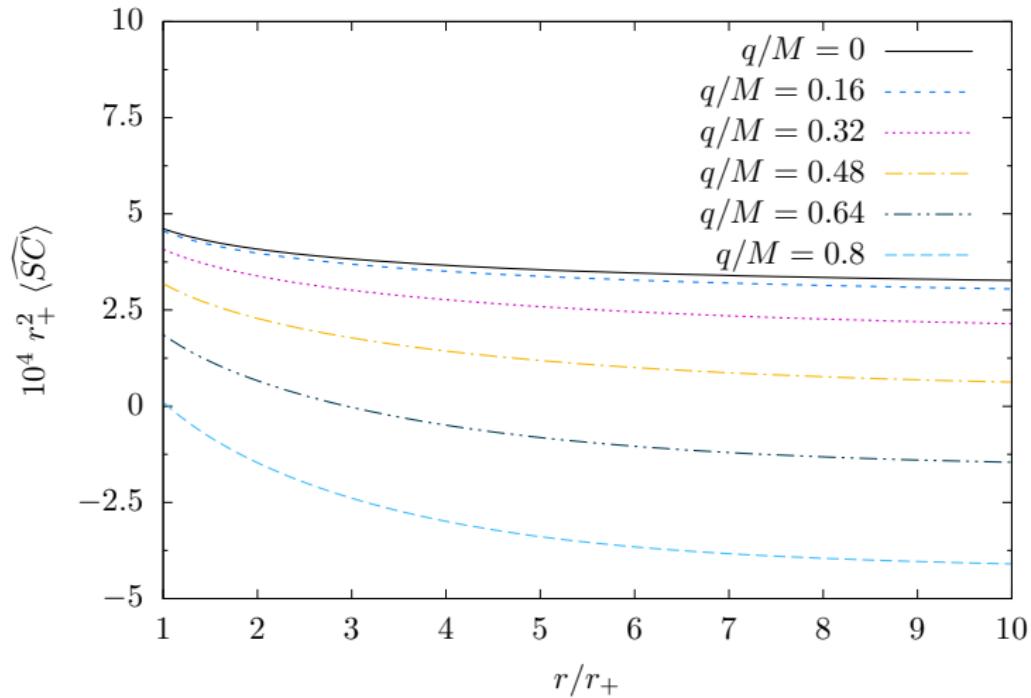
$$\mathcal{K}_H = 0 \quad \mathcal{L}_H = 0$$

$$\langle H | \widehat{SC} | H \rangle - \langle U | \widehat{SC} | U \rangle$$

$$= \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \int_0^{\infty} d\omega \left\{ \frac{|\phi_{\omega\ell m}^{\text{in}}|^2}{\exp\left(\frac{\tilde{\omega}}{T_H}\right) - 1} + \frac{|\phi_{-\omega\ell m}^{\text{in}}|^2}{\exp\left(\frac{\bar{\omega}}{T_H}\right) - 1} \right\}$$

# “Hartle-Hawking”-like state on RN

$$\langle H | \widehat{SC} | H \rangle - \langle U | \widehat{SC} | U \rangle$$



[ Bernar, Balakumar & EW PRD **106** 125013 (2022) ]

# Quantum states for a neutral scalar field

# Quantum states for a neutral scalar field

Schwarzschild

# Quantum states for a neutral scalar field

## Schwarzschild

- Unruh  $|U\rangle$  - Hawking flux at  $\mathcal{I}^+$

# Quantum states for a neutral scalar field

## Schwarzschild

- Unruh  $|U\rangle$  - Hawking flux at  $\mathcal{I}^+$
- Boulware  $|B\rangle$  - empty as possible at infinity

# Quantum states for a neutral scalar field

## Schwarzschild

- Unruh  $|U\rangle$  - Hawking flux at  $\mathcal{I}^+$
- Boulware  $|B\rangle$  - empty as possible at infinity
- Hartle-Hawking  $|H\rangle$  - equilibrium, regular at horizon

