Entanglement harvesting from conformal vacuums between two Unruh-DeWitt detectors moving along null paths

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Reference

- S. Barman, D. Barman, B. R. Majhi, "Entanglement harvesting from conformal vacuums between two Unruh-DeWitt detectors moving along null paths", *JHEP* 09 (2022) 106, arXiv:2112.01308 [gr-qc].

Motivation and outline

- Black holes (BHs) are astrophysical objects on which escape velocity is equal to the speed of light.
- Classically BHs only swallow particles and do not emit any.
- Using QFT in BH spacetime Stephen Hawking[1] showed that BHs can also emit particles, which has a Planckian spectrum.
- This spectrum only depends on the final BH parameters, like mass, charge, angular momentum, etc.
- There is no other information about the matter that collapsed into the BH, and this leads to the BH information loss paradox[2] once the black hole is completely evaporated.

[1] S. W. Hawking, *Comm. Math. Phys.* 43, 199 (1975).

[2] S. Chakraborty, K. Lochan, Universe 3 (2017) 3, 55.

Motivation and outline

- There is no classical way to probe the inside structure, if any, of a BH.
- Here comes 'Quantum Entanglement', a fascinating phenomenon that distinguishes a quantum event from classical.
- Entanglement helps to frame the BH information loss problem[3] in a mathematical manner.
- As measurements of a physical observable on entangled particles are not independent of each other, one can allow one particle of the entangled pair to move into the BH and take measurements on the other.

[3] S. D. Mathur, *Class. Quantum Gravity* 26(22), 224001 (2009).

Motivation and outline

- We investigate the entanglement harvesting from conformal vacuums with Unruh-DeWitt detectors in null paths.

- We have considered the $(1+1)$ dimensional Schwarzschild, $(1+1)$ and $(3+1)$ dimensional FLRW spacetimes, that are conformally flat.

- In FLRW spacetime also the particular de Sitter era is studied.

- For brevity here we discuss only the $(1+1)$ dimensional de Sitter background, and $(1+1)$ dimensional Schwarzschild case that provide a concise picture.

Entanglement harvesting

Model set-up:

- The initial state is $|\,in\rangle = |\,0\rangle\,|\,E_0^A\rangle\,|\,E_0^B\rangle$, the Hamiltonian is $H = H_A + H_F + H_{int}$.

  - H_A denotes the detector Hamiltonian
  - H_F denotes the free field Hamiltonian
  - The interaction Hamiltonian is $H_{int} = \sum_{j=1}^{2} c_j\,\kappa_j(\tau_j)\,m^j(\tau_j)\,\Phi\big(X_j(\tau_j)\big)$

- The final state will be $|\,out\rangle = U\,|\,in\rangle$, with $U = \mathcal{T}\ \exp\left\{-i\int_{-\infty}^{\infty}\sum_{j=1}^{2} d\tau_j\ c_j\,\kappa_j(\tau_j)\,m^j(\tau_j)\,\Phi\big(X_j(\tau_j)\big)\right\}$.

Model set-up:

- One can get the final reduced detector density matrix as

$$\rho_{AB} = \begin{bmatrix} 0 & 0 & 0 & c_a c_b \mathscr{E} \\ 0 & c_a^2 P_A & c_a c_b P_{AB} & c_a^2 W_A^{(N)} + c_a c_b W_A^{(S)} \\ 0 & c_a c_b P_{AB}^* & c_b^2 P_B & c_b^2 W_B^{(N)} + c_a c_b W_B^{(S)} \\ c_a c_b \mathscr{E}^* & c_a^2 W_A^{(N)*} + c_a c_b W_A^{(S)*} & c_b^2 W_B^{(N)*} + c_a c_b W_B^{(S)*} & 1 - (c_a^2 P_A + c_b^2 P_B) \end{bmatrix} + \mathcal{O}(c^4) \, ,$$

here $P_j = |\langle E_1^j | m_j(0) | E_0^j \rangle|^2 \, \mathcal{I}_j$, and $\mathscr{E} = \langle E_1^B | m_B(0) | E_0^B \rangle \langle E_1^A | m_A(0) | E_0^A \rangle \mathcal{I}_\varepsilon$.

- The quantities for $\kappa_j(\tau_j) = 1$: $\mathcal{I}_j = \int_{-\infty}^{\infty} d\tau_j' \int_{-\infty}^{\infty} d\tau_j \, e^{-i\Delta E^j(\tau_j' - \tau_j)} \, G_W(X_j', X_j)$, and $\mathcal{I}_\varepsilon = -i \int_{-\infty}^{\infty} d\tau_B' \int_{-\infty}^{\infty} d\tau_A \, e^{i(\Delta E^B \tau_B' + \Delta E^A \tau_A)} \, G_F(X_B', X_A)$, with $\Delta E^j = E_1^j - E_0^j > 0$.

Entanglement harvesting condition:

- Entanglement harvesting is possible only when the partial transposition of the reduced density matrix has negative eigenvalues[4,5].

- The entanglement harvesting condition is $P_A P_B < |\mathscr{E}|^2$ which turns into $\mathscr{I}_A \mathscr{I}_B < |\mathscr{I}_\varepsilon|^2$.

