“Quantum Effects in Gravitational Fields” @ Leipzig University

Quantum effect unique to gravity based on the weak equivalence principle

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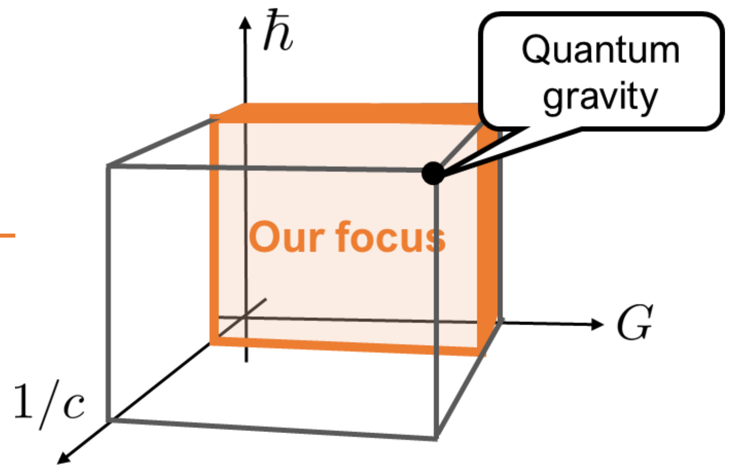
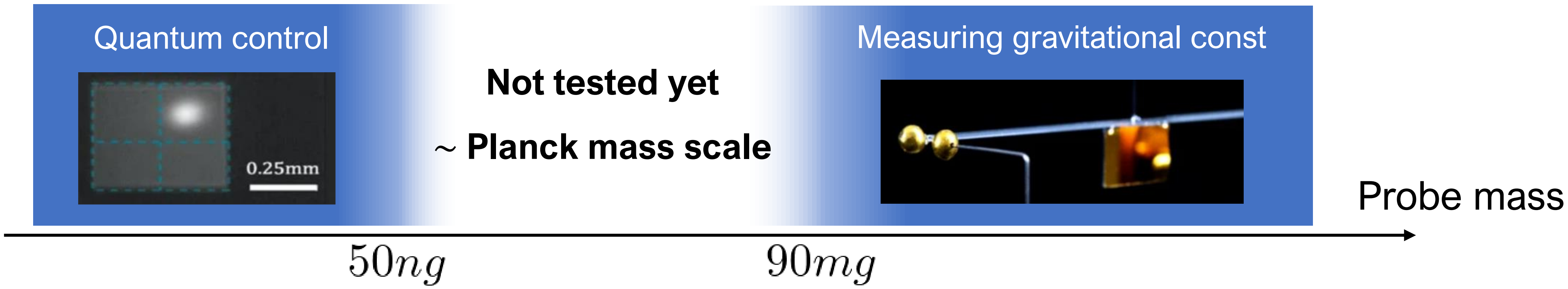
Phys. Rev. D 106(2022)12, 126005, [arXiv: 2207.11848]

Introduction

Introduction

- We need experimental evidence to explore quantum gravity
- Quantum gravity theory $\ni$ non-relativistic gravity in QM regime
  Our focus
- Regarding the recent progress of quantum experiment in mesoscopic scale, it is expected to be realized in near future.





Let's consider how to test the quantum nature of non-relativistic gravity targeting on mesoscopic object!

Quantum Gravity witness

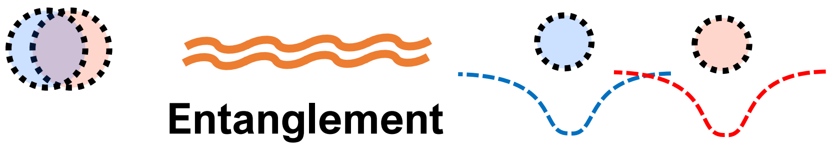
Feynman (1957)
Marletto, Vedral (2017), Bose et al. (2017)

“Is the gravitational field superposed when the mass source is in quantum superposition?”

If Yes, Quantized Gravity (QG)

$$V_{\rm QG}(\hat{x},\hat{X}) = -\frac{GmM}{|\hat{x}-(\hat{X}+d)|}$$ **Operator** of the source system

Particle: $m,\ \hat{x}$ Mass source: $M,\ \hat{X}$



Entanglement

Gravitational field

If No, Semiclassical Gravity (CG)

$$V_{\rm CG}(\hat{x}) = -\left\langle \frac{GmM}{|\hat{x}-(\hat{X}+d)|} \right\rangle_{\rm source}$$ **c-number** of the source system

Particle: $m,\ \hat{x}$ Mass source: $M,\ \hat{X}$

No entanglement

Gravitational field

Therefore, creation of entanglement may be good criteria to judge superposed gravity.

However, since QG as well as other quantum interaction can also create entanglement,

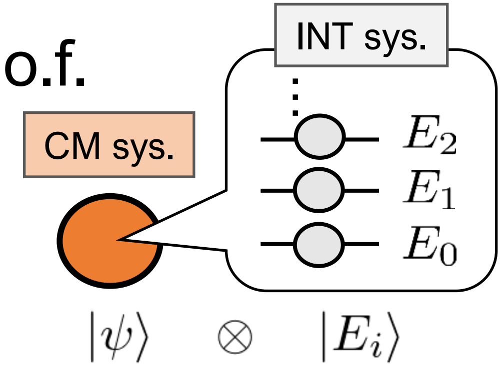
it is better if we can pick out quantum feature unique to gravity!!

Quantum clock and Ramsey interference

Quantum clock Particle which has energy levels as its internal d.o.f.

- Particle state is defined by "the CM state" ⊗ "the internal energy state"
- Total rest mass depends on internal energy: $m(\hat{E}) = \underline{m_0} + \hat{E}/c^2$

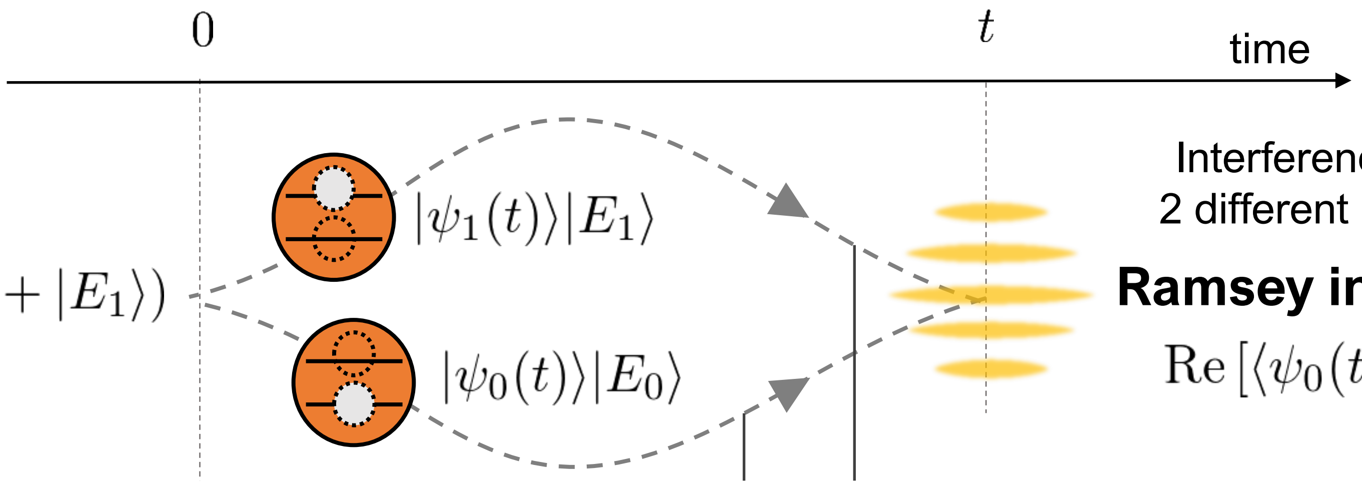
Mass of the CM system



Ramsey interferometry Measurement of an energy gap of the quantum clock

Superpose the int. energy state

$$|\psi_{\mathrm{ini}}\rangle|E_0\rangle \longrightarrow \frac{1}{\sqrt{2}}|\psi_{\mathrm{ini}}\rangle(|E_0\rangle + |E_1\rangle)$$



Interference between 2 different energy state

Ramsey interference

$\mathrm{Re}\left[\langle\psi_0(t)|\psi_1(t)\rangle\right]$

Different time evolution with different rest mass $\hat{H}(\hat{E}, \hat{x}, \hat{p})$

Gravity induced decoherence of the Ramsey interference

Pikovski, et. al. (2015) Haustein et. al. (2019)

- Background spacetime $ds^2 \simeq -(1+2\Phi(\hat{x})/c^2)dt^2 + (1-2\Phi(\hat{x}/c^2))dx^2$

An arbitrary weak gravitational field

Note Total rest mass is $m(\hat{E}) = m_0 + \hat{E}/c^2$

- Hamiltonian of freefalling quantum clock

$$H_{\mathrm{free}}(\hat{E},\hat{x},\hat{p}) = \sqrt{-g_{00}(\hat{m}^2c^4 + c^2\hat{p}^2)} \simeq \underbrace{m_0c^2 + \frac{\hat{p}^2}{2m_0} + m_0\Phi(\hat{x})}_{\text{CM Hamiltonian}} + \underbrace{\hat{E}\left(1+\frac{\Gamma(\hat{x},\hat{p})}{c^2}\right)}_{\text{INT Hamiltonian}}$$

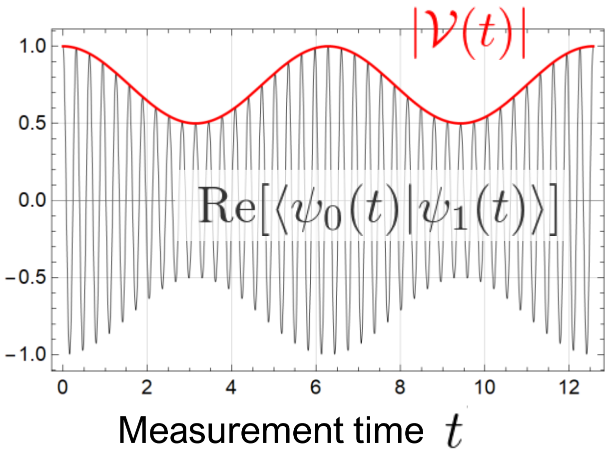
where $\Gamma(\hat{x},\hat{p}) = -\frac{\hat{p}^2}{2m_0^2} + \Phi(\hat{x})$: SR/ GR redshift factor

- Visibility: The absolute value of the Ramsey interference

$$|\mathcal{V}(t)| := |\langle\psi_0(t)|\psi_1(t)\rangle| = \left|\langle\psi_{\mathrm{ini}}|e^{i\hat{H}(E_0)t}e^{-i\hat{H}(E_1)t}|\psi_{\mathrm{ini}}\rangle\right|$$

$$\simeq \left|\langle\psi_{\mathrm{ini}}|e^{i(E_0-E_1)\,\Gamma(\hat{x},\hat{p})t}|\psi_{\mathrm{ini}}\rangle\right| \leq 1$$

Decoherence in homogeneous gravity



The Ramsey interference is damped down due to SR/GR redshift.

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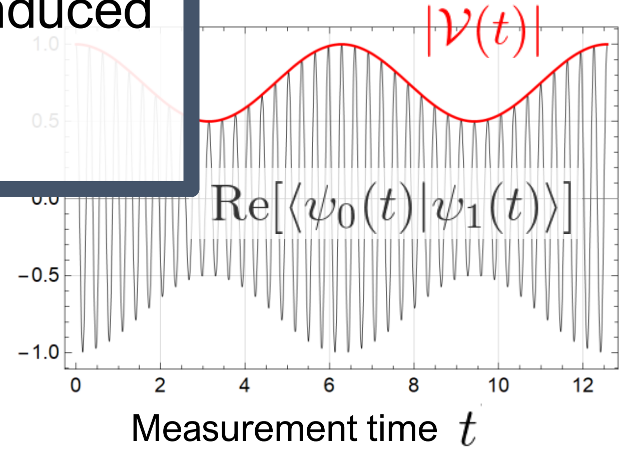
Decoherence in homogeneous gravity



What we are going to do?

- Consider QG/CG as the background spacetime, and focus on the time dependence of the visibility.
- We may be able capture quantumness unique to the gravity by observing the decoherence induced by the gravitational redshift.

