w/ Y. Kaku, A. Matsumura & Y. Michimura

based on arXiv: 2308.14552

Tomohiro Fujita
(Waseda U & Tokyo. U)

Aug 30th @ Leipzig

Towards the Optimal Experiment of Gravity-induced Quantum Entanglement

Outline

1. Introduction
2. Previous Proposals
3. General Analysis
4. Our Proposal
5. Summary

Outline

1. Introduction
2. Previous Proposals
3. General Analysis
4. Our Proposal
5. Summary

Key Question

Is Gravity Quantum?

Physics celebrities said...

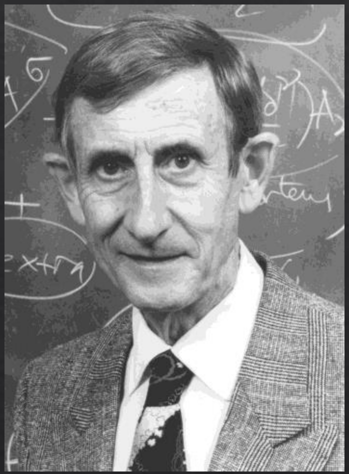
Ricard Feynman

"Maybe we should not try to quantize gravity."

Roger Penrose

"Quantum theory fits most uncomfortably with the curved space-time notion of the general relativity."

Freeman Dyson



"Should quantum mechanics and GR be unified?
I don't think so.
Maybe, they should not be unified..."

Key Question

Is Gravity Quantum?

We aren't sure.

Common sense?

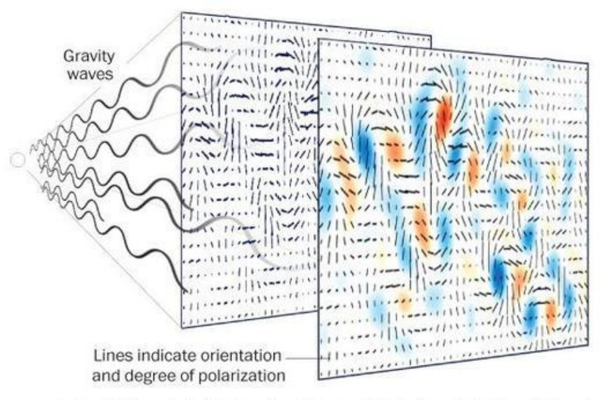
Nakayama+[0804.1827]

We often take **Quantum Gravity** for granted.

① Grav. fields can be in quantum superposition

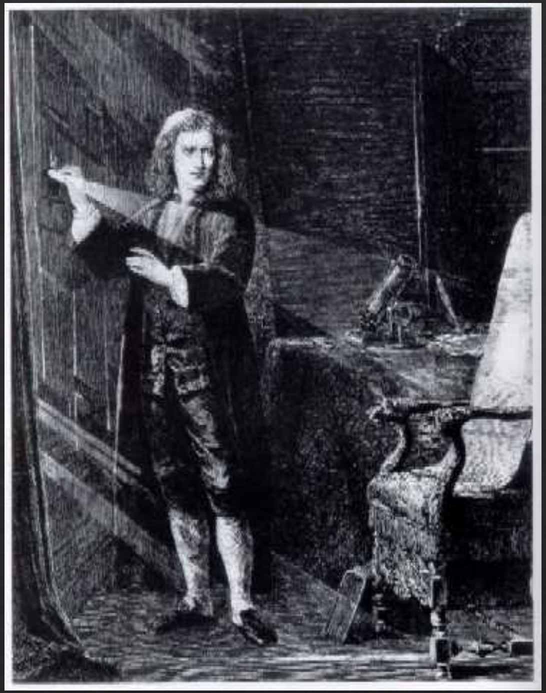
② Graviton are quantized like QED.

(As a cosmologist, I often assume ② in my work)



Their validity has **never** been confirmed.

That’s science



Let’s test it with experiments!

Key Question

"Is Gravity Quantum?"

⇩ Testable

① Do weak gravitational fields become quantum superposition?

② graviton is (far) future step.

Key Question

Is Gravity Quantum?

We aren't sure.

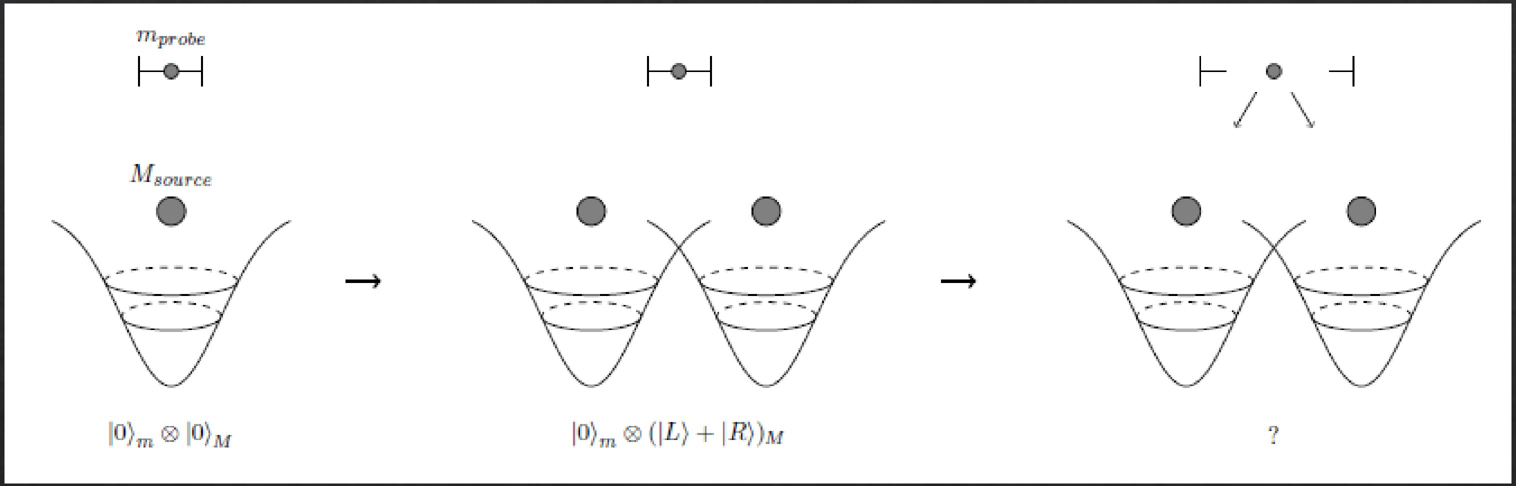
Let's test it!

Outline

1. Introduction
2. Previous Proposals
3. General Analysis
4. Our Proposal
5. Summary

Sketch of idea

Carney+[1807.11494]



Does quantum superposition of a source mass lead to the superposition of gravitational fields?

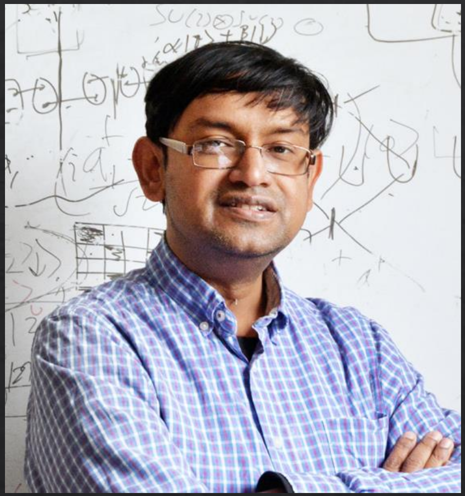
⇨ We can check it with entanglement.

Proposers

Bose+ PRL119.240401(2017)

Marletto&Vedral PRL119.240402(2017)

Sougato Bose et al.



