Infrared Aspects of QFT and Quantum Gravity: Scattering and Coherence

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Princeton University

K.Prabhu, G.S., & R.M. Wald, Phys. Rev. D 106, 066005 (2022) [arXiv:2203.14334]

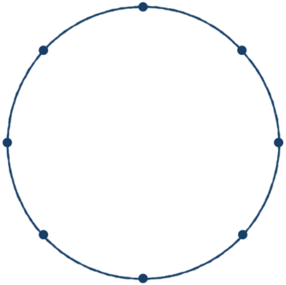
G.S., D. Danielson, & R.M. Wald (in prep.)

K. Prabhu & G.S. (in prep.)

Quantum Effects in Gravitational Fields

August 30, 2023

Gravitational and Electromagnetic Memory Effects



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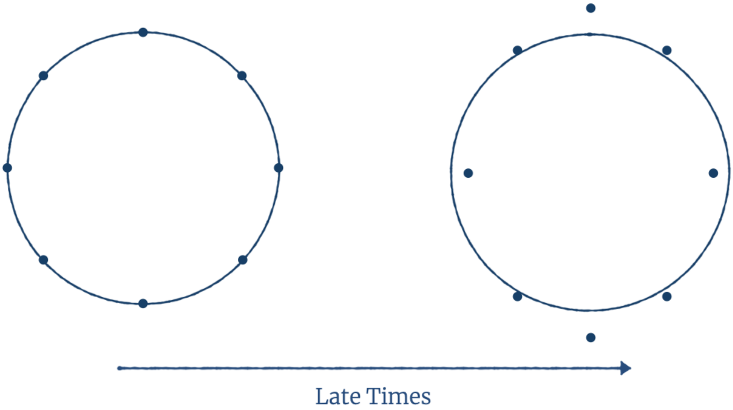

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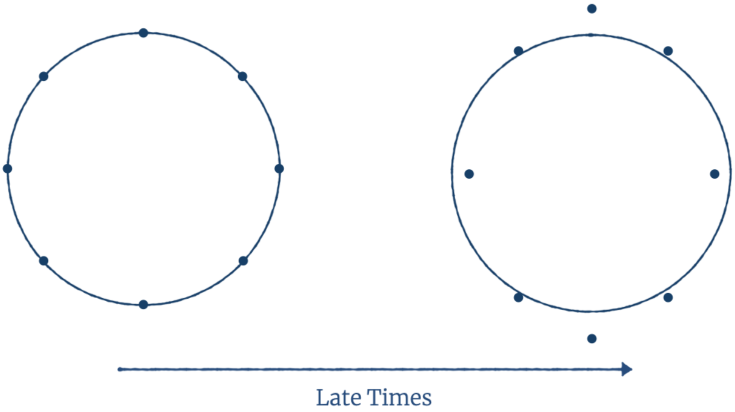
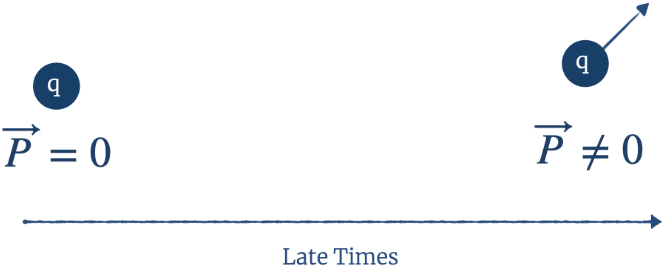


relative displacement

$$\Delta\xi_a = \frac{\Delta_{ab}\xi^b_{(0)}}{r}$$

Gravitational Memory

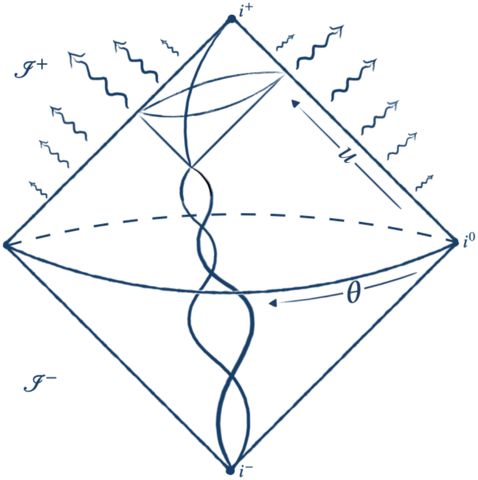
Gravitational and Electromagnetic Memory Effects