Our work

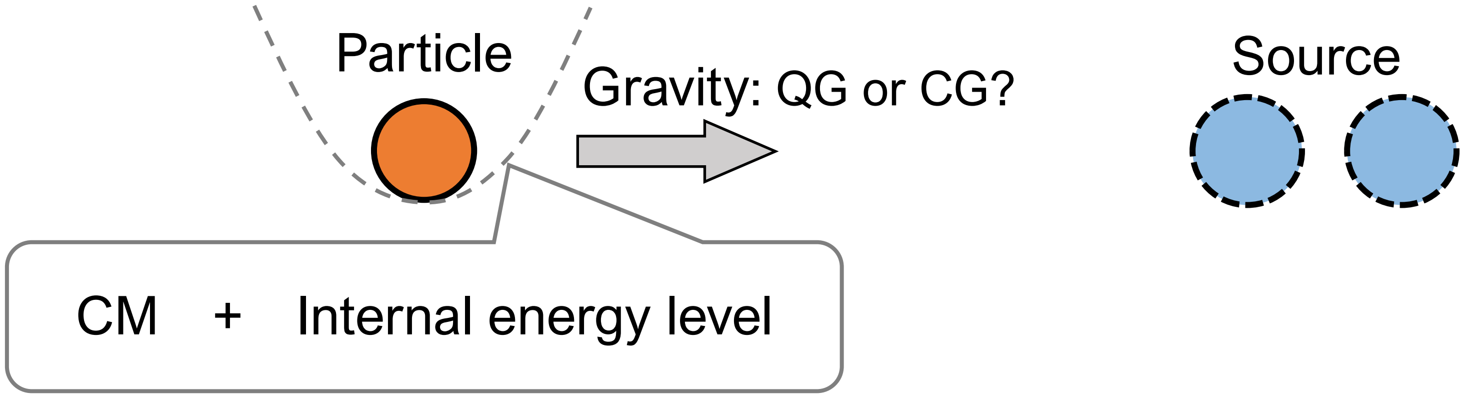
Quantumness of gravity in harmonically trapped particles

Phys. Rev. D 106(2022)12, 126005, [arXiv: 2207.11848]

Quantumness of Gravity in harmonically trapped particles

Main idea

- Ramsey interferometry of a particle gravitationally interacting with superposed mass source.



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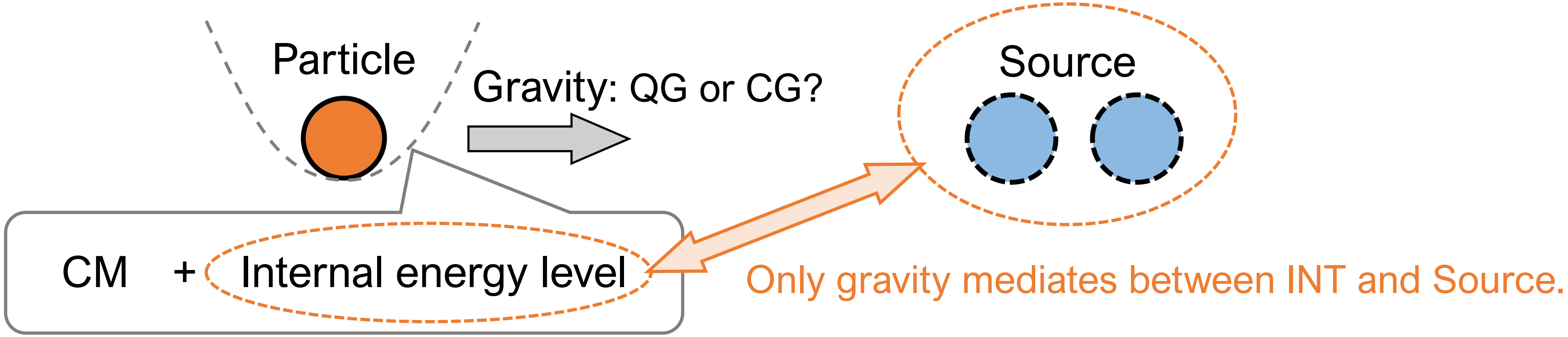
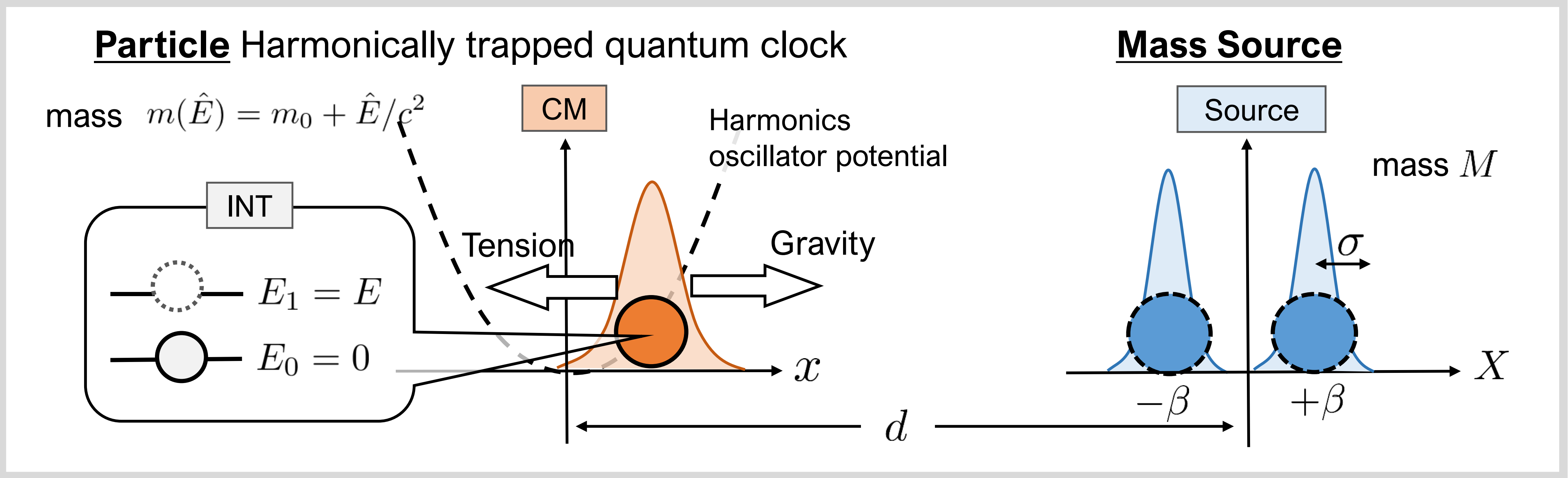


- According to the weak equivalence principle, only gravity couples to mass.
  Also, by referring to mass-energy equivalence, only gravity couples to energy.
- Entanglement between INT and Source systems can be only created by gravity!

If we can test the existence of INT: Source entanglement in the Ramsey interferometry,

we will be able to test the quantum nature unique to gravity!

Setup



- Hamiltonian $\hat{H} \simeq \hat{m}c^2 + \dfrac{\hat{p}^2}{2\hat{m}} + \dfrac{1}{2}k\hat{x}^2 + \boxed{\hat{V}}$ Consider both QG and CG cases
- Initial state $|\Psi_{\text{ini}}\rangle = \underbrace{|\psi_{\text{ini}}\rangle}_{\text{Equilibrium state}} \otimes \underbrace{|E_0\rangle}_{\text{Ground state}} \otimes \underbrace{|\varphi\rangle}_{\text{Superposition of Gaussian state}}$

How does the decoherence of Ramsey visibility $|\mathcal{V}(t)| := |\langle\psi_0(t)|\psi_1(t)\rangle|$ change w.r.t. QG and CG?

Visibility (QG case)

$$\left(\sqrt{m_0^3/k}=10,\ \sqrt{m_0/m_1}=0.5,\ Gm_0M/(kd^3)=0.015,\ m_0^{1/4}d=10,\ \beta/d=0.01,\ \sigma/d=0.001\right)$$

$$V_{\rm QG}(\hat{E},\hat{x},\hat{X}) = -\frac{GM\,m(\hat{E})}{|\hat{x}-(\hat{X}+d)|}$$

INT and Source systems entangle through QG

- The Ramsey interference visibility is given by

$$|\mathcal{V}(t)| = |\mathcal{V}_Q(t)\mathcal{V}_C(t)|$$

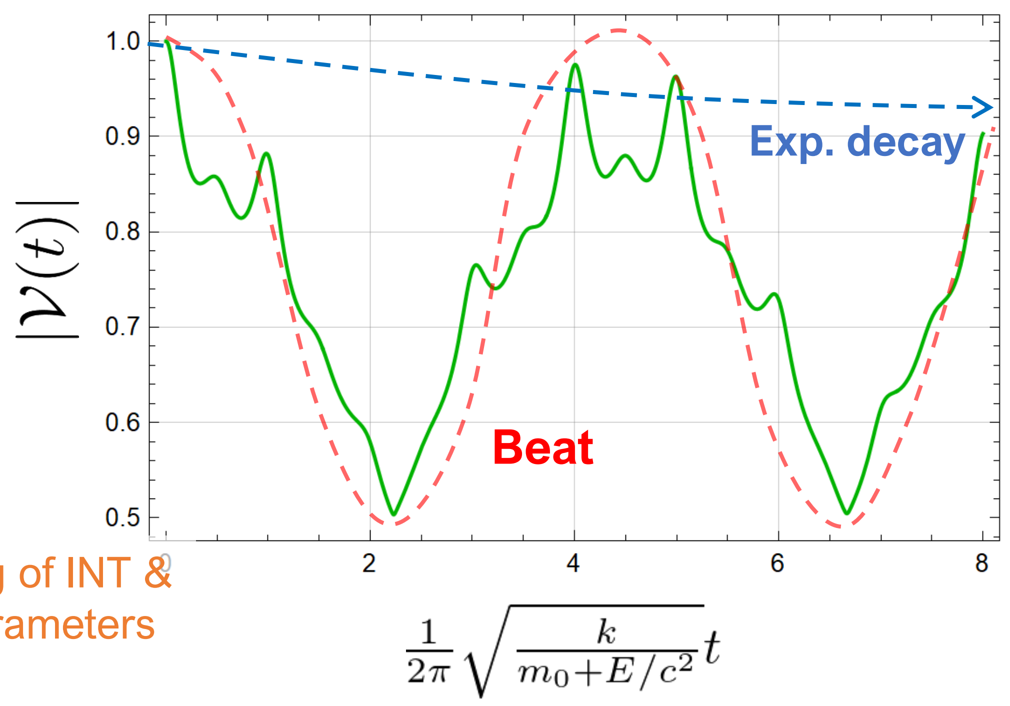
$$\begin{cases} |\mathcal{V}_C(t)| : \boxed{2\pi \text{ periodic func.}} \\ |\mathcal{V}_Q(t)| = \dfrac{2}{N}\left(\boxed{\cos\left[\dfrac{GME}{c^2|d|}\dfrac{\beta}{d}\dfrac{t}{\hbar}\right]} + e^{-\beta^2/\sigma^2}\right) \\ \qquad \times \boxed{\exp\left[-\dfrac{GME}{2c^2|d|}\dfrac{\sigma}{d}\dfrac{t}{\hbar}\right]} \end{cases}$$

Coupling of 2 modes leads to **beat**

Coupling of INT & Src. parameters

Exp. decay

Time dependence of Visibility



$|\mathcal{V}_Q(t)|$, which results from INT-Src. entanglement, leads the visibility to be non-revival.

Visibility (CG case)

$$\sqrt{m_0^3/k} = 10,\ \sqrt{m_0/m_1} = 0.5,\ Gm_0M/(kd^3) = 0.015,$$
$$m_0^{1/4}d = 10,\ \beta/d = 0.01,\ \sigma/d = 0.001$$

$$V_{\mathrm{CG}}(\hat{E},\hat{x}) = -\left\langle \frac{GM\,m(\hat{E})}{|\hat{x} - (\hat{X}+d)|} \right\rangle_{\mathrm{source}}$$

No entanglement between INT and Source

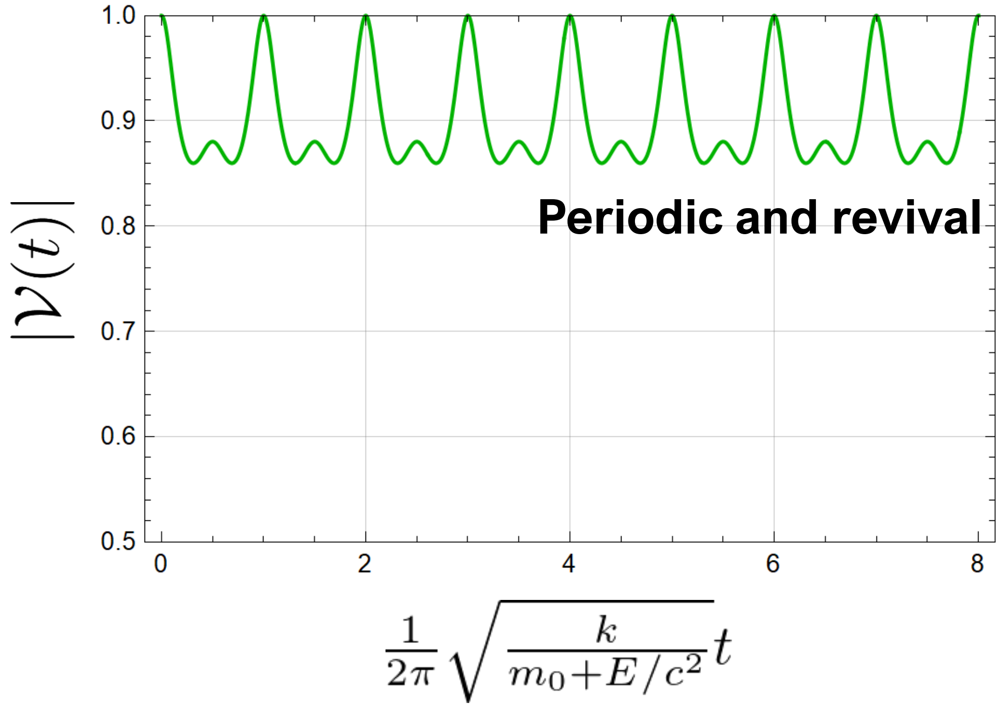
- The Ramsey interference visibility is given by

$$|\mathcal{V}(t)| = |\cancel{\mathcal{V}_Q(t)}\mathcal{V}_C(t)|$$

$|\mathcal{V}_Q(t)|$ disappears since there is no entanglement between INT and Source.