- Concurrence is an entanglement measure defined as $\mathscr{C}(\rho_{AB}) = max\left[0,\ 2c^2\left(|\mathscr{E}| - \sqrt{P_A P_B}\right) + \mathscr{O}(c^4)\right]$.

- We consider a simplified form of it, $\mathscr{C}_{\mathscr{I}} = \left(|\mathscr{I}_\varepsilon| - \sqrt{\mathscr{I}_A \mathscr{I}_B}\right)^4$.

[4] A. Peres, *Phys. Rev. Lett.* 77 (1996) 1413-1415.
[5] S. Barman, D. Barman, B. R. Majhi, *JHEP* 09 (2022) 106.

Considered background: (1+1) dimensional FLRW spacetime

- The $(1+1)$ dimensional Friedman-Lemaitre-Robertson-Walker (FLRW) spacetime is given by line element $ds^2 = -dt^2 + a^2(t)\,dx^2$;

- In de Sitter era the scale factor is $a(t) = e^{t/\alpha_d}$.

- With a change of variables $\eta = -\alpha_d\, e^{-t/\alpha_d}$ the metric becomes conformally flat: $ds^2 = a^2(\eta)(-d\eta^2 + dx^2)$.

- Along outgoing and ingoing null paths $u = \eta - x$ and $v = \eta + x$ are constants respectively.

- They are called outgoing and ingoing null coordinates or retarded and advanced time coordinates.

- The scalar field is decomposed as

$$\Phi = \int_0^\infty \frac{d\omega_k}{\sqrt{4\pi\omega_k}}\left[\hat{a}_k^D e^{-i\omega_k u} + \hat{a}_k^{D\dagger} e^{i\omega_k u} + \hat{b}_k^D e^{-i\omega_k v} + \hat{b}_k^{D\dagger} e^{i\omega_k v}\right].$$

(1+1) de Sitter spacetime

- The lowering operators annihilate the conformal vacuum $|0\rangle_D$, i.e., $\hat{a}_k^D|0\rangle_D = 0 = \hat{b}_k^D|0\rangle_D$.

- Then the Witghtman function is

$$G_D^+(X_j, X_l) = {}_D\langle 0|\Phi(X_j)\Phi(X_l)|0\rangle_D = \int_0^\infty \frac{d\omega_k}{4\pi\omega_k}\left[e^{-i\omega_k(u_j-u_l)} + e^{-i\omega_k(v_j-v_l)}\right].$$

- We choose the outgoing trajectories for Alice as $u = d$ $(i.e., \eta = x + d)$, while for Bob as $u = 0$ $(i.e., \eta = x)$.

- Detector trajectories are given in terms of (t, x).

- Then $v_j - v_l = 2\alpha_d(e^{-t_l/\alpha_d} - e^{-t_j/\alpha_d}) - d(\delta_{jA}\delta_{lB} - \delta_{jB}\delta_{lA})$ and $u_j - u_l = d(\delta_{jA}\delta_{lB} - \delta_{jB}\delta_{lA})$.

Results:

- First we have $\mathcal{I}_j = \int_{-\infty}^{\infty} d\tau_j' \int_{-\infty}^{\infty} d\tau_j \; e^{-i\Delta E^j(\tau_j'-\tau_j)} \; G_W(X_j', X_j) = \int_0^{\infty} d\omega_k/(4\pi\omega_k)\, \mathcal{I}_{j_{\omega_k}}$.

- Now using $\tau_A = t_A$, and $\tau_B = t_B$, we have $\mathcal{I}_{j_{\omega_k}} = \dfrac{2\pi\alpha_d}{\Delta E^j} \dfrac{1}{e^{2\pi\alpha_d \Delta E^j} - 1}$.

- Again $\mathcal{I}_{\varepsilon} = -\,\mathcal{I}_{\varepsilon}^W - \mathcal{I}_{\varepsilon}^R$,

where $\mathcal{I}_{\varepsilon}^W = \int_{-\infty}^{\infty} d\tau_B \int_{-\infty}^{\infty} d\tau_A \; e^{i(\Delta E^B \tau_B + \Delta E^A \tau_A)} G_W(X_B, X_A) = \int_0^{\infty} \dfrac{d\omega_k}{4\pi\omega_k} \; \mathcal{I}_{\varepsilon_{\omega_k}}^W$

and $\mathcal{I}_{\varepsilon}^R = \int_{-\infty}^{\infty} d\tau_B \int_{-\infty}^{\infty} d\tau_A \; e^{i(\Delta E^B \tau_B + \Delta E^A \tau_A)} \theta(T_A - T_B) \times \left[G_W\left(X_A, X_B\right) - G_W\left(X_B, X_A\right)\right] = \int_0^{\infty} \dfrac{d\omega_k}{4\pi\omega_k} \; \mathcal{I}_{\varepsilon_{\omega_k}}^R$.

Results:

- Then one can express $\mathscr{I}_{\epsilon} = -\int_0^\infty d\omega_k/(4\pi\omega_k)\,\mathscr{I}_{\epsilon_{\omega_k}}$, where $\mathscr{I}_{\epsilon_{\omega_k}} = \mathscr{I}^W_{\epsilon_{\omega_k}} + \mathscr{I}^R_{\epsilon_{\omega_k}}$.