Electromagnetic Memory

$$\Delta P_a = \frac{\Delta_a}{r}$$

Classical Scattering, Radiation and Memory



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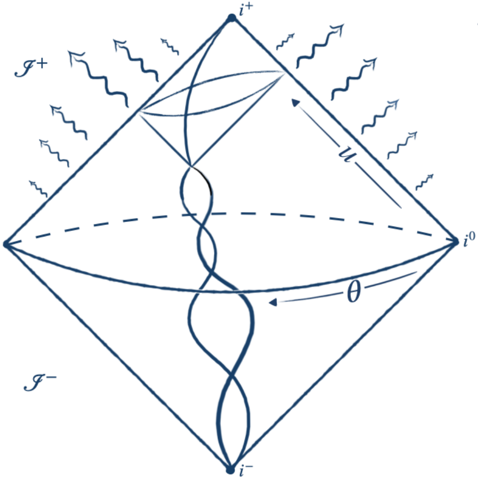
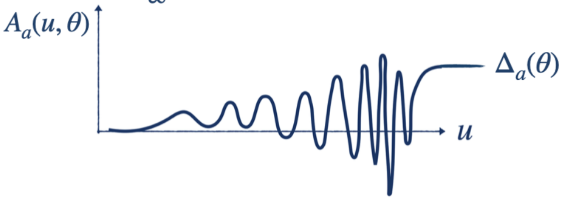


$$\Delta_a(\theta) = -\int_{-\infty}^{\infty} du\; E_a(u,\theta) \qquad E_a(u,\theta) = \partial_u A_a$$



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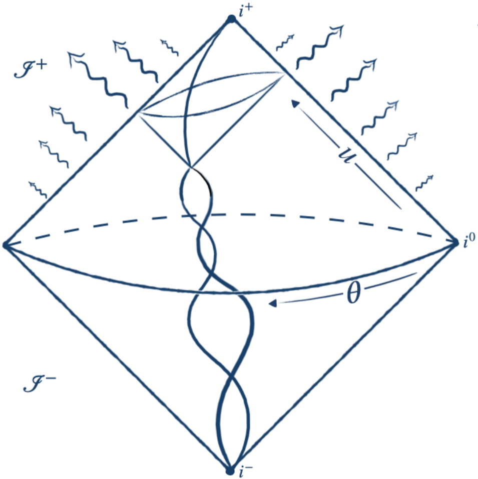

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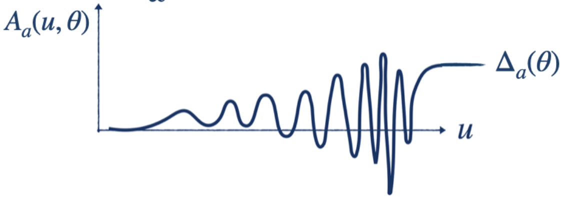

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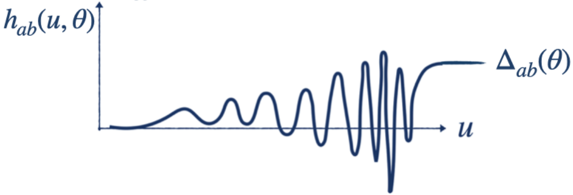
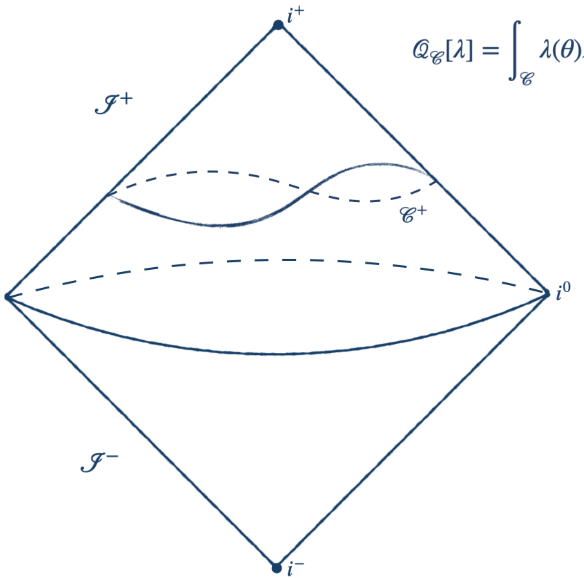


