Trace anomaly and compact stars

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Quantum effects in gravitational fields
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The Planck scale

Classical gravity with matter,

$$T_{\mu\nu} = (8\pi G)^{-1}\left(R_{\mu\nu} - \frac{R}{2}g_{\mu\nu}\right)$$

Now introduce quantum corrections for matter

$$\langle : \hat{T}_{\mu\nu} : \rangle \sim \hbar R^2 g_{\mu\nu} + \ldots$$

Comparing components,

$$T_{\mu\nu} \approx \langle : \hat{T}_{\mu\nu} : \rangle \quad \Rightarrow \qquad R^{-1} \sim G\hbar \quad \leftrightarrow \quad \rho \sim \rho_P = M_P^4$$

We can neglect quantum effects of curvature below Planck scale.

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Outline

1) When does QFTCS become macroscopic?

2) Application: compact stars

Derivation

Trace Anomaly

Trace anomaly

QFT curved spacetime:

$$T_{\mu\nu} = T^{(m)}_{\mu\nu} + T^{(A)}_{\mu\nu}$$

break conformal symmetry classically

not explicit dependence

$g_{\mu\nu}$

anomaly

explicitly dependent

Weyl anomaly: $$T^{(A)} = cW^2 - aE$$

$$c, a > 0$$

$$\int E = \chi$$

Assumptions

Much below the Planck density $\rho_P = M_p^4$:

(A1) Semi-classical approximation is valid

$$G_{\mu\nu} = 8\pi G \left(T^{(m)}_{\mu\nu} + T^{(A)}_{\mu\nu} \right)$$

(A2) Matter equation of state is conformal at leading order

$$T^{(m)} = -\rho + 3p \underset{m^4 \ll \rho \ll \rho_P}{\approx} -m^{4(1-\alpha)} \rho^{\alpha} \qquad \alpha < 1$$

(A3) Individual components

$$|T^{(A)}_{\mu\nu}| \ll |T^{(m)}_{\mu\nu}|$$

Evaluation

Restrict to regions where $W^2 \approx 0$

Ricci decomposition

$$E = W^2 + 2\left(\frac{R^2}{3} - R_{\mu\nu}R^{\mu\nu}\right)$$

Evaluate on background:

$$R = -GT^{(m)} = -Gm^{4(1-\alpha)}\rho^{\alpha} \qquad R_{\mu\nu}R^{\mu\nu} \sim G^2\rho^2$$

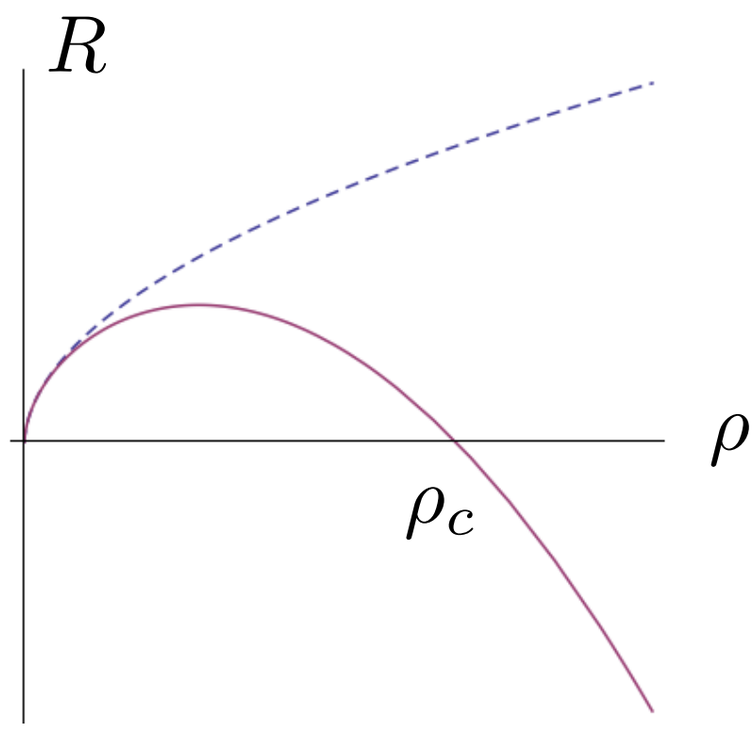
$$T^{(A)} = -aE \sim G^2\rho^2$$

Anomalous field equations

Trace of semi-classical equations:

$$R = -8\pi G \left(T^{(m)} + T^{(A)} \right)$$

We find:



$$R \approx M_p^{-2} \left(m^{4(1-\alpha)} \rho^{\alpha} - M_p^{-4} \rho^2 \right)$$

$$\rho_c \sim (m/M_p)^{\frac{4(1-\alpha)}{2-\alpha}} \rho_p$$

If you ask: when does $|T^{(A)}_{\mu\nu}| \sim |T^{(m)}_{\mu\nu}|$

the answer is* $\rho \sim \rho_p$

*Arrechea's talk

If you ask: when does $|T^{(A)}| \sim |T^{(m)}|$

the answer is $\rho_c \sim (m/M_p)^{\frac{4(1-\alpha)}{2-\alpha}} \rho_p \ll \rho_p$

Physical implications?