Therefore, the visibility of CG is simply given by 2π periodic function $|\mathcal{V}_C(t)|$.

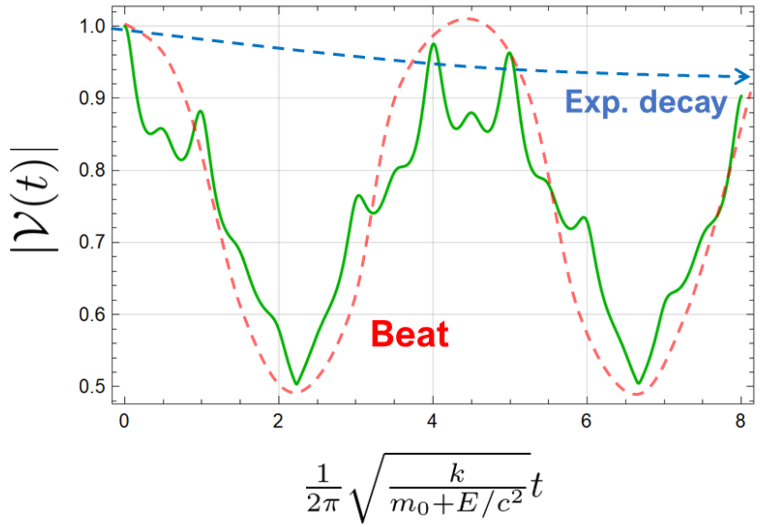
Time dependence of Visibility



Summary 1

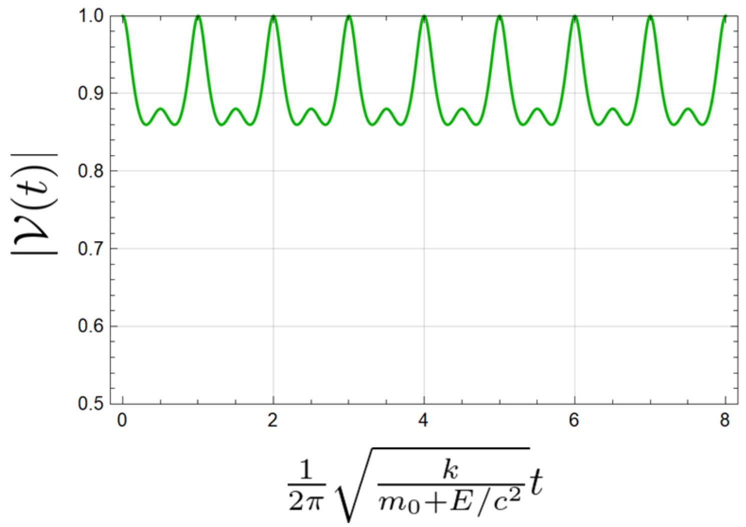
- The Ramsey interference Visibility

QG $|\mathcal{V}(t)| = |\mathcal{V}_Q(t)\mathcal{V}_C(t)|$



- INT and Src. systems entangle.
- This leads the visibility to be non-revival.

CG $|\mathcal{V}(t)| = |\cancel{\mathcal{V}_Q(t)}\mathcal{V}_C(t)|$



- INT and Src. systems are separable.
- This leads the visibility to be revival.

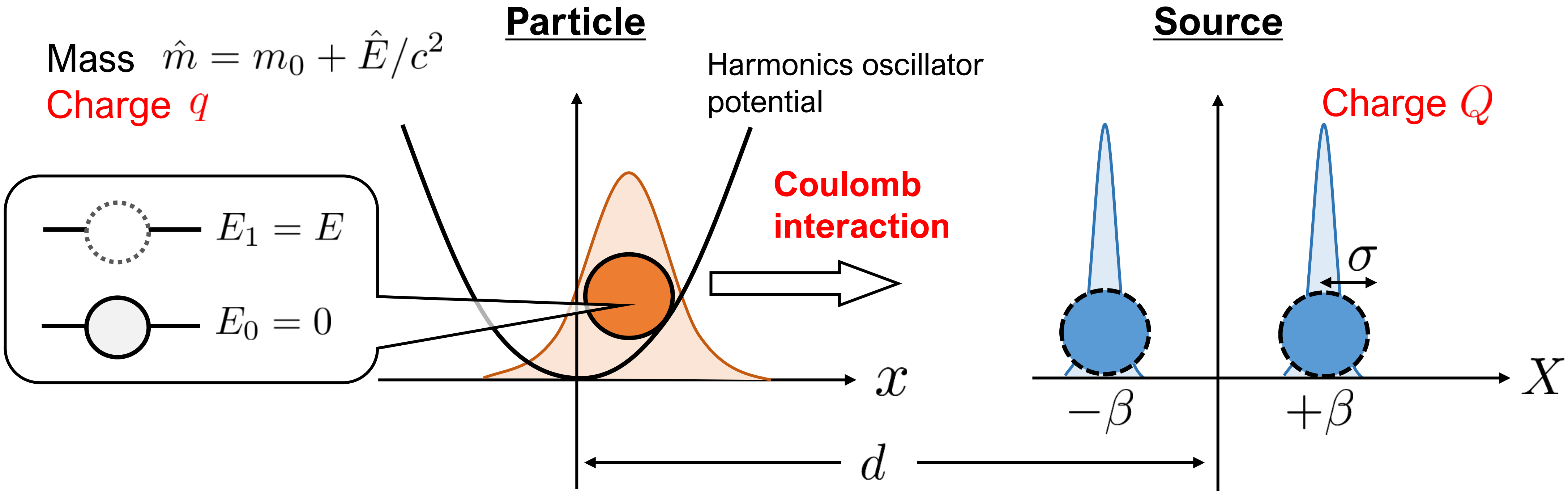
We can test the quantumness of gravity by testing non-revival feature of the visibility!

Discussion & Conclusion

QG and Coulomb interaction

- In the previous proposal, the quantum effect of QG and other interaction were degenerated.
- Did we verify the quantum effect unique to the QG in our proposal?

Let us consider the similar calculation for the Coulomb interaction instead of QG.



QG and Coulomb interaction

- Hamiltonian $$\hat{H}(\hat{E},\hat{X}) \simeq \hat{m}c^2 + \frac{\hat{p}^2}{2\hat{m}} + \frac{1}{2}k\hat{x}^2 + \frac{qQ}{4\pi\epsilon_0\left|\hat{x}-(\hat{X}+d)\right|} + \mathcal{O}(c^{-2})$$

$m(\hat{E})$ **and** $\hat{X}$ **does not couple for the Coulomb interaction**[1] **(c.f. weak equivalence principle)**

- By calculation, we get π periodic function for the visibility[2]

Interaction	The visibility behavior	INT-Source entanglement
CG	2π periodic	×
QG	non-revival	○
Coulomb	π periodic	×

The quantum effect unique to QG is reflected to the time period of the visibility.

Note 1. INT-Source systems entangle not only by QG, but also by spin-photon interaction.
Note 2. We considered only up to the lowest order of Taylor expansion.
(The visibility is not revival also for the Coulomb case when we treat higher orders.)

Conclusion

- We considered the Ramsey interferometry using the harmonically trapped quantum clock.

Interaction	The visibility behavior	INT-Source entanglement
CG	2π periodic	×
QG	non-revival	○
Coulomb	π periodic	×

- INT-Source systems entangle and the Ramsey interference visibility does not revive only when QG is mediated in the system.
- We can verify the quantumness of gravity by analyzing the time period of the visibility.
- When we consider the Coulomb interaction in the system instead of QG, we can distinguish its quantum feature from QG respecting weak equivalence principle.

Danke! 🇩🇪

Backup

Introduction

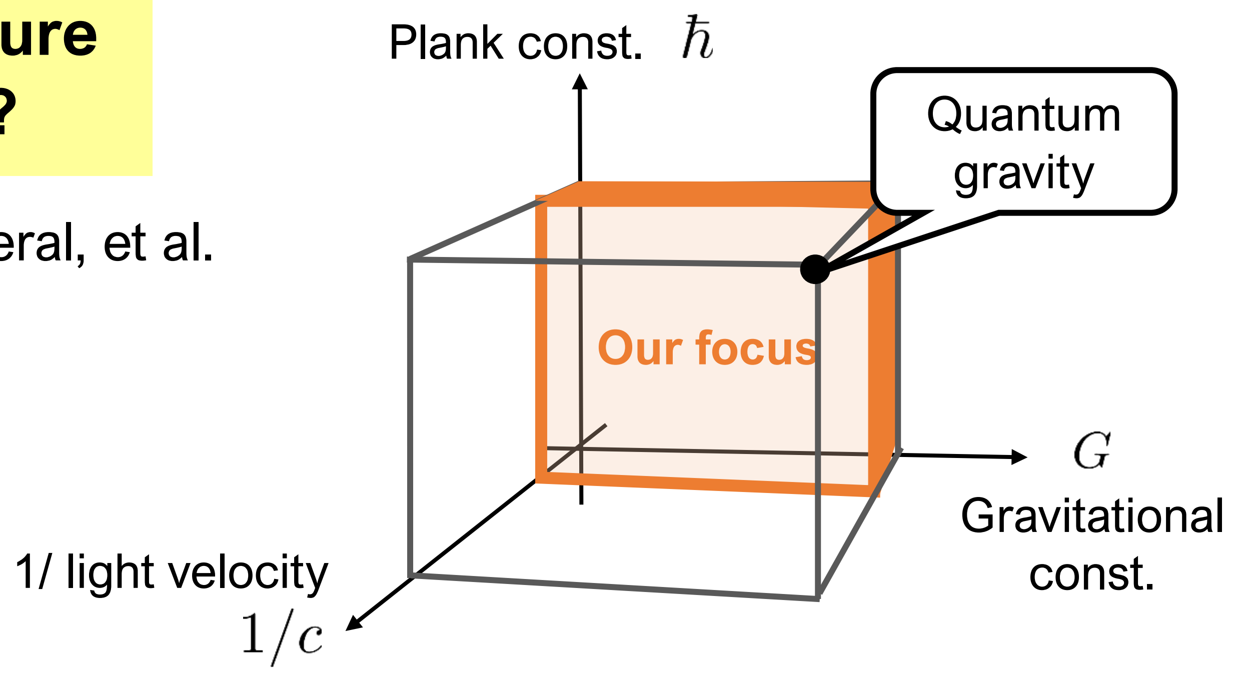
- We need experimental evidence to explore quantum gravity
- Quantum gravity theory $\ni$ non-relativistic gravity in QM regime

Our focus

How to test the quantum nature of non-relativistic gravity?

First proposed by **B**ose, **M**arletto, **V**ederal, et al.

$\Rightarrow$ BMV proposal



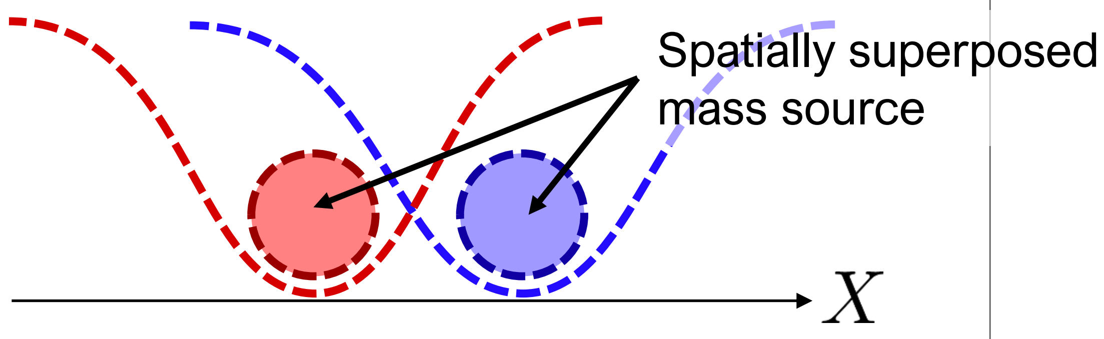
Backup

Purpose of quantum gravity witness

Feynman (1957), Zeh (2008)

We want to clarify if **superposition of gravity is realized or not**.