Memory and Charges



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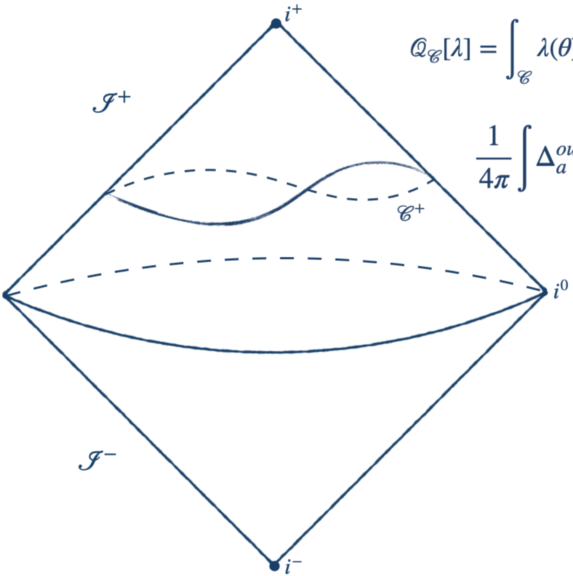

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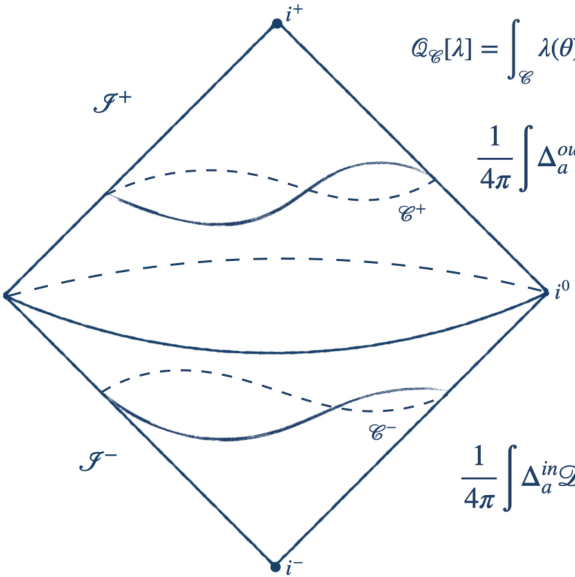
massless charge-current flux

$$\frac{1}{4\pi} \int \Delta^{out}_{a} \mathscr{D}^{a} \lambda \; d\Omega = \mathcal{Q}_{i^+}[\lambda] - \mathcal{Q}_{i^0}[\lambda] + \int_{\mathscr{I}^+} J_{out} \lambda$$



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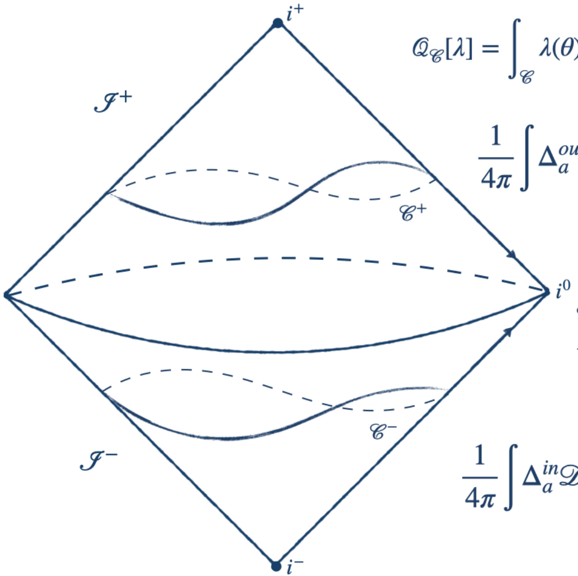

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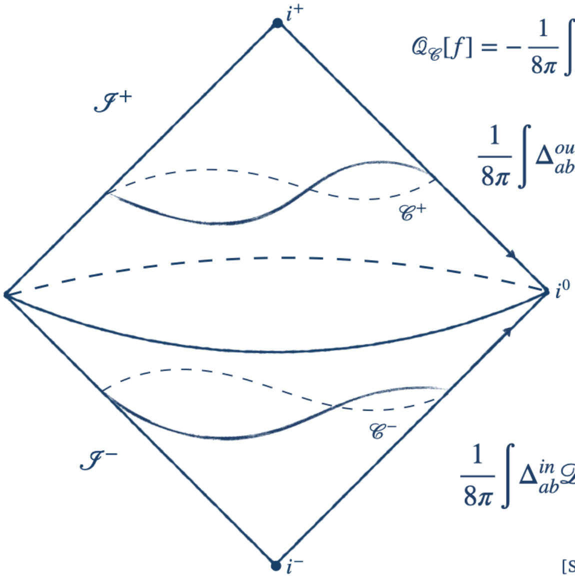
[Campiglia et al. 2017], [Prabhu, 2018] ...



$$\mathcal{Q}_{\mathscr{C}}[f] = -\frac{1}{8\pi}\int_{\mathscr{C}} f(\theta) C^{(3)}_{urur}(u,\theta)\, d\Omega \quad \text{when } N_{AB} = 0$$

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[Strominger et al. 2013], [Troessaert, 2018], [Prabhu, 2018] ...