- Furthermore, it is now convenient to define $\mathscr{C}_{\mathscr{I}_{\omega_k}} = |\mathscr{I}_{\epsilon_{\omega_k}}| - \sqrt{\mathscr{I}_{A_{\omega_k}}\mathscr{I}_{B_{\omega_k}}}$, as a quantity that signifies concurrence corresponding to a certain field mode frequency[6].

[6] M. O. Scully, S. Fulling, D. Lee, D. N. Page, W. Schleich, and A. Svidzinsky, Proc. Nat. Acad. Sci. 115, 8131 (2018), arXiv:1709.00481.
S. Kolekar, T. Padmanabhan, Phys. Rev. D 89, 064055 (2014), arXiv:1309.4424.

Results:

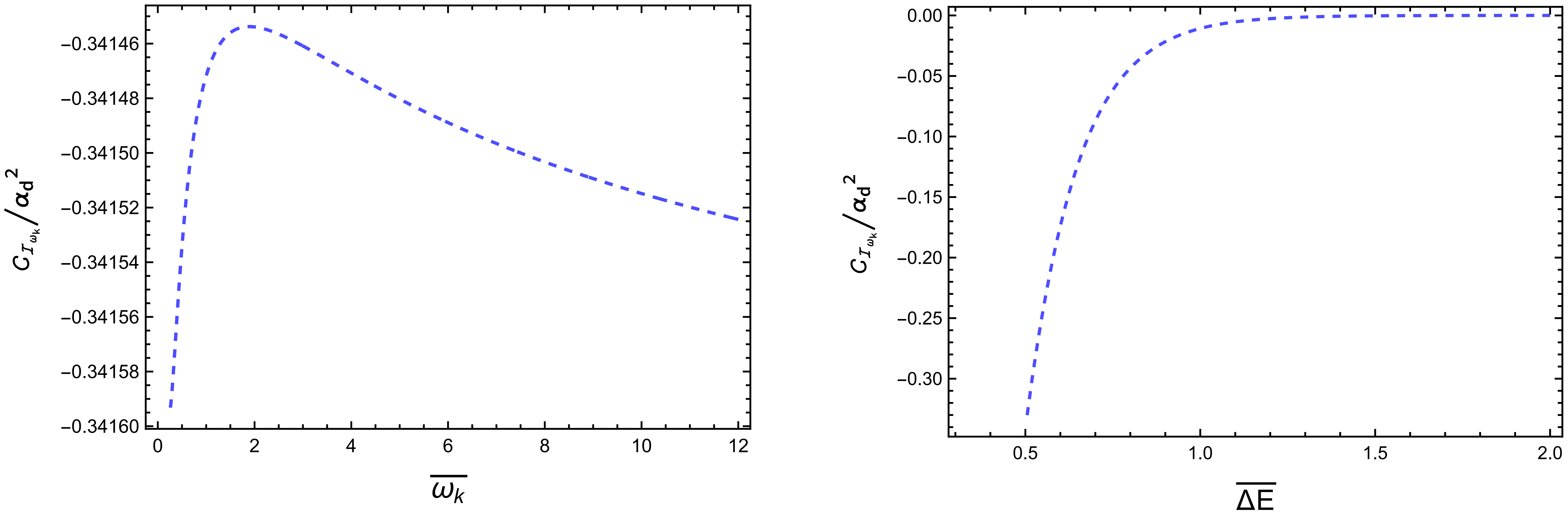


FIG. 2: $\mathscr{C}_{\mathscr{I}_{\omega_k}}/\alpha_d^2$ is plotted as functions of dimensionless frequency $\overline{\omega}_k = \omega_k\,\alpha_d$ and transition energy $\overline{\Delta E} = \Delta E\,\alpha_d$. We have fixed $\overline{\Delta E} = 0.5, \overline{\omega}_k = 0.2$, and $d/\alpha_d = 0$.

- Two co-moving outgoing detectors do not harvest any entanglement.

Results:

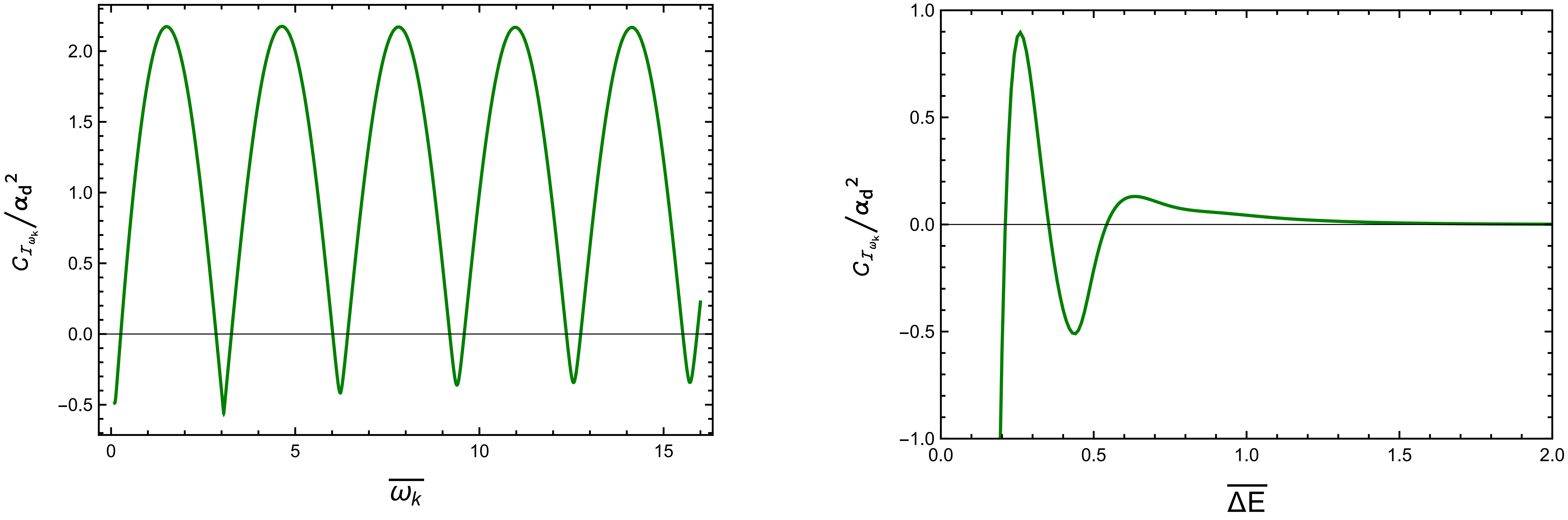


FIG. 3: $\mathscr{C}_{\mathcal{I}_{\omega_k}}/\alpha_d^2$ is plotted as functions of dimensionless frequency $\overline{\omega}_k = \omega_k\, \alpha_d$ and transition energy $\overline{\Delta E} = \Delta E\, \alpha_d$. We have fixed $\overline{\Delta E} = 0.5, \overline{\omega}_k = 0.2$, and $d/\alpha_d = 1$.

- Two outgoing detectors in different null trajectories may harvest entanglement.

Results:

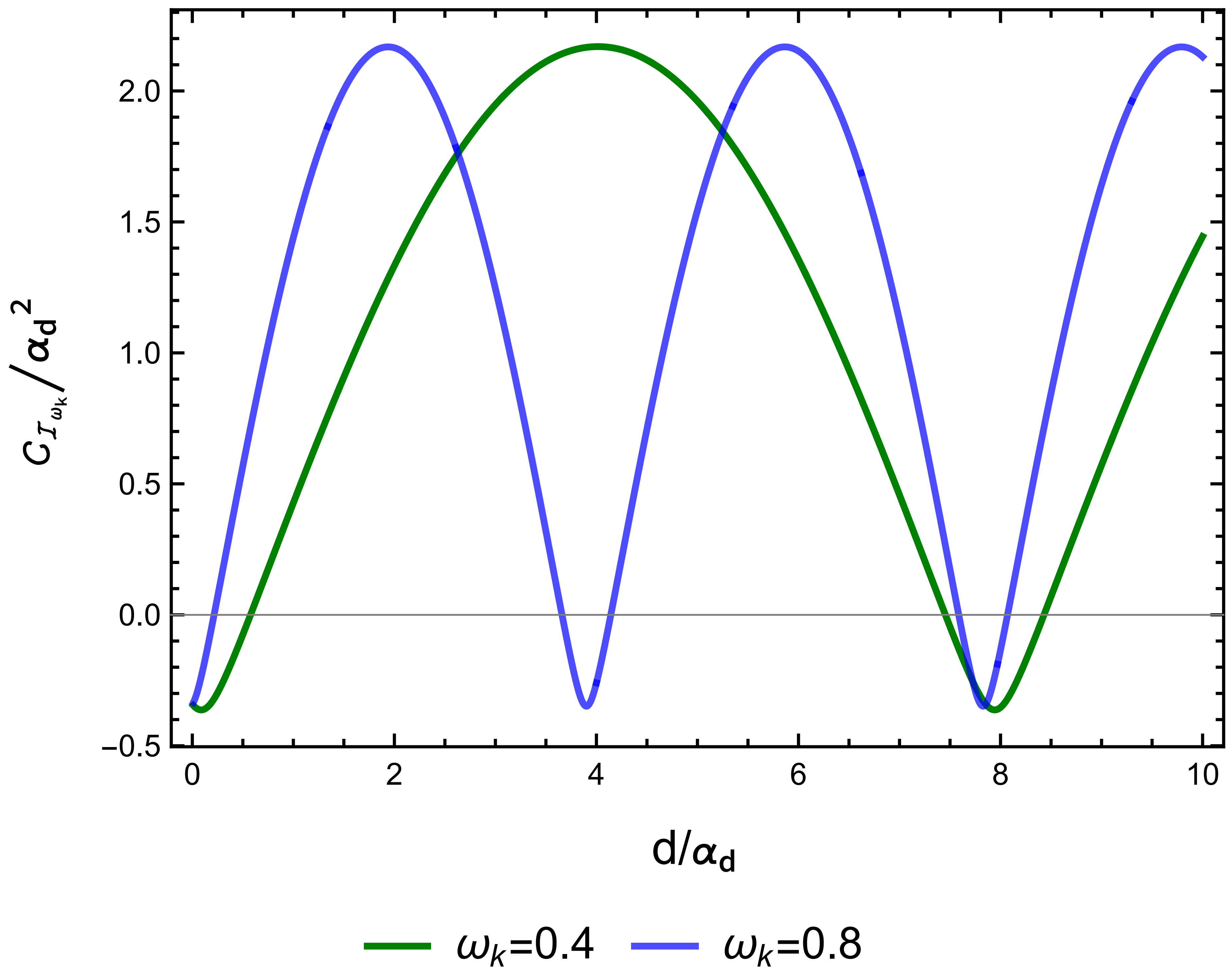


FIG. 4: $\mathscr{C}_{\mathcal{I}_{\omega_k}}/\alpha_d^2$ is plotted as a function of the dimensionless distance d/α_d. We have fixed $\overline{\omega}_k$ and $\overline{\Delta E} = 0.5$.