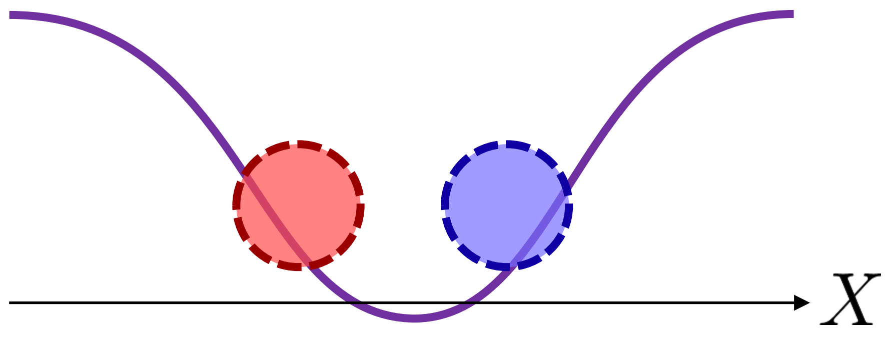
If Yes → **Quantized gravity (QG)**



Newtonian potential at x act as **operator** of the source system.

$$\Phi(\hat{x}, \hat{X}) = -\frac{GM}{|\hat{x} - \hat{X}|}$$

If No → **Semi-classical gravity (CG)**



Newtonian potential at x act as **c-number** of the source system.

$$\Phi(\hat{x}) = \left\langle -\frac{GM}{|\hat{x} - \hat{X}|} \right\rangle_{\mathrm{src}}$$

Backup

Review of BMV proposals

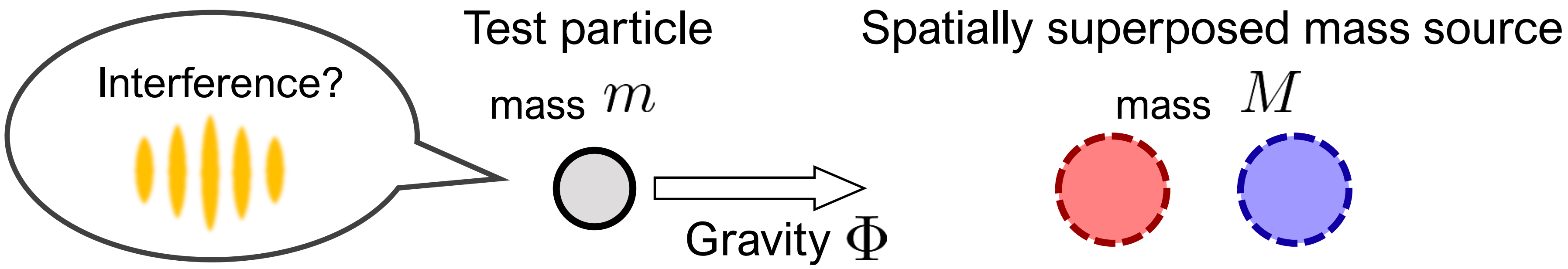
Bose, Marletto, Vedral (2017)

Main idea

Verify quantum entanglement created by superposed gravitational field through gravity induced interference of a test particle.

Setup

Measure a position of the test particle and see its interference pattern after the time evolution under the external gravity from spatially superposed mass source.



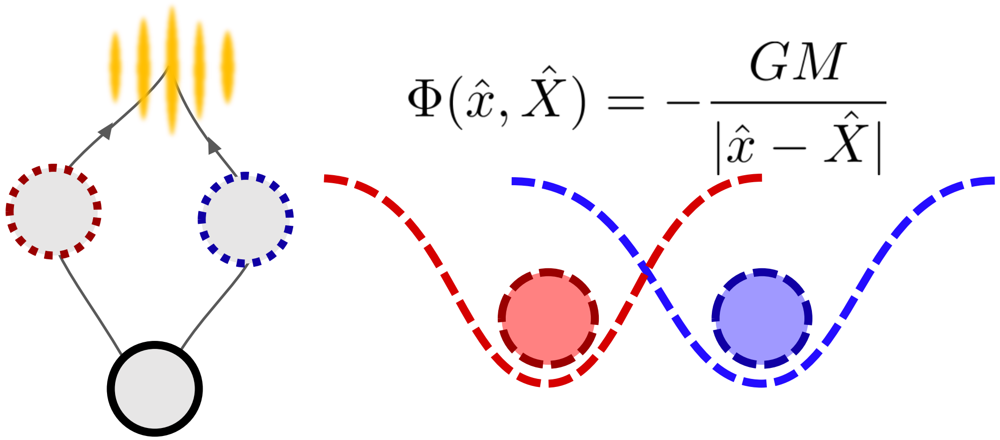
Initial state: $|\psi\rangle \quad \otimes \quad (|L\rangle + |R\rangle)$ **Separable state**

Backup

Review of BMV proposals

Bose, Marletto, Vedral (2017)

QG



$$\Phi(\hat{x},\hat{X}) = -\frac{GM}{|\hat{x}-\hat{X}|}$$

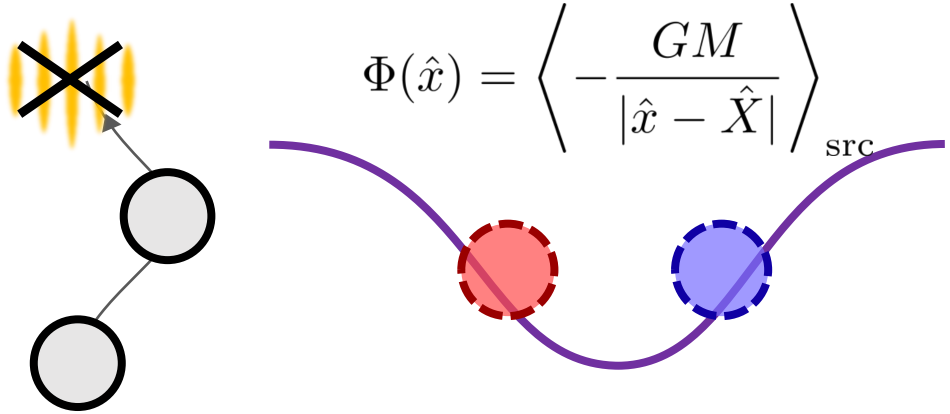
$$e^{-im\Phi t/\hbar}|\psi\rangle(|L\rangle + |R\rangle)$$

$$= e^{\phi_L}|\psi\rangle|L\rangle + e^{\phi_R}|\psi\rangle|R\rangle$$ **Entangled state**

$$(\phi_j = -im\Phi(\hat{x},j)t/\hbar)$$

→ QG induced interference

CG



$$\Phi(\hat{x}) = \left\langle -\frac{GM}{|\hat{x}-\hat{X}|} \right\rangle_{\mathrm{src}}$$

$$e^{-im\Phi t/\hbar}|\psi\rangle(|L\rangle + |R\rangle)$$

$$= e^{\phi}|\psi\rangle(|L\rangle + |R\rangle)$$ **Separable state**

$$(\phi = -im\Phi(\hat{x})t/\hbar)$$

→ No interference fringe

Backup

Review of BMV proposals

Bose, Marletto, Vedral (2017)

Conclusion of BMV proposals

- **QG creates the entanglement and induce a test particle interference.**
- We may be able to test a quantum nature of gravity from the existence of the interference.

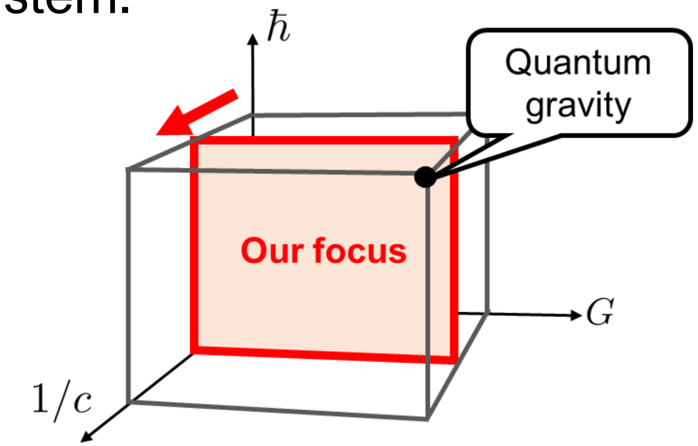
Issues

- However, other quantum interaction can also create the entanglement and the interference.

QG: $\Phi(\hat{x}, \hat{X}) = -\dfrac{GM}{|\hat{x} - \hat{X}|}$ Coulomb potential: $A_t(\hat{x}, \hat{X}) = \dfrac{Q}{4\pi\epsilon_0} \dfrac{1}{|\hat{x} - \hat{X}|}$

- BMV proposals doesn't work when Coulomb force is mixed in the system.

We consider new proposal to **test the quantum nature unique to gravity by stepping into a slightly relativistic region.**



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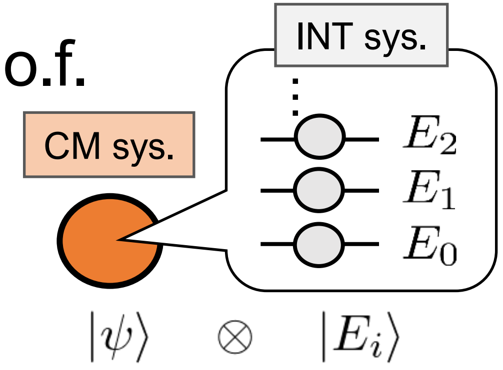
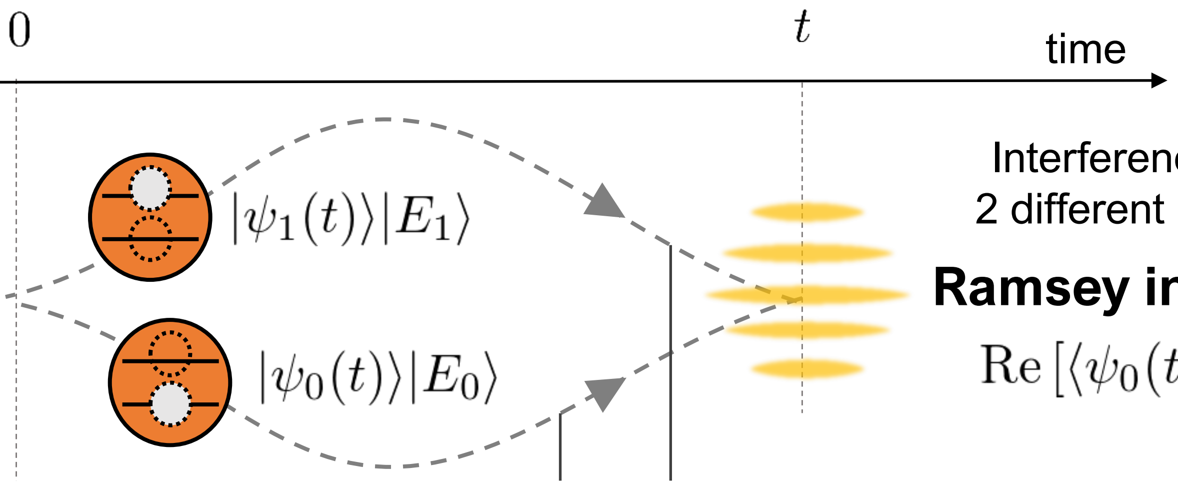
Mass of the CM system



Ramsey interferometry Measurement of an energy gap of the quantum clock

Superpose the int. energy state

$$|\psi_{\mathrm{ini}}\rangle|E_0\rangle \longrightarrow \frac{1}{\sqrt{2}}|\psi_{\mathrm{ini}}\rangle(|E_0\rangle + |E_1\rangle)$$



Interference between 2 different energy state

Ramsey interference

$\mathrm{Re}\left[\langle\psi_0(t)|\psi_1(t)\rangle\right]$

Different time evolution with different rest mass $\hat{H}(\hat{E}, \hat{x}, \hat{p})$

Backup

Ramsey Interference

Ramsey (1950)

Method

Consider a particle which has 2 energy levels as internal d.o.f.
We will apply 4 operations to the initial state $|\Psi_{\text{ini}}\rangle = |\psi_{\text{ini}}\rangle \otimes |E_0\rangle$ as shown in the right figure.

- Pulse operator: Induces an internal energy transition

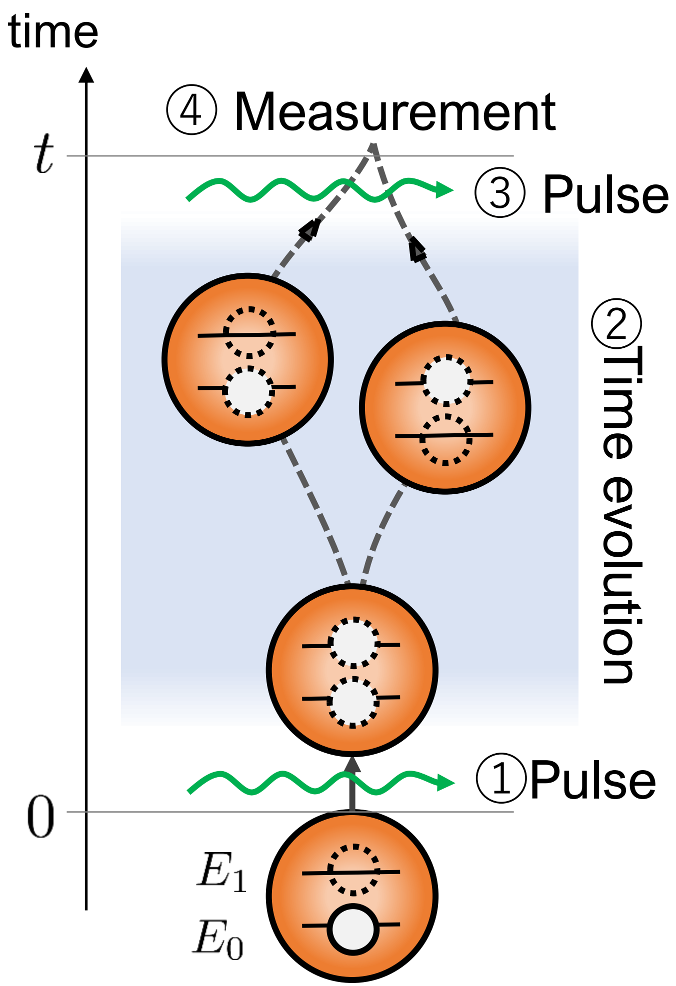
$$\hat{G} = \hat{I} \otimes e^{i\hat{\sigma}_y \pi/4} = \hat{I} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

e.g. $|E_0\rangle \xrightarrow{\hat{G}} \frac{|E_0\rangle + |E_1\rangle}{\sqrt{2}} \xrightarrow{\hat{G}} |E_1\rangle \xrightarrow{\hat{G}} \frac{-|E_0\rangle + |E_1\rangle}{\sqrt{2}} \xrightarrow{\hat{G}} -|E_0\rangle \xrightarrow{\hat{G}} \cdots$