Chiara Marletto

Vlatko Vedral

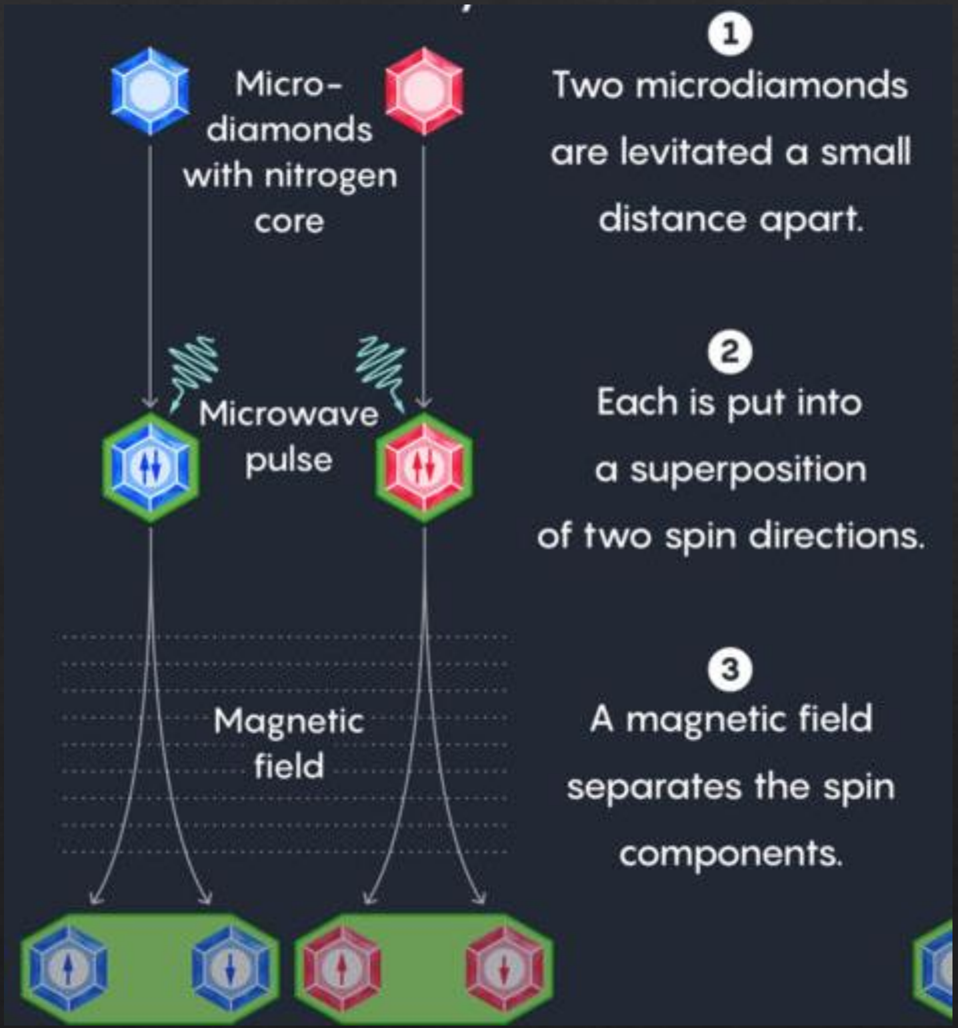
2 papers were published in PRL on the same day.

= BMV proposal

BMV experiment

Bose+ PRL119.240401(2017)

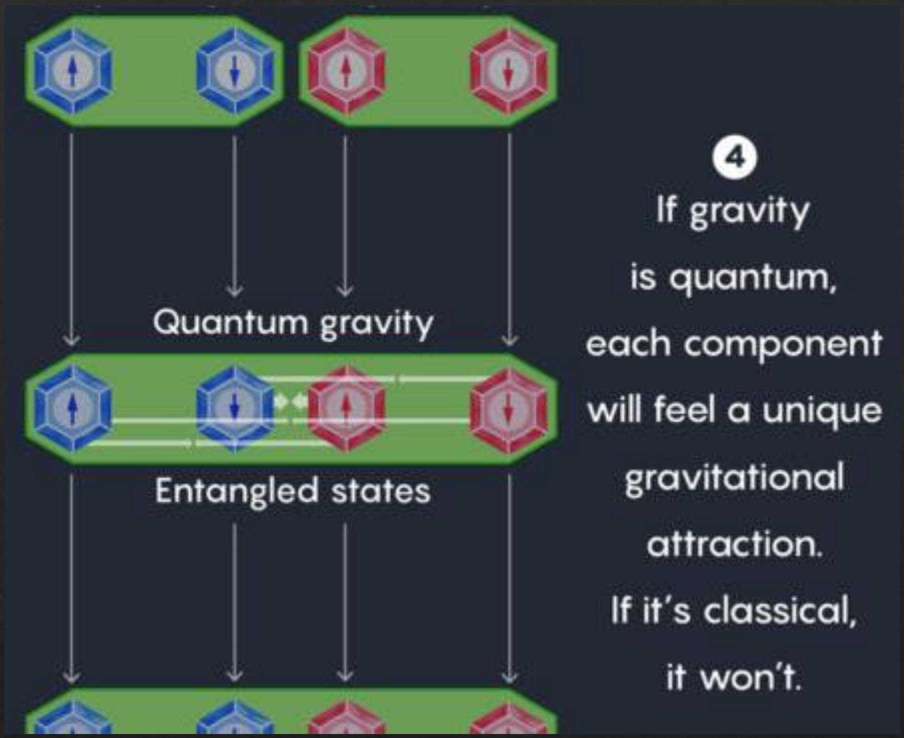
Marletto&Vedral PRL119.240402(2017)



BMV experiment

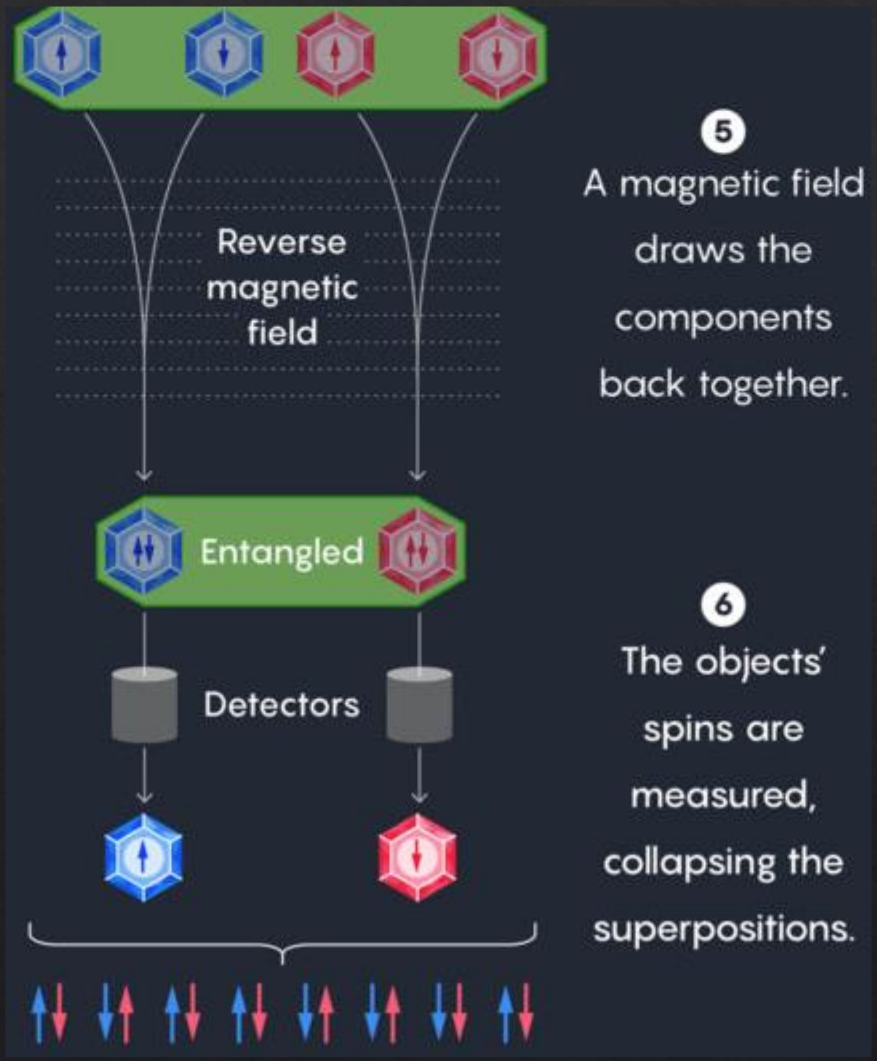
Bose+ PRL119.240401(2017)

