Towards a Probabilistic Foundation of Relativistic Quantum Theory

The One-Body Born Rule in Curved Spacetime

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Motivation

- The Standard Model is our currently most successful theory of particle physics. It is an example of a 'perturbative' quantum field theory (QFT).
- Following Fraser [2], perturbative QFT has three central problems:
  - the lack of mathematical rigor
  - inconsistency due to Haag's theorem
  - questionable, ad hoc reasoning (e.g. renormalization)

  Perturbative QFT is therefore more of "a method for producing approximations" than a fundamental theory of physics.
- Axiomatic approaches to QFT such as Algebraic QFT [1, 3, 4] have made progress in remedying this problematic situation.
- Yet problematic concepts like renormalization have remained part of those approaches. A more conceptual approach is needed.

The Born rule as a novel approach to relativistic quantum theory

- Our approach to the language of relativistic particle physics is to generalize the Born rule of non-relativistic QM to the curved spacetime setting.
- prior relativistic generalizations of the Born rule exist [5, 6], but are somewhat restrictive
- advantages of our approach:
  - probabilistic by construction
  - It can accommodate a wide array of dynamical models.
  - It respects the general principle of relativity and does not rely on the symmetries of Minkowski spacetime (see also [4]).
  - On Minkowski spacetime the theory is fully compatible with the Dirac equation.

Summary of results

- The 1-body generalization turns out to be a special case of the theory of the general-relativistic continuity equation.
- Based on contributions by Eckart and Ehlers, we provide a comprehensive development of this theory in the smooth case.
- As in the non-relativistic analog time evolution gives rise to an Eulerian and Lagrangian picture. The development of the Langrangian picture is our main contribution to the mathematical physics literature.
- The theory overcomes overly restrictive assumptions of prior approaches.

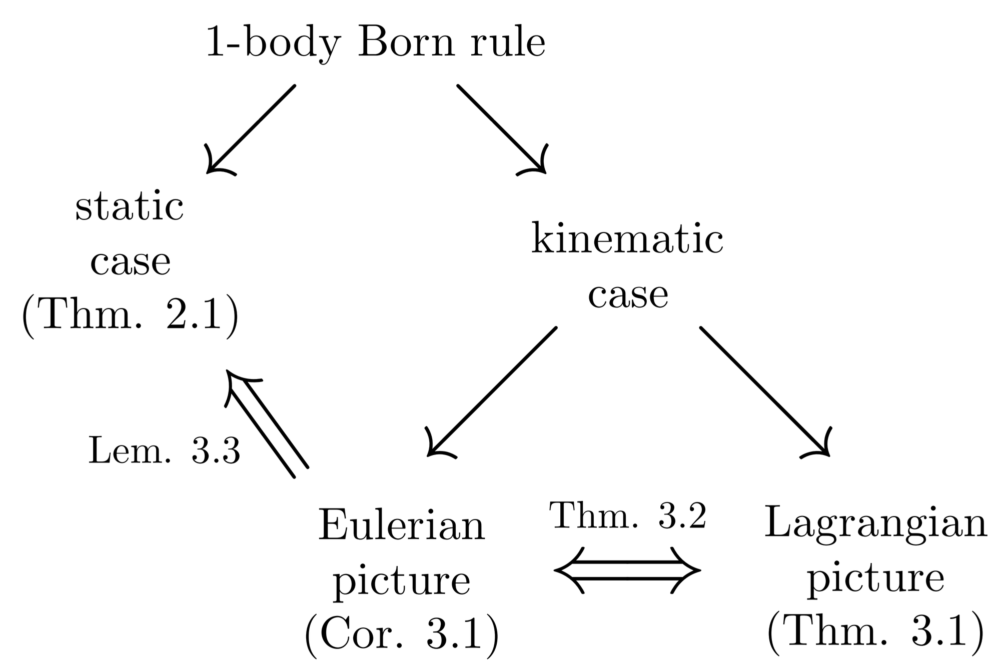

Figure 1. The general structure of the theory laid out in our work [7] along with respective main theorems.