- Time operator:

$$\hat{U}(t) = e^{-i\hat{H}t}$$

When the energy level changes…
⇒ The total mass changes
⇒ The CM motion and the world line change



Backup Ramsey Interference

Ramsey (1950)

Method

The final state after step ③

$$
\begin{aligned}
&|\Psi(t)\rangle \\
&= \hat{G}\,\hat{U}(t)\,\hat{G}\,|\Psi_{\mathrm{ini}}\rangle \\
&= \frac{1}{\sqrt{2}}\hat{G}\,\hat{U}(t)\,|\psi_{\mathrm{ini}}\rangle(|E_0\rangle + |E_1\rangle) \\
&= \frac{1}{\sqrt{2}}\hat{G}\,\underline{(|\psi_0(t)\rangle|E_0\rangle + |\psi_1(t)|E_1\rangle)} \\
&= \frac{1}{2}\{|\psi_0(t)\rangle(|E_0\rangle + |E_1\rangle) + |\psi_1(t)\rangle(-|E_0\rangle + |E_1\rangle)\}
\end{aligned}
$$

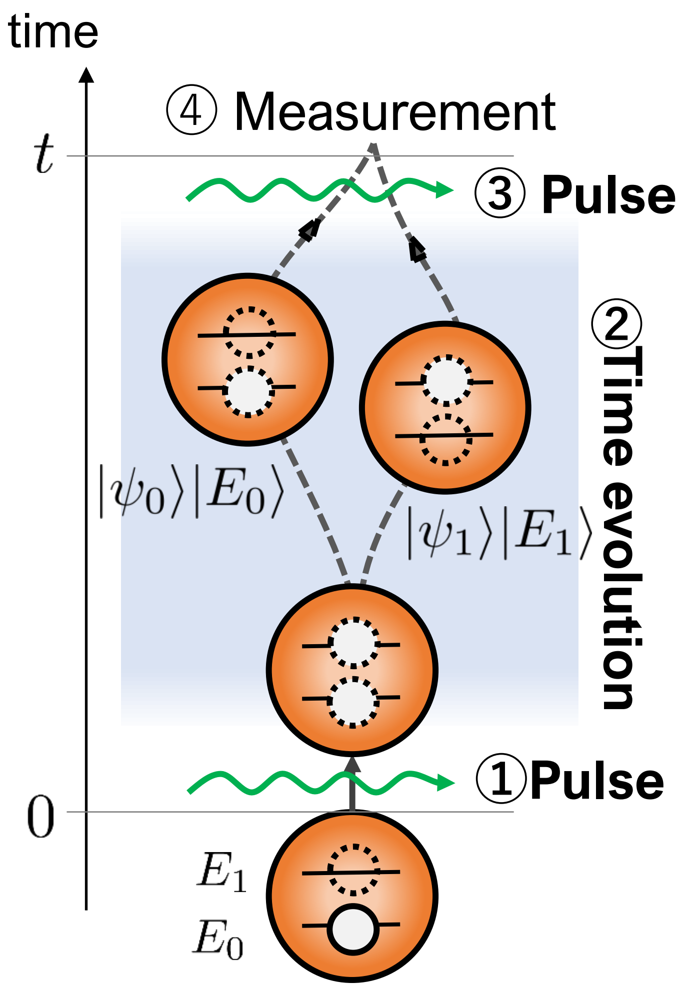
CM-INT entanglement

where $|\psi_j(t)\rangle = \langle E_j|\hat{U}(t)|E_j\rangle|\psi_{\mathrm{ini}}\rangle$

Note

$|\Psi_{\mathrm{ini}}\rangle = |\psi_{\mathrm{ini}}\rangle \otimes |E_0\rangle$

$\hat{G} = \hat{I} \otimes e^{i\hat{\sigma}_y\pi/4} = \hat{I} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$



Backup Ramsey Interference

Ramsey (1950)

Method

Finally, we will measure

a transition late of $E_0 \to E_1$

Note

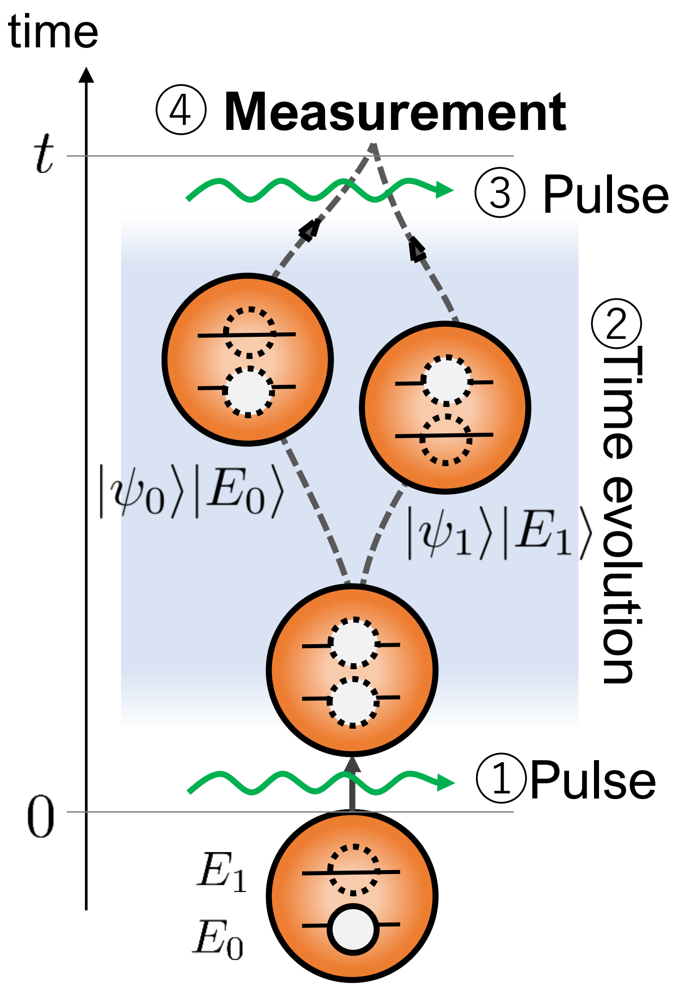
$$|\Psi(t)\rangle = \frac{1}{2}\{|\psi_0(t)\rangle(|E_0\rangle + |E_1\rangle) + |\psi_1(t)\rangle(-|E_0\rangle + |E_1\rangle)\}$$

$$\begin{aligned} P(t) &= \mathrm{Tr}\left[\langle E_1|\Psi(t)\rangle\langle\Psi(t)|E_1\rangle\right] \\ &= \frac{1}{4}\left(2 + \langle\psi_0(t)|\psi_1(t)\rangle + \langle\psi_1(t)|\psi_0(t)\rangle\right) \\ &= \frac{1}{2}\left(1 + \underline{\mathrm{Re}\left[\langle\psi_0(t)|\psi_1(t)\rangle\right]}\right) \\ &= \frac{1}{2}\left(1 + |\mathcal{V}(t)|\cos\Theta(t)\right) \end{aligned}$$

Interference term between 2 world lines with different internal energy level

where $|\mathcal{V}(t)| := |\langle\psi_0(t)|\psi_1(t)\rangle|$: the visibility

$\Theta(t) := \mathrm{Arg}\left[\langle\psi_0(t)|\psi_1(t)\rangle\right]$



Backup

Gravity induced decoherence of the Ramsey interference

Pikovski, et. al. (2015) Haustein et. al. (2019)

- Background spacetime $$ds^2 \simeq -(1 + 2\underline{\Phi(\hat{x})}/c^2)dt^2 + (1 - 2\Phi(\hat{x}/c^2))dx^2$$

An arbitrary weak gravitational field

Note: Total rest mass is $\hat{m} = m_0 + \hat{E}/c^2$

- Hamiltonian of freefalling quantum clock

$$H_{\text{free}}(\hat{E}, \hat{x}, \hat{p}) = \sqrt{-g_{00}(\hat{m}^2 c^4 + c^2 \hat{p}^2)} \simeq \underbrace{m_0 c^2 + \frac{\hat{p}^2}{2m_0} + m_0 \Phi(\hat{x})}_{\text{CM Hamiltonian}} + \underbrace{\hat{E}\left(1 + \frac{\Gamma(\hat{x}, \hat{p})}{c^2}\right)}_{\text{INT Hamiltonian}}$$

where $\Gamma(\hat{x}, \hat{p}) = -\frac{\hat{p}^2}{2m_0^2} + \Phi(\hat{x})$: SR/ GR redshift factor

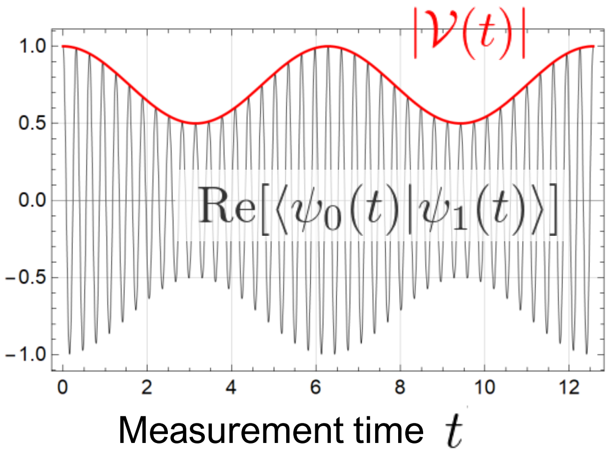
- Visibility: The absolute value of the Ramsey interference

$$|\mathcal{V}(t)| := |\langle \psi_0(t)|\psi_1(t)\rangle| = \left|\langle \psi_{\text{ini}}|e^{i\hat{H}(E_0)t} e^{-i\hat{H}(E_1)t}|\psi_{\text{ini}}\rangle\right|$$

$$\simeq \left|\langle \psi_{\text{ini}}|e^{i(E_0 - E_1)\,\Gamma(\hat{x},\hat{p})t}|\psi_{\text{ini}}\rangle\right| \boxed{\leq 1}$$

The Ramsey interference is damped down due to SR/GR redshift.

Decoherence in homogeneous gravity



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CM Hamiltonian

INT Hamiltonian

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$$\simeq \left|\langle\psi_{\text{ini}}|e^{i(E_0 - E_1)\Gamma(\hat{x},\hat{p})t}|\psi_{\text{ini}}\rangle\right| \leq 1$$

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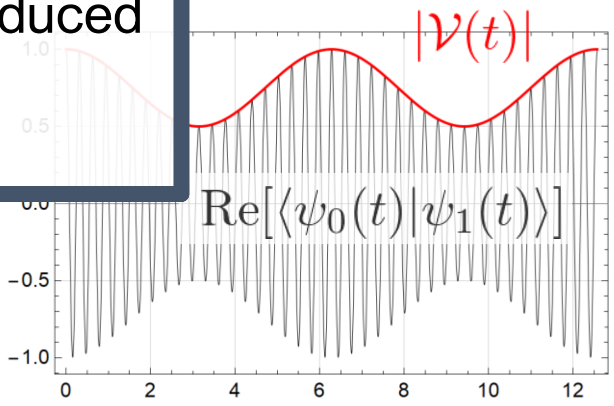


Measurement time t

What we are going to do?

- Consider QG/CG as the background spacetime, and focus on the time dependence of the visibility.
- We may be able capture quantumness unique to the gravity by observing the decoherence induced by the gravitational redshift.