Marletto&Vedral PRL119.240402(2017)



BMV experiment

Bose+ PRL119.240401(2017)
Marletto&Vedral PRL119.240402(2017)

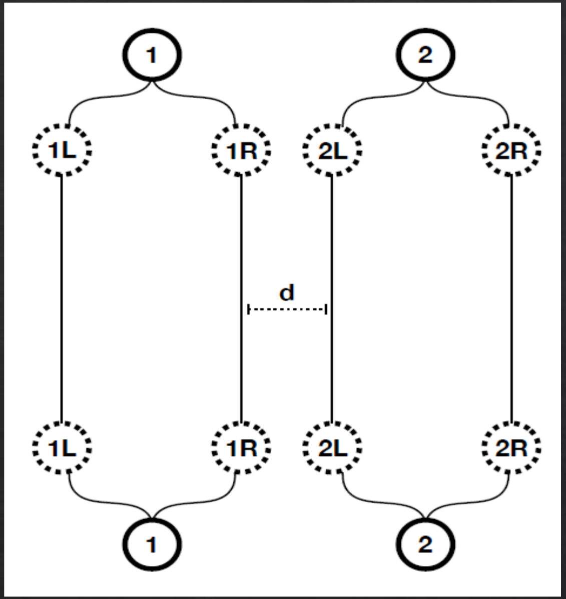


Quantum state

Christodoulou & Rovelli, PLB 792 (2019)[1808.05842]

The initial state is

$$|\Psi_1\rangle = \frac{1}{2}\Big(|\psi_1^L\rangle + |\psi_1^R\rangle\big) \otimes \big(|\psi_2^L\rangle + |\psi_2^R\rangle\Big) \otimes |g\rangle.$$
$$= \frac{1}{2}\Big(|LL\rangle + |RR\rangle + |LR\rangle + |RL\rangle\Big) \otimes |g\rangle.$$



if GFs can be quantum superposition

$$|\Psi_2\rangle = \frac{1}{2}\Big(|LL\rangle \otimes |g_{d_{LL}}\rangle + |RR\rangle \otimes |g_{d_{RR}}\rangle + |LR\rangle \otimes |g_{d_{LR}}\rangle + |RL\rangle \otimes |g_{d_{RL}}\rangle\Big),$$

Only the nearest pair |RL> gains a significant phase factor

$$|\Psi_3\rangle = \frac{1}{2}\Big(|LL\, g_{d_{LL}}\rangle + |RR\, g_{d_{RR}}\rangle + |LR\, g_{d_{LR}}\rangle + e^{i\frac{Gm^2 t}{\hbar d}}|RL\, g_{d_{RL}}\rangle\Big).$$

Another proposal

Krisnanda et al., npj Quant. Inf. **6**,12 (2020)

Tanjung Krisnanda

Simple Procedure:

1. Trap two masses in a harmonic potential

Another proposal

Krisnanda et al., npj Quant. Inf. **6**,12 (2020)

Tanjung Krisnanda

Simple Procedure:

1. Trap two masses in a harmonic potential
2. Release and let them grav. interact

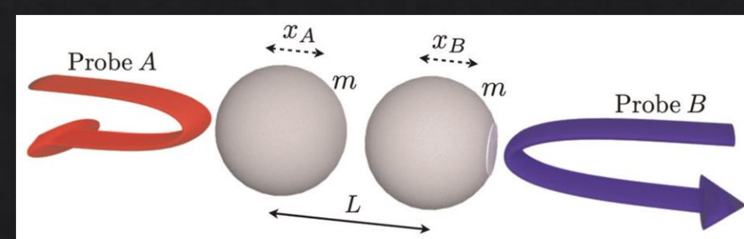
Another proposal

Krisnanda et al., npj Quant. Inf. 6,12 (2020)

Tanjung Krisnanda

Simple Procedure:

1. Trap two masses in a harmonic potential
2. Release and let them grav. interact
3. Measure the positions and momenta



Another proposal

Krisnanda et al., npj Quant. Inf. 6,12 (2020)

Tanjung Krisnanda

Simple Procedure:

1. Trap two masses in a harmonic potential

Wavefunction

Another proposal

Krisnanda et al., npj Quant. Inf. 6,12 (2020)

Tanjung Krisnanda

Simple Procedure:

1. Trap two masses in a harmonic potential
2. Release and let them grav. interact

Spread out

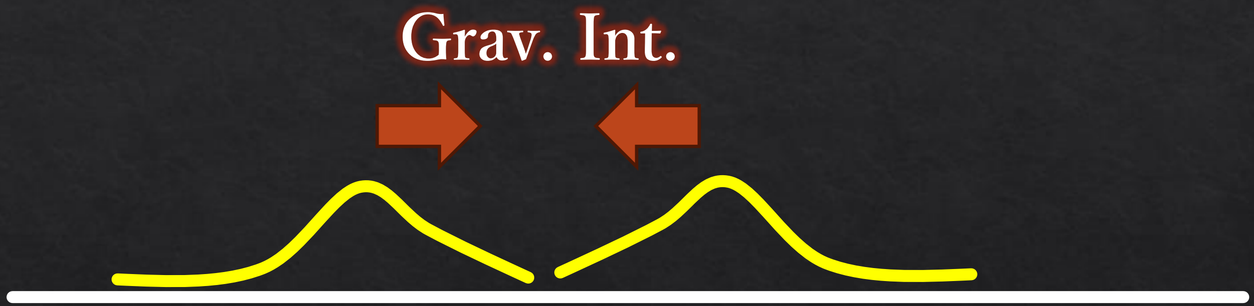
Another proposal

Krisnanda et al., npj Quant. Inf. **6**,12 (2020)

Tanjung Krisnanda

Simple Procedure:

1. Trap two masses in a harmonic potential
2. Release and let them grav. interact
3. Measure the positions and momenta



Entangled

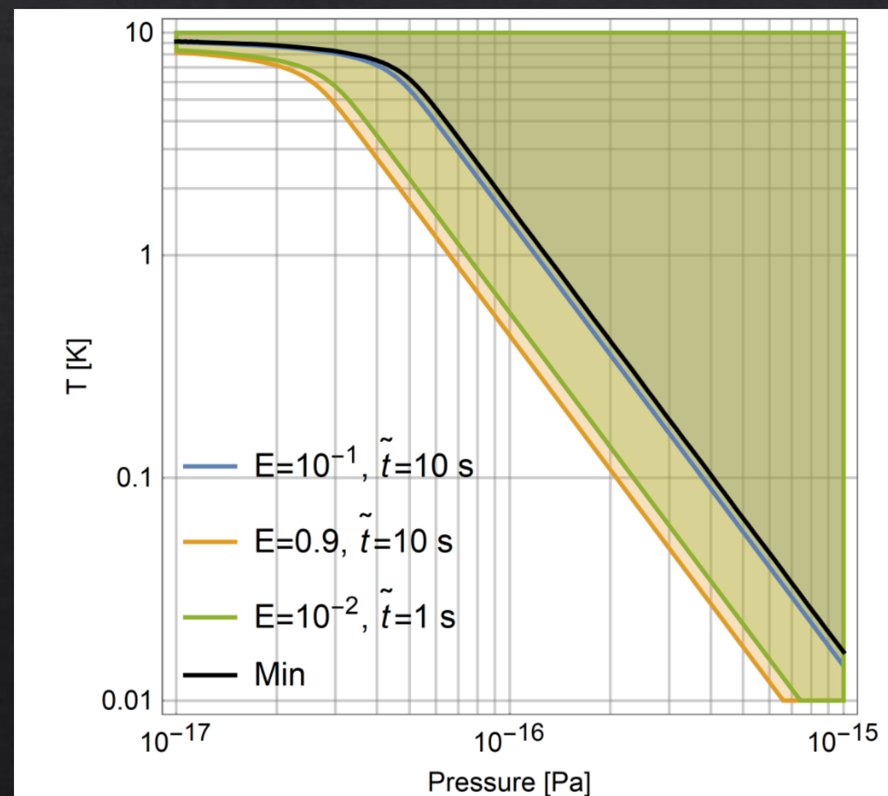
Feasibility

Rijavec et al., New J. Phys. **23** 043040 (2021)

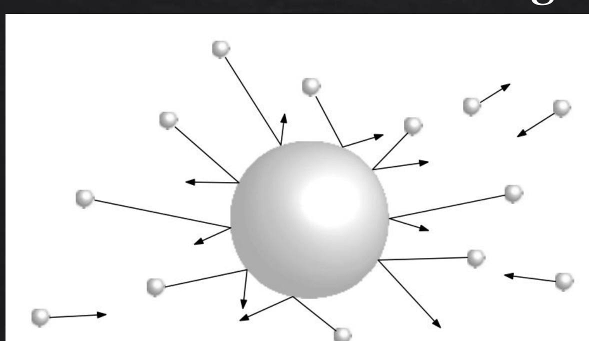
Simone Rijavec

For a real experiment,

1. Ultra-high vacuum to avoid decoherence



Air molecule Scattering



Feasibility

Rijavec et al., New J. Phys. **23** 043040 (2021)

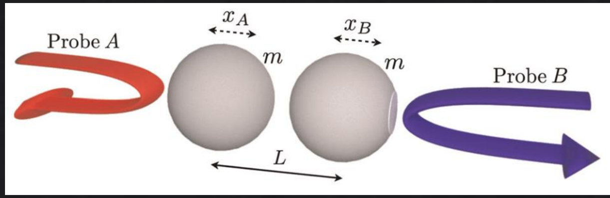
Simone Rijavec

For a real experiment,

1. Ultra-high vacuum to avoid decoherence
2. Free-fall problem

Free-fall

40m down for 3 sec



Outline

1. Introduction
2. Previous Proposals
3. General Analysis
4. Our Proposal
5. Summary

General Hamiltonian

TF. et al. (2023) [2308.14552]

Our quadratic Hamiltonian:

$$H = \frac{p_1^2}{2m} + \frac{1}{2}k_1 x_1^2 + \frac{p_2^2}{2m} + \frac{1}{2}k_2 x_2^2 - \frac{Gm^2}{d^3}(x_1 - x_2)^2,$$

oscillator1 oscillator2 Grav. Int.

$(d \gg |x_1 - x_2|)$

The system is quadratic.
⇨ Exactly Solvable!

General Hamiltonian

TF. et al. (2023) [2308.14552]

Our quadratic Hamiltonian:

$$H = \frac{p_1^2}{2m} + \frac{1}{2}k_1 x_1^2 + \frac{p_2^2}{2m} + \frac{1}{2}k_2 x_2^2 - \frac{Gm^2}{d^3}(x_1 - x_2)^2,$$

Spring constant k_i

$\Downarrow$

Potential parameter: $\lambda_i \equiv k_i/m\omega^2$

General Hamiltonian

TF. et al. (2023) [2308.14552]

Our quadratic Hamiltonian:

$$H = \frac{p_1^2}{2m} + \frac{1}{2}k_1 x_1^2 + \frac{p_2^2}{2m} + \frac{1}{2}k_2 x_2^2 - \frac{Gm^2}{d^3}(x_1 - x_2)^2,$$

Spring constant k_i

Potential parameter: $\lambda_i \equiv k_i/m\omega^2$

$\lambda = 1$: Harmonic

General Hamiltonian

TF. et al. (2023) [2308.14552]

Our quadratic Hamiltonian:

$$H = \frac{p_1^2}{2m} + \frac{1}{2}k_1 x_1^2 + \frac{p_2^2}{2m} + \frac{1}{2}k_2 x_2^2 - \frac{Gm^2}{d^3}(x_1 - x_2)^2,$$

Potential parameter: $\lambda_i \equiv k_i / m\omega^2$

$\lambda = 1$: Harmonic

$\lambda = 0$: Free mass

General Hamiltonian

Our quadratic Hamiltonian:

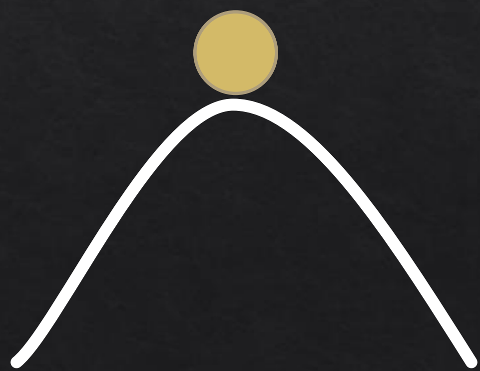
$$H = \frac{p_1^2}{2m} + \frac{1}{2}k_1 x_1^2 + \frac{p_2^2}{2m} + \frac{1}{2}k_2 x_2^2 - \frac{Gm^2}{d^3}(x_1 - x_2)^2,$$

Potential parameter: $\lambda_i \equiv k_i / m\omega^2$

$\lambda = 1$: Harmonic

$\lambda = 0$: Free mass

$\lambda = -1$: Inverted



Experimental Goal

Good indicator of entanglement:

Logarithmic Negativity E_N

- $E_N > 0 \quad \Leftrightarrow$ Two oscillators are entangled
- Larger E_N indicates larger entanglement
- $E_N = 0.01$ is experimentally detectable.

$$E_N \equiv \max\left[0, -\log_2\left(2\tilde{\nu}_{\min}\right)\right]$$

$$\tilde{\nu}_{\min} \equiv \left[\frac{1}{2}\left(\tilde{\Sigma} - \sqrt{\tilde{\Sigma}^2 - 4\det\sigma}\right)\right]^{1/2}$$

$$u_i(t) = \Big(X_1(t), P_1(t), X_2(t), P_2(t)\Big),$$
$$\sigma_{ij}(t) = \frac{1}{2}\langle u_i(t)u_j(t) + u_j(t)u_i(t)\rangle\,.$$

$$\sigma(t) = \begin{bmatrix} \sigma_1 & \sigma_3 \\ \sigma_3^{\mathrm{T}} & \sigma_2 \end{bmatrix}$$

$$\tilde{\Sigma} \equiv \det\sigma_1 + \det\sigma_2 - 2\det\sigma_3$$

Calculation

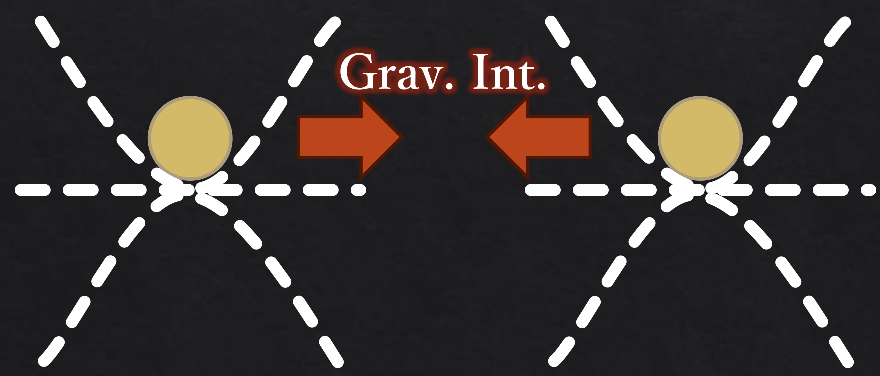
TF. et al. (2023) [2308.14552]