The Langrangian picture

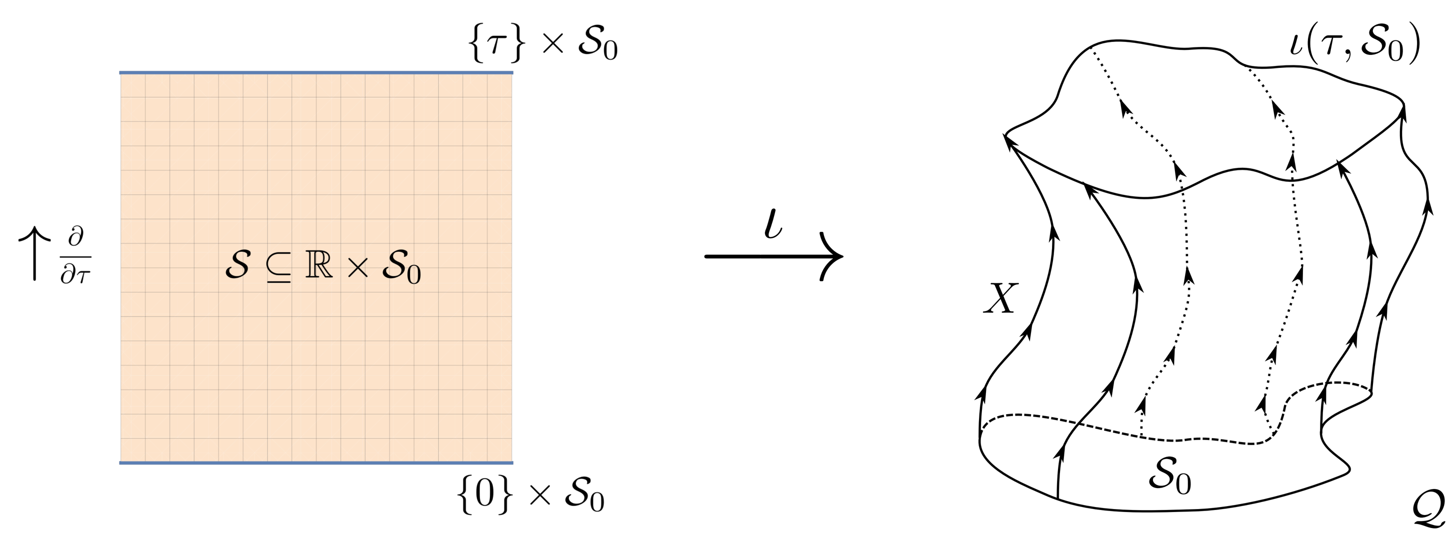

Figure 2. In the Lagrangian picture the map ι trivializes the time evolution of the initial hypersurface $\mathcal{S}_0$ on the spacetime $\mathcal{Q}$. X is its (proper) time derivative.

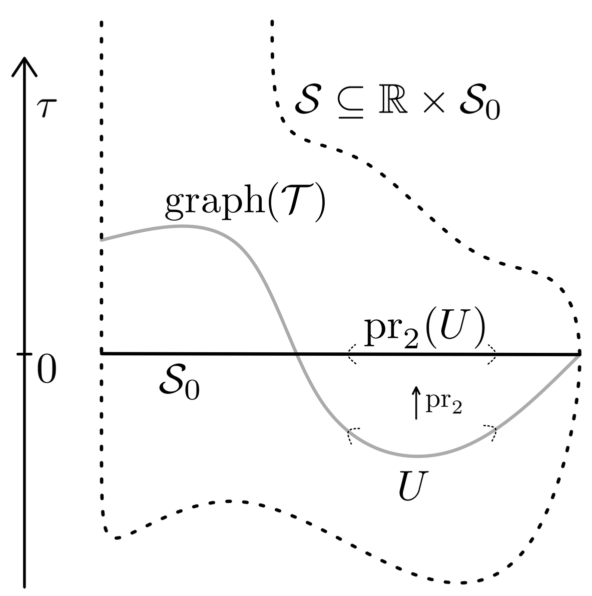

Figure 3. This sketch illustrates the Born rule in the Lagrangian picture. The flow domain $\mathcal{S}$ is indicated by the dashed line. As in the static case, we formulate the Born rule on an embedded hypersurface—yet here this hypersurface is the graph of a 'timeshift' $\mathcal{T}$. Assuming probability conservation, for a given 'region' $U \subseteq \mathrm{graph}(\mathcal{T})$ the probability to find the body in a subset $\iota(U)$ of $\iota(\mathrm{graph}(\mathcal{T}))$ is the same as to find it in $\iota_0(\mathrm{pr}_2(U))$ of $\iota_0(\mathcal{S}_0)$.

Outlook for further research

- the Lagrangian picture offers a path to the construction of an N-body generalization and a generalization to a varying number of bodies
- the approach is mostly focused on kinematics, so the question of (possibly non-linear) dynamics needs to be addressed
- ultimately, the regularity conditions on the relevant probability densities and mappings needs to be loosened (Sobolev spaces)

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Acknowledgments

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Check out the preprint

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