Calculation: the QG case

- QG potential: $$\Phi(\hat{x},\hat{X}) = -\frac{GM}{|\hat{x}-(\hat{X}+d)|}$$

Perform Taylor expansion for $d \gg x, X$

$$\hat{m}\hat{\Phi} \simeq -\frac{G\hat{m}M}{|d|}\left(1+\frac{\hat{x}-\hat{X}}{d}\right)$$

Then, the Hamiltonian leads to

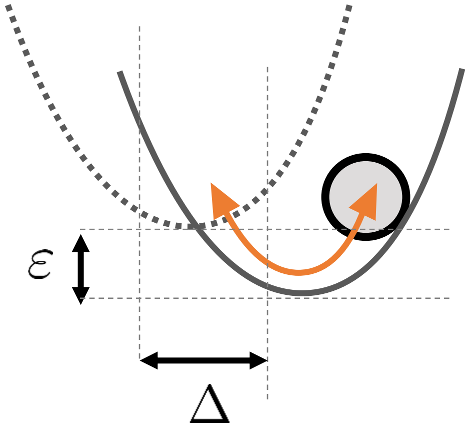
$$\hat{H}(\hat{E},\hat{X}) \simeq \varepsilon(\hat{E},\hat{X}) + \frac{\hat{p}^2}{2\hat{m}} + \frac{1}{2}k(\hat{x}-\Delta(\hat{E}))^2 \equiv \varepsilon(\hat{E},\hat{X}) + \hat{h}(\hat{E})$$

$$\Delta(\hat{E}) = \frac{G\hat{m}M}{kd^2},\ \varepsilon(\hat{E},\hat{X}) = \frac{\hat{E}}{c^2}\left(\frac{GM}{|d|}\frac{\hat{X}}{d} + \text{const.}\right)$$

Coupling of INT-Source operators

Time evolution of the state associated with E_j is given by

$$|\Psi_j\rangle \sim \sum_{s=\pm\beta} e^{-i\hat{H}(E_j,s)t/\hbar}|\psi_{\text{ini}}\rangle|\varphi_s\rangle$$



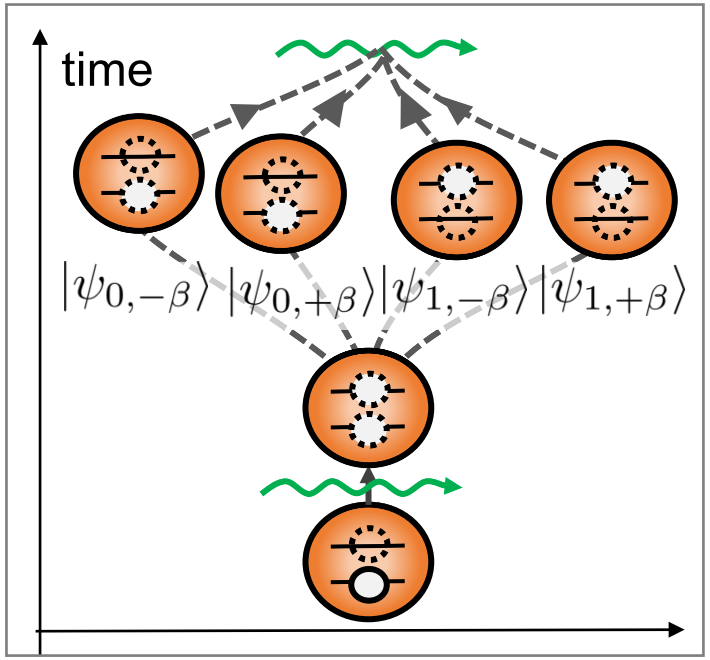


Image of the worldlines

Backup

Calculation: the QG case

$$V_{\mathrm{QG}}(\hat{E},\hat{x},\hat{X}) = -\frac{G\hat{m}M}{|\hat{x}-(\hat{X}+d)|}$$

INT and Source systems entangle through QG

- Hamiltonian of QG

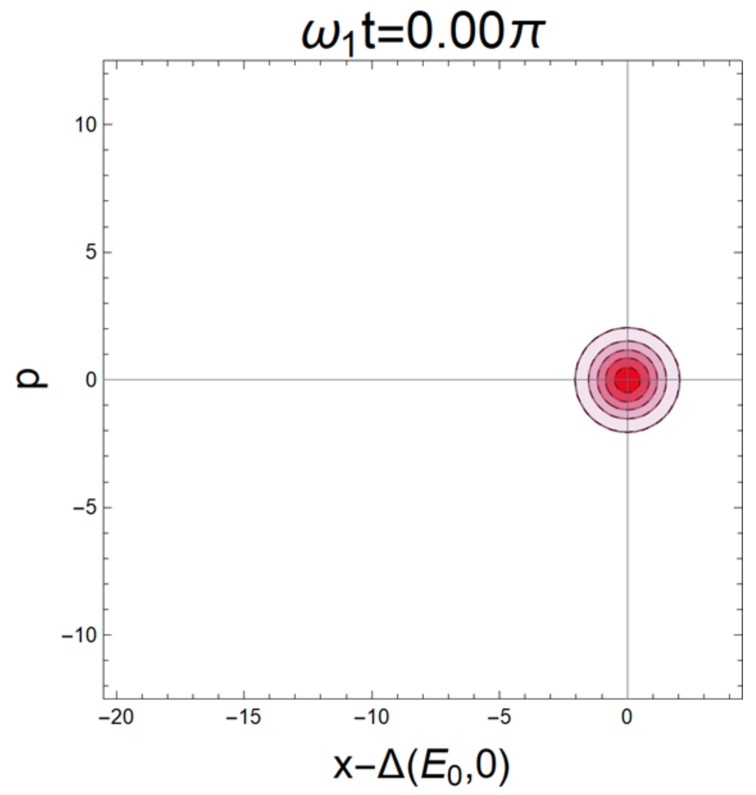
$$\hat{H}_{\mathrm{QG}}(\hat{E},\hat{X}) = \hat{m}c^2 + \frac{\hat{p}^2}{2\hat{m}} + \frac{1}{2}k\hat{x}^2 + V_{\mathrm{QG}}(\hat{E},\hat{x},\hat{X})$$

- How does the particle evolve?

$$\langle\hat{E}\rangle = \{E_0, E_1\}, \quad \langle\hat{X}\rangle \sim \pm\beta$$

$\rightarrow$ 4 different patterns for the particle state!

4 different CM states in the phase space



- Visibility $|\mathcal{V}(t)| \sim |\langle\psi_0(t)|\psi_1(t)\rangle|$ corresponds to the innerproduct between and

$$\left(\begin{array}{l}\text{Red: } \langle\hat{E}\rangle = E_0 \text{ with } \langle\hat{X}\rangle = \pm\beta, \\ \text{Blue: } \langle\hat{E}\rangle = E_1 \text{ with } \langle\hat{X}\rangle = \pm\beta\end{array}\right)$$

Calculation: the QG case

$$V_{\mathrm{QG}}(\hat{E},\hat{x},\hat{X}) = -\frac{G\hat{m}M}{|\hat{x}-(\hat{X}+d)|}$$

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$$\hat{H}_{\mathrm{QG}}(\hat{E},\hat{X}) = \hat{m}c^2 + \frac{\hat{p}^2}{2\hat{m}} + \frac{1}{2}k\hat{x}^2 + V_{\mathrm{QG}}(\hat{E},\hat{x},\hat{X})$$

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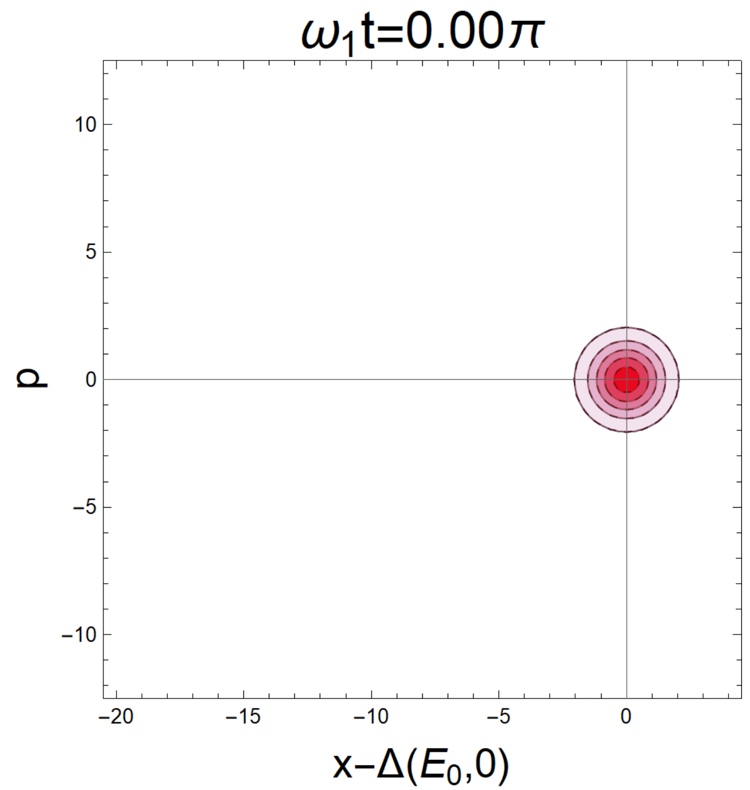

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Backup

Calculation: the QG case

Calculation of the Ramsey interference visibility $|\mathcal{V}(t)|$

$$|\mathcal{V}(t)| = \left| \int dX |\varphi(X)|^2 \langle \psi_{\rm ini}| e^{i\hat{H}(E_0,X)t/\hbar} e^{-i\hat{H}(E_1,X)t/\hbar} |\psi_{\rm ini}\rangle \right|$$

$$= \left| \int dX |\varphi(X)|^2 e^{i(\varepsilon(E_0,X)-\varepsilon(E_1,X))t/\hbar} \right| \times \left| \langle \psi_{\rm ini}| e^{i\hat{h}(E_0)t/\hbar} e^{-i\hat{h}(E_1)t/\hbar} |\psi_{\rm ini}\rangle \right|$$

Note

$$\hat{H}(\hat{E},\hat{X}) \equiv \varepsilon(\hat{E},\hat{X}) + \hat{h}(\hat{E})$$

$$\varepsilon(\hat{E},\hat{X}) = \frac{\hat{E}}{c^2}\left(\frac{GM}{|d|}\frac{\hat{X}}{d} + \text{const.}\right)$$

$|\psi_{\rm ini}\rangle$ is an equilibrium state of $\hat{h}(E_0)$

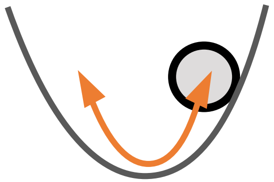
$$|\mathcal{V}_Q(t)| = \left| \int dX |\varphi(X)|^2 e^{i\frac{E}{c^2}\frac{GM}{|d|}\frac{X}{d}\frac{t}{\hbar}} \right|$$

$$= \frac{2}{N} \exp\left[-\left(\frac{GME}{2c^2|d|}\frac{\sigma}{d}\frac{t}{\hbar} \right)^2 \right] \left(\cos\left[\frac{GME}{c^2|d|}\frac{\beta}{d}\frac{t}{\hbar} \right] + e^{-\beta^2/\sigma^2} \right)$$

$$|\mathcal{V}_C(t)| = \left| e^{i\omega_0 t/2} \langle \psi_{\rm ini}| e^{-i\hat{h}(E_1)t/\hbar} |\psi_{\rm ini}\rangle \right|$$

$$= \left| \langle \psi_{\rm ini}| e^{-i\hat{h}(E_1)t/\hbar} |\psi_{\rm ini}\rangle \right|$$

2π periodic function for $\sqrt{\frac{k}{m_0 + E_1/c^2}}\,t$



Hence, $|\mathcal{V}(t)| = |\mathcal{V}_Q(t)\mathcal{V}_C(t)|$

Backup

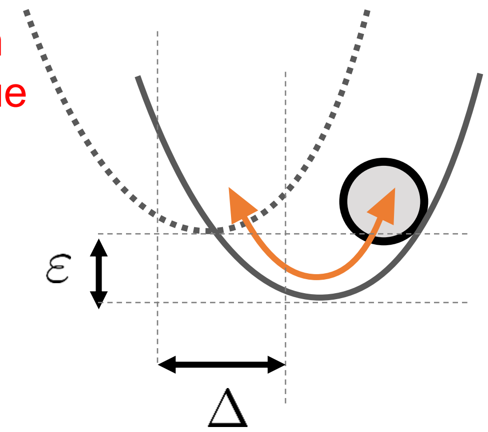
Calculation: the CG case

- QG potential:

$$\Phi(\hat{x}) = \left\langle -\frac{GM}{|\hat{x}-\hat{X}|} \right\rangle_{\mathrm{src}}$$

Replacing the source operators emerged from QG to its expectation value

$$\hat{X} \to \langle X \rangle = 0$$



Perform Taylor expansion for $d \gg x, X$

$$\hat{m}\hat{\Phi} \simeq -\frac{G\hat{m}M}{|d|}\left(1+\frac{\hat{x}-\cancel{\hat{X}}}{d}\right)$$

Then, the Hamiltonian leads to

$$\hat{H}(\hat{E},\cancel{\hat{X}}) \simeq \varepsilon(\hat{E},\cancel{\hat{X}}) + \frac{\hat{p}^2}{2\hat{m}} + \frac{1}{2}k(\hat{x}-\Delta(\hat{E}))^2 \equiv \varepsilon(\hat{E},\cancel{\hat{X}}) + \hat{h}(\hat{E})$$

$$\Delta(\hat{E}) = \frac{G\hat{m}M}{kd^2}, \quad \varepsilon(\hat{E},\cancel{\hat{X}}) = \frac{\hat{E}}{c^2}\left(\frac{GM}{|d|}\frac{\cancel{\hat{X}}}{d} + \mathrm{const.}\right)$$

No entanglement between INT-Source

Time evolution of the state associated with E_j is given by

$$|\Psi_j\rangle = e^{-i\hat{H}(E_j)t/\hbar}|\psi_{\mathrm{ini}}\rangle|\varphi\rangle$$

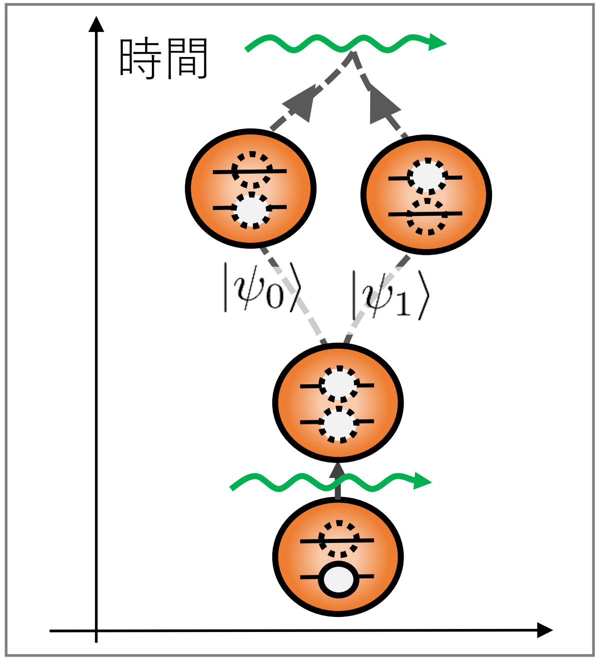


Image of the worldlines

Backup

Calculation: the CG case

$$V_{\rm CG}(\hat{E},\hat{x}) = -\left\langle \frac{G\hat{m}M}{|\hat{x}-(\hat{X}+d)|} \right\rangle_{\rm source}$$