We compute E_N when

At $t = 0$,
they're in the
ground state
w/o gravity

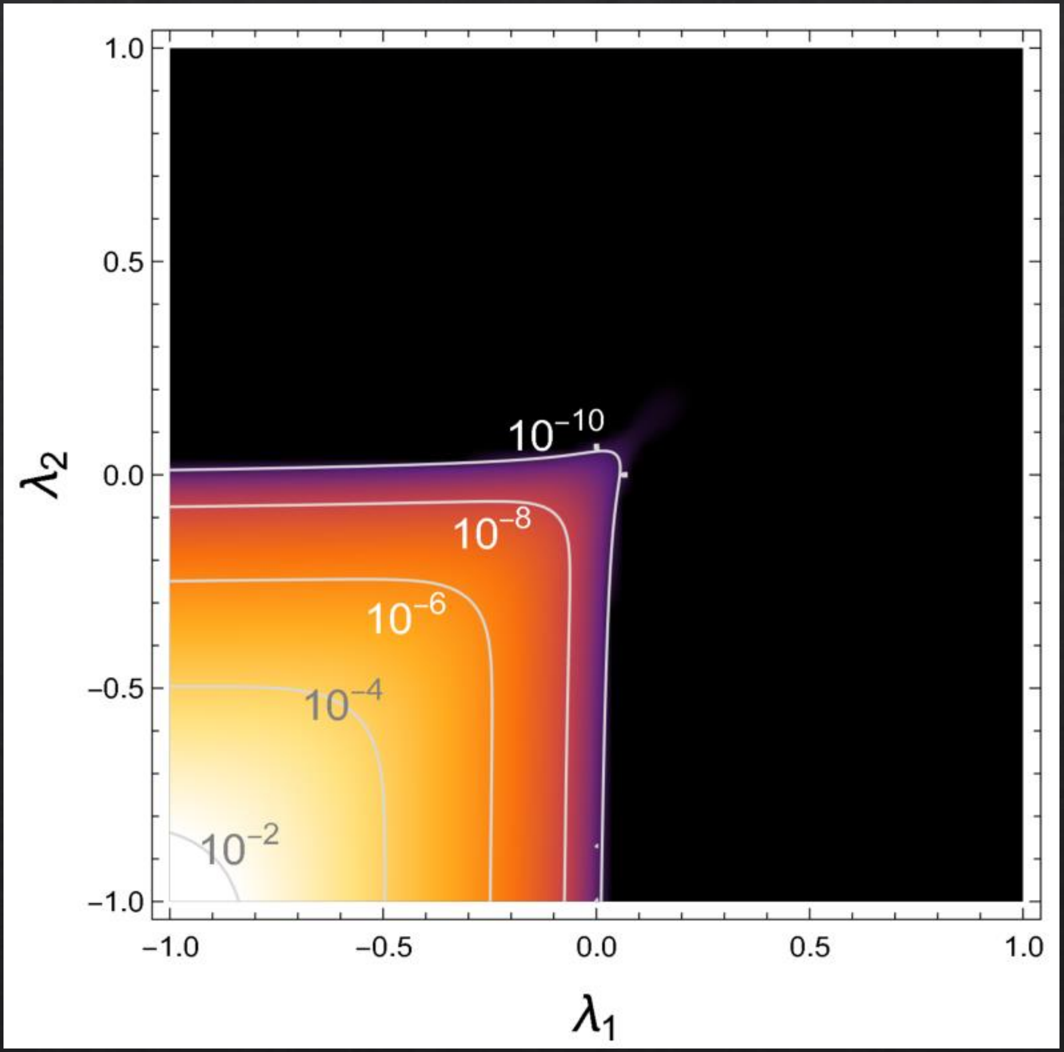
ω harmonic potential

For $t > 0$,
they evolve
in the λ_i potential
w/ gravity



Result of Entanglement

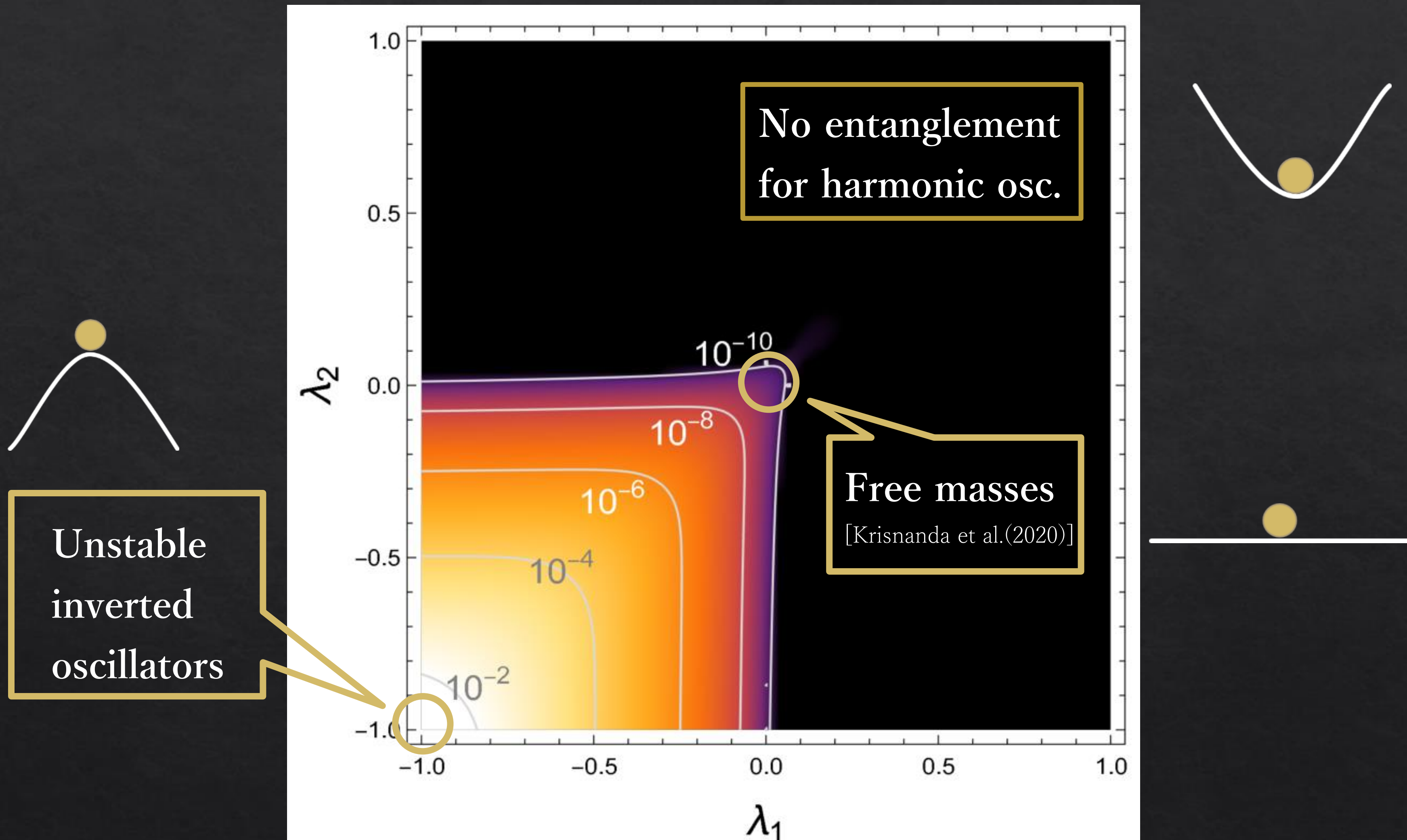
TF. et al. (2023) [2308.14552]