# Quantum states for a neutral scalar field

## Schwarzschild

- Unruh  $|U\rangle$  - Hawking flux at  $\mathcal{I}^+$
- Boulware  $|B\rangle$  - empty as possible at infinity
- Hartle-Hawking  $|H\rangle$  - equilibrium, regular at horizon

## Kerr

# Quantum states for a neutral scalar field

## Schwarzschild

- Unruh  $|U\rangle$  - Hawking flux at  $\mathcal{I}^+$
- Boulware  $|B\rangle$  - empty as possible at infinity
- Hartle-Hawking  $|H\rangle$  - equilibrium, regular at horizon

## Kerr

- Classical superradiance

# Quantum states for a neutral scalar field

## Schwarzschild

- Unruh  $|U\rangle$  - Hawking flux at  $\mathcal{I}^+$
- Boulware  $|B\rangle$  - empty as possible at infinity
- Hartle-Hawking  $|H\rangle$  - equilibrium, regular at horizon

## Kerr

- Classical superradiance
- No Hartle-Hawking state

# Quantum states for a neutral scalar field

## Schwarzschild

- Unruh  $|U\rangle$  - Hawking flux at  $\mathcal{I}^+$
- Boulware  $|B\rangle$  - empty as possible at infinity
- Hartle-Hawking  $|H\rangle$  - equilibrium, regular at horizon

## Kerr

- Classical superradiance
- No Hartle-Hawking state
- $|CCH\rangle$  - regular, not equilibrium

# Quantum states for a neutral scalar field

## Schwarzschild

- Unruh  $|U\rangle$  - Hawking flux at  $\mathcal{I}^+$
- Boulware  $|B\rangle$  - empty as possible at infinity
- Hartle-Hawking  $|H\rangle$  - equilibrium, regular at horizon

## Kerr

- Classical superradiance
- No Hartle-Hawking state
- $|CCH\rangle$  - regular, not equilibrium
- $|FT\rangle$  - equilibrium, divergent almost everywhere

# Quantum states for a charged scalar field on RN

# Quantum states for a charged scalar field on RN

Unruh state  $|U\rangle$

# Quantum states for a charged scalar field on RN

## Unruh state $|U\rangle$

- Hawking flux at  $\mathcal{I}^+$

# Quantum states for a charged scalar field on RN

Unruh state  $|U\rangle$

- Hawking flux at  $\mathcal{I}^+$

“Boulware”-like states

# Quantum states for a charged scalar field on RN

## Unruh state $|U\rangle$

- Hawking flux at  $\mathcal{I}^+$

## "Boulware"-like states

- $|B_1\rangle$  - superradiant flux at  $\mathcal{I}^+$

# Quantum states for a charged scalar field on RN

## Unruh state $|U\rangle$

- Hawking flux at  $\mathcal{I}^+$

## "Boulware"-like states

- $|B_1\rangle$  - superradiant flux at  $\mathcal{I}^+$
- Proposed state  $|B\rangle$  - empty as possible at infinity

# Quantum states for a charged scalar field on RN

## Unruh state $|U\rangle$

- Hawking flux at  $\mathcal{I}^+$

## "Boulware"-like states

- $|B_1\rangle$  - superradiant flux at  $\mathcal{I}^+$
- Proposed state  $|B\rangle$  - empty as possible at infinity

## "Hartle-Hawking"-like states

# Quantum states for a charged scalar field on RN

## Unruh state $|U\rangle$

- Hawking flux at  $\mathcal{I}^+$

## "Boulware"-like states

- $|B_1\rangle$  - superradiant flux at  $\mathcal{I}^+$
- Proposed state  $|B\rangle$  - empty as possible at infinity

## "Hartle-Hawking"-like states

- $|H_1\rangle$  - regular, not equilibrium

# Quantum states for a charged scalar field on RN

## Unruh state $|U\rangle$

- Hawking flux at  $\mathcal{I}^+$

## "Boulware"-like states

- $|B_1\rangle$  - superradiant flux at  $\mathcal{I}^+$
- Proposed state  $|B\rangle$  - empty as possible at infinity

## "Hartle-Hawking"-like states

- $|H_1\rangle$  - regular, not equilibrium
- $|H_2\rangle$  - not equilibrium, divergent