No entanglement between INT and Source

- Hamiltonian of CG

$$\hat{H}_{\rm CG}(\hat{E}) = \hat{m}c^2 + \frac{\hat{p}^2}{2\hat{m}} + \frac{1}{2}k\hat{x}^2 + V_{\rm CG}(\hat{E},\hat{x})$$

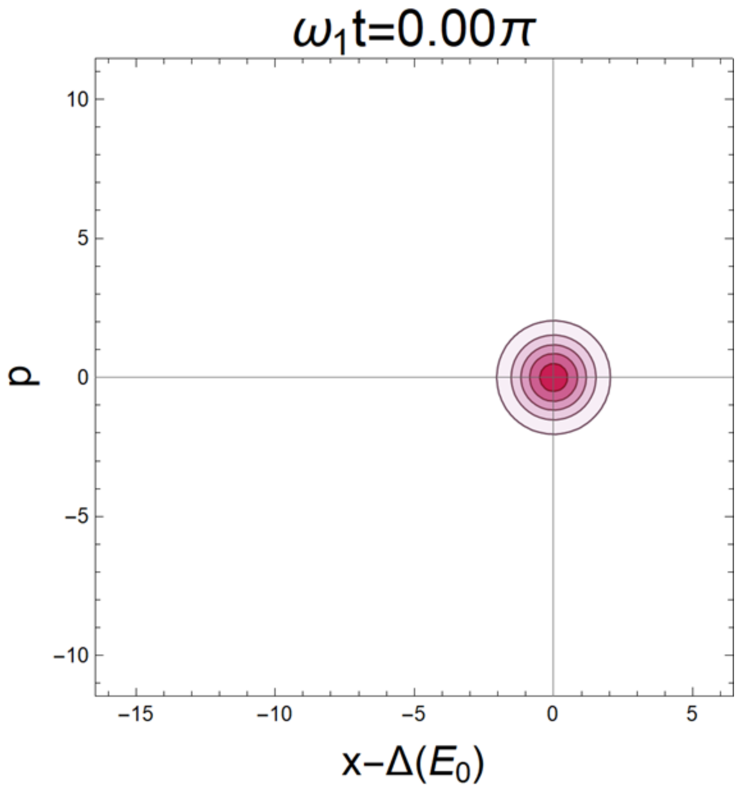
- How does the particle evolve?

$$\langle \hat{E} \rangle = \{E_0, E_1\}$$

→ 2 different patterns for the particle state!

- Visibility $|\mathcal{V}(t)| \sim |\langle \psi_0(t)|\psi_1(t)\rangle|$ corresponds to the innerproduct between  and 

4 different CM states in the phase space



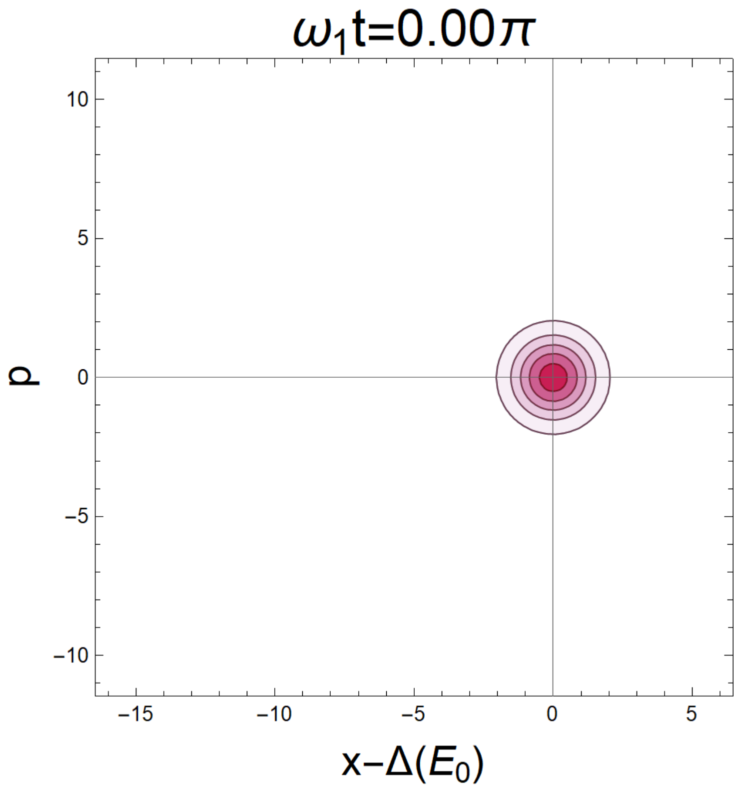
$$\left(\begin{array}{l} \text{Red: } \langle \hat{E} \rangle = E_0 \\ \text{Blue: } \langle \hat{E} \rangle = E_1 \end{array} \right)$$

Calculation: the CG case

$$V_{\mathrm{CG}}(\hat{E},\hat{x}) = -\left\langle \frac{G\hat{m}M}{|\hat{x}-(\hat{X}+d)|} \right\rangle_{\text{source}}$$

No entanglement between INT and Source

4 different CM states in the phase space



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Backup

Calculation: the CG case

Note

$$\hat{H}(\hat{E},\hat{X}) \equiv \varepsilon(\hat{E},\hat{X}) + \hat{h}(\hat{E})$$

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$|\psi_{\rm ini}\rangle$ is an equilibrium state of $\hat{h}(E_0)$

$$|\mathcal{V}_Q(t)| = \left|\int dX|\varphi(X)|^2 e^{i\cancel{\frac{E}{c^2}\frac{GM}{|d|}\frac{X}{d}\frac{t}{\hbar}}}\right|$$

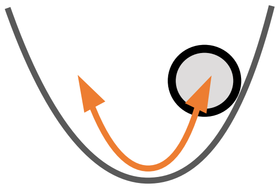
1

$$= \cancel{\frac{2}{N}\exp\left[-\left(\frac{GME}{2c^2|d|}\frac{\sigma}{d}\frac{t}{\hbar}\right)^2\right]\left(\cos\left[\frac{GME}{c^2|d|}\frac{\beta}{d}\frac{t}{\hbar}\right]+e^{-\beta^2/\sigma^2}\right)}$$

$$|\mathcal{V}_C(t)| = \left|e^{i\omega_0 t/2}\langle\psi_{\rm ini}|e^{-i\hat{h}(E_1)t/\hbar}|\psi_{\rm ini}\rangle\right|$$

$$= \left|\langle\psi_{\rm ini}|e^{-i\hat{h}(E_1)t/\hbar}|\psi_{\rm ini}\rangle\right|$$

2π periodic function for $\sqrt{\frac{k}{m_0+E_1/c^2}}t$



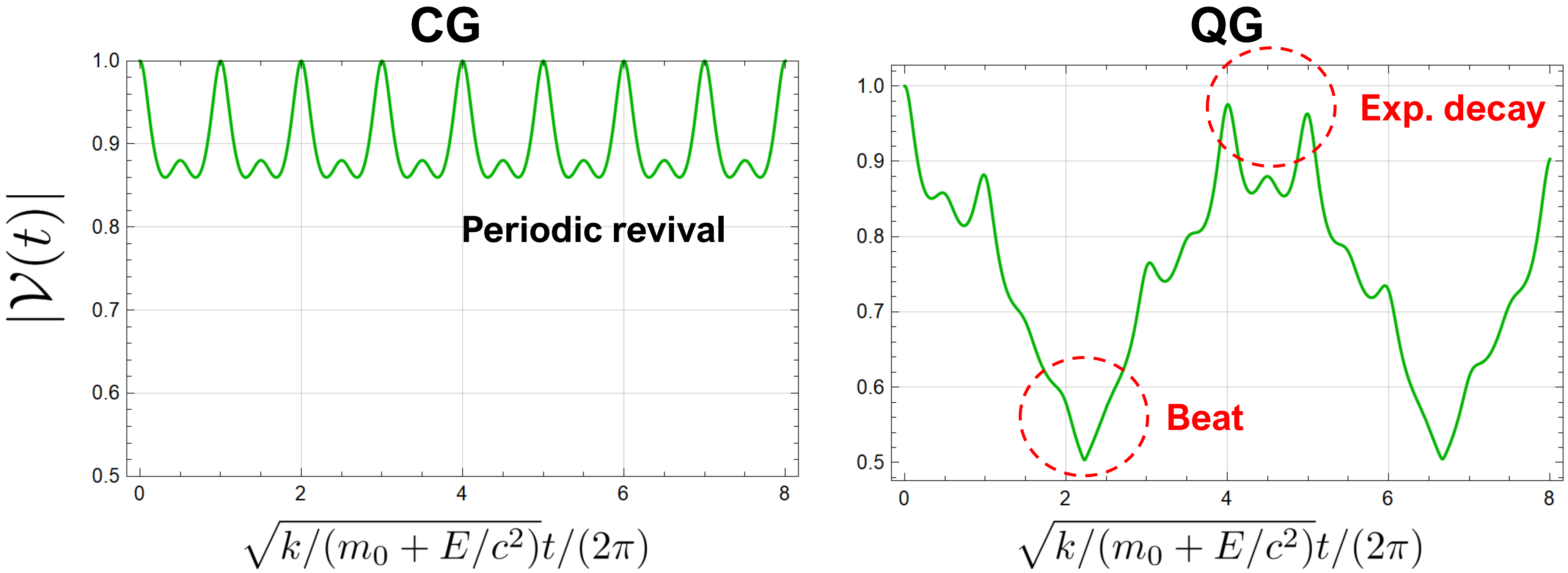
Hence, $|\mathcal{V}(t)| = |\cancel{\mathcal{V}_Q}(t)\mathcal{V}_C(t)|$

Backup

Result: the plot of the Ramsey visibility

The time dependence of the Ramsey visibility (considering up to the higher order)

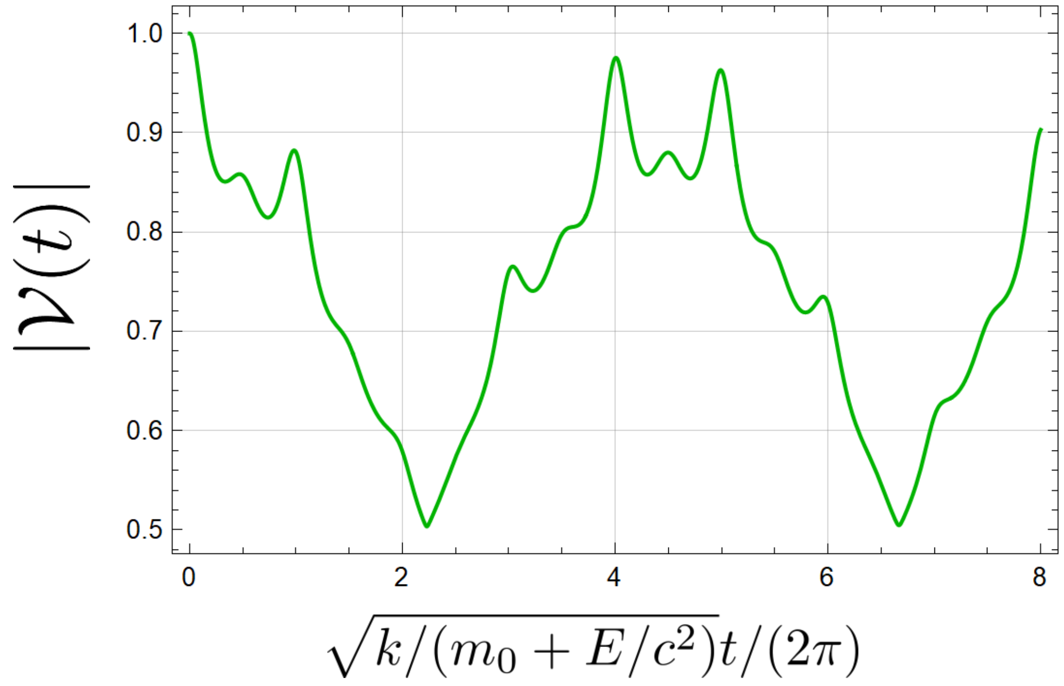
Parameters are set to $\sqrt{m_0^3/k} = 10,\ \sqrt{m_0/m_1} = 0.5,\ Gm_0M/(kd^3) = 0.015,\ m_0^{1/4}d = 10,\ \beta/d = 0.01,\ \sigma/d = 0.001$