- Therefore, there are periodic entanglement harvesting shadow regions in $(1+1)$ dimensional de Sitter spacetime with respect to the distance d between the outgoing null trajectories. This shadow region decreases as $\overline{\omega}_k$ increases.

Considered background: (1+1) dimensional Schwarzschild spacetime

- The line-element in (1+1) dimensional Schwarzschild background
$$ds^2 = -\left(1 - \frac{r_H}{r}\right)dt_s^2 + \left(1 - \frac{r_H}{r}\right)^{-1} dr^2 \ ,$$

- In terms of tortoise coordinates $r_\star$ defined as $dr_\star = dr/(1 - r_H/r)$, the metric becomes conformally flat

$$ds^2 = \left(1 - \frac{r_H}{r}\right)\left[-dt_s^2 + dr_\star^2\right] \ .$$

- Outgoing and ingoing null paths are obtained when $u = t_s - r_\star$ or $v = t_s + r_\star$ is constant.

- Scalar field in terms of the Boulware modes expressed in terms of the null coordinates
$$\Phi = \int_0^\infty \frac{d\omega_k}{\sqrt{4\pi\omega_k}}\left[\hat{a}_k^B e^{-i\omega_k u} + \hat{a}_k^{B\dagger} e^{i\omega_k u} + \hat{b}_k^B e^{-i\omega_k v} + \hat{b}_k^{B\dagger} e^{i\omega_k v}\right] \ .$$

(1+1) Schwarzschild spacetime

- The annihilation operators annihilate the Boulware vacuum $|0\rangle_B$, i.e., $\hat{a}_k^B|0\rangle_B = 0 = \hat{b}_k^B|0\rangle_B$.
- Using the expression of the field and these operator actions one gets
$$G_B^+(X_j, X_l) = {}_B\langle 0|\Phi(X_j)\Phi(X_l)|0\rangle_B = \int_0^\infty \frac{d\omega_k}{4\pi\omega_k}\left[e^{-i\omega_k(u_j-u_l)} + e^{-i\omega_k(v_j-v_l)}\right],$$
- For outgoing detector the detector trajectory is defined by the the Eddington-Finkelstein (EF) coordinates (t, r) with $t + r = t_s + r_\star$.
- For outgoing null path $t = r + 2r_H \ln\left[\frac{r}{r_H} - 1\right] + d$.
- We consider for detector A with a non zero d, and for detector B with $d = 0$.
- Then $v_j - v_l = 2(r_j - r_l) + 2r_H \ln\left[\frac{r_j - r_H}{r_l - r_H}\right] + d\,(\delta_{jA}\,\delta_{lB} - \delta_{jB}\,\delta_{lA})$ and $u_j - u_l = d\,(\delta_{jA}\,\delta_{lB} - \delta_{jB}\,\delta_{lA})$.