Fock Quantization

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$$||h||^2 = 16\pi \int_0^\infty \int_{\mathbb{S}^2} d\omega d\Omega \; \omega |\tilde{h}_{ab}(\omega, \theta)|^2$$

where $\tilde{h}_{ab}(\omega, \theta)$ is Fourier transform of $h_{ab}(u, \theta)$.

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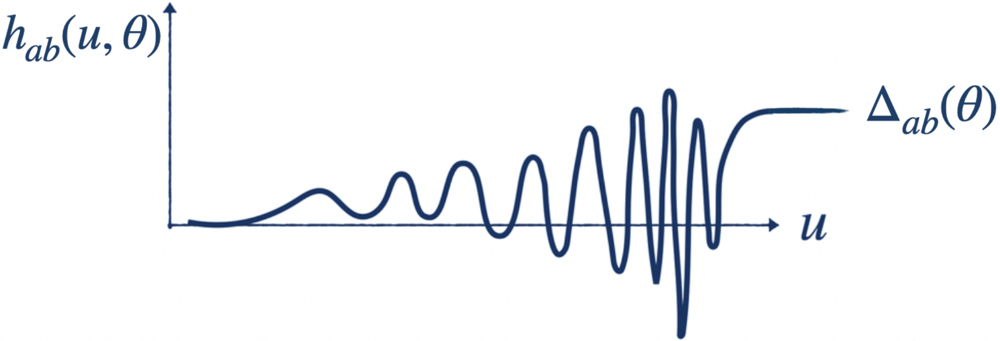

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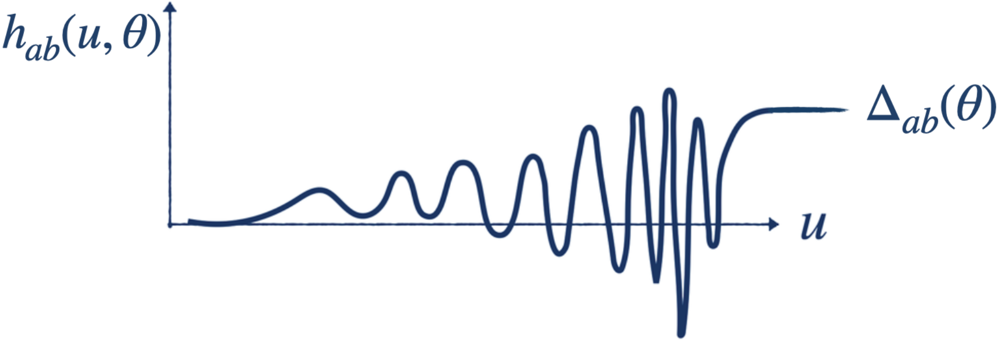

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- There are an *uncountably infinite* number of "in/out" Fock spaces labeled by all possibles memories $\Delta^{\text{in/out}}$. Memory is not conserved. To go beyond "inclusive cross sections" and have a well-defined S-matrix one needs to include states with memory.

Massive QED - Faddeev-Kulish Hilbert Space

$$\mathcal{Q}_{i^0}(\lambda) = \mathcal{Q}_{i^-}(\lambda) - \frac{1}{4\pi}\int_{\mathbb{S}^2} \Delta_a^{\text{in}} \mathscr{D}^a \lambda$$

- Key Idea: The charge at spatial infinity is conserved. Therefore "in" Hilbert space of eigenstates of the charge $\boldsymbol{\mathcal{Q}}_{i^0}(\lambda)$ with eigenvalue $\mathcal{Q}_{i^0}(\lambda)$ will will map to an "out" Hilbert space of eigenstates with eigenvalue $\mathcal{Q}_{i^0}(\tilde{\lambda})$ [Faddeev & Kulish, 70']...

