THE VACUUM STRIKES BACK: BLACK STARS

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Meditations
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Quantum effects in gravitational fields

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INTRODUCTION

Black holes (BHs) are the most well-accepted candidates for the dark and compact objects observed. Among the reasons:

- **Observational fits (EHT, GWs...)**
- **Theoretical arguments (Penrose theorem, maximum mass...)**

Buchdahl limit

There are no regular stars beyond certain compactness $\left(\frac{2M_R}{R} < \frac{8}{9}\right)$

Can semiclassical effects overcome this limit?

SEMICLASSICAL GRAVITY

Theory that takes into account the contribution of zero-point energies of quantum fields on a classical spacetime

$$G_{\mu\nu} = 8\pi \left(T_{\mu\nu} + \langle \hat{T}_{\mu\nu} \rangle \right)$$

Effectively classical SET *Expectation value* ***in vacuum*** *of the RSET*

- **The RSET is a function of the metric, field modes, and their derivatives**
- **It encodes both vacuum polarization and particle creation effects**
- **We search for self-consistent solutions**

SEMICLASSICAL GRAVITY

Q1: How do stars polarize the vacuum?

Obtaining accurate RSETs in *fixed backgrounds*

- Exact results for many field parameters
- Computationally expensive
- Only perturbative backreaction

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Q1: How do stars polarize the vacuum?

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Q2: How does the vacuum backreact on stars?

Searching for approximate analytic RSETs and find their *backreaction*

- Full self-consistent backreaction
- Broad range of application
- Accuracy of approximations unclear

The approach we adopt

SEMICLASSICAL GRAVITY

Assumptions:

- **Spherical symmetry, staticity, asymptotic flatness**

Misner-Sharp mass

Redshift function

$$ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2 d\Omega^2 \qquad C(r) = 1 - h(r)^{-1} = \frac{2m(r)}{r}$$

Compactness function

- **We model the SET as an isotropic perfect fluid with $\rho(r) \equiv \rho = \text{const}$.**

  Saturates the Buchdahl limit

- **Massless scalar field in the *Boulware vacuum***

  The natural vacuum for stars

We developed two approximations…

Regularized Polyakov RSET

Dimensional reduction

Order-Reduced AHS-RSET

Perturbative reduction of order

…based on different physical principles

THE REGULARIZED POLYAKOV RSET

Relate the 2D RSET of a massless scalar with a 4D RSET →

$$\langle \hat{T}^{\mu}_{\nu} \rangle^{\rm P} = F(r)\delta^{\mu}_{a}\delta^{b}_{\nu}\langle \hat{T}^{a}_{b} \rangle^{\rm 2D} + \langle \hat{T}^{\mu}_{\nu} \rangle_{\rm AC}$$

The components are

$$\langle \hat{T}^{t}_{t} \rangle^{\rm P} = \frac{F}{96\pi h}\left[\frac{2f'h'}{fh} + 3\left(\frac{f'}{f}\right)^2 - \frac{4f''}{f}\right]$$

$$\langle \hat{T}^{r}_{r} \rangle^{\rm P} = -\frac{F}{96\pi h}\left(\frac{f'}{f}\right)^2$$

$$\langle \hat{T}^{\theta}_{\theta} \rangle^{\rm P} = -\frac{(2F+rF')}{192\pi h}\left(\frac{f'}{f}\right)^2$$

We must regularize this RSET

$$F = 1/r^2, \qquad r \geq r_{core}$$

$$F = F_{reg}, \qquad r < r_{core}$$

We match the solution for $r > r_{core}$ with an analytic pressure ansatz for $r < r_{core}$

[Sci. Rep. 12 (2022) 1, 15958]