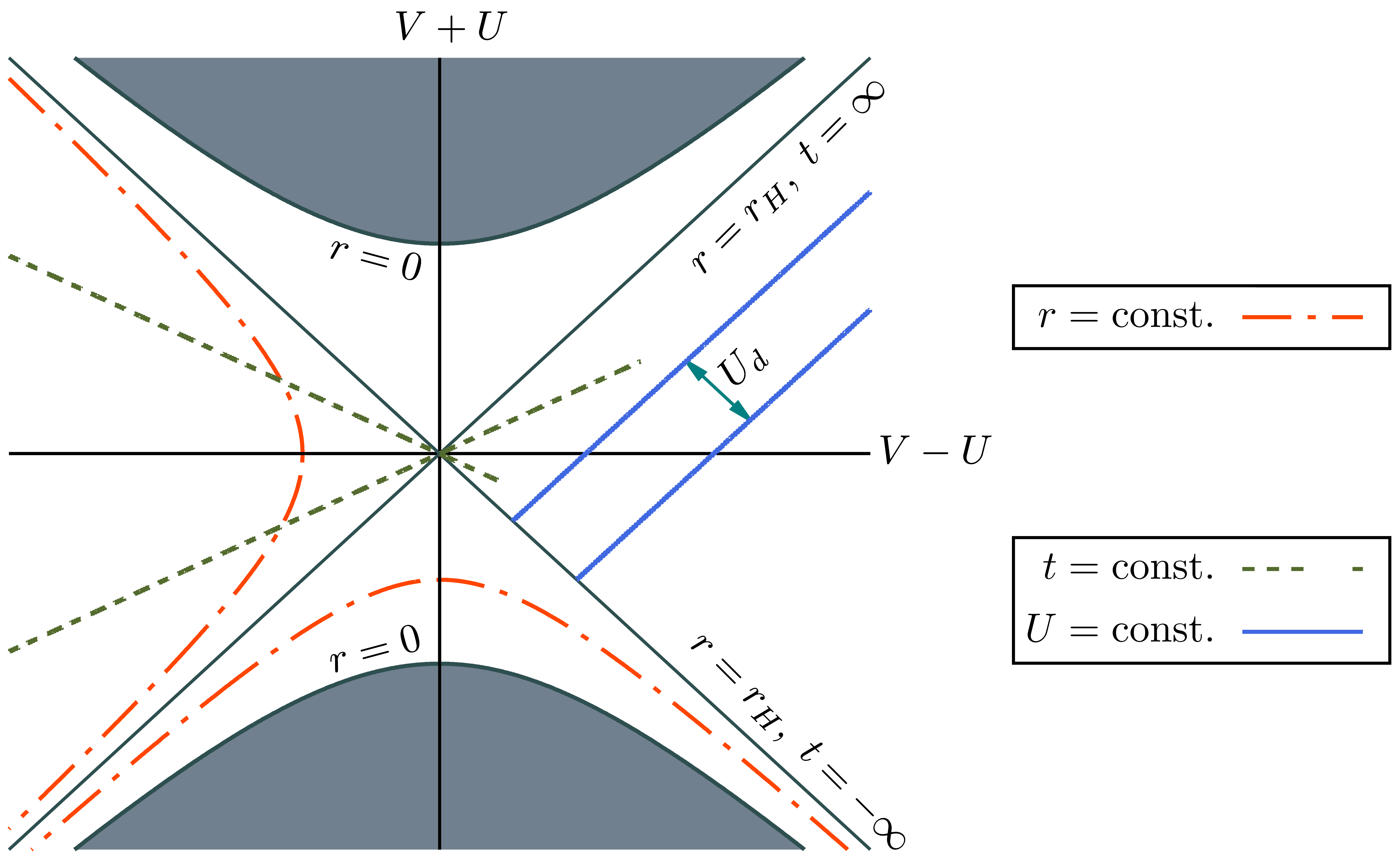


FIG. 5: Schematic representation of two detectors in outgoing null paths in a Schwarzschild background is depicted in a Kruskal diagram. In Eddington-Finkelstein coordinates the two detectors are separated as $u_A - u_b = d$, while in Kruskal coordinates they are separated as $U_d = U_B - U_A = r_H(1 - e^{-d/2r_H})$, where $U_j = -\,2r_H\ e^{-u_j/2r_H}$.

Results:

- Here also we have $\mathcal{I}_j = \int_{-\infty}^{\infty} d\tau_j' \int_{-\infty}^{\infty} d\tau_j \, e^{-i\Delta E^j(\tau_j'-\tau_j)} \, G_W(X_j', X_j) = \int_0^{\infty} d\omega_k/(4\pi\omega_k) \, \mathcal{I}_{j_{\omega_k}}$.

- Local terms in the concurrence $\mathcal{I}_{j_{\omega_k}} = \dfrac{4\pi r_H \omega_k^2}{(\Delta E^j + \omega_k)(\Delta E^j + 2\omega_k)^2} \times \dfrac{1}{e^{4\pi r_H(\Delta E^j + \omega_k)} - 1}$.

- $\mathcal{I}_\epsilon = -\int_0^{\infty} d\omega_k/(4\pi\omega_k) \, \mathcal{I}_{\epsilon_{\omega_k}}$, with $\mathcal{I}_{\epsilon_{\omega_k}} = \mathcal{I}^W_{\epsilon_{\omega_k}} + \mathcal{I}^R_{\epsilon_{\omega_k}}$ and we investigate $\mathcal{C}_{\mathcal{I}_{\omega_k}} = |\mathcal{I}_{\epsilon_{\omega_k}}| - \sqrt{\mathcal{I}_{A_{\omega_k}} \mathcal{I}_{B_{\omega_k}}}$ as concurrence corresponding to a certain field mode frequency.

Results:

- Let us first understand the features of $\mathscr{I}_{j_{\omega_k}}$:

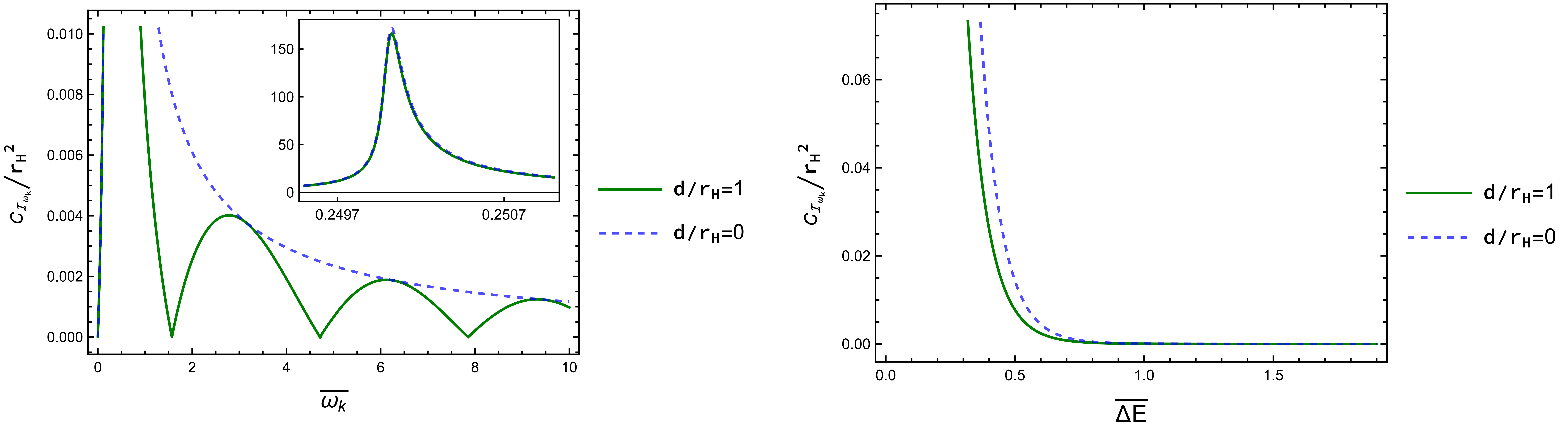
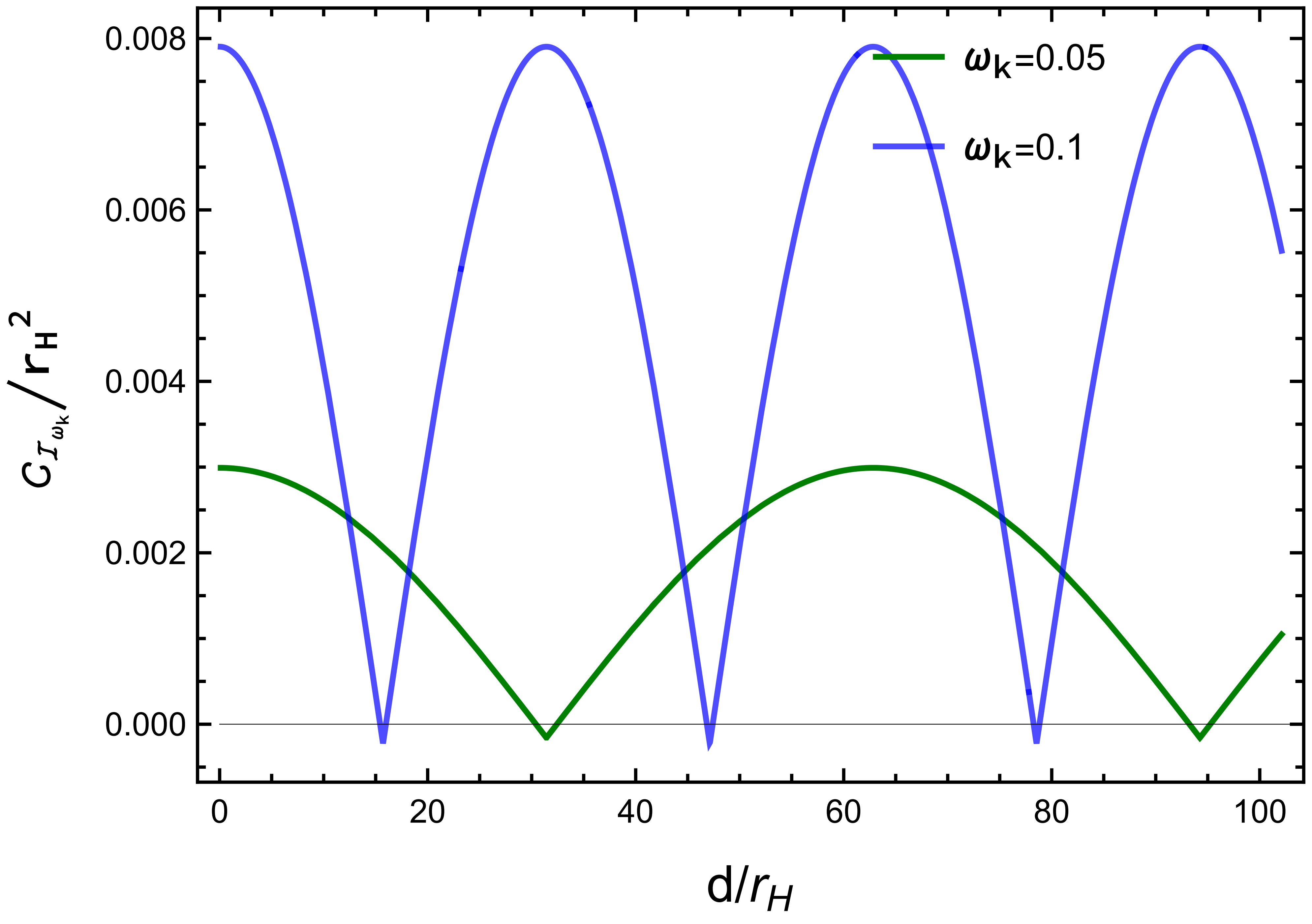


FIG. 7: In left and right $\mathscr{C}_{\mathscr{I}_{\omega_k}}/r_H^2$ is plotted for two outgoing null detectors in a (1+1) dimensional Schwarzschild black hole spacetime with respect to $\overline{\omega}_k$ and the dimensionless transition energy $\overline{\Delta E} = r_H \Delta E$ of the detectors. We have fixed $\overline{\Delta E} = 0.5$ and $\overline{\omega}_k = 1$.