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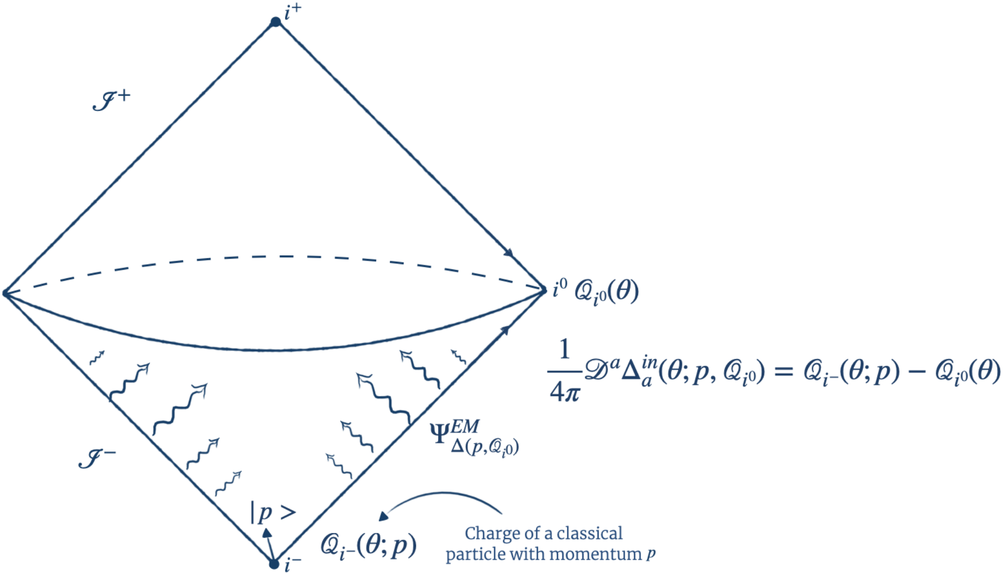

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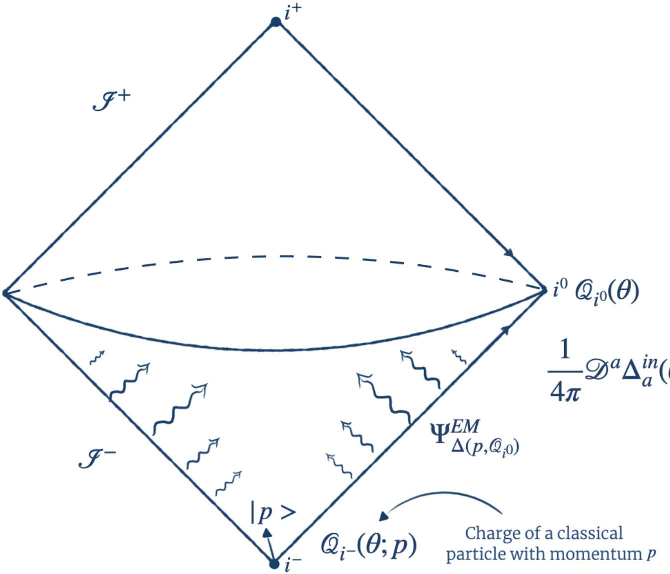

$$\frac{1}{4\pi}\mathscr{D}^a\Delta^{in}_a(\theta;p,\mathcal{Q}_{i^0}) = \mathcal{Q}_{i^-}(\theta;p) - \mathcal{Q}_{i^0}(\theta)$$

$$|p> \otimes \Psi^{EM}_{\Delta(p,\mathcal{Q}_{i^0})}$$

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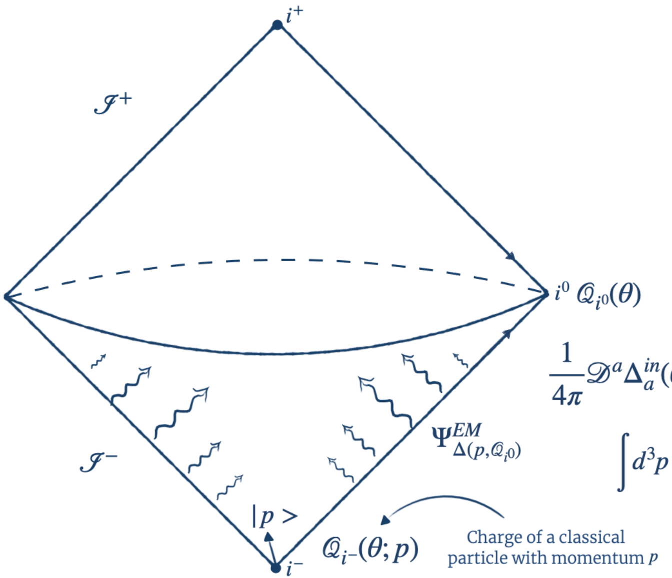

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- This construction fails in *all* other theories including quantum gravity.