THE REGULARIZED POLYAKOV RSET

We find whole families of F_{reg} compatible with regular super-Buchdahl stars

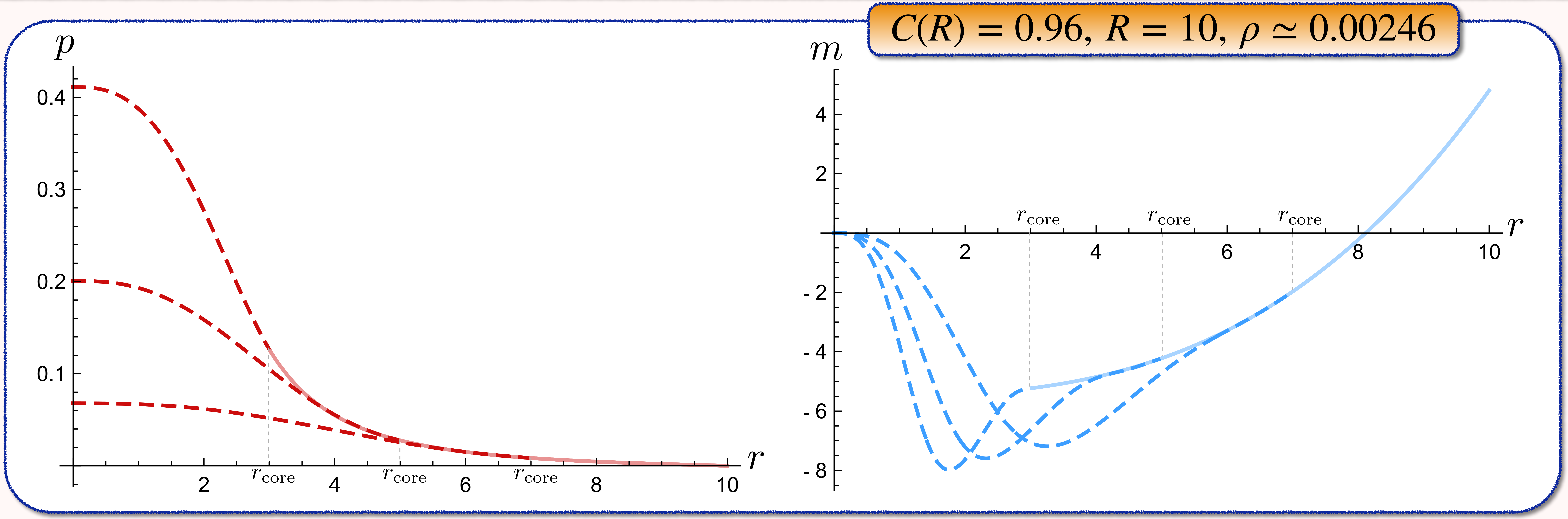

Fig: Regular super-Buchdahl semiclassical stars

- **These exist for any r_{core} and for a simple polynomial pressure**
- **The negative mass interior (vacuum polarization) supports the structure**

THE REGULARIZED POLYAKOV RSET

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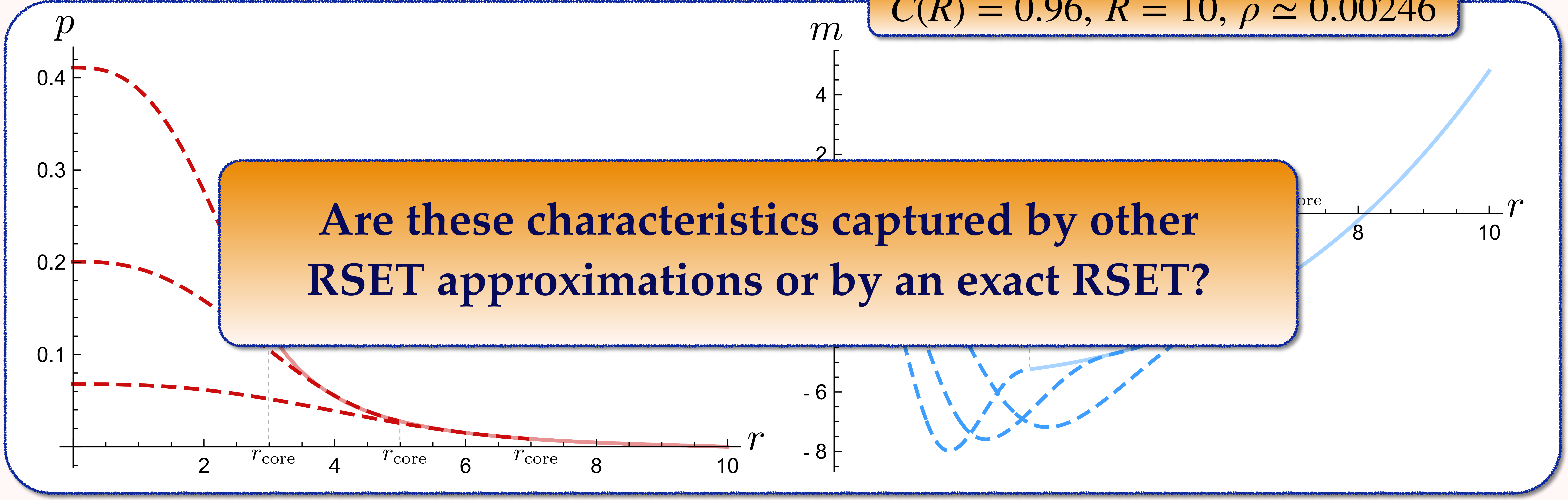
$C(R) = 0.96,\ R = 10,\ \rho \simeq 0.00246$