Contour of E_N $(\omega t = 13,\ \eta = 2\mu = 10^{-12})$

Result of Entanglement

TF. et al. (2023) [2308.14552]



Contour of E_N $(\omega t = 13,\ \eta = 2\mu = 10^{-12})$

Including Decoherence

TF. et al. (2023) [2308.14552]

Heisenberg-Langevin eqs:

$$\dot{X}_i = \omega P_i , \quad \dot{P}_i = -\lambda\omega X_i + \omega\eta(X_i - X_j) + \xi_i ,$$

ξ_i: random noise force $\Rightarrow$ decoherence

$$\frac{1}{2}\langle \xi_i(t)\xi_j(t') + \xi_i(t')\xi_j(t) \rangle = \mu\omega\delta(t-t')\delta_{ij} \, .$$

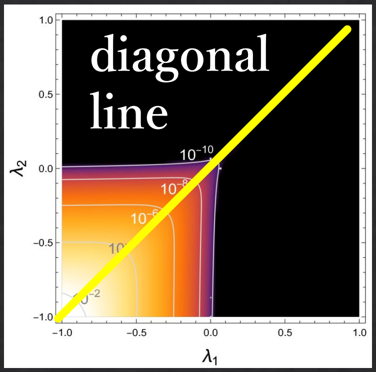
μ: size of env. fluctuation

η: grav. coupling constant

$$\eta \equiv \frac{2Gm}{\omega^2 d^3} = 2.7 \times 10^{-13} \, \omega_{\mathrm{kHz}}^{-2} \left(\frac{m/d^3}{2 \ \mathrm{g/cm^3}} \right)$$

Analytic Solution

TF. et al. (2023) [2308.14552]



For the identical oscillators $(\lambda_1 = \lambda_2)$

Logarithmic Negativity reads

$$E_N \simeq 3(\eta - \mu) f_{\rm gra} \quad (\lambda \leq 0)$$

Grav. coupling constant

Random noise parameter

$$f_{\rm gra} \simeq \begin{cases} \cancel{\frac{1}{2}|\sin(\omega t)| \quad (\lambda = 1)} \\ \frac{1}{6}(\omega t)^3 \quad (\lambda = 0) \\ \frac{1}{8}e^{2\omega t} \quad (\lambda = -1) \end{cases}$$

Power-law

Exponential

Time scale

The time required to generate observable $E_N = 0.01$

$$\boxed{\text{w/o } \mu} \quad \tau_{\rm ent} \simeq \begin{cases} 4.2\,\omega_{\rm kHz}^{-1/3}\ {\rm sec} & (\lambda = 0) \\ 1.3 \times 10^{-2}\,\omega_{\rm kHz}^{-1}\ {\rm sec} & (\lambda = -1) \end{cases}$$

300 times faster!

Including decoherence parameter μ

$$\tau_{\rm ent} \simeq \begin{cases} 4.2\,\omega_{\rm kHz}^{-1/3}\,[\eta/(\eta - \mu)]^{1/3}\ {\rm sec} & (\lambda = 0) \\ 1.3 \times 10^{-2}\,\omega_{\rm kHz}^{-1}\ {\rm sec} + \log[\eta/(\eta-\mu)]/(2\omega) & (\lambda = -1) \end{cases}$$

$$\simeq \mathcal{O}(10^{-3})\,\omega_{\rm kHz}^{-1}\,{\rm sec}$$

Time scale

The time required to generate observable $E_N = 0.01$

The inverted oscillators generate the gravity-induced entanglement most quickly and are most resistant to decoherence.

$$\simeq \mathcal{O}(10^{-3})\omega_{\mathrm{kHz}}^{-1}\mathrm{sec}$$

Outline

1. Introduction
2. Previous Proposals
3. General Analysis
4. Our Proposal
5. Summary

Optomechanics

Rahman & Barker (2020)