Vacuum Gravity - Failure of Faddeev-Kulish Hilbert Space

$$\mathcal{Q}^{\mathrm{GR}}_{i^0}(f) = -\frac{1}{8\pi}\int_{\mathbb{S}^2} \Delta^{\mathrm{in}}_{ab}\mathscr{D}^a\mathscr{D}^b f(\theta) + \int_{\mathscr{I}^-} f(\theta) N^2$$

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There does not appear to be *any* "preferred" Hilbert space for scattering in QG ("Non-Faddeev-Kulish" representations also fail)

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- Given any state $|\Psi\rangle$ in a Hilbert space $\mathscr{H}$ one can express that state as a list of correlation functions of operators in an Algebra $\mathscr{A}$. For example,

$$\langle\phi(x)\rangle_\Psi\,, \langle\phi(x_1)\phi(x_2)\rangle_\Psi\,, \ldots, \langle\phi(x_1)\ldots\phi(x_n)\rangle_\Psi\,, \ldots$$

  Conversely, given a list of correlation functions on $\mathscr{A}$ (satisfying commutation relations, positivity, ...) one can construct (by GNS) a Hilbert space where this list of correlation functions is packaged as a vector $|\Psi\rangle \in \mathscr{H}$. Thus viewing a state as a list of correlation functions on $\mathscr{A}$ or as a vector in a Hilbert space are essentially equivalent. [Witten, 2022],[Hollands & Wald, 2014]

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- However, by considering states as lists of correlation functions one is now freed from choosing in advance a particular Hilbert space!

Algebraic Scattering Theory

- Given a set of correlation functions Ψ_{in} on the "in" algebra $\mathscr{A}_{\text{in}}$ (i.e. thus specifying the "in" state) then what is the expected value of any "out" observable in $\mathscr{A}_{\text{out}}$ (which would then specify the "out" state)?

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- The (perturbative) formulation of algebraic scattering theory for a massive scalar field coupled to a massless scalar field can be straightforwardly constructed and one can compute the correlation functions of any "out" observables (fields, memory, charges, ...) to any order in perturbation theory [**G.S.**, K. Prabhu, in prep.].

Bad Things Happen to “Good” Scattering Data

- In any gauge theory, the charges $\mathcal{Q}_{i^0}(\lambda)$ have serious implications for coherence. This ultimately comes from the charges “superselect”. Any local gauge invariant observable $\mathcal{O}$ commutes with all of the charges

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- A more familiar case of superselection is the total electric charge $\mathcal{Q}(1)$. Given states Ψ_{q_1} and Ψ_{q_2} with total charge q_1 and q_2, a standard argument shows that the matrix element $\langle \Psi_{q_1} | \boldsymbol{O} | \Psi_{q_2} \rangle$ for any local gauge invariant observable $\boldsymbol{O}$ must vanish

$$\langle \Psi_{q_1} | [\mathcal{Q}(1), \boldsymbol{O}] | \Psi_{q_2} \rangle = (q_1 - q_2) \langle \Psi_{q_1} | \boldsymbol{O} | \Psi_{q_2} \rangle = 0$$

  Therefore, if $q_1 \neq q_2$ then $\langle \Psi_{q_1} | \boldsymbol{O} | \Psi_{q_2} \rangle = 0$. In other words, for any local gauge invariant observable $\boldsymbol{O}$, a superposition of Ψ_{q_1} and Ψ_{q_2} is an *incoherent* superposition — these states cannot interfere.

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- An incoming electron Ψ_f has definite total electric charge but is a superposition of incoming momenta and therefore superposition of large gauge charges.

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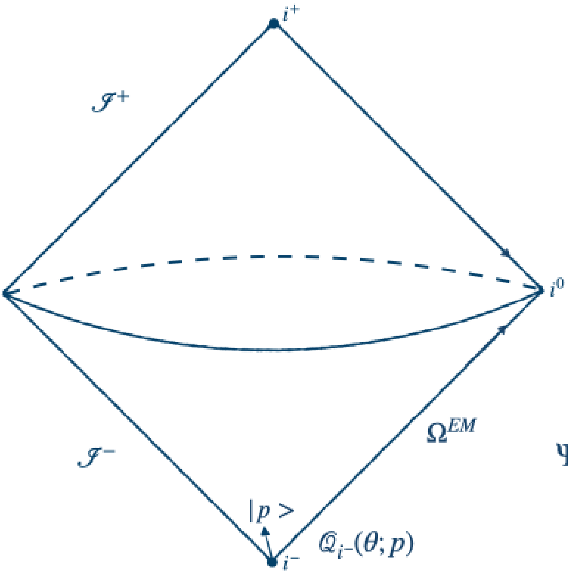