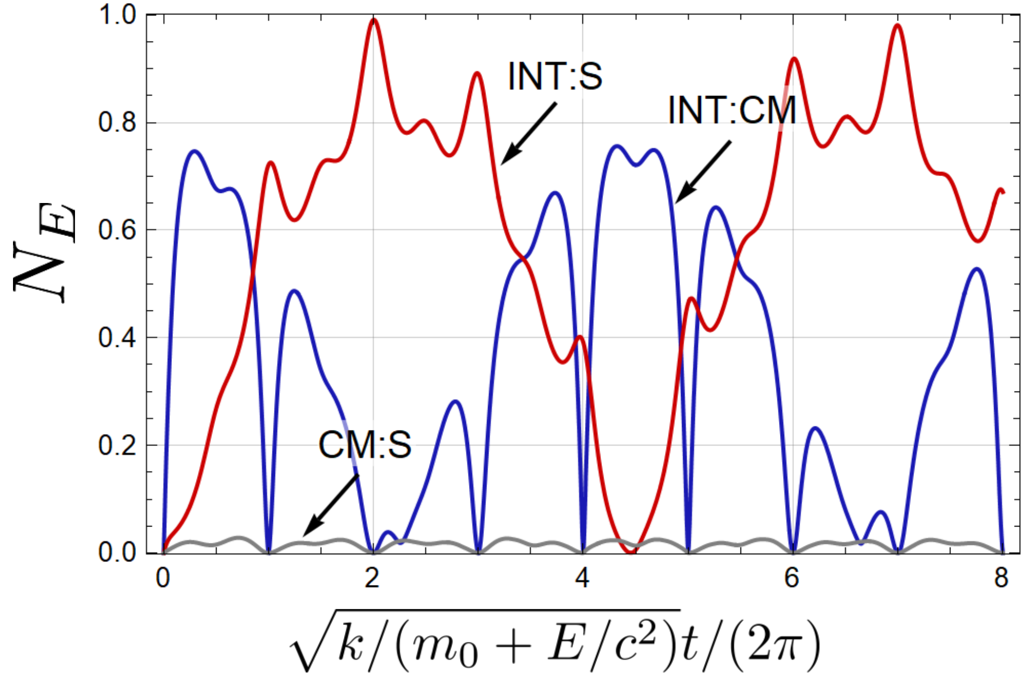
Backup Decoherence and Entanglement (QG)

<u>Visibility and Logarithmic Negativity in QG case</u> with better approximation than the last page

Visibility of QG



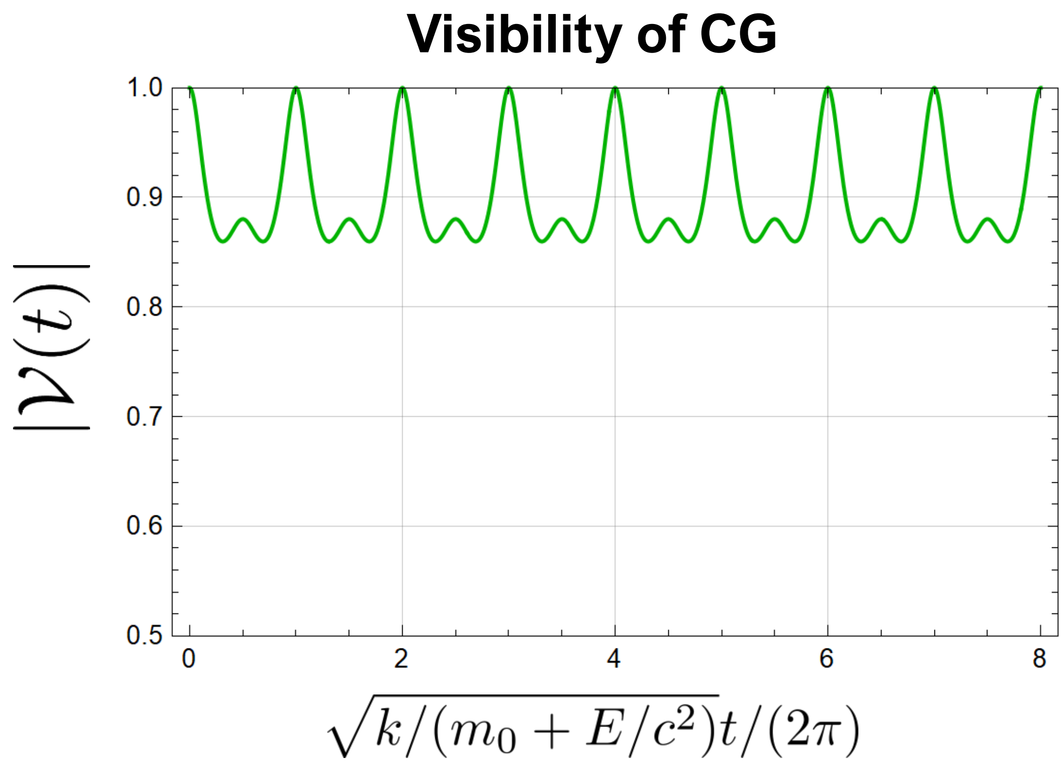
Log Negativity of QG

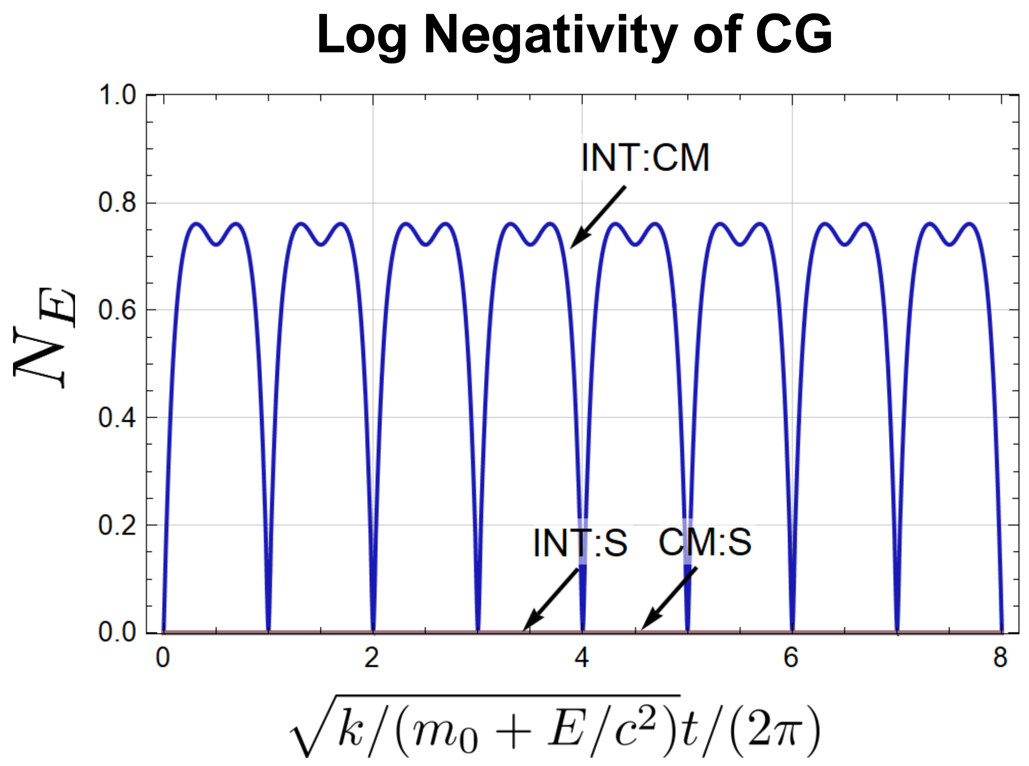


- **Whole 3 systems share entanglement.**
- **Higher the INT:Src. negativity is, more the visibility decoheres.**

Backup Decoherence and Entanglement (CG)

<u>Visibility and Logarithmic Negativity in CG case</u> with better approximation than the last page





- **Only INT and CM systems entangle periodically.**
- **The higher the log negativity is, more the visibility decoheres.**

Entanglement structure

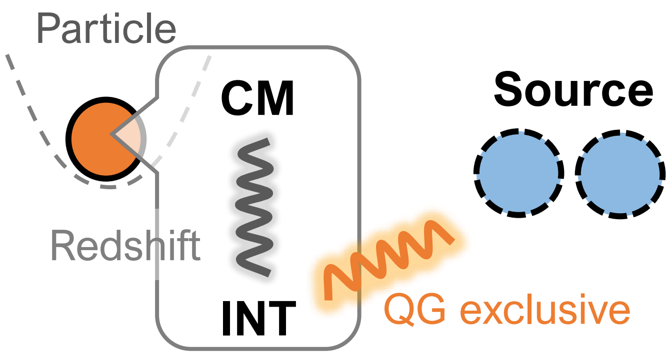
Let us calculate the negativity under approximation that the CM and Src. state is well localized.

QG case ($|\mathcal{V}(t)| = |\mathcal{V}_Q(t)\mathcal{V}_C(t)|$)

$$\mathcal{N}(\rho_{\mathrm{INT:S}}) \sim \frac{1}{4}\left(-1+|\mathcal{V}_Q(t)|+\sqrt{(1+|\mathcal{V}_Q(t)|)^2-4|\mathcal{V}_C(t)|^2|\mathcal{V}_Q(t)|}\right)$$

$$\mathcal{N}(\rho_{\mathrm{INT:CM}}) \sim \frac{1}{4}\left(-1+|\mathcal{V}_C(t)|+\sqrt{(1+|\mathcal{V}_C(t)|)^2-4|\mathcal{V}_Q(t)|^2|\mathcal{V}_C(t)|}\right)$$

$$\mathcal{N}(\rho_{\mathrm{CM:S}}) \sim 0$$

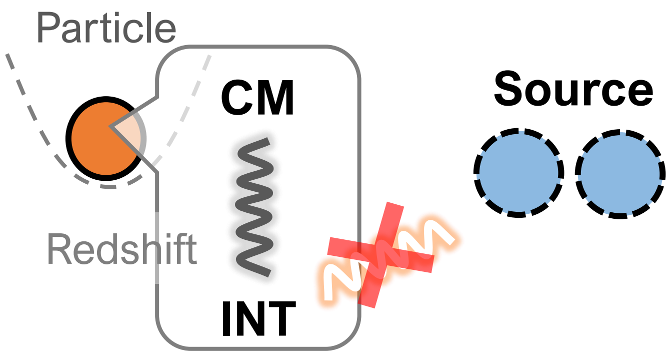


CG case ($|\mathcal{V}(t)| = |\mathcal{V}_C(t)|$)

$$\boxed{\mathcal{N}(\rho_{\mathrm{INT:S}}) \sim 0}$$

$$\mathcal{N}(\rho_{\mathrm{INT:CM}}) \sim \frac{1}{2}|\mathcal{V}_C(t)|$$

$$\mathcal{N}(\rho_{\mathrm{CM:S}}) \sim 0$$



Visibility is non-revival $\leftrightarrow$ $|\mathcal{V}_Q(t)| < 1$ $\leftrightarrow$ $\mathcal{N}(\rho_{\mathrm{INT:S}}) > 0$

Backup

Feasibility of detection

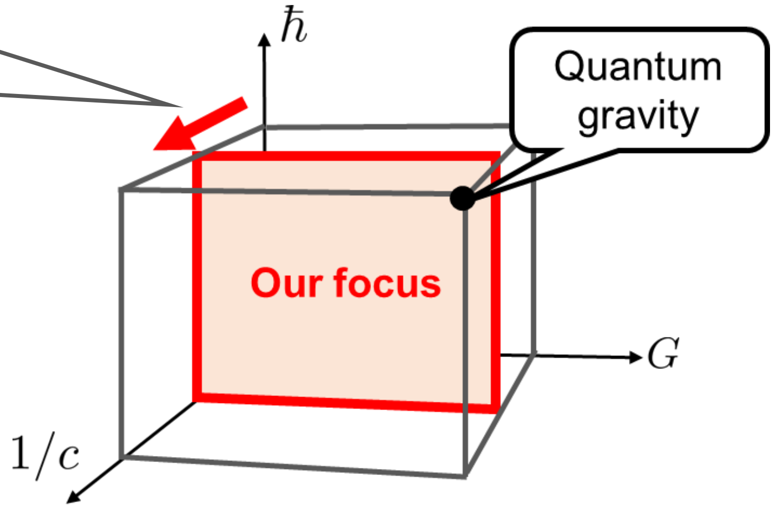
The evaluation of the decoherence factor for the QG case were as follows.

$$\left(\frac{GME}{2c^2|d|}\frac{\sigma}{d}\frac{t}{\hbar}\right)^2 = 1.7\times10^{-34}\left(\frac{M}{10\,\mathrm{ng}}\right)^2\left(\frac{\lambda}{267\,\mathrm{nm}}\right)^{-2}\left(\frac{d}{200\,\mu\mathrm{m}}\right)^{-4}\left(\frac{\sigma}{1\,\mu\mathrm{m}}\right)^2\left(\frac{t}{20\,\mathrm{sec}}\right)^2,$$

$$\left(\frac{GME}{c^2|d|}\frac{\beta}{d}\frac{t}{\hbar}\right)^2 = 6.8\times10^{-32}\left(\frac{M}{10\,\mathrm{ng}}\right)^2\left(\frac{\lambda}{267\,\mathrm{nm}}\right)^{-2}\left(\frac{d}{200\,\mu\mathrm{m}}\right)^{-4}\left(\frac{\beta}{10\,\mu\mathrm{m}}\right)^2\left(\frac{t}{20\,\mathrm{sec}}\right)^2.$$

Since the observation uncertainty is about 10^{-22}, it is difficult to detect at present.

We are looking into the higher order of $1/c$ expansion to capture uniqueness of gravity!



Note

$$|\mathcal{V}_Q(t)| = \frac{2}{N}\exp\left[-\left(\frac{GME}{2c^2|d|}\frac{\sigma}{d}\frac{t}{\hbar}\right)^2\right]\left(\cos\left[\frac{GME}{c^2|d|}\frac{\beta}{d}\frac{t}{\hbar}\right]+e^{-\beta^2/\sigma^2}\right)$$