# Quantum states for a charged scalar field on RN

## Unruh state $|U\rangle$

- Hawking flux at  $\mathcal{I}^+$

## "Boulware"-like states

- $|B_1\rangle$  - superradiant flux at  $\mathcal{I}^+$
- Proposed state  $|B\rangle$  - empty as possible at infinity

## "Hartle-Hawking"-like states

- $|H_1\rangle$  - regular, not equilibrium
- $|H_2\rangle$  - not equilibrium, divergent
- Proposed state  $|H\rangle$  - equilibrium, regular

# Outlook

# Outlook

## Quantum fields on stationary black holes

# Outlook

## Quantum fields on stationary black holes

- Classical superradiance only for bosonic fields

# Outlook

## Quantum fields on stationary black holes

- Classical superradiance only for bosonic fields
- Quantum superradiance for fields of *all* spins

# Outlook

## Quantum fields on stationary black holes

- Classical superradiance only for bosonic fields
- Quantum superradiance for fields of *all* spins
- Bosonic fields

[ Casals & Ottewill *PRD* **71** 124016 (2005), Gérard *RMP* **33** 2150028 (2021),  
Klein *Ann. Inst. Henri Poincaré* **24** 2401 (2023), Iuliano & Zahn 2307.07467 ]

# Outlook

## Quantum fields on stationary black holes

- Classical superradiance only for bosonic fields
- Quantum superradiance for fields of *all* spins
- Bosonic fields
  - [ Casals & Ottewill *PRD* **71** 124016 (2005), Gérard *RMP* **33** 2150028 (2021), Klein *Ann. Inst. Henri Poincaré* **24** 2401 (2023), Iuliano & Zahn 2307.07467 ]
- Fermionic fields
  - [ Casals et al *PRD* **87** 064027 (2013), Gérard, Häfner & Wrochna 2008.10995 ]

# Outlook

## Quantum fields on stationary black holes

- Classical superradiance only for bosonic fields
- Quantum superradiance for fields of *all* spins
- Bosonic fields
  - [ Casals & Ottewill *PRD* **71** 124016 (2005), Gérard *RMP* **33** 2150028 (2021), Klein *Ann. Inst. Henri Poincaré* **24** 2401 (2023), Iuliano & Zahn 2307.07467 ]
- Fermionic fields
  - [ Casals et al *PRD* **87** 064027 (2013), Gérard, Häfner & Wrochna 2008.10995 ]

## Charged scalar on charged black holes

# Outlook

## Quantum fields on stationary black holes

- Classical superradiance only for bosonic fields
- Quantum superradiance for fields of *all* spins
- Bosonic fields
  - [ Casals & Ottewill *PRD* **71** 124016 (2005), Gérard *RMP* **33** 2150028 (2021), Klein *Ann. Inst. Henri Poincaré* **24** 2401 (2023), Iuliano & Zahn 2307.07467 ]
- Fermionic fields
  - [ Casals et al *PRD* **87** 064027 (2013), Gérard, Häfner & Wrochna 2008.10995 ]

## Charged scalar on charged black holes

- Renormalized expectation values
  - [ Klein, Zahn & Hollands *PRL* **127** 23 (2021), Klein & Zahn *PRD* **104** 025009 (2021) ]

# Outlook

## Quantum fields on stationary black holes

- Classical superradiance only for bosonic fields
- Quantum superradiance for fields of *all* spins
- Bosonic fields
  - [ Casals & Ottewill *PRD* **71** 124016 (2005), Gérard *RMP* **33** 2150028 (2021), Klein *Ann. Inst. Henri Poincaré* **24** 2401 (2023), Iuliano & Zahn 2307.07467 ]
- Fermionic fields
  - [ Casals et al *PRD* **87** 064027 (2013), Gérard, Häfner & Wrochna 2008.10995 ]

## Charged scalar on charged black holes

- Renormalized expectation values
  - [ Klein, Zahn & Hollands *PRL* **127** 23 (2021), Klein & Zahn *PRD* **104** 025009 (2021) ]
- Analogue of Kay-Wald theorem for a charged scalar field