Results:



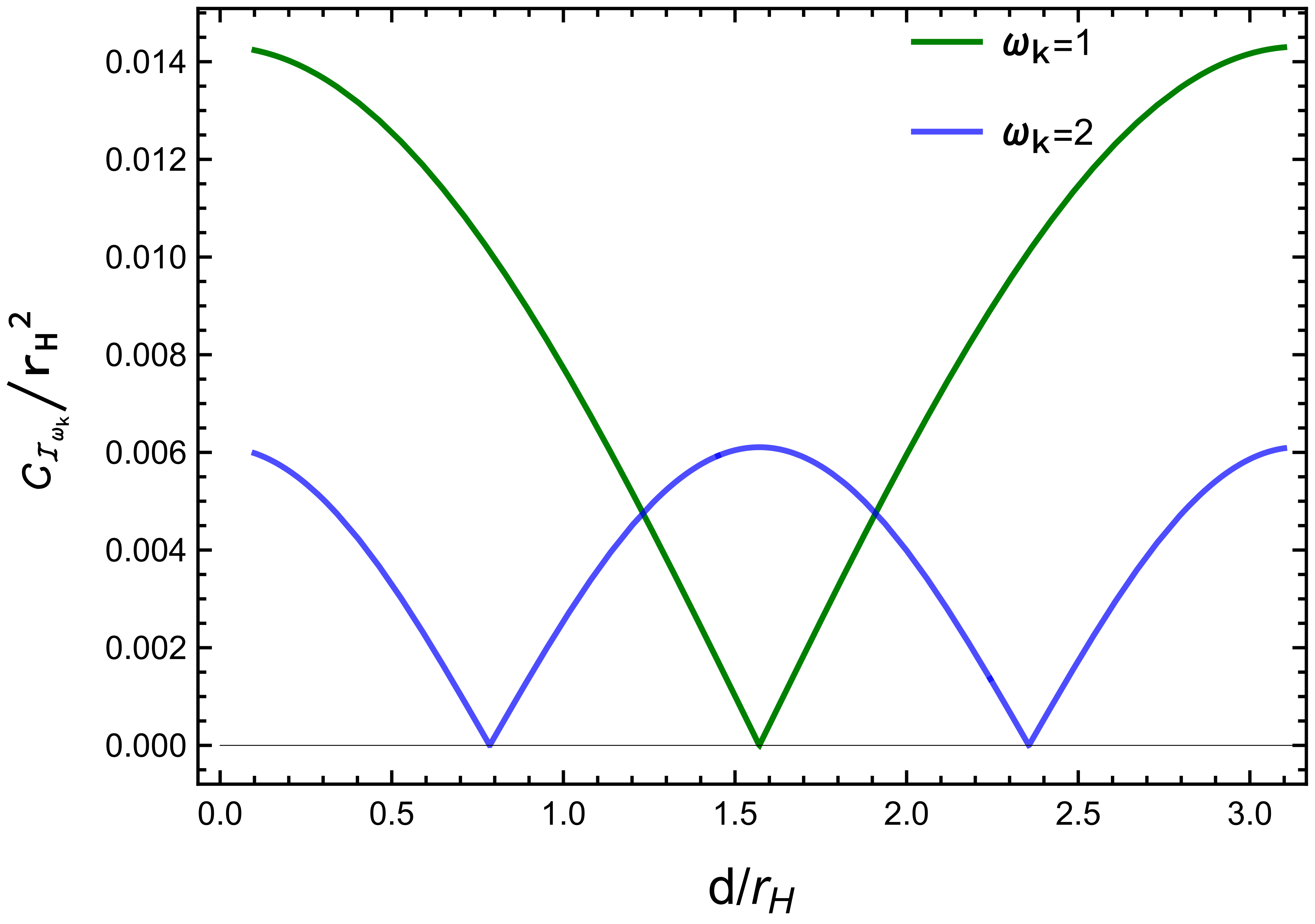


FIG. 8: In left and right $\mathscr{C}_{\mathscr{I}_{\omega_k}}/r_H^2$ is plotted for two outgoing null detectors in a (1+1) dimensional Schwarzschild black hole spacetime with respect to d/r_H. Fixed parameters $\overline{\omega}_k$ and $\overline{\Delta E} = 0.5$. With increasing $\overline{\omega}_k$, shadow region becomes smaller.

Conclusion

Entanglement harvesting: (1+1) de Sitter

- Two Unruh-DeWitt detectors co-moving in outgoing null paths do not harvest any entanglement from the $(1+1)$ dimensional de Sitter background.

- However entanglement harvesting is possible for non zero distance between the two detectors.

- In fact the Harvesting periodically depends on the distance d between the null paths.

- There are periodic entanglement harvesting shadow regions with respect to the distance d and $\overline{\omega}_k$.

- With increasing $\overline{\omega}_k$, this shadow region becomes smaller.

Conclusion

Entanglement harvesting: (1+1) Schwarzschild

- Two Unruh-DeWitt detectors co-moving in outgoing null paths can harvest entanglement from the Boulware vacuum of $(1+1)$ dimensional Schwarzschild background unlike the de Sitter case.

- Here also harvesting periodically depends on the distance d between the null paths.

- There are periodic entanglement harvesting shadow regions with respect to the distance d.

- With increasing $\overline{\omega}_k$, this shadow region becomes smaller and eventually vanishes.

THANK YOU FOR LISTENING!