Are these characteristics captured by other RSET approximations or by an exact RSET?

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ORDER-REDUCED SEMICLASSICAL GRAVITY

Anderson-Hiscock-Samuel (AHS) obtained the RSET of a scalar field in four dimensions *[AHS1995]*

$$\langle \hat{T}_{\mu\nu} \rangle = \langle \hat{T}_{\mu\nu} \rangle^{\text{AHS}} + \langle \hat{T}_{\mu\nu} \rangle^{\text{num}}$$

- **The AHS-RSET exhibits higher-order derivatives of the metric**
- **Higher-derivatives introduce large number of boundary conditions and spurious solutions**

We apply a perturbative order reduction to the AHS-RSET

ORDER-REDUCED SEMICLASSICAL GRAVITY

Let us derive a Matter-Order-Reduced RSET for constant density fluid spheres

$$\frac{h(1-h)-rh'}{h^2r^2} = -8\pi\rho + \mathcal{O}(\hbar) \quad \rightarrow \quad h^{(n)} = \mathcal{H}_n\left(h,f,p,\rho,r\right) + \mathcal{O}(\hbar)$$

$$\frac{rf'+f-fh}{fhr^2} = 8\pi p + \mathcal{O}(\hbar) \quad \rightarrow \quad f^{(n)} = \mathcal{F}_n\left(h,f,p,\rho,r\right) + \mathcal{O}(\hbar)$$

where we have used:

- $\nabla_\mu T^\mu_r = p' + \frac{f'}{2f}\left(\rho + p\right) = 0$
- $\rho(r) =$ **const.**
- $\nabla_\mu \langle \hat{T}^\mu_\nu \rangle = 0$

$$\langle \hat{T}_{\mu\nu} \rangle^{\mathrm{AHS}} \rightarrow \langle \hat{T}_{\mu\nu} \rangle^{\mathrm{MOR}}$$