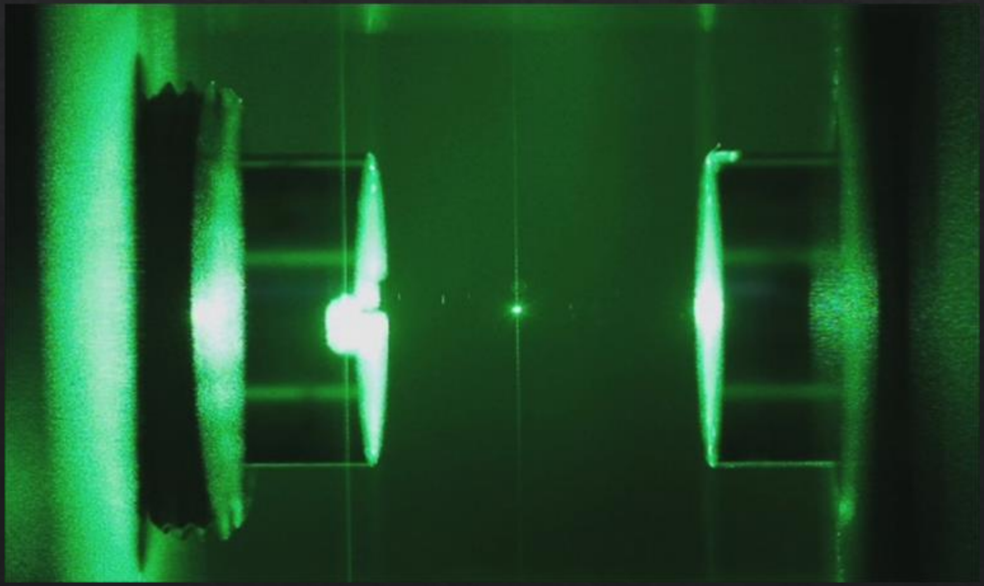
- Free fall problem

Free-fall

40m down for 3 sec

- Optical levitation

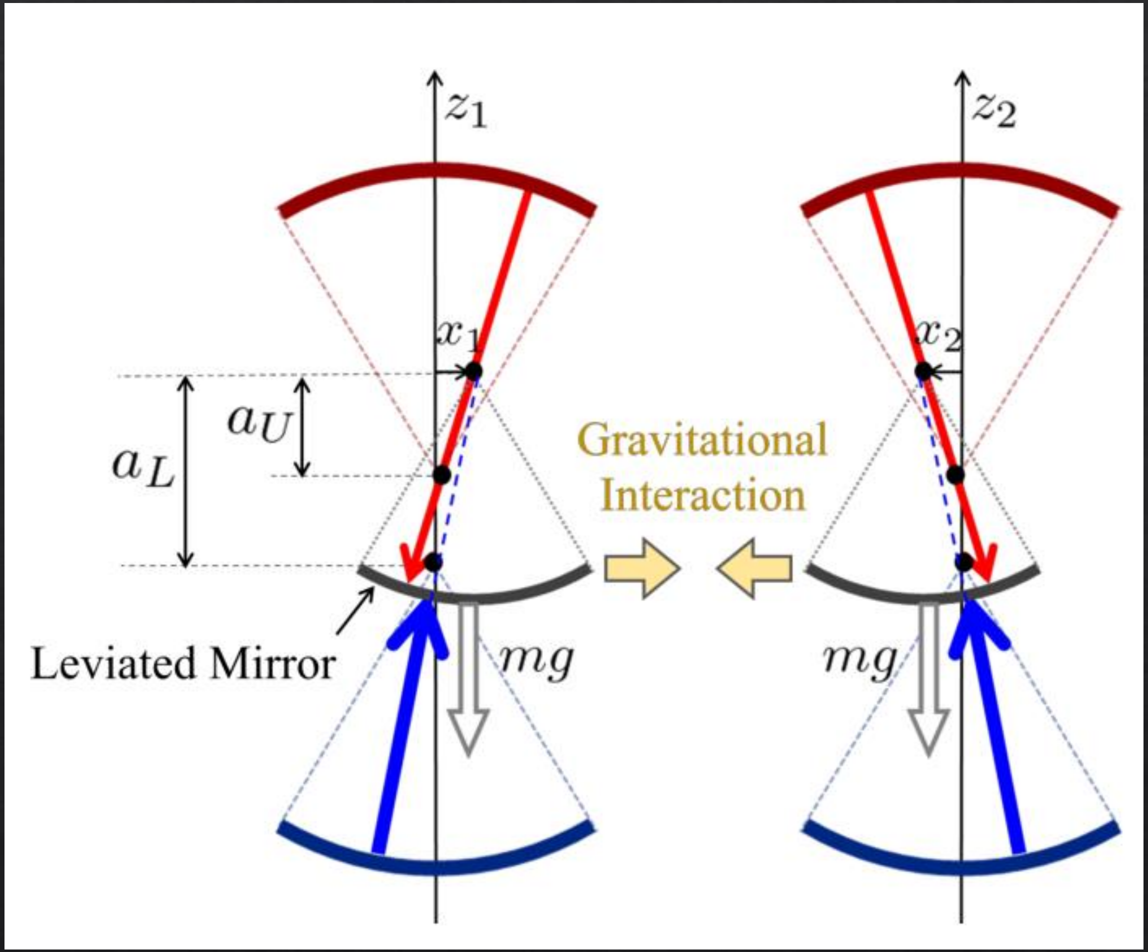
demonstrated to levitate small
particle by laser pressure
w/o mechanical support



Sandwich Setup

Michimura. et al. (2017)

Inverted Oscillator ⇔ Anti-spring effect

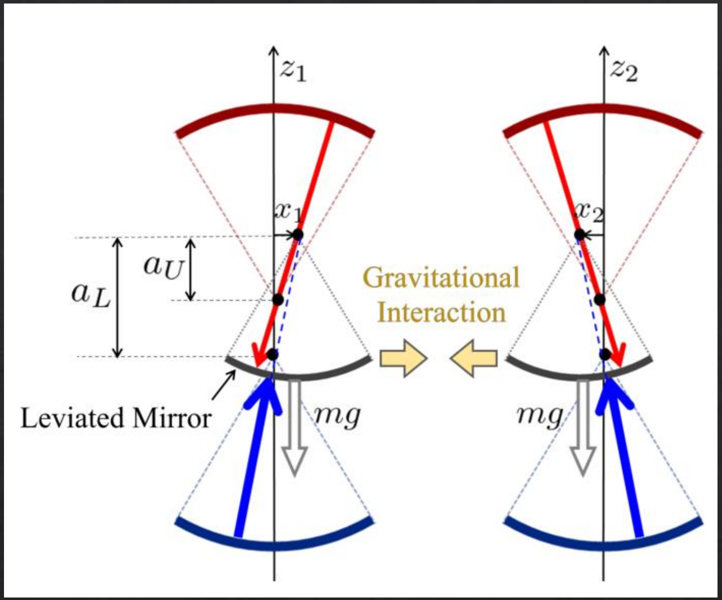


Sandwich Setup

Michimura. et al. (2017)

We can realize high frequency inverted oscillator

$$\omega_{\rm hor}^2 = \frac{2}{mc}\left(\frac{P_{\rm U}}{a_U} - \frac{P_{\rm L}}{a_L}\right) = \frac{2(a_L - a_U)}{mc\, a_U a_L} P_L - \frac{g}{a_U},$$
$$\simeq -(1{\rm kHz})^2 \left(\frac{m}{0.1{\rm mg}}\right)^{-1} \left(\frac{P_L}{30{\rm kW}}\right)\left(\frac{a_L}{2{\rm mm}}\right)^{-1},$$



while suppressing decoherence due to photon shot noise.

$$\mu_{\rm shot,hor} = \frac{16\omega_\ell P_L}{m\omega^2 c^2 T_{\rm in}}\left(\frac{\Delta x}{a_L}\right)^2 \simeq \frac{8\omega_\ell \Delta x^2}{c a_L T_{\rm in}},$$
$$= 2.5\times 10^{-14}\,\omega_{\rm kHz}^3 \left(\frac{a_L}{2{\rm mm}}\right)^{-1}\left(\frac{m}{0.1{\rm mg}}\right)^{-1}\left(\frac{\omega_{\rm in}}{1{\rm MHz}}\right)^{-2}$$

Summary

- Lack of experimental verification of quantum gravity.
  Not even sure if grav. fields can be quantum superposition.

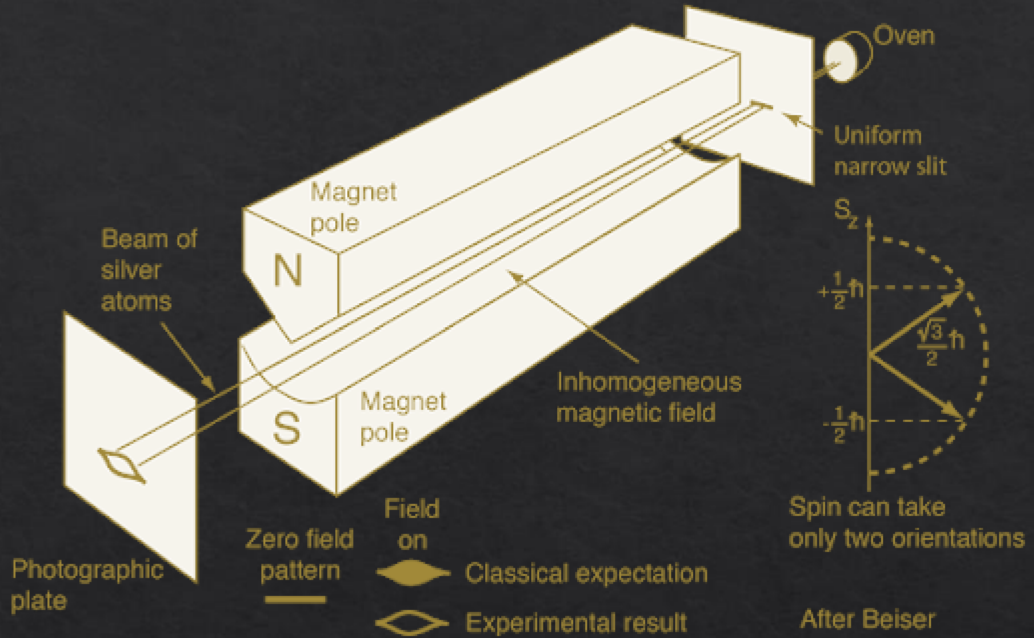
- Many proposals to test gravity-induced entanglement. “Trap & release” masses generates entanglement. (free-fall problem)

- We analyzed two general quadratic oscillators coupled by gravity and found inverted oscillators exponentially generate entanglement and resistant to decoherence.

- As an experimental implementation, we proposed levitated mirror with anti-spring effect in a sandwich configuration.

Thank you

Sketch of idea



Stern-Gerlach experiment enables us to prepare the quantum superposition of a mass at two different locations.

Superposition

Pure state: $|\Psi\rangle = c_1|\phi_1\rangle + c_2|\phi_2\rangle$

quantum superposition

It's undetermined whether $\Psi = \phi_1$ or ϕ_2 (c.f. Schrodinger's cat)

quantum $\Updownarrow$ classical

It's pre-determined whether $\Psi = \phi_1$ or ϕ_2

The probabilities of each realization p_1 and p_2 are known.