$$\mathcal{Q}_{i^0}(\theta) = \mathcal{Q}_{i^-}(\theta; p)$$

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The expected value $\langle \boldsymbol{O}(x) \rangle_{\Psi_f}$ is spacetime translation invariant for any gauge invariant observable $\boldsymbol{O}$ *[D. Danielson,* ***G.S.*** *& R. M. Wald, in prep.]*

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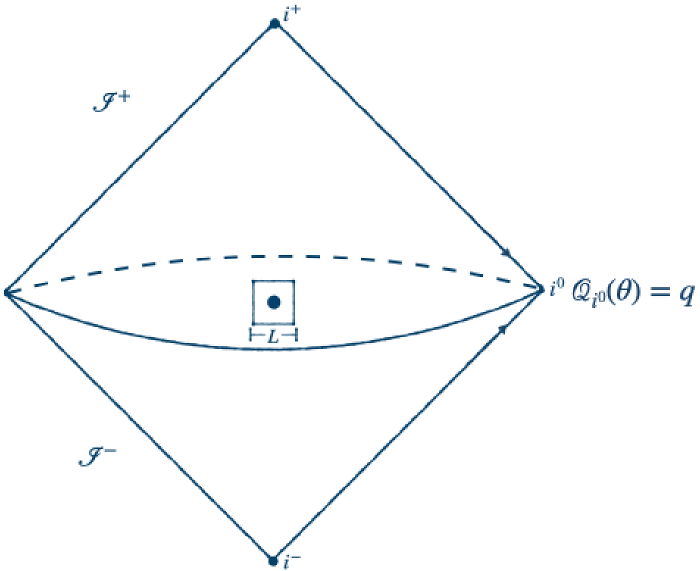

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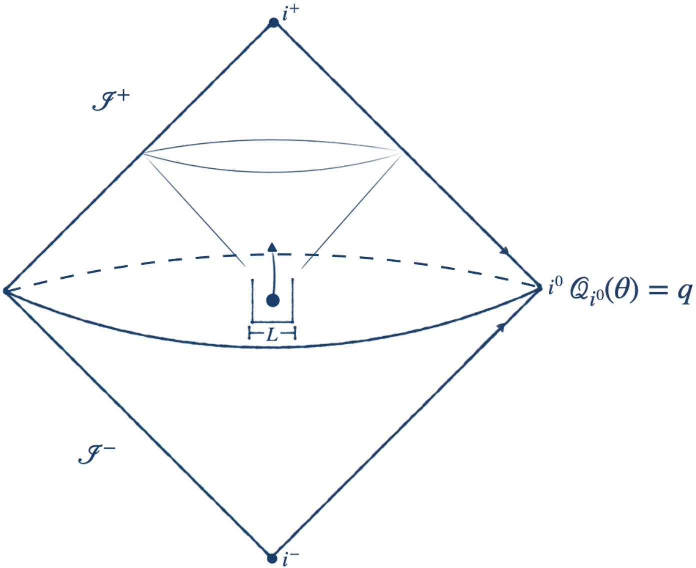

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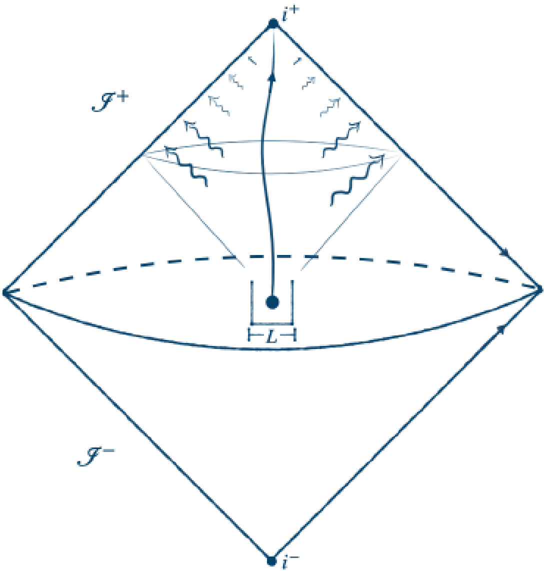