Application:

Compact stars

Regular star

Static, spherically symmetric isotropic perfect fluid

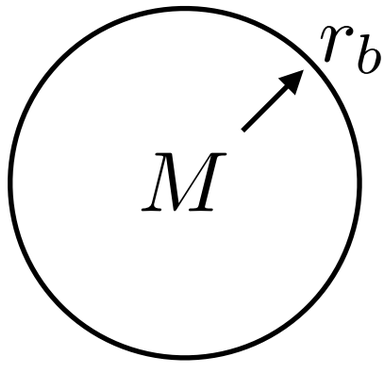
$$T^{(m)\mu}{}_{\nu} = \mathrm{diag}(-\rho, p, p, p)$$

$$ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2 d\Omega^2$$

Regularity at the center

$$h(0) = 1 \quad , \quad f'(0) = 0 = h'(0)$$

$$\Rightarrow \quad W^2(0) = 0$$



Conservation

Since anomaly is identically conserved $\nabla_\mu T^{(A)\mu\nu} = 0$

$$\nabla_\mu T^{(m)\mu\nu} = 0 \qquad \frac{f'}{f} = -2\frac{p'}{p+\rho}$$

Integrating inside the sphere

$$\log\frac{f(r_b)}{f(0)} = -2\int_{p(0)\to\infty}^{p(r_b)=0}\frac{dp}{p+\rho} \approx 2\int^{p(0)\to\infty}\frac{dp}{4p+m^2\sqrt{p}} \sim \frac{1}{2}\log\frac{p(0)}{m^4}$$

$$\frac{f(0)}{f(r_b)} \sim \frac{m^2}{\sqrt{\rho(0)}}$$

(generalized) Buchdahl limit

Scalar linear perturbations

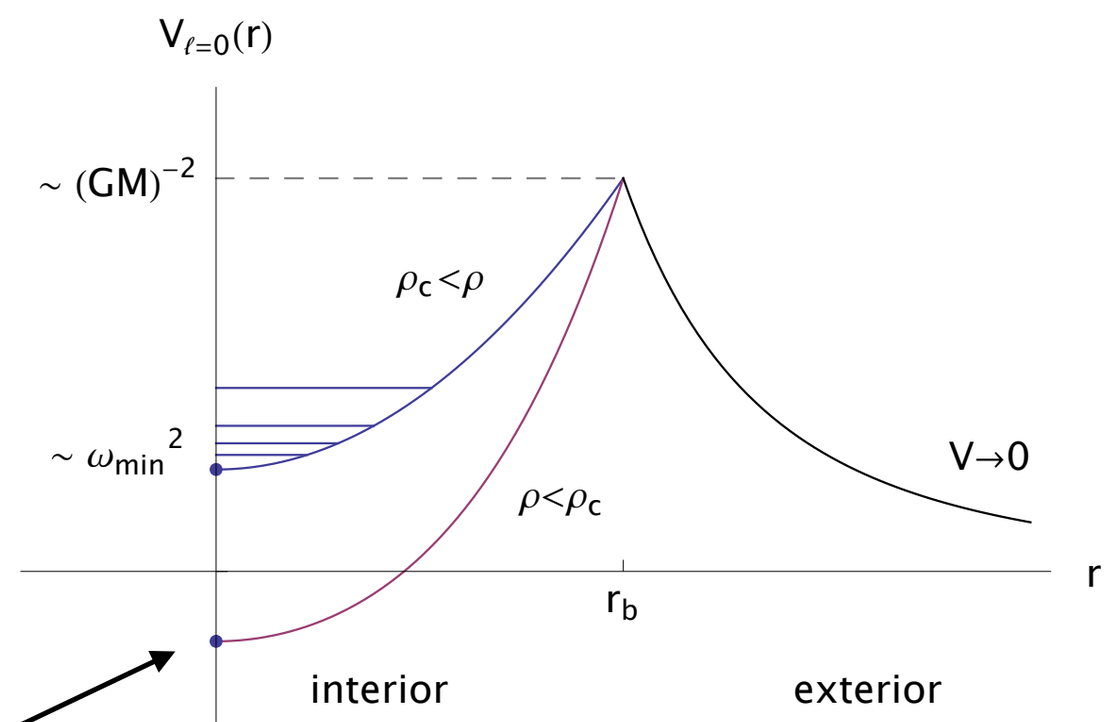
$$\Box\Phi = 0 \qquad\qquad \Phi = \frac{u(r)}{r} Y^{\ell}_{m}(\theta,\varphi) e^{-i\omega t}$$

$$-\partial^2_{r_*} u + V(r_*) u = \omega^2 u$$

$$V_{\ell=0}(0) = -\frac{1}{6} f(0) R(0)$$

$$\sim -\frac{m^2}{\sqrt{\rho}} M_P^{-2} \left(m^2 \sqrt{\rho} - M_P^{-4} \rho^2 \right)$$

$$\sim -\left(\frac{m^2}{M_P} \right)^2$$



Mass of the star?

The condition

$$V_{\ell=0}(0) \sim V_{\ell=0}(r_b)$$

Close to the crossover this is

$$\frac{m^2}{M_P} \sim \frac{M_P^2}{M}$$

$$M \sim \frac{M_P^3}{m^2}$$

Conclusions

The effects of the trace anomaly become macroscopic at

$$\rho_c \sim (m/M_P)^{\frac{4(1-\alpha)}{2-\alpha}} \rho_P \ll \rho_P$$

$$\alpha < 1$$

$$m \ll M_P$$

and the curvature becomes negative $R < 0$

For scalar perturbations of stars close to Buchdahl limit, the anomaly controls the low frequency spectrum, and the object becomes 'less attractive'.

Outlook

- Spin perturbations
- Gravitational collapse