The MOR-RSET is regular at $r=0$, unambiguous and valid for all couplings

ORDER-REDUCED SEMICLASSICAL GRAVITY

As $C(R)$ is increased, a *negative mass interior* emerges

$C(R) = \{0.89, 0.9, 0.93, 0.96\}$

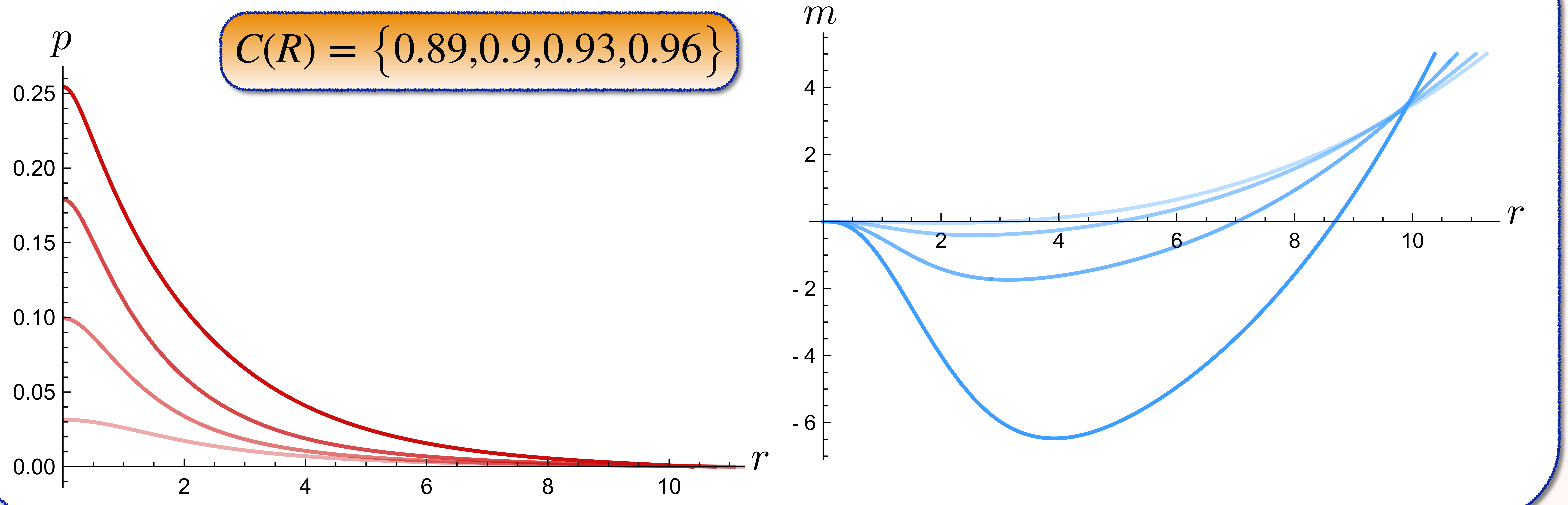

Fig: Classical pressure and Misner-Sharp mass of semiclassical stars surpassing the Buchdahl limit