$$\Psi_{FK} = \int d^3p\, f(p)\,|p> \otimes \Psi^{EM}_{\Delta(q,p)}$$

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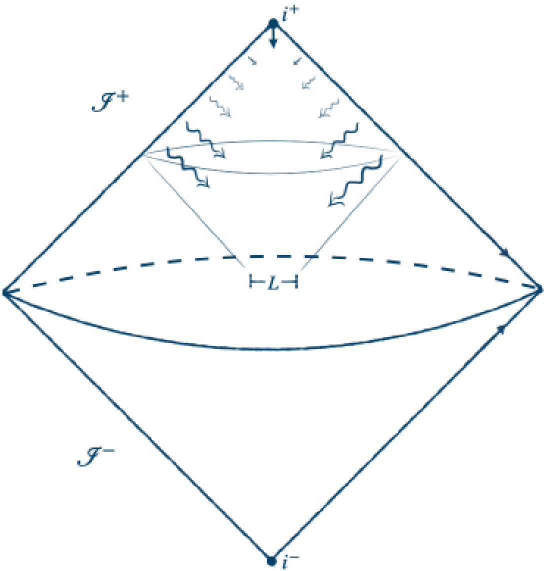

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- Physical states can be obtained by starting with a localised electron in the bulk.

- The (extremely fine tuned!) entanglement and absorption of “soft radiation” of the FK state is responsible for the localization of the electron in the bulk!

Summary

- IR divergences arise from sticking a state in a Hilbert space to which it doesn't belong.
- In massive QED the Faddeev-Kulish representation is a preferred representation but, as opposed to a "proof of principle" it is actually a "fluke"!
- Non-Kulish-Faddeev representations don't work
- A well-defined (IR-finite) scattering theory can be constructed by simply evolving "in" correlation functions to "out" correlation functions.
- Due to the infrared properties of the theory, the space of asymptotic states in QED (or Yang Mills) which correspond to physical "bulk" states are highly fine-tuned and there are many states that are "junk"!

Failure of FK: Massless QED and Linearized Gravity

$$\mathcal{Q}_{i^0}(\lambda) = \mathcal{J}(\lambda) - \frac{1}{4\pi}\int_{\mathbb{S}^2} \Delta^{\text{in}}_a \mathscr{D}^a \lambda$$

- In massless QED, the analogous construction is to pair eigenstates of the incoming charge-current flux with memory. However, the eigenvalue is now a δ-function on $\mathbb{S}^2$. The required "dressings" have "collinear divergences" and therefore have infinite energy! All "FK states" are unphysical except the vacuum

  [Kinoshita,'62],[Lee & Nauenberg, '64]

$$\mathcal{Q}^{\text{GR}}_{i^0}(f) = -\frac{1}{8\pi}\int_{\mathbb{S}^2} \Delta^{\text{in}}_{ab}\mathscr{D}^a\mathscr{D}^b f(\theta) + \int_{\mathscr{I}^-} f(\theta) T_{vv}(v,\theta)$$

- In linearized quantum gravity one can again repeat the FK construction. [Akhoury & Choi, 2017] In this case there are no collinear divergences so the "dressings" are not singular. However, we cannot set $\mathcal{Q}^{\text{GR}}_{i^0} = 0$ since this would set the total four-momenum to zero! (*Can't hide mass behind the moon!*) All "FK states" have undefined angular momentum except the vacuum

Memory Representations

- States with memory are perfectly legitimate states and a Hilbert space of states with memory Δ_{ab} can be constructed by starting with $\mathscr{F}_0^{\mathscr{I}}$ and performing the field redefinition:

$$\boldsymbol{N}_{ab}(u,\theta) \to \boldsymbol{N}_{ab}(u,\theta) + N_{ab}(u,\theta)\mathbf{1} \text{ where } \int_{-\infty}^{\infty} du\; N_{ab}(u,\theta) = \Delta_{ab}(\theta)$$

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$$\boldsymbol{\Delta}_{ab}(\theta)\left|\Psi_\Delta^{\mathscr{I}}\right\rangle = \Delta_{ab}(\theta)\left|\Psi_\Delta^{\mathscr{I}}\right\rangle \quad \forall \left|\Psi_\Delta^{\mathscr{I}}\right\rangle \in \mathscr{F}_\Delta^{\mathscr{I}}$$

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- There are an *uncountably infinite* number of of "in" and "out" Fock spaces $\mathscr{F}_\Delta^{\mathscr{I}^\pm}$ labeled by the "in/out" memory $\Delta^{in/out}$. The memory is *not* conserved and so the "standard" S-matrix does not exist! Need to include states with memory.