Its QM description = **Mixed State**

Density matrix

$\hat{\rho}$ gives the probability and the expected value

$$p_i = \mathrm{Tr}\left[\hat{P}_i \hat{\rho}\right] \qquad \langle \hat{O} \rangle = \mathrm{Tr}\left[\hat{O}\hat{\rho}\right]$$

quantum

$$\hat{\rho}_{\mathrm{pure}} = |\Psi\rangle\langle\Psi| = \Sigma_i |c_i|^2 \, |\phi_i\rangle\langle\phi_i| + \Sigma_{i \neq j} c_i c_j^* \, \left|\phi_i\right\rangle \left\langle\phi_j\right|$$

Interference term

classical

$$\hat{\rho}_{\mathrm{mix}} = \Sigma_i p_i |\phi_i\rangle\langle\phi_i|$$

The essential difference btw quantum and classical state appears in the interference term in the density matrix.

Entanglement

A quantum system consists of subsystem A & B.

General state $|\Psi\rangle = \Sigma_{ij} c_{ij} |\psi_i\rangle_A \otimes |\phi_j\rangle_B$

If $|\psi\rangle_A = \Sigma_i a_i |\psi_i\rangle_A$ and $|\phi\rangle_B = \Sigma_j b_j |\phi_j\rangle_B$ independently,

Separable state state $|\Psi\rangle = \Sigma_{ij} a_i b_j |\psi_i\rangle_A \otimes |\phi_j\rangle_B$

Non separable = **Entangled state**

Interaction btw the subsystems can induce entanglement.

Trace out

Remember $\langle \hat{O} \rangle = \mathrm{Tr}[\hat{O}\hat{\rho}]$

If we only consider observables of the subsystem A, $\hat{O}_A$, we take the trace of $\hat{\rho}$ over the subsystem B,

Reduced density matrix: $\hat{\rho}_A = \mathrm{Tr}_B[\hat{\rho}]$

This operation won't change anything in A, if A & B are separable.

Decoherence

Pure entangled state

$$|\Psi\rangle = \Sigma_i c_i |\psi_i\rangle_A \otimes |\phi_i\rangle_B$$

Density matrix

$$\hat{\rho} = |\Psi\rangle\langle\Psi| = \Sigma_{ik} c_i c_k^* |\psi_i\rangle\langle\psi_k| \otimes |\phi_i\rangle\langle\phi_k|$$

Trace out:

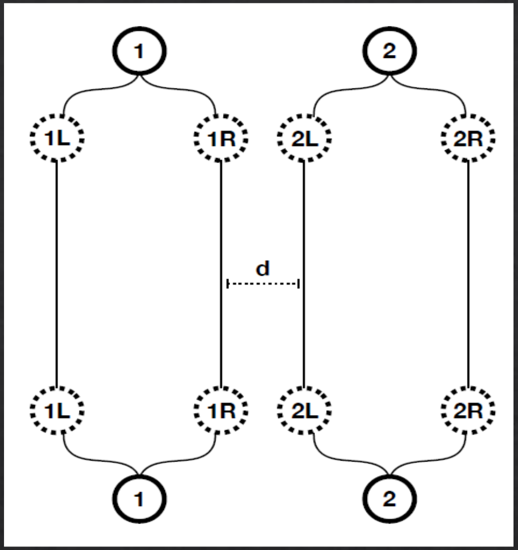
$$\mathrm{Tr}_B[\hat{\rho}] = \Sigma_l \langle\phi_l|\hat{\rho}|\phi_l\rangle = \Sigma_i |c_i|^2 |\psi_i\rangle\langle\psi_i|$$

When the original state is entangled,
tracing out it into a mixed state.

Decoherence = interference terms (quantum-ness) vanish

Phase from potential

Christodoulou & Rovelli, PLB 792 (2019)[1808.05842]



Schrodinger eq.: $i\partial_t|\psi\rangle = \widehat{H}|\psi\rangle$

$\Rightarrow$Phase from mass energy $\mathrm{e}^{-imt}|\psi\rangle$

GR replaces t by the proper time s

Newtonian: $\mathrm{d}s^2 = [1+2\Phi]\mathrm{d}t^2, \qquad \Phi = -\dfrac{\mathrm{Gm}}{\mathrm{d}}$

c.f. $m_P \approx 2\times10^{-5}\mathrm{g}$

$c * \sec \approx 3\times10^{8}\mathrm{m}$

The relative phase that |RL> gains is

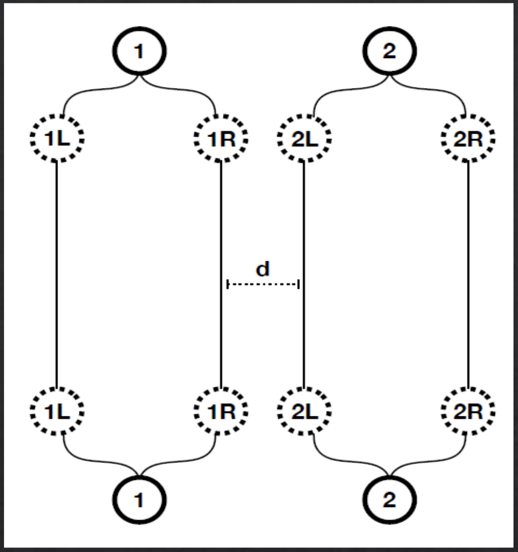
Phase: $\phi = \dfrac{Gm^2}{d}t \approx 2\pi\left(\dfrac{t}{1\sec}\right)\left(\dfrac{d}{1\mathrm{mm}}\right)^{-1}\left(\dfrac{m}{10^{-11}\mathrm{g}}\right)^2$

Small mass compensated by long time

Quantum state

Christodoulou & Rovelli, PLB 792 (2019)[1808.05842]

$$|\Psi_3\rangle = \frac{1}{2}\Big(|LL\, g_{d_{LL}}\rangle + |RR\, g_{d_{RR}}\rangle + |LR\, g_{d_{LR}}\rangle + e^{i\frac{Gm^2 t}{\hbar d}}|RL\, g_{d_{RL}}\rangle\Big).$$



Bring them back by inverse-SG

$$|\psi_4\rangle = \frac{1}{2}\left[|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle + |\downarrow\uparrow\rangle + e^{i\phi}|\uparrow\downarrow\rangle\right]$$

The entangled state is tested by Bell inequality

$$\mathcal{W} = |\langle\sigma_x^{(1)} \otimes \sigma_z^{(2)}\rangle - \langle\sigma_y^{(1)} \otimes \sigma_z^{(2)}\rangle|$$

If $\mathcal{W} > 1$, the state is entangled and GFs are superposed.

General Hamiltonian

TF. et al. (2023) [2308.14552]
]

Our quadratic Hamiltonian:

$$H = \frac{p_1^2}{2m} + \frac{1}{2}k_1 x_1^2 + \frac{p_2^2}{2m} + \frac{1}{2}k_2 x_2^2 - \frac{Gm^2}{d^3}(x_1 - x_2)^2,$$

dimentionless form

$$H = \frac{\omega}{2}\left[P_1^2 + \lambda_1 X_1^2 + P_2^2 + \lambda_2 X_2^2 - \eta(X_1 - X_2)^2\right]$$

oscillator1 oscillator2 Grav. Int.

Variable: $P_i \equiv p_i/\sqrt{\hbar m\omega}$ $X_i \equiv \sqrt{m\omega/\hbar}x_i$

Coupling constant:

$$\eta \equiv \frac{2Gm}{\omega^2 d^3} = 2.7 \times 10^{-13}\,\omega_{\rm kHz}^{-2}\left(\frac{m/d^3}{2\ {\rm g/cm^3}}\right)$$

Gravity is weak