Similarity with results *à la Polyakov* is striking

ORDER-REDUCED SEMICLASSICAL GRAVITY

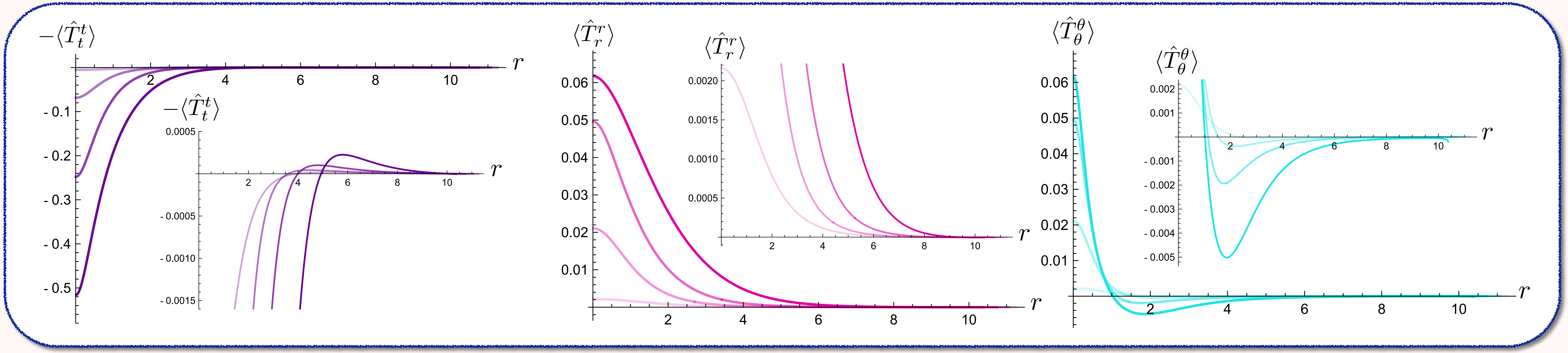

Fig: MOR-RSET of semiclassical stars surpassing the Buchdahl limit

The magnitude of the components is comparable to the classical SET

The MOR-RSET allows to explore non-minimal coupling

Future work…

SEMICLASSICAL BLACK STARS

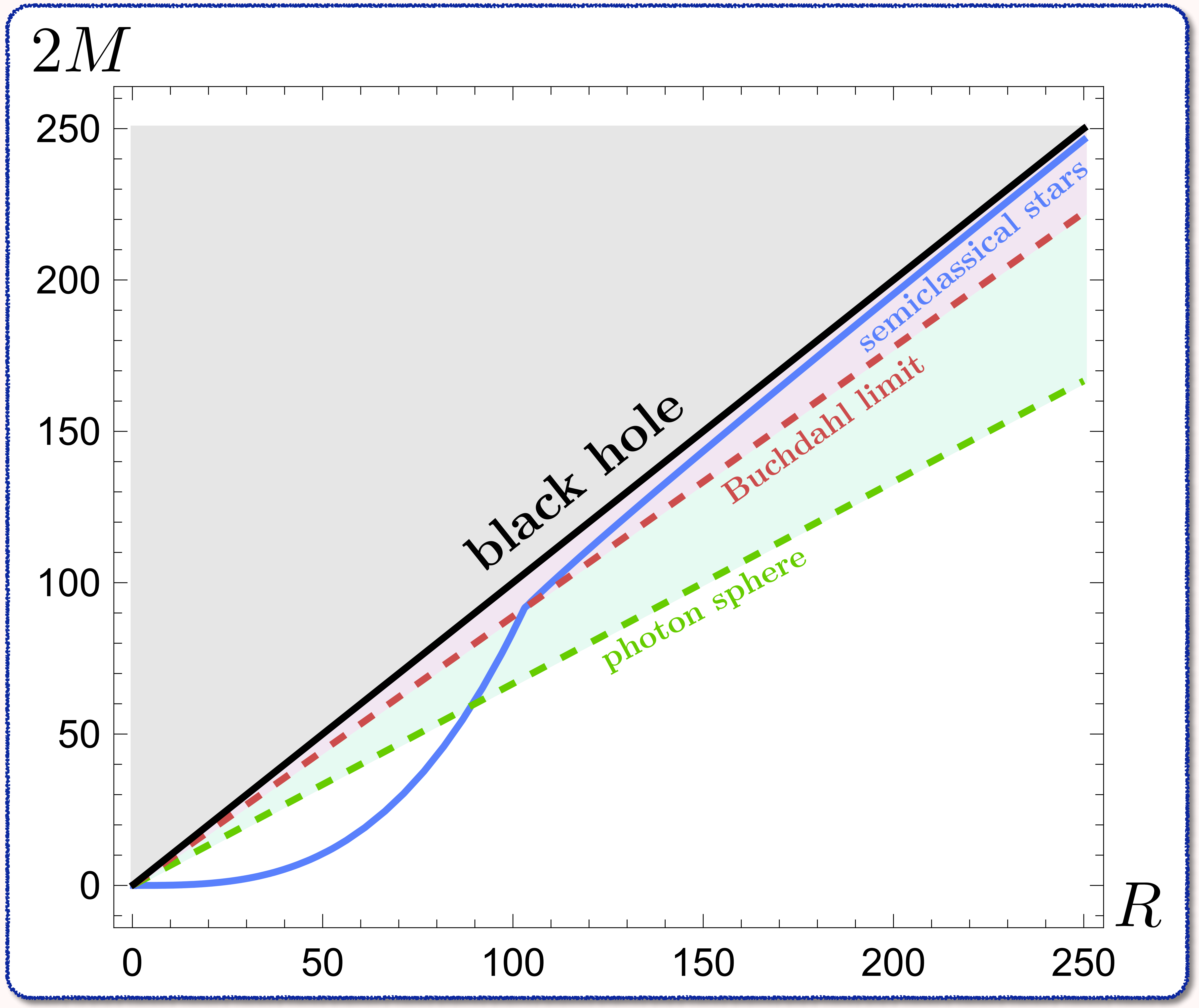

Fig: Mass-to-Radius diagram of semiclassical stars

Three regimes:

- **Sub-Buchdahl: perturbatively corrected constant-density stars**
- **Buchdahl: negative energies build up near the center and support the structure**
- **Super-Buchdahl: stars with negative mass and large redshift interiors**

CONCLUSIONS

The key property that allows to surpass the Buchdahl limit is the *negative energy densities* generated at the interior

- Buchdahl theorem requires the total energy density to decrease outwards
- Once this assumption is broken, there is no further compactness bound
- This phenomenon is predicted by two independent modelings of the RSET

Thank you for your attention

Image credit:
Walking your path
Alina Mir